4.2 $\lim\limits_{n\to\infty} \frac{2n^2-1}{6n^2+1} = \lim\limits_{n\to\infty} \frac{n^2\left(2-\frac{1}{n^2}\right)}{n^2\left(6-\frac{1}{n^2}\right)} = \lim\limits_{n\to\infty} \frac{2-\frac{1}{n^2}\to 0}{6-\frac{1}{n^2}\to 0} = \frac{2}{6} = \frac{1}{3}.$

4.5 $\lim\limits_{n\to\infty} \frac{\sin n}{\sqrt{n}} = \lim\limits_{n\to\infty}\left(\sin n \cdot \frac{1}{\sqrt{n}}\right) = O(1)\cdot o(1) = o(1).$

4.7 $\lim\limits_{n\to\infty} \frac{\ln(1+n^2)}{2^n} = \lim\limits_{n\to\infty} \ln(1+n^2) \cdot \lim\limits_{n\to\infty} \frac{1}{2^n} = O(1)\cdot o(1) = o(1).$

4.17 $x_n = \sum\limits_{k=1}^{n} \frac{2k+1}{(2k+2)!!} = \sum\limits_{k=1}^{n}\left(\frac{2k+2}{(2k+2)!!} - \frac{1}{(2k+2)!!}\right) = \sum\limits_{k=1}^{n}\left(\frac{1}{2k!!} - \frac{1}{(2k+2)!!}\right) =$

$= \frac{1}{2} - \cancel{\frac{1}{4!!}} + \cancel{\frac{1}{4!!}} - \cancel{\frac{1}{6!!}} + \cancel{\frac{1}{6!!}} \dots - \frac{1}{(2n+2)!!} = \frac{1}{2} - \frac{1}{(2n+2)!!} \to \frac{1}{2}$

4.25 $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n}} \operatorname{arctg} \frac{n^3}{2n+1} + \frac{\sin n - n}{1-4n}\right) =$

$= \lim\limits_{n\to\infty}\left(\frac{\operatorname{arctg}\left(\frac{n^3}{2n+1}\right)}{\sqrt{n}}\right) + \lim\limits_{n\to\infty}\left(\frac{\sin n - n}{1-4n}\right) =$

$= \lim\limits_{n\to\infty}\left(O(1)\cdot o(1)\right) + \lim\limits_{n\to\infty}\left(\frac{\cancel{n}\left(\frac{\sin n}{n}-1\right)}{\cancel{n}\left(\frac{1}{n}-4\right)}\right) =$ $\left(\frac{\sin n}{n} \to O(1)\cdot o(1) = o(1)\right)$, $\left(\frac{1}{n} \to o(1)\right)$

$= \frac{-1}{-4} = \frac{1}{4}.$

4.26 $\lim\limits_{n\to\infty}\left(\frac{\operatorname{arctg} n}{\sqrt{n}-\sqrt[3]{n}} + \frac{\{n^3-\frac{2}{3}n\}}{\ln n} + 1\right) =$

$= \lim\limits_{n\to\infty}\left(\frac{\operatorname{arctg} n}{\sqrt{n}-\sqrt[3]{n}}\right) + \lim\limits_{n\to\infty}\left(\frac{\{n^3-\frac{2}{3}n\}}{\ln n}\right) + 1 =$

$= \frac{\frac{\pi}{2}}{+\infty} = 0 + \frac{\frac{2}{3}}{+\infty} = 0 + 1 = 1.$

4.27 $\lim\limits_{n\to\infty}\left(\frac{n}{2n^2-1}\cdot\cos\frac{n+1}{2n-1} - \frac{n}{1-2n}\cdot\frac{n\cdot(-1)^n}{n^2+1}\right) =$

$= 0\cdot\frac{1}{2} - \left(-\frac{1}{2}\right)\cdot 0 = 0.$

4.34 $\lim\limits_{n\to\infty} \sqrt[n]{a} = 1 \quad (a>0)$

$\sqrt[n]{a} = a^{\frac{1}{n}\to 0} \Rightarrow a \to 1 \Rightarrow \lim\limits_{n\to\infty} \sqrt[n]{a} = 1$

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