
```

% Optimization of two-link robot arm tracking
clear; clc;

% Define desired trajectory and Middle Points
qDes = [ -0.4986    2.5681;
         0.5371    1.5108 ];

qMid = [inverse_kinematics(0.4, 0.6, 1, 1), ...
        inverse_kinematics(0.4, 0.7, 1, 1), ...
        inverse_kinematics(0.4, 0.8, 1, 1), ...
        inverse_kinematics(0.4, 0.9, 1, 1), ...
        inverse_kinematics(0.4, 1.0, 1, 1), ...
        inverse_kinematics(0.4, 1.1, 1, 1), ...
        inverse_kinematics(0.4, 1.2, 1, 1)];

% Parameters
time = [10 20];           % time
wn = [2 1.5];             % Prefilter Omega
kj = [40 25 40 25];       % Spring [t1q1 t1q2 t2q1 t2q2]
bj = [10 30 10 30];       % Damping [t1q1 t1q2 t2q1 t2q2]
wt = [400, .5, 1800];     % weights [qDes, Time, qMid]

% Optimization setup
initParams = [time wn bj kj]; % Initial guess for [time, wn, bj, kj]

[init_T, init_Y] = ode45(@(t, x) myTwolinkwithprefilter(t, x, wn,
initParams(1:2), qDes, bj, kj), [0 initParams(2)], zeros(8, 1));

% Lower and upper boundaries
lb = [0 0    1.5  1.5    10  10  10  10    2  2  2  2    ]; % Lower
bounds
ub = [2 4    10   10    100 100 100 100    200 200 200 200 ]; % Upper
bounds

% Objective Function
objectiveFunc = @(params) objectiveFunction(params, qDes, wt, qMid);

% Run optimization
options = optimset('Display', 'iter', 'TolFun', 1e-6, 'MaxIter', 400);
optimalParams = fmincon(objectiveFunc, initParams, [], [], [], [], lb, ub,
[], options);

% Simulate with optimal parameters and plot results
[t, y] = ode45(@(t, x) myTwolinkwithprefilter(t, x, optimalParams(3:4),
optimalParams(1:2), qDes, optimalParams(5:8), optimalParams(9:12)), [0
optimalParams(2)], zeros(8, 1));

% Output
xAct = forward_kinematics(y(:, 5), y(:, 6), 1, 1);
xDes = forward_kinematics(qDes(:, 1), qDes(:, 2), 1, 1);
xInit = forward_kinematics(init_Y(:, 5), init_Y(:, 6), 1, 1);

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% Plotting
% Desired, Actual and Optimised Data
figure(1); hold on; grid on;
plot(xInit(:, 1), xInit(:, 2), '-');
plot(xAct(:, 1), xAct(:, 2), '-');
plot(xDes(:, 1), xDes(:, 2), 'o-');
plot(0.4,0.6, '*',0.4,0.7, '*',0.4,0.8, '*',0.4,0.9, '*',0.4,1.0,
      '*',0.4,1.1, '*',0.4,1.2, '*');
xlabel('X axis'); ylabel('Y axis');
legend('Initial','Optimised','Desired');
title('Optimized Trajectory Tracking');
disp(['Opt Time: ', num2str(optimalParams(1:2))])
disp(['Opt Wn  : ', num2str(optimalParams(3:4))])
disp(['Opt bj  : ', num2str(optimalParams(5:8))])
disp(['Opt kj  : ', num2str(optimalParams(9:12))])

% Mid points in joint space
figure(2); plot(y(:,5), y(:,6), qMid(1,:), qMid(2,:), 'o');
xlabel('Joint 1 position')
ylabel('Joint 2 position')

title('joint space of a (near) optimal staight line in cartesian space')

% joint space plot
figure(3); grid on; hold on;
plot(t, y(:,5:6));
xlabel('Time (s)')
ylabel('Position (rad)')
legend('Q1','Q2')
title('Joint position (rad)')

% cartesian space plot
figure(4); hold on; grid on;
plot(xAct(:,1), xAct(:,2))
xlabel('X axis')
ylabel('Y axis')
legend('X','Y')
title('Cartesian Position (m)')

% x/y vs time
figure(5); grid on; hold on;
plot(t, xAct(:,1:2))
xlabel('Time (s)')
ylabel('Position')
legend('X','Y')
title('Cartesian Position vs Time')

% publish('simOpt.m','pdf');
% disp(sprintf('KY %s \t %s \t %s',mfilename,pwd,datetime("now")));

% Objective function
function error = objectiveFunction(params, qDes, wt, qMid)

    % Initial conditions

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x0 = zeros(8, 1);
x0(1:2) = [qDes(1, 1); qDes(1, 2)];

% Simulate the system
[t, y] = ode45(@(t, x) myTwolinkwithprefilter(t, x, params(3:4),
params(1:2), qDes, params(5:8), params(9:12)), [0 params(2)], x0);

% Calculate the error metric
distto1 = min(sum((y(:, 5:6) - qDes(1,:)).^2,2) + sum((params(1) -
t).^2,2) );
distto2 = min(sum((y(:, 5:6) - qDes(2,:)).^2,2) + sum((params(2) -
t).^2,2) );

distMid1 = min(sum((y(:, 5:6) - qMid(:,1)').^2,2));
distMid2 = min(sum((y(:, 5:6) - qMid(:,2)').^2,2));
distMid3 = min(sum((y(:, 5:6) - qMid(:,3)').^2,2));
distMid4 = min(sum((y(:, 5:6) - qMid(:,4)').^2,2));
distMid5 = min(sum((y(:, 5:6) - qMid(:,5)').^2,2));
distMid6 = min(sum((y(:, 5:6) - qMid(:,6)').^2,2));
distMid7 = min(sum((y(:, 5:6) - qMid(:,7)').^2,2));

% time1 = min(sum((params(1) - t).^2,2));
% time2 = min(sum((params(2) - t).^2,2));
time1 = params(1);
time2 = params(2);

error = wt(1) * distto1 + wt(1) * distto2 + ... % Desired
        wt(2) * time1 + wt(2) * time2 + ... % time
        wt(3) * distMid1 + wt(3) * distMid2 + ... % Mid-point
        wt(3) * distMid3 + wt(3) * distMid4 + ... % Mid-point
        wt(3) * distMid5 + wt(3) * distMid6 + ... % Mid-point
        wt(3) * distMid7; % Mid-point

% distto5 = 5000 * sum((y(:, 5:6) - qMid3'),2) + w2 *
(sum( ( (time(1) + (time(2) - time(1))/2 ) - t).^2 ,2));

end

% myTwolinkwithprefilter function
function dxdt = myTwolinkwithprefilter(t, x, wn, time, qDes, bj, kj)
    zeta = 1;
    A1 = [zeros([2 2]) eye(2); -eye(2)*wn(1)^2 -eye(2)*2*zeta*wn(1)];
    B1 = [0 0; 0 0; wn(1)^2 0; 0 wn(1)^2];

    A2 = [zeros([2 2]) eye(2); -eye(2)*wn(2)^2 -eye(2)*2*zeta*wn(2)];
    B2 = [0 0; 0 0; wn(2)^2 0; 0 wn(2)^2];
    % Actual position and velocity
    q = x(5:6);
    qd = x(7:8);
    q1p = x(7); q2p = x(8);
    q1 = x(5); q2 = x(6);

    % Robot constants

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L_1 = 1; L_2 = 1; m_1 = 1; m_2 = 1;
ka = L_2^2 * m_2;
kb = L_1 * L_2 * m_2;
kc = L_1^2 * (m_1 + m_2);

M = [ka + 2*kb*cos(q2) + kc, ka + kb*cos(q2);
      ka + kb*cos(q2), ka];
V = ka*sin(q2)*([0 -1; 1 0] * [q1p^2; q2p^2] + [-2*q1p*q2p; 0]);

% Numerator = V + [-bj(1) 0; 0 -bj(2)]*qd + [-kj(1) 0; 0 -kj(2)]*(q -
x(1:2));
% qdd = M\Numerator;
if t < time(1)
    Numerator = V + [-bj(1) 0; 0 -bj(2)]*qd + [-kj(1) 0; 0 -kj(2)]*(q -
x(1:2));
    qdd = M\Numerator;
    dotx = A1*x(1:4) + B1*qDes(1, :)';
else
    Numerator = V + [-bj(3) 0; 0 -bj(4)]*qd + [-kj(3) 0; 0 -kj(4)]*(q -
x(1:2));
    qdd = M\Numerator;
    dotx = A2*x(1:4) + B2*qDes(2, :)';
end
dxdt = [dotx; qd; qdd];
end

```

*Initial point X0 is not between bounds LB and UB;
 FMINCON shifted X0 to strictly satisfy the bounds.*

Iter	F-count	f(x)	Feasibility	First-order optimality	Norm of step
0	13	1.435564e+03	0.000e+00	3.742e+02	
1	26	2.201101e+02	0.000e+00	2.225e+02	2.388e+01
2	39	4.684079e+01	0.000e+00	1.988e+02	4.224e+00
3	52	2.291971e+01	0.000e+00	2.196e+02	4.946e+00
4	65	2.220989e+01	0.000e+00	6.802e+01	1.253e-01
5	80	1.829532e+01	0.000e+00	6.824e+01	2.936e+00
6	99	1.785620e+01	0.000e+00	1.483e+01	3.543e-01
7	113	1.748082e+01	0.000e+00	3.345e+00	5.850e-01
8	127	1.670010e+01	0.000e+00	1.211e+01	9.053e-01
9	140	1.441588e+01	0.000e+00	2.210e+01	9.550e-01
10	161	1.430170e+01	0.000e+00	2.126e+00	1.456e-01
11	182	1.427675e+01	0.000e+00	9.470e-01	9.154e-03
12	196	1.426723e+01	0.000e+00	3.823e+00	1.412e-02
13	226	1.426720e+01	0.000e+00	2.901e+00	4.344e-06
14	241	1.426720e+01	0.000e+00	2.901e+00	1.370e-05
15	258	1.426700e+01	0.000e+00	2.879e+00	3.041e-05
16	272	1.426687e+01	0.000e+00	2.878e+00	2.628e-04
17	287	1.415640e+01	0.000e+00	5.316e+00	2.300e-01
18	302	1.386612e+01	0.000e+00	3.226e+00	1.048e+00
19	315	1.312882e+01	0.000e+00	1.127e+01	3.225e+00
20	333	1.244487e+01	0.000e+00	1.060e+01	3.340e+00
21	346	1.230746e+01	0.000e+00	1.991e+00	1.325e+00
22	360	1.217994e+01	0.000e+00	2.874e+00	2.445e+00

23	373	1.112840e+01	0.000e+00	1.034e+01	9.327e+00
24	386	1.123582e+01	0.000e+00	4.679e+00	3.812e+00
25	399	1.097960e+01	0.000e+00	1.920e+00	3.934e+00
26	412	1.021450e+01	0.000e+00	4.959e+00	8.181e+00
27	425	8.157047e+00	0.000e+00	8.583e+00	4.149e+01
28	438	7.879307e+00	0.000e+00	6.380e+00	1.175e+02
29	451	6.964305e+00	0.000e+00	4.066e+00	1.217e+01
30	464	6.525154e+00	0.000e+00	5.403e+00	4.577e+00

<i>Iter</i>	<i>F-count</i>	<i>f(x)</i>	<i>Feasibility</i>	<i>First-order optimality</i>	<i>Norm of step</i>
31	478	6.364799e+00	0.000e+00	1.942e+01	2.389e+00
32	491	5.983465e+00	0.000e+00	1.436e+01	6.719e+00
33	517	5.951188e+00	0.000e+00	3.593e+00	5.872e-03
34	540	5.945809e+00	0.000e+00	2.886e+00	5.322e-04
35	553	5.935179e+00	0.000e+00	1.334e+00	7.862e-01
36	569	5.899038e+00	0.000e+00	1.081e+01	2.281e-01
37	583	5.885097e+00	0.000e+00	9.115e+00	2.814e-01
38	596	5.889310e+00	0.000e+00	6.693e+00	1.666e+00
39	610	5.849384e+00	0.000e+00	2.762e+00	4.799e-01
40	635	5.848818e+00	0.000e+00	2.335e+00	2.296e-04
41	649	5.848714e+00	0.000e+00	2.316e+00	2.564e-03
42	662	5.832734e+00	0.000e+00	7.758e+00	4.591e-01
43	676	5.876316e+00	0.000e+00	3.689e+00	2.075e+00
44	701	5.861968e+00	0.000e+00	1.321e+00	1.726e-03
45	719	5.845735e+00	0.000e+00	1.678e+00	5.831e-03
46	743	5.845145e+00	0.000e+00	1.318e+00	3.800e-04
47	757	5.844948e+00	0.000e+00	1.312e+00	3.419e-03
48	778	5.844513e+00	0.000e+00	1.010e+00	3.461e-04
49	792	5.844104e+00	0.000e+00	1.010e+00	2.961e-03
50	821	5.844103e+00	0.000e+00	1.007e+00	1.236e-06
51	834	5.675282e+00	0.000e+00	3.081e+00	1.954e+00
52	847	5.547220e+00	0.000e+00	3.658e+00	2.194e+00
53	861	5.531441e+00	0.000e+00	1.083e+00	6.423e-01
54	875	5.529843e+00	0.000e+00	7.527e+00	3.982e-01
55	890	5.488433e+00	0.000e+00	1.398e+01	2.040e+00
56	903	5.390490e+00	0.000e+00	7.863e+00	1.077e+00
57	917	5.241316e+00	0.000e+00	2.415e+00	7.129e+00
58	935	5.092270e+00	0.000e+00	1.957e+00	2.567e+00
59	955	5.086614e+00	0.000e+00	1.923e+00	6.534e-01
60	973	5.072686e+00	0.000e+00	1.742e+00	3.269e-01

<i>Iter</i>	<i>F-count</i>	<i>f(x)</i>	<i>Feasibility</i>	<i>First-order optimality</i>	<i>Norm of step</i>
61	986	4.917467e+00	0.000e+00	1.851e+01	3.202e+00
62	1000	4.812662e+00	0.000e+00	1.269e+01	3.445e+00
63	1022	4.787421e+00	0.000e+00	1.943e+00	1.336e-01
64	1035	4.758598e+00	0.000e+00	1.705e+00	2.178e+00
65	1065	4.758497e+00	0.000e+00	1.374e+00	7.141e-05
66	1079	4.758482e+00	0.000e+00	1.375e+00	9.813e-04
67	1102	4.758460e+00	0.000e+00	1.356e+00	1.582e-05
68	1116	4.758455e+00	0.000e+00	1.355e+00	2.156e-04
69	1130	4.742448e+00	0.000e+00	3.601e+00	4.773e+00
70	1143	4.700137e+00	0.000e+00	2.442e+00	3.688e+00

71	1156	4.623466e+00	0.000e+00	6.489e+00	1.347e+00
72	1170	4.601237e+00	0.000e+00	5.554e+00	1.008e+00
73	1185	4.579377e+00	0.000e+00	1.296e+07	5.625e-01
74	1202	4.564270e+00	0.000e+00	2.196e+01	4.096e-02
75	1217	4.506464e+00	0.000e+00	1.044e+01	3.521e-02
76	1234	4.475379e+00	0.000e+00	1.955e+00	4.870e-02
77	1252	4.468708e+00	0.000e+00	1.689e+00	6.604e-02
78	1280	4.467976e+00	0.000e+00	1.610e+00	1.209e-04
79	1299	4.466647e+00	0.000e+00	1.445e+00	2.703e-04
80	1312	4.457330e+00	0.000e+00	1.228e+01	1.786e+00
81	1326	4.422030e+00	0.000e+00	5.136e+00	1.579e+00
82	1339	4.405408e+00	0.000e+00	9.572e+00	1.421e+00
83	1363	4.392107e+00	0.000e+00	6.376e+00	2.012e-03
84	1376	4.367898e+00	0.000e+00	4.163e-01	2.089e+00
85	1389	4.322757e+00	0.000e+00	1.852e+00	3.957e+00
86	1411	4.310231e+00	0.000e+00	2.328e+00	5.102e-02
87	1425	4.306148e+00	0.000e+00	5.587e-01	3.586e-01
88	1438	4.303808e+00	0.000e+00	6.906e+00	2.139e+00
89	1453	4.282216e+00	0.000e+00	3.299e+00	1.939e+00
90	1466	4.264330e+00	0.000e+00	6.021e+00	5.134e-01

<i>Iter</i>	<i>F-count</i>	<i>f(x)</i>	<i>Feasibility</i>	<i>First-order optimality</i>	<i>Norm of step</i>
91	1484	4.239597e+00	0.000e+00	7.848e+00	3.082e-01
92	1505	4.232595e+00	0.000e+00	3.205e+00	1.342e-01
93	1520	4.229167e+00	0.000e+00	1.616e-01	1.308e-01
94	1545	4.227782e+00	0.000e+00	9.493e-01	3.774e-03
95	1560	4.227080e+00	0.000e+00	8.926e-01	1.712e-02
96	1575	4.185855e+00	0.000e+00	2.595e+00	5.131e-01
97	1603	4.185314e+00	0.000e+00	2.231e+00	2.388e-04
98	1617	4.185169e+00	0.000e+00	2.224e+00	3.782e-03
99	1637	4.184419e+00	0.000e+00	1.600e+00	4.109e-04
100	1650	4.083505e+00	0.000e+00	2.365e+00	2.577e+00
101	1665	4.038388e+00	0.000e+00	3.274e+00	3.680e+00
102	1678	4.026301e+00	0.000e+00	1.355e+01	2.483e+00
103	1693	3.780690e+00	0.000e+00	1.092e+01	1.252e+01
104	1709	3.778464e+00	0.000e+00	7.565e+00	3.260e-01
105	1726	3.719588e+00	0.000e+00	7.120e+00	6.544e-01
106	1743	3.700321e+00	0.000e+00	3.359e+00	1.476e+00
107	1756	3.598661e+00	0.000e+00	3.576e+00	6.289e+01
108	1770	3.474408e+00	0.000e+00	1.175e+00	5.762e+00
109	1785	3.456396e+00	0.000e+00	4.876e+00	1.722e+00
110	1801	3.439921e+00	0.000e+00	3.581e+00	6.645e-01
111	1824	3.437818e+00	0.000e+00	3.334e+00	8.224e-03
112	1849	3.437536e+00	0.000e+00	3.237e+00	2.094e-04
113	1863	3.437513e+00	0.000e+00	3.234e+00	1.526e-03
114	1882	3.436484e+00	0.000e+00	2.883e+00	7.096e-04
115	1895	3.419829e+00	0.000e+00	4.003e+00	3.279e-01
116	1911	3.416564e+00	0.000e+00	5.030e-01	1.108e-01
117	1935	3.415492e+00	0.000e+00	1.232e+00	1.779e-03
118	1950	3.415424e+00	0.000e+00	1.262e+00	6.079e-03
119	1964	3.396581e+00	0.000e+00	4.655e+00	2.588e+00
120	1994	3.396515e+00	0.000e+00	4.621e+00	1.437e-05

<i>Iter</i>	<i>F-count</i>	<i>f(x)</i>	<i>Feasibility</i>	<i>First-order optimality</i>	<i>Norm of step</i>
121	2017	3.396507e+00	0.000e+00	4.601e+00	1.569e-06
122	2032	3.396507e+00	0.000e+00	4.614e+00	8.524e-06
123	2054	3.396506e+00	0.000e+00	4.606e+00	3.432e-07
124	2067	3.377520e+00	0.000e+00	3.199e+00	1.695e+00
125	2080	3.342641e+00	0.000e+00	1.606e+00	2.009e+00
126	2107	3.342573e+00	0.000e+00	1.547e+00	1.817e-04
127	2122	3.342558e+00	0.000e+00	1.542e+00	5.938e-04
128	2141	3.342387e+00	0.000e+00	1.427e+00	3.009e-04
129	2159	3.341866e+00	0.000e+00	9.966e-01	1.042e-03
130	2176	3.337293e+00	0.000e+00	1.930e+00	7.720e-03
131	2204	3.337259e+00	0.000e+00	1.911e+00	1.736e-05
132	2218	3.337251e+00	0.000e+00	1.898e+00	1.915e-04
133	2240	3.337237e+00	0.000e+00	1.895e+00	7.555e-06
134	2254	3.337236e+00	0.000e+00	1.930e+00	7.382e-05
135	2275	3.337223e+00	0.000e+00	1.854e+00	6.609e-06
136	2289	3.337222e+00	0.000e+00	1.840e+00	6.491e-05
137	2313	3.337221e+00	0.000e+00	1.910e+00	7.237e-07
138	2329	3.337220e+00	0.000e+00	1.902e+00	1.893e-06
139	2342	3.314594e+00	0.000e+00	5.288e-01	4.239e+00
140	2360	3.289038e+00	0.000e+00	1.585e+00	1.142e+00
141	2382	3.286653e+00	0.000e+00	7.881e-01	1.427e-02
142	2397	3.286624e+00	0.000e+00	7.881e-01	6.959e-03
143	2418	3.285858e+00	0.000e+00	2.224e+00	9.209e-04
144	2432	3.285867e+00	0.000e+00	2.125e+00	5.607e-03
145	2445	3.284321e+00	0.000e+00	2.599e+00	1.630e-01
146	2458	3.274713e+00	0.000e+00	7.937e-01	7.143e-01
147	2484	3.274630e+00	0.000e+00	7.845e-01	1.901e-04
148	2498	3.274599e+00	0.000e+00	7.753e-01	1.306e-03
149	2518	3.274446e+00	0.000e+00	7.013e-01	3.122e-04
150	2532	3.274405e+00	0.000e+00	6.796e-01	2.347e-03

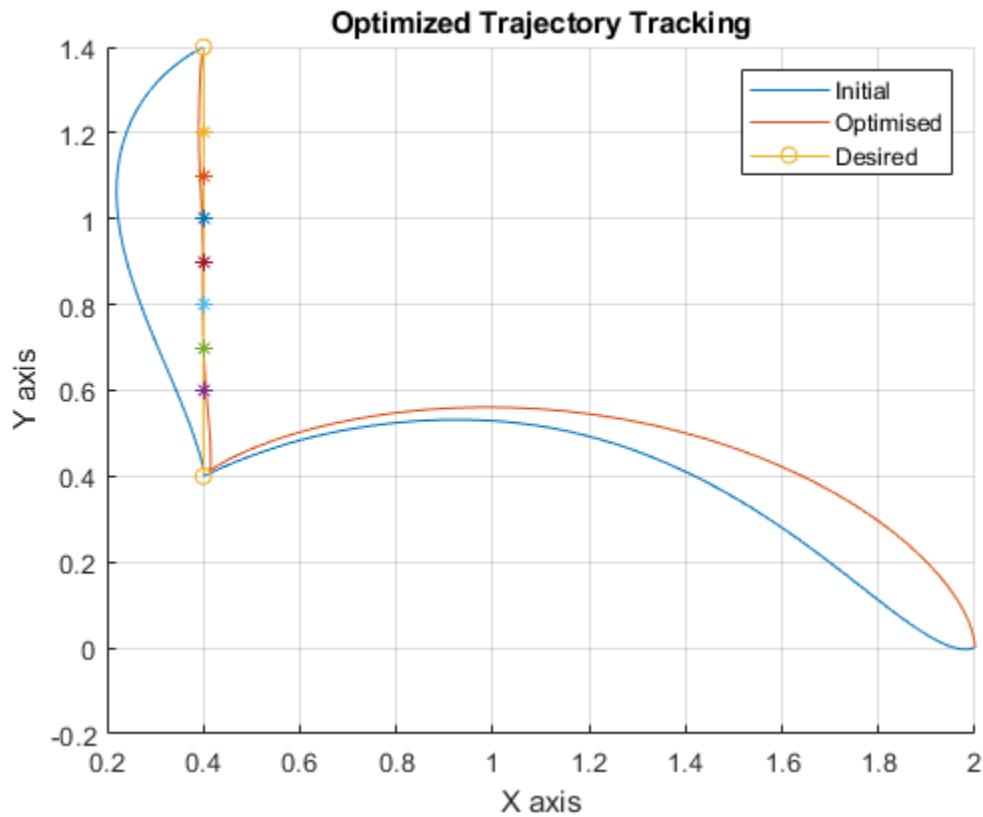
<i>Iter</i>	<i>F-count</i>	<i>f(x)</i>	<i>Feasibility</i>	<i>First-order optimality</i>	<i>Norm of step</i>
151	2551	3.273919e+00	0.000e+00	4.939e-01	1.152e-03
152	2569	3.272554e+00	0.000e+00	4.058e-01	3.988e-03
153	2594	3.272510e+00	0.000e+00	4.031e-01	1.158e-04
154	2609	3.272502e+00	0.000e+00	4.039e-01	3.920e-04
155	2623	3.266508e+00	0.000e+00	1.611e+01	3.180e+00
156	2639	3.259234e+00	0.000e+00	5.557e+00	4.123e-01
157	2653	3.230421e+00	0.000e+00	1.387e+01	2.329e-01
158	2670	3.208279e+00	0.000e+00	9.095e-01	5.563e-03
159	2690	3.202602e+00	0.000e+00	9.844e+00	3.355e-02
160	2705	3.192585e+00	0.000e+00	2.533e+00	1.049e-01
161	2729	3.187232e+00	0.000e+00	3.676e+00	1.850e-03
162	2745	3.187084e+00	0.000e+00	3.664e+00	2.919e-03
163	2758	3.167077e+00	0.000e+00	4.192e+00	3.005e+00
164	2781	3.153975e+00	0.000e+00	9.564e-01	5.087e-03
165	2805	3.153885e+00	0.000e+00	8.958e-01	2.800e-04
166	2819	3.153825e+00	0.000e+00	8.961e-01	1.906e-03
167	2839	3.153613e+00	0.000e+00	8.200e-01	4.819e-04
168	2854	3.153554e+00	0.000e+00	8.200e-01	1.691e-03
169	2872	3.152918e+00	0.000e+00	8.197e-01	1.693e-03

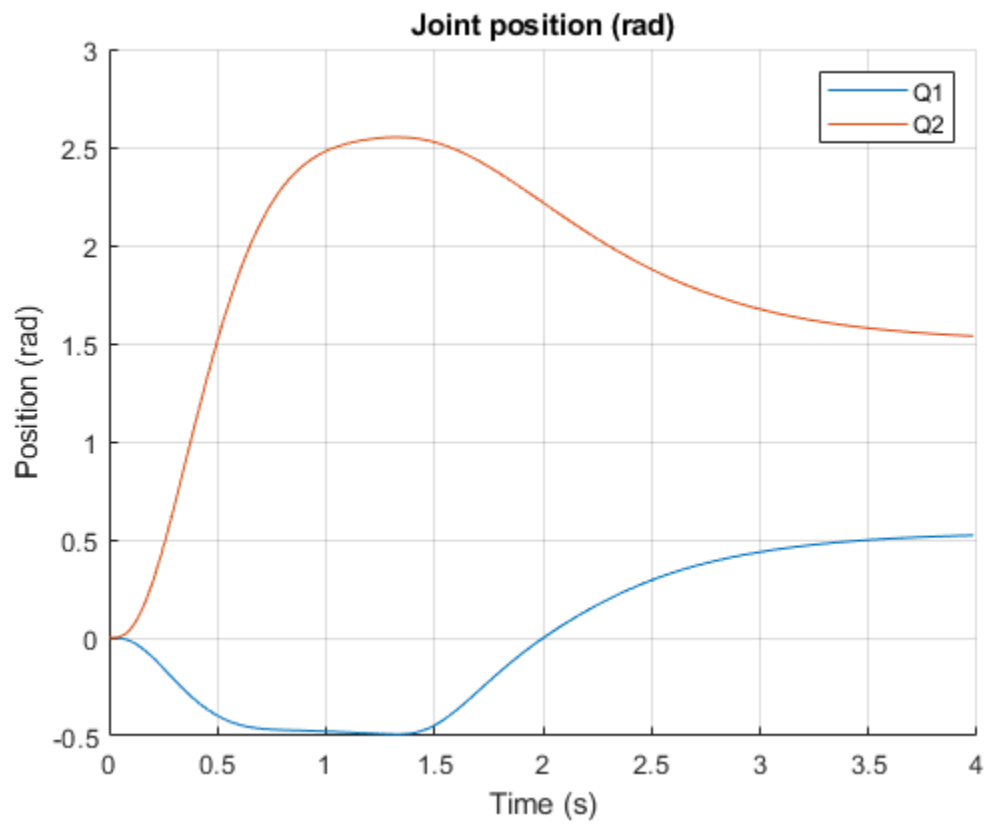
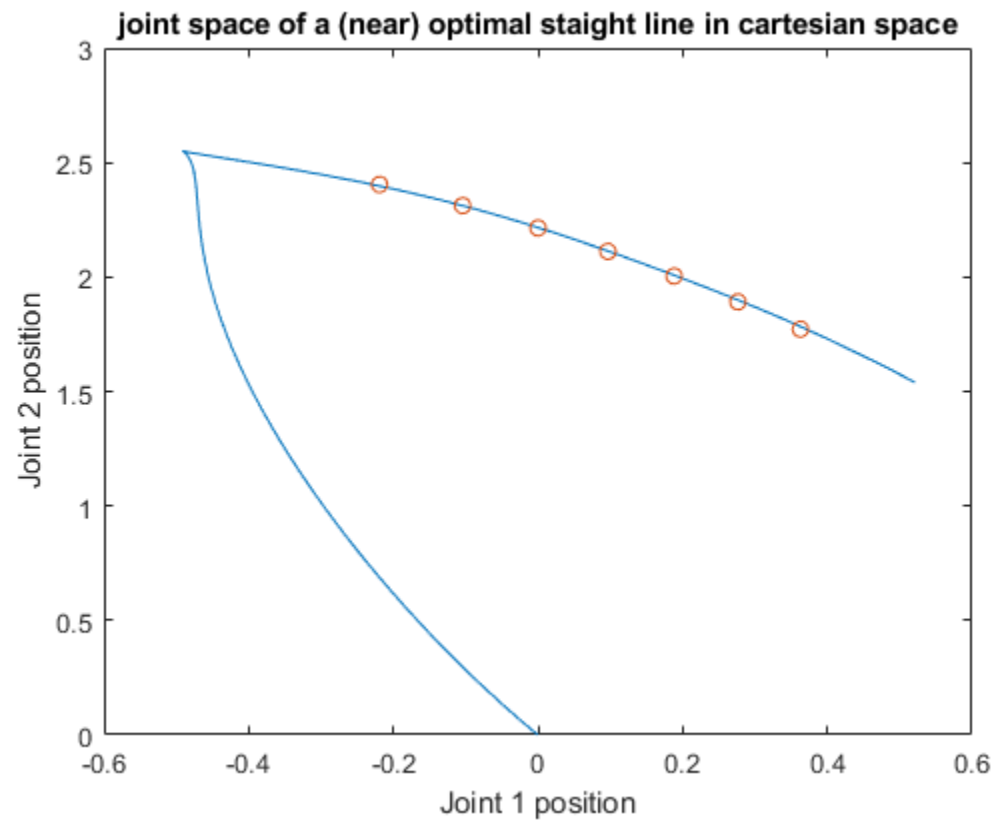
170	2888	3.152819e+00	0.000e+00	8.196e-01	2.998e-03
171	2905	3.151500e+00	0.000e+00	4.017e-01	5.902e-03
172	2928	3.151421e+00	0.000e+00	4.020e-01	6.502e-04
173	2943	3.151347e+00	0.000e+00	4.017e-01	2.297e-03
174	2961	3.151119e+00	0.000e+00	4.024e-01	2.310e-03
175	2985	3.151103e+00	0.000e+00	4.063e-01	1.261e-04
176	2999	3.151073e+00	0.000e+00	4.010e-01	8.827e-04

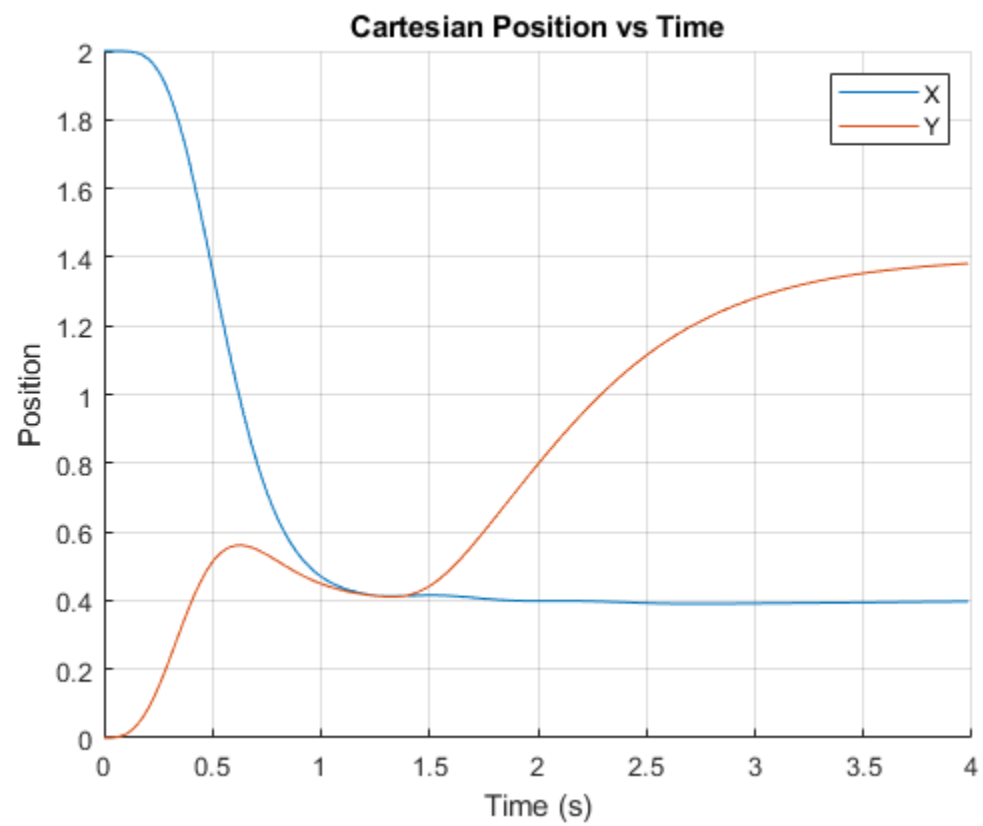
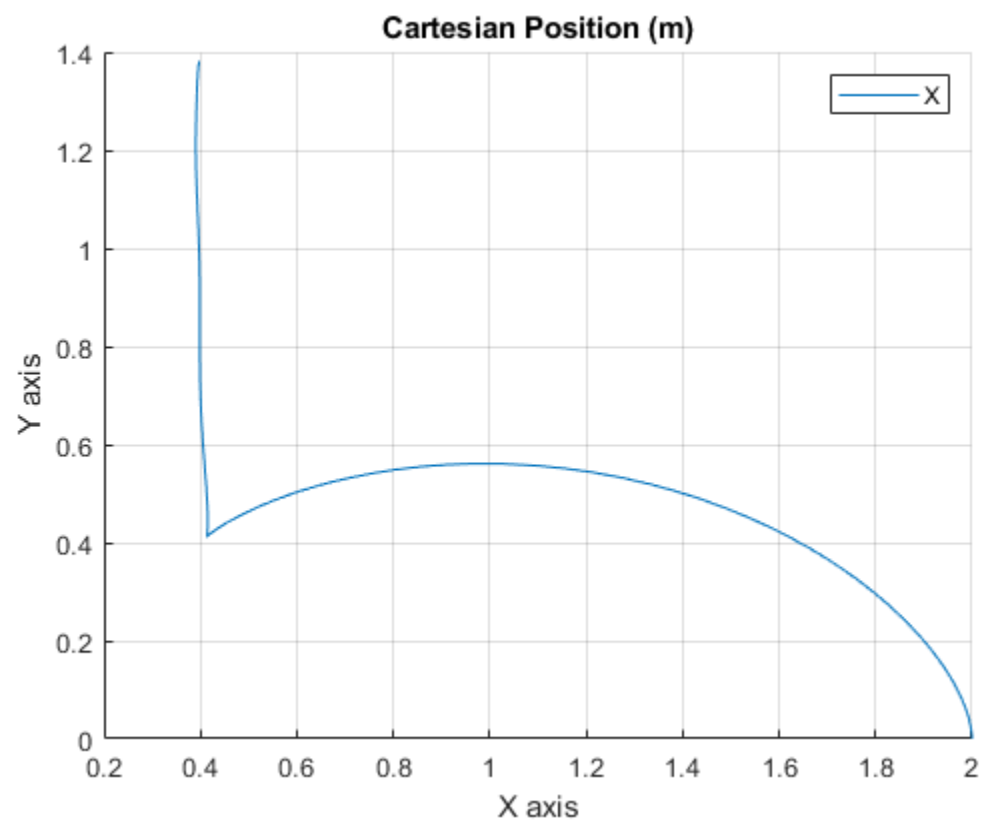
Solver stopped prematurely.

*fmincon stopped because it exceeded the function evaluation limit,
options.MaxFunctionEvaluations = 3.000000e+03.*

Opt Time: 1.2437 3.9825
 Opt Wn : 6.3031 2.3182
 Opt bj : 23.0489 15.223 12.491 50.7821
 Opt kj : 117.097 90.71263 169.3678 197.7183
 Warning: Ignoring extra legend entries.







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