# ELEN 21, COEN 21: Logic Design Winter 2021

# Homework

**due Wednesday 20 January 2021 11:59 pm**

1. (20 points) For each of the four functions below:

$f_1(a,b,c) = \sum m(0,4,5,6)$ $\quad f_3(a,b,c) = a'b' + a'c + abc + ab'c'$

$f_2(a,b,c) = \prod M(0,1,5,6)$ $\quad f_4(a,b,c) = (b' + c')(a' + b + c')(a + b + c)$

   a. Show the function on a K-map.
   b. Identify all pairs of minterms that can be combined into a single product based on the logical adjacency theorem (14a in the text). Identify all the products for the minterm pairs.
   c. Identify the all the minterms that do not have a logically adjacent minterm in the function.
   d. For a SOP implementation using all the products in (b) and (c), what is the cost of the implementation in terms of the number of gates and the number of inputs?
   e. Identify all the essential products from (b) and (c) that must be included in any correct implementation of the function. Then, from the remaining products, select any addition products needed to complete the function. How does this cost compare to the implementation in (d)?

2. (20 points) Repeat the previous problem for a POS implementation using maxterms instead of minterms (based on Theorem 14.b instead of 14.a).

3. (15 points) The exclusive-or function $f(x,y) = x \oplus y = x'y + xy' = (x + y)(x' + y')$ is very important for arithmetic circuits and error detection and correction in communications. Show each of the five functions below on a K-map, and find the two functions that have the same map. Algebraically show that the two with the same K-map are equivalent functions.

a.) $f(a,b,c) = a \oplus b$ b.) $f(a,b,c) = a \oplus c$ c.) $f(a,b,c) = b \oplus c$

d.) $f(a,b,c) = (a \oplus b) \oplus (a \oplus c)$ e.) $f(a,b,c) = (a \oplus b) \oplus (c)$

4. (15 points) For each logic function, show the circuit for an AND-OR or OR-AND implementation as indicated, and then use DeMorgan's Law to redraw the circuit using only NANDs or NORs as indicated. Note that a single variable can be considered as both a product of one variable and also as a sum of one variable.
    a. Show the circuit for $f_1(a,b,c) = a'b + ab'c$ in SOP form using an AND-OR implementation. Show the circuit implementation using only NANDs.
    b. Show the circuit for $f_2(a,b,c) = a + b'c + bc'$ in SOP form using an AND-OR implementation. Show the circuit implementation using only NANDs.
    c. Show the circuit for $f_3(a,b,c) = b \cdot (a+c) \cdot (a'+c')$ in POS form using an OR-AND implementation. Show the circuit implementation using only NORs.

5. (10 points) Write a structural Verilog module to implement the AND-OR circuit in Problem 4b following the example in Figure 2.36 and 2.37 in the text (or the full adder example from class notes). Name all the internal wires (nets) before you write the module.