Homework 9

1. Claim: Regular_{TM} is not recognizable and not corecogizable.

Proof:

From Theorem 5.3, $A_{TM} \leq_m \text{REGULAR}_{TM}$ with reduction $f(\langle M, w\rangle) = \langle M_w\rangle$. Thus $\overline{A_{TM}} \leq_m \overline{\text{REGULAR}_{TM}}$ and $\overline{A_{TM}}$ is not recognizable since A_{TM} is recognizable and A_{TM} is not decidable. So REGULAR_{TM} is not corecognizable since $\overline{\text{REGULAR}_{TM}}$ is not recognizable by inheritance of $\overline{A_{TM}} \leq \overline{\text{REGULAR}_{TM}}$.

Also from $A_{TM} \leq_m \text{REGULAR}_{TM} \iff \overline{A_{TM}} \leq_m \overline{\text{REGULAR}_{TM}}$, $\overline{A_{TM}} \leq_m \text{REGULAR}_{TM}$ and REGULAR_{TM} inherits $\overline{A_{TM}}$'s unrecognizability and thus REGULAR_{TM} is also not recognizable.

2. $\Rightarrow$ Assume A is recognizable by M. Then $A \leq_m A_{TM}$ with reduction $f(w) = \langle M, w\rangle$ because the assumption that M recognizes A means $w \in A$ iff $w \in L(M)$.

$\Leftarrow$ Assume $A \leq A_{TM}$. Then we can run the universal turing machine on reduction f, $f(x) = \langle M, w\rangle$ to recognize A. $x \in A$ iff an A_{TM} recognizer accepts $f(x) = \langle M, w\rangle$. This A is recognizable

3. Not recognizable. $f(\langle M, w\rangle) = 1\langle M, w\rangle$ is a mapping reduction for $\overline{A_{TM}} \leq_m J$ because $f(x) \in J \iff x \in \overline{A_{TM}}$, thus J inherits $\overline{A_{TM}}$'s unrecognizability

Not corecognizable. $f(\langle M, w\rangle) = 0\langle M, w\rangle$ is a mapping reduction for $\overline{A_{TM}} \leq_m \overline{J}$ because $f(x) \in \overline{J} \iff x \in \overline{A_{TM}}$ and $\overline{J}$ then inherits $\overline{A_{TM}}$'s unrecognizability.

4. Claim: EQ_{LBA} is corecognizable and not recognizable, therefore not decidable

Proof:

Corecognizable

Build recognizer for $\overline{EQ_{LBA}}$ as follows

On input $\langle M_1, M_2 \rangle$:

If $\langle M_1, M_2 \rangle$ are not a pair of LBAs: accept

For $l = 0, 1, 2, \ldots$

For each string w of length $|w| = l$

Check if M_1 M_2 accept w after $\leq |Q_1| \, l \, |\Gamma|^l \leq |Q_2| \, l \, |\Gamma|^l$

If one accepts and not the other: accept

Not Decidable

Suppose EQ_{LBA} is decidable. Since every CFG can be represented by LBA, create two LBAs M_1 and M_2 equivalent to two CFGs. Check if $\langle M_1, M_2 \rangle$ are in EQ_{LBA}. If yes: accept. Else reject

Contradiction: EQ_{CFG} is not decidable, thus EQ_{LBA} is not decidable

Not recognizable: Since EQ_{LBA} is not decidable and is corecognizable, EQ_{LBA} must not be recognizable.

5a. We cannot prove undecidability of EQ_{CFG} by reduction from E_{CFG} because E_{CFG} is decidable

b) The mapping reduction that shows $ALL_{CFG} \leq_m EQ_{CFG}$ is

Let M_1 be a TM that decides EQ_{CFG} and we construct M_2 to decide ALL_{CFG}, as follows

M_2 = On input $\langle G \rangle$ where G is a CFG

1. Run M_1 on input $\langle G, G_0 \rangle$, G_0 is CFG with $L(G_0) = \Sigma^*$
2. If M_1 accepts: accept. If M_1 rejects: reject.

c. It is a mapping reduction $\overline{A_{TM}} \leq ALL_{CFG}$

Citation: Used Class Notes, textbook, online lecture slides from Stanford and University of Iowa