20.26 $\lim\limits_{x\to 0} \dfrac{\int\limits_0^{x^2} \operatorname{arctg} t^2 dt}{\int\limits_{x^3}^{x^5} \operatorname{arctg} t\, dt} = \left[\frac{0}{0}\right] = \lim\limits_{x\to 0} \dfrac{\frac{d}{dx} \int\limits_0^{x^2} \operatorname{arctg} t^2 dt}{\frac{d}{dx}\left(\int\limits_0^{x^5} \operatorname{arctg} t\, dt - \int\limits_0^{x^3} \operatorname{arctg} t\, dt\right)} =$

$= \lim\limits_{x\to 0} \dfrac{\operatorname{arctg} x^4}{\operatorname{arctg} x^5 - \operatorname{arctg} x^3} = \lim\limits_{x\to 0} \dfrac{x^4 + o(x^5)}{x^5 - x^3 + o(x^5)} = \lim\limits_{x\to 0} \dfrac{x}{x^2 - 1} = 0$ //

20.29 $I_1 = \int\limits_1^2 \ln^2 x\, dx$? $I_2 = \int\limits_1^2 \ln x\, dx$

$\ln^2 x < \ln x$, оскільки $\ln x < 1$, $x \in (1, 2)$

$\Rightarrow I_1 < I_2$ //

20.30 $I_1 = \int\limits_0^1 2^{x^2} dx$? $I_2 = \int\limits_0^1 2^{x^3} dx$

$x^2 > x^3$, оскільки $x \in (0, 1)$

$\Rightarrow I_1 > I_2$ //

20.38 $\int\limits_{\pi}^{2\pi} \sin x^2 dx$ За I теоремою про середнє

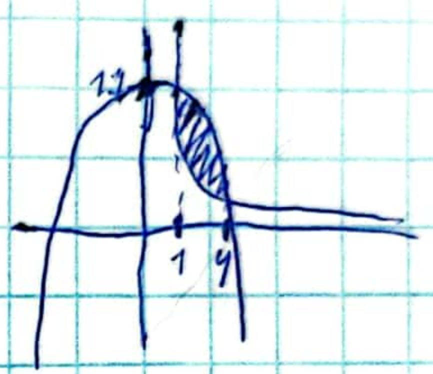
$I = \sin \xi^2 \int\limits_{\pi}^{2\pi} dx = \sin \xi^2 \cdot \pi$

$M = \sup\limits_{\xi \in [\pi, 2\pi]} \sin \xi^2 = 1$, $m = \inf\limits_{\xi \in [\pi, 2\pi]} \sin \xi^2 = -1$

$-1 \le \sin \xi^2 \cdot \pi \le 1$ $I \in [-1; 1]$

21.4 $y = \frac{16}{x^2}, y = 17 - x^2, x > 0$

$\frac{16}{x^2} = 17 - x^2; \; x^4 - 17x^2 + 16 = 0$



$x_1 = 1 \qquad x_2 = -1 \Big\} < 0$

$x_3 = 4 \qquad x_4 = -4$

$S = \int_1^4 \left(17 - x^2 - \frac{16}{x^2}\right) dx = \left(17x - \frac{x^3}{3} + \frac{16}{x}\right)\Big|_1^4 = 18 \,//$

21.12 $\rho = 2\cos\varphi, \rho \geq 1$

$2\cos\varphi \geq 1 \Rightarrow \cos\varphi \geq \frac{1}{2}$

$2\pi k - \frac{\pi}{3} \leq \varphi \leq \frac{\pi}{3} + 2\pi k$

$S_1 = \frac{1}{2}\int_0^{\pi/3} \rho^2(\varphi)\, d\varphi = \frac{1}{2}\int_0^{\pi/3} 4\cos^2\varphi\, d\varphi = 2\int_0^{\pi/3} \cos^2\varphi\, d\varphi =$

$= \int_0^{\pi/3} (1 + \cos 2\varphi)\, d\varphi = \frac{\pi}{3} + \frac{1}{2}\left(\sin\frac{2\pi}{3} - \sin 0\right) = \frac{\pi}{3} + \frac{\sqrt{3}}{4}$

$S = \frac{2\pi}{3} + \frac{\sqrt{3}}{2} \,//$