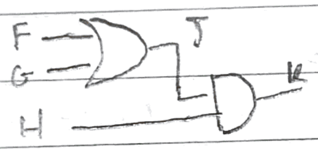
Homework 1

1a. 

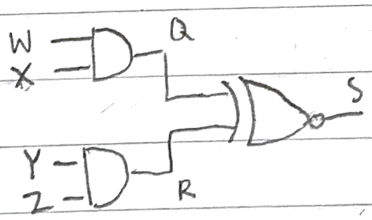
$E = (A \cdot B) + C$

A	B	C	D	E
0	0	0	0	0
0	0	1	0	1
0	1	0	0	0
0	1	1	0	1
1	0	0	0	0
1	0	1	0	1
1	1	0	1	1
1	1	1	1	1

1b. 

$K = (F + G) \cdot H$

F	G	H	J	K
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

1c. 

$S = (W \cdot X)\ \overline{\oplus}\ (Y \cdot Z)$

$2^4 = 16$

W	X	Y	Z	Q	R	S
0	0	0	0	0	0	1
0	0	0	1	0	0	1
0	0	1	0	0	0	1
0	0	1	1	0	1	0
0	1	0	0	0	0	1
0	1	0	1	0	0	1
0	1	1	0	0	0	1
0	1	1	1	0	1	0
1	0	0	0	0	0	1
1	0	0	1	0	0	1
1	0	1	0	0	0	1
1	0	1	1	0	1	0
1	1	0	0	1	0	0
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	1	1

2a. $f_1(a,b) = ab' + b$

a	b	b'	$f_1(a,b)$
0	0	1	0
0	1	0	1
1	0	1	1
1	1	0	1

2b. $f_2(a,b) = (a+b')\,b$

a	b	b'	$a+b'$	$f_2(a,b)$
0	0	1	1	0
0	1	0	0	0
1	0	1	1	0
1	1	0	1	1

2c. $f_3(a,b) = (a'+b)\,b'$

a	b	a'	b'	$(a'+b)$	$f_3(a,b)$
0	0	1	1	1	1
0	1	1	0	1	0
1	0	0	1	0	0
1	1	0	0	1	0

$f_3(a,b)$ is a complement of $f_1(a,b)$

2d. $f_4(a,b) = a'b + b$

a	b	a'	$a'b$	$f_4(a,b)$
0	0	1	0	0
0	1	1	1	1
1	0	0	0	0
1	1	0	0	1

2e. $f_5(a,b) = (a)\ XOR\ (ab)$

a	b	ab	$f_5(a,b)$
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	0

3a. $f_1(x,y,z) = xz + yz' + xy$ $\quad f_2(x,y,z) = xz + yz'$

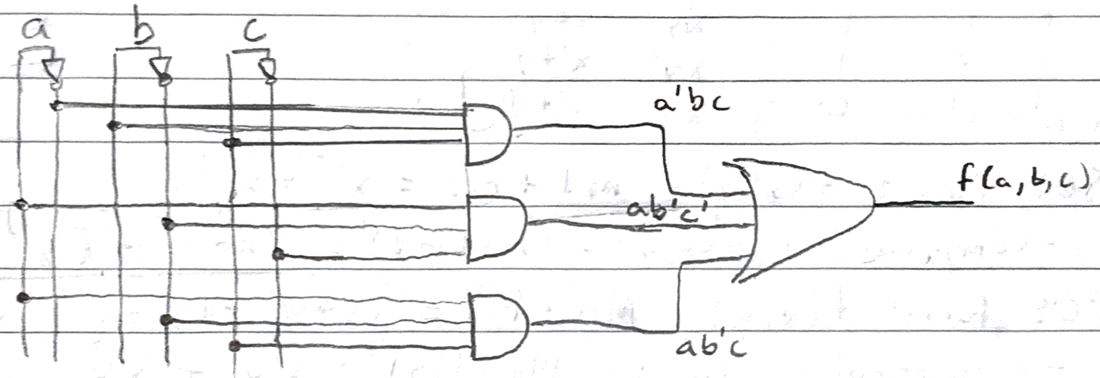
x	y	z	z'	xz	yz'	xy	$f_1(x,y,z)$	$f_2(x,y,z)$
0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0
0	1	0	1	0	1	0	1	1
0	1	1	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0
1	0	1	0	1	0	0	1	1
1	1	0	1	0	1	1	1	1
1	1	1	0	1	0	1	1	1

3b. $(x+z)(y+z')(x+y) = (x+z)(y+z')$

4a. $f(a,b,c) = m3 + m4 + m5$

	a	b	c	product
0	0	0	0	$a'b'c'$
1	0	0	1	$a'b'c$
2	0	1	0	$a'bc'$
3	0	1	1	$a'bc$
4	1	0	0	$ab'c'$
5	1	0	1	$ab'c$
6	1	1	0	abc'
7	1	1	1	abc

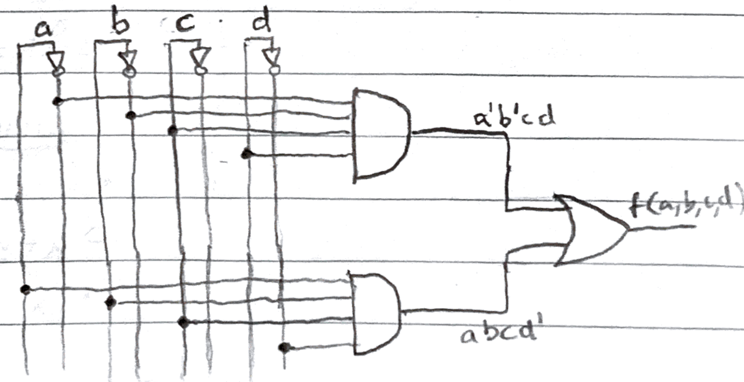
$f(a,b,c) = a'bc + ab'c' + ab'c$



4b. $f(a,b,c,d) = m3 + m14$

	a	b	c	d	product
0	0	0	0	0	$a'b'c'd'$
1	0	0	0	1	$a'b'c'd$
2	0	0	1	0	$a'b'cd'$
3	0	0	1	1	$a'b'cd$
4	0	1	0	0	$a'bc'd'$
5	0	1	0	1	$a'bc'd$
6	0	1	1	0	$a'bcd'$
7	0	1	1	1	$a'bcd$
8	1	0	0	0	$ab'c'd'$
9	1	0	0	1	$ab'c'd$
10	1	0	1	0	$ab'cd'$
11	1	0	1	1	$ab'cd$
12	1	1	0	0	$abc'd'$
13	1	1	0	1	$abc'd$
14	1	1	1	0	$abcd'$
15	1	1	1	1	$abcd$

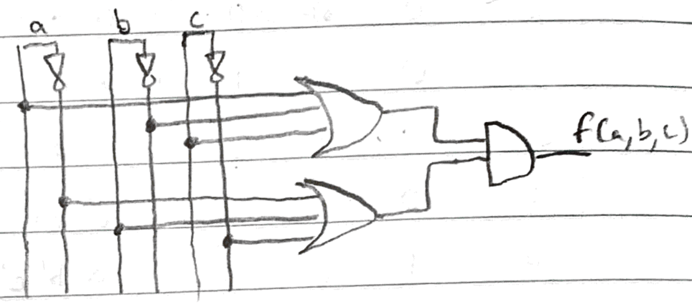
$f(a,b,c,d) = a'b'cd + abcd'$



4c. $f(a,b,c) = (M2)(M5)$

	a	b	c	sum
0	0	0	0	$a+b+c$
1	0	0	1	$a+b+c'$
2	0	1	0	$a+b'+c$
3	0	1	1	$a+b'+c'$
4	1	0	0	$a'+b+c$
5	1	0	1	$a'+b+c'$
6	1	1	0	$a'+b'+c$
7	1	1	1	$a'+b'+c'$

$f(a,b,c) = (a+b'+c)\cdot(a'+b+c')$



5a. $f(x,y) = x \oplus y$

x	y	f(x,y)	SOP	POS
0	0	0	$x'y'$	$x+y$
0	1	1	$x'y$	$x+y'$
1	0	1	xy'	$x'+y$
1	1	0	xy	$x'+y'$

SOP form: $f(x,y) = m1 + m2 = \boxed{x'y + xy'}$

- complement: $f'(x,y) = (m1+m2)' = \boxed{(x+y')\cdot(x'+y)}$ or $M_1 \cdot M_2$

POS form: $f(x,y) = M0 \cdot M3 = \boxed{(x+y)\cdot(x'+y')}$

- complement: $f'(x,y) = (M0 \cdot M3)' = \boxed{x'y' + xy}$ or $(m_0 + m_3)$

5b. $f(x,y,z) = xz + yz' = xyz + xy'z + xyz' + x'yz'$

	x	y	z	f(x,y,z)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

SOP form

$f(x,y,z) = \Sigma m(2,5,6,7)$

$= x'yz' + xy'z + xyz' + xyz$

Complement of SOP

$f'(x,y,z) = (m2+m5+m6+m7)'$

$= (x+y'+z)\cdot(x'+y+z')\cdot(x'+y'+z)\cdot(x'+y'+z')$

or

$M2 \cdot M5 \cdot M6 \cdot M7$

POS form

$f(x,y,z) = \Pi M(0,1,3,4)$

$= (x+y+z)\cdot(x+y+z')\cdot(x+y'+z')\cdot(x'+y+z)$

Complement of POS

$f'(x,y,z) = (M_0 \cdot M_1 \cdot M_3 \cdot M_4)'$

$= (x'y'z') + (x'y'z) + (x'yz) + (xy'z')$

or

$m_0 + m_1 + m_3 + m_4$

5c. $f(x,y,z) = xz + yz' + xy = xyz' + xy'z + xyz' + x'yz' + xyz + xyz'$

	x	y	z	f(x,y,z)
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

SOP form

$f(x,y,z) = \Sigma m(2,5,6,7)$

$= x'yz' + xy'z + xyz' + xyz$

Complement of SOP

$f'(x,y,z) = (m2+m5+m6+m7)'$

$= (x+y'+z)\cdot(x'+y+z')\cdot(x'+y'+z)\cdot(x'+y'+z')$

or

$M2 \cdot M5 \cdot M6 \cdot M7$

POS form

$f(x,y,z) = \Pi M(0,1,3,4)$

$= (x+y+z)\cdot(x+y+z')\cdot(x+y'+z')\cdot(x'+y+z)$

Complement of POS

$f'(x,y,z) = (M0 \cdot M1 \cdot M3 \cdot M4)'$

$= x'y'z' + x'y'z + x'yz + xy'z'$

or

$m0 + m1 + m3 + m4$

5d. $f(x,y,z) = (x+y')(x'+z) = (x+y'+z)(x+y'+z')(x'+y+z)(x+y'+z)$

	x	y	z	f(x,y,z)
0	0	0	0	1
1	0	0	1	1
2	0	1	0	0
3	0	1	1	0
4	1	0	0	0
5	1	0	1	1
6	1	1	0	0
7	1	1	1	1

SOP form

$f(x,y,z) = \Sigma m(0,1,5,7)$

$= x'y'z' + x'y'z + xy'z + xyz$

Complement of SOP

$f'(x,y,z) = (m0+m1+m5+m7)'$

$= (x+y+z)\cdot(x+y+z')\cdot(x'+y+z')\cdot(x'+y'+z')$

or

$M0 \cdot M1 \cdot M5 \cdot M7$

POS form

$f(x,y,z) = \Pi M(2,3,4,6)$

$= (x+y'+z)\cdot(x+y'+z')\cdot(x'+y+z)\cdot(x'+y'+z)$

Complement of POS

$f'(x,y,z) = (M2 \cdot M3 \cdot M4 \cdot M6)'$

$= x'yz' + x'yz + xy'z' + xyz'$

or

$m2 + m3 + m4 + m6$

6a. $f_1(b,c) = b+c$ $f_2(b,c) = bc$

b	c	f_1	f_2
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

f_1

c \ b	0	1
0	0	1
1	1	1

f_2

c \ b	0	1
0	0	0
1	0	1

6b. $f_1(b,c)$: SOP form

$f_1(b,c) = \Sigma m(1,2,3) = b'c + bc' + bc$

POS form

$f_1(b,c) = M0 = b+c$

$f_2(b,c)$: SOP form

$f_2(b,c) = m3 = bc$

POS form

$f_2(b,c) = \Pi M(0,1,2) = (b+c)\cdot(b+c')\cdot(b'+c)$

6c. $f_3(a,b,c) = a' \cdot f_1(b,c) + a \cdot f_2(b,c)$

$= a' \cdot (b+c) + a \cdot bc$

$= a'b + a'c + abc$

6d.

f_3: $c \backslash ab$	00	01	11	10
0	0	1	0	0
1	1	1	1	0

$a'c$ $a'b$ bc

	a	b	c	$a'b$	$a'c$	abc	$f_3(a,b,c)$
0	0	0	0	0	0	0	0
1	0	0	1	0	1	0	1
2	0	1	0	1	0	0	1
3	0	1	1	1	1	0	1
4	1	0	0	0	0	0	0
5	1	0	1	0	0	0	0
6	1	1	0	0	0	0	0
7	1	1	1	0	0	1	1

6e. $f_3(a,b,c) = a'c + a'b + bc$