# Trapped Ion Notes - Part 2

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I recommend downloading the latest version from the following link: https://GitHub.com/Ke-Sun96/Notes.git

## Motional state phonon number statistic

Prob of motional number $n$:

$$P(n) = \frac{e^{-nh\nu/k_BT} \, e^{-\frac{1}{2}h\nu/k_BT}}{\sum_i e^{-n_i h\nu/k_BT} \cdot e^{-\frac{1}{2}h\nu/k_BT}}$$

$$= \frac{e^{-nh\nu/k_BT}}{\sum_i e^{-n_i h\nu/k_BT}} \longrightarrow$$

$q = e^{-\beta h\nu}$, $a_1 = 1$

$$S_\infty = \frac{a_1}{1-q} = \frac{1}{1-e^{-\beta h\nu}}$$

$\beta \equiv 1/k_BT$

$$= \frac{e^{-n\beta h\nu}}{(1-e^{-\beta h\nu})^{-1}}$$

$$= (1-e^{-\beta h\nu})\, e^{-n\beta h\nu}$$

$\Rightarrow$ For ground state, $n=0$, $P(0) = 1-e^{-\beta h\nu}$

excited state, $n>0$, $P(n>0) = 1-P(0) = e^{-\beta h\nu}$

$$\frac{P_{RSB}}{P_{BSB}} = \frac{P(n>0)}{P(n>0)} = \frac{1}{e^{-\beta h\nu}} = e^{\beta h\nu} \equiv r$$

Bose-Einstein distribution:

$$\boxed{\bar{n} = \frac{1}{e^{\beta h\nu}-1} = \frac{1}{r-1} = \frac{1}{\frac{P_{BSB}}{P_{RSB}}-1} = \frac{P_{RSB}}{P_{BSB}-P_{RSB}}}$$

If we go back to consider the population of each state:

$$P(n) = P(0) \cdot (e^{-\beta h\nu})^n = P(0) \cdot \underbrace{\left(\frac{1}{r}\right)}_{q}{}^n$$

The BSB transition of a thermal state will be

$$P(t) = \sum_n P(n) \cdot \sin(\sqrt{n+1}\,\Omega \cdot t)^2$$

$$= \sum_n P(0) \cdot q^n \cdot \sin(\sqrt{n+1}\,\Omega\, t)^2$$

1 2 3

Fitting variables.

Constrain: $\sum_n P(n) = 1 \Rightarrow \sum_n P(0) \cdot q^n = \frac{P(0)}{1-q} = 1$

$$\bar{n} = \frac{1}{e^{\beta h\nu}-1} = \frac{1}{\frac{1}{q}-1} = \frac{q}{1-q}$$

$$\frac{\partial \bar{n}}{\partial q} = \frac{(1-q)-q(-1)}{(1-q)^2} = \frac{1}{(1-q)^2}$$

$$\sigma_{\bar{n}} = \sqrt{\left(\frac{\partial \bar{n}}{\partial q}\right)^2 \cdot \sigma_q^2} = \frac{1}{(1-q)^2}\sigma_q$$

# EIT cooling

Before diving into EIT cooling, we should study dressed state first.

## Dressed state: Light-atom interaction

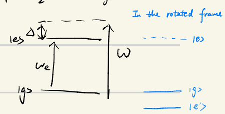
$$H = H_A + H_{int}$$

in an appropriate rotating frame:

$$= -\Delta |e\rangle\langle e| + \frac{\Omega}{2}\left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

$$= \begin{pmatrix} 0 & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta \end{pmatrix}$$

$$H_0 = \omega_e |e\rangle\langle e| + \frac{\Omega}{2} e^{i\omega t} |g\rangle\langle e| + \frac{\Omega}{2} e^{-i\omega t} |e\rangle\langle g|$$




$$H_D = \omega |e\rangle\langle e|$$

$$e^{iH_D t} = e^{i\omega t} |e\rangle\langle e|$$

$$\tilde{H}_I = e^{iH_D t} V e^{-iH_D t}$$

$$= \frac{\Omega}{2} \left[ e^{i\omega t} |e\rangle\langle e| \left( \frac{\Omega}{2} e^{i\omega t} |g\rangle\langle e| + \frac{\Omega}{2} e^{-i\omega t} |e\rangle\langle g| \right) e^{-i\omega t} |e\rangle\langle e| \right]$$

$$= e^{i(\omega-\omega)t} |g\rangle\langle e| + e^{i(\omega-\omega)t} |e\rangle\langle g| = \left(|g\rangle\langle e| + |e\rangle\langle g|\right)\frac{\Omega}{2}$$

$$\tilde{H} = H_0 - H_D + \tilde{H}_I = -\Delta |e\rangle\langle e| + \frac{\Omega}{2}\left(|g\rangle\langle e| + |e\rangle\langle g|\right)$$

Eigensolver:

$$|H - EI| = \begin{vmatrix} -E & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta - E \end{vmatrix} = E^2 + \Delta E - \frac{\Omega^2}{4} = 0$$

$$\boxed{E_\pm = \frac{-\Delta \pm \sqrt{\Delta^2 + \Omega^2}}{2}}$$

$$H|\psi\rangle = E|\psi\rangle \Rightarrow \begin{pmatrix} 0 & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \left( \frac{\sqrt{\Delta^2 + \Omega^2}}{2} - \frac{\Delta}{2} \right) \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\Rightarrow \begin{cases} \frac{\Omega}{2} b = E_\pm a \\ \frac{\Omega}{2} a - \Delta b = E_\pm b \end{cases}$$

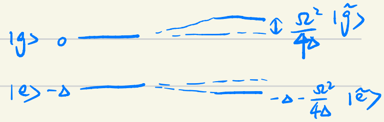
$$\frac{b}{a} = \frac{E_\pm}{\Omega/2} = \frac{\pm\sqrt{\Delta^2 + \Omega^2} - \Delta}{\Omega}.$$

If we don't care about the normalization:

$$|\psi_+\rangle = \begin{pmatrix} 1 \\ \frac{\Omega' - \Delta}{\Omega} \end{pmatrix} \qquad |\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-\Omega' - \Delta}{\Omega} \end{pmatrix}$$

$\Omega' \equiv \sqrt{\Delta^2+\Omega^2}$

For weak laser: $\Omega \ll \Delta$




$$\Omega' = \sqrt{\Omega^2+\Delta^2} = \Delta\left(1+\frac{\Omega^2}{\Delta^2}\right)^{1/2} \approx \Delta + \frac{1}{2}\frac{\Omega}{\Delta}\cdot\Omega.$$

$$|\psi_+\rangle = \begin{pmatrix} 1 \\ \underbrace{\frac{\Omega}{2\Delta}}_{\sim 0} \end{pmatrix} \qquad |\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-2\Delta}{\Omega} \end{pmatrix} \rightarrow \gg 1$$

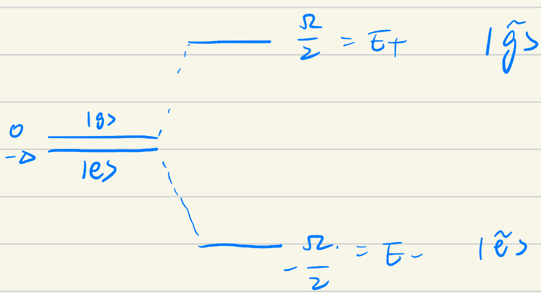
$$E_+ = \frac{-\Delta+\Omega'}{2} = \frac{-\Delta+\Delta+\frac{\Omega^2}{2\Delta}}{2} = \frac{\Omega^2}{4\Delta}, \qquad E_- = \frac{-\Delta-\Omega'}{2} = \frac{-2\Delta-\frac{\Omega^2}{2\Delta}}{2} = -\Delta-\frac{\Omega^2}{4\Delta}$$

For strong laser: $\Omega \gg \Delta$

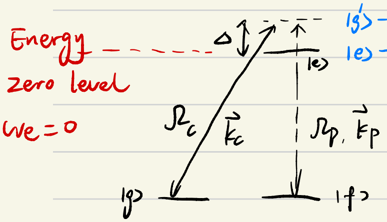
$$\Omega' = \Omega\left(1+\frac{\Delta^2}{\Omega^2}\right)^{1/2} \approx \Omega + \frac{1}{2}\frac{\Delta^2}{\Omega}$$

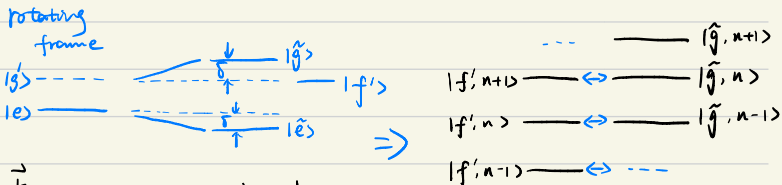
$$|\psi_+\rangle = \begin{pmatrix} 1 \\ \frac{\Omega+\frac{1}{2}\frac{\Delta^2}{\Omega}-\Delta}{\Omega} \end{pmatrix} = \begin{pmatrix} 1 \\ 1-\frac{\Delta}{\Omega}+\frac{1}{2}\frac{\Delta^2}{\Omega^2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1-O(\frac{\Delta}{\Omega}) \end{pmatrix}, \qquad E_+ \approx \frac{\Omega}{2}-\frac{\Delta}{2}+\frac{1}{4}\frac{\Delta^2}{\Omega}$$

$$|\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-\Omega-\frac{1}{2}\frac{\Delta^2}{\Omega}-\Delta}{\Omega} \end{pmatrix} = \begin{pmatrix} 1 \\ -1-\frac{\Delta}{\Omega}-\frac{1}{2}\frac{\Delta^2}{\Omega^2} \end{pmatrix} = \begin{pmatrix} 1 \\ -1-O(\frac{\Delta}{\Omega}) \end{pmatrix}, \qquad E_- \approx -\frac{\Omega}{2}-\frac{\Delta}{2}-\frac{1}{4}\frac{\Delta^2}{\Omega}$$



EIT cooling:



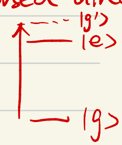

$\Omega_c$: coupling beam

$\Omega_p$: probe beam.

$$\hat{H}/\hbar = \omega_f |f\rangle\langle f| + \omega_g |g\rangle\langle g| + \left( \frac{\Omega_p}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i(\vec{k}_c\cdot\vec{r} - \omega_c t)} |g\rangle\langle e| + h.c. \right)$$

Rotating frame: $H_D/\hbar = -\omega_c |g\rangle\langle g| - \omega_p |f\rangle\langle f|$

( Rotate the Bloch sphere towards the inversed direction, to increase the freq )




$$\tilde{V} = e^{iH_D t/\hbar} V e^{-iH_D t/\hbar}$$

$$e^{iH_D t/\hbar} = |e\rangle\langle e| + e^{-i\omega_c t} |g\rangle\langle g| + e^{-i\omega_p t} |f\rangle\langle f|$$

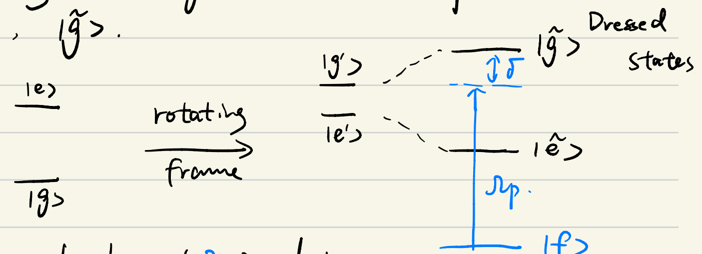
$$= \left(|e\rangle\langle e| + e^{-i\omega_c t} |g\rangle\langle g| + e^{-i\omega_p t} |f\rangle\langle f|\right) \left( \frac{\Omega_p}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i(\vec{k}_c\cdot\vec{r} - \omega_c t)} |g\rangle\langle e| + h.c. \right) \left(|e\rangle\langle e| + e^{+i\omega_c t} |g\rangle\langle g| + e^{+i\omega_p t} |f\rangle\langle f|\right)$$

$$= \left( \frac{\Omega_p}{2} e^{-i\vec{k}_p\cdot\vec{r}} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i\vec{k}_c\cdot\vec{r}} |g\rangle\langle e| \right) + h.c.$$

$$H_0 - H_D + \tilde{V} = \Delta_c |g\rangle\langle g| + \Delta_p |f\rangle\langle f| + \left[ \left( \frac{\Omega_p}{2} e^{-i\vec{k}_p\cdot\vec{r}} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i\vec{k}_c\cdot\vec{r}} |g\rangle\langle e| \right) + h.c. \right]$$

$$= \begin{array}{c} \begin{pmatrix} 0 & \frac{\Omega_c}{2} e^{+i\vec{k}_c \cdot \vec{r}} & \frac{\Omega_p}{2} e^{+i\vec{k}_p \cdot \vec{r}} \\ \frac{\Omega_c}{2} e^{-i\vec{k}_c \cdot \vec{r}} & \Delta_c & \\ \frac{\Omega_p}{2} e^{-i\vec{k}_p \cdot \vec{r}} & & \Delta_p \end{pmatrix} \end{array} \begin{array}{c} |e\rangle \\ |g\rangle \\ |f\rangle \end{array}$$

Intuitively: ① Consider the coupling beam and probe beam separately, since $\Omega_c \gg \Omega_p$. In this case, the coupling beam generate a set of dressed state $|\tilde{e}\rangle$, $|\tilde{g}\rangle$.




② The probe beam ($\Omega_p$) drives the RSB transition to cool the ion.

Light-shifted : $\delta = \frac{1}{2}\left(\sqrt{\Omega_c^2 + \Delta^2} - |\Delta|\right)$

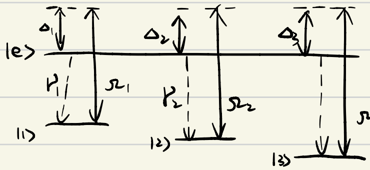
For EIT cooling, $\Delta > 0$ so that one can tune $\delta$ to be $\nu$ to accomplish the cooling process).

Intense 370 nm beam $\Omega_c$ is used to generate the dressed states: $|\tilde{g}\rangle$ & $|\tilde{e}\rangle$,

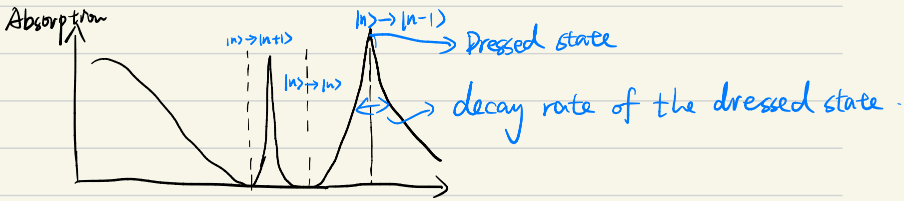
The weak 370 nm beam $\Omega_p$ is used to drive the cooling transitions: $|f, n\rangle \Longleftrightarrow |\tilde{g}, n-1\rangle$.

# Double EIT:

Jö RG. Evers & Christoph. H. Keitel.

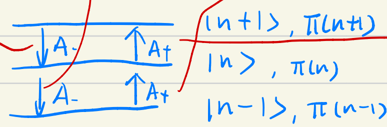



$$H = \hbar\omega a^\dagger a + \hbar\sum_{j=1}^{3}\Delta_j |j\rangle\langle j|$$
$$+\hbar\sum_{j=1}^{3}\frac{\Omega_j}{2}\left(e^{-ik_j\cos(\phi_j)x}|e\rangle\langle j| + h.c.\right)$$




Population of motional mode: $\Pi(n)$

$$\frac{d}{dt}\Pi(n) = A_-\left[(n+1)\Pi(n+1) - n\Pi(n)\right] + A_+\left[n\Pi(n-1) - (n+1)\Pi(n)\right]$$




$$\langle n\rangle = \sum_{k=0}^{\infty}\Pi(k)\cdot k$$

$$\langle \dot n\rangle = \sum_k \frac{d}{dt}\Pi(k)\cdot k$$

$$= \sum_k A_-\left[k(k+1)\Pi(k+1) - k^2\Pi(k)\right] + A_+\left[k^2\Pi(k-1) - k(k+1)\Pi(k)\right]$$

$$= \sum_k A_- k\cdot(k+1)\Pi(k+1) = \sum_{k=0}^{\infty} A_-(k-1)\,k\Pi(k)$$

$$- \sum_k A_- k\cdot k\,\Pi(k) + \sum_k A_+ k^2\Pi(k-1) - \sum (k+1)\,k\Pi(k)A_+$$

$$= \sum_{k=1} A_+ k^2\Pi(k-1)$$

$$= \sum_{k=0} A_+(k+1)^2\Pi(k) \qquad \sum_{k=0} A_+(k+1)\Pi(k)$$

$$-\sum_{k=0} A_- k\Pi(k)$$

$$= -A_-\langle n\rangle + A_+\langle n\rangle + A_+\sum_{k=0}\Pi(k)$$

$$= -(A_- - A_+)\langle n\rangle + A_+$$

$\Rightarrow$ cooling rate: $W = A_- - A_+$

cooling limit: $\langle \dot{n} \rangle = 0$

$$= -W\langle n \rangle + A_+$$

$$\Rightarrow \langle n \rangle_{ss} = \frac{A_+}{A_- - A_+}$$

$$\boxed{\frac{A_\pm}{\eta^2} = \frac{\Omega_1^2}{\Omega_1^2+\Omega_3^2} \, \frac{\Gamma_3 \nu^2 \Omega_3^2}{4\{[(\Omega_1^2+\Omega_3^2)/4 - \nu(\nu \mp \Delta)] + \varepsilon_\pm\}^2 + \Gamma_3^2 \nu^2}}$$

This part is only valid for the structure mentioned in previous page.

$$\varepsilon_\pm = \mp \frac{\nu \Omega_2^2}{4(\Delta - \Delta_2 \mp \nu)}$$

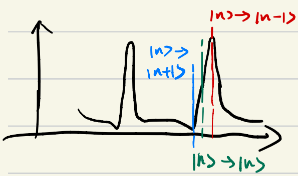
Optimal condition:

$$\boxed{\begin{aligned} &\Delta_3 = \Delta_1 = \Delta \quad \Delta_2 = \Delta_1 - \nu \\ &\nu = \frac{1}{2}\left(\sqrt{\Delta_1^2 + \Omega_1^2 + \Omega_2^2/2 + \Omega_3^2} - \Delta_1\right) \end{aligned}}$$

The derivation of $A_\pm$ here doesn't fit our exp setup which use the following structure

$$\Rightarrow \varepsilon_+ = -\frac{\nu\Omega_2^2}{4(\Delta - \Delta + \nu - \nu)} = \infty. \Rightarrow \boxed{A_+ = 0}$$

$$\varepsilon_- = \frac{\nu\Omega_2^2}{4(\Delta - \Delta + \nu + \nu)} = \frac{\nu\Omega_2^2}{8\nu} = \frac{\Omega_2^2}{8}$$




$$A_- = \eta^2 \frac{\Omega_1^2}{\Omega_1^2+\Omega_3^2} \, \frac{\Gamma_3 \nu^2 \Omega_3^2}{4\{[(\Omega_1^2+\Omega_3^2)/4 - \nu(\nu+\Delta) + \frac{\Omega_2^2}{8}\}^2 + \Gamma_3^2\nu^2}$$

$$= \frac{\eta^2\Omega_1^2}{\Omega_1^2+\Omega_3^2} \, \frac{\Gamma_3 \nu^2 \Omega_3^2}{[\underbrace{(\Omega_1^2+\Omega_3^2+\frac{\Omega_2^2}{2}) - \nu^2 - \Delta\nu}_{=0}]^2 + \Gamma_3^2\nu^2}$$

$$(\nu + \frac{1}{2}\Delta)^2 = \frac{1}{4}\Delta^2 + \Omega_1^2 + \Omega_3^2 + \frac{\Omega_2^2}{2}$$

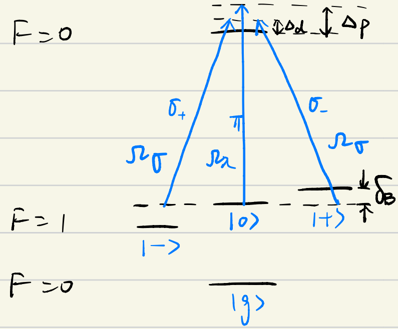
$$\Rightarrow \Omega_1^2 + \Omega_3^2 + \frac{\Omega_2^2}{2} = \nu^2 + \nu\Delta$$

$$= \frac{\eta^2\Omega_1^2}{\Omega_1^2+\Omega_3^2} \, \frac{\Gamma_3\nu^2\Omega_3^2}{\Gamma_3^2\nu^2} = \boxed{\frac{\eta^2\Omega_1^2\Omega_3^2}{\Gamma_3(\Omega_1^2+\Omega_3^2)}}$$

Hamiltonian of EIT
for $^{171}Yb^+$ ion.

$\Omega_\sigma$ : driving (coupling) beam.
$\Omega_\pi$ : probe beam.

$\boxed{\text{This } \hat{H} \text{ is suitable for Red chamber}}$




set $w_{|e\rangle} = 0$, $w_{|+\rangle}, w_{|0\rangle}, w_{|-\rangle}$ are negative

$$\hat{H}/\hbar = w_{|+\rangle} |+\rangle\langle+| + w_{|0\rangle} |0\rangle\langle 0| + w_{|-\rangle} |-\rangle\langle-|$$
$$+ \Big( \frac{\Omega_\sigma}{2} e^{-i(\vec{k}_d \cdot \vec{r} - w_d t)} |+\rangle\langle e| - \frac{\Omega_\pi}{2} e^{-i(\vec{k}_p \vec{r} - w_p t)} |0\rangle\langle e|$$
$$+ \frac{\Omega_\sigma}{2} e^{-i(\vec{k}_d \cdot \vec{r} - w_d t)} |-\rangle\langle e| + h.c. \Big)$$

Rotating frame: $H_D/\hbar = -w_d |+\rangle\langle+| - w_p |0\rangle\langle 0| - w_d |-\rangle\langle -|$

$$\hat{H}_I = e^{iH_0 t/\hbar} V e^{-iH_0 t/\hbar}$$

$$e^{iH_0 t/\hbar} = \mathbb{1} |e\rangle\langle e| + e^{-iw_d t} |+\rangle\langle+| + e^{-iw_p t} |0\rangle\langle 0| + e^{-iw_d t} |-\rangle\langle -|$$

$$= \mathbb{1} |e\rangle\langle e| + e^{-iw_d t} |+\rangle\langle+| + e^{-iw_p t} |0\rangle\langle 0| + e^{-iw_d t} |-\rangle\langle -|$$
$$\Big( \frac{\Omega_\sigma}{2} e^{-i(\vec{k}_d \cdot \vec{r} - w_d t)} |+\rangle\langle e| - \frac{\Omega_\pi}{2} e^{-i(\vec{k}_p \vec{r} - w_p t)} |0\rangle\langle e|$$
$$+ \frac{\Omega_\sigma}{2} e^{-i(\vec{k}_d \cdot \vec{r} - w_d t)} |-\rangle\langle e| + h.c. \Big)$$
$$\mathbb{1} |e\rangle\langle e| + e^{iw_d t} |+\rangle\langle+| + e^{iw_p t} |0\rangle\langle 0| + e^{iw_d t} |-\rangle\langle -|$$

$$= \frac{\Omega_\sigma}{2} e^{-i\vec{k}_d\cdot\vec{r}} |+\rangle\langle e| - \frac{\Omega_\pi}{2} e^{-i\vec{k}_p\cdot\vec{r}} |0\rangle\langle e|$$

$$+ \frac{\Omega_\sigma}{2} e^{-i\vec{k}_d\cdot\vec{r}} |-\rangle\langle e| + h.c.$$

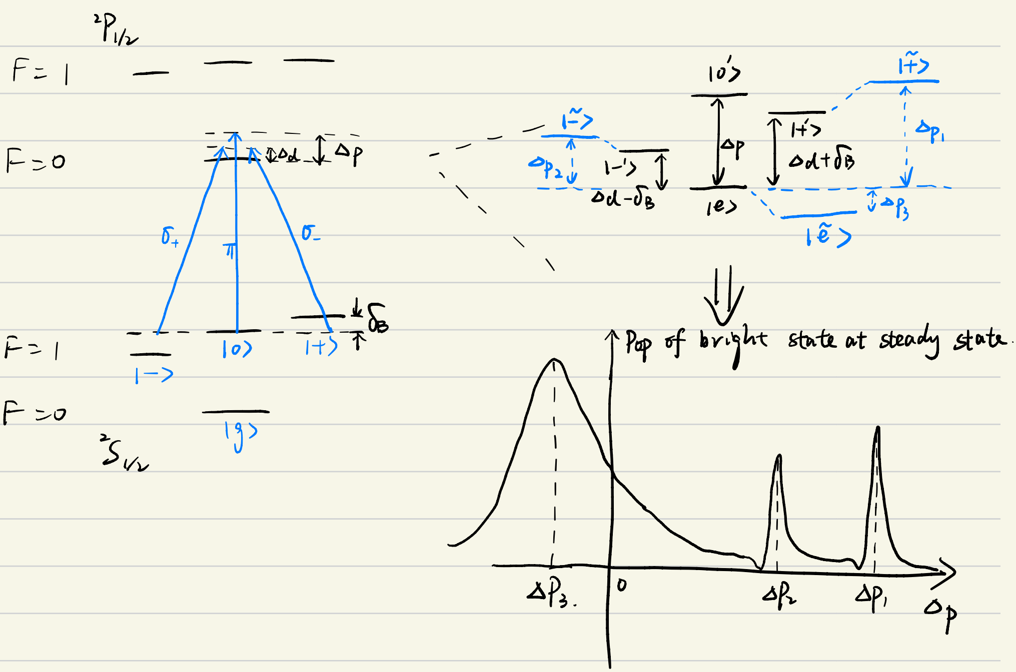
Diagonal term: $H_0 - H_D = (\Delta d + \delta_B)\,|+\rangle\langle+| + \Delta p\,|0\rangle\langle 0|$

$$+ (\Delta d - \delta_B)\,|-\rangle\langle-|$$

Full Hamiltonian:

$$\tilde{H} = H_0 - H_D + \tilde{H}_I$$

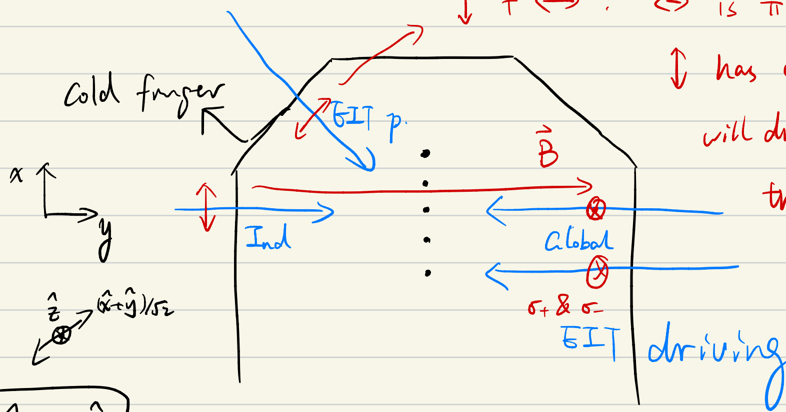
Energy levels in the Rotating frame:



# Polarization configurations for $\boxed{\text{Cryo}}$

$\updownarrow + \leftrightarrow$ : $\leftrightarrow$ is $\pi$, which is needed

$\updownarrow$ has components of $\sigma_+$ & $\sigma_-$, will drive unwanted transition




The only difference of cryo and Red chamber is the unwanted $\updownarrow$ transition

$$\boxed{\hat{x} = \hat{y}}$$

$$\vec{E}_p = \left(\frac{\sqrt{2}}{2} E_0 \, \hat{x} + \frac{\sqrt{2}}{2} E_0 \, \hat{y}\right) \cos(kz - \omega t)$$

$$= \frac{\sqrt{2}}{2} E_0 \, \hat{\pi} + \frac{\sqrt{2}}{2} E_0 \left(\frac{\hat{\sigma}^+ + \hat{\sigma}^-}{2}\right)$$

$$\vec{E}_c = E_1 \, \hat{z} \cdot \cos(kz - \omega t)$$

$$= \frac{E_1}{2} \hat{z} \cos(kz - \omega t) + \frac{E_1}{2} \hat{x} \sin(kz - \omega t)$$

$$+ \frac{E_1}{2} \hat{z} \cos(kz - \omega t) - \frac{E_1}{2} \hat{x} \sin(kz - \omega t)$$

$$= \frac{E_1}{2} (\hat{\sigma}_+ + \hat{\sigma}_-)$$

# Hamiltonian for Cryo chamber.

$$\hat{H}/\hbar = \omega_{|+\rangle} |+\rangle\langle+| + \omega_{|0\rangle} |0\rangle\langle 0| + \omega_{|-\rangle} |-\rangle\langle -|$$

$$+\Big\{ \Big[\frac{\Omega_{1+}}{2} e^{-i(\vec{k}_d\cdot\vec{r}-\omega_d t)} + \frac{\Omega_{2+}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i(\vec{k}_d\cdot\vec{r}-\omega_d t)} + \frac{\Omega_{2-}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)} |0\rangle\langle e| + h.c. \Big\}$$

Extra terms compared with Red chamber

Rotating frame: $H_0/\hbar = -\omega_d |+\rangle\langle+| - \omega_p |0\rangle\langle 0| - \omega_d |-\rangle\langle -|$

$$\tilde{V}_I = e^{iH_0 t/\hbar}\, V\, e^{-iH_0 t/\hbar}$$

$$e^{iH_0 t/\hbar} = \mathbb{1}\, |e\rangle\langle e| + e^{-i\omega_d t} |+\rangle\langle+| + e^{-i\omega_p t} |0\rangle\langle 0| + e^{-i\omega_d t} |-\rangle\langle -|$$

$$= \Big(\mathbb{1}\, |e\rangle\langle e| + e^{-i\omega_d t} |+\rangle\langle+| + e^{-i\omega_p t} |0\rangle\langle 0| + e^{-i\omega_d t} |-\rangle\langle -|\Big)$$

$$\Big\{ \Big[\frac{\Omega_{1+}}{2} e^{-i(\vec{k}_d\cdot\vec{r}-\omega_d t)} + \frac{\Omega_{2+}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i(\vec{k}_d\cdot\vec{r}-\omega_d t)} + \frac{\Omega_{2-}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i(\vec{k}_p\cdot\vec{r}-\omega_p t)} |0\rangle\langle e| + h.c. \Big\}$$

$$\Big(\mathbb{1}\, |e\rangle\langle e| + e^{i\omega_d t} |+\rangle\langle+| + e^{i\omega_p t} |0\rangle\langle 0| + e^{i\omega_d t} |-\rangle\langle -|\Big)$$

$$= \Big[\frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}} \cdot e^{-i(\omega_d-\omega_p)t}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}} |0\rangle\langle e| + h.c. \Big\}$$

$$\frac{\tilde{H}_I}{\hbar} = \left(H - H_D + \tilde{U}_I\right)/\hbar$$

$$= (\Delta d + \delta_B)\,|+\rangle\langle+| + \Delta p\,|0\rangle\langle 0| + (\Delta d - \delta_B)\,|-\rangle\langle-|$$

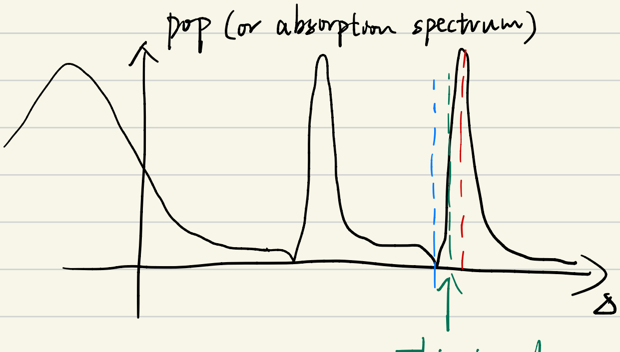
$$\left[\frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \underbrace{\frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}}\cdot e^{-i(\omega_d-\omega_p)t}}\right]|+\rangle\langle e|$$

$$+\left[\frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \underbrace{\frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t}}\right]|-\rangle\langle e|$$

$$-\frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}}\,|0\rangle\langle e| + h.c. \Big\}$$

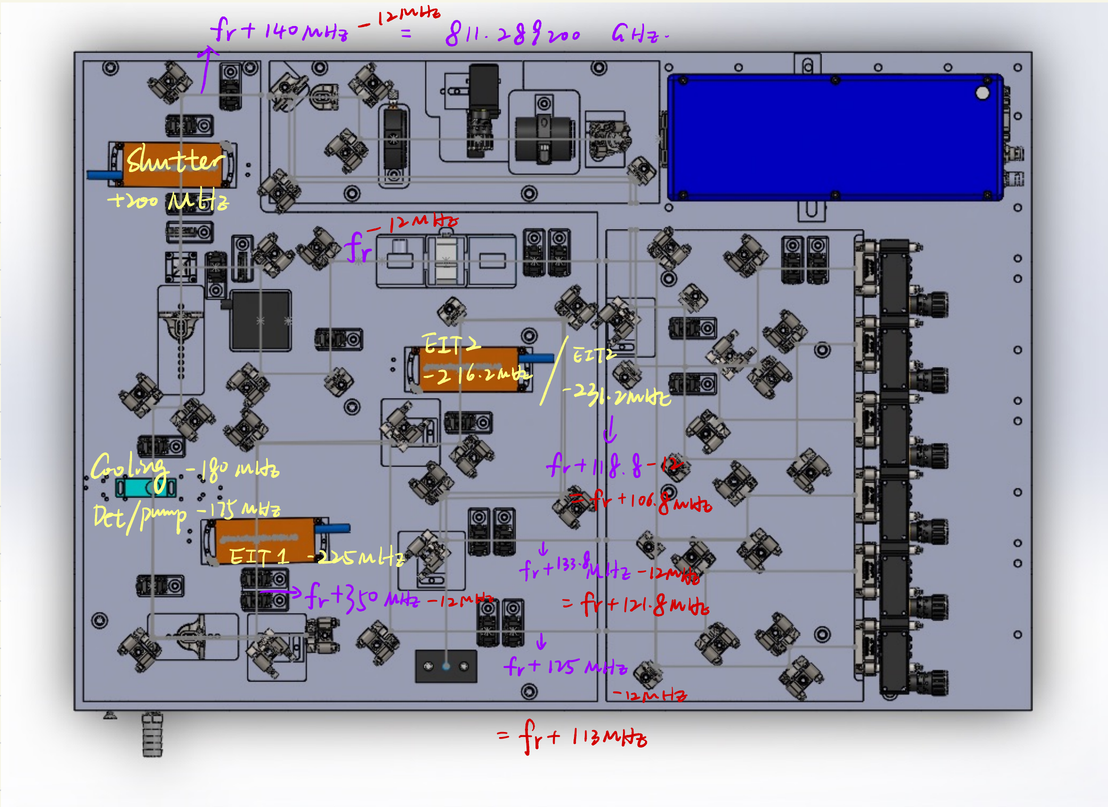
$$\omega_d - \omega_p = (\omega_{e0} + \Delta d) - (\omega_{e0} + \Delta p) = \Delta d - \Delta p.$$

$$= \begin{pmatrix} 0 & \frac{\Omega_{1+}}{2} e^{i\vec{k}_d\vec{r}} + \frac{\Omega_{2+}}{2} e^{i\vec{k}_p\cdot\vec{r}} e^{i(\omega_d-\omega_p)t} & -\frac{\Omega_{2\pi}}{2} e^{i\vec{k}_p\cdot\vec{r}} & \frac{\Omega_{1-}}{2} e^{i\vec{k}_d\vec{r}} + \frac{\Omega_{2-}}{2} e^{i\vec{k}_p\vec{r}} e^{i(\omega_d-\omega_p)t} \\ \frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\vec{r}} + \frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t} & \Delta d + \delta_B & 0 & 0 \\ -\frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}} & 0 & \Delta p & 0 \\ \frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t} & 0 & 0 & \Delta d - \delta_B \end{pmatrix}$$




This is where we set $\Delta p$ for EIT cooling

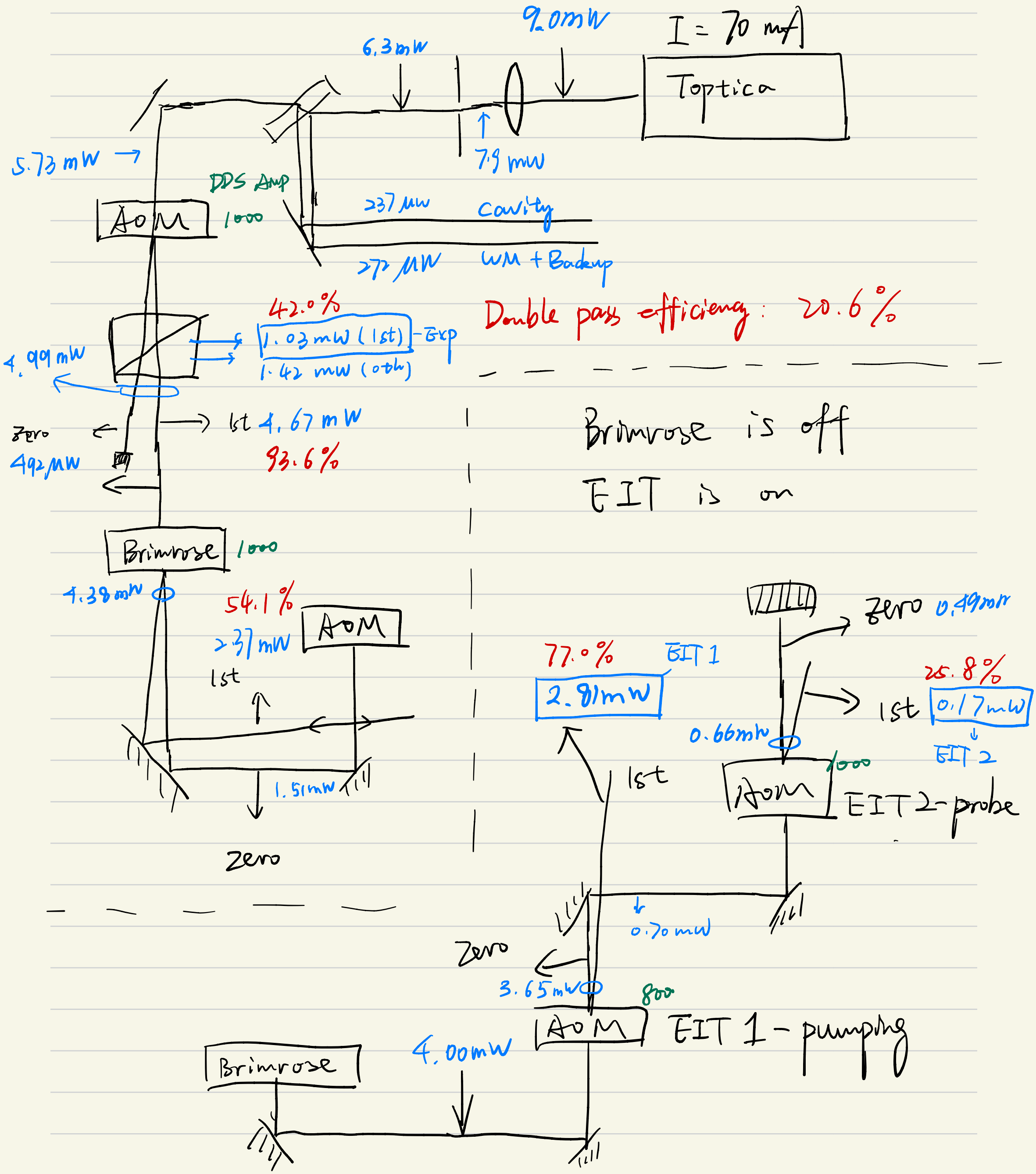
# EIT Plate Design (Only for Cryo)




## Reasons of −12 MHz:

When detecting, the 370 should be on resonance. But due to the laser broadening, it's better to be red detunned. So a good point is −12 MHz away from resonance.

# Power budget



$\boxed{\text{CPMG}}$ Ref: Ye Wang's thesis.

Hamiltonian: $H = \frac{\hbar}{2}(\omega_0 + \beta(t))\sigma_z$, $\omega_0$ is the splitting of the qubit.

$$\beta(t) = \beta_B(t) + \beta_{LO}(t)$$

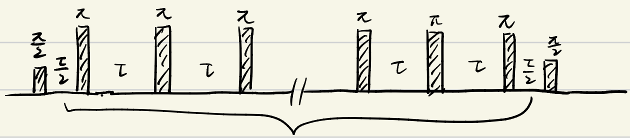
$\beta_B(t)$ ↓ magnetic field fluctuation

$\beta_{LO}(t)$ ↳ phase noise of qubit operational microwave.

Dynamical decoupling (DD):

preserve the qubit coherence against random phase noise.

CPMG pulses:



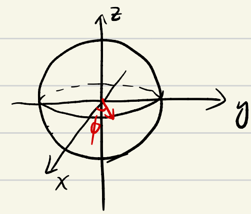

N · π pulses.

Calculation of the filter function:

All the rotation axis used are in x-y plane, the rotation is

$$D_\phi(\gamma) = D_z(\phi)\, D_x(\gamma)\, D_z(-\phi)$$

$$= e^{-\frac{i}{2}\sigma_z\phi} \cdot e^{-\frac{i}{2}\sigma_x\gamma} \cdot e^{\frac{i}{2}\sigma_z\phi}$$




For CPMG. $\gamma = \pi$

$$D_x(\gamma) = e^{-\frac{i}{2}\cdot\pi\sigma_x}$$

$$= \cos(\pi/2)\,\mathbb{1} + i\sigma_x \sin(\pi/2) = -i\sigma_x$$

$$D_\phi(\pi) = e^{-\frac{i}{2}\sigma_z\phi}(-i\sigma_x)\cdot e^{\frac{i}{2}\sigma_z\phi}$$

Initial state $|\psi(0)\rangle = |\downarrow\rangle$

↓ CPMG.

Final state $|\psi(T)\rangle = D_x(\pi/2)\cdot\tilde{R}(T)\, D_x(\pi/2)\,|\psi(0)\rangle$

During the waiting time: $e^{-i\int_{t_1}^{t_2}\beta(\tau)\,\sigma_z\,d\tau}$

$$\Rightarrow \tilde{R}(T) = \underbrace{e^{-i\sigma_z\int_{T_N}^{T_{N+1}}\beta(t)dt}}_{\text{Last } \frac{T}{2} \text{ waiting}}\cdot D_{\phi_N}(\pi)\cdot$$

$$e^{-i\sigma_z\int_{T_{N-1}}^{T_N}\beta(t)dt}\cdot D_{\phi_{N-1}}(\pi)\cdot \;\cdots$$

$$e^{-i\sigma_z\int_{T_1}^{T_2}\beta(t)\,dt}\; D_{\phi_1}(\pi)\cdot \underbrace{e^{-i\sigma_z\int_{T_0}^{T_1}\beta(t)\,dt}}_{\text{First } T/2 \text{ waiting}}$$

$$= (-i)^N\, e^{-i\sigma_z\int_{T_N}^{T_{N+1}}\beta(t)dt}\cdot e^{-\frac{i}{2}\sigma_z\phi_N}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_N}\cdot$$

$$e^{-i\sigma_z\int_{T_{N-1}}^{T_N}\beta(t)dt}\cdot e^{-\frac{i}{2}\sigma_z\phi_{N-1}}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_N}\cdot$$

$$\cdots$$

$$e^{-i\sigma_z\int_{T_1}^{T_2}\beta(t)\,dt}\cdot e^{-\frac{i}{2}\sigma_z\phi}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_1}\cdot$$

$$e^{-i\sigma_z\int_{T_0}^{T_1}\beta(t)\,dt}.$$

$$= (-i)^N\, e^{-i\sigma_z\left(\int_{T_N}^{T_{N+1}}\beta(t)dt+\phi_N/2\right)}\cdot\sigma_x$$

global parameter

$$e^{-i\sigma_z\left(\int_{T_{N-1}}^{T_N}\beta(t)dt-\phi_N/2+\phi_{N-1}/2\right)}\cdot\sigma_x$$

$$\cdots$$

$$e^{-i\sigma_z\left(\int_{T_1}^{T_2}\beta(t)dt-\phi_2/2+\phi_1/2\right)}\;\sigma_x\cdot$$

$$e^{-i\sigma_z\left(\int_{T_0}^{T_1}\beta(t)dt-\phi_1/2\right)}$$

Because $N$ is even, and only $\sigma_x$ can flip the state $|\downarrow\rangle$, $\sigma_z$ only add phases.

$$\Rightarrow \begin{cases} \tilde{R}(T)\,|\downarrow\rangle = e^{-iF_N(T)}\,|\downarrow\rangle \\ \tilde{R}(T)\,|\uparrow\rangle = e^{+iF_N(T)}\,|\uparrow\rangle \\ \tilde{R}(T) = e^{iF_N(T)\sigma_z}. \end{cases}$$

where

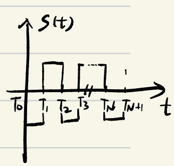
$$\begin{aligned} F_N(T) = & -\left(\int_{T_0}^{T_1}\beta(t)dt-\phi_1/2\right) \\ & +\left(\int_{T_1}^{T_2}\beta(t)\,dt+\phi_1/2-\phi_2/2\right) \\ & \cdots + (-1)^n\left(\int_{T_{n-1}}^{T_n}\beta(t)dt+\phi_{n-1}/2-\phi_n/2\right) \\ & + (-1)^{n+1}\left(\int_{T_n}^{T_{n+1}}\beta(t)dt+\phi_n/2-\phi_{n+1}/2\right) \\ = & \sum_{i=0}^{N}(-1)^{i+1}\int_{T_i}^{T_{i+1}}\beta(t)\,dt + \sum_{i=1}^{N}(-1)^{i+1}\phi_i. \end{aligned}$$

For CPMG, $\phi_i = 0$, $\sum_{i=1}^{N}(-1)\,\phi_i = 0$.

$$F_N(T) = \sum_{i=0}^{N}(-1)^{i+1}\int_{T_i}^{T_{i+1}}\beta(t)\,dt.$$

$$\equiv \boxed{\int_{-\infty}^{\infty} s_N(t')\,\beta(t')\,dt'}$$

where $s_N(t) = \begin{cases} 0 & t \le 0 \\ (-1)^j & T_j < t < T_{j+1} \\ 0 & t > T \end{cases}$



Measured result:

$$\left\langle \langle \psi(T) | \sigma_z | \psi(T) \rangle \right\rangle = \left\langle \langle \psi(0) | D_x(\pi/2)^{+} \tilde{R}(T)^{+} D_x(\pi/2)^{+} \sigma_z D_x(\pi/2) \tilde{R}(T) D_x(\pi/2) | \psi(0) \rangle \right\rangle$$

$$= \left\langle \langle \downarrow | e^{+\frac{i\pi}{4}\sigma_x} e^{-iF_N(T)\sigma_z} e^{\frac{i\pi}{4}\sigma_x} \sigma_z e^{-\frac{i\pi}{4}\sigma_x} e^{iF_N(T)\sigma_z} \underbrace{\underbrace{\underbrace{e^{-\frac{i\pi}{4}\sigma_x} | \downarrow \rangle}_{\frac{1}{\sqrt{2}}(|\downarrow\rangle - i|\uparrow\rangle)}}_{\frac{1}{\sqrt{2}}(e^{-iF_N(T)}|\downarrow\rangle - i e^{iF_N(T)}|\uparrow\rangle)}}_{\frac{1}{2}\left(e^{-iF_N(T)}(|\downarrow\rangle - i|\uparrow\rangle - i e^{iF_N(T)}(|\uparrow\rangle - i|\downarrow\rangle)\right)} \right\rangle$$

$$= \frac{1}{2}\left( -2i\sin(F_N(T))|\downarrow\rangle - 2i\cos(F_N(T))|\uparrow\rangle \right)$$

$$= -i\left( \sin(F_N(T))|\downarrow\rangle + \cos(F_N(T))|\uparrow\rangle \right)$$

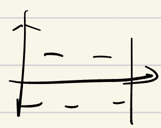
$$= \left\langle \left[ \sin(F_N(T)) \langle \downarrow | + \cos(F_N(T)) \langle \uparrow | \right] \left[ -\sin F_N(T) |\downarrow\rangle + \cos F_N(T) |\uparrow\rangle \right] \right\rangle$$

$$= \left\langle -\sin^2 + \cos^2 \right\rangle = \langle \cos(2F_N(T)) \rangle$$

$$\approx e^{2\langle F_N(T)^2 \rangle}$$

Calculate the Fourier transform of $F_N(T)$:

$$\tilde{y}(\omega, T) \equiv \mathcal{F}(S_N(t')) = \int_{-\infty}^{\infty} S_N(t') e^{i\omega t'} dt'$$



$$= \int_{T_0}^{T_1} -e^{i\omega t'} dt' + \int_{T_1}^{T_2} e^{i\omega t'} dt'$$

$$+ (-1) \cdot \int_{T_2}^{T_3} e^{i\omega t'} dt' + \int_{T_3}^{T_4} e^{i\omega t'} dt'$$

$$+ \cdots + (-1)^{N+1} \int_{T_N}^{T_{N+1}} e^{i\omega t'} dt'$$

$$= \frac{1}{i\omega} (-1)^1 \cdot \left(e^{i\omega T_1} - e^{i\omega T_0}\right) + \frac{1}{i\omega}\left(e^{i\omega T_2} - e^{i\omega T_1}\right) + \cdots$$

$$= \frac{1}{i\omega}\left[ 1 - e^{i\omega T_1} - e^{i\omega T_1} + e^{i\omega T_2} + e^{i\omega T_2} + \cdots + (-1)^N e^{i\omega T_N} \times 2 + (-1)^{N+1} e^{i\omega T_{N+1}} \right.$$

$$= \frac{1}{i\omega}\left[ 1 + \sum_{n=1}^{N} 2 \cdot (-1)^n e^{i\omega T_n} + (-1)^{N+1} e^{i\omega T_{N+1}} \right]$$

$$\Rightarrow \langle F_N(T)^2 \rangle = \langle f(F_N(T)^2) \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} S_\beta(\omega) \cdot |\tilde{y}(\omega, T)|^2 d\omega.$$

$$\boxed{S_\beta(\omega) \equiv \int_{-\infty}^{\infty} \beta(t') e^{i\omega t'} dt'}$$

The noise spectrum, what we expect to know.

$|\tilde{y}(\omega, T)|^2$ : filter function.

~~If~~ Assume the noise is purely coherent:

$$\tilde{\beta}(\omega) = \sum_{k=1}^{n} \beta_k \delta(\omega - \omega_k)$$

Then what we measure is

$$\langle \cos(2F_N(T)) \rangle = \prod_k^n J_0(|\beta_k \tilde{y}(\omega_k, T)|)$$

n Eqns.

$$\begin{cases} J_0(|\beta_1 \cdot \tilde{y}(\omega_1, T_1)|) \cdot J_0(\beta_2 \cdot \tilde{y}(\omega_2, T_1)|) \cdots J_0(\beta_n \cdot \tilde{y}(\omega_n, T_1)) = \langle M_z(T_1) \rangle \\ J_0(|\beta_1 \tilde{y}(\omega_1, T_2)|) \cdot J_0(\beta_2 \tilde{y}(\omega_2, T_2)|) \cdots J_0(\beta_n \tilde{y}(\omega_n, T_2)) = \langle M_z(T_2) \rangle \\ \vdots \\ J_0(|\beta_1 \tilde{y}(\omega_1, T_n)|) \cdot J_0(\beta_2 \tilde{y}(\omega_2, T_n)|) \cdots J_0(\beta_n \tilde{y}(\omega_n, T_n)|) = \langle M_z(T_n) \rangle \end{cases}$$

Assume $\beta(t) = \beta_0 \cos(\omega_0 t + \phi)$ ← One example coherent noise.

$$S_\beta(\omega) = \int_{-\infty}^{\infty} \beta_0 \cos(\omega_0 t + \phi)\, e^{i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{\beta_0}{2} \left(e^{i(\omega_0 t + \phi)} + e^{-i(\omega_0 t + \phi)}\right) e^{i\omega t}\, dt$$

$$= \frac{\beta_0}{2} \int_{-\infty}^{\infty} e^{i[(\omega_0 + \omega)t + \phi]} + e^{i[(\omega - \omega_0)t - \phi]}\, dt$$

$$= \frac{\beta_0}{2} \left[ \frac{1}{i(\omega_0 + \omega)} e^{i[(\omega_0 + \omega)t + \phi]} \Big|_{-\infty}^{\infty} + \frac{1}{i(\omega - \omega_0)} e^{i[(\omega - \omega_0)t - \phi]} \Big|_{-\infty}^{\infty} \right]$$

$$= \frac{\beta_0}{2} \left[ \delta(\omega + \omega_0) + \delta(\omega - \omega_0) \right]$$

$$F_N(T) = \sum_{i=0}^{N} (-1)^{i+1} \int_{T_i}^{T_{i+1}} \beta(t)\, dt + \sum_{i=1}^{N} (-1)^{i+1} \phi_i$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \int_{T_i}^{T_{i+1}} \beta_0 \cos(\omega_0 t + \phi)\, dt + \underbrace{\sum_{i=1}^{N} (-1)^{i+1} \phi_i}_{\phi_i = 0 \text{ for CPMG.}}$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \frac{\beta_0}{\omega_0} \sin(\omega_0 t + \phi) \Big|_{T_i}^{T_{i+1}}$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \frac{\beta_0}{\omega_0} \left[ \sin(\omega_0 T_{i+1} + \phi) - \sin(\omega_0 T_i + \phi) \right]$$

where $T_0 = 0$, $T_1 = \frac{\tau}{2}$, $T_2 = \frac{3}{2}\tau$, ---- $T_n = (n - \frac{1}{2})\tau$, ---

$T_N = (N - \frac{1}{2})\tau$, $T_{N+1} = N\tau$.

$$= \frac{\beta_0}{\omega_0} \Big\{ -\left[ \sin(\omega_0 T_1 + \phi) - \sin(\cancelto{0}{\omega_0 T_0} + \phi) \right]$$
$$+ \left[ \sin(\omega_0 T_2 + \phi) - \sin(\omega_0 T_1 + \phi) \right]$$
$$- - -$$
$$+ (-1)^{N+1} \left[ \sin(\omega_0 T_{N+1} + \phi) - \sin(\omega_0 T_N + \phi) \right] \Big\}$$

N is even

$$= \frac{\beta_0}{\omega_0} \Big\{ \sin\phi + \sin(\omega_0 \cdot N\tau + \phi) + \sum_{i=1}^{N} 2 \cdot (-1)^i \sin\left[\omega_0 (i - \tfrac{1}{2})\tau + \phi\right] \Big\}$$

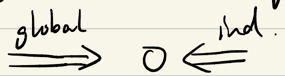
$\langle \cos(2F_N(\tau)) \rangle$ = average over phase $\phi$

## MS Gate

MS gate theory can be found everywhere. Here I just put some tips when running experiments.

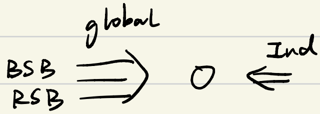
### 1. Freq calibration.

Normal BSB / RSB calibration is implemented by fixing the ind freq while scanning the global beam.



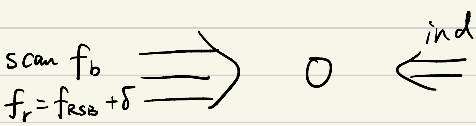

①

However, when implementing MS gate / kick operation, the pulses those are used are as follows, which has 3 tones in total.




②

These two configurations (① & ②) have different Stark shift, therefore, the calibrated BSB / RSB freq is INCORRECT when running MS gate. Thus, when scanning BSB / RSB for MS gate / kick operations, we should:




— Apply 3 tones when scanning BSB (for example) rather than only two tones

- Individual
- Global tone 0: scan $f_b$
- Global tone 1: Apply a far detuned RSB $\delta \sim 300kHz$.

The amplitude of these 3 tones should be the same as MS gate.

E.g. — [H] — [Detuned SQ gate] — [Detuned Kick] — [H] —

Laser: $\delta_{ss\text{-}sq}$ $\quad$ $\delta_{ss\text{-}sq}+\delta_{det\text{-}sq}$ $\quad$ $\delta_{cal\text{-}B/R}+\delta_{det\text{-}kick}$ $\quad$ $\delta_{ss\text{-}sq}$

Qubit: $\delta_{ss\text{-}sq}$ $\quad$ $\cancel{\delta_{det\text{-}ss\text{-}sq}}$ $\delta_{ss\text{-}sq}$ $\quad$ $\cancel{\delta_{det\text{-}B/R}}$ $\delta_{ss\text{-}B/R}$ $\quad$ $\delta_{ss\text{-}sq}$.

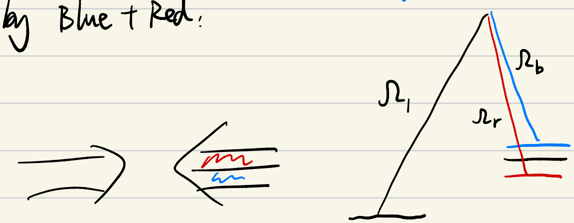
↑ Stark shift w/ detuned SQ gate

Found extra Stark shift due to detuning is negligible.

↑ Stark shift w/ detuned BSB/RSB transitions.

→ Target: Make sure calibrated $\delta_{cal\_B/R}=\delta_{ss\text{-}B/R}$.

SS by Blue + Red:




2-photon:

$$\delta_{2s}=\frac{\Omega_b^2}{4(\Delta-\omega_{HF}-\nu)}+\frac{\Omega_r^2}{4(\Delta-\omega_{HF}+\nu)}-\frac{\Omega_1^2}{4\Delta}$$

$$=\frac{1}{4\Delta}\left(\frac{\Omega_b^2}{1-\frac{\omega_{HF}+\nu}{\Delta}}+\frac{\Omega_r^2}{1-\frac{\omega_{HF}-\nu}{\Delta}}-\Omega_1^2\right)$$

$$=\frac{1}{4\Delta}\left(\Omega_b^2+\Omega_r^2-\Omega_1^2+\Omega_b^2\frac{\omega_{HF}-\nu}{\Delta}+\Omega_r^2\frac{\omega_{HF}+\nu}{\Delta}\right)$$

$$\approx\frac{1}{4\Delta}\left(\Omega_c^2-\Omega_1^2+\Omega_c^2\frac{\omega_{HF}}{\Delta}\right)$$

since $\Omega^2\propto I$, $I_b+I_r=I_c$, ($\uparrow$ carrier)

$$\Omega_b^2=\Omega_r^2\simeq\frac{\Omega_c^2}{2}$$

2-photon SS for carrier:

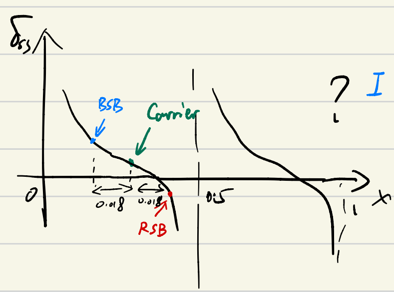
$$\delta_{2s\text{-}c}=\frac{\Omega_c^2}{4(\Delta-\omega_{HF})}-\frac{\Omega_1^2}{4\Delta}=\frac{1}{4\Delta}\left(\frac{\Omega_c^2}{1-\frac{\omega_{HF}}{\Delta}}-\Omega_1^2\right)$$

$$=\frac{1}{4\Delta}\left(\Omega_c^2-\Omega_1^2+\Omega_c^2\frac{\omega_{HF}}{\Delta}\right)$$

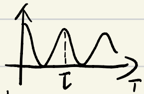
$$=\delta_{2s\text{-}c}$$

4-photon: Single beam $\delta_{4s}\approx 0$ for linear polarization

Inter beam:



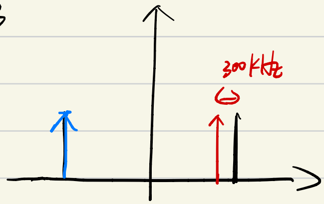

? I guess it should follow this logic but I didn't calculate it clearly. So I measured it in the next page.

Method: Ramsey $\boxed{\pi/2}$ — $\boxed{\text{wait } T}$ — $\boxed{\pi/2}$ $\Rightarrow$  , $|\delta| = \frac{1}{\tau}$

where $\boxed{\pi/2}$ are the pulses listed below

Blue/Red arrows represent the tones applied when implementing $\pi/2$

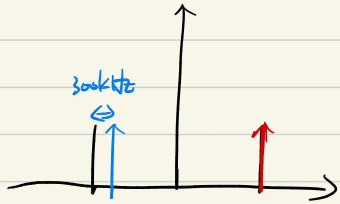
① BSB




$\delta_{BSB} = \delta_{B\text{-}res} + \underbrace{\delta_{R\text{-}det}}_{}$ $= 1.57$ kHz

$-4.31$ kHz

② RSB




$\delta_{RSB} = \underbrace{\delta_{B\text{-}det}}_{} + \delta_{R\text{-}res}$ $= 2.6$ kHz

5.6 kHz

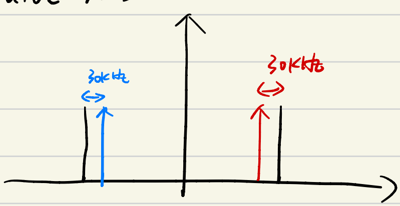
③ BSB only

$\delta_B = \delta_{B\text{-}res}$ $= 5.88$ kHz

④ RSB only

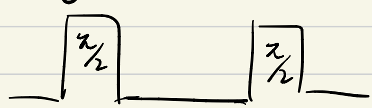
$\delta_R = \delta_{R\text{-}res}$. $= -3$ kHz.

⑤ Naive MS




$\delta_{MS} = \delta_{B\text{-}ms} + \delta_{R\text{-}ms}$

Ramsey:




Laser: $\delta_{ss}$ $\delta_{ss}$ $\delta_{ss}$

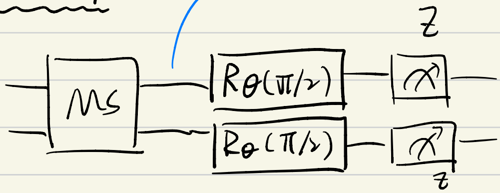
Ion: $\delta_{ss}$ $0$ $\delta_{ss}$

So we can meas $\delta_{ss}$.

Laser tone keeps oscillating during both $\pi/2$ and wait at $f = f_0 + \delta_{ss}$, while the ion oscillates at $f = f_0 + \delta_{ss}$ when $\pi/2$ but $f = f_0$ when waiting.

$|\psi\rangle = \frac{\sqrt{2}}{2}\left(|00\rangle + i|11\rangle\right)$, $\rho = \frac{1}{2}\begin{pmatrix} 1 & & & i \\ & 0 & 0 & \\ & 0 & 0 & \\ -i & & & 1 \end{pmatrix}$

2. parity scan tips.




Fidelity:

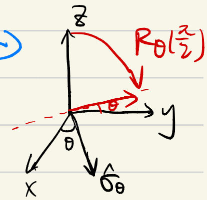
$$F = \frac{\rho_{|00\rangle\langle 00|} + \rho_{|11\rangle\langle 11|}}{2} + \left|\rho_{|00\rangle\langle 11|}\right|$$

↓ population ↓ parity scan.

$\langle ZZ \rangle$

$$= \mathrm{Tr}\left[ Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{(2)}\left(\tfrac{\pi}{2}\right) \rho \cdot R_\theta^{\dagger(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{\dagger(2)}\left(\tfrac{\pi}{2}\right) \right]$$

$$= \mathrm{Tr}\left[ \rho\, R_\theta^{\dagger(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{\dagger(2)}\left(\tfrac{\pi}{2}\right) Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{(2)}\left(\tfrac{\pi}{2}\right) \right]$$




$$= \left\langle \left(\cos\theta\, \hat{Y}^{(1)} - \sin\theta\, \hat{X}^{(1)}\right)\left(\cos\theta\, \hat{Y}^{(2)} - \sin\theta\, \hat{X}^{(2)}\right) \right\rangle$$

$$= \left\langle \cos^2\theta\, Y_1 Y_2 - \sin\theta\cos\theta\left(Y_1 X_2 + X_1 Y_2\right) + \sin^2\theta\, X_1 X_2 \right\rangle$$

$$= \mathrm{Tr}\left[ \sin^2\theta \begin{pmatrix} \rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & \rho_{|11\rangle\langle 00|} \end{pmatrix} + \cos^2\theta \begin{pmatrix} -\rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & -\rho_{|11\rangle\langle 00|} \end{pmatrix} - \sin 2\theta \begin{pmatrix} i\rho_{|00\rangle\langle 11|} & \\ & -i\rho_{|11\rangle\langle 00|} \end{pmatrix} \right]$$

$P = \left|\rho_{|00\rangle\langle 11|}\right|$

$$= 2\cos 2\theta\, \underbrace{\mathrm{Re}\left(\rho_{|00\rangle\langle 11|}\right)}_{\alpha} + 2\sin 2\theta\, \underbrace{\mathrm{Im}\left(\rho_{|00\rangle\langle 11|}\right)}_{\beta} + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 10|}\right)$$

$\sqrt{\alpha^2+\beta^2} = P \Rightarrow \alpha \equiv \sin\varphi \cdot P$

$$= 2\cos 2\theta\, P \cdot \sin\varphi + 2\sin 2\theta \cdot P \cdot \cos\varphi + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 01|}\right)$$

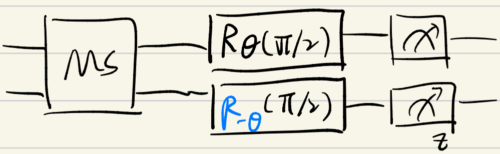
$$= 2P\sin(2\theta + \varphi) + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 01|}\right)$$

← Contrast.

$\boxed{\text{Parity scan } -\theta}$

If preparing other Bell state,

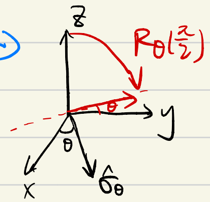
e.g. $\rho = \begin{pmatrix} 0 & & & 0 \\ & 1 & 1 & \\ & 1 & 1 & \\ 0 & & & 0 \end{pmatrix}$




parity scan needs to capture $|\rho_{|01\rangle\langle 10|}|$

$$\mathrm{Tr}\left[\, Z^{(1)} Z^{(2)}\, R_\theta^{(1)}(\tfrac{\pi}{2})\, R_{-\theta}^{(2)}(\tfrac{\pi}{2})\, \rho \cdot R_\theta^{\dagger(1)}(\tfrac{\pi}{2})\, R_{-\theta}^{\dagger(2)}(\tfrac{\pi}{2}) \right]$$

$$= \mathrm{Tr}\left[\, \rho\, \underline{R_\theta^{\dagger(1)}(\tfrac{\pi}{2})\, R_{-\theta}^{(2)}(\tfrac{\pi}{2})\, Z^{(1)} Z^{(2)}\, R_\theta^{(1)}(\tfrac{\pi}{2})\, R_{-\theta}^{(2)}(\tfrac{\pi}{2})} \right]$$




$$= \langle (\cos\theta\, \hat{Y}^{(1)} - \sin\theta\, \hat{X}^{(1)})(\cos\theta\, \hat{Y}^{(2)} + \sin\theta\, \hat{X}^{(2)}) \rangle$$

$$= \langle \cos^2\theta\, Y_1 Y_2 + \sin\theta\cos\theta\, (Y_1 X_2 - X_1 Y_2) - \sin^2\theta\, X_1 X_2 \rangle$$

$$= \mathrm{Tr}\left[ -\sin^2\theta \begin{pmatrix} \rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & \rho_{|11\rangle\langle 00|} \end{pmatrix} + \cos^2\theta \begin{pmatrix} -\rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & -\rho_{|11\rangle\langle 00|} \end{pmatrix} + \sin 2\theta \begin{pmatrix} 0 & & & \\ & i\rho_{|01\rangle\langle 10|} & & \\ & & -i\rho_{|10\rangle\langle 01|} & \\ & & & 0 \end{pmatrix} \right]$$

$$= -2\,\mathrm{Re}(\rho_{|00\rangle\langle 11|}) + \cos 2\theta\, \mathrm{Re}(\rho_{|01\rangle\langle 10|}) + \sin 2\theta\, \mathrm{Im}(\rho_{|01\rangle\langle 10|})$$

$$= -2\,\mathrm{Re}(\rho_{|00\rangle\langle 11|}) + P \cdot \sin(2\theta + \varphi),$$ where $P = |\rho_{|01\rangle\langle 10|}|$

$$\sin\varphi = \mathrm{Re}(\rho_{|01\rangle\langle 10|})/P$$

Calculation supplementary:

Because we want to know the trace of them, so only calculate the diagonal terms

$$\rho \cdot X_1 X_2 = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \rho_{|00\rangle\langle 11|} \\ & \rho_{|01\rangle\langle 01|} & \rho_{|01\rangle\langle 10|} & \\ & \rho_{|10\rangle\langle 01|} & \rho_{|10\rangle\langle 10|} & \\ \rho_{|11\rangle\langle 00|} & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}$$

$$= \begin{pmatrix} \rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & \rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot Y_1 Y_2 = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & \rho_{|01\rangle\langle 01|} & & \\ & & \rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} \begin{pmatrix} & & & -1 \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}$$

$$= \begin{pmatrix} -\rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & -\rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot \frac{(X_1 Y_2 + X_2 Y_1)}{2} = \begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} & & & -i \\ & & 0 & \\ & 0 & & \\ i & & & \end{pmatrix} = \begin{pmatrix} i \cdot \rho_{|00\rangle\langle 11|} & & & \\ & 0 & & \\ & & 0 & \\ & & & -i\rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot \frac{(Y_1 X_2 - X_1 Y_2)}{2} = \begin{pmatrix} & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} & & & 0 \\ & & -i & \\ & i & & \\ 0 & & & \end{pmatrix} = \begin{pmatrix} 0 & & & \\ & i\rho_{|01\rangle\langle 10|} & & \\ & & -i\rho_{|10\rangle\langle 01|} & \\ & & & 0 \end{pmatrix}$$

$$Z_1 Z_2 \cdot \rho = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & \rho_{|01\rangle\langle 01|} & & \\ & & \rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & -\rho_{|01\rangle\langle 01|} & & \\ & & -\rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix}$$

$$Z_1 Y_2 \rho = \begin{pmatrix} & -i & & \\ i & & & \\ & & & i \\ & & -i & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} -i\rho_{|01\rangle\langle 00|} & & & \\ & i\rho_{|00\rangle\langle 01|} & & \\ & & i\rho_{|11\rangle\langle 10|} & \\ & & & -i\rho_{|10\rangle\langle 11|} \end{pmatrix}$$

$$Z_1 X_2 \rho = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & -1 \\ & & -1 & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} \rho_{|01\rangle\langle 00|} & & & \\ & \rho_{|00\rangle\langle 01|} & & \\ & & -\rho_{|11\rangle\langle 10|} & \\ & & & -\rho_{|10\rangle\langle 11|} \end{pmatrix}$$

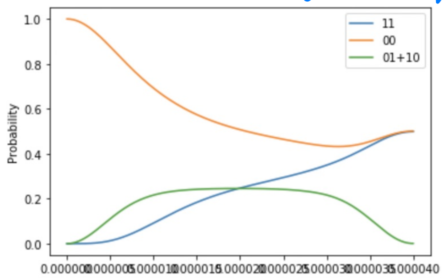
$$Y_1 Z_2 \rho = \begin{pmatrix} & & -i & \\ & & & i \\ i & & & \\ & -i & & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} -i\rho_{|10\rangle\langle 00|} & & & \\ & i\rho_{|11\rangle\langle 01|} & & \\ & & i\rho_{|00\rangle\langle 10|} & \\ & & & -i\rho_{|01\rangle\langle 11|} \end{pmatrix}$$

$$X_1 Z_2 \rho = \begin{pmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} \rho_{|10\rangle\langle 00|} & & & \\ & -\rho_{|11\rangle\langle 01|} & & \\ & & \rho_{|00\rangle\langle 10|} & \\ & & & -\rho_{|01\rangle\langle 11|} \end{pmatrix}$$

```
delta =  25000.0
t_gate =  4e-05
```

MS Gate time scan

$t \in [0, t_g]$



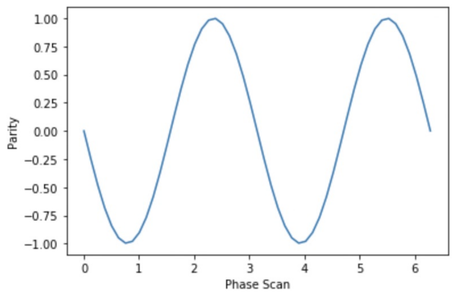

```
0:00:00.540298
Max parity =  0.99699977466192
Min parity =  -0.9952544323520293
Parity contrast =  0.9961271035069746
Prob_11 =  0.49859747106299007
Prob_00 =  0.5004713911571109
Prob_01+10 =  0.0009311377798986452
```

Out[62]: Quantum object: dims = [[2, 2], [2, 2]], shape = (4, 4), type = oper, isherm = True

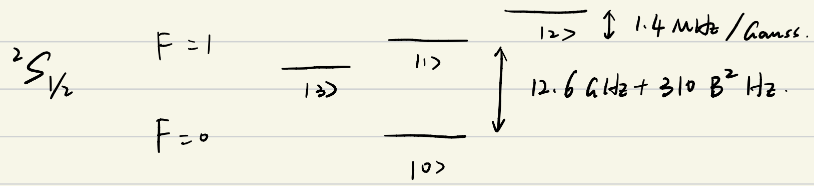
$$\begin{pmatrix} 0.499 & 0.0 & 0.0 & (4.656 \times 10^{-04} - 0.499j) \\ 0.0 & 4.656 \times 10^{-04} & 4.656 \times 10^{-04} & 0.0 \\ 0.0 & 4.656 \times 10^{-04} & 4.656 \times 10^{-04} & 0.0 \\ (4.656 \times 10^{-04} + 0.499j) & 0.0 & 0.0 & 0.500 \end{pmatrix}$$

This term is the degree of entanglement. when it's 0.5j. it's maximal entangled. When it becomes 0, it means it becomes an entire mixed state

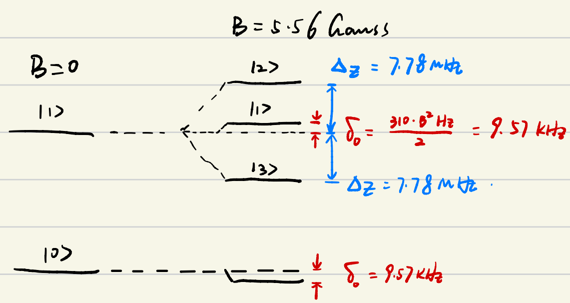
Parity scan



# Zeeman transitions




$$f_{Zeeman} = 7.78 \text{ MHz} \Rightarrow B = 5.56 \text{ Gauss}$$




$$\Rightarrow \begin{aligned} W_{12} &= \Delta_z - \delta_0 \\ W_{13} &= \Delta_z + \delta_0 \end{aligned}$$

$\Rightarrow$ Energy difference between $|1\rangle \to |2\rangle$ and $|1\rangle \to |3\rangle$ transition is $\boxed{\delta = 2\delta_0 = 19.15 \text{ kHz.}}$

Interaction picture:

$$H = \begin{array}{c} \\ \\ \end{array} \begin{array}{cccc|c} |0\rangle & |1\rangle & |2\rangle & |3\rangle & \end{array}$$

$$H = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_1^* & \Omega_2^* \\ 0 & \Omega_1 & 2\delta_0 & 0 \\ 0 & \Omega_2 & 0 & 0 \end{pmatrix} \begin{array}{c} |0\rangle \\ |1\rangle \\ |2\rangle \\ |3\rangle \end{array}$$

$$\begin{pmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1 & \delta_1 & 0 \\ \Omega_2 & 0 & \delta_2 \end{pmatrix} \to \begin{vmatrix} -\lambda & \Omega_1 & \Omega_2 \\ \Omega_1 & \delta_1-\lambda & 0 \\ \Omega_2 & 0 & \delta_2-\lambda \end{vmatrix}$$

$$\Rightarrow \underbrace{\delta_2\Omega_1^2+\delta_1\Omega_2^2}_{d} + \underbrace{(\delta_1\delta_2-\Omega_1^2-\Omega_2^2)}_{c}\lambda + \underbrace{(-\delta_1-\delta_2)}_{b}\lambda^2 + \underbrace{\lambda^3}_{a=1} = 0$$

For $ax^3+bx^2+cx+d=0$.

$$p = -\frac{b}{3a} = \frac{\delta_1+\delta_2}{3}$$

$$q = p^3 + \frac{bc-3ad}{6a^2} = \left(\frac{\delta_1+\delta_2}{3}\right)^3 + \frac{-(\delta_1+\delta_2)(\delta_1\delta_2-\Omega_1^2-\Omega_2^2) - 3(\delta_2\Omega_1^2+\delta_1\Omega_2^2)}{6}$$

$$= \frac{\delta_1^3+3\delta_1^2\delta_2+3\delta_1\delta_2^2+\delta_2^3}{27} + \frac{-\delta_1^2\delta_2+\delta_1\Omega_1^2+\cancel{\delta_1\Omega_2^2}-\delta_1\delta_2^2+\cancel{\delta_2\Omega_1^2}+\delta_2\Omega_2^2-\cancelto{2}{3}\delta_2\Omega_1^2-\cancelto{2}{3}\delta_1\Omega_2^2}{6}$$

$$= \frac{1}{54}\Big[\underline{2\delta_1^3}+\cancel{6\delta_1^2\delta_2}+\cancel{6\delta_1\delta_2^2}+2\delta_2^3-\cancelto{3}{9}\delta_1^2\delta_2+\underline{9\delta_1\Omega_1^2}-\cancelto{3}{9}\underline{\delta_1\delta_2^2}+9\delta_2\Omega_2^2$$
$$-18\delta_2\Omega_1^2-\underline{18\delta_1\Omega_2^2}\Big]$$

$$= \frac{1}{54}\Big[2\delta_1^3+3\delta_1(-\delta_2^2+3\Omega_1^2-6\Omega_2^2)+3\delta_2(-\delta_1^2+3\Omega_2^2-6\Omega_1^2)$$
$$+2\delta_2^3\Big]$$

$$r = \frac{c}{3a} = \frac{1}{3}(\delta_1\delta_2-\Omega_1^2-\Omega_2^2)$$

Roots: $\beta_1$, $\beta_2$, $\beta_3$

$$|1'\rangle = \left[ -\frac{\delta_2 - \beta_1}{w_2},\ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \cdot \beta_1 - \beta_1^2\right),\ 1 \right]$$

$$|2'\rangle = \left[ -\frac{\delta_2 - \beta_2}{w_2},\ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \beta_2 - \beta_2^2\right),\ 1 \right]$$

$$|3'\rangle = \left[ -\frac{\delta_2 - \beta_3}{w_2},\ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \beta_3 - \beta_3^2\right),\ 1 \right]$$

when $\delta_1 \sim \delta_2$ and $|\delta_{1,2}| \gg w_{1,2}$

$$\beta_1 = \delta_1 + \varepsilon,\quad \beta_2 = \delta_1,\quad \beta_3 \approx -\varepsilon \ll w_{12}$$

$$|1'\rangle = \left[ -\frac{\varepsilon}{w},\ -1,\ -1 \right] \qquad \beta_1 = \delta$$

$$|2'\rangle = \left[ 0,\ -1,\ 1 \right] \qquad \beta_2 = \delta$$

$$|3'\rangle = \left[ \infty,\ 1,\ 1 \right] \qquad \beta_3 = 0$$

J. Randall 2015. PRA paper.

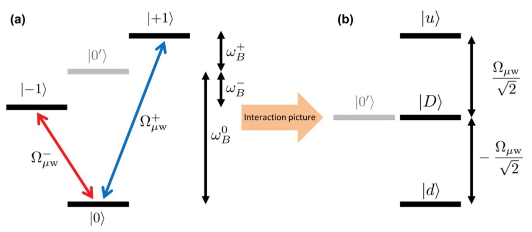



FIG. 1. (Color online) (a) Energy level diagram of the $^2S_{1/2}$ ground state hyperfine manifold of $^{171}Yb^+$ where the degeneracy of the Zeeman states has been lifted by a static magnetic field. Here the transitions $|0\rangle \leftrightarrow |+1\rangle$ and $|0\rangle \leftrightarrow |-1\rangle$ are coupled with resonant microwave radiation to obtain the three dressed states $|d\rangle, |D\rangle$, and $|u\rangle$. (b) The resultant energy level diagram in the dressed basis when the Rabi frequencies of the dressing fields are set to be equal ($\Omega^+_{\mu w} = \Omega^-_{\mu w} = \Omega_{\mu w}$). The energy difference between the dressed-states in this case is given by $\pm\hbar\Omega_{\mu w}/\sqrt{2}$.

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|{+1}\rangle - |{-1}\rangle\right)$$
$$|u\rangle = \frac{1}{2}|{+1}\rangle + \frac{1}{2}|{-1}\rangle + \frac{1}{\sqrt{2}}|0\rangle$$
$$|d\rangle = \frac{1}{2}|{+1}\rangle + \frac{1}{2}|{-1}\rangle - \frac{1}{\sqrt{2}}|0\rangle$$

$$H_{\mu w} = \frac{\hbar\Omega_{\mu w}}{2}\left(|{+1}\rangle\langle 0| + |{-1}\rangle\langle 0| + H.c.\right)$$

$\downarrow$ diagonalize

$$H_{\mu w} = \frac{\hbar\Omega_{\mu w}}{\sqrt{2}}\left(|u\rangle\langle u| - |d\rangle\langle d|\right)$$

Reasons of insensitive to B field:

Perturbation up to first order of B field.

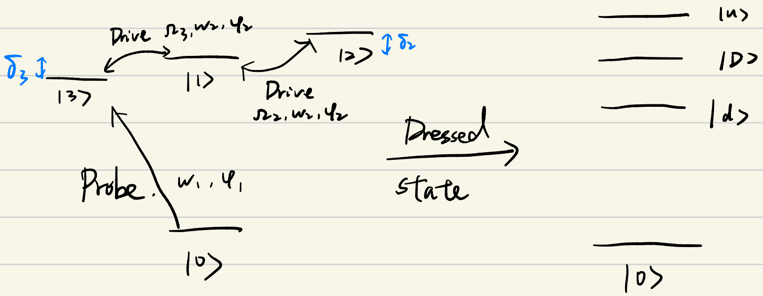
$$H_p = \hbar\lambda_0(t)\left(|{+1}\rangle\langle{+1}| - |{-1}\rangle\langle{-1}|\right)$$

$$\xrightarrow[\text{basis}]{\text{Dressed state}} H_p = \frac{\hbar\lambda_0(t)}{\sqrt{2}}\left(|D\rangle\langle u| + |D\rangle\langle d| + H.c.\right).$$

B field fluctuations will try to drive population between $|D\rangle$, $|u\rangle$ and $|d\rangle$. But these states are separated by $\hbar\Omega_{\mu w}/\sqrt{2}$ So only B field fluctuating with a frequency at or near $\Omega_{\mu w}/\sqrt{2}$ can cause transitions between dressed states.

- Dressed state.

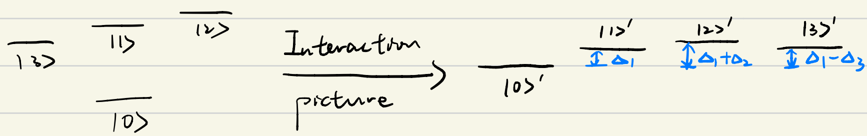
Two lasers:




Assume $\omega_0 = 0$

$$H_0 = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$$
$$+ \left[\Omega_{03} e^{i(\omega_1 t + \varphi_1)} |0\rangle\langle 3| + \Omega_{02} e^{i(\omega_1 t + \varphi_1)} |0\rangle\langle 2| + \Omega_{01} e^{i(\omega_1 t + \varphi_1)} |0\rangle\langle 1| + h.c.\right]$$
$$+ \left[\Omega_{12} e^{i(\omega_2 t + \varphi_2)} |1\rangle\langle 2| + \Omega_{13} e^{i(\omega_2 t + \varphi_2)} |3\rangle\langle 1| + h.c.\right]$$

Let $H_D = \omega_1 |1\rangle\langle 1| + \omega_1 |2\rangle\langle 2| + \omega_1 |3\rangle\langle 3| + \omega_2 |2\rangle\langle 2| - \omega_2 |3\rangle\langle 3|$




Def $\Delta_1 = \omega_{01} - \omega_1$, $\Delta_2 = \delta_2 - \omega_2$, $\Delta_3 = \delta_3 - \omega_2$

$$H_I = H_0 - H_D = (\Delta_1 - \Delta_3)|3\rangle\langle 3| + \Delta_1 |1\rangle\langle 1| + (\Delta_1 + \Delta_2)|2\rangle\langle 2|$$
$$+ [\ \cdots\ ]$$

$$\tilde{H}_I = e^{iH_D t} H_I e^{-iH_D t}$$
$$= (\Delta_1 - \Delta_3)|3\rangle\langle 3| + \Delta_1 |1\rangle\langle 1| + (\Delta_1 + \Delta_2)|2\rangle\langle 2|$$
$$+ \Big[\Omega_{03} \cdot e^{i(\omega_1 t + \varphi_1)} |0\rangle\langle 3| e^{-i(\omega_1 - \omega_2)t} + \Omega_{02} e^{i(\omega_1 t + \varphi)} |0\rangle\langle 2| e^{-i(\omega_1 + \omega_2)t}$$
$$+ \Omega_{01} e^{i(\omega_1 t + \varphi_1)} |0\rangle\langle 1| e^{-i\omega_1 t} + h.c.\Big]$$
$$+ \Big[\Omega_{12} e^{i(\omega_2 t + \varphi_2)} \cdot e^{i\omega_1 t} |1\rangle\langle 2| e^{-i(\omega_1 + \omega_2)t}$$
$$+ \Omega_{13} e^{i(\omega_2 t + \varphi_2)} e^{i(\omega_1 - \omega_2)t} |3\rangle\langle 1| e^{-i\omega_1 t}$$

$$= \Delta_1 |1\rangle\langle 1| + (\Delta_1+\Delta_2)|2\rangle\langle 2| + (\Delta_1-\Delta_3)|3\rangle\langle 3|$$
$$+ [\Omega_{03} e^{i(+\omega_2 t+\varphi_1)} |0\rangle\langle 3| + \Omega_{02} e^{i(-\omega_2 t+\varphi_1)} |0\rangle\langle 2| + \Omega_{01} e^{i\varphi_1} |0\rangle\langle 1| + h.c.]$$
$$+ [\Omega_{12} e^{i\varphi_2} |1\rangle\langle 2| + \Omega_{13} e^{-i\varphi_2} |1\rangle\langle 3| + h.c.]$$

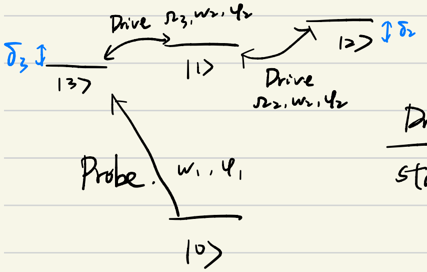
$$= \begin{bmatrix} 0 & \Omega_{01} e^{i\varphi_1} & \Omega_{02} e^{-i\omega_2 t} e^{i\varphi_1} & \Omega_{03} e^{+i\omega_2 t} e^{i\varphi_1} \\ \Omega_{01}^* e^{-i\varphi_1} & \Delta_1 & \Omega_{12} e^{i\varphi_2} & \Omega_{13} e^{-i\varphi_2} \\ \Omega_{02}^* e^{i\omega_2 t} e^{-i\varphi_1} & \Omega_{12}^* e^{-i\varphi_2} & \Delta_1+\Delta_2 & 0 \\ \Omega_{03}^* e^{-i\omega_2 t} e^{-i\varphi_1} & \Omega_{13}^* e^{i\varphi_2} & 0 & \Delta_1-\Delta_3 \end{bmatrix}$$

if $\varphi_1 = 0$, $\varphi_2 = 0$

$$= \begin{bmatrix} 0 & \Omega_{01} & \Omega_{02} e^{-i\omega_2 t} & \Omega_{03} e^{+i\omega_2 t} \\ \Omega_{01}^* & \Delta_1 & \Omega_{12} & \Omega_{13} \\ \Omega_{02}^* e^{i\omega_2 t} & \Omega_{12}^* & \Delta_1+\Delta_2 & 0 \\ \Omega_{03} e^{-i\omega_2 t} & \Omega_{13}^* & 0 & \Delta_1-\Delta_3 \end{bmatrix}$$

# Another choice of interaction picture

## Two lasers:




Dressed state $\longrightarrow$

$|u\rangle = \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle + \frac{1}{\sqrt{2}}|1\rangle$

$|D\rangle = \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|3\rangle$

$|d\rangle = \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle - \frac{1}{\sqrt{2}}|1\rangle$

$\Updownarrow$

$|1\rangle = \frac{1}{\sqrt{2}}(|u\rangle - |d\rangle)$

$|2\rangle = \frac{1}{\sqrt{2}}|D\rangle + \frac{1}{2}|u\rangle + \frac{1}{2}|d\rangle$

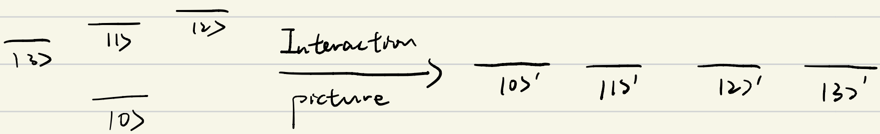
$|3\rangle = -\frac{1}{\sqrt{2}}|D\rangle + \frac{1}{2}|u\rangle + \frac{1}{2}|d\rangle$

$|0\rangle$

Assume $\omega_0 = 0$

$$H_0 = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$$
$$+ \left[\Omega_{03} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3| + \Omega_{02} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 2| + \Omega_{01} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1| + h.c.\right]$$
$$+ \left[\Omega_{12} e^{i(\omega_2 t + \varphi_2)}|1\rangle\langle 2| + \Omega_{13} e^{i(\omega_2 t + \varphi_2)}|3\rangle\langle 1| + h.c.\right]$$

Let $H_D = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$




$H_I = H_0 - H_D = $ [ Interaction terms only ]

$$\tilde{H}_I = e^{iH_D t} H_I e^{-iH_D t} = \Big[\Omega_{03} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3| e^{-i(\omega_{01} - \delta_3)t} + \Omega_{02} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 2| e^{-i(\omega_{01} + \delta_2)t}$$
$$+ \Omega_{01} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1| e^{-i\omega_{01} t} + \Omega_{12} e^{i(\omega_2 t + \varphi_2)} e^{i\omega_{01} t}|1\rangle\langle 2| e^{-i(\omega_{01} + \delta_2)t}$$
$$+ \Omega_{13} e^{i(\omega_2 t + \varphi_2)} e^{i(\omega_{01} - \delta_3)t}|3\rangle\langle 1| e^{-i\omega_{01} t}\Big] + h.c.$$

$$= \underbrace{\Omega_{03} e}_{A_1}{}^{i[(\omega_1 - \omega_{01} + \delta_3)t + \varphi_1]}|0\rangle\langle 3| + \underbrace{\Omega_{02} e}_{A_2}{}^{i[(\omega_1 - \omega_{01} - \delta_2)t + \varphi_1]}|0\rangle\langle 2|$$
$$+ \underbrace{\Omega_{01} e}_{A_3}{}^{i[(\omega_1 - \omega_{01})t + \varphi_1]}|0\rangle\langle 1| + \underbrace{\Omega_{12} e}_{B_1}{}^{i[(\omega_2 - \delta_2)t + \varphi_2]}|1\rangle\langle 2|$$
$$+ \underbrace{\Omega_{13} e}_{B_2}{}^{-i[(\omega_2 - \delta_3)t + \varphi_2]}|1\rangle\langle 3| + h.c.$$

Assumption:

$A_i \ll B_i$

change $|1\rangle, |2\rangle, |3\rangle$ to $|u\rangle, |D\rangle, |d\rangle$

$$= A_1 |0\rangle\left(\tfrac{1}{2}\langle u| + \tfrac{1}{2}\langle d| - \tfrac{1}{\sqrt{2}}\langle D|\right) + A_2 |0\rangle\left(\tfrac{1}{2}\langle u| + \tfrac{1}{2}\langle d| + \tfrac{1}{\sqrt{2}}\langle D|\right)$$
$$+ A_3 |0\rangle\left(\tfrac{1}{\sqrt{2}}\langle u| - \tfrac{1}{\sqrt{2}}\langle d|\right) + B_1\left(\tfrac{1}{\sqrt{2}}|u\rangle - \tfrac{1}{\sqrt{2}}|d\rangle\right)\left(\tfrac{1}{2}\langle u| + \tfrac{1}{2}\langle d| + \tfrac{1}{\sqrt{2}}\langle D|\right)$$
$$+ B_2\left(\tfrac{1}{\sqrt{2}}|u\rangle - \tfrac{1}{\sqrt{2}}|d\rangle\right)\left(\tfrac{1}{2}\langle u| + \tfrac{1}{2}\langle d| - \tfrac{1}{\sqrt{2}}\langle D|\right) + h.c.$$

change $|1\rangle, |2\rangle, |3\rangle$ to $|u\rangle, |D\rangle, |d\rangle$

$$= A_1 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + A_2 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$
$$+ A_3 |0\rangle\left(\frac{1}{\sqrt{2}}\langle u| - \frac{1}{\sqrt{2}}\langle d|\right) + B_1\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$
$$+ B_2\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + h.c.$$
$$= \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 + \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle u| + \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 - \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle d|$$
$$+ \left(\frac{1}{\sqrt{2}}A_2 - \frac{1}{\sqrt{2}}A_1\right)|0\rangle\langle D| + \frac{1}{2\sqrt{2}}(B_1+B_2)\left(|u\rangle\langle u| + |u\rangle\langle d| - |d\rangle\langle u| - |d\rangle\langle d|\right)$$
$$+ \frac{1}{2}(B_1 - B_2)|u\rangle\langle D| + \frac{1}{2}(B_2 - B_1)|d\rangle\langle D| + h.c.$$

A second interaction picture: $H_D^{(2)} = \frac{1}{2\sqrt{2}}(B_1+B_2)\left(|u\rangle\langle u| - |d\rangle\langle d|\right)$

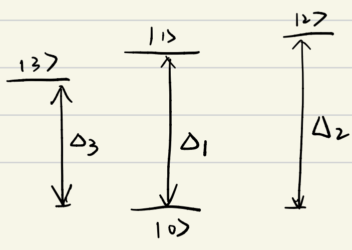
$$\tilde{H}_I^{(2)} = e^{iH_D^{(2)}t}\,\tilde{H}_I\,e^{-iH_D^{(2)}t}$$
$$= \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 + \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle u|\,e^{-i\frac{1}{2\sqrt{2}}(B_1+B_2)t} + \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 - \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle d|\,e^{+i\frac{1}{2\sqrt{2}}(B_1+B_2)t}$$
$$+ \left(\frac{1}{\sqrt{2}}A_2 - \frac{1}{\sqrt{2}}A_1\right)|0\rangle\langle D| + \frac{1}{2\sqrt{2}}(B_1+B_2)\,|u\rangle\langle d|\,e^{\frac{i}{\sqrt{2}}(B_1+B_2)t}$$
$$- \frac{1}{2\sqrt{2}}(B_1+B_2)\,|d\rangle\langle u|\,e^{-\frac{i}{\sqrt{2}}(B_1+B_2)t}$$
$$+ \frac{1}{2}(B_1 - B_2)\,e^{\frac{i}{2\sqrt{2}}(B_1+B_2)t}\,|u\rangle\langle D| + \frac{1}{2}(B_2 - B_1)\,e^{-\frac{i}{2\sqrt{2}}(B_1+B_2)t}\,|d\rangle\langle D| + h.c.$$

Discussion: ① Set the laser s.t. $\varphi_1 = \varphi_2 = 0$

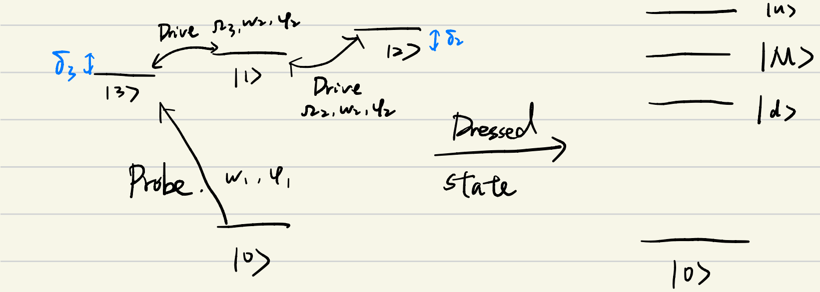
② when $\omega_{12} = \omega_{13} = \omega_2$, i.e. the Driving laser is resonance with both $|1\rangle \to |2\rangle$ & $|1\rangle \to |3\rangle$ simultaneously. $B_1 = B_2 = \Omega_d$ $\mathrm{Im}(B_1) = \mathrm{Im}(B_2) = 0$.

③ Define $\Delta_1 = \omega_{01}$, $\Delta_2 = \omega_{01} + \delta_2$, $\Delta_3 = \omega_{03} - \delta_3$ are the energy levels of $|1\rangle, |2\rangle, |3\rangle$ from $|0\rangle$

$$\tilde{H}_I^{(2)} = \left\{\frac{1}{2}\Omega_{03}e^{i[\omega_1 - (\Delta_3 + \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{2}\Omega_{02}e^{i[\omega_1 - (\Delta_2 + \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{\sqrt{2}}\Omega_{01}e^{i[\omega_1 - (\Delta_1 + \frac{1}{\sqrt{2}}\Omega_d)]t}\right\}|0\rangle\langle u|$$
$$+ \left\{\frac{1}{2}\Omega_{03}e^{i[\omega_1 - (\Delta_3 - \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{2}\Omega_{02}e^{i[\omega_1 - (\Delta_2 - \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{\sqrt{2}}\Omega_{01}e^{i[\omega_1 - (\Delta_1 - \frac{1}{\sqrt{2}}\Omega_d)]t}\right\}|0\rangle\langle d|$$
$$+ \left\{-\frac{1}{\sqrt{2}}\Omega_{03}e^{i[\omega_1 - \Delta_3]t} + \frac{1}{\sqrt{2}}\Omega_{02}e^{i[\omega_1 - \Delta_2]t}\right\}|0\rangle\langle D| + h.c.$$



# Analyze the dressed states.




If $\delta_2 = \delta_3$. $\quad w_1 = w_2 = 1$

| | | | |
|---|---|---|---|
| | $\|u\rangle$ | $\lambda_1 = \sqrt{2}$ | $\|u\rangle = \frac{1}{\sqrt{2}}\|1\rangle + \frac{1}{2}\|2\rangle + \frac{1}{2}\|3\rangle$ |
| $\sqrt{2}$ | | | |
| | $\|M\rangle$ | $\lambda_2 = 0$ | $\|M\rangle = \frac{1}{\sqrt{2}}\|2\rangle - \frac{1}{\sqrt{2}}\|3\rangle$ |
| $\sqrt{2}$ | | | |
| | $\|d\rangle$ | $\lambda_3 = -\sqrt{2}$ | $\|d\rangle = \frac{1}{\sqrt{2}}\|1\rangle - \frac{1}{2}\|2\rangle - \frac{1}{2}\|3\rangle$ |

If $\delta_2 - \delta_3 = 0.25$, $\quad w_1 = w_2 = 1$

| | | |
|---|---|---|
| $\|u\rangle$ | $\lambda_1 = 1.48$ | $\|u\rangle = 0.69\|1\rangle + 0.56\|2\rangle + 0.46\|3\rangle$ |
| $\|M\rangle$ | $\lambda_2 = 0.12$ | $\|M\rangle = 0.09\|1\rangle - 0.70\|2\rangle + 0.71\|3\rangle$ |
| $\|d\rangle$ | $\lambda_3 = -1.36$ | $\|d\rangle = -0.72\|1\rangle + 0.45\|2\rangle + 0.53\|3\rangle$ |

This is nontrivial due to ① $310 B^2$ Hz.

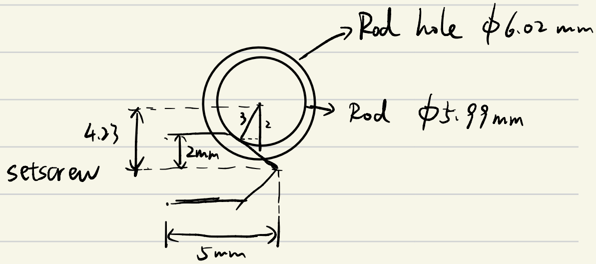
② AC - Stark shift.

This is what we can use!

# Some design tips

① Setscrew: SS4MS5C or SS4MS5

Use a setscrew to fix a rod in a customized hole.




② Coupling lens: Fiber MFD = 2.3 μm

Beam diameter = 0.8 mm = D

Laser wavelength = 369 nm = λ

$$f = \frac{\pi \cdot D \cdot MFD}{4\lambda} = 3.92 \text{ mm}$$

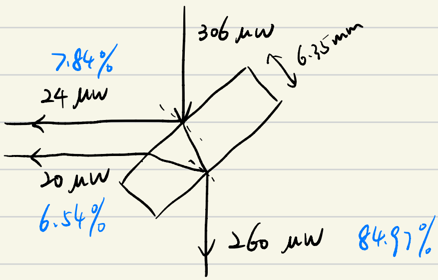
Choose Thorlabs: C610TME-A

EFL = 4.00 mm

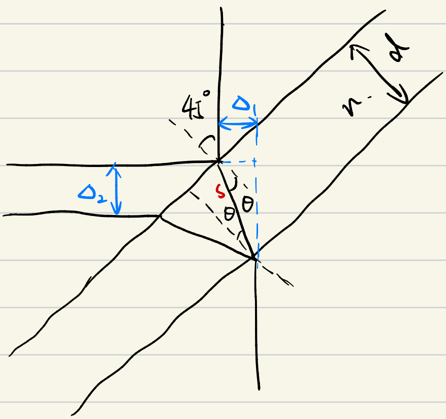
Glass: ECO-550. transimission rate @ 370 nm is higher than D-ZK3 > D-ZLaF52LA.

Window, uncoated:

Measured PW1-1025-UV (CVI Laser Optics)




$(260 + 24 + 20)/306 = 99.35\%$




$$\sin 45^\circ = n \sin\theta$$

$$s = d/\cos\theta$$

$$\Delta_1 = s \cdot \sin(45^\circ - \theta) = \frac{d}{\cos\theta} \cdot \sin(45^\circ - \theta)$$

$$\Delta_2 = s \cdot \sin(\theta) \cdot 2 \cdot \sin 45^\circ = \sqrt{2}\,\frac{d}{\cos\theta} \cdot \sin\theta = \sqrt{2}\, d \tan\theta$$

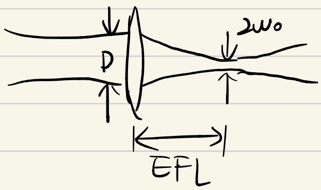
For UV-Fused sillica: $n = 1.4740$ @ 369 nm, $\phi\, 1'' \rightarrow d = 5$ mm

$$\Rightarrow \theta = \arcsin\left[\frac{\sin 45^\circ}{n}\right] = \arcsin\left(\frac{\sqrt{2}}{2 \cdot 1.4740}\right) = 28.6671^\circ$$

$$\Delta_1 = 1.60252 \text{ mm}$$

$$\Delta_2 = 3.86602 \text{ mm}$$

Focused beam size :




Diameter : $2w_0 = \frac{4M^2}{\pi} \cdot \frac{\lambda f}{D}$

where $M^2$ is the beam qulity parameter, normally 1

$\lambda$ : wavelength
$f$ : Effective focal length
$D$ : input beam diameter.

Focused diffraction limited spot size:

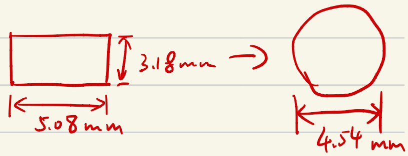
Diameter: $d_{lim} = 2 \cdot \frac{\lambda f}{D} = \frac{2\pi}{4M} \cdot (2w_0)$

$\approx 1.57 \cdot (2w_0)$

$d_{lim}$ : contains 99% beam power
$2w_0$ : $1/e^2$ beam size, contains 87.5% power

- O-ring:

slot:




Company: Globaloring and seal

Measured with Solidworks:

O-ring length − 5456.77 mm

Crosssection diameter $d = 2 \times \sqrt{\frac{3.18 \times 5.08}{\pi}} = 4.54\,mm$

Actually used dimension:

length − 5377.00 mm

( 80 mm shorter, 1.5% shorter)

diameter − $d = 4.5$ mm.