Homework 4

1. $S_0 \to 1S_01 \mid S\# \mid \#S$ CFG $G_1 = (\{S_0, S\}, \{1, \#\}, R, S_0)$
$S \to 1S \mid 1$ w/ $R = \{S_0 \to 1S_01, S_0 \to S\#, S_0 \to \#S, S \to 1S, S \to 1\}$

2. $S \to 0S0 \mid 1S0 \mid 0X1 \mid 1X1$
$X \to 0X0 \mid 1X0 \mid 0X1 \mid 1X1 \mid 0 \mid 1 \mid \varepsilon$

CFG $G_2 = (\{S, X\}, \{0,1\}, R, S)$
w/ $R = \{S \to 0S0, S \to 1S0, S \to 0X1, S \to 1X1, X \to 0X0, X \to 1X0, X \to 0X1, X \to 1X1, X \to 0, X \to 1, X \to \varepsilon\}$

3. 2. $R = (R_1 \circ R_2)$ This is equivalent to the CFG that consists of all rules and variables from each of G_1 and G_2 with a new start variable S and one additional rule $S \to S_1S_2$.
Equivalence is because S derives the set of strings that have a prefix derived by S_1 and a suffix derived by S_2. This is the language of $(R_1 \circ R_2)$

3. $R = (R_1^*)$ This is equivalent to the CFG that consists of all rules and variables from G_1 with a new start variable S and one additional rule $S \to SS_1 \mid \varepsilon$.
Equivalence is because S derives any string that consists of a non-negative number of substrings which are derived by S_1.
This is the language of R_1^*.

4a. The grammar generates all strings that consist of positive number of substrings that are a^nb^n with $n > 0$.

b. The grammar is ambiguous because it can derive ababab in two distinct leftmost derivations

1st $S \to SS \to SSS \to TSS \to abSS \to abTS \to ababS \to ababT \to ababab$
2nd $S \to SS \to TS \to abS \to abSS \to abTS \to ababS \to ababT \to ababab$

c. $S \to SS \mid T$

$T \to aTb \mid ab$

step 1

$S_0 \to S$

$S \to SS \mid T$

$T \to aTb \mid ab$

step 2

$S_0 \to SS$

$S \to SS \mid T$

$T \to aTb \mid ab$

step 3

$S_0 \to SS \mid aTb \mid ab$

$S \to SS \mid aTb \mid ab$

$T \to aTb \mid ab$

Step 4

$S_0 \to SS \mid AZ \mid AB$

$S \to SS \mid AZ \mid AB$

$T \to AZ \mid AB$

$Z \to TB$

$A \to a$

$B \to b.$

CNF

$S_0 \to SS \mid AZ \mid AB$

$S \to SS \mid AZ \mid AB$

$T \to AZ \mid AB$

$Z \to TB$

$A \to a$

$B \to b.$

Citations: Used Class Notes & Textbook