Білет №8

1. A і B – незалежні, $P(A)=0,45$, $P(A\cap B)=0,18$

$P(B)$, $P(A\cup B)$, $P(\bar{A}\cup\bar{B})$ – ?

$P(A)P(B)=P(A\cap B)\Rightarrow P(B)=\frac{P(A\cap B)}{P(A)}$

$P(B)=\frac{0,18}{0,45}=0,4$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)=0,45+0,4-0,18=0,67$

$P(\bar{A}\cup\bar{B})=P(\overline{A\cap B})=1-P(A\cap B)=1-0,18=0,82$

2. $[0;4]$, $\eta = 3\xi + 9$

Знайдемо математичне сподівання та дисперсію для ξ

$M\xi = \frac{0+4}{2} = 2$;

$D\xi = \frac{(4-0)^2}{12} = \frac{4}{3}$;

Використовуючи властивості маємо:

$M\eta = 3M\xi + 9 = 3 \cdot 2 + 9 = 15$ //

$D\eta = 3^2 D\xi = 9 \cdot \frac{4}{3} = 12$ //

$cov(\xi, \eta) = cov(\xi, 3\xi + 9) = 3D\xi = 4$ //

$r_{\xi,\eta} = \frac{cov(\xi,\eta)}{\sqrt{D\xi D\eta}} = \frac{4}{\sqrt{\frac{4}{3} \cdot 12}} = 1$ //

3. $f(x;\alpha)=\begin{cases}0, & x<0\\ \frac{1}{\alpha}e^{-\frac{1}{\alpha}}, & x\geq 0\end{cases}$, $\hat{\alpha}=\bar{\xi}=\frac{1}{n}\sum_{i=1}^{n}\xi_i$

$$L(x,\alpha)=\prod_{i=1}^{n}f(x_i,\alpha)=\frac{1}{\alpha^n}e^{-\frac{1}{\alpha}}\Rightarrow$$

$$\Rightarrow \ln L(x,\alpha)=-n\ln\alpha-\frac{n}{\alpha}$$

$$\frac{\partial}{\partial\alpha}\ln L(x,\alpha)=-\frac{n}{\alpha}+\frac{n}{\alpha^2}\Rightarrow\frac{\partial^2}{\partial\alpha^2}=\frac{n}{\alpha^2}-\frac{2n}{\alpha^3}$$

$$I(\alpha)=-M\frac{\partial^2}{\partial\alpha^2}\ln L(\xi,\alpha)=\frac{2n}{\alpha^3}-\frac{n}{\alpha^2}=\frac{n(2\alpha-1)}{\alpha^3}$$

$\sum_{i=1}^{n}\xi_i=\gamma_n$ – розподіл Ерланга, $f_{\gamma_n}(x)=\frac{\alpha^n x^{n-1}}{(n-1)!}e^{-\alpha x}, x\geq 0$

$$M\hat{\alpha}=M\frac{1}{n}\gamma_n=\int_0^\infty\frac{x}{n}\cdot\frac{\alpha^n x^{n-1}}{(n-1)!}e^{-\alpha x}dx=\left|\begin{matrix}t=\alpha x\\ dt=\alpha dx\end{matrix}\right|=$$

$$=\frac{1}{\alpha n!}\int_0^\infty t^n e^{-t}dt=\frac{\Gamma(n+1)}{\alpha n!}=\frac{n!}{\alpha n!}=\frac{1}{\alpha}=$$

$=\alpha+\frac{1-\alpha^2}{\alpha}\Rightarrow$ оцінка зміщена, $b(\alpha)=\frac{1-\alpha^2}{\alpha}$

$$M\hat{\alpha}^2=M\left(\frac{\gamma_n}{n}\right)^2=\int_0^\infty\frac{x^2}{n^2}\cdot\frac{\alpha^n x^{n-1}}{(n-1)!}e^{-\alpha x}dx=\left|\begin{matrix}t=\alpha x\\ dt=\alpha dx\end{matrix}\right|=$$

$$=\frac{1}{\alpha^2 n\cdot n!}\int_0^\infty \alpha^{n+1}x^{n+1}e^{-t}dt=\frac{\Gamma(n+2)}{n\alpha^2 n!}=\frac{(n+1)!}{n\alpha^2 n!}=\frac{n+1}{n\alpha^2};$$

$$D\hat{\alpha}=M\hat{\alpha}^2-(M\hat{\alpha})^2=\frac{n+1}{n\alpha^2}-\frac{1}{\alpha^2}=\frac{1}{n\alpha^2}$$

$$\frac{(1+b'(\alpha))^2}{I(\alpha)}=\frac{\left(-\frac{1}{\alpha^2}\right)^2\alpha^3}{n(2\alpha-1)}=\frac{1}{n\alpha(2\alpha-1)}$$

$$D\hat{\alpha}\neq\frac{(1+b'(\alpha))^2}{I(\alpha)}$$