Exploring equations of motion using neural network

An Exploration of Double Pendulum

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Abstract

We introduce the latest neural network structure: Neural Ordinary Differential Equations (Neural ODE), Hamiltonian Neural Network (HNN) and Hamiltonian Generative Network (HGN) to simulate the particle motion. Whereas related work like Lagrangian Neural Network (LNN) and Deep Lagrangian Network (DeLaN) are introduced by group 8. [which encode some extent of symmetrics to boost the accuracy and efficiency for the simulation of equation of motion and discuss their application in field theory.]

Introduction

Neural Ordinary Differential Equation (Neural ODE)

If we use traditional method like recurrent neural network or residual neural network, to simulate every point of the particle in time would take infinite layers which is impossible to implement in practice.

However, in the continuous limit, we can parameterize the dynamics of hidden units using an ordinary differential equation (ODE) specified by a neural network:

$$\frac{d\mathbf{h}}{dt} = f(\mathbf{h}(\mathbf{t}), \mathbf{t}, \theta) \tag{1}$$

Starting from the input layer $\mathbf{h}(\mathbf{0})$, we can define the output layer $\mathbf{h}(\mathbf{T})$ to be the slution to this ODE initial value problem at some time T. This value can be unit dynamics f wherever necessary to determine the solution with the desired accuracy.

Treating the ODE solver as a black box, and compute gradients using the adjoint sensitivity method. This approach computes gradients by solving a second, augmented ODE backwards in time, and is applicable all Ode solvers. This approach scales linearly the problem size, has low memory cost, and explicitly controls numerical error.

Main Objectives

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2. Nullam at mi nisl. Vestibulum est purus, ultricies cursus volutpat sit amet, vestibulum eu.
3. Praesent tortor libero, vulputate quis elementum a, iaculis.
4. Phasellus a quam mauris, non varius mauris. Fusce tristique, enim tempor varius porta, elit purus commodo velit, pretium mattis ligula nisl nec ante.
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Materials and Methods

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Mathematical Section

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$$E = mc^2 \tag{1}$$

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Vestibulum ac diam a odio tempus congue. Vivamus id enim nisi:

$$\begin{aligned} &\cos\bar{\phi}_k Q_{j,k+1,t} + Q_{j,k+1,x} + \frac{\sin^2\bar{\phi}_k}{T\cos\bar{\phi}_k} Q_{j,k+1} = \\ &\quad -\cos\phi_k Q_{j,k,t} + Q_{j,k,x} - \frac{\sin^2\phi_k}{T\cos\phi_k} Q_{j,k} \end{aligned} \tag{2}$$

and

$$\begin{aligned} &\cos\bar{\phi}_j Q_{j+1,k,t} + Q_{j+1,k,y} + \frac{\sin^2\bar{\phi}_j}{T\cos\bar{\phi}_j} Q_{j+1,k} = \\ &\quad -\cos\phi_j Q_{j,k,t} + Q_{j,k,y} - \frac{\sin^2\phi_j}{T\cos\phi_j} Q_{j,k}. \end{aligned} \tag{3}$$

Nulla sed arcu arcu. Duis et ante gravida orci venenatis tincidunt. Fusce vitae lacinia metus. Pellentesque habitant morbi. $\mathbf{A}\underline{\xi} = \underline{\beta}$ Vim $\underline{\xi}$ enum nidi $3(P+2)^2$ lacina. Id feugain $\mathbf{A}$ nun quis; magno.

Results

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Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 1: Table caption

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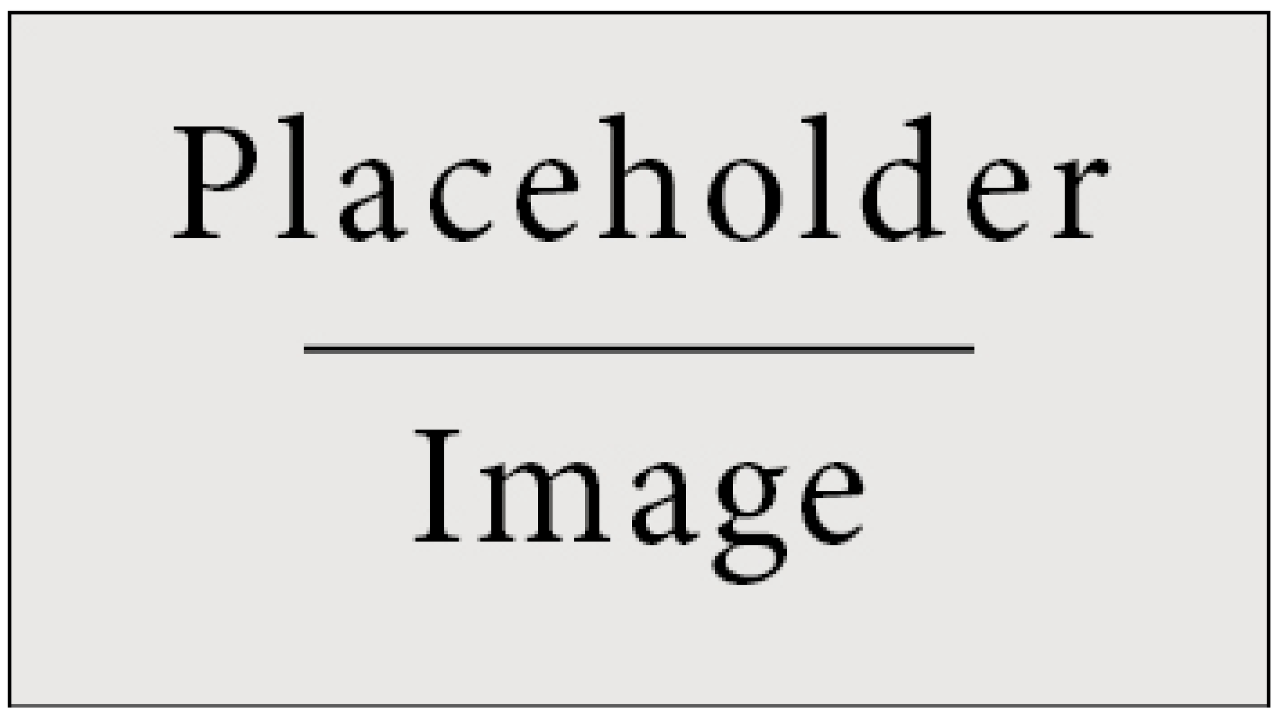

Figure 1: Figure caption

In hac habitasse platea dictumst. Etiam placerat, risus ac.
Adipiscing lectus in magna blandit:

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table 2: Table caption

Vivamus sed nibh ac metus tristique tristique a vitae ante. Sed lobortis mi ut arcu fringilla et adipiscing ligula rutrum. Aenean turpis velit, placerat eget tincidunt nec, ornare in nisl. In placerat.

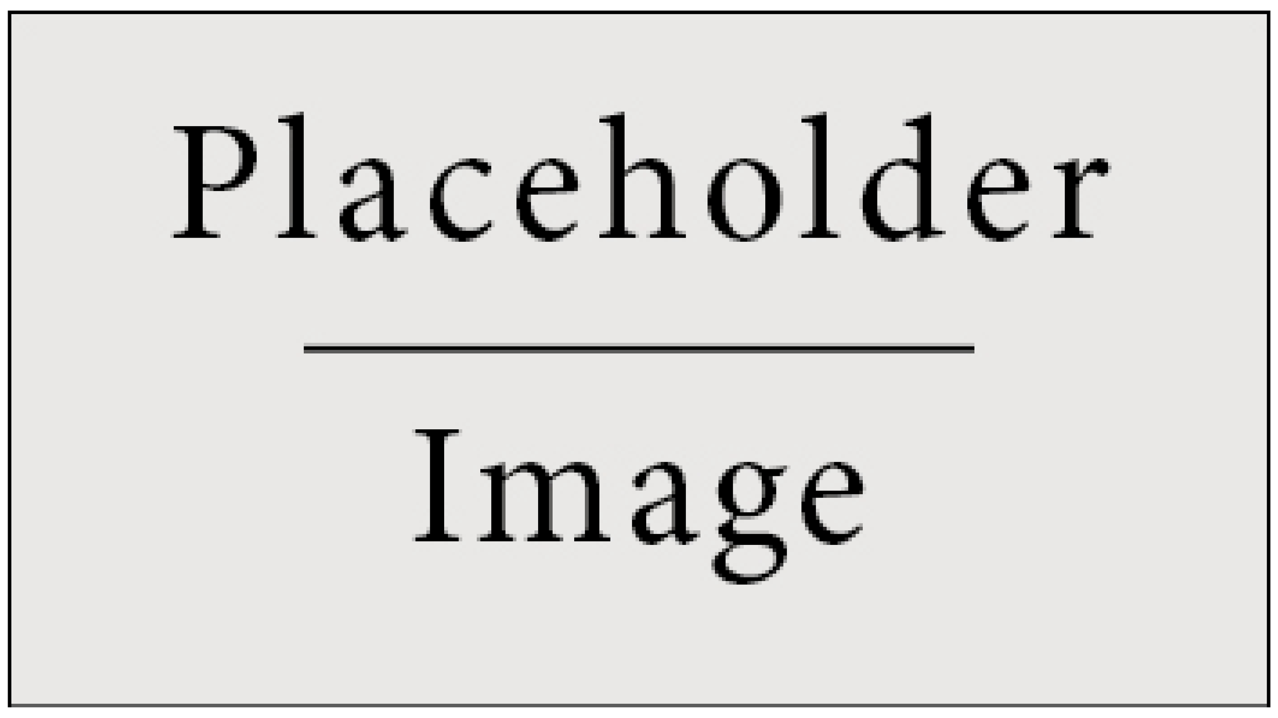

Figure 2: Figure caption

Conclusions

- Pellentesque eget orci eros. Fusce ultricies, tellus et pellentesque fringilla, ante massa luctus libero, quis tristique purus urna nec nibh. Phasellus fermentum rutrum elementum. Nam quis justo lectus.
- Vestibulum sem ante, hendrerit a gravida ac, blandit quis magna.
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- Nunc at convallis urna. isus ante. Pellentesque condimentum dui. Etiam sagittis purus non tellus tempor volutpat. Donec et dui non massa tristique adipiscing.

Forthcoming Research

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Acknowledgements

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