# Homework 8

1. Claim: $EQ_{DR}$ is decidable

Proof:

Let $EQ_{DR} = \{\langle D, R\rangle \mid D$ is a DFA, $R$ is a regex, and $L(D) = L(R)\}$

Define a TM $M_{EQDR}$ that decides $EQ_{DR}$ as follows:

$M_{EQDR}$ = On Input $\langle D, R\rangle$

1. Convert $R$ into an equivalent DFA $D_R$
2. Construct DFA $D_p$ that recognizes $L(D) \oplus L(R)$. To construct this modify $D$ and $D_R$ to recognize the corresponding complement, intersection and union languages
3. Mark the start state in $D_p$ and mark every state reachable from start state
4. If final state in $D_p$ is marked: reject
   Else: accept

2. Claim: $SS_{Rex} = \{\langle R, S\rangle \mid R, S$ are regular expressions and $L(R) \subseteq L(S)\}$ is decidable

Proof:

Define a TM $M_{SSREX}$ that decides $SS_{REX}$

$M_{SSREX}$ = On input $\langle R, S\rangle$

1. Convert both R and S into equivalent DFAs $D_R$ and $D_S$
2. Construct a new DFA $D$ that recognizes $L(D_R) \cap \overline{L(D_S)}$ using DFA complement and intersection construction
3. Mark the start state of $D$ and mark every state reachable from start state
4. If any final state is marked: reject
   Else: accept.

3. Claim: $A_{\varepsilon CFG} = \{\langle G\rangle \mid G$ is a CFG and $\varepsilon \in L(G)\}$ is decidable

Proof:

Define a TM $M_{A_{\varepsilon CFG}}$ that decides $A_{\varepsilon CFG}$ as follows:

$M_{A_{\varepsilon CFG}}$ = on Input $\langle G\rangle$

1. Convert G to Chomsky Normal Form with start variable S
2. If S has the rule $S \to \varepsilon$ : accept
   Else: reject

4. Claim: $IMB_{DFA} = \{\langle D\rangle \mid D$ is a DFA whose language includes a string with more 1's than 0's$\}$ is decidable

Proof:

Define a TM $M_{IMB_{DFA}}$ that decides $IMB_{DFA}$ as follows:

$M_{IMB_{DFA}}$ = on Input $\langle D\rangle$

1. Create a CFG $G_{IMB}$ that generates all strings with more 1's than 0's. $S \to SS \mid 1S0 \mid 0S1 \mid 1S \mid \varepsilon$
2. Then create a corresponding PDA $P_{IMB}$ such that $L(P_{IMB}) = L(G_{IMB})$.
3. Next construct another PDA P where $L(P) = L(D) \cap L(P_{IMB})$
4. Convert P to a CFG G
5. Mark all terminals RHS of any rule of G. and mark each occurance of a variable on RHS for any variable that has all-marked RHS
6. If the start variable in G is marked: accept
   Else: reject

5. Claim: $EQ_{CFG}$ is co-Turing-recognizable

Proof:

Define a $TM_{EQ_{CFG}}$ that shows $EQ_{CFG}$ is co-Turing-recognizable

$TM_{EQ_{CFG}}$ = On Input $\langle G_1, G_2 \rangle$, where $G_1$ and $G_2$ are CFGs

0. If $G_1$ or $G_2$ are not properly formatted: accept
1. Convert $G_1$ and $G_2$ in Chomsky Normal Form $G_1'$ and $G_2'$ respectively
2. For every w of increasing size
   Run CYK for $G_1$ and $G_2$ on w
   If w is shown to be generated by exactly one grammar: accept.

6. Claim: $REV_{TM} = \{\langle M \rangle \mid M$ is a TM with $w \in L(M) \Rightarrow w^R \in L(M)\}$ is undecidable

Proof:

Define TM D as follows

D = On Input $\langle M, w \rangle$

1. Create another TM M' that simulates M on w and accepts 01 if M accepts w. Then on input 10, accepts and any other input rejects
2. Run a TM that decides $REV_{TM}$ on $\langle M' \rangle$
3. If H accepts M': accept
   Else: reject

Since D decides $A_{TM}$ as shown, there is a contradiction. $A_{TM}$ is undecidable, then $REV_{TM}$ is undecidable

Citations: Used Class Notes, Textbook and Online Examples.