Trapped Ion Notes

By Ke Sun and Yichao Yu

I recommend downloading the latest version from the following link: https://GitHub.com/Ke-Sun96/Notes.git

$\boxed{\text{SPAM}}$ Error budget

a. $|0\rangle$ state
- PMT dark count. ①
- SPAM related
  - optical pumping ②
  - $|0\rangle \to |1\rangle$ leakage. ③

b. $|1\rangle$ state
- Gate error (μ-wave / Raman) ④
- SPAM related
  - optical pumping ②
  - $|1\rangle \to |0\rangle$ leakage. ③

a. Measure the overall error of detecting $|0\rangle$ state.

Method. – Calibrate pump / detection time

– Run the circuit repeatly: pump → detect, to get m points, each points contains n experiments. So the total sampling number $N = mn$.

– The average prob of these N samplings is the error

$$\varepsilon_{|0\rangle} = \frac{\Sigma P_{mn-|0\rangle}}{N}$$

b. Measure the overall error of detecting $|1\rangle$ state.

Method. – Calibrate pump / detection time (This should be the same as those when measuring $|0\rangle$ state error)

– Calibrate gate π-time

– Run the circuit repeatly: pump → π-gate (prep $|1\rangle$) → detect to get $N = mn$ samples.

– The error $\varepsilon_{|1\rangle} = 1 - \dfrac{\Sigma P_{mn-|1\rangle}}{N}$

① PMT dark count:

Method: - Run the EXACT same scan as measuring $|0\rangle$ error but only dumpping the ion.

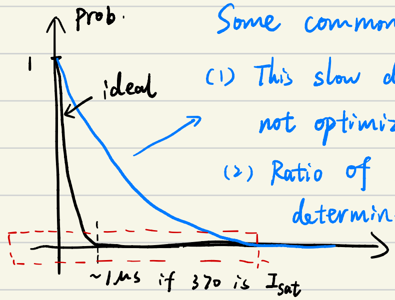
- The error $\varepsilon_{PMT} = \frac{\sum P_{\text{un-PMT}}}{N}$

If ε_{PMT} is large, we should check it in detail $\begin{cases} \text{turn off 370/355} \\ \text{unplug PMT} \\ \text{block PMT} \\ \text{etc.} \end{cases}$

② Optical pumping.

This is influenced by $\begin{cases} 370 \begin{cases} \text{polarization, frequency, power.} \\ \text{2.1 GHz intensity. frequency.} \end{cases} \\ 935 \begin{cases} \text{Power, polarization, freq} \\ \text{3.1 GHz sideband intensity, freq.} \end{cases} \end{cases}$

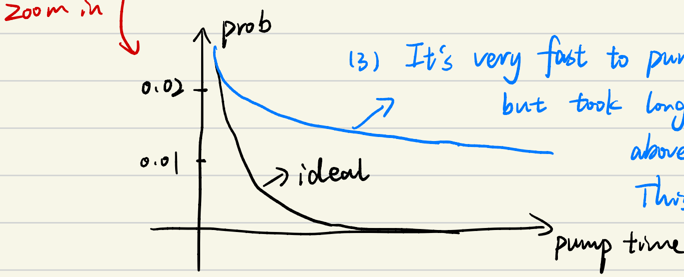
The typical way to examine: run pump time scan:



Some common problems:

(1) This slow decay indicates the 370 beam is not optimized.

(2) Ratio of 370 sideband (2.1GHz) and carrier determines the number of photons during OP. [carrier should be minimized]



(3) It's very fast to pump down to 0.01~0.02, but took long time to 0, or stays above 0.01 for >100 μs. This indicates that 935 beam has some problems.

③ Leakage error : $|1\rangle \to |0\rangle$

what can be measured

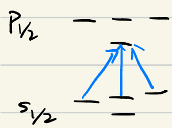
P_{det} : PMT count when detection on only

Method: Prep $|1\rangle$, measure with detection on for T_1, get the average phonon number from histogram. Change detection time ($T_2, T_3 \cdots$)

Repeat, get avg phonon number N, $P_{det} = \frac{N}{T}$.

τ : Decay rate when turning on detection beam after prep $|1\rangle$

T : Detection time.



Note: The detection beam off-resonately couples $|1\rangle \to |P_{1/2}, F=1\rangle$ (2.1 GHz) and $|0\rangle \to |P_{1/2}, F=1\rangle$ (14.7 GHz), which results in leakage as well as a dark state $\alpha|0\rangle + \beta|1\rangle$, where $|\alpha|^2 = a$, $|\beta|^2 = b$

Based on Rachel M. Noek's thesis (Duke), page 71.

The rate at which an ion initially in the $|1\rangle$ state will pump to $|0\rangle$ state when detection is

$$R_d \approx \left(\frac{2}{3}\right)\left(\frac{1}{3}\right)\left(\frac{\Gamma}{2}\right)\left(\frac{2\Omega^2}{\Gamma^2}\right)\left(\frac{\Gamma}{2\Delta_{HFP}}\right)^2$$

$\Delta_{HFP} = 2.1$ GHz.

The first factor of $\left(\frac{2}{3}\right)$ is due to the fact that one out of three states in the $^2S_{1/2}|F=1\rangle$ manifold is a coherent dark state at any given time.

The factor of $\left(\frac{1}{3}\right)$ is the branching ratio of the $^2P_{1/2}|F=1\rangle$ states decaying into $|0\rangle$ state.

The rate for $|0\rangle$ to pump into one of the bright $^2S_{1/2}|F=1\rangle$ states is

$$R_b \approx \left(\frac{2}{3}\right)\left(\frac{\Gamma}{2}\right)\left(\frac{2\Omega^2}{\Gamma^2}\right)\cdot\left(\frac{\Gamma}{2(\Delta_{HFP}+\Delta_{HFS})}\right)^2$$

$\Delta_{HFS} = 12.6$ GHz

The factor of $\left(\frac{2}{3}\right)$ is the branching ratio of the $^2P_{1/2}|F=1\rangle$ states decaying into $^2S_{1/2}|F=1\rangle$ states.

For $^{171}Yb^+$, $R_d/R_b = \frac{1}{3}\left(\frac{2.1+12.6}{2.1}\right)^2 = 16.3$.

Therefore, when applying DETCTION beam, the population of bright and dark state ($P_b(t)$ & $P_d(t)$) involves as follows.

$$\frac{dP_b}{dt} = -R_d \cdot P_b(t) + R_b \cdot P_d(t)$$

$$\frac{dP_d}{dt} = -R_b \cdot P_d(t) + R_d \cdot P_b(t)$$

At equilibrium, $\frac{dP_b}{dt} = \frac{dP_d}{dt} = 0$

$$\Rightarrow \begin{cases} \frac{P_b}{P_d} = \frac{R_b}{R_d} = \frac{1}{16.3} \\ P_b + P_d = 1 \end{cases} \Rightarrow P_b \approx 0.058.$$

Assume the solution is $P_b = c_1 + c_2 e^{\beta t}$, $P_d = (1-c_1) + c_3 e^{\beta t}$

$$\Rightarrow \begin{cases} c_2 \beta e^{\beta t} = -R_d \cdot (c_1 + c_2 e^{\beta t}) + R_b \cdot [(1-c_1) + c_3 e^{\beta t}] \\ c_3 \beta e^{\beta t} = -R_b [(1-c_1) + c_3 e^{\beta t}] + R_d (c_1 + c_2 e^{\beta t}) \end{cases}$$

$$\Rightarrow \begin{cases} -R_d c_1 + R_b (1-c_1) = 0 \\ c_2 \beta = -R_d c_2 + R_b c_3 \\ c_3 \beta = -R_b c_3 + R_d c_2 \end{cases} \Rightarrow \begin{cases} c_1 = \frac{R_b}{R_b+R_d} \approx 0.058 \\ c_2 = -c_3 = c, \\ \beta = -(R_b + R_d) \end{cases}$$

$\Rightarrow P_b(t) = 0.058 + c \cdot e^{-(R_b+R_d)t}$, where c is determined by the initial condition.

if $P_b(t=0) = 1$, $P_b(t) = 0.058 + 0.942\, e^{-(R_b+R_d)t}$ Prep bright

if $P_b(t=0) = 0$, $P_b(t) = 0.058 - 0.058\, e^{-(R_b+R_d)t}$ Prep dark.

For simplicity, Let's assign $a = 0.942$, $b = 0.058$, $\tau = \frac{1}{R_b+R_d}$

If prep bright, $P_b(t) = a e^{-t/\tau} + b$

To get the leakage error, we need to calculate how much error it will cause if the leakage happens.

Firstly, if the ion is at $|1\rangle$ state, the prob of detecting more than 1 photon (determine it's $|1\rangle$ state) during detection time t is

$$P_0(t) = a \cdot [1 - \exp(-P_{det} t)] + b$$

- The part that doesn't decay to $|0\rangle$ during T: $P_b(T) = a \exp(-\frac{T}{\tau}) + b$

Prob of measuring $|1\rangle$: $P_\alpha(T) = [a \exp(-\frac{T}{\tau}) + b] \cdot P_0(T)$

- The part that decays to $|0\rangle$ during T:

Prob that it still stays at $|1\rangle$ at t: $P_b(t) = a \exp(-\frac{t}{\tau}) + b$

Within $(t, t+dt)$, the portion that decays is

$$dp_{|1\rangle} = \frac{dP_{|1\rangle}}{dt} = -\frac{a}{\tau} \exp(-\frac{t}{\tau})\, dt$$

During detection time T, the prob we can still measure $|1\rangle$:

$$P_\beta(T) = \int_0^T \underbrace{|dP_{|1\rangle}(t)| \cdot P_0(t)}$$

$|dP_{|1\rangle}(t)|$

The detection success rate before it decays.

So, the prob that we can measure $|1\rangle$ is

$$P(T) = P_\alpha(T) + P_\beta(T)$$

For $|0\rangle \to |1\rangle$, Prob of getting $|1\rangle$ is $P_{|1\rangle} = -b\exp(\frac{-t}{\tau}) + b$

$$\frac{dP_{|1\rangle}}{dt} = \frac{b}{\tau} \exp(\frac{-t}{\tau})$$

Prob of measuring $|1\rangle$ state is

$$P(T) = \int_0^T |dP_{|1\rangle}(t)|\, P(T-t)$$

④ Gate error:

Two methods: (1) composed gates like SK1.

(2) Normal gates.

Method: Scan the gate number and fit the gate error

Assuming gate error is linear. Linear curve is enough.

(1) SK1 is robust to gate over-rotation error, but gate time is 5× longer.

(2) Normal gate is fast, but the over-rotation error becomes very significant when the gate number increases.

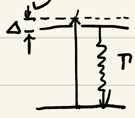
- To mitigate this error, it's important to use multiple gates to calibrate the gate time.

  eg. - Scan π-time with 1 gate first, get T_π, error σ_π

  - Then scan between $[100.5\, T_\pi,\ 101.5\, T_\pi]$ to get $T_{101\pi}$, the more precise π-time becomes $T_\pi' = \frac{T_{101\pi}}{101}$, error also decreases to $\sigma_\pi' = \frac{\sigma_\pi}{101}$

Doppler cooling

Doppler cooling uses a laser detuned to form an optical transition between two levels: Δ



$|e\rangle$

Γ : natural radiative linewidth of decay.

$|g\rangle$

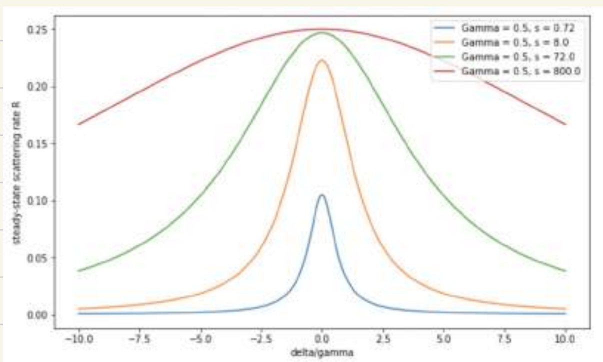
The photon scattering rate $\Gamma_s = \Gamma \cdot \rho_{ee} = \dfrac{s \cdot \frac{\Gamma}{2}}{1 + s + 4\frac{\Delta^2}{\Gamma^2}}$.

where $s = I/I_s$, $I_s = \pi h c / 3\lambda^3 \tau$ is the saturation intensity.

λ is the wavelength of light that resonants with the transition.

τ is the lifetime of excited state. $\tau = 1/\Gamma$.

Γ is the natural linewidth.



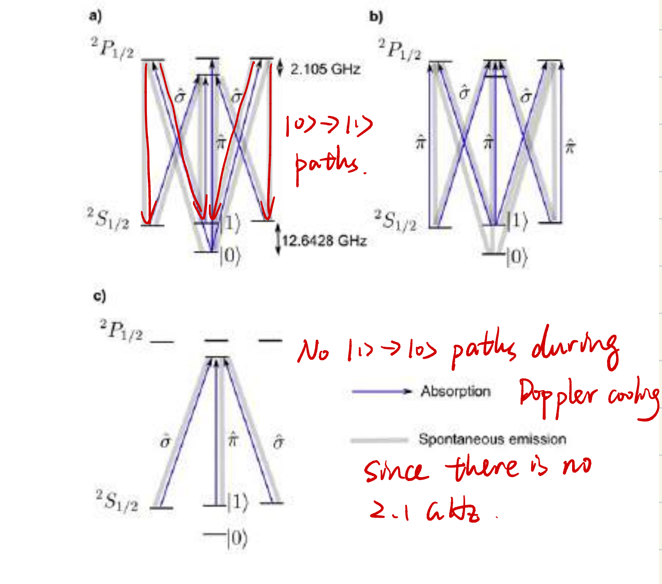


Figure 2.5: $^{171}Yb^+$ **Doppler cooling, optical pumping and state detection schemes.** The solid arrows indicate applied frequency and polarization components in the 369.5 nm beams and gray lines show the possible routes for spontaneous emission to the ground state manifold. a) Doppler cooling frequencies and polarization. b) Optical pumping frequencies and polarization required for qubit state preparation in $|0\rangle$, c) Frequency and polarization components for qubit state detection .

Doppler cooling:

Ensure every ion (both in $|0\rangle$ and $|1\rangle$ states) can be stimulated.

So we need:

$$\begin{cases} 1.\ 370\text{ nm (both } \hat{\pi} \ \& \ \hat{\sigma}) \\ 2.\ 370\text{ nm} + 14.7\text{ GHz}. \end{cases}$$

$\Rightarrow$ After Doppler cooling, the ion should be at a mixed state $\rho = \{p_0 : |0\rangle\langle 0|,\ p_1 |1\rangle\langle 1|\}$, with $p_1 \gg p_0$, i.e. $p_1 \approx 1$, $p_0 \approx 0$. Because the speed of $|1\rangle \to |0\rangle$ is much slower than $|0\rangle \to |1\rangle$. As a result, most transitions happens btw $\{^2S_{1/2}, F=1\} \Leftrightarrow \{^2P_{1/2}, F=0\}$ during Doppler cooling

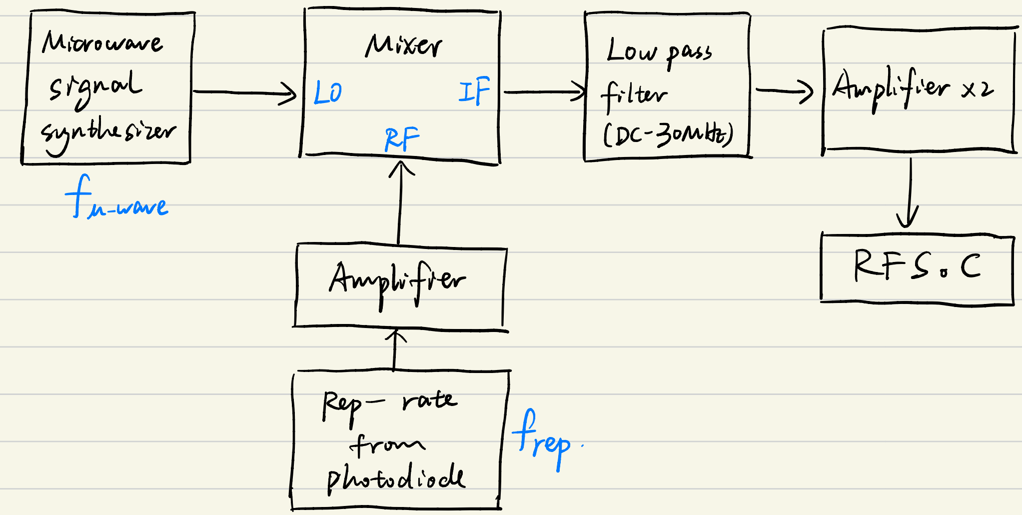
Doppler cooling limit : $k_B T \sim \hbar\Gamma/2$

$$\begin{cases} ^2P_{3/2} : \Gamma_2 = 2\pi \times 25.9\text{ MHz} \\ ^2P_{1/2} : \Gamma_1 = 2\pi \times 19.7\text{ MHz}. \end{cases}$$

$$n_{th} = \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1} = \frac{1}{e^{2\times 1.8/19.7} - 1} \approx 4.99.$$

Additionally, 0.5% $^2P_{1/2}$ is decayed to $^3D_{3/2}$. Therefore, a 935.2 nm & 932.5 nm + 3.07 GHz beam

Rep-rate lock



NCO mixer Frequency

Harmonic divider (Red labels are the names in RFsoC)

$$f_{diff} = f_{\mu\text{-wave}} - n_h \cdot f_{rep}$$

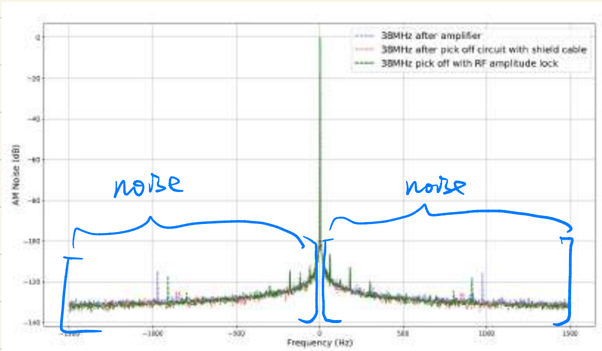
Detection Freq shift

The selection of $f_{\mu\text{-wave}}$ & n_h should make sure

1. $|f_{diff}|$ is the lowest freq among all harmonics.
2. The nearest harmonic can be suppressed by the LPF.
3. n_h is better to be 2^k (eg, 8, 32...)

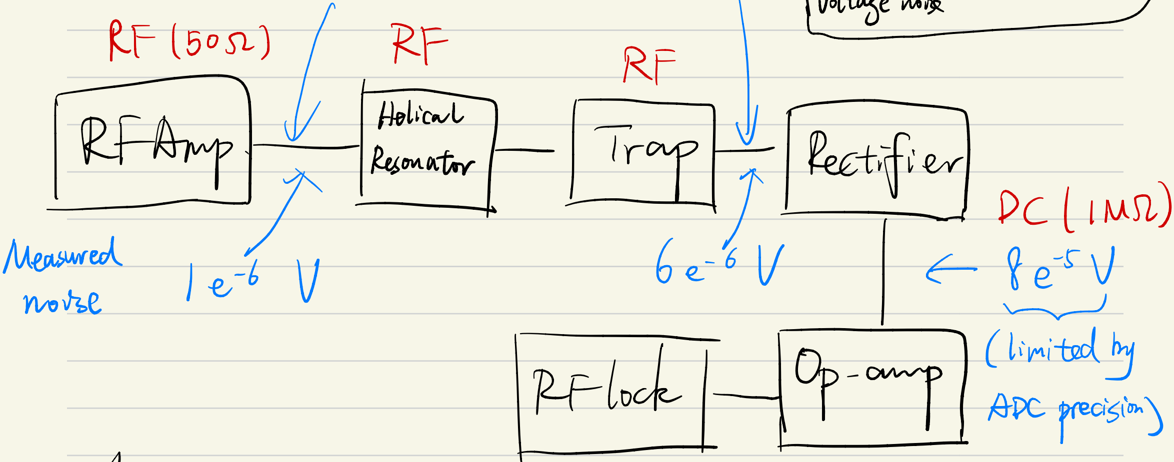
RF Lock circuit

Methods of measuring noise (V_{rms})

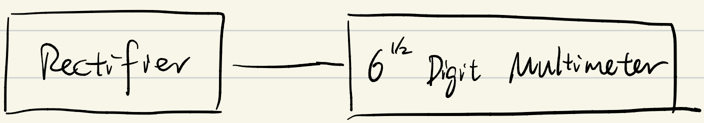


The bandwidth of this data is 1 Hz. So summing over all the spectrum other than the central 1 Hz will give us the noise level in dB. (x dB).

$$\boxed{\text{The rms voltage noise} = \sqrt{10^{\left(\frac{x}{10}\right)} \cdot 50\Omega}}$$



When

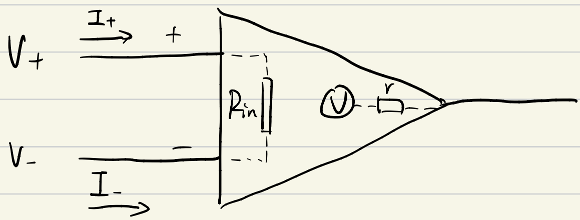


noise $V_{pp} \approx 20\ \mu V$ @ 600 kHz BW.

$$V_{rms} = \frac{V_{pp}}{6.6} \cdot \sqrt{\frac{50k}{600}} \approx 28\ \mu V$$

The only part that we can optimize is the op-amps after Rectifier.

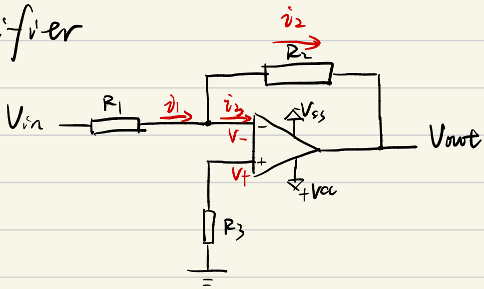
Op-amp:



Principles: Virtual short circuit: $V_+ \approx V_-$

Virtual open circuit: $I_+ = I_- = 0$

1. Inverting amplifier



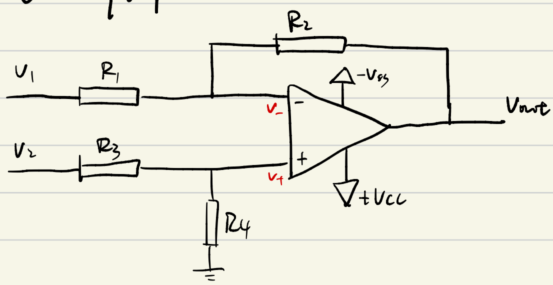
virtual open: $i_3 = 0$. $\Rightarrow i_1 = i_2$.

$$V_- = \frac{R_2}{R_1+R_2} \cdot V_{in} + \frac{R_1}{R_1+R_2} \cdot V_{out}$$

virtual short: $V_- = V_+ = 0\,V$

$$\Rightarrow \boxed{V_{out} = -\frac{R_2}{R_1} V_{in}}$$

2. Differential amplifier.



$$V_- = V_1 \cdot \frac{R_2}{R_1+R_2} + V_{out} \cdot \frac{R_1}{R_1+R_2}$$

$$V_+ = V_2 \cdot \frac{R_4}{R_3+R_4}$$

$$V_- = V_+$$

$$\Rightarrow \quad V_{out} \cdot \frac{R_1}{R_1+R_2} + V_1 \frac{R_2}{R_1+R_2} = V_2 \frac{R_4}{R_3+R_4}$$

$$V_{out} \cdot \frac{R_1}{R_1+R_2} = V_2 \frac{R_4}{R_3+R_4} - V_1 \frac{R_2}{R_1+R_2}$$

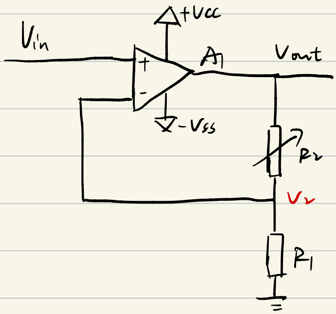
$$\boxed{V_{out} = \frac{R_1+R_2}{R_3+R_4} \cdot \frac{R_4}{R_1} \cdot V_2 - \frac{R_2}{R_1} V_1}$$

when $R_1 = R_3$, $R_2 = R_4$

$$\Rightarrow V_{out} = \frac{R_2}{R_1}(V_2 - V_1)$$

3. Non-inverting amplifier circuit.

(In general, it's only used to amplify the DC voltage).



$$
\begin{cases} V_2 = \dfrac{R_1}{R_1 + R_2} \cdot V_{out} \\ V_2 = V_{in} \end{cases}
$$

$$
\Rightarrow \quad V_{out} = \left(1 + \frac{R_2}{R_1}\right) V_{in}
$$

RF lock noise analysis.

Noise figure (NF)

$$NF\,(dB) = 10 \log (F)$$

$$F = \frac{SNR_I}{SNR_O}$$ → input SNR → output SNR.

Output signal $S_O = S_I \times G$

Input signal

Gain

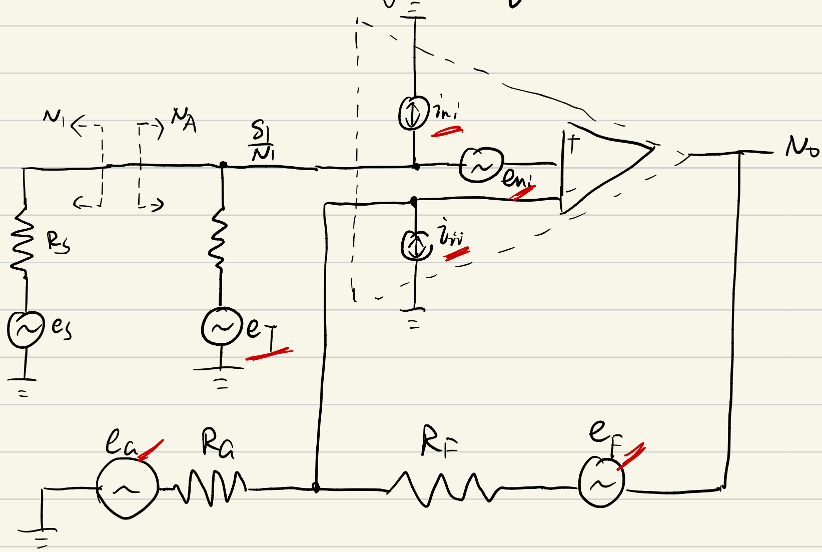
Output noise $N_o = (N_I + N_A) \times G$

noise from input

noise of the device

$$\Rightarrow F = \frac{SNR_I}{SNR_O} = \frac{S_I / N_I}{S_o / N_o} = 1 + \frac{N_A}{N_I}$$

1. Non-inverting noise analysis diagram



Input noise: voltage noise: $\sqrt{4kTR_S}$

Adding terminating resistor: R_T $(= R_S)$

$$N_1 = 4kTR_S\left(\frac{R_T}{R_S+R_T}\right)^2 = kTR_S$$

Device noise: $N_A = C_1 e_{ni}^2 + C_2 i_{ni}^2 + C_3 i_{vi}^2 + C_4 e_T^2 + C_5 e_a^2 + C_6 e_F^2$

where C_i are scaling factors.

- $C_1 = 1$ b.c. voltage directly at the amplifier's input

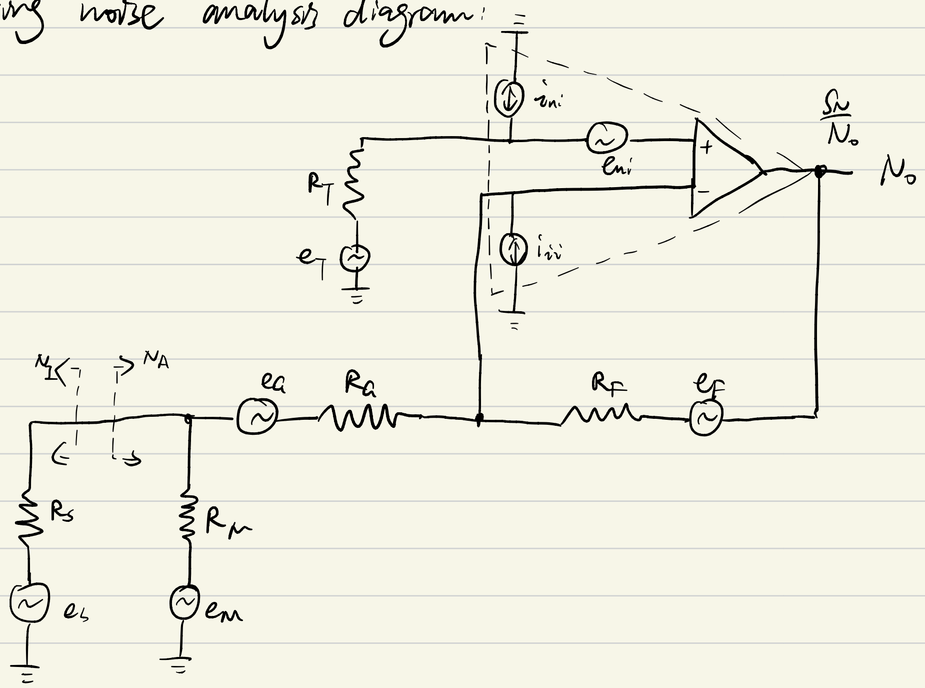
- $C_2 i_{ni}^2 = i_{ni}^2\left(\frac{R_S R_T}{R_S+R_T}\right)^2$

- $C_3\ \bar{i}_{ii}^2 = \bar{i}_{ii}^2 \left(\dfrac{R_F R_a}{R_F + R_a}\right)^2$
- $C_4\ e_T^2 = 4kTR_T \left(\dfrac{R_s}{R_s + R_T}\right)^2 \xlongequal{R_s = R_T} kTR_T$
- $C_5\ e_a^2 = 4kTR_a \cdot \left(\dfrac{R_F}{R_F + R_a}\right)^2$
- $C_6\ e_F^2 = 4kTR_F \left(\dfrac{R_a}{R_F + R_a}\right)^2$

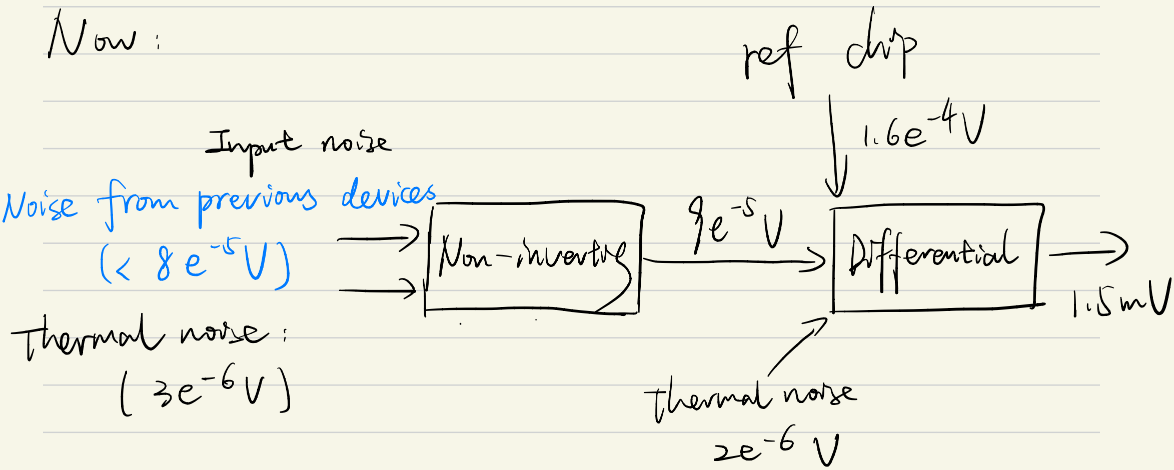
Unit: kTR : $eV \cdot \Omega = C \cdot V \cdot \dfrac{V}{I} = C \cdot V \cdot \dfrac{V}{C/s} = V^2 \cdot s$

e^2 : V^2

Inverting noise analysis diagram:

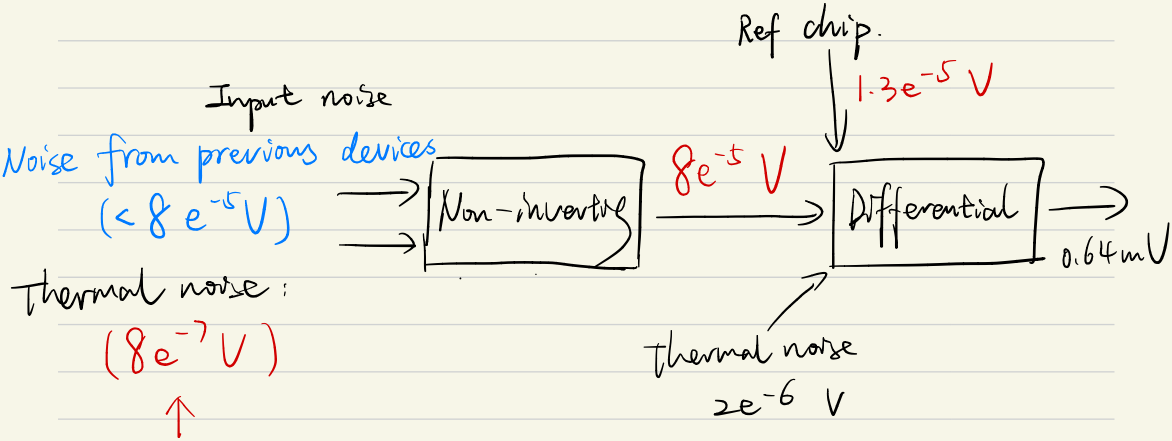


Now:



NSR = 4.09×10^{-5}

Optimize: { ref chip: change to a low-noise one (V_{rms}: $1.6e^{-4}V \to 1.3e^{-5}V$)
Resistors: Reduce thermal noise



By changing the resistance of R_1 and R_2

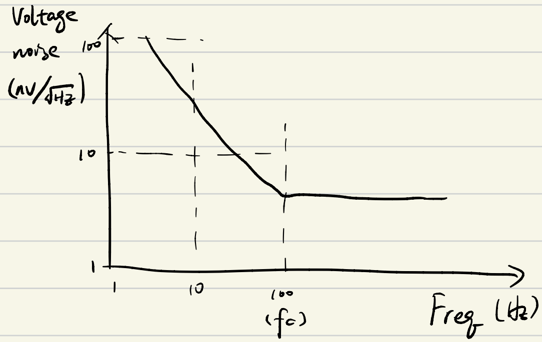
NSR = 1.76×10^{-5}

Tips:

1. Detailed optimizations and calculation codes can be found in github.com/Ke-Sun96/Ion-simulation, under the "RF_lock-noise" folder.

2. Calculate the input noise of a device based on datasheet.

Input noise calculation:



eg: $V_n = \frac{100}{\sqrt{f}}$ $\quad f < 10k$

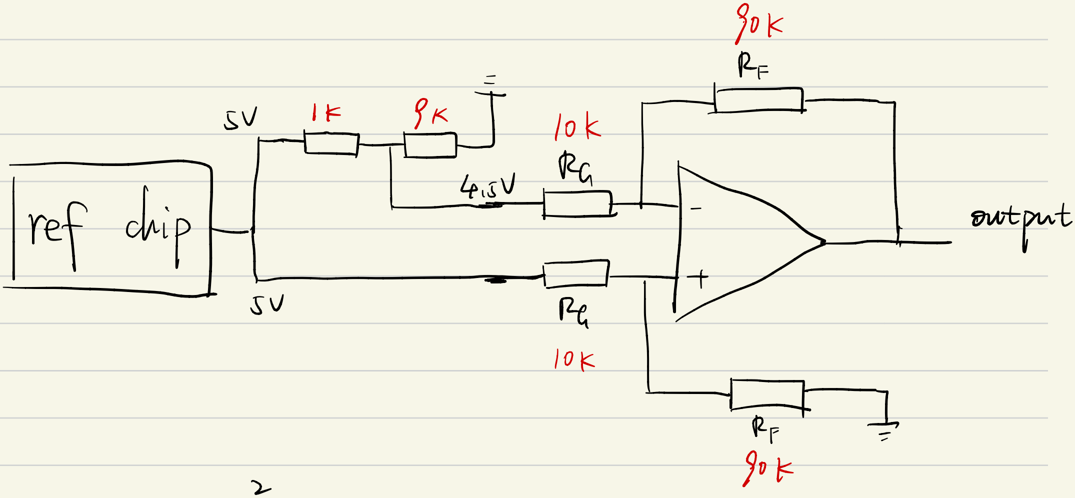
$V_n = 2.7$ $\quad f > 10k$

Integral: $f < 10k$ $\quad V_{nI\_1}^2 = \int_1^{10000} \left(\frac{100}{\sqrt{f}}\right)^2 df = 92103.4\ nV^2$

$f > 10k$ $\quad V_{nI\_2} = \int_{10000}^{50000} (2.7)^2 df = 291600\ nV^2$

$$V_{nI} = \sqrt{V_{nI-1}^2 + V_{nI-2}^2} = 619.438\ nV$$

3. Measure the noise of a Ref chip by yourself.



$$\left(V_{noise-out}\right)^2 = \left(V_{chip}^2 + \left(\frac{4.5}{5} \cdot V_{chip}\right)^2\right) \times gain$$

$$V_{noise-out} = 4.03 \cdot V_{chip}$$

$$\Rightarrow V_{chip} = \frac{V_{noise-out}}{4.03} = 0.027\ mV.$$

measured value: 0.108 mV

$\boxed{\text{DC solutions}}$

Mode frequency:

The potential $\phi = ax^2 + by^2 + cz^2$. (general expression)

the coefficients a, b, c contains both DC & RF terms.

$$\begin{cases} a = a_{DC} + a_{RF} \\ b = b_{DC} + b_{RF} \\ c = c_{DC} + c_{RF} \end{cases}$$

According to Maxwell equation: $\nabla^2 \phi = 0$

$$\nabla^2 (a_{DC} x^2 + b_{DC} y^2 + c_{DC} z^2) = 0$$

$$\Rightarrow a_{DC} + b_{DC} + c_{DC} = 0$$

$$\Rightarrow a + b + c = a_{RF} + b_{RF} + c_{RF}$$

In a particular case, where $\hat{x}$-direction has no confinement.

$a_{RF} = 0$, $b_{RF} = c_{RF} = \frac{1}{2} m w_{sec}^2$, where w_{sec} is the secular frequency.

$$\Rightarrow a + b + c = 2 \cdot \frac{1}{2} m w_{sec}^2$$

On the other hand, $a = \frac{1}{2} m w_x^2$, $b = \frac{1}{2} m w_y^2$, $c = \frac{1}{2} m w_z^2$

$$\Rightarrow w_x^2 + w_y^2 + w_z^2 = 2 \cdot w_{sec}^2$$

Mathematical derivation (Radial direction)

Potential with Rotation (yz) and squeeze (y^2-z^2)

$$\phi = a(y^2+z^2) + \underbrace{2b\,yz}_{\text{Rotation}} + \underbrace{c(y^2-z^2)}_{\text{Squeeze}}$$

$$= (y,\ z)\,\underbrace{\left(a\hat{I} + b\hat{\sigma}_x + c\hat{\sigma}_z\right)}\begin{pmatrix} y \\ z \end{pmatrix}$$

$$= \begin{pmatrix} a+c & b \\ b & a-c \end{pmatrix}$$

$\downarrow$ diagonalize.

Eigen values:

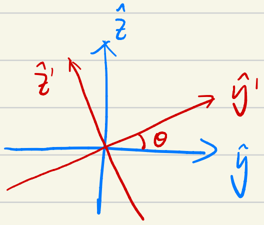
$$w_1^2 = a - \sqrt{b^2+c^2}$$

$$w_2^2 = a + \sqrt{b^2+c^2}$$

Eigenvectors

$$v_1 = \left\{ \frac{c-\sqrt{b^2+c^2}}{b},\ 1 \right\}$$

$$v_2 = \left\{ \frac{c+\sqrt{b^2+c^2}}{b},\ 1 \right\}.$$

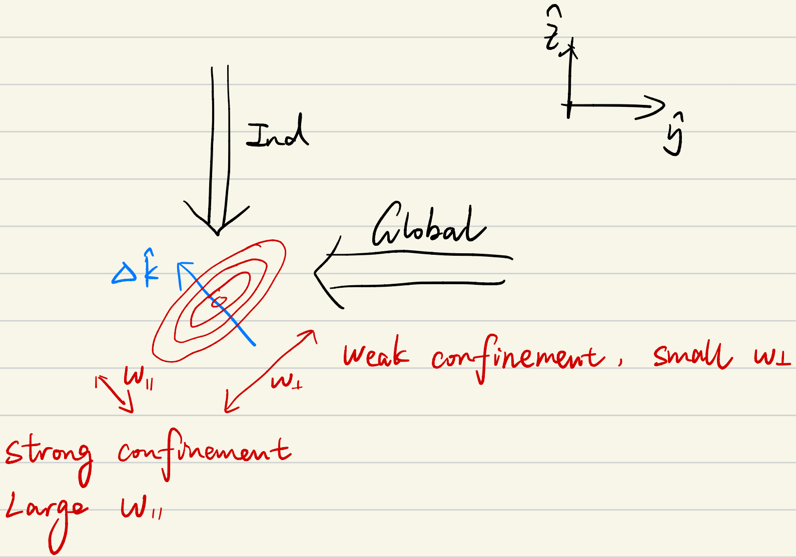


$$\tan\theta = \frac{b}{c+\sqrt{b^2+c^2}}$$

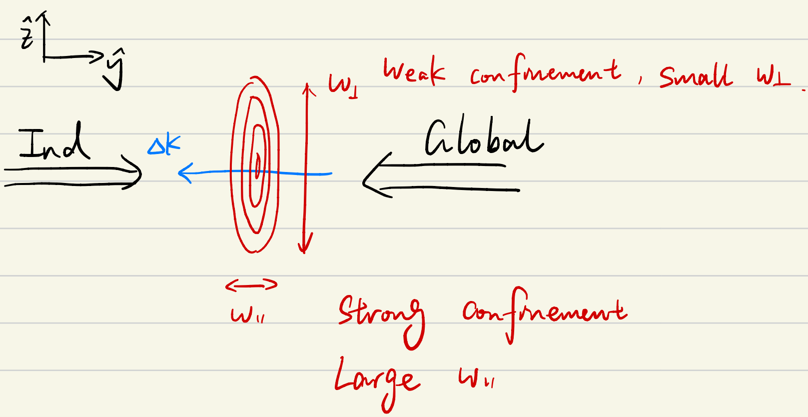
As long as $c=0$, $\tan\theta = 1$, $\theta = 45°$

Or $b \gg c$, $\tan\theta \approx \dfrac{1}{1+\frac{c}{b}+\frac{c^2}{2b^2}} \approx 1-\frac{c}{b}-\frac{c^2}{2b^2}$

In red chamber:



In Cryo:



We want $\tan\theta = 0 \Rightarrow b = 0$.

Only squeeze is required, no rotation is required.

Calibration steps:

1. Adjust X1 (x comp) to move the DC null point, s.t. when
A little bit Y1 & Z1.
adjusting Trap solution the ion doesn't move.

— By the end of this step, "Trap solution" will be independent to the X1 (Y1 & Z1 are hard b.c. you need to adjust Y1 & Z1 for μ-motion)

For Red chamber: 2. Adjust Rotation (YZ):

① When tweaking the "SimpleRot", the ion will move. This is because at DC null point, it's indep. to X1, but not necessarily to YZ. So we need to either

a. Quantify how much "Simple Rot" is related to "Y comp" and "Z comp", eg. α·"Simple Rot" + β"Y comp" + γ"Z comp" makes sure the ion goes back to the original position (PMT counts stay the same). Then define a new rotation operation "Rot_new" = α"simpleRot" + β"Y comp" + γ"Z comp"

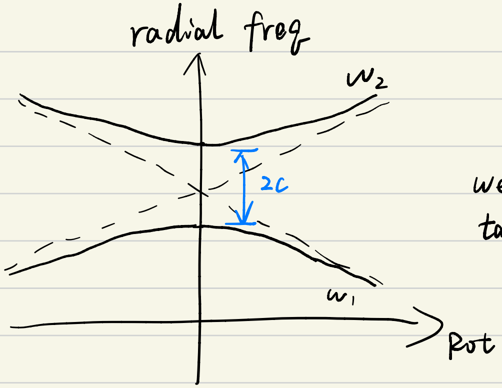
or b. Adjust "SimpleRot", the ion will move away from DC null, Adjust "Y comp" and "Z comp" to move the ion back

② Scan the sideband frequency.

③ Repeat ① & ② to get desired radial mode frequency.
Normally, the desired SB freq is to make sure the Raman only couples to one of the two directions, whichever has higher frequency.

3. Adjust Squeeze ($y^2 - z^2$)

When scanning Rotation, one will find the following relation:



By adjusting squeezing

we want to make sure $c=0$, s.t. $\tan\theta = 1$, $\theta = 45°$.

Recall: Eigenvalues:

$$w_1^2 = a - \sqrt{b^2+c^2}$$

$$w_2^2 = a + \sqrt{b^2+c^2}$$

Eigenvectors

$$v_1 = \left\{ \frac{c-\sqrt{b^2+c^2}}{b},\ 1 \right\}$$

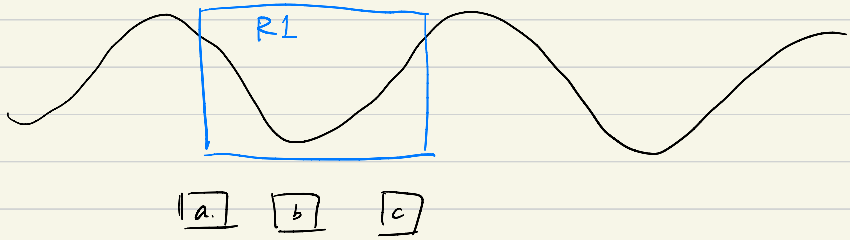
$$v_2 = \left\{ \frac{c+\sqrt{b^2+c^2}}{b},\ 1 \right\}.$$

$$\tan\theta = \frac{b}{c+\sqrt{b^2+c^2}}$$

* Important: Step 2 & 3 only applies for Red chamber since $\angle(\Delta\vec{k}, \vec{B}) = 45°$

For Cryo and STAQ, $\angle(\Delta\vec{k}, \vec{B}) = 0°$, the influence of Rotation and Squeeze are flipped. But it's easy to analyze.

Ideas of generating DC solutions.



1. (1) Calculate the potential generated by electrode α ($\alpha = a, b, c$) in R1, and get V_α.

(2) Fit $V_\alpha = \sum_{ijk} C_{ijk}^{(\alpha)} x^i y^j z^k$. Store $C_{ijk}^{(\alpha)}$

2. Fit: Desired potential V.

$$V = \sum_\alpha b_\alpha V_\alpha$$

$$= \sum_\alpha b_\alpha \sum_{ijk} C_{ijk}^{(\alpha)} x^i y^j z^k = \sum_{ijk} \left(\sum_\alpha b_\alpha \cdot C_{ijk}^{(\alpha)} \right) x^i y^j z^k$$

$$= \sum_{ijk} \beta_{ijk} x^i y^j z^k$$

Def: $\#_i \cdot \#_j \cdot \#_k = m$

$\#_\alpha = n$

$$\Rightarrow \sum_\alpha b_\alpha C_{ijk}^{(\alpha)} = \beta_{ijk}$$

$$\Rightarrow \begin{pmatrix} & & \\ & C & \\ & & \end{pmatrix}_{m \cdot n} \begin{bmatrix} b \end{bmatrix}_{n \times 1} \simeq \begin{bmatrix} \beta \end{bmatrix}_{m \times 1}$$

Normally $m \geq n$. over-determined.

Step 1. Find b_0 s.t. $C \cdot b_0 \simeq \beta$

2. Find b_i's s.t. $C \cdot b_i = 0$

3. $b = b_0 + \sum_i t_i \cdot b_i$ is also a solution for

$$\hat{C} \cdot b \simeq \hat{\beta}.$$

4. Optimize t_i s.t. the maximum absolute value of b is the smallest

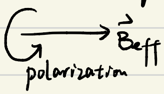
maximum voltage on the electrode.

Co-propagation

An easy way to determine the polarization that is needed to drive co-prop transition is as follows:

necessary condition.

The laser with circular polarization will generate an effective magnetic field $\vec{B}_{eff}$ of which the direction follows the right hand rule:



Actually, the direction of $\vec{B}_{eff}$ is $\vec{\varepsilon}_{B_{eff}} = \vec{\varepsilon}_E^* \times \vec{\varepsilon}_E$.

Thus, if the laser is linear polarization, $\vec{\varepsilon}_E^* \times \vec{\varepsilon}_E = 0$

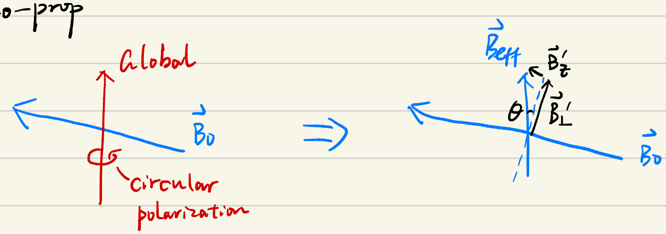
If it's circularly polarized, $\vec{\varepsilon}_E = [a_1\cos(kz-\omega t), i a_2 \sin(kz-\omega t), 0]$

$$\vec{\varepsilon}_E^* \times \vec{\varepsilon}_E = \begin{pmatrix} \vec{e}_x & \vec{e}_y & \vec{e}_z \\ \alpha^*(t) & \beta^*(t) & 0 \\ \alpha(t) & \beta(t) & 0 \end{pmatrix} = (\alpha^*\beta - \alpha\beta^*)\,\vec{e}_z = i a_1 a_2 \sin[2(kz-\omega t)]\,\vec{e}_z$$

*Ref: Madjarov, Ivaylo Sashkov Thesis, Caltech, Sec. 2.3.2.

Red chamber

Global co-prop



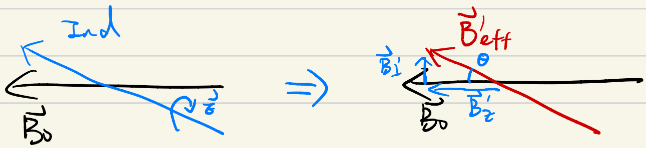
$\vec{B}_z'$ has the same effect of the static $\vec{B}_0$, driving $|0,0\rangle \to |1,0\rangle$

$\vec{B}_\perp'$ can drive the transition $|0,0\rangle \to |1,-1\rangle$ & $|1,+1\rangle$

$\vec{B}_\perp' = \vec{B}_{\sigma+}' + \vec{B}_{\sigma-}'$. $|\vec{B}_\perp'|^2 = |\vec{B}_{\sigma+}'|^2 + |\vec{B}_{\sigma-}'|^2$

$$|\tan\theta| = \frac{B_z'}{B_\perp'} = \frac{\mu B_z'}{\mu B_\perp'} = \frac{\Omega_{|0,0\rangle \to |1,0\rangle}}{\sqrt{2}\,\Omega_{|0,0\rangle \to |1,-1\rangle}}$$

For Individual beam (also apply to Global beam in Purple system)



The $|0,0\rangle \xrightarrow{\vec{B}'_z} |1,0\rangle$ transition should be faster than $|0,0\rangle \xrightarrow{\vec{B}'_\perp} \begin{cases} |1,-1\rangle \\ |1,+1\rangle \end{cases}$

At non-linear polarization, we can get

$$|\tan\theta| = \frac{\sqrt{2}\,\Omega_{|0,0\rangle \to |1,-1\rangle}}{\Omega_{|0,0\rangle \to |1,0\rangle}}$$

Frequencies for Co-prop:

Global: $\underbrace{\text{Tone 0}: f_0, \quad \text{Tone 1}: f_1}_{\text{Aom freq settings.}}$

$$f_1 - f_0 + n_{rep} \cdot f_{rep} = f_{Target}$$

$$\Rightarrow f_1 = f_0 + f_{Target} - n_{rep} \cdot f_{rep}.$$

f_{rep}: rep-rate $\begin{cases} \text{measured – no fb.} \\ \text{set : with feedback} \end{cases}$

f_1: Scan this freq.

f_0: Set as a constant

f_{Target}: Can be $\begin{cases} f_{HF} \\ f_{HF} \pm f_{Zeeman} \end{cases}$

n_{rep}: multiple of rep-rate, need to be correctly set in the RFsoC, normally it should be a negtive value

Individual: double pass in upstream

$$2f_1 - 2f_0 + n_{rep} \cdot f_{rep} = f_{Target}$$

$$\Rightarrow f_1 = f_0 + \tfrac{1}{2} f_{Target} - \frac{n_{rep}}{2} f_{rep}$$

Note that when setting the # of Harmonic for fb in RFsoC, the value should be $\frac{-n_{rep}}{2}$.

Counter-prop Raman transition

1. Freq: $f_{ind} - f_{global} + n_{rep} \cdot f_{rep} = f_{HF}$

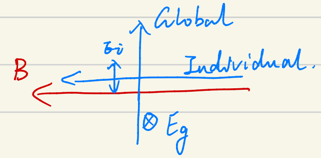
$f_{global} \to f_g$

For Red chamber $(2 f_{ind} + f_{shutter})$ $n_{rep} = 104$ $f_{rep} \sim 117.801$ MHz.

$f_{ind} = 195.5$ MHz. $f_{shutter} = 210$ MHz. $f_g \sim 209.516$ MHz

2. Adjust the individual & global waveplates to make sure they follow lin-perp-lin configuration.

For Red chamber:



① Use PBS to ensure Eg is linear and vertical.

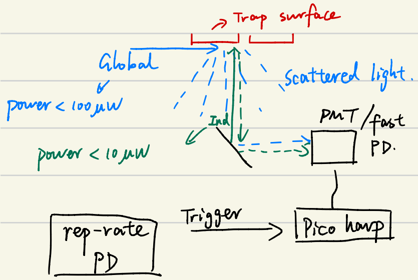
② Use individual co-prop to ensure that ind beam is linear (minimize co-prop Rabi frequency.

Optional: Use global co-prop to optimize QWP of global further more (Minimize global co-prop Rabi freq)

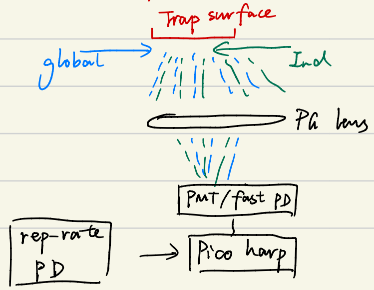
③ Drive counter-prop, Rotate HWP of individual to maximize counter-prop Rabi freq.

3. Beam path match.

For Red chamber:



For Purple chamber:



Tips of using Picoharp:

1. PMT / PD should be fast enough (fast than f-reprate)
2. Important: Add sufficient proper filters before the PMT / PD.
3. PMT has readout delay (ns) if the optical power is too large. ⇒ Need to adjust the power settings and make sure it's small enough

Raman transition theory

1. Pulsed laser

- Why not use cw laser?

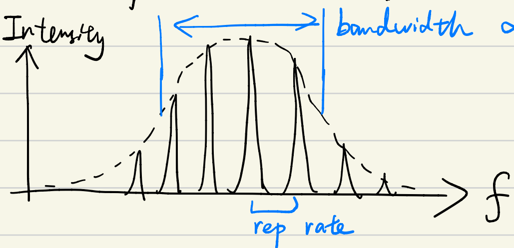
  A: We need two beams separated by 12.6 GHz, one could modulate that by AOM or EOM, but both of which is difficult to implement at the required freq, due to the limited bandwidth of these devices.

- Pulsed laser: pulse duration $\sim 10\ ps$.

  full bandwidth $\delta_{BW} = 200\ GHz$.

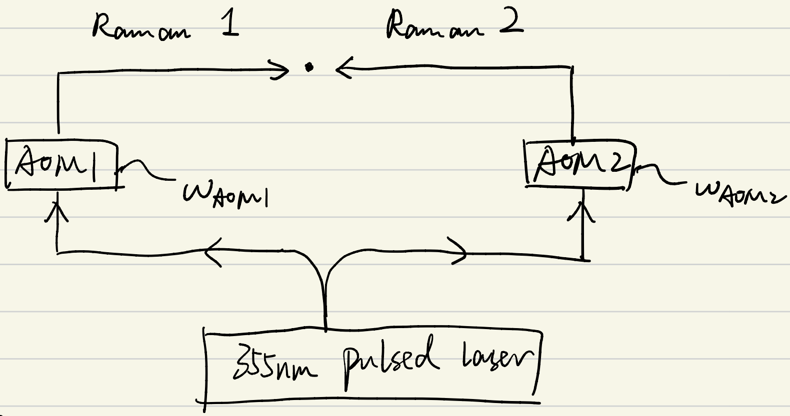
Frequency domain.

bandwidth $\propto \frac{1}{\text{pulse duration}}$



- It's not straightforward to implement Raman transitions:
  - a) the central wavelength of the laser (355 nm) couples the ground state to $^2P_{1/2}$ & $^2P_{3/2}$ state with a detuning of $\Delta = 33\ THz$ and $\Delta = -66\ THz$, respectively.
  - b) In the freq comb regime there are multiple frequency components in the optical spectrum that can potentially drive resonant and off-resonant Raman trancitions between the qubit levels.

2. Freq-comb picture:



E-field at ion:

$$\vec{E}_i(t) = \vec{E}_{0i} \sum_{n=0}^{N} f(t-nT)\, e^{-i(\omega_c + \omega_{AOM1})t + \phi}$$

$(i = 1, 2)$

where $f(t)$ is the pulse shape in time domain.

T is the time separation between pulses.

ω_c is the center freq of 355 laser

ϕ is the spatially dependent phase of the field.

$\Rightarrow$ Freq domain: freq combs separated by $\omega_r = 2\pi/T$

(Assuming large N)

$$E = E_0 \sum_{j=-\infty}^{\infty} \bar{f}(j\omega_r)\, e^{-i(j\omega_r + \omega_c + \omega_{AOM1})t}\, e^{i(k_j^0 x + \phi_0)}$$

where $\bar{f}(\omega)$ is the Fourier trans of pulse shape $f(t)$

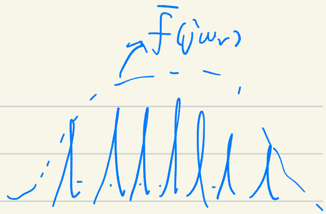
k_j^0 is the wave vector for j-th freq component.

The E field at ion will be:

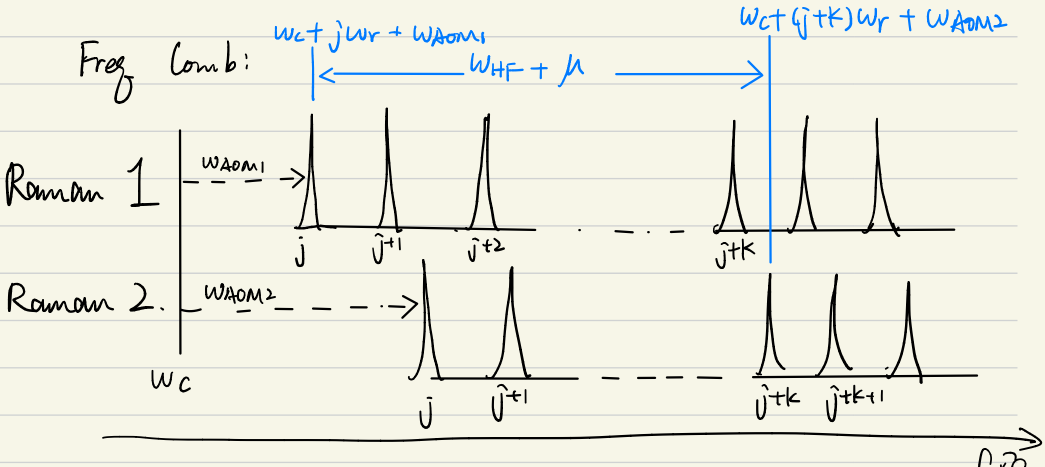
$$E_T = \bar{E}_0 \sum_{j=-\infty}^{+\infty} a_j e^{-i(j\omega_r + \omega_c + \omega_{AOM1})t} + \bar{E}_1 \sum_{j=-\infty}^{\infty} b_j e^{-i(j\omega_r + \omega_c + \omega_{AOM2})t}$$

$$a_j = \bar{f}(j\omega_r)\, e^{i(k_j^0 \cdot x + \phi_0)}$$ where

$$b_j = \bar{f}(j\omega_r)\, e^{i(k_j^1 x + \phi_1)}$$



Freq Comb:



$\omega_r \approx 119\,\text{MHz}$. $\omega_{HF} = 12.6\,\text{GHz}$

3.1 State evolution: (Not considering pulse laser)

Details see S. Debnath's paper.

$$i\begin{pmatrix} \dot{C}_0(t) \\ \dot{C}_1(t) \end{pmatrix} = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} C_0(t) \\ C_1(t) \end{pmatrix}$$

Hermitian requires $\beta = \gamma^*$

Assumptions: $k\omega_r + \omega_{AOM2} - \omega_{AOM1} = \omega_{HF} + \mu$.

$$\underset{\text{kHz}}{\mu} \ll \underset{\text{MHz}}{\omega_r}\,,\ \omega_{AOM1,2} \ll \underset{\text{GHz}}{\omega_{HF}} \ll \underset{\text{THz}}{\Delta}$$

$$\Rightarrow \alpha = \frac{1}{4}\left(|\bar{d}_{oe}\cdot\bar{E}_0|^2 \sum_j \frac{|a_j|^2}{\Delta + j\omega_r + \omega_{Aom1}} + |\bar{d}_{oe}\cdot\bar{E}_1|^2 \sum_j \frac{|b_j|^2}{\Delta + j\omega_r + \omega_{Aom2}} \right)$$

$$\delta = \frac{1}{4}\left(|\bar{d}_{ie}\cdot\bar{E}_0|^2 \sum \frac{|a_j|^2}{\Delta - \omega_{HF} + j\omega_r + \omega_{Aom1}} + |\bar{d}_{ie}\cdot\bar{E}_1|^2 \sum_j \frac{|b_j|^2}{\Delta - \omega_{HF} + j\omega_r + \omega_{Aom2}} \right)$$

Taylor expansion in terms of $\frac{j\omega_r}{\Delta}$, $\frac{\omega_{HF} + j\omega_r}{\Delta}$

$$\Longrightarrow$$

$$\alpha = \frac{1}{4}\Big[|\bar{d}_{oe}\bar{E}_0|^2 \frac{1}{\Delta} \underbrace{\sum_j |a_j|^2}_{=1} \left(1 + \frac{j\omega_r}{\Delta} + \frac{\omega_{Aom1}}{\Delta}\right)^{-1}$$

$$+ |\bar{d}_{oe}\bar{E}_1|^2 \frac{1}{\Delta} \underbrace{\sum_j |b_j|^2}_{=1} \left(1 + \frac{j\omega_r}{\Delta} + \frac{\omega_{Aom2}}{\Delta}\right)^{-1} \Big]$$

$$= \frac{1}{4}\Big[|\bar{d}_{oe}\cdot\bar{E}_0|^2 \frac{1}{\Delta}\Big(1 - \cancel{\frac{\omega_{Aom1}}{\Delta}}^{\,0} - \cancel{\underbrace{\sum_{j=-\infty}^{+\infty} |a_j|^2 \frac{j\omega_r}{\Delta}}}^{\,0}\Big)$$

$$+ |\bar{d}_{oe}\bar{E}_1|^2 \frac{1}{\Delta}\Big(1 - \cancel{\frac{\omega_{Aom2}}{\Delta}}^{\,0} - \cancel{\sum_{j=-\infty}^{\infty} |b_j|^2 \frac{j\omega_r}{\Delta}}^{\,0}\Big)\Big]$$

$\Leftarrow \left\{ \sum_{j=-\infty}^{\infty} j\omega_r = 0, \quad |a_j| \approx |a_{j+1}| \approx \cdots \approx |a_{j+k}| \right.$

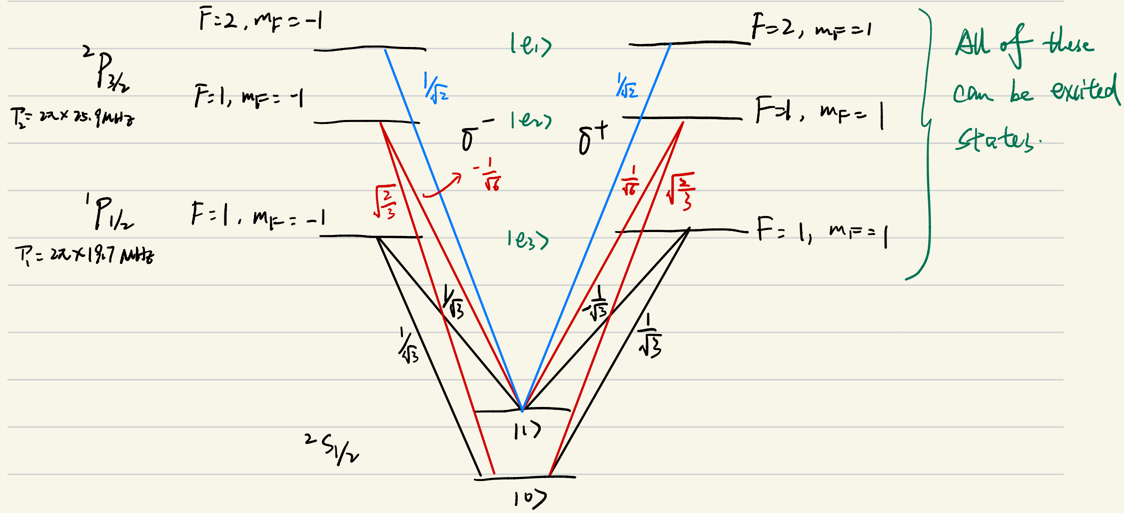
$$\Rightarrow \quad \alpha = \frac{1}{4}\left(|\bar{d}_{oe}\cdot\bar{E}_0|^2 + |\bar{d}_{oe}\cdot\bar{E}_1|^2\right)\frac{1}{\Delta}$$

Similarly, $\delta = \frac{1}{4}\left(|\bar{d}_{ie}\cdot\bar{E}_0|^2 + |\bar{d}_{ie}\cdot\bar{E}_1|^2\right)\frac{1}{\Delta}\left(1 + \frac{\omega_{HF}}{\Delta}\right)$

$$\beta = \frac{e^{i\mu t}}{4}\left(\bar{d}_{ie}^{*}\cdot\bar{E}_1^{*}\right)\left(\bar{d}_{oe}\cdot\bar{E}_0\right)\sum_j a_j b_{j+k}^{*} \frac{1}{\Delta + j\omega_r + \omega_{Aom1} + \mu}$$

$$\gamma = \beta^{*}$$

3.2. Raman transitions via multiple excited states.



$$d_{0e_1}\cdot \bar{E}_0 = \underbrace{\sqrt{I_0}}_{I_0=|E_0|^2 \text{ intensity}} \ \underbrace{Cg(0,e_1)}_{\text{CG coeff}} \ \underbrace{(\hat{a}_0\cdot\hat{\sigma}_\pm)}_{\text{unit vector of E-field}} \ \frac{\Gamma}{\sqrt{2I_{sat}}}$$

Γ → natural radiative linewidth

I_{sat} → saturation intensity.

$I_{sat} = \pi hc/3\lambda^3\tau$

$\tau = 1/\Gamma$

$$\Rightarrow \alpha = \frac{1}{4\Delta_1}\left(\frac{1}{3}\right)(I_0+I_1)\frac{\Gamma_1^2}{2I_{sat1}} + \frac{1}{4\Delta_2}\left(\frac{2}{3}\right)(I_0+I_1)\frac{\Gamma_2^2}{2I_{sat2}}$$

$$\delta = \frac{1}{4\Delta_1}\left(1+\frac{\omega_{HF}}{\Delta_1}\right)\left(\frac{1}{3}\right)(I_0+I_1)\frac{\Gamma_1^2}{2I_{sat1}} + \frac{1}{4\Delta_2}\left(\frac{2}{3}\right)\left(1+\frac{\omega_{HF}}{\Delta_2}\right)(I_0+I_1)\frac{\Gamma_2^2}{2I_{sat2}}.$$

$$\beta = \frac{\sqrt{I_0 I_1}}{12}e^{i\mu t}\left(\frac{\Gamma_1^2 D_{k,\Delta_1}}{I_{sat1}} - \frac{\Gamma_2^2 D_{k,\Delta_2}}{I_{sat2}}\right)\left(-\sigma_{0+}\sigma_{1+}^* + \sigma_{0-}\sigma_{1-}^*\right)$$

where $\Delta_1 = 33$ THz, $\Delta_2 = -67$ THz are detuning of 355 nm from $^2P_{1/2}$ and $^2P_{3/2}$ states. $D_{k,\Delta_i} = \sum_j \frac{a_j b_{j+k}^*}{\Delta_i + j\omega_r + \omega_{AOM1}}$, $E_i = \sqrt{I_i}\left(\sigma_{i+}\hat{\sigma}_+ + \sigma_{i-}\hat{\sigma}_- + \pi_i\hat{\pi}_i\right)$

So, Stark shift is

$$\Delta_s = \delta - \alpha$$

$$= \frac{1}{4\Delta_1} \frac{\omega_{HF}}{\Delta_1} \cdot \frac{1}{3} (I_0 + I_1) \frac{\Gamma_1^2}{2 I_{sat_1}} + \frac{1}{4\Delta_2} \frac{\omega_{HF}}{\Delta_2} \frac{2}{3} (I_0 + I_1) \frac{\Gamma_2^2}{2 I_{sat_2}}$$

$$= \frac{\omega_{HF}}{24 \Delta_1^2} \frac{(I_0 + I_1) \Gamma_1^2}{I_{sat_1}} + \frac{\omega_{HF}}{12 \Delta_2^2} \frac{(I_0 + I_1) \Gamma_2^2}{I_{sat_2}}$$

$$\approx 24. \text{ Hz.}$$

This value is only the scaler term of Stark shift. (part of Two-photon SS)

Not considering
- vector term
- Four-photon SS
- pulsed laser.

See Stark shift part for details.

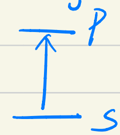
$\boxed{\text{Stark shift}}$ $\delta = \overbrace{\alpha_s I_g}^{①} + \overbrace{\alpha_v |\vec{B}_0 + \vec{B}' I_g|^2}^{②} + \overbrace{\alpha_4 I_g^2}^{③}$

Two-photon Stark shift:

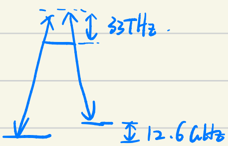
• This comes from diagnolizing the matrix of a single beam:

$$\begin{array}{cc} |0\rangle & |e\rangle \end{array}$$
$$\begin{pmatrix} 0 & \Omega \\ \Omega & \delta \end{pmatrix}$$

$\Omega \propto I_g$



① Scalar term of the laser. This term comes from the detuning difference between $|0\rangle$ and $|1\rangle$ state



Scalar term $\delta_s = \frac{\Omega^2}{\Delta} - \frac{\Omega^2}{\Delta + w_{HF}}$

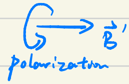
$= \frac{\Omega^2}{\Delta}\left(1 - \frac{1}{1 + \frac{w_{HF}}{\Delta}}\right)$

$\Omega_{eff} = \frac{\Omega^2}{\Delta} \gtrsim 25\,\text{kHz}$ $\approx \frac{\Omega^2}{\Delta} \cdot \frac{w_{HF}}{\Delta}$

(pi time < 20 μs) $= 25\,\text{kHz} \cdot \frac{12.6\,\text{GHz}}{33\,\text{THz}}$

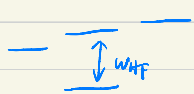
A more detailed calculation of this term can be referred to "Raman transition Theory" section. ← $= \frac{10^4 \cdot 10^{10}}{33 \cdot 10^{12}} \sim 10\,\text{Hz}$.

② Vector term: The laser with circular polarization will generate an effective magnetic field $\vec{B}'$, which follows the right-hand rule:



This $\vec{B}'$ will generate frequency shift the same as a real magnetic field:

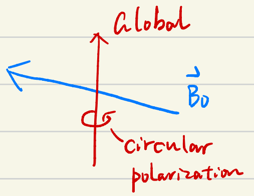
eg.



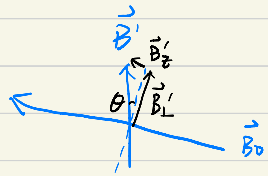
$^2S_{1/2}, F=1$: $\Delta_{Zeeman} = \pm 1.4\,(\text{MHz}/G) \cdot B$

or $w_{HF} = 12.6\,\text{GHz} + 310\,(\text{Hz}/G^2) \cdot B^2$

Red chamber:



$\Longrightarrow$



$\vec{B}_z'$ has the same effect of the static $\vec{B}_0$, driving $|0,0\rangle \to |1,0\rangle$

$\vec{B}_\perp'$ can drive the transition $|0,0\rangle \to |1,-1\rangle$ & $|1,+1\rangle$

$$\vec{B}_\perp' = \vec{B}_{\sigma+}' + \vec{B}_{\sigma-}' . \qquad |\vec{B}_\perp'|^2 = |\vec{B}_{\sigma+}'|^2 + |\vec{B}_{\sigma-}'|^2$$

$$|\tan\theta| = \frac{B_z'}{B_\perp'} = \frac{\mu B_z'}{\mu B_\perp'} = \frac{\Omega_{|0,0\rangle\to|1,0\rangle}}{\sqrt{2}\,\Omega_{|0,0\rangle\to|1,-1\rangle}}$$

- α_v in $\alpha_v |\vec{B}_0 + \vec{B}' I_g|^2$ is $\alpha_v = 310\ B^2$ Hz (B in Gauss)

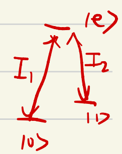
③ Four photon Stark shift:

$$\delta_4 = \alpha_4 I_g^2$$

$\Omega_{eff} \propto I_1 I_2$

This comes from diagnolizing the Raman transition (Two beams)

$$\begin{array}{ccc} & |0\rangle & |1\rangle \\ & \begin{pmatrix} 0 & \Omega_{eff} \\ \Omega_{eff} & \delta \end{pmatrix} & \end{array}$$



- α_4 is a function of
  - polarization : P
  - Rep-rate : f_{rep}
  - Detuning : δ

$$\alpha_4 \propto P \cdot \beta(f_{rep}, \delta)$$

β $\downarrow$ function calculated in M. Goldman's Note (not published)

For global beam only } we want the beam
Ind beam only }

to be lin-perp-lin, meaning that both the beams are linearly polarized.

$$\Rightarrow P = 0. \quad \Rightarrow \delta_4 = 0.$$

Summary:

- For global only (Consider polarization and intensity only)

$$\delta = \alpha_s I_g + \alpha_v |\vec{B_0} + \vec{B}'(P)\, I_g|^2 + \alpha_4(P)\, I_g^2$$

$$= \alpha_s I_g + \alpha_v |\vec{B_0}|^2 + \alpha_v |\vec{B}'|^2 I_g^2 + \alpha_4(P)\, I_g^2$$

$$+ \alpha_v\, |\vec{B_0}|\, |\vec{B}'| \cos\left(\frac{\pi}{2} - \theta\right) I_g$$

$$= \alpha_v |\vec{B_0}|^2 + \left(\alpha_s + \alpha_v \vec{B_0} \cdot \underbrace{\vec{B_z}'(P)}_{=0}\right) I_g + \left(\alpha_v \underbrace{|\vec{B}'(P)|^2}_{=0} + \underbrace{\alpha_v(P)}_{=0}\right) I_g^2$$

This term exists even no light, so it's not Stark shift

→ for linear polarization.

$$\delta_{stark\_shift} = \alpha_s I_g$$

For circular polarization:

$$\delta_{stark\text{-}shift} = \left[\alpha_s + \alpha_v \vec{B_0} \cdot \vec{B_z}(\varphi)\right] I_g + \left[\alpha_v |\vec{B}'(\varphi)|^2 + \alpha_4(\varphi)\right] I_g^2$$

What we can get:

① At linear polarization, get $\delta_{stark\ drift}$ vs. I_g.

$$\delta_{ss} = k_1 \cdot I_g. \qquad k_1 = \alpha_s$$

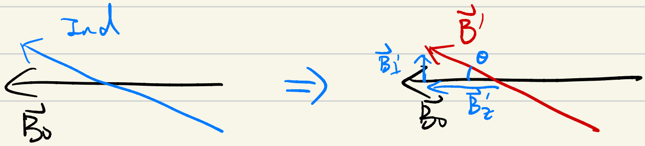
② At non-linear polarization:

- Calibrate the Global beam intensity for $|0,0\rangle \rightarrow \begin{cases} |1,-1\rangle \\ |1,0\rangle \\ |1,-1\rangle \end{cases}$

. We can get the

$$|\tan\theta| = \frac{\Omega_{|0,0\rangle \rightarrow |1,0\rangle}}{\sqrt{2}\,\Omega_{|0,0\rangle \rightarrow |1,-1\rangle}}$$

For Individual beam:



The $|0,0\rangle \to |1,0\rangle$ transition should be faster than $|0,0\rangle \to \begin{cases} |1,-1\rangle \\ |1,+1\rangle \end{cases}$

Similar to the global only case.

① At linear polarization, we can get

$$\delta_{ss} = k_i \cdot I_i \quad , \quad k_i = \alpha_s \quad (\text{scalar term})$$

② At non-linear polarization, we can get

$$|\tan\theta| = \frac{\sqrt{2}\,\Omega_{|0,0\rangle \to |1,-1\rangle}}{\Omega_{|0,0\rangle \to |1,0\rangle}}$$

For counter-prop:

$$\delta_{ss} = \delta\_\text{global only} + \delta\_\text{ind only} + \delta_{g\&i}.$$

Conditions: lin-perp-lin

$$\Rightarrow \delta\_\text{global only} = \alpha_{sg} \cdot I_g$$

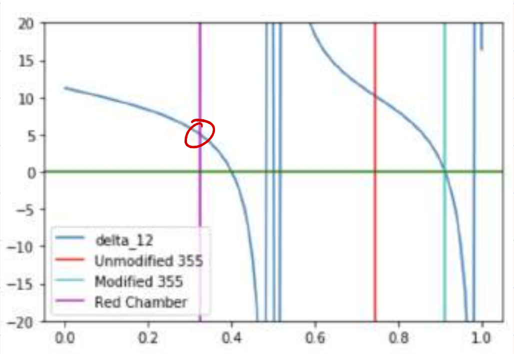
$$\delta\text{- ind only} = \alpha_{si} \cdot I_i.$$

$\delta_{g\&i}$ is 4-photon stark shift, assuming the detuning $\delta=0$

$$\delta_{g\&i} = P \cdot \delta_0(f\text{-rep}) \cdot I_g I_i$$

Polarization term, For lin-perp-lin, P is a non-zero constant.

$\delta_0(f\text{-rep})$:



Total Stark shift for Raman transition is:

$$\delta_{ss} = \alpha_{sg} I_g + \alpha_{si} I_i + \underbrace{P\,\delta_0(f\text{-rep})}_{\alpha_4} I_g \cdot I_i$$

By measurement, $\alpha_{sg} = \alpha_{si} < 0$.

$\alpha_4 > 0$.

• M. Goldman's Note

(Not published, but people in EURIQA project should has a copy).

– For the specific case of driving Raman transitions between two clock states ($m_F = 0$) via $^2P_{1/2}$ and $^2P_{3/2}$, the Rabi frequency is

$$\Omega = P_{1,2} \frac{g_1 g_2}{3} \frac{\omega_p}{\Delta(\Delta - \omega_p)} \qquad (1)$$

where $\hbar\omega_p$ is the energy difference between $^2P_{1/2}$ & $^2P_{3/2}$

Δ is the detuning from $^2P_{1/2}$ ($\Delta \gg \omega_0$)

$g_i = \frac{\varepsilon_i |\vec{d}_0 \cdot \hat{\sigma}_+|}{\hbar}$, $\vec{d}_0$ is the dipole operator between

$P_{1,2} = i \cdot (\hat{\epsilon}_2^* \times \hat{\epsilon}_1) \cdot \hat{B}$,

where $E_i(t) = \varepsilon_i e^{i\omega_i t} \hat{\epsilon}_i$. $\vec{B}$ is the direction of ion's quantization axis.

– Pulsed laser:

$$\varepsilon_i(t) = \sum_{n=1}^{\tilde{N}} p(t - n/f_{rep}) \qquad (2)$$

If the pulse width $\tau \ll 1/f_{rep}$. Fourier trans of (2) is

$$\tilde{E}_i(\omega) = \sum_k \tilde{E}_{i,k}(\omega) = \sum_k \delta(\omega - \omega_k) \tilde{p}_i(\omega - \omega_i) \hat{\epsilon}_0 . \qquad (3)$$

where $\delta(\omega)$ is a sharply peaked function of width $\sim \frac{f_{rep}}{N}$

$\tilde{p}(\omega)$ is the envelope function.

if $p_i(t) = \varepsilon_i \sqrt{\pi/2}\ \mathrm{sech}(\pi t/\tau)$

$\tilde{p}_i(\omega) = \varepsilon_i \tau \sqrt{\pi/2}\ \mathrm{sech}(\omega\tau/2)$

$\Rightarrow$ Time-averaged E-field amplitude

$$\underset{\substack{\text{ith comb} \\ \text{kth tooth}}}{\mathcal{E}_{i,k}} = f_{rep} \cdot \int_{-\infty}^{\infty} |\vec{E}_{i,k}(\omega)| d\omega$$

$$= f_{rep} \cdot \tilde{p}_i(\omega_k - \omega_i) \qquad (4)$$

☆ Condition: pulse width τ is short enough s.t. BW of freq comb is larger than the qubit splitting ω_0.

$$|f_A + n f_{rep}| = \omega_0 / 2\pi \qquad (5)$$

$$(4) \Rightarrow g_{i,k} = \frac{\mathcal{E}_{i,k} \, |\vec{d}_0 \cdot \hat{\sigma}_+|}{\hbar} = \frac{|\vec{d}_0 \cdot \hat{\sigma}_+|}{\hbar} \cdot f_{rep} \, \tilde{p}_i(\omega_k - \omega_i) \qquad (6)$$

(6) insert into (1) $\Rightarrow$

$$\Omega_n = \sum_k P_{1,2} \frac{g_{1,k} \, g_{2,k+n}}{3} \, \frac{\omega_p}{\Delta(\Delta - \omega_p)}$$

$$= \sum_k P_{1,2} \frac{|\vec{d}_0 \cdot \hat{\sigma}_+|^2}{3\hbar^2} f_{rep}^2 \cdot \tilde{p}_1(\omega_k - \omega_1) \cdot \tilde{p}_2(\omega_{k+n} - \omega_2) \cdot \frac{\omega_p}{\Delta(\Delta - \omega_p)}$$

$$= \underbrace{P_{1,2} \cdot \mathcal{E}_1 \mathcal{E}_2 \frac{|\vec{d}_0 \cdot \hat{\sigma}_+|^2}{3\hbar^2} \frac{\omega_p}{\Delta(\Delta - \omega_p)}}_{\equiv \Omega_{1,2}} \cdot \underbrace{\sum_k f_{rep}^2 \cdot \left(\tau \cdot \sqrt{\tfrac{\pi}{2}}\right)^2 \operatorname{sech}\left(\frac{(\omega_k - \omega_1)\tau}{2}\right) \cdot \operatorname{sech}\left(\frac{(\omega_{k+n} - \omega_2)\tau}{2}\right)}_{\equiv h_n = \sum_k h_{k,n}} \qquad (7)$$

$$\mathcal{F}[\operatorname{sech}(x)] = \pi \operatorname{sech}\left(\frac{\pi k}{2}\right)$$

So if $p = \sqrt{\frac{\pi}{2}} \, \xi_0 \operatorname{sech}\left(\frac{\pi t}{\tau}\right)$

$$\int \operatorname{sech}(ax) e^{i\omega x} dx \overset{y=ax}{=} \int_{-\infty}^{\infty} \frac{1}{a} \operatorname{sech}(y) e^{i\omega \frac{y}{a}} dy = \frac{1}{a} \int_{-\infty}^{\infty} \operatorname{sech}(y) e^{i\frac{\omega}{a} y} \cdot dy = \frac{1}{a} \pi \operatorname{sech}\left(\frac{\pi\omega}{2a}\right)$$

if $a = \frac{\pi}{\tau}$,

$$\tilde{p}(\omega) = \sqrt{\frac{\pi}{2}} \, \xi_0 \, \frac{\tau}{\pi} \cdot \pi \operatorname{sech}\left(\frac{\pi \omega \tau}{2\pi}\right) = \sqrt{\frac{\pi}{2}} \, \xi_0 \, \tau \operatorname{sech}\left(\frac{\omega\tau}{2}\right) \quad \checkmark$$

Let $(\omega_k - \omega_1) = 2\pi f_{rep} \cdot k$. $\xrightarrow[\text{to discrete}]{\text{continuous}}$ $\tilde{p}_k = \sqrt{\frac{\pi}{2}} \, \tau \, \xi_0 \operatorname{sech}(k\pi f_{rep} \tau)$

So $h_k = f_{rep}^2 \cdot \left(\tau \sqrt{\frac{\pi}{2}}\right)^2 \operatorname{sech}(k\pi f_{rep} \tau) \operatorname{sech}[(k+n)\pi f_{rep} \tau]$

So $h_n = \sum_k f_{rep}^2 \cdot \left(\tau\sqrt{\frac{\pi}{2}}\right)^2 \operatorname{sech}(k\pi f_{rep}\tau) \operatorname{sech}[(k+n)\pi f_{rep}\tau]$

$$= f_{rep}^2 \left(\tau\sqrt{\frac{\pi}{2}}\right)^2 \sum_k \frac{1}{\cosh(k\pi f_{rep}\tau)} \cdot \frac{1}{\cosh[(k+n)\pi f_{rep}\tau]}$$

$$\underline{\underline{\theta = \pi f_{rep}\tau}} \sum_k \frac{1}{\frac{e^{k\theta}+e^{-k\theta}}{2}} \cdot \frac{1}{\frac{e^{(k+n)\theta}+e^{-(k+n)\theta}}{2}}$$

$$= \sum_k \frac{4}{e^{(2k+n)\theta} + e^{-(2k+n)\theta} + e^{n\theta} + e^{-n\theta}}$$

$$= \sum_k \frac{2}{\cosh[(2k+n)\theta] + \cosh(n\theta)}$$

$$= \sum_k \frac{2}{\cosh(2k\theta)\cosh(n\theta) + \sinh(2k\theta)\sinh(n\theta) + \cosh(n\theta)}$$

$$= \sum_k \frac{2}{[\cosh(2k\theta)+1]\cosh(n\theta) + \sinh(2k\theta)\sinh(n\theta)}$$

when $n\theta \ll 1$, $= \sum_k \frac{2}{[\cosh(2k\theta)+1]\cosh(n\theta)}$

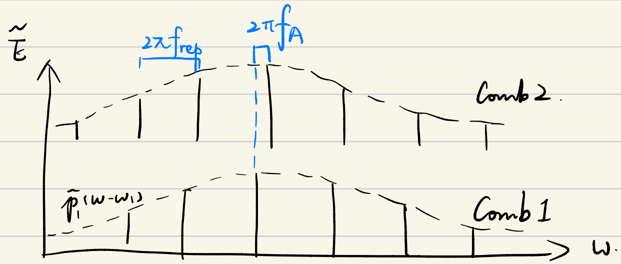
$$\approx \frac{1}{\cosh(n\theta)} \cdot \frac{2.00408}{\theta}$$

when $n\theta \gtrsim 1$ $= \sum_k \frac{2}{[\cosh(2k\theta)+1+\sinh(2k\theta)]\sinh(n\theta)}$

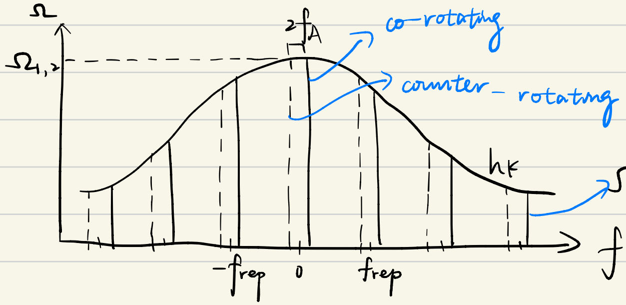
$$= \sum_k^N \frac{2}{(e^{2k\theta}+1)\sinh(n\theta)}$$

$$= 2N \frac{1}{\sinh(n\theta)}$$

$$= \begin{cases} f_{rep}\tau / \cosh(n\pi f_{rep}\tau), & n\theta \ll 1 \\ k f_{rep}^2 \tau^2 \pi / \sinh(n\theta), & n\theta \gtrsim 1 \end{cases}$$



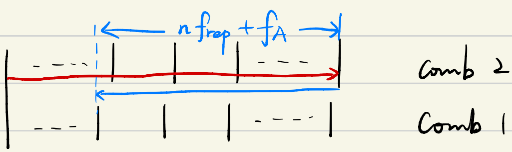
Beatnote:



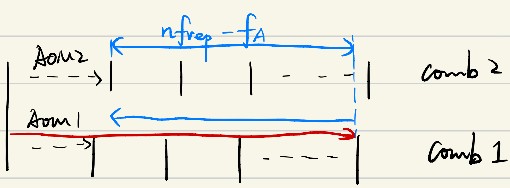
$\Omega_K = \Omega_{1,2} \cdot h_K$.

every tooth is one transition

Co-rotating:



Counter-rotating

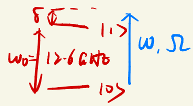


Define $\quad x \equiv \dfrac{\omega_0/2\pi \mod f_{rep}}{f_{rep}} \in [0,1)$.

The resonance condition in $|f_A + nf_{rep}| = \omega_0/2\pi$
is $f_A = (k \pm x) f_{rep}$.

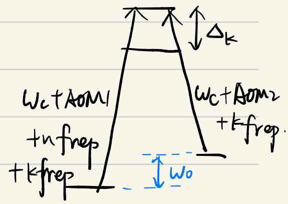
→ This stark shift is the effective single-photon stark shift:

$$\delta_{AC} = -\frac{|\Omega|^2}{\delta}$$



The Stark shift due to the kth comb tooth is

$$\delta_k = -\frac{|\Omega_k|^2}{\Delta_k} \quad \text{(Actually it's } \delta_k\text{)}$$



Summing over all the δ_k:

$$\delta = \sum_k \delta_k = -\sum_k \frac{|\Omega_k|^2}{\Delta_k} \rightarrow \delta_k = \begin{cases} (k\cdot f_{rep} + f_A) - (n+x) f_{rep} & \text{co-rotating} \\ (k\cdot f_{rep} - f_A) - (n+x) f_{rep} & \text{counter-rotating} \end{cases}$$

$$= -\sum_k \frac{|\Omega_{1,2}\cdot h_k|^2}{\Delta_k}$$

$$= \underbrace{-\frac{|\Omega_{1,2}|^2}{2\pi f_{rep}}}_{\delta_{1,2}} \sum_k h_k^2 \cdot \left(\frac{1}{k-n-x+f_A/f_{rep}} + \frac{1}{k-n-x-f_A/f_{rep}} \right) \tag{8}$$

Before calculating specific Stark shift, we define some independent parameters:

1. Raman Rabi frequency:

$$\Omega_0 \equiv \frac{\Omega_{ij}}{P_{ij}\sqrt{I_i I_j}}$$

independent of $\begin{cases} \text{intensity } I_{ij} \\ \text{polarization } P_{ij} \end{cases}$

2. Stark shift:

$$\delta_0 \equiv \frac{\delta_{ij}}{P_{ij}^2 I_i I_j} = -\frac{|\Omega_0|^2}{2\pi f_{rep}}.$$

3. Polarization: Beam 1 before QWP:

$$\vec{\varepsilon} = \varepsilon_0 (\hat{x} + \hat{y}) e^{i\omega t}$$

After QWP which has an angle ϕ relevant to $\hat{y}$:

$$\hat{\varepsilon} = \varepsilon_x \hat{x} e^{i\omega t} + \varepsilon_y \hat{y} e^{i(\omega t + 2\phi)}$$

$$\hat{\varepsilon}^* \times \hat{\varepsilon} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \varepsilon_x e^{-i\omega t} & \varepsilon_y e^{-i\omega t - i2\phi} & \\ \varepsilon_x e^{i\omega t} & \varepsilon_y e^{i\omega t + i2\phi} & \end{vmatrix} = \hat{z} \cdot \varepsilon_x \varepsilon_y e^{i2\phi} - \varepsilon_x \varepsilon_y e^{-i2\phi}$$

$$= \hat{z} \cdot \varepsilon_x \varepsilon_y \, 2i \sin 2\phi$$

$$\propto \sin 2\phi$$

Define $S_1 = \hat{\varepsilon}_1^* \times \hat{\varepsilon}_1$

$$= -\sin(2\phi)$$

$$\boxed{|P_{1,2}| = \sqrt{1 - S_1^2/2} \qquad |P_{1,1}| = |S_1|}$$

$\Rightarrow$ ① Dual-beam transitions are most strongly driven when both beams are linearly and orthogonally polarized. $(S_1 = 0)$ $(S_1 = 0)$ ($\hat{\varepsilon}_1^* \times \hat{\varepsilon}_2$ is max when ortho)

② We can drive single-beam transition only when the beam has some degree of circular polarization. $(S_1 \neq 0)$

4. Intensity imbalance: $\alpha = \sqrt{I_{1b}/I_{1r}}$

If one beam goes into one AOM with two RF signals: $I_{1b} + I_{1r} = I_1$, when $\alpha = 1$. $\sqrt{I_{1b} I_{1r}} = I_1/2$.

① Stark shift in single-qubit rotation with one comb.

Eqn (8): $\delta = \delta_{1,2} \sum_k h_k^2 \left(\frac{1}{k-n-x+f_A/f_{rep}} + \frac{1}{k-n-x-f_A/f_{rep}} \right)$

For one comb situation: $f_A = 0$

$f_A = 0$. one term within the sum should be dropped.

$$\delta_{11} = -\frac{|\Omega_{1,1}|^2}{2\pi f_{rep}} \rightarrow \Omega_{1,1} = |S_1|\, \varepsilon_1^2 \underbrace{\frac{|d_0 \cdot \sigma_T|^2}{3\hbar^2} \cdot \frac{\omega_P}{\Delta(\Delta-\omega_P)}}_{|\Omega_0|}$$

Define $\xi_1(x) = \sum_k h_k^2 \frac{1}{k-n-x}$

$$\Rightarrow \boxed{\delta^{(1)} = \delta_{1,1}\, \xi_1(x)}$$

$$\delta_0 = -\frac{|\Omega_0|^2}{2\pi f_{rep}}$$

$$\delta^{(1,res)} = \underset{\underset{P_{ij}^2}{\downarrow}}{S_1^2} I_1^2 \cdot \frac{\Omega_{ij}^2}{P_{ij}^2 I_i I_j} \cdot \frac{1}{2\pi f_{rep}}$$

$$= \frac{I_1^2}{I_i I_j} \frac{\Omega_{ij}^2}{2\pi f_{rep}}$$

Because $|\Omega^{(1,res)}| = |\Omega_{1,1}|\, h_n$

$= |S_1| \cdot I_1 |\Omega_0| h_n.$

$$\boxed{\begin{aligned} \delta^{(1,res)} &= \delta_{1,1} \cdot \xi_1(0) \\ &= S_1^2 I_1^2 \delta_0 \cdot \xi_1(0) \end{aligned}}$$

② Stark-shift in single-qubit rotation with two combs.

$$f_A = \begin{cases} \pm x\, f_{rep} & \text{if } 0 \le x < 0.5 \\ (1 \mp x) f_{rep} & \text{if } 0.5 \le x < 1 \end{cases}$$

Two tones to a single AOM, one beam: I_{1r} & I_{1b}.

$$|\Omega^{(2)}| = |S_1| \cdot \sqrt{I_r I_b} \cdot |\Omega_0|\, h_n$$

$$= \frac{1}{2} |S_1|\, I_1\, |\Omega_0|\, h_n.$$

$$\delta^{(2)} = \underbrace{\delta_{1b,1r} \cdot \frac{2}{\Delta} \cdot \xi_2(x, \overset{f_A = x f_{rep}}{x f_{rep}})}_{\text{inter-comb}} + \underbrace{(\delta_{1b,1b} + \delta_{1r,1r})\, \xi_1(x)}_{\text{intra-comb}}$$

$$= |P_{1,1}|^2 \delta_0 \left[I_{1b} I_{1r} \cdot 2 \xi_2(x, x f_{rep}) + (I_{1b}^2 + I_{1r}^2)\, \xi_1(x) \right]$$

$$= |S_1|^2 \delta_0 \cdot \underbrace{\frac{I_1^2}{4}}_{\text{Assume } \alpha \approx 1} \left[2\, \xi_2(x, x f_{rep}) + (\alpha^2 + \alpha^{-2})\, \xi_1(x) \right]$$

$$\overset{\alpha \approx 1}{\approx} \frac{1}{2} |S_1|^2 \delta_0 I_1^2 \left[\xi_2(x, x f_{rep}) + \xi_1(x) \right]$$

where $\xi_2(x, f_A) = \frac{1}{2} \sum_k h_k^2 \left(\frac{1}{k-n-x+f_A/f_{rep}} + \frac{1}{k-n-x-f_A/f_{rep}} \right)$

$$\xi_1(x) = \sum_k h_k^2 \frac{1}{k-n-x}$$

③ Stark shift for dual-qubit operations.

$$H = \frac{\hbar}{2} \sum_{i \in \{a,b\}} \eta |\uparrow_i\rangle\langle\downarrow_i| (\Omega_r \hat{a} e^{-i\delta t} + \Omega_b a^\dagger e^{i\delta t}) + h.c.$$

Effective Rabi frequency:

$$\Omega_{MS} = \frac{\eta^2 \Omega_{1b,2}^* \Omega_{1r,2} h_n^2}{\delta}$$

$$|\Omega_0| = \frac{\Omega_{i,j}}{|P_{i,j}| \cdot \sqrt{I_i I_j}}$$

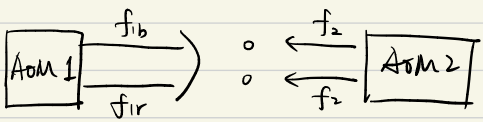
Define : $\Omega_{MS,0} \equiv \frac{\Omega_{MS}}{|P_{1,2}|^2 \sqrt{I_{1b} I_{1r}}\, I_2} = \frac{\eta^2 |\Omega_0|^2 h_n^2}{\delta}$

On the contrary

$$|\Omega_{MS}| = |P_{1,2}|^2 \sqrt{I_{1b} I_{1r}}\, I_2\, |\Omega_{MS,0}|\, \eta_n^2$$

$$= \frac{1}{2}\sqrt{1-S_1^2/2}\; I_1 I_2\, \underbrace{|\Omega_{MS,0}|}_{\text{independent of experimental paras}}\, \eta_n^2$$

- Resonance condition:



Define $f_{Ab} = f_{1b} - f_2$, $\quad f_{Ar} = f_{1r} - f_2$

phase-insensitive: Drive blue by causing the ion to absorb one ion from beam 1 and emit a photon into beam 2. Red is the opposite.

So
$$\left.\begin{aligned} k f_{rep} \pm f_{Ab} &= \omega_0/2\pi + f_m \\ k f_{rep} \mp f_{Ar} &= \omega_0/2\pi - f_m \end{aligned}\right\}$$

$\xrightarrow[\text{the two eqns}]{\text{subtract}}$ $f_{Ab} + f_{Ar} = k f_{rep} \pm 2 f_m$ (phase-insensitive)

$\left(f_{Ab} - f_{Ar} = k f_{rep} \pm 2 f_m \right.$ for phase-sensitive $\left.\right)$

$\xrightarrow[\text{the two eqns}]{\text{Add}}$ $(k_1+k_2) f_{rep} + f_{Ab} - f_{Ar} = 2\omega_0/2\pi$

$$f_{Ab} - f_{Ar} = 2\omega_0/2\pi - (\hat{k}_1 + \hat{k}_2) f_{rep}$$
$$= 2x f_{rep}$$

To minimize f_{Ab} & f_{Ar}
Let $k_1 = k_2 = n$.

$$|f_{Ab} - f_{Ar}| = \begin{cases} 2x\, f_{rep} & 0 \le x < 0.5 \\ 2(1-x)\, f_{rep} & 0.5 \le x < 1 \end{cases}$$

– Stark shift: intra-beam shift $\delta^{(MS,11)}$
inter-beam shift $\delta^{(MS,12)}$

$$\delta^{(MS,11)} = \underbrace{\delta_{1b,1r}\cdot\frac{2}{\Delta}\cdot\xi_2(x, 2x\overbrace{f_{rep}}^{|f_{Ab}-f_{Ar}|})}_{\text{inter-comb}} + \underbrace{(\delta_{1b,1b}+\delta_{1r,1r})\,\xi_1(x)}_{\text{intra-comb}}$$

$$= \frac{1}{4}|S_1|^2 I_1^2 \delta_0 \left[2\xi(x, 2xf_{rep}) + (\alpha^2+\alpha^{-2})\xi_1(x)\right]$$

$$= \frac{1}{2}|S_1|^2 I_1^2 \delta_0 \left[\xi(x, 2xf_{rep}) + \xi_1(x)\right]$$

☆ $\delta^{(MS,11)}$ is very sensitive to x.
but not sensitive to α

$$\delta^{(MS,12)} = \delta_{1b,2}\, 2\cdot\xi_2(x, f_{Ab}) + \delta_{1r,2}\cdot 2\xi_2(x, f_{Ar})$$

$$= 2|P_{1,2}|^2 I_2 \delta_0 \left[I_{1b}\xi_2(x, f_{Ab}) + I_1\xi_2(x, f_{Ar})\right]$$

$$= (1-S_1^2/2) I_1 I_2 \delta_0 \left[\alpha\,\xi_2(x, f_{Ab}) + \alpha^{-1}\xi_2(x, f_{Ar})\right]$$

Re-express this equation by assuming:

$$\begin{cases} f_{Ab} = \pm(xf_{rep}+f_m) \\ f_{Ar} = \mp(xf_{rep}-f_m) \end{cases}$$

$$f_m \ll x f_{rep}$$

$$\Rightarrow \delta^{(MS,12)} = (1-S_1^2/2) I_1 I_2 \delta \left[(\alpha+\alpha^{-1})\xi_2(x, xf_{rep}) + (\alpha-\alpha^{-1})\, h_n^2 \frac{f_{rep}}{f_m}\right]$$

drop high order of $\frac{f_m}{f_{rep}} \ll 1$, Assume $\alpha \approx 1$

$$= (1-S_1^2/2) I_2 \delta_0 \left[2I_1\xi_2(x, xf_{rep}) + (I_{1b}-I_{1r})\, h_n^2 \frac{f_{rep}}{f_m}\right]$$

For generality:

① Stark shift $\delta^{(1)}$ due to a single freq comb:

$$\frac{\delta^{(1)}}{|\Omega^{(1)}|^2}\cdot \omega_{rep} = \frac{S_1^2 I_1^2 \delta_0 \, \xi(x)}{|S_1|^2 I_1^2 \, |\Omega_0|^2 \, h_n^2} \cdot 2\pi f_{rep}$$

$$= \frac{\delta_0}{|\Omega_0|^2} \, \frac{\xi_1(x)}{h_n^2} \cdot 2\pi f_{rep}$$

$$= -\frac{|\Omega_0|^2}{2\pi f_{rep}} \Big/ |\Omega_0|^2 \cdot \frac{\xi_1(x)}{h_n^2}$$

$$= -\frac{\xi_1(x)}{h_n^2}$$

② Stark shift $\delta^{(2)}$ due to two freq combs.

$$\frac{\delta^{(2)}}{|\Omega^{(2)}|^2}\cdot \omega_{rep} = \frac{\frac{1}{4}|S_1|^2 I_1^2 \delta_0 \left[2\xi_2(x, x f_{rep}) + (\alpha^2+\alpha^{-2})\,\xi_1(x)\right]}{\frac{1}{4}|S_1|^2 I_1^2 |\Omega_0|^2 h_n^2} \times 2\pi f_{rep}$$

$$= \frac{\delta_0}{|\Omega_0|^2} \cdot 2\pi f_{rep} \cdot \frac{\left[2\xi_2(x, x f_{rep}) + (\alpha^2+\alpha^{-2})\,\xi_1(x)\right]}{h_n^2}$$

$$= -\frac{\left[2\xi_2(x, x f_{rep}) + (\alpha^2+\alpha^{-2})\,\xi_1(x)\right]}{h_n^2}$$

MS gate stark shift:

$$\frac{\delta^{(MS,11)}}{|\Omega^{(2)}|^2} \times \omega_{rep} \overset{\alpha\approx 1}{\approx} \frac{\frac{1}{2}|S_1|^2 I_1^2 \underbrace{\delta_0}_{-\frac{|\Omega_0|^2}{2x f_{rep}}} [\xi_2(x, 2x f_{rep}) + \xi_1(x)]}{\left(\frac{1}{2}|S_1| I_1 |\Omega_0| h_n\right)^2} \cdot \omega_{rep}$$

global beam

$$= \frac{-2[\xi_2(x,2x f_{rep}) + \xi_1(x)]}{h_n^2}$$

But the absolute value of $\delta^{(MS,11)}$ should be very small because we use lin-perp-lin, so $|S_1| \approx 0$.

$$\frac{\delta^{(MS,12)}}{|\Omega^{(MS)}|}\cdot\frac{\omega_{rep}\eta^2}{\delta} = \frac{(1-S_1^2/2)\, I_2 I_1 \underbrace{\delta_0}_{-\frac{|\Omega_0|^2}{2x f_{rep}}} \left[(\alpha+\alpha^{-1})\xi_2(x, x f_{rep}) + (\alpha-\alpha^{-1}) h_n^2 \frac{f_{rep}}{f_m}\right]}{\frac{1}{2}(1-S_1^2/2) I_1 I_2 h_n^2 \cdot \eta^2 |\Omega_0|^2/\delta} \cdot \frac{\omega_{rep}\cdot\eta^2}{\delta}$$

$$= \frac{-2\left[(\alpha+\alpha^{-1})\xi_2(x, x f_{rep}) + (\alpha-\alpha^{-1}) h_n^2 \frac{f_{rep}}{f_m}\right]}{h_n^2}$$

where $\alpha \equiv \sqrt{I_{1b}/I_{1r}}$

$$\frac{\delta^{(MS,11)}}{\delta^{(MS,12)}} \overset{\alpha=1}{\approx} \frac{-2[\xi_2(x,2x f_{rep}) + \xi_1(x)]}{-2[2\xi_2(x, x f_{rep})]} \cdot \frac{|\Omega^{(2)}|^2/\omega_{rep}}{|\Omega^{(MS)}|\cdot\delta/\omega_{rep}\eta^2}$$

$$= \frac{\frac{1}{4}|S_1|^2 I_1^2 |\Omega_0|^2 h_n^2}{\frac{1}{2}(1-S_1^2/2) I_1 I_2 h_n^2\, \eta^2 |\Omega_0|^2/\delta} \cdot \frac{\delta}{\eta^2}$$

$$= \frac{1}{2}\frac{S_1^2}{1-S_1^2/2}\frac{I_1 \rightarrow \text{global}}{I_2 \rightarrow \text{Ind}}$$

$$= \frac{[\xi_2(x,2x f_{rep}) + \xi_1(x)]}{2\xi_2(x, x f_{rep})} \cdot \frac{1}{2}\frac{S_1^2}{1-S_1^2/2}\frac{I_1}{I_2}$$

If no approximation:

$$\frac{\delta^{(MS,12)}}{|\Omega^{(MS)}|} \cdot \frac{w_{rep}[\,]^2}{\delta} = \frac{-2\left[\alpha\, \xi_2(x, f_{Ab}) + \alpha^{-1} \xi_2(x, f_{Ar})\right]}{h_m^2}$$

if $f_{Ab} = [(1-x) f_{rep} + f_m]$, $f_{Ar} = -[(1-x) f_{rep} - f_m]$

$$= -(1-x) f_{rep} + f_m$$

Motional state phonon number statistic

Prob of motional number n:

$$P(n) = \frac{e^{-nh\nu/k_BT}\; e^{-\frac{1}{2}h\nu/k_BT}}{\sum_i e^{-n_i h\nu/k_BT} \cdot e^{-\frac{1}{2}h\nu/k_BT}}$$

$$= \frac{e^{-nh\nu/k_BT}}{\sum_i e^{-n_i h\nu/k_BT}} \longrightarrow$$

$q = e^{-\beta h\nu}$, $a_1 = 1$

$$S_\infty = \frac{a_1}{1-q} = \frac{1}{1-e^{-\beta h\nu}}$$

$\beta \equiv 1/k_BT$

$$= \frac{e^{-n\beta h\nu}}{(1-e^{-\beta h\nu})^{-1}}$$

$$= (1-e^{-\beta h\nu})\, e^{-n\beta h\nu}$$

$\Rightarrow$ For ground state, $n=0$, $P(0) = 1-e^{-\beta h\nu}$

excited state, $n>0$, $P(n>0) = 1-P(0) = e^{-\beta h\nu}$

$$\frac{P_{BSB}}{P_{RSB}} = \frac{P(n\geq 0)}{P(n>0)} = \frac{1}{e^{-\beta h\nu}} = e^{\beta h\nu} \equiv r$$

Bose-Einstein distribution:

$$\boxed{\bar{n} = \frac{1}{e^{\beta h\nu}-1} = \frac{1}{r-1} = \frac{1}{\frac{P_{BSB}}{P_{RSB}}-1} = \frac{P_{RSB}}{P_{BSB}-P_{RSB}}}$$

If we go back to consider the population of each state:

$$P(n) = P(0) \cdot (e^{-\beta h\nu})^n = P(0) \cdot \underbrace{\left(\frac{1}{r}\right)}_{q}{}^n$$

The BSB transition of a thermal state will be

$$P(t) = \sum_n P(n) \cdot \sin(\sqrt{n+1}\,\Omega \cdot t)^2$$

$$= \sum_n \underset{①}{P(0)} \cdot \underset{②}{q^n} \cdot \sin(\sqrt{n+1}\,\underset{③}{\Omega}\, t)^2$$

Fitting variables.

Constrain: $\sum_n P(n) = 1 \Rightarrow \sum_n P(0) \cdot q^n = \frac{P(0)}{1-q} = 1$

$$\bar{n} = \frac{1}{e^{\beta h\nu} - 1} = \frac{1}{\frac{1}{q} - 1} = \frac{q}{1-q}$$

$$\frac{\partial \bar{n}}{\partial q} = \frac{(1-q) - q(-1)}{(1-q)^2} = \frac{1}{(1-q)^2}$$

$$\sigma_{\bar{n}} = \sqrt{\left(\frac{\partial \bar{n}}{\partial q}\right)^2 \cdot \sigma_q^2} = \frac{1}{(1-q)^2}\sigma_q$$

EIT cooling

Before diving into EIT cooling, we should study dressed state first.

Dressed state: Light-atom interaction

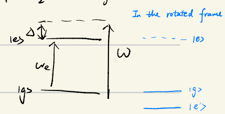
$$H = H_A + H_{int}$$

in an appropriate rotating frame:

$$= -\Delta |e\rangle\langle e| + \frac{\Omega}{2}\left(|e\rangle\langle g| + |g\rangle\langle e|\right)$$

$$= \begin{pmatrix} 0 & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta \end{pmatrix}$$

$$H_0 = \omega_e |e\rangle\langle e| + \frac{\Omega}{2} e^{i\omega t} |g\rangle\langle e| + \frac{\Omega}{2} e^{-i\omega t} |e\rangle\langle g|$$



$$H_D = \omega |e\rangle\langle e|$$

$$e^{iH_D t} = e^{i\omega t} |e\rangle\langle e|$$

$$\tilde{H}_I = e^{iH_D t} V e^{-iH_D t}$$

$$= \frac{\Omega}{2} \left[e^{i\omega t} |e\rangle\langle e| \left(\frac{\Omega}{2} e^{i\omega t} |g\rangle\langle e| + \frac{\Omega}{2} e^{-i\omega t} |e\rangle\langle g| \right) e^{-i\omega t} |e\rangle\langle e| \right]$$

$$= e^{i(\omega-\omega)t} |g\rangle\langle e| + e^{i(\omega-\omega)t} |e\rangle\langle g| = \left(|g\rangle\langle e| + |e\rangle\langle g|\right)\frac{\Omega}{2}$$

$$\tilde{H} = H_0 - H_D + \tilde{H}_I = -\Delta |e\rangle\langle e| + \frac{\Omega}{2}\left(|g\rangle\langle e| + |e\rangle\langle g|\right)$$

Eigensolver:

$$|H - EI| = \begin{vmatrix} -E & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta - E \end{vmatrix} = E^2 + \Delta E - \frac{\Omega^2}{4} = 0$$

$$\boxed{E_\pm = \frac{-\Delta \pm \sqrt{\Delta^2 + \Omega^2}}{2}}$$

$$H|\psi\rangle = E|\psi\rangle \Rightarrow \begin{pmatrix} 0 & \frac{\Omega}{2} \\ \frac{\Omega}{2} & -\Delta \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \left(\frac{\sqrt{\Delta^2+\Omega^2}}{2} - \frac{\Delta}{2}\right) \begin{pmatrix} a \\ b \end{pmatrix}$$

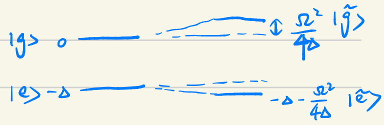
$$\Rightarrow \begin{cases} \frac{\Omega}{2} b = E_\pm a \\ \frac{\Omega}{2} a - \Delta b = E_\pm b \end{cases}$$

$$\frac{b}{a} = \frac{E_\pm}{\Omega/2} = \frac{\pm\sqrt{\Delta^2+\Omega^2} - \Delta}{\Omega}.$$

If we don't care about the normalization:

$$\Omega' \equiv \sqrt{\Delta^2+\Omega^2} \qquad |\psi_+\rangle = \begin{pmatrix} 1 \\ \frac{\Omega'-\Delta}{\Omega} \end{pmatrix} \qquad |\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-\Omega'-\Delta}{\Omega} \end{pmatrix}$$

For weak laser: $\Omega \ll \Delta$



$$\Omega' = \sqrt{\Omega^2+\Delta^2} = \Delta\left(1+\frac{\Omega^2}{\Delta^2}\right)^{1/2} \approx \Delta + \frac{1}{2}\frac{\Omega}{\Delta}\cdot\Omega.$$

$$|\psi_+\rangle = \begin{pmatrix} 1 \\ \underset{\sim 0}{\frac{\Omega}{2\Delta}} \end{pmatrix} \qquad |\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-2\Delta}{\Omega} \end{pmatrix} \gg 1$$

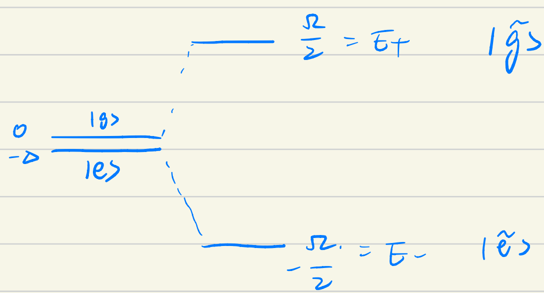
$$E_+ = \frac{-\Delta+\Omega'}{2} = \frac{-\Delta+\Delta+\frac{\Omega^2}{2\Delta}}{2} = \frac{\Omega^2}{4\Delta}, \qquad E_- = \frac{-\Delta-\Omega'}{2} = \frac{-2\Delta-\frac{\Omega^2}{2\Delta}}{2} = -\Delta-\frac{\Omega^2}{4\Delta}$$

For strong laser: $\Omega \gg \Delta$

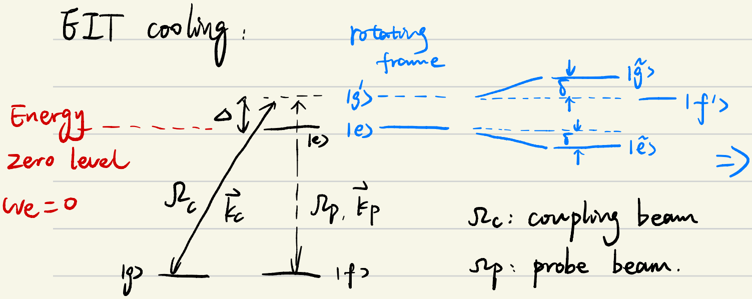
$$\Omega' = \Omega\left(1+\frac{\Delta^2}{\Omega^2}\right)^{1/2} \approx \Omega + \frac{1}{2}\frac{\Delta^2}{\Omega}$$

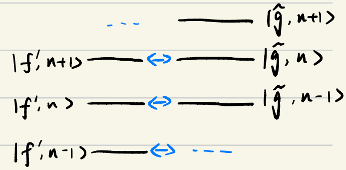
$$|\psi_+\rangle = \begin{pmatrix} 1 \\ \frac{\Omega+\frac{1}{2}\frac{\Delta^2}{\Omega}-\Delta}{\Omega} \end{pmatrix} = \begin{pmatrix} 1 \\ 1-\frac{\Delta}{\Omega}+\frac{1}{2}\frac{\Delta^2}{\Omega^2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1-O(\frac{\Delta}{\Omega}) \end{pmatrix}, \quad E_+ \approx \frac{\Omega}{2}-\frac{\Delta}{2}+\frac{1}{4}\frac{\Delta^2}{\Omega}$$

$$|\psi_-\rangle = \begin{pmatrix} 1 \\ \frac{-\Omega-\frac{1}{2}\frac{\Delta^2}{\Omega}-\Delta}{\Omega} \end{pmatrix} = \begin{pmatrix} 1 \\ -1-\frac{\Delta}{\Omega}-\frac{1}{2}\frac{\Delta^2}{\Omega^2} \end{pmatrix} = \begin{pmatrix} 1 \\ -1-O(\frac{\Delta}{\Omega}) \end{pmatrix}, \quad E_- \approx -\frac{\Omega}{2}-\frac{\Delta}{2}-\frac{1}{4}\frac{\Delta^2}{\Omega}$$



EIT cooling:





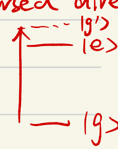
Ω_c: coupling beam

Ω_p: probe beam.

$$\hat{H}/\hbar = \omega_f |f\rangle\langle f| + \omega_g |g\rangle\langle g| + \left(\frac{\Omega_p}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i(\vec{k}_c\cdot\vec{r} - \omega_c t)} |g\rangle\langle e| + h.c.\right)$$

Rotating frame: $H_D/\hbar = -\omega_c |g\rangle\langle g| - \omega_p |f\rangle\langle f|$

(Rotate the Bloch sphere towards the inversed direction, to increase the freq)



$$\tilde{V} = e^{iH_D t/\hbar}\, V\, e^{-iH_D t/\hbar}$$

$$e^{iH_D t/\hbar} = |e\rangle\langle e| + e^{-i\omega_c t} |g\rangle\langle g| + e^{-i\omega_p t} |f\rangle\langle f|$$

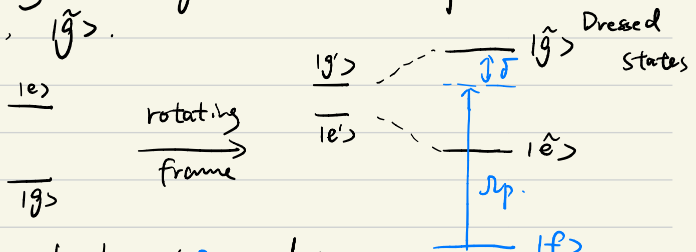
$$= \left(|e\rangle\langle e| + e^{-i\omega_c t} |g\rangle\langle g| + e^{-i\omega_p t} |f\rangle\langle f|\right) \left(\frac{\Omega_p}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i(\vec{k}_c\cdot\vec{r} - \omega_c t)} |g\rangle\langle e| + h.c.\right) \left(|e\rangle\langle e| + e^{+i\omega_c t} |g\rangle\langle g| + e^{+i\omega_p t} |f\rangle\langle f|\right)$$

$$= \left(\frac{\Omega_p}{2} e^{-i\vec{k}_p\cdot\vec{r}} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i\vec{k}_c\cdot\vec{r}} |g\rangle\langle e|\right) + h.c.$$

$$H_0 - H_D + \tilde{V} = \Delta_c |g\rangle\langle g| + \Delta_p |f\rangle\langle f| + \left[\left(\frac{\Omega_p}{2} e^{-i\vec{k}_p\cdot\vec{r}} |f\rangle\langle e| + \frac{\Omega_c}{2} e^{-i\vec{k}_c\cdot\vec{r}} |g\rangle\langle e|\right) + h.c.\right]$$

$$
= \begin{array}{c}\begin{pmatrix} 0 & \frac{\Omega_c}{2} e^{+i\vec{k}_c \cdot \vec{r}} & \frac{\Omega_p}{2} e^{+i\vec{k}_p \cdot \vec{r}} \\ \frac{\Omega_c}{2} e^{-i\vec{k}_c \cdot \vec{r}} & \Delta_c & \\ \frac{\Omega_p}{2} e^{-i\vec{k}_p \cdot \vec{r}} & & \Delta_p \end{pmatrix}\end{array} \begin{array}{c} |e\rangle \\ |g\rangle \\ |f\rangle \end{array}
$$

Intuitively: ① Consider the coupling beam and probe beam separately, since $\Omega_c \gg \Omega_p$. In this case, the coupling beam generate a set of dressed state $|\tilde{e}\rangle$, $|\tilde{g}\rangle$.



② The probe beam (Ω_p) drives the RSB transition to cool the ion.

Light-shifted : $\delta = \frac{1}{2}\left(\sqrt{\Omega_c^2 + \Delta^2} - |\Delta|\right)$

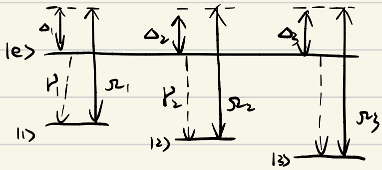
For EIT cooling, $\Delta > 0$ so that one can tune δ to be ν to accomplish the cooling process).

Intense 370 nm beam Ω_c is used to generate the dressed states: $|\tilde{g}\rangle$ & $|\tilde{e}\rangle$,

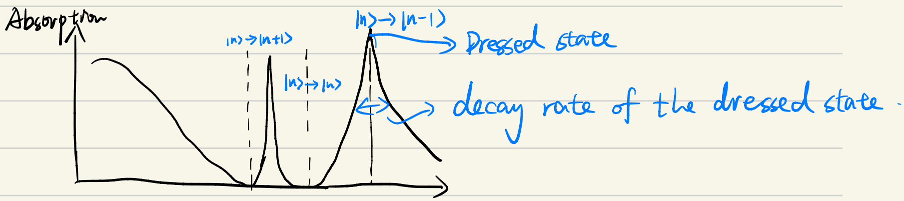
The weak 370 nm beam Ω_p is used to drive the cooling transitions: $|f, n\rangle \longleftrightarrow |\tilde{g}, n-1\rangle$.

Double EIT:

Jörg. Evers & Christoph. H. Keitel.

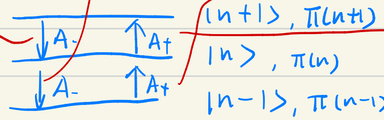


$$H = \hbar\omega a^\dagger a + \hbar\sum_{j=1}^{3} \Delta_j |j\rangle\langle j|$$
$$+ \hbar\sum_{j=1}^{3} \frac{\Omega_j}{2}\left(e^{-ik_j\cos(\phi_j)x}|e\rangle\langle j| + h.c.\right)$$



Population of motional mode: $\Pi(n)$

$$\frac{d}{dt}\Pi(n) = A_-\left[(n+1)\Pi(n+1) - n\Pi(n)\right] + A_+\left[n\Pi(n-1) - (n+1)\Pi(n)\right]$$



$$\langle n\rangle = \sum_{k=0}^{\infty} \Pi(k)\cdot k$$

$$\langle \dot n\rangle = \sum_k \frac{d}{dt}\Pi(k)\cdot k$$

$$= \sum_k A_-\left[k(k+1)\Pi(k+1) - k^2\Pi(k)\right] + A_+\left[k^2\Pi(k-1) - k(k+1)\Pi(k)\right]$$

$$= \underbrace{\sum_k A_- k\cdot(k+1)\Pi(k+1)}_{= \sum_{k=0}^{\infty} A_-(k-1)k\Pi(k)}$$
$$- \underbrace{\sum_k A_- k\cdot k\,\Pi(k)} + \underbrace{\sum_k A_+ k^2\Pi(k-1)}_{\substack{= \sum_{k=1} A_+ k^2\Pi(k-1) \\ = \sum_{k=0} A_+(k+1)^2\Pi(k)}} - \sum (k+1)k\Pi(k)A_+$$

$$-\sum_{k=0} A_- k\Pi(k) \qquad \sum_{k=0} A_+(k+1)\Pi(k)$$

$$= -A_-\langle n\rangle + A_+\langle n\rangle + A_+\sum_{k=0}\Pi(k)$$

$$= -(A_- - A_+)\langle n\rangle + A_+$$

$\Rightarrow$ cooling rate: $W = A_- - A_+$

Cooling limit: $\langle \dot{n} \rangle = 0$

$$= -W\langle n\rangle + A_+$$

$$\Rightarrow \langle n \rangle_{ss} = \frac{A_+}{A_- - A_+}$$

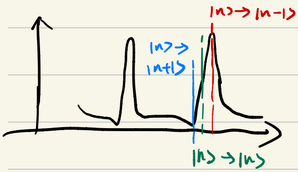
$$\boxed{\frac{A_\pm}{\eta^2} = \frac{\Omega_1^2}{\Omega_1^2+\Omega_3^2} \frac{\Gamma_3 \nu^2 \Omega_3^2}{4\{[(\Omega_1^2+\Omega_3^2)/4 - \nu(\nu\mp\Delta)] + \varepsilon_\pm\}^2 + \Gamma_3^2\nu^2}}$$

This part is only valid for the structure mentioned in previous page.

$$\varepsilon_\pm = \mp \frac{\nu\Omega_2^2}{4(\Delta-\Delta_2\mp\nu)}$$

Optimal condition: $\boxed{\begin{array}{l}\Delta_3 = \Delta_1 = \Delta \quad \Delta_2 = \Delta_1 - \nu \\ \nu = \frac{1}{2}\left(\sqrt{\Delta_1^2+\Omega_1^2+\Omega_2^2/2+\Omega_3^2} - \Delta_1\right)\end{array}}$

The derivation of $A_\pm$ here doesn't fit our exp setup which use the following structure



$$\Rightarrow \varepsilon_+ = -\frac{\nu\Omega_2^2}{4(\Delta-\Delta+\nu-\nu)} = \infty. \Rightarrow \boxed{A_+ = 0}$$

$$\varepsilon_- = \frac{\nu\Omega_2^2}{4(\Delta-\Delta+\nu+\nu)} = \frac{\nu\Omega_2^2}{8\nu} = \frac{\Omega_2^2}{8}$$

$$A_- = \eta^2 \frac{\Omega_1^2}{\Omega_1^2+\Omega_3^2} \frac{\Gamma_3\nu^2\Omega_3^2}{4\{[(\Omega_1^2+\Omega_3^2)/4 - \nu(\nu+\Delta) + \frac{\Omega_2^2}{8}\}^2 + \Gamma_3^2\nu^2}$$

$$= \frac{\eta^2\Omega_1^2}{\Omega_1^2+\Omega_3^2} \frac{\Gamma_3\nu^2\Omega_3^2}{[\underbrace{(\Omega_1^2+\Omega_3^2+\frac{\Omega_2^2}{2}) - \nu^2 - \Delta\nu}_{=0}]^2 + \Gamma_3^2\nu^2}$$

$$(\nu+\tfrac{1}{2}\Delta)^2 = \tfrac{1}{4}\Delta^2+\Omega_1^2+\Omega_3^2+\tfrac{\Omega_2^2}{2}$$

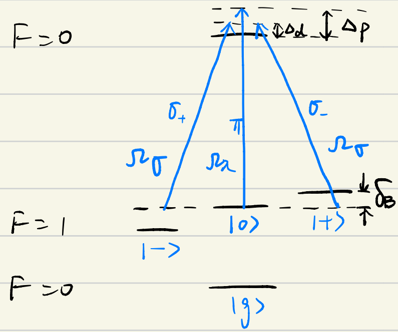
$$\Rightarrow \Omega_1^2+\Omega_3^2+\frac{\Omega_2^2}{2} = \nu^2+\nu\Delta$$

$$= \frac{\eta^2\Omega_1^2}{\Omega_1^2+\Omega_3^2} \frac{\Gamma_3\nu^2\Omega_3^2}{\Gamma_3^2\nu^2} = \boxed{\frac{\eta^2\Omega_1^2\Omega_3^2}{\Gamma_3(\Omega_1^2+\Omega_3^2)}}$$

Hamiltonian of EIT
for $^{171}Yb^+$ ion.

Ω_σ: driving (coupling) beam.
Ω_π: probe beam.

$\boxed{\text{This } \hat{H} \text{ is suitable for Red chamber}}$



set $w_{|e\rangle} = 0$, $W_{|+\rangle}, w_{|0\rangle}, w_{|-\rangle}$ are negative

$$\hat{H}/\hbar = W_{|+\rangle}|+\rangle\langle+| + W_{|0\rangle}|0\rangle\langle 0| + w_{|-\rangle}|-\rangle\langle-|$$
$$+\Big(\frac{\Omega_\sigma}{2}e^{-i(\vec{k}_d\cdot\vec{r}-w_d t)}|+\rangle\langle e| - \frac{\Omega_\pi}{2}e^{-i(\vec{k}_p\vec{r}-w_p t)}|0\rangle\langle e|$$
$$+\frac{\Omega_\sigma}{2}e^{-i(\vec{k}_d\cdot\vec{r}-w_d t)}|-\rangle\langle e| + h.c.\Big)$$

Rotating frame: $H_p/\hbar = -W_d\,|+\rangle\langle+| - w_p|0\rangle\langle 0| - w_d|-\rangle\langle-|$

$$\tilde{H}_I = e^{iH_0 t/\hbar}\,V\,e^{-iH_0 t/\hbar}$$

$$e^{iH_0 t/\hbar} = \mathbb{1}\,|e\rangle\langle e| + e^{-iw_d t}|+\rangle\langle+| + e^{-iw_p t}|0\rangle\langle 0| + e^{-iw_d t}|-\rangle\langle-|$$

$$= \mathbb{1}\,|e\rangle\langle e| + e^{-iw_d t}|+\rangle\langle+| + e^{-iw_p t}|0\rangle\langle 0| + e^{-iw_d t}|-\rangle\langle-|$$
$$\Big(\frac{\Omega_\sigma}{2}e^{-i(\vec{k}_d\cdot\vec{r}-w_d t)}|+\rangle\langle e| - \frac{\Omega_\pi}{2}e^{-i(\vec{k}_p\vec{r}-w_p t)}|0\rangle\langle e|$$
$$+\frac{\Omega_\sigma}{2}e^{-i(\vec{k}_d\cdot\vec{r}-w_d t)}|-\rangle\langle e| + h.c.\Big)$$
$$\mathbb{1}\,|e\rangle\langle e| + e^{iw_d t}|+\rangle\langle+| + e^{iw_p t}|0\rangle\langle 0| + e^{iw_d t}|-\rangle\langle-|$$

$$= \frac{\Omega_\sigma}{2} e^{-i\vec{k}_d\cdot\vec{r}} |+\rangle\langle e| - \frac{\Omega_\pi}{2} e^{-i\vec{k}_p\cdot\vec{r}} |0\rangle\langle e|$$

$$+ \frac{\Omega_\sigma}{2} e^{-i\vec{k}_d\cdot\vec{r}} |-\rangle\langle e| + h.c.$$

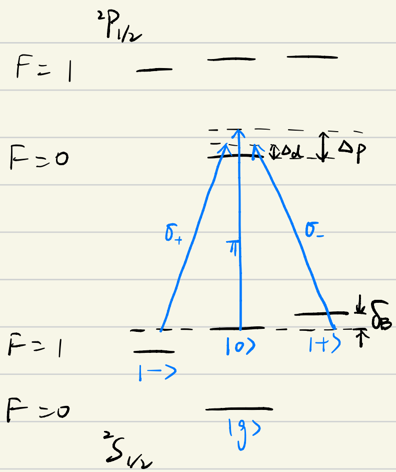
Diagonal term: $H_0 - H_D = (\Delta_d + \delta_B) |+\rangle\langle+| + \Delta p |0\rangle\langle 0|$

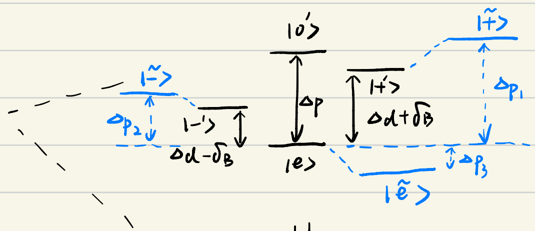
$$+ (\Delta_d - \delta_B) |-\rangle\langle-|$$

Full Hamiltonian:

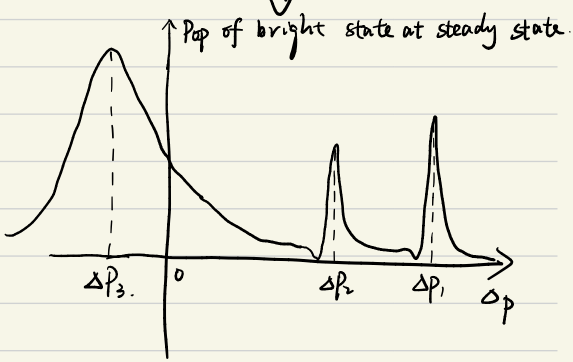
$$\tilde{H} = H_0 - H_D + \tilde{H}_I$$

Energy levels in the Rotating frame:

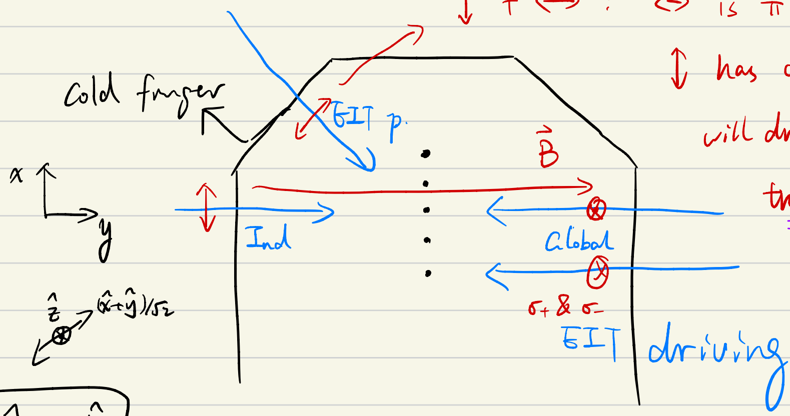




$\Downarrow$



Polarization configurations for $\boxed{\text{Cryo}}$



$\updownarrow + \leftrightarrow$: $\leftrightarrow$ is π, which is needed

$\updownarrow$ has components of σ_+ & σ_-, will drive unwanted transition

The only difference of cryo and Red chamber is the unwanted $\updownarrow$ transition

$$\boxed{\hat{x} = \hat{y}}$$

$$\vec{E}_p = \left(\frac{\sqrt{2}}{2} E_0\,\hat{x} + \frac{\sqrt{2}}{2} E_0\,\hat{y}\right)\cos(kz-\omega t)$$

$$= \frac{\sqrt{2}}{2} E_0\,\hat{x} + \frac{\sqrt{2}}{2} E_0 \left(\frac{\hat{\sigma}^+ + \hat{\sigma}^-}{2}\right)$$

$$\vec{E}_c = E_1\,\hat{z}\cdot\cos(kz-\omega t)$$

$$= \frac{E_1}{2}\hat{z}\cos(kz-\omega t) + \frac{E_1}{2}\hat{x}\sin(kz-\omega t)$$

$$+ \frac{E_1}{2}\hat{z}\cos(kz-\omega t) - \frac{E_1}{2}\hat{x}\sin(kz-\omega t)$$

$$= \frac{E_1}{2}(\hat{\sigma}_+ + \hat{\sigma}_-)$$

Hamiltonian for Cryo chamber.

$$\hat{H}/\hbar = \omega_{|+\rangle} |+\rangle\langle+| + \omega_{|0\rangle} |0\rangle\langle 0| + \omega_{|-\rangle} |-\rangle\langle-|$$

$$+\Big\{ \Big[\frac{\Omega_{1+}}{2} e^{-i(\vec{k}_d\cdot\vec{r} - \omega_d t)} + \underbrace{\frac{\Omega_{2+}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)}}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i(\vec{k}_d\cdot\vec{r} - \omega_d t)} + \underbrace{\frac{\Omega_{2-}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)}}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |0\rangle\langle e| + h.c. \Big\}$$

Extra terms compared with Red chamber

Rotating frame: $H_0/\hbar = -\omega_d |+\rangle\langle+| - \omega_p |0\rangle\langle 0| - \omega_d |-\rangle\langle-|$

$$\tilde{V}_I = e^{iH_0 t/\hbar} V e^{-iH_0 t/\hbar}$$

$$e^{iH_0 t/\hbar} = \mathbb{1} |e\rangle\langle e| + e^{-i\omega_d t} |+\rangle\langle+| + e^{-i\omega_p t} |0\rangle\langle 0| + e^{-i\omega_d t} |-\rangle\langle-|$$

$$= \mathbb{1} |e\rangle\langle e| + e^{-i\omega_d t} |+\rangle\langle+| + e^{-i\omega_p t} |0\rangle\langle 0| + e^{-i\omega_d t} |-\rangle\langle-|$$

$$\Big\{ \Big[\frac{\Omega_{1+}}{2} e^{-i(\vec{k}_d\cdot\vec{r} - \omega_d t)} + \frac{\Omega_{2+}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i(\vec{k}_d\cdot\vec{r} - \omega_d t)} + \frac{\Omega_{2-}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i(\vec{k}_p\cdot\vec{r} - \omega_p t)} |0\rangle\langle e| + h.c. \Big\}$$

$$\mathbb{1} |e\rangle\langle e| + e^{i\omega_d t} |+\rangle\langle+| + e^{i\omega_p t} |0\rangle\langle 0| + e^{i\omega_d t} |-\rangle\langle-|$$

$$= \Big[\frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\vec{r}} + \underbrace{\frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}} \cdot e^{-i(\omega_d - \omega_p)t}}\Big] |+\rangle\langle e|$$

$$+ \Big[\frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \underbrace{\frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\vec{r}} e^{-i(\omega_d - \omega_p)t}}\Big] |-\rangle\langle e|$$

$$- \frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}} |0\rangle\langle e| + h.c. \Big\}$$

$$\frac{\hat{H}_I}{\hbar} = (H - H_D + \hat{V}_I)/\hbar$$

$$= (\Delta_d + \delta_B)|+\rangle\langle+| + \Delta_p |0\rangle\langle 0| + (\Delta_d - \delta_B)|-\rangle\langle-|$$

$$\Big[\frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\vec{r}} + \frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}} \cdot e^{-i(\omega_d-\omega_p)t}\Big]|+\rangle\langle e|$$

$$+\Big[\frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t}\Big]|-\rangle\langle e|$$

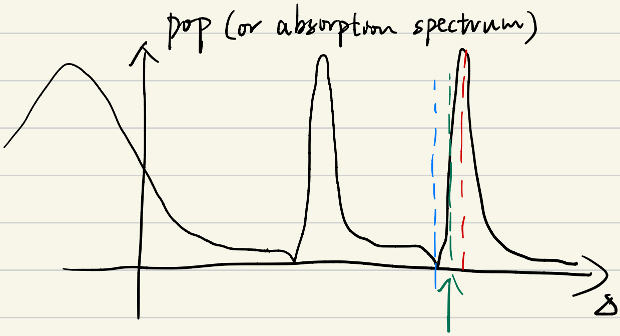
$$- \frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}} |0\rangle\langle e| + h.c. \Big\}$$

$\omega_d - \omega_p$

$= (\omega_{e0} + \Delta_d) - (\omega_{e0} + \Delta_p$

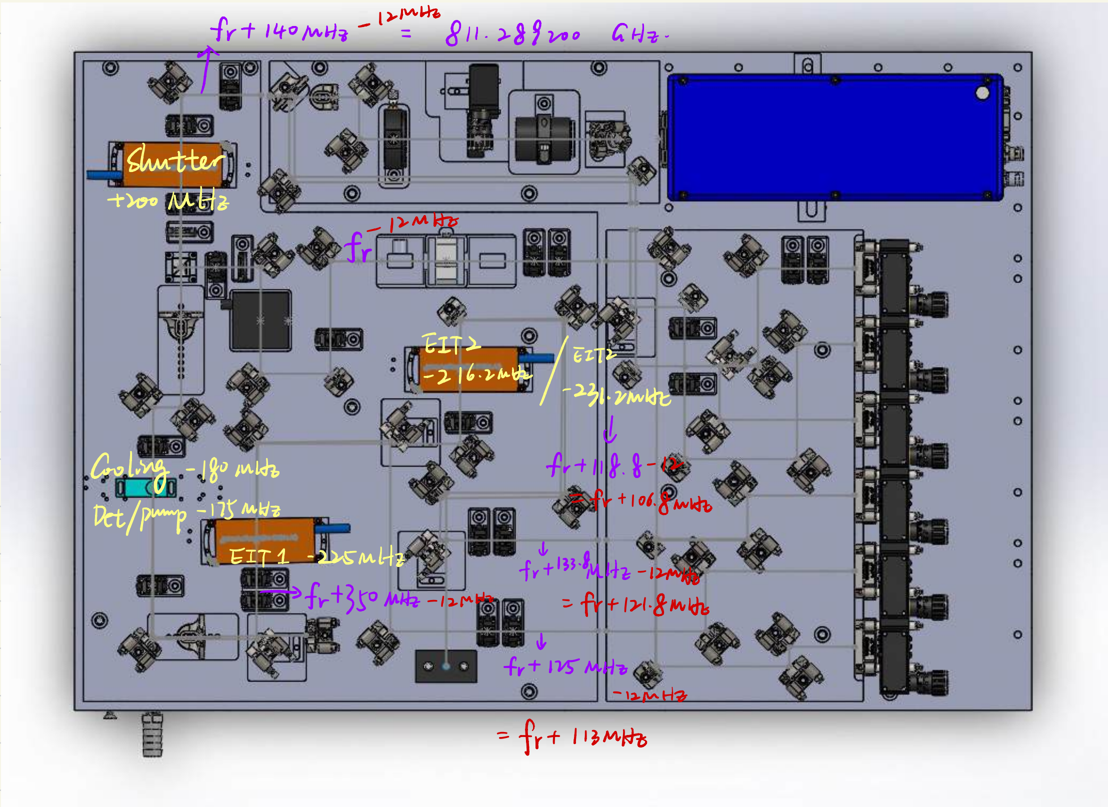
$= \Delta_d - \Delta_p.$

$$= \begin{pmatrix} 0 & \frac{\Omega_{1+}}{2} e^{i\vec{k}_d\vec{r}} + \frac{\Omega_{2+}}{2} e^{i\vec{k}_p\cdot\vec{r}} e^{i(\omega_d-\omega_p)t} & -\frac{\Omega_{2\pi}}{2} e^{i\vec{k}_p\cdot\vec{r}} & \frac{\Omega_{1-}}{2} e^{i\vec{k}_d\vec{r}} + \frac{\Omega_{2-}}{2} e^{i\vec{k}_p\vec{r}} e^{i(\omega_d-\omega_p)t} \\ \frac{\Omega_{1+}}{2} e^{-i\vec{k}_d\vec{r}} + \frac{\Omega_{2+}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t} & \Delta_d + \delta_B & 0 & 0 \\ -\frac{\Omega_{2\pi}}{2} e^{-i\vec{k}_p\cdot\vec{r}} & 0 & \Delta_p & 0 \\ \frac{\Omega_{1-}}{2} e^{-i\vec{k}_d\cdot\vec{r}} + \frac{\Omega_{2-}}{2} e^{-i\vec{k}_p\cdot\vec{r}} e^{-i(\omega_d-\omega_p)t} & 0 & 0 & \Delta_d - \delta_B \end{pmatrix}$$



This is where we set Δp. for EIT cooling

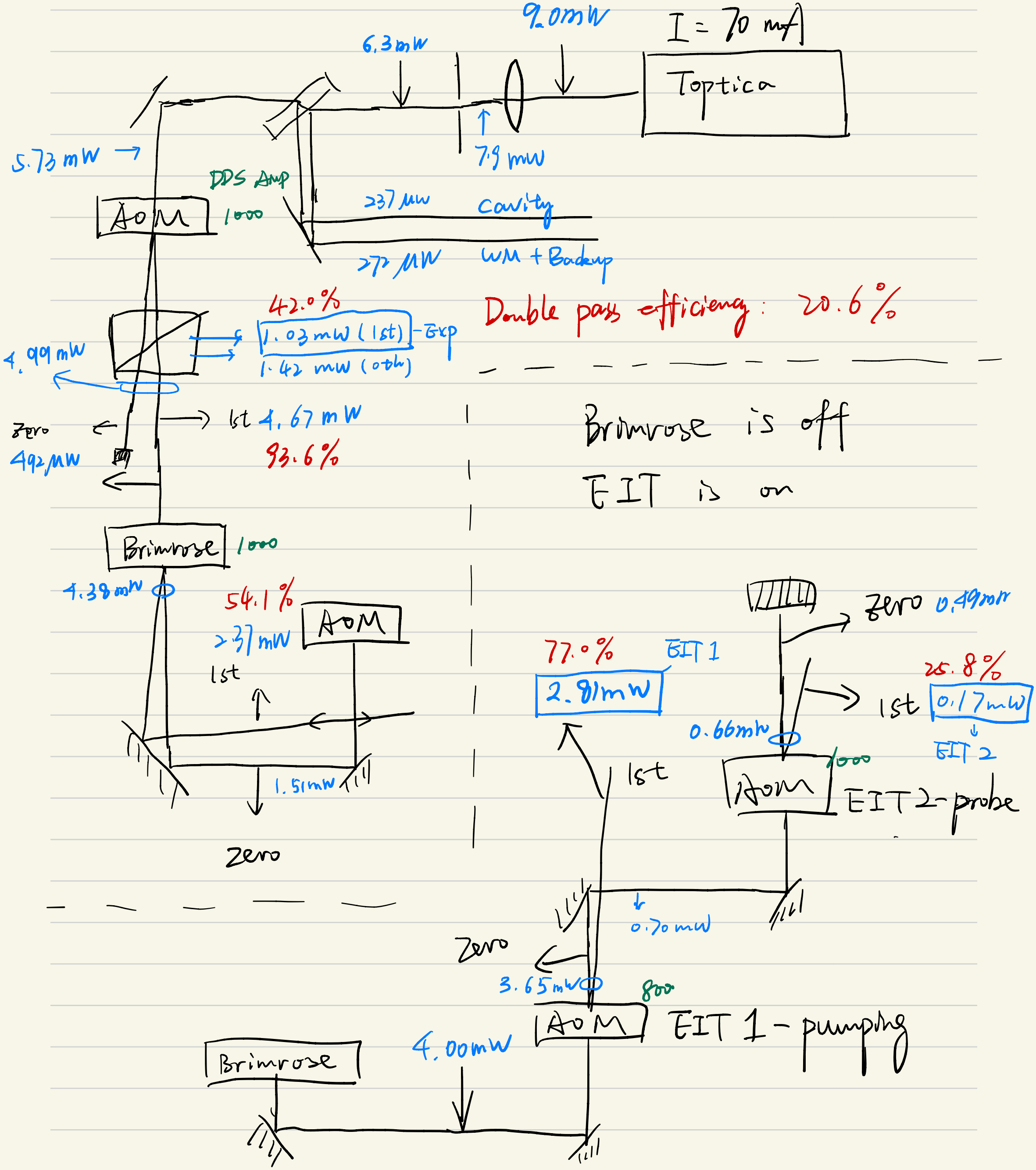
EIT Plate Design (Only for Cryo)



Reasons of −12MHz:

When detecting, the 370 should be on resonance. But due to the laser broadening, it's better to be red detunned. So a good point is −12MHz away from resonance.

Power budget



$\boxed{\text{CPMG}}$ Ref: Ye Wang's thesis.

Hamiltonian: $H = \frac{\hbar}{2}(\omega_0 + \beta(t))\,\sigma_z$, ω_0 is the splitting of the qubit.

$\beta(t) = \beta_B(t) + \beta_{LO}(t)$

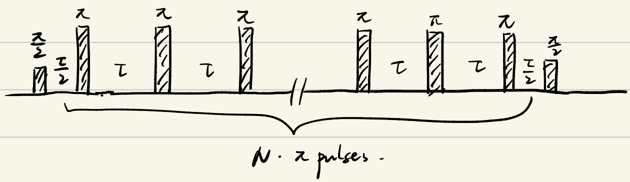
$\beta_{LO}(t)$ ↳ phase noise of qubit operational microwave.

$\beta_B(t)$ ↓ magnetic field fluctuation

Dynamical decoupling (DD):

preserve the qubit coherence against random phase noise.

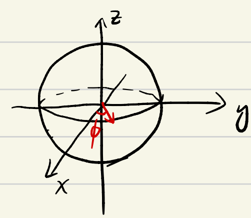
CPMG pulses:



Calculation of the filter function:

All the rotation axis used are in x-y plane, the rotation is $D_\phi(\gamma) = D_z(\phi)\, D_x(\gamma)\, D_z(-\phi)$

$= e^{-\frac{i}{2}\sigma_z\phi} \cdot e^{-\frac{i}{2}\sigma_x\gamma} \cdot e^{\frac{i}{2}\sigma_z\phi}$



For CPMG, $\gamma = \pi$

$D_x(\gamma) = e^{-\frac{i}{2}\cdot\pi\sigma_x}$

$= \cos(\pi/2)\,\mathbb{1} + i\sigma_x \sin(\pi/2) = -i\sigma_x$

$D_\phi(\pi) = e^{-\frac{i}{2}\sigma_z\phi}(-i\sigma_x) \cdot e^{\frac{i}{2}\sigma_z\phi}$

Initial state $|\psi(0)\rangle = |\downarrow\rangle$

↓ CPMG.

Final state $|\psi(T)\rangle = D_x(\pi/2) \cdot \tilde{R}(T)\, D_x(\pi/2)\, |\psi(0)\rangle$

During the waiting time: $e^{-i\int_{t_1}^{t_2}\beta(\tau)\sigma_z d\tau}$

$$\Rightarrow \tilde{R}(T) = e^{-i\sigma_z\int_{T_N}^{T_{N+1}}\beta(t)dt}\cdot D_{\phi_N}(\pi)\cdot$$

Last $\frac{T}{2}$ waiting

$$e^{-i\sigma_z\int_{T_{N-1}}^{T_N}\beta(t)dt}\cdot D_{\phi_{N-1}}(\pi)\cdot \cdots$$

$$e^{-i\sigma_z\int_{T_1}^{T_2}\beta(t)dt}\, D_{\phi_1}(\pi)\cdot \underbrace{e^{-i\sigma_z\int_{T_0}^{T_1}\beta(t)dt}}_{\text{First } T/2 \text{ waiting}}$$

$$= (-i)^N\, e^{-i\sigma_z\int_{T_N}^{T_{N+1}}\beta(t)dt}\cdot e^{-\frac{i}{2}\sigma_z\phi_N}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_N}\cdot$$

$$e^{-i\sigma_z\int_{T_{N-1}}^{T_N}\beta(t)dt}\cdot e^{-\frac{i}{2}\sigma_z\phi_{N-1}}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_N}\cdot$$

$$\cdots$$

$$e^{-i\sigma_z\int_{T_1}^{T_2}\beta(t)dt}\cdot e^{-\frac{i}{2}\sigma_z\phi}\,\sigma_x\, e^{\frac{i}{2}\sigma_z\phi_1}.$$

$$e^{-i\sigma_z\int_{T_0}^{T_1}\beta(t)dt}.$$

$$= (-i)^N\, e^{-i\sigma_z\left(\int_{T_N}^{T_{N+1}}\beta(t)dt+\phi_N/2\right)}\cdot\sigma_x$$

global parameter

$$e^{-i\sigma_z\left(\int_{T_{N-1}}^{T_N}\beta(t)dt-\phi_N/2+\phi_{N-1}/2\right)}\cdot\sigma_x$$

$$\cdots$$

$$e^{-i\sigma_z\left(\int_{T_1}^{T_2}\beta(t)dt-\phi_2/2+\phi_1/2\right)}\,\sigma_x\,\cdot$$

$$e^{-i\sigma_z\left(\int_{T_0}^{T_1}\beta(t)dt-\phi_1/2\right)}$$

Because N is even, and only σ_x can flip the state $|\downarrow\rangle$, σ_z only add phases.

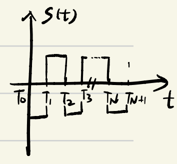
$$\Rightarrow \begin{cases} \tilde{R}(T)\,|\downarrow\rangle = e^{-iF_N(T)}\,|\downarrow\rangle \\ \tilde{R}(T)\,|\uparrow\rangle = e^{+iF_N(T)}\,|\uparrow\rangle \\ \tilde{R}(T) = e^{iF_N(T)\sigma_z}. \end{cases}$$

where
$$F_N(T) = -\left(\int_{T_0}^{T_1}\beta(t)dt-\phi_1/2\right) + \left(\int_{T_1}^{T_2}\beta(t)dt+\phi_1/2-\phi_2/2\right) \cdots + (-1)^n\left(\int_{T_{n-1}}^{T_n}\beta(t)dt+\phi_{n-1}/2-\phi_n/2\right) + (-1)^{n+1}\left(\int_{T_n}^{T_{n+1}}\beta(t)dt+\phi_n/2-\phi_{n+1}/2\right)$$
$$= \sum_{i=0}^{N}(-1)^{i+1}\int_{T_i}^{T_{i+1}}\beta(t)dt + \sum_{i=1}^{N}(-1)^{i+1}\phi_i.$$

For CPMG, $\phi_i=0$, $\sum_{i=1}^{N}(-1)\,\phi_i = 0$.

$$F_N(T) = \sum_{i=0}^{N}(-1)^{i+1}\int_{T_i}^{T_{i+1}}\beta(t)dt.$$
$$\equiv \boxed{\int_{-\infty}^{\infty} s_N(t')\,\beta(t')\,dt'}$$

where $s_N(t) = \begin{cases} 0 & t\le 0 \\ (-1)^j & T_j<t<T_{j+1} \\ 0 & t>T \end{cases}$



Measured result:

$$\left\langle \langle \psi(T) | \sigma_z | \psi(T) \rangle \right\rangle = \left\langle \langle \psi(0) | D_x(\pi/2)^{+} \tilde{R}(T)^{+} D_x(\pi/2)^{+} \sigma_z D_x(\pi/2) \tilde{R}(T) D_x(\pi/2) | \psi(0) \rangle \right\rangle$$

$$= \left\langle \langle \downarrow | e^{+\frac{i\pi}{4}\sigma_x} e^{-iF_N(T)\sigma_z} e^{\frac{i\pi}{4}\sigma_x} \sigma_z e^{-\frac{i\pi}{4}\sigma_x} \underbrace{e^{iF_N(T)\sigma_z} \underbrace{e^{-\frac{i\pi}{4}\sigma_x} |\downarrow\rangle}_{\frac{1}{\sqrt{2}}(|\downarrow\rangle - i|\uparrow\rangle)}}_{\frac{1}{\sqrt{2}}(e^{-iF_N(T)}|\downarrow\rangle - i e^{iF_N(T)}|\uparrow\rangle)} \right\rangle$$

$$\frac{1}{2}\left(e^{-iF_N(T)}(|\downarrow\rangle - i|\uparrow\rangle - i e^{iF_N(T)}(|\uparrow\rangle - i|\downarrow\rangle)\right)$$

$$= \frac{1}{2}\left(-2i\sin(F_N(T))|\downarrow\rangle - 2i\cos(F_N(T))|\uparrow\rangle \right)$$

$$= -i\left(\sin(F_N(T))|\downarrow\rangle + \cos(F_N(T))|\uparrow\rangle \right)$$

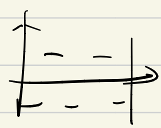
$$= \left\langle \left[\sin(F_N(T))\langle\downarrow| + \cos(F_N(T))\langle\uparrow| \right] \left[-\sin F_N(T)|\downarrow\rangle + \cos F_N(T)|\uparrow\rangle \right] \right\rangle$$

$$= \left\langle -\sin^2 + \cos^2 \right\rangle = \langle \cos(2F_N(T)) \rangle$$

$$\approx e^{2\langle F_N(T)^2 \rangle}$$

Calculate the Fourier transform of $F_N(T)$:

$$\tilde{y}(w,T) \equiv \mathcal{F}(S_N(t')) = \int_{-\infty}^{\infty} S_N(t') e^{iwt'} dt'$$



$$= \int_{T_0}^{T_1} -e^{iwt'} dt' + \int_{T_1}^{T_2} e^{iwt'} dt'$$

$$+ (-1)\cdot \int_{T_2}^{T_3} e^{iwt'} dt' + \int_{T_3}^{T_4} e^{iwt'} dt'$$

$$+ \cdots + (-1)^{N+1} \int_{T_N}^{T_{N+1}} e^{iwt'} dt'$$

$$= \frac{1}{iW}(-1)^1 \cdot \left(e^{iWT_1} - e^{iWT_0}\right) + \frac{1}{iW}\left(e^{iWT_2} - e^{iWT_1}\right) + \cdots$$

$$= \frac{1}{iW}\left[1 - e^{iWT_1} - e^{iWT_1} + e^{iWT_2} + e^{iWT_2} + \cdots + (-1)^N e^{iWT_N} \times 2 + (-1)^{N+1} e^{iWT_{N+1}} \right.$$

$$= \frac{1}{iW}\left[1 + \sum_{n=1}^{N} 2\cdot(-1)^n e^{iWT_n} + (-1)^{N+1} e^{iWT_{N+1}} \right]$$

$$\Rightarrow \langle F_N(T)^2 \rangle = \langle f(F_N(T)^2) \rangle = \frac{1}{\pi} \int_{-\infty}^{\infty} S_\beta(\omega) \cdot |\tilde{y}(\omega, T)|^2 d\omega .$$

$$\boxed{S_\beta(\omega) \equiv \int_{-\infty}^{\infty} \beta(t') e^{i\omega t'} dt'}$$

The noise spectrum, what we expect to know.

$|\tilde{y}(\omega, T)|^2$: filter function.

If Assume the noise is purely coherent:

$$\tilde{\beta}(\omega) = \sum_{k=1}^{n} \beta_k \delta(\omega - \omega_k)$$

Then what we measure is

$$\langle \cos(2F_N(T)) \rangle = \prod_k^n J_0(|\beta_k \tilde{y}(\omega_k, T)|)$$

n Eqns.

$$\begin{cases} J_0(|\beta_1 \cdot \tilde{y}(\omega_1, T_1)|) \cdot J_0(\beta_2 \cdot \tilde{y}(\omega_2, T_1)|) \cdots J_0(\beta_n \cdot \tilde{y}(\omega_n, T_1)) = \langle M_z(T_1) \rangle \\ J_0(|\beta_1 \tilde{y}(\omega_1, T_2)|) \cdot J_0(\beta_2 \tilde{y}(\omega_2, T_2)|) \cdots J_0(\beta_n \tilde{y}(\omega_n, T_2)) = \langle M_z(T_2) \rangle \\ \vdots \\ J_0(|\beta_1 \tilde{y}(\omega_1, T_n)|) \cdot J_0(\beta_2 \tilde{y}(\omega_2, T_n)|) \cdots J_0(\beta_n \tilde{y}(\omega_n, T_n)|) = \langle M_z(T_n) \rangle \end{cases}$$

Assume $\beta(t) = \beta_0 \cos(\omega_0 t + \phi)$ $\leftarrow$ One example coherent noise.

$$S_\beta(\omega) = \int_{-\infty}^{\infty} \beta_0 \cos(\omega_0 t + \phi)\, e^{i\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} \frac{\beta_0}{2} \left(e^{i(\omega_0 t+\phi)} + e^{-i(\omega_0 t + \phi)}\right) e^{i\omega t}\, dt$$

$$= \frac{\beta_0}{2} \int_{-\infty}^{\infty} e^{i[(\omega_0+\omega)t+\phi]} + e^{i[(\omega-\omega_0)t-\phi]}\, dt$$

$$= \frac{\beta_0}{2} \left[\frac{1}{i(\omega_0+\omega)} e^{i[(\omega_0+\omega)t+\phi]}\Big|_{-\infty}^{\infty} + \frac{1}{i(\omega-\omega_0)} e^{i[(\omega-\omega_0)t-\phi]}\Big|_{-\infty}^{\infty} \right]$$

$$= \frac{\beta_0}{2} \left[\delta(\omega+\omega_0) + \delta(\omega-\omega_0) \right]$$

$$F_N(T) = \sum_{i=0}^{N} (-1)^{i+1} \int_{T_i}^{T_{i+1}} \beta(t)\, dt + \sum_{i=1}^{N} (-1)^{i+1} \phi_i$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \int_{T_i}^{T_{i+1}} \beta_0 \cos(\omega_0 t + \phi)\, dt + \underbrace{\sum_{i=1}^{N} (-1)^{i+1} \phi_i}_{\phi_i = 0 \text{ for CPMG.}}$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \frac{\beta_0}{\omega_0} \sin(\omega_0 t + \phi)\Big|_{T_i}^{T_{i+1}}$$

$$= \sum_{i=0}^{N} (-1)^{i+1} \frac{\beta_0}{\omega_0} \left[\sin(\omega_0 T_{i+1} + \phi) - \sin(\omega_0 T_i + \phi)\right]$$

where $T_0 = 0$, $T_1 = \frac{\tau}{2}$, $T_2 = \frac{3}{2}\tau$, ---- $T_n = (n-\frac{1}{2})\tau$, ---
$T_N = (N-\frac{1}{2})\tau$, $T_{N+1} = N\tau$.

$$= \frac{\beta_0}{\omega_0} \Big\{ -[\sin(\omega_0 T_1 + \phi) - \sin(\cancel{\omega_0 T_0}^{\,0} + \phi)]$$
$$+ [\sin(\omega_0 T_2 + \phi) - \sin(\omega_0 T_1 + \phi)]$$
$$---$$
$$+ (-1)^{N+1} [\sin(\omega_0 T_{N+1} + \phi) - \sin(\omega_0 T_N + \phi)] \Big\}$$

N is even

$$= \frac{\beta_0}{\omega_0} \Big\{ \sin\phi + \sin(\omega_0 \cdot N\tau + \phi) + \sum_{i=1}^{N} 2 \cdot (-1)^i \sin[\omega_0 (i-\tfrac{1}{2})\tau + \phi] \Big\}$$

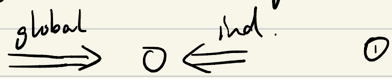
$\langle \cos(2F_N(\tau)) \rangle$ = average over phase ϕ

MS Gate

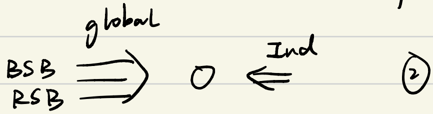
MS gate theory can be found everywhere. Here I just put some tips when running experiments.

1. Freq calibration.

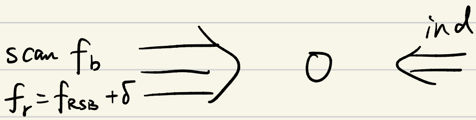
Normal BSB / RSB calibration is implemented by fixing the ind freq while scanning the global beam.



However, when implementing MS gate / kick operation, the pulses those are used are as follows, which has 3 tones in total.



These two configurations (① & ②) have different Stark shift, therefore, the calibrated BSB / RSB freq is INCORRECT when running MS gate. Thus, when scanning BSB / RSB for MS gate / kick operations, we should:



— Apply 3 tones when scanning BSB (for example) rather than only two tones

- Individual
- Global tone 0: scan f_b
- Global tone 1: Apply a far detunned RSB $\delta \sim 300\text{kHz}$.

The amplitude of these 3 tones should be the same as MS gate.

E.g. —[H]—[Detuned SQ gate]—[Detuned Kick]—[H]—

Laser: δ_{ss-sq} $\quad$ $\delta_{ss-sq}+\delta_{det-sq}$ $\quad$ $\delta_{cal-B/R}+\delta_{det-kick}$ $\quad$ δ_{ss-sq}

Qubit: δ_{ss-sq} $\quad$ $\cancel{\delta_{det-ss-sq}}$ δ_{ss-sq} $\quad$ $\cancel{\delta_{det-B/R}}$ $\delta_{ss-B/R}$ $\quad$ δ_{ss-sq}.

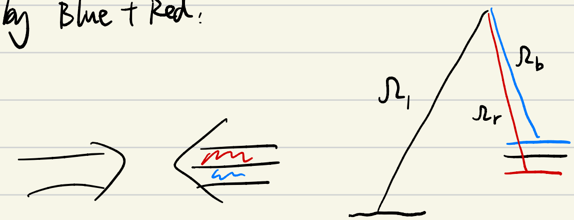
Stark shift w/ detuned SQ gate

Stark shift w/ detuned BSB/RSB transitions.

Target: Make sure calibrated $\delta_{cal\_B/R} = \delta_{ss-B/R}$.

Found extra stark shift due to detuning is negligible.

SS by Blue + Red:



2-photon:

$$\delta_{2s} = \frac{\Omega_b^2}{4(\Delta-\omega_{HF}-\nu)} + \frac{\Omega_r^2}{4(\Delta-\omega_{HF}+\nu)} - \frac{\Omega_1^2}{4\Delta}$$

$$= \frac{1}{4\Delta}\left(\frac{\Omega_b^2}{1-\frac{\omega_{HF}+\nu}{\Delta}} + \frac{\Omega_r^2}{1-\frac{\omega_{HF}-\nu}{\Delta}} - \Omega_1^2\right)$$

$$= \frac{1}{4\Delta}\left(\Omega_b^2+\Omega_r^2-\Omega_1^2 + \Omega_b^2\frac{\omega_{HF}-\nu}{\Delta} + \Omega_r^2\frac{\omega_{HF}+\nu}{\Delta}\right)$$

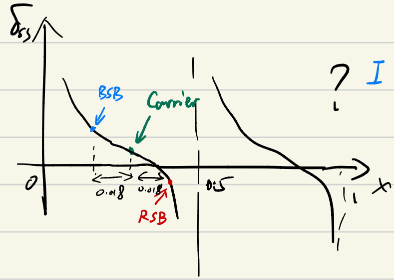
since $\Omega^2 \propto I$, $I_b + I_r = I_c$ (carrier), $\Omega_b^2 = \Omega_r^2 \simeq \frac{\Omega_c^2}{2}$

$$\approx \frac{1}{4\Delta}\left(\Omega_c^2 - \Omega_1^2 + \Omega_c^2\frac{\omega_{HF}}{\Delta}\right)$$

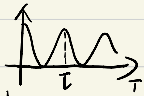
2-photon SS for carrier: $$\delta_{2s-c} = \frac{\Omega_c^2}{4(\Delta-\omega_{HF})} - \frac{\Omega_1^2}{4\Delta} = \frac{1}{4\Delta}\left(\frac{\Omega_c^2}{1-\frac{\omega_{HF}}{\Delta}} - \Omega_1^2\right)$$
$$= \frac{1}{4\Delta}\left(\Omega_c^2-\Omega_1^2+\Omega_c^2\frac{\omega_{HF}}{\Delta}\right)$$ $= \delta_{2s-c}$

4-photon: Single beam $\delta_{4s} \approx 0$ for linear polarization

Inter beam:



? I guess it should follow this logic but I didn't calculate it clearly. So I measured it in the next page.

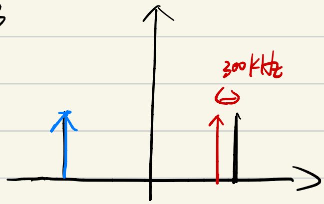
Method: Ramsey $\boxed{\pi/2}$ — $\boxed{\text{wait } T}$ — $\boxed{\pi/2}$ $\Rightarrow$ 

, $|\delta| = \frac{1}{\tau}$

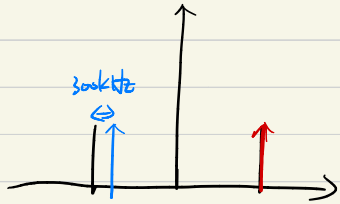
where $\boxed{\pi/2}$ are the pulses listed below

Blue/Red arrows represent the tones applied when implementing $\pi/2$

① BSB



② RSB



$\delta_{BSB} = \delta_{B\text{-}res} + \underbrace{\delta_{R\text{-}det}}_{-4.31\ \text{kHz}}$ $= 1.57$ kHz

$\delta_{RSB} = \underbrace{\delta_{B\text{-}det}}_{5.6\ \text{kHz}} + \delta_{R\text{-}res}$ $= 2.6$ kHz

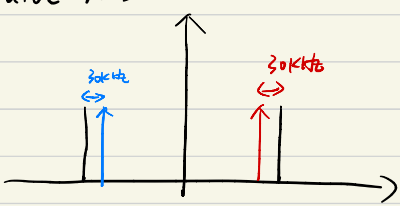
③ BSB only

$\delta_B = \delta_{B\text{-}res}$ $= 5.88$ kHz

④ RSB only

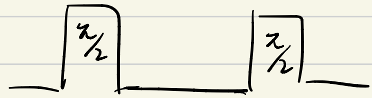
$\delta_R = \delta_{R\text{-}res}$. $= -3$ kHz.

⑤ Naive MS



$\delta_{MS} = \delta_{B\text{-}MS} + \delta_{R\text{-}MS}$

Ramsey:



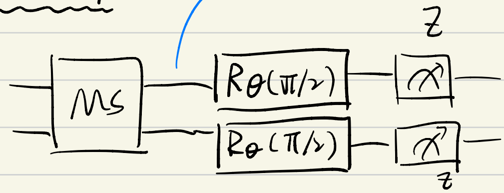
Laser:	δ_{ss}	δ_{ss}	δ_{ss}
Ion:	δ_{ss}	0	δ_{ss}.

So we can meas δ_{ss}.

Laser tone keeps oscillating during both $\pi/2$ and wait at $f = f_0 + \delta_{ss}$, while the ion oscillates at $f = f_0 + \delta_{ss}$ when $\pi/2$ but $f = f_0$ when waiting.

$|\psi\rangle = \frac{\sqrt{2}}{2}\left(|00\rangle + i|11\rangle\right)$, $\rho = \frac{1}{2}\begin{pmatrix} 1 & & & i \\ & 0 & 0 & \\ & 0 & 0 & \\ -i & & & 1 \end{pmatrix}$

2. parity scan tips.



Fidelity:

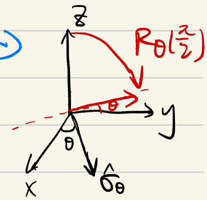
$F = \frac{\rho_{|00\rangle\langle 00|} + \rho_{|11\rangle\langle 11|}}{2} + \left|\rho_{|00\rangle\langle 11|}\right|$

↓ population ↓ parity scan.

$\langle ZZ \rangle$

$= \mathrm{Tr}\left[Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{(2)}\left(\tfrac{\pi}{2}\right) \rho \cdot R_\theta^{+(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{+(2)}\left(\tfrac{\pi}{2}\right) \right]$

$= \mathrm{Tr}\left[\rho \, R_\theta^{+(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{+(2)}\left(\tfrac{\pi}{2}\right) Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_\theta^{(2)}\left(\tfrac{\pi}{2}\right) \right]$



$= \left\langle \left(\cos\theta\, \hat{Y}^{(1)} - \sin\theta\, \hat{X}^{(1)}\right)\left(\cos\theta\, \hat{Y}^{(2)} - \sin\theta\, \hat{X}^{(2)}\right)\right\rangle$

$= \left\langle \cos^2\theta\, Y_1 Y_2 - \sin\theta\cos\theta\left(Y_1 X_2 + X_1 Y_2\right) + \sin^2\theta\, X_1 X_2 \right\rangle$

$= \mathrm{Tr}\left[\sin^2\theta \begin{pmatrix} \rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & \rho_{|11\rangle\langle 00|} \end{pmatrix} + \cos^2\theta \begin{pmatrix} -\rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & -\rho_{|11\rangle\langle 00|} \end{pmatrix} - \sin 2\theta \begin{pmatrix} i\rho_{|00\rangle\langle 11|} & & & \\ & & & \\ & & & \\ & & & -i\rho_{|11\rangle\langle 00|} \end{pmatrix} \right]$

$P = \left|\rho_{|00\rangle\langle 11|}\right|$

$= 2\cos 2\theta \underbrace{\mathrm{Re}\left(\rho_{|00\rangle\langle 11|}\right)}_{\alpha} + 2\sin 2\theta \underbrace{\mathrm{Im}\left(\rho_{|00\rangle\langle 11|}\right)}_{\beta} + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 10|}\right)$

$\sqrt{\alpha^2+\beta^2} = P \Rightarrow \alpha \equiv \sin\varphi \cdot P$

$= 2\cos 2\theta \, P \cdot \sin\varphi + 2\sin 2\theta \cdot P \cdot \cos\varphi + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 10|}\right)$

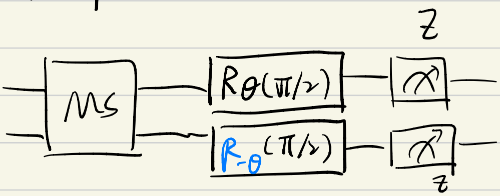
$= 2P \sin(2\theta + \varphi) + 2\mathrm{Re}\left(\rho_{|01\rangle\langle 10|}\right)$

← Contrast.

$\boxed{\text{Parity scan } -\theta}$

If preparing other Bell state,

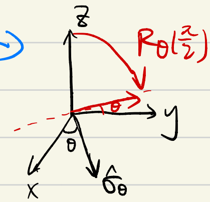
e.g. $\rho = \begin{pmatrix} 0 & & & 0 \\ & 1 & 1 & \\ & 1 & 1 & \\ 0 & & & 0 \end{pmatrix}$



parity scan needs to capture $|\rho_{|01\rangle\langle10|}|$

$$\mathrm{Tr}\left[\, Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_{-\theta}^{(2)}\left(\tfrac{\pi}{2}\right) \rho \cdot R_\theta^{\dagger(1)}\left(\tfrac{\pi}{2}\right) R_{-\theta}^{\dagger(2)}\left(\tfrac{\pi}{2}\right) \right]$$

$$= \mathrm{Tr}\left[\, \rho\, \underline{R_\theta^{\dagger(1)}\left(\tfrac{\pi}{2}\right) R_{-\theta}^{(2)}\left(\tfrac{\pi}{2}\right) Z^{(1)} Z^{(2)} R_\theta^{(1)}\left(\tfrac{\pi}{2}\right) R_{-\theta}^{(2)}\left(\tfrac{\pi}{2}\right)} \right]$$



$$= \langle (\cos\theta\, \hat{Y}^{(1)} - \sin\theta\, \hat{X}^{(1)})(\cos\theta\, \hat{Y}^{(2)} + \sin\theta\, \hat{X}^{(2)}) \rangle$$

$$= \langle \cos^2\theta\, Y_1 Y_2 + \sin\theta\cos\theta\, (Y_1 X_2 - X_1 Y_2) - \sin^2\theta\, X_1 X_2 \rangle$$

$$= \mathrm{Tr}\left[-\sin^2\theta \begin{pmatrix} \rho_{|00\rangle\langle11|} & & & \\ & \rho_{|01\rangle\langle10|} & & \\ & & \rho_{|10\rangle\langle01|} & \\ & & & \rho_{|11\rangle\langle00|} \end{pmatrix} + \cos^2\theta \begin{pmatrix} -\rho_{|00\rangle\langle11|} & & & \\ & \rho_{|01\rangle\langle10|} & & \\ & & \rho_{|10\rangle\langle01|} & \\ & & & -\rho_{|11\rangle\langle00|} \end{pmatrix} \right.$$

$$\left. + \sin 2\theta \begin{pmatrix} 0 & & & \\ & i\rho_{|01\rangle\langle10|} & & \\ & & -i\rho_{|10\rangle\langle01|} & \\ & & & 0 \end{pmatrix} \right]$$

$$= -2\,\mathrm{Re}(\rho_{|00\rangle\langle11|}) + \cos 2\theta\, \mathrm{Re}(\rho_{|01\rangle\langle10|}) + \sin 2\theta\, \mathrm{Im}(\rho_{|01\rangle\langle10|})$$

$$= -2\,\mathrm{Re}(\rho_{|00\rangle\langle11|}) + P \cdot \sin(2\theta + \varphi),$$ where $P = |\rho_{|01\rangle\langle10|}|$

$$\sin\varphi = \mathrm{Re}(\rho_{|01\rangle\langle10|})/P$$

Calculation supplementary:

Because we want to know the trace of them, so only calculate the diagonal terms

$$\rho \cdot X_1 X_2 = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \rho_{|00\rangle\langle 11|} \\ & \rho_{|01\rangle\langle 01|} & \rho_{|01\rangle\langle 10|} & \\ & \rho_{|10\rangle\langle 01|} & \rho_{|10\rangle\langle 10|} & \\ \rho_{|11\rangle\langle 00|} & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} \begin{pmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{pmatrix}$$

$$= \begin{pmatrix} \rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & \rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot Y_1 Y_2 = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & \rho_{|01\rangle\langle 01|} & & \\ & & \rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} \begin{pmatrix} & & & -1 \\ & & 1 & \\ & 1 & & \\ -1 & & & \end{pmatrix}$$

$$= \begin{pmatrix} -\rho_{|00\rangle\langle 11|} & & & \\ & \rho_{|01\rangle\langle 10|} & & \\ & & \rho_{|10\rangle\langle 01|} & \\ & & & -\rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot \frac{(X_1 Y_2 + X_2 Y_1)}{2} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} & & & -i \\ & & 0 & \\ & 0 & & \\ i & & & \end{pmatrix} = \begin{pmatrix} i \cdot \rho_{|00\rangle\langle 11|} & & & \\ & 0 & & \\ & & 0 & \\ & & & -i\rho_{|11\rangle\langle 00|} \end{pmatrix}$$

$$\rho \cdot \frac{(Y_1 X_2 - X_1 Y_2)}{2} = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} & & & 0 \\ & & -i & \\ & i & & \\ 0 & & & \end{pmatrix} = \begin{pmatrix} 0 & & & \\ & i\rho_{|01\rangle\langle 10|} & & \\ & & -i\rho_{|10\rangle\langle 01|} & \\ & & & 0 \end{pmatrix}$$

$$Z_1 Z_2 \cdot \rho = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & 1 \end{pmatrix} \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & \rho_{|01\rangle\langle 01|} & & \\ & & \rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix} = \begin{pmatrix} \rho_{|00\rangle\langle 00|} & & & \\ & -\rho_{|01\rangle\langle 01|} & & \\ & & -\rho_{|10\rangle\langle 10|} & \\ & & & \rho_{|11\rangle\langle 11|} \end{pmatrix}$$

$$Z_1 Y_2 \rho = \begin{pmatrix} & -i & & \\ i & & & \\ & & & i \\ & & -i & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} -i\rho_{|01\rangle\langle 00|} & & & \\ & i\rho_{|00\rangle\langle 01|} & & \\ & & i\rho_{|11\rangle\langle 10|} & \\ & & & -i\rho_{|10\rangle\langle 11|} \end{pmatrix}$$

$$Z_1 X_2 \rho = \begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & -1 \\ & & -1 & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} \rho_{|01\rangle\langle 00|} & & & \\ & \rho_{|00\rangle\langle 01|} & & \\ & & -\rho_{|11\rangle\langle 10|} & \\ & & & -\rho_{|10\rangle\langle 11|} \end{pmatrix}$$

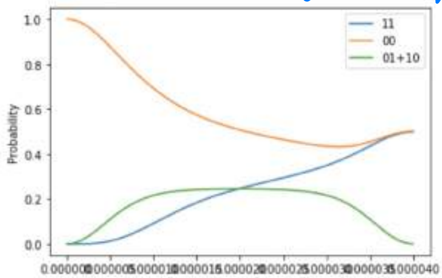
$$Y_1 Z_2 \rho = \begin{pmatrix} & & -i & \\ & & & i \\ i & & & \\ & -i & & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} -i\rho_{|10\rangle\langle 00|} & & & \\ & i\rho_{|11\rangle\langle 01|} & & \\ & & i\rho_{|00\rangle\langle 10|} & \\ & & & -i\rho_{|01\rangle\langle 11|} \end{pmatrix}$$

$$X_1 Z_2 \rho = \begin{pmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{pmatrix} \begin{pmatrix} & & \\ & \rho & \\ & & \end{pmatrix} = \begin{pmatrix} \rho_{|10\rangle\langle 00|} & & & \\ & -\rho_{|11\rangle\langle 01|} & & \\ & & \rho_{|00\rangle\langle 10|} & \\ & & & -\rho_{|01\rangle\langle 11|} \end{pmatrix}$$

```
delta =  25000.0
t_gate =  4e-05
```

MS Gate time scan

$t \in [0, t_g]$



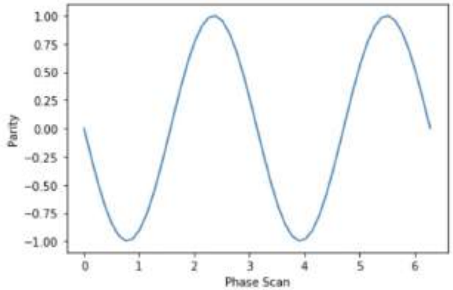

```
0:00:00.540298
Max parity =  0.99699977466192
Min parity =  -0.9952544323520293
Parity contrast =  0.9961271035069746
Prob_11 =  0.49859747106299007
Prob_00 =  0.5004713911571109
Prob_01+10 =  0.0009311377798986452
```

Out[62]: Quantum object: dims = [[2, 2], [2, 2]], shape = (4, 4), type = oper, isherm = True

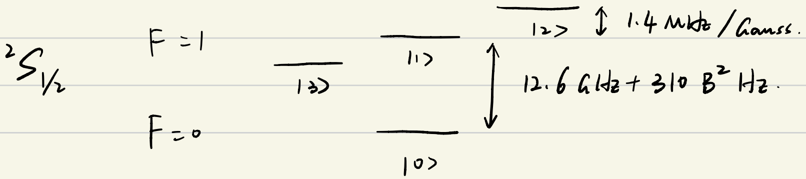
$$\begin{pmatrix} 0.499 & 0.0 & 0.0 & (4.656\times10^{-04} - 0.499j) \\ 0.0 & 4.656\times10^{-04} & 4.656\times10^{-04} & 0.0 \\ 0.0 & 4.656\times10^{-04} & 4.656\times10^{-04} & 0.0 \\ (4.656\times10^{-04} + 0.499j) & 0.0 & 0.0 & 0.500 \end{pmatrix}$$

This term is the degree of entanglement. when it's 0.5j. it's maximal entangled. When it becomes 0, it means it becomes an entire mixed State

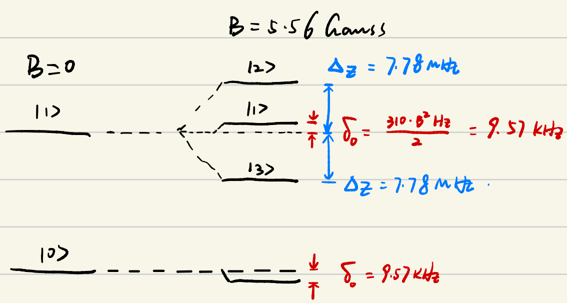
Parity scan



$\boxed{\text{Zeeman transitions}}$



$$f_{Zeeman} = 7.78 \text{ MHz} \Rightarrow B = 5.56 \text{ Gauss}$$



$$\Rightarrow \omega_{12} = \Delta_Z - \delta_0$$
$$\omega_{13} = \Delta_Z + \delta_0$$

$\Rightarrow$ Energy difference between $|1\rangle \to |2\rangle$ and $|1\rangle \to |3\rangle$ transition is

$$\boxed{\delta = 2\delta_0 = 19.15 \text{ kHz}.}$$

Interaction picture:

$$H = \begin{array}{c} \\ \\ \\ \\ \\ \end{array}\begin{array}{c} |0\rangle \quad |1\rangle \quad |2\rangle \quad |3\rangle \\ \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \Omega_1^* & \Omega_2^* \\ 0 & \Omega_1 & 2\delta_0 & 0 \\ 0 & \Omega_2 & 0 & 0 \end{pmatrix} \end{array} \begin{array}{c} \\ |0\rangle \\ |1\rangle \\ |2\rangle \\ |3\rangle \end{array}$$

$$\begin{pmatrix} 0 & \Omega_1 & \Omega_2 \\ \Omega_1 & \delta_1 & 0 \\ \Omega_2 & 0 & \delta_2 \end{pmatrix} \rightarrow \begin{vmatrix} -\lambda & \Omega_1 & \Omega_2 \\ \Omega_1 & \delta_1-\lambda & 0 \\ \Omega_2 & 0 & \delta_2-\lambda \end{vmatrix}$$

$$\Rightarrow \underbrace{\delta_2\Omega_1^2+\delta_1\Omega_2^2}_{d} + \underbrace{(\delta_1\delta_2-\Omega_1^2-\Omega_2^2)}_{c}\lambda + \underbrace{(-\delta_1-\delta_2)}_{b}\lambda^2 + \underbrace{\lambda^3}_{a=1} = 0$$

For $ax^3+bx^2+cx+d=0$.

$$p = -\frac{b}{3a} = \frac{\delta_1+\delta_2}{3}$$

$$q = p^3 + \frac{bc-3ad}{6a^2} = \left(\frac{\delta_1+\delta_2}{3}\right)^3 + \frac{-(\delta_1+\delta_2)(\delta_1\delta_2-\Omega_1^2-\Omega_2^2) - 3(\delta_2\Omega_1^2+\delta_1\Omega_2^2)}{6}$$

$$= \frac{\delta_1^3+3\delta_1^2\delta_2+3\delta_1\delta_2^2+\delta_2^3}{27} + \frac{-\delta_1^2\delta_2+\delta_1\Omega_1^2+\cancel{\delta_1\Omega_2^2}-\delta_1\delta_2^2+\cancel{\delta_2\Omega_1^2}+\delta_2\Omega_2^2-\overset{2}{\cancel{3}}\delta_2\Omega_1^2-\overset{2}{\cancel{3}}\delta_1\Omega_2^2}{6}$$

$$= \frac{1}{54}\Big[2\delta_1^3+\cancel{6\delta_1^2\delta_2}+\cancel{6\delta_1\delta_2^2}+2\delta_2^3-\overset{3}{\cancel{9}}\delta_1^2\delta_2+9\delta_1\Omega_1^2-\overset{3}{\cancel{9}}\delta_1\delta_2^2+9\delta_2\Omega_2^2$$
$$-18\delta_2\Omega_1^2-18\delta_1\Omega_2^2\Big]$$

$$= \frac{1}{54}\Big[2\delta_1^3+3\delta_1(-\delta_2^2+3\Omega_1^2-6\Omega_2^2)+3\delta_2(-\delta_1^2+3\Omega_2^2-6\Omega_1^2)$$
$$+2\delta_2^3\Big]$$

$$r = \frac{c}{3a} = \frac{1}{3}(\delta_1\delta_2-\Omega_1^2-\Omega_2^2)$$

Roots: $\beta_1, \beta_2, \beta_3$

$$|1'\rangle = \left[-\frac{\delta_2 - \beta_1}{w_2}, \ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \cdot \beta_1 - \beta_1^2\right), \ 1 \right]$$

$$|2'\rangle = \left[-\frac{\delta_2 - \beta_2}{w_2}, \ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \beta_2 - \beta_2^2\right), \ 1 \right]$$

$$|3'\rangle = \left[-\frac{\delta_2 - \beta_3}{w_2}, \ -\frac{1}{w_1 w_2}\left(w_2^2 + \delta_2 \beta_3 - \beta_3^2\right), \ 1 \right]$$

when $\delta_1 \sim \delta_2$ and $|\delta_{1,2}| \gg w_{1,2}$

$$\beta_1 = \delta_1 + \varepsilon, \quad \beta_2 = \delta_1, \quad \beta_3 \approx -\varepsilon \ll w_{12}$$

$$|1'\rangle = \left[-\frac{\varepsilon}{w}, \ -1, \ -1 \right] \qquad \beta_1 = \delta$$

$$|2'\rangle = \left[0, \ -1, \ 1 \right] \qquad \beta_2 = \delta$$

$$|3'\rangle = \left[\infty, \ 1, \ 1 \right] \qquad \beta_3 = 0$$

J. Randall 2015. PRA paper.

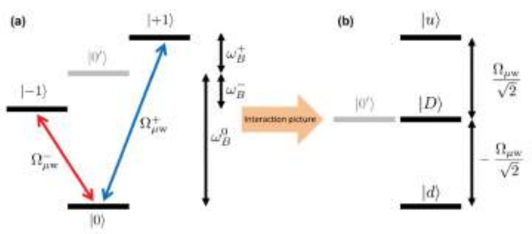


FIG. 1. (Color online) (a) Energy level diagram of the $^2S_{1/2}$ ground state hyperfine manifold of $^{171}Yb^+$ where the degeneracy of the Zeeman states has been lifted by a static magnetic field. Here the transitions $|0\rangle \leftrightarrow |+1\rangle$ and $|0\rangle \leftrightarrow |-1\rangle$ are coupled with resonant microwave radiation to obtain the three dressed states $|d\rangle, |D\rangle$, and $|u\rangle$. (b) The resultant energy level diagram in the dressed basis when the Rabi frequencies of the dressing fields are set to be equal ($\Omega^+_{\mu w} = \Omega^-_{\mu w} = \Omega_{\mu w}$). The energy difference between the dressed-states in this case is given by $\pm\hbar\Omega_{\mu w}/\sqrt{2}$.

$$|D\rangle = \frac{1}{\sqrt{2}}\left(|+1\rangle - |-1\rangle\right)$$

$$|u\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle + \frac{1}{\sqrt{2}}|0\rangle$$

$$|d\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle - \frac{1}{\sqrt{2}}|0\rangle$$

$$H_{\mu w} = \frac{\hbar\Omega_{\mu w}}{2}\left(|+1\rangle\langle 0| + |-1\rangle\langle 0| + H.c.\right)$$

$$\Big\downarrow \text{diagonalize}$$

$$H_{\mu w} = \frac{\hbar\Omega_{\mu w}}{\sqrt{2}}\left(|u\rangle\langle u| - |d\rangle\langle d|\right)$$

Reasons of insensitive to B field:

Perturbation up to first order of B field.

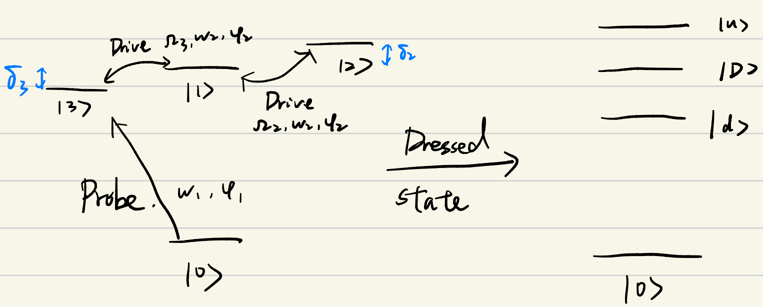
$$H_p = \hbar\lambda_0(t)\left(|+1\rangle\langle +1| - |-1\rangle\langle -1|\right)$$

$$\xrightarrow[\text{basis}]{\text{Dressed state}} H_p = \frac{\hbar\lambda_0(t)}{\sqrt{2}}\left(|D\rangle\langle u| + |D\rangle\langle d| + H.c.\right).$$

B field fluctuations will try to drive population between $|D\rangle$, $|u\rangle$ and $|d\rangle$. But these states are separated by $\hbar\Omega_{\mu w}/\sqrt{2}$. So only B field fluctuating with a frequency at or near $\Omega_{\mu w}/\sqrt{2}$ can cause transitions between dressed states.

- Dressed state.

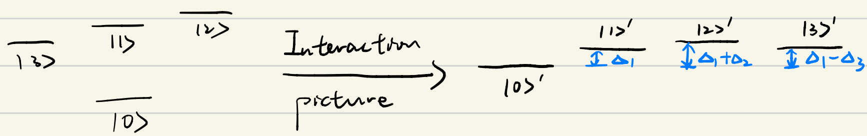
Two lasers:



Assume $\omega_0 = 0$

$$H_0 = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$$
$$+ \left[\Omega_{03}\, e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3| + \Omega_{02}\, e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 2| + \Omega_{01}\, e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1| + h.c.\right]$$
$$+ \left[\Omega_{12}\, e^{i(\omega_2 t + \varphi_2)}|1\rangle\langle 2| + \Omega_{13}\, e^{i(\omega_2 t + \varphi_2)}|3\rangle\langle 1| + h.c.\right]$$

Let $H_D = \omega_1|1\rangle\langle 1| + \omega_1|2\rangle\langle 2| + \omega_1|3\rangle\langle 3| + \omega_2|2\rangle\langle 2| - \omega_2|3\rangle\langle 3|$



Def $\Delta_1 = \omega_{01} - \omega_1$, $\Delta_2 = \delta_2 - \omega_2$, $\Delta_3 = \delta_3 - \omega_2$

$$H_I = H_0 - H_D = (\Delta_1 - \Delta_3)|3\rangle\langle 3| + \Delta_1|1\rangle\langle 1| + (\Delta_1 + \Delta_2)|2\rangle\langle 2|$$
$$+ [\cdots]$$

$$\tilde{H}_I = e^{iH_D t} H_I e^{-iH_D t}$$
$$= (\Delta_1 - \Delta_3)|3\rangle\langle 3| + \Delta_1|1\rangle\langle 1| + (\Delta_1 + \Delta_2)|2\rangle\langle 2|$$
$$+ [\Omega_{03} \cdot e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3|\, e^{-i(\omega_1 - \omega_2)t} + \Omega_{02}\, e^{i(\omega_1 t + \varphi)}|0\rangle\langle 2|\, e^{-i(\omega_1 + \omega_2)t}$$
$$+ \Omega_{01}\, e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1|\, e^{-i\omega_1 t} + h.c.]$$
$$+ [\Omega_{12}\, e^{i(\omega_2 t + \varphi_2)} \cdot e^{i\omega_1 t}|1\rangle\langle 2|\, e^{-i(\omega_1 + \omega_2)t}$$
$$+ \Omega_{13}\, e^{i(\omega_2 t + \varphi_2)}\, e^{i(\omega_1 - \omega_2)t}|3\rangle\langle 1|\, e^{-i\omega_1 t}$$

$$= \Delta_1 |1\rangle\langle 1| + (\Delta_1+\Delta_2)|2\rangle\langle 2| + (\Delta_1-\Delta_3)|3\rangle\langle 3|$$
$$+ \left[\Omega_{03}\, e^{i(+\omega_2 t+\varphi_1)} |0\rangle\langle 3| + \Omega_{02}\, e^{i(-\omega_2 t+\varphi_1)} |0\rangle\langle 2| + \Omega_{01}\, e^{i\varphi_1} |0\rangle\langle 1| + h.c.\right]$$
$$+ \left[\Omega_{12}\, e^{i\varphi_2} |1\rangle\langle 2| + \Omega_{13}\, e^{-i\varphi_2} |1\rangle\langle 3| + h.c.\right]$$

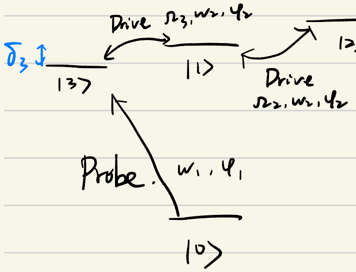
$$= \begin{bmatrix} 0 & \Omega_{01} e^{i\varphi_1} & \Omega_{02} e^{-i\omega_2 t} e^{i\varphi_1} & \Omega_{03} e^{+i\omega_2 t} e^{i\varphi_1} \\ \Omega_{01}^* e^{-i\varphi_1} & \Delta_1 & \Omega_{12} e^{i\varphi_2} & \Omega_{13} e^{-i\varphi_2} \\ \Omega_{02}^* e^{i\omega_2 t} e^{-i\varphi_1} & \Omega_{12}^* e^{-i\varphi_2} & \Delta_1+\Delta_2 & 0 \\ \Omega_{03}^* e^{-i\omega_2 t} e^{-i\varphi_1} & \Omega_{13}^* e^{i\varphi_2} & 0 & \Delta_1-\Delta_3 \end{bmatrix}$$

if $\varphi_1 = 0$, $\varphi_2 = 0$

$$= \begin{bmatrix} 0 & \Omega_{01} & \Omega_{02} e^{-i\omega_2 t} & \Omega_{03} e^{+i\omega_2 t} \\ \Omega_{01}^* & \Delta_1 & \Omega_{12} & \Omega_{13} \\ \Omega_{02}^* e^{i\omega_2 t} & \Omega_{12}^* & \Delta_1+\Delta_2 & 0 \\ \Omega_{03} e^{-i\omega_2 t} & \Omega_{13}^* & 0 & \Delta_1-\Delta_3 \end{bmatrix}$$

Another choice of interaction picture

Two lasers:



$\xrightarrow[\text{state}]{\text{Dressed}}$

$$|u\rangle = \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
$$|D\rangle = \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|3\rangle$$
$$|d\rangle = \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

$\Updownarrow$

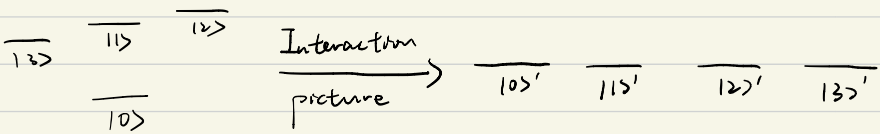
$$|1\rangle = \frac{1}{\sqrt{2}}(|u\rangle - |d\rangle)$$
$$|2\rangle = \frac{1}{\sqrt{2}}|D\rangle + \frac{1}{2}|u\rangle + \frac{1}{2}|d\rangle$$
$$|3\rangle = -\frac{1}{\sqrt{2}}|D\rangle + \frac{1}{2}|u\rangle + \frac{1}{2}|d\rangle$$

$|0\rangle$

Assume $\omega_0 = 0$

$$H_0 = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$$
$$+ \left[\Omega_{03} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3| + \Omega_{02} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 2| + \Omega_{01} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1| + h.c.\right]$$
$$+ \left[\Omega_{12} e^{i(\omega_2 t + \varphi_2)}|1\rangle\langle 2| + \Omega_{13} e^{i(\omega_2 t + \varphi_2)}|3\rangle\langle 1| + h.c.\right]$$

Let $H_D = (\omega_{01} - \delta_3)|3\rangle\langle 3| + \omega_{01}|1\rangle\langle 1| + (\omega_{01} + \delta_2)|2\rangle\langle 2|$



$H_I = H_0 - H_D = $ [Interaction terms only]

$$\tilde{H}_I = e^{iH_D t} H_I e^{-iH_D t} = \Big[\Omega_{03} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 3| e^{-i(\omega_{01} - \delta_3)t} + \Omega_{02} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 2| e^{-i(\omega_{01} + \delta_2)t}$$
$$+ \Omega_{01} e^{i(\omega_1 t + \varphi_1)}|0\rangle\langle 1| e^{-i\omega_{01}t} + \Omega_{12} e^{i(\omega_2 t + \varphi_2)} e^{i\omega_{01}t}|1\rangle\langle 2| e^{-i(\omega_{01} + \delta_2)t}$$
$$+ \Omega_{13} e^{i(\omega_2 t + \varphi_2)} e^{i(\omega_{01} - \delta_3)t}|3\rangle\langle 1| e^{-i\omega_{01}t}\Big] + h.c.$$
$$= \underbrace{\Omega_{03} e}_{A_1}{}^{i[(\omega_1 - \omega_{01} + \delta_3)t + \varphi_1]}|0\rangle\langle 3| + \underbrace{\Omega_{02} e}_{A_2}{}^{i[(\omega_1 - \omega_{01} - \delta_2)t + \varphi_1]}|0\rangle\langle 2|$$
$$\underbrace{+ \Omega_{01} e}_{A_3}{}^{i[(\omega_1 - \omega_{01})t + \varphi_1]}|0\rangle\langle 1| + \underbrace{\Omega_{12} e}_{B_1}{}^{i[(\omega_2 - \delta_2)t + \varphi_2]}|1\rangle\langle 2|$$
$$\underbrace{+ \Omega_{13} e}_{B_2}{}^{-i[(\omega_2 - \delta_3)t + \varphi_2]}|1\rangle\langle 3| + h.c.$$

Assumption:

$A_i \ll B_i$

change $|1\rangle, |2\rangle, |3\rangle$ to $|u\rangle, |D\rangle, |d\rangle$

$$= A_1 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + A_2 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$
$$+ A_3 |0\rangle\left(\frac{1}{\sqrt{2}}\langle u| - \frac{1}{\sqrt{2}}\langle d|\right) + B_1\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$
$$+ B_2\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + h.c.$$

change $|1\rangle, |2\rangle, |3\rangle$ to $|u\rangle, |D\rangle, |d\rangle$

$$= A_1 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + A_2 |0\rangle\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$

$$+ A_3 |0\rangle\left(\frac{1}{\sqrt{2}}\langle u| - \frac{1}{\sqrt{2}}\langle d|\right) + B_1\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| + \frac{1}{\sqrt{2}}\langle D|\right)$$

$$+ B_2\left(\frac{1}{\sqrt{2}}|u\rangle - \frac{1}{\sqrt{2}}|d\rangle\right)\left(\frac{1}{2}\langle u| + \frac{1}{2}\langle d| - \frac{1}{\sqrt{2}}\langle D|\right) + h.c.$$

$$= \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 + \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle u| + \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 - \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle d|$$

$$+ \left(\frac{1}{\sqrt{2}}A_2 - \frac{1}{\sqrt{2}}A_1\right)|0\rangle\langle D| + \frac{1}{2\sqrt{2}}(B_1+B_2)\left(|u\rangle\langle u| + |u\rangle\langle d| - |d\rangle\langle u| - |d\rangle\langle d|\right)$$

$$+ \frac{1}{2}(B_1 - B_2)|u\rangle\langle D| + \frac{1}{2}(B_2 - B_1)|d\rangle\langle D| + h.c.$$

A second interaction picture: $H_D^{(2)} = \frac{1}{2\sqrt{2}}(B_1+B_2)\left(|u\rangle\langle u| - |d\rangle\langle d|\right)$

$$\tilde{H}_I^{(2)} = e^{iH_D^{(2)}t}\,\tilde{H}_I\,e^{-iH_D^{(2)}t}$$

$$= \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 + \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle u|\,e^{-i\frac{1}{2\sqrt{2}}(B_1+B_2)t} + \left(\frac{1}{2}A_1 + \frac{1}{2}A_2 - \frac{1}{\sqrt{2}}A_3\right)|0\rangle\langle d|\,e^{+i\frac{1}{2\sqrt{2}}(B_1+B_2)t}$$

$$+ \left(\frac{1}{\sqrt{2}}A_2 - \frac{1}{\sqrt{2}}A_1\right)|0\rangle\langle D| + \frac{1}{2\sqrt{2}}(B_1+B_2)\,|u\rangle\langle d|\,e^{\frac{i}{\sqrt{2}}(B_1+B_2)t}$$

$$- \frac{1}{2\sqrt{2}}(B_1+B_2)\,|d\rangle\langle u|\,e^{-\frac{i}{\sqrt{2}}(B_1+B_2)t}$$

$$+ \frac{1}{2}(B_1 - B_2)\,e^{\frac{i}{2\sqrt{2}}(B_1+B_2)t}\,|u\rangle\langle D| + \frac{1}{2}(B_2 - B_1)\,e^{-\frac{i}{2\sqrt{2}}(B_1+B_2)t}\,|d\rangle\langle D| + h.c.$$

Discussion: ① Set the laser s.t. $\varphi_1 = \varphi_2 = 0$

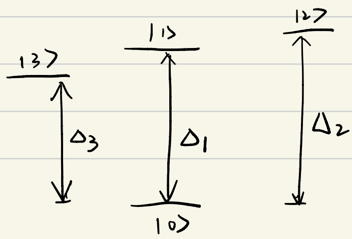
② when $\omega_{12} = \omega_{13} = \omega_2$, i.e. the Driving laser is resonance with both $|1\rangle \to |2\rangle$ & $|1\rangle \to |3\rangle$ simultaneously. $B_1 = B_2 = \Omega_d$ $\mathrm{Im}(B_1) = \mathrm{Im}(B_2) = 0$.

③ Define $\Delta_1 = \omega_{01}$, $\Delta_2 = \omega_{01} + \delta_2$, $\Delta_3 = \omega_{03} - \delta_3$ are the energy levels of $|1\rangle, |2\rangle, |3\rangle$ from $|0\rangle$

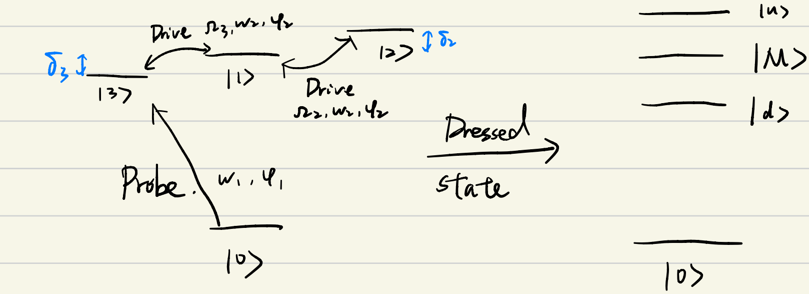
$$\tilde{H}_I^{(2)} = \left\{\frac{1}{2}\Omega_{03}\,e^{i[\omega_1 - (\Delta_3 + \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{2}\Omega_{02}\,e^{i[\omega_1 - (\Delta_2 + \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{\sqrt{2}}\Omega_{01}\,e^{i[\omega_1 - (\Delta_1 + \frac{1}{\sqrt{2}}\Omega_d)]t}\right\}|0\rangle\langle u|$$

$$+ \left\{\frac{1}{2}\Omega_{03}\,e^{i[\omega_1 - (\Delta_3 - \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{2}\Omega_{02}\,e^{i[\omega_1 - (\Delta_2 - \frac{1}{\sqrt{2}}\Omega_d)]t} + \frac{1}{\sqrt{2}}\Omega_{01}\,e^{i[\omega_1 - (\Delta_1 - \frac{1}{\sqrt{2}}\Omega_d)]t}\right\}|0\rangle\langle d|$$

$$+ \left\{-\frac{1}{\sqrt{2}}\Omega_{03}\,e^{i[\omega_1 - \Delta_3]t} + \frac{1}{\sqrt{2}}\Omega_{02}\,e^{i[\omega_1 - \Delta_2]t}\right\}|0\rangle\langle D| + h.c.$$



Analyze the dressed states.



If $\delta_2 = \delta_3$. $\quad w_1 = w_2 = 1$

$|u\rangle \quad \lambda_1 = \sqrt{2} \quad |u\rangle = \frac{1}{\sqrt{2}}|1\rangle + \frac{1}{2}|2\rangle + \frac{1}{2}|3\rangle$

$\sqrt{2}$

$|M\rangle \quad \lambda_2 = 0 \quad |M\rangle = \frac{1}{\sqrt{2}}|2\rangle - \frac{1}{\sqrt{2}}|3\rangle$

$\sqrt{2}$

$|d\rangle \quad \lambda_3 = -\sqrt{2} \quad |d\rangle = \frac{1}{\sqrt{2}}|1\rangle - \frac{1}{2}|2\rangle - \frac{1}{2}|3\rangle$

If $\delta_2 - \delta_3 = 0.25$, $\quad w_1 = w_2 = 1$

$|u\rangle \quad \lambda_1 = 1.48 \quad |u\rangle = 0.69|1\rangle + 0.56|2\rangle + 0.46|3\rangle$

$|M\rangle \quad \lambda_2 = 0.12 \quad |M\rangle = 0.09|1\rangle - 0.70|2\rangle + 0.71|3\rangle$

$|d\rangle \quad \lambda_3 = -1.36 \quad |d\rangle = -0.72|1\rangle + 0.45|2\rangle + 0.53|3\rangle$

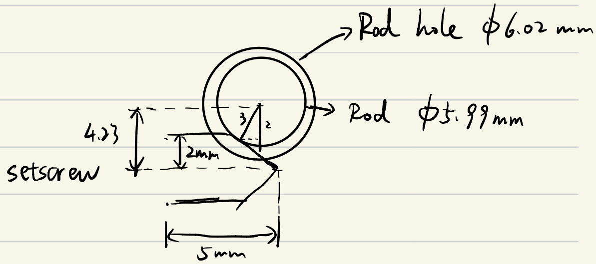
This is nontrivial due to ① $310 B^2$ Hz.
② AC - Stark shift.

This is what we can use!

Some design tips

① Setscrew: SS4MS5C or SS4MS5

Use a setscrew to fix a rod in a customized hole.



② Coupling lens: Fiber MFD = 2.3 μm

Beam diameter = 0.8 mm = D

Laser wavelength = 369 nm = λ

$$f = \frac{\pi \cdot D \cdot MFD}{4\lambda} = 3.92 \text{ mm}$$

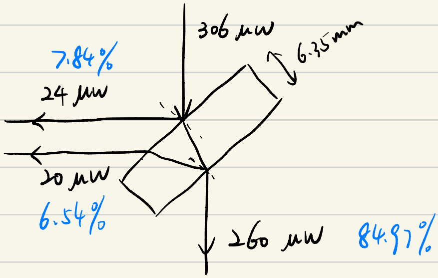
Choose Thorlabs: C610TME-A

EFL = 4.00 mm

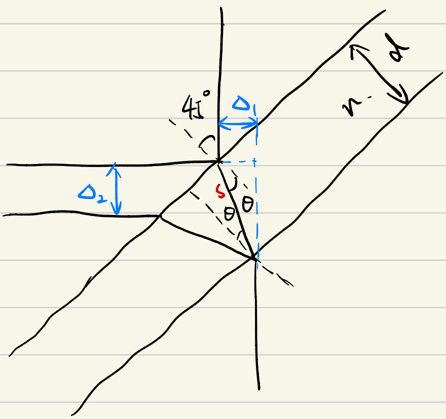
Glass: ECO-550. transimission rate @370 nm is higher than D-ZK3 > D-ZLaF52LA.

Window, uncoated:

Measured PWI-1025-UV (CVI Laser Optics)



$(260+24+20)/306 = 99.35\%$



$\sin 45^\circ = n \sin\theta$

$s = d/\cos\theta$

$\Delta_1 = s \cdot \sin(45^\circ - \theta)$

$= \frac{d}{\cos\theta} \cdot \sin(45^\circ - \theta)$

$\Delta_2 = s \cdot \sin(\theta) \cdot 2 \cdot \sin 45^\circ$

$= \sqrt{2}\, \frac{d}{\cos\theta} \cdot \sin\theta$

$= \sqrt{2}\, d \tan\theta$

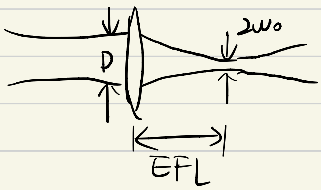
For UV-Fused sillica: $n = 1.4740$ @ 369 nm, $\phi 1'' \rightarrow d = 5$ mm

$\Rightarrow \theta = \arcsin\left[\frac{\sin 45^\circ}{n}\right] = \arcsin\left(\frac{\sqrt{2}}{2 \cdot 1.4740}\right) = 28.6671^\circ$

$\Delta_1 = 1.60252$ mm

$\Delta_2 = 3.86602$ mm

Focused beam size :



Diameter : $2w_0 = \frac{4M^2}{\pi} \cdot \frac{\lambda f}{D}$

where M^2 is the beam qulity parameter, normally 1

λ : wavelength
f : Effective focal length
D : input beam diameter.

Focused diffraction limited spot size:

Diameter: $d_{lim} = 2 \cdot \frac{\lambda f}{D} = \frac{2\pi}{4M} \cdot (2w_0)$

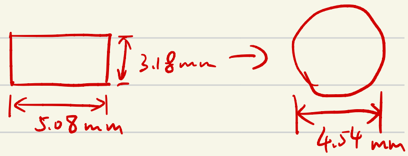
$\approx 1.57 \cdot (2w_0)$

d_{lim} : contains 99% beam power
$2w_0$: $1/e^2$ beam size, contains 87.5% power

- O-ring:

Company: Globaloring and seal

slot:



Measured with Solidworks:

O-ring length — 5456.77 mm

Crosssection diameter $d = 2 \times \sqrt{\frac{3.18 \times 5.08}{\pi}} = 4.54\ \text{mm}$

Actually used dimension:

length — 5377.00 mm

(80 mm shorter, 1.5% shorter)

diameter — $d = 4.5$ mm.