

# Homework #5

$$1. \quad V_1 \rightarrow V_2 V_2 \mid V_4 V_3 \mid V_4 V_5 \mid \epsilon$$
$$V_2 \rightarrow V_2 V_2 \mid -V_4 V_3 \mid V_4 V_5 \mid$$
$$V_3 \rightarrow V_2 V_5$$
$$V_u \rightarrow ($$
$$V_5 \rightarrow )$$

(	(	)	)	(	(	)	(	)	)
1	2	3	4	5	6	7	8	9	10

1 (88)

[illegible]

a.  $P[2, 2]$ ,  $P[2, 6]$ ,  $P[2, 8]$ ,  $P[4, 1]$ ,  $P[4, 6]$  &  $P[10, 1]$  contains  $V_1$

dii  $P[1,4]$  contains  $V_5$  True

$$\begin{array}{ccccccc} & & (1,1,1) & & & & \\ \sqrt{4} & ( & ( & ( & ( & ( & ( \\ \sqrt{2} & (1,1) & 1,1) & \sqrt{2}(1) & & & \end{array}$$

Q III  $P[2, 3]$  contains  $V_4$  False

$$\begin{array}{l} V_2 \\ V_4 V_3 \\ V_4 V_2 V_3 \\ V_4 V_4 V_3 V_5 \end{array}$$

Q10  $PL[5,6]$  contains  $V_3$  True

a)  $P[6, 5]$  contains  $V_2$  True

QVI  $PI[9, 27]$  contains  $\sqrt{3}$  False

The diagram illustrates a hierarchical tree structure with nodes labeled  $V_1$  through  $V_5$ . The structure is as follows:

- $V_1$  is the root node, branching into  $V_2$  and  $V_3$ .
- $V_2$  branches into  $V_4$  and  $V_5$ .
- $V_3$  branches into  $V_4$  and  $V_5$ .
- $V_4$  branches into  $V_2$  and  $V_3$ .
- $V_5$  branches into  $V_4$  and  $V_5$ .

The diagram is drawn on lined paper, with the nodes and edges clearly marked.

2a.  $S \rightarrow |S| | + X |$   
 $X \rightarrow |X| | =$

b.

b.

Diagram illustrating a DFA (Deterministic Finite Automaton) with states and transitions. The states are represented by circles, and the transitions are labeled with input/output pairs.

Transitions shown:

- Top-left state to Top-right state:  $b, c \rightarrow a, \epsilon$
- Top-left state to Top-middle state:  $a, \epsilon \rightarrow c$
- Top-middle state to Top-right state:  $c, \epsilon \rightarrow \epsilon$  and  $\epsilon, \epsilon \rightarrow c$
- Top-middle state to Bottom-middle state:  $\epsilon, c \rightarrow \epsilon$
- Top-right state to Bottom-right state:  $a, \epsilon \rightarrow a, c$ ,  $b, \epsilon \rightarrow b, c$ , and  $c, \epsilon \rightarrow c$
- Bottom-right state to Bottom-left state:  $a, \epsilon \rightarrow a, c$ ,  $b, \epsilon \rightarrow b, c$ , and  $c, \epsilon \rightarrow c$
- Bottom-left state to Top-middle state:  $\epsilon, c \rightarrow \epsilon$
- Bottom-left state to itself:  $a, a \rightarrow \epsilon$ ,  $b, b \rightarrow \epsilon$ , and  $c, c \rightarrow \epsilon$
- Bottom-middle state to Top-middle state:  $\epsilon, c \rightarrow \epsilon$
- Bottom-middle state to Bottom-right state:  $a, \epsilon \rightarrow a, c$ ,  $b, \epsilon \rightarrow b, c$ ,  $c, \epsilon \rightarrow c$ ,  $a, \epsilon \rightarrow \epsilon$ ,  $b, \epsilon \rightarrow \epsilon$ , and  $c, \epsilon \rightarrow \epsilon$

4a.  $L(P) = \{a^{n+m}b^n c^{2m} \mid n, m \geq 0\}$

b. Let  $p > 0$  and pick  $S = a^{2p}b^p c^{2p}$ . Consider any decomposition  $S = uvxyz$  with  $|vy| > 0$ ,  $|vxy| \leq p$ .

case 1: If  $v$  or  $y$  has an  $a$ , then  $v$  and  $y$  cannot have a  $c$ , thus the balance of  $a$ 's and  $c$ 's falls off.

case 2: If  $v$  or  $y$  has a  $c$ , then  $v$  and  $y$  cannot have an  $a$ , thus the balance of  $a$ 's and  $c$ 's falls off.

case 3: If  $v$  and  $y$  are  $b$ 's, then  $uxz$  has ~~few~~ too few  $b$ 's relative to  $a$ 's and  $c$ 's since the amount of  $a$ 's and  $c$ 's are not 2 times the amount of  $b$ 's.

Citations: Used Class Notes and Textbook