1533. $A = \begin{pmatrix} 6 & 6 & -15 \\ 1 & 5 & -5 \\ 1 & 2 & -2 \end{pmatrix}$

$$|A-\lambda E| = \begin{vmatrix} 6-\lambda & 6 & -15 \\ 1 & 5-\lambda & -5 \\ 1 & 2 & -2-\lambda \end{vmatrix} = \begin{vmatrix} 6-\lambda & -9 & -15 \\ 1 & -\lambda & -5 \\ 1 & -\lambda & -2-\lambda \end{vmatrix} = \begin{vmatrix} 6-\lambda & -9 & -15 \\ 1 & -\lambda & -5 \\ 0 & 0 & 3-\lambda \end{vmatrix} =$$

$$= (3-\lambda)\begin{vmatrix} 6-\lambda & -9 \\ 1 & -\lambda \end{vmatrix} = (3-\lambda)(-6\lambda+\lambda^2+9) = (3-\lambda)^3$$

$\lambda_1 = 3$ кр. 3

$B = \begin{pmatrix} 3 & 6 & -15 \\ 1 & 2 & -5 \\ 1 & 2 & -5 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & -5 \end{pmatrix}$ x_2, x_3 — с. з.

ФСР

x_1	x_2	x_3
2	-1	0
5	0	1

$a_1 = (2, -1, 0)$

$a_2 = (5, 0, 1)$

$$B^2 = \begin{pmatrix} 3 & 6 & -15 \\ 1 & 2 & -5 \\ 1 & 2 & -5 \end{pmatrix}\begin{pmatrix} 3 & 6 & -15 \\ 1 & 2 & -5 \\ 1 & 2 & -5 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

ДФСР

x_1	x_2	x_3
1	0	0

$a_3 = (1, 0, 0)$

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$a_3)$ $\varphi(a_3) = Ba_3 = \begin{pmatrix} 3 & 6 & -15 \\ 1 & 2 & -5 \\ 1 & 2 & -5 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$

$f_1 = \varphi(a_3) = (3, 1, 1)$ $\sim \begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$

$f_2 = a_3 = (1, 0, 0)$ $\quad A_j = \left(\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 3 & 0 \\ \hline 0 & 0 & 3 \end{array}\right)$

$f_3 = a_1 = (2, -1, 0) \sim (3)$

1534.

$$A=\begin{pmatrix}0&1&-1&1\\-1&2&-1&1\\-1&1&1&0\\-1&1&0&1\end{pmatrix}$$

$$|A-\lambda E|=\begin{vmatrix}-\lambda&1&-1&1\\-1&2-\lambda&-1&1\\-1&1&1-\lambda&0\\-1&1&0&1-\lambda\end{vmatrix}=\begin{vmatrix}1-\lambda&1&-1&1\\1-\lambda&2-\lambda&-1&1\\0&1&1-\lambda&0\\0&1&0&1-\lambda\end{vmatrix}=\begin{vmatrix}1-\lambda&1&-1&1\\0&1-\lambda&0&0\\0&1&1-\lambda&0\\0&1&0&1-\lambda\end{vmatrix}=$$

$$=(1-\lambda)\begin{vmatrix}1-\lambda&0&0\\1&1-\lambda&0\\1&0&1-\lambda\end{vmatrix}=(1-\lambda)^4$$

$\lambda=1$ кр. 4

$$B=\begin{pmatrix}-1&1&-1&1\\-1&1&-1&1\\-1&1&0&0\\-1&1&0&0\end{pmatrix}\sim\begin{pmatrix}-1&1&-1&1\\-1&1&0&0\end{pmatrix}$$

ФСР

x_1	x_2	x_3	x_4
1	1	0	0
0	0	1	1

$a_1=(1,1,0,0)$

$a_2=(0,0,1,1)$

$$B^2=\begin{pmatrix}-1&1&-1&1\\-1&1&-1&1\\-1&1&0&0\\-1&1&0&0\end{pmatrix}\begin{pmatrix}-1&1&-1&1\\-1&1&-1&1\\-1&1&0&0\\-1&1&0&0\end{pmatrix}=\begin{pmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix}$$

ДФСР

x_1	x_2	x_3	x_4
1	0	0	0
0	0	1	0

$a_3=(1,0,0,0)$

$a_4=(0,0,1,0)$

$$a_3,\ \psi(a_3)=Ba_3=\begin{pmatrix}-1&1&-1&1\\-1&1&-1&1\\-1&1&0&0\\-1&1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\0\\0\end{pmatrix}=\begin{pmatrix}-1\\-1\\-1\\-1\end{pmatrix}$$

$$a_4,\ \psi(a_4)=Ba_4=\begin{pmatrix}-1&1&-1&1\\-1&1&-1&1\\-1&1&0&0\\-1&1&0&0\end{pmatrix}\begin{pmatrix}0\\0\\1\\0\end{pmatrix}=\begin{pmatrix}-1\\-1\\0\\0\end{pmatrix}$$

$$\left.\begin{aligned}f_1&=\psi(a_3)=(-1,-1,-1,-1)\\f_2&=a_3=(1,0,0,0)\end{aligned}\right\}\begin{pmatrix}1&1\\0&1\end{pmatrix}$$

$$\left.\begin{aligned}f_3&=\psi(a_4)=(-1,-1,0,0)\\f_4&=a_4=(0,0,1,0)\end{aligned}\right\}\begin{pmatrix}1&1\\0&1\end{pmatrix}$$

$$A_j=\left(\begin{array}{cc|cc}1&1&0&0\\0&1&0&0\\\hline0&0&1&1\\0&0&0&1\end{array}\right)$$

//

1090.

$$A=\begin{pmatrix}0&1&0\\-4&4&0\\-2&-1&2\end{pmatrix}$$

$$|A-\lambda E|=\begin{vmatrix}-\lambda&1&0\\-4&4-\lambda&0\\-2&-1&2-\lambda\end{vmatrix}=-\lambda(4-\lambda)(2-\lambda)+4(2-\lambda)=$$

$$=(-4\lambda+\lambda^2+4)(2-\lambda)=(2-\lambda)^3$$

$\lambda_1=2$ кр. 3

$$B=\begin{pmatrix}-2&1&0\\-4&2&0\\-2&-1&0\end{pmatrix}\sim\begin{pmatrix}-2&1&0\\0&-2&0\end{pmatrix}$$

ФРС:

x_1	x_2	x_3
1	2	0
0	0	1

$a_1=(1,2,0)$
$a_2=(0,0,1)$

$$B^2=\begin{pmatrix}-2&1&0\\-4&2&0\\-2&-1&0\end{pmatrix}\begin{pmatrix}-2&1&0\\-4&2&0\\-2&-1&0\end{pmatrix}=(2\ -1\ 0)$$

ФРС:

x_1	x_2	x_3
1	2	0

$a_3=(1,2,0)$

$$a_3,\ \varphi(a_3)=Ba_3=\begin{pmatrix}-2&1&0\\-4&2&0\\-2&-1&0\end{pmatrix}\begin{pmatrix}1\\2\\0\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

$f_1=a_2=(0,0,1)\}\ (2)$

$f_2=\varphi(a_3)=(0,0,0)$
$f_3=a_3=(1,2,0)$ $\Big\}\begin{pmatrix}2&1\\0&2\end{pmatrix}$

$$A_j=\left(\begin{array}{c|cc}2&0&0\\\hline 0&2&1\\0&0&2\end{array}\right)$$

1096. $A = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}$

$$|A-\lambda E| = \begin{vmatrix} 4-\lambda & -5 & 2 \\ 5 & -7-\lambda & 3 \\ 6 & -9 & 4-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & -5 & 2 \\ 1-\lambda & -7-\lambda & 3 \\ 1-\lambda & -9 & 4-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & -5 & 2 \\ 0 & -2-\lambda & 1 \\ 0 & -4 & 2-\lambda \end{vmatrix} =$$

$$= (1-\lambda)\begin{vmatrix} -2-\lambda & 1 \\ -4 & 2-\lambda \end{vmatrix} = (1-\lambda)\left((-2-\lambda)(2-\lambda)+4\right) =$$

$$= (1-\lambda)\lambda^2$$

$\lambda_1 = 1$ кр. 1

$\lambda_2 = 0$ кр. 2

$\underline{\lambda_1 = 1}$

$$B_1 = \begin{pmatrix} 3 & -5 & 2 \\ 5 & -8 & 3 \\ 6 & -9 & 3 \end{pmatrix} \sim \begin{pmatrix} 3 & -5 & 2 \\ 2 & -3 & 1 \\ 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 2 & -3 & 1 \\ 0 & 1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & -1 \\ \cancel{0} & \cancel{1} & \cancel{-1} \end{pmatrix} \quad r = 1, 3.$$

ФРС:

x_1	x_2	x_3
1	1	1

$a_1 = (1, 1, 1)$

$f_1 = a_1 = (1, 1, 1) \sim (1)$

$\underline{\lambda_2 = 0}$

$$B_2 = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 4 & -5 & 2 \\ 6 & -9 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 \\ 0 & 3 & -2 \\ \cancel{0} & \cancel{3} & \cancel{-2} \end{pmatrix}$$

ФРС:

x_1	x_2	x_3
1	2	3

$a_2 = (1, 2, 3)$

$$B_2^2 = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}\begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix} = \begin{pmatrix} 3 & -3 & 1 \\ \cancel{3} & \cancel{-3} & \cancel{1} \\ \cancel{3} & \cancel{-3} & \cancel{1} \end{pmatrix} \sim (3, -3, 1)$$

ДФРС:

x_1	x_2	x_3
1	1	0

$a_3 = (1, 1, 0)$ $\quad \varphi_2(a_3) = \begin{pmatrix} 4 & -5 & 2 \\ 5 & -7 & 3 \\ 6 & -9 & 4 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \end{pmatrix}$

$f_1 = a_1 = (1, 1, 1) \; \} \; (1)$

$f_2 = \varphi_2(a_3) = (-1, -2, -3) \; \} \; \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$f_3 = a_3 = (1, 1, 0)$

$$A_f = \left(\begin{array}{c|cc} 1 & 0 & 0 \\ \hline 0 & 0 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

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1097. $A = \begin{pmatrix} 5 & -3 & 2 \\ 6 & -4 & 4 \\ 4 & -4 & 5 \end{pmatrix}$

$$|A - \lambda E| = \begin{vmatrix} 5-\lambda & -3 & 2 \\ 6 & -4-\lambda & 4 \\ 4 & -4 & 5-\lambda \end{vmatrix} = (5-\lambda)^2(-4-\lambda) - 48 - 48 - 8(-4-\lambda) +$$

$$+ 12(5-\lambda) + 16(5-\lambda) = -\lambda^3 + 6\lambda^2 - 11\lambda + 6 =$$

$$= (\lambda-1)(\lambda-2)(\lambda-3)$$

$\lambda_1 = 1$ кр. 1

$\lambda_2 = 2$ кр. 1

$\lambda_3 = 3$ кр. 1

$\lambda_1 = 1$

$$B_1 = \begin{pmatrix} 4 & -3 & 2 \\ 6 & -5 & 4 \\ 4 & -4 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ \cancel{0} & \cancel{1} & \cancel{-2} \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	2	1

$a_1 = (1, 2, 1)$

$\lambda_2 = 2$

$$B_2 = \begin{pmatrix} 3 & -3 & 2 \\ \cancel{6} & \cancel{-6} & \cancel{4} \\ 4 & -4 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	1	0

$a_2 = (1, 1, 0)$

$\lambda_3 = 3$

$$B_3 = \begin{pmatrix} 2 & -3 & 2 \\ 6 & -7 & 4 \\ 4 & -4 & 2 \end{pmatrix} \sim \begin{pmatrix} 2 & -3 & 1 \\ 0 & -1 & 1 \\ \cancel{0} & \cancel{2} & \cancel{-2} \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	2	2

$a_3 = (1, 2, 2)$

$f_1 = a_1 = (1, 2, 1) \sim (1)$

$f_2 = a_2 = (1, 1, 0) \sim (2)$

$f_3 = a_3 = (1, 2, 2) \sim (3)$

$$A_J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

1107. $A = \begin{pmatrix} 3 & -4 & 0 & 2 \\ 4 & -5 & -2 & 4 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$

$|A - \lambda E| = \begin{vmatrix} 3-\lambda & -4 & 0 & 2 \\ 4 & -5-\lambda & -2 & 4 \\ 0 & 0 & 3-\lambda & -2 \\ 0 & 0 & 2 & -1-\lambda \end{vmatrix} = (3-\lambda)(-1-3\lambda-3\lambda^2-\lambda^3)+2(2+4\lambda+2\lambda^2)$

$= 1 - 2\lambda^2 + \lambda^4 = ((1-\lambda)(1+\lambda))^2$

$\lambda_1 = 1$ кр. 2

$\lambda_2 = -1$ кр. 2

$\underline{\lambda_1 = 1}$

$B_1 = \begin{pmatrix} 2 & -4 & 0 & 2 \\ 4 & -6 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ \cancel{0} & \cancel{0} & \cancel{2} & \cancel{-2} \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 2 & -2 & 0 \\ 0 & 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 & 1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$

ФРС:

x_1	x_2	x_3	x_4
1	1	1	1

$a_1 = (1, 1, 1, 1)$

$B_1^2 = \begin{pmatrix} 2 & -4 & 0 & 2 \\ 4 & -6 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} 2 & -4 & 0 & 2 \\ 4 & -6 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & -2 \end{pmatrix} = \begin{pmatrix} -12 & 16 & 12 & -16 \\ -16 & 20 & 16 & -20 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{pmatrix}$

2 ФРС:

x_1	x_4	x_2	x_4
1	0	1	0

$a_2 = (1, 0, 1, 0)$

$\varphi_1(a_2) = \begin{pmatrix} 2 & -4 & 0 & 2 \\ 4 & -6 & -2 & 4 \\ 0 & 0 & 2 & -2 \\ 0 & 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$

$\underline{\lambda_2 = -1}$

$B_2 = \begin{pmatrix} 4 & -4 & 0 & 2 \\ 4 & -4 & -2 & 4 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 2 & -2 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \end{pmatrix} \sim \begin{pmatrix} 2 & -2 & 0 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

ФРС:

x_1	x_2	x_3	x_4
1	1	0	0

$a_3 = (1, 1, 0, 0)$

$B_2^2 = \begin{pmatrix} 4 & -4 & 0 & 2 \\ 4 & -4 & -2 & 4 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 2 & -2 \end{pmatrix} \begin{pmatrix} 4 & -4 & 0 & 2 \\ 4 & -4 & -2 & 4 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 3 & -2 \\ 0 & 0 & 2 & -1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2 ФРС:

x_1	x_2	x_3	x_4
0	0	1	1

$a_4 = (0, 0, 1, 1)$ $\quad \varphi_2(a_4) = \begin{pmatrix} 4 & -4 & 0 & 2 \\ 4 & -4 & -2 & 4 \\ 0 & 0 & 4 & -2 \\ 0 & 0 & 2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$

$f_1 = \varphi(a_2) = (2, 2, 2, 2)$ $\quad f_3 = \varphi(a_4) = (2, 2, 2, 2)$

$f_2 = a_2 = (1, 0, 1, 0)$ $\quad \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\quad f_4 = a_4 = (0, 0, 1, 1)$ $\quad \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

$A_j = \left(\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -1 \end{array}\right)$