11.8 $f(x) = 10^{\frac{x}{\log_x 3}} = 10^{\frac{x\log_{10}x}{\log_{10}3}}$, $x>0$

$f'(x) = 10^{\frac{x}{\log_x 3}} \cdot \ln 10 \cdot \left(\frac{x\log_{10}x}{\log_{10}3}\right)' =$

$= 10^{\frac{x}{\log_x 3}} \cdot \cancel{\ln 10} \cdot \frac{\ln 10 \cdot \log_{10}x + 1}{\cancel{\ln 10}\cdot\log_{10}3} = 10^{\frac{x}{\log_x 3}} \cdot \frac{\ln x + 1}{\log_{10}3}$, $x>0$ //

11.10 $f(x) = \operatorname{arctg}\frac{1+x}{1-x}$, $x \neq 1$

$f'(x) = \left(\operatorname{arctg}\frac{1+x}{1-x}\right)' \cdot \left(\frac{1+x}{1-x}\right)' = \frac{1}{1+\left(\frac{1+x}{1-x}\right)^2} \cdot \frac{2}{(1-x)^2} =$

$= \frac{2}{(1-x)^2+(1+x)^2} = \frac{2}{2+2x^2} = \frac{1}{1+x^2}$, $x \neq 1$ //

11.13 $f(x) = 2^{\operatorname{tg}\frac{1}{x}}$, $x_0 = \frac{1}{\pi}$

$f'(x) = 2^{\operatorname{tg}\frac{1}{x}} \cdot \ln 2 \cdot \left(\operatorname{tg}\frac{1}{x}\right)' = 2^{\operatorname{tg}\frac{1}{x}} \cdot \ln 2 \cdot \left(\operatorname{tg}\frac{1}{x}\right)' \cdot \left(\frac{1}{x}\right)' =$

$= 2^{\operatorname{tg}\frac{1}{x}} \cdot \ln 2 \cdot \frac{1}{\cos^2\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)$

$f'(x_0) = f'\left(\frac{1}{\pi}\right) = \underbrace{2^{\overbrace{\operatorname{tg}\pi}^{=0}}}_{1} \cdot \ln 2 \cdot \underbrace{\frac{1}{\cos^2\pi}}_{1} \cdot (-\pi^2) = -\pi^2 \ln 2$ //

11.14 $f(x) = \operatorname{sgn} x$, $x_0 = 0$

$f(x) = \operatorname{sgn} x = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ 0, & x = 0 \end{cases}$

$f'_п(0) = \lim_{\Delta x \to 0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim_{\Delta x \to 0} \frac{1}{\Delta x}$

$f'_л(0) = \lim_{\Delta x \to 0} \frac{f(0) - f(0-\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-1}{\Delta x}$

не диференційовна //

11.15 $f(x) = \min\{x, \sin x\}$, 1) $x_0 = -\pi$, 2) $x_0 = 0$

1) $f'_п(-\pi) = \lim_{\Delta x \to 0} \frac{f(-\pi+\Delta x) - f(-\pi)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\pi + \Delta x + \pi}{\Delta x} = 1$ $\quad f'_п = f'_л \Rightarrow$

$f'_л(-\pi) = \lim_{\Delta x \to 0} \frac{f(-\pi) - f(-\pi-\Delta x)}{\Delta x} = \lim_{\Delta x \to 0} \frac{-\pi + \pi + \Delta x}{\Delta x} = 1$ $\quad \rightarrow$ диференційовна //

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2) $f'_п(0) = \lim\limits_{\Delta x \to 0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{0+\Delta x - 0}{\Delta x} = 1$

$f'_л(0) = \lim\limits_{\Delta x \to 0} \frac{f(0) - f(0-\Delta x)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{0 - 0 + \Delta x}{\Delta x} = 1$

$f'_п(0) = f'_л(0) \Rightarrow$ диференційовна //

11.16 $f(x) = \begin{cases} \frac{x}{1+e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ 1) $x_0 = 0$, 2) $x_0 = \frac{1}{2}$, 3) $x_0 = 2$

1) $f'_п(0) = \lim\limits_{\Delta x \to 0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{\frac{\Delta x}{1+e^{\frac{1}{\Delta x}}} - 0}{\Delta x} = \frac{1}{1+e^{\frac{1}{\Delta x}}} = \frac{1}{\infty} = 0$

$f'_л(0) = \lim\limits_{\Delta x \to 0} \frac{f(0) - f(0-\Delta x)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{0 - \frac{-\Delta x}{1+e^{-\frac{1}{\Delta x}}}}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{1}{1+e^{-\frac{1}{\Delta x}}} = 1$

$f'_п(0) \neq f'_л(0) \Rightarrow$ не диференційовна //

При $x \neq 0$

$f'(x) = \frac{1+e^{\frac{1}{x}} - x(e^{\frac{1}{x}})(-\frac{1}{x^2})}{(1+e^{\frac{1}{x}})^2} = \frac{1+(\frac{1}{x}+1)e^{\frac{1}{x}}}{(1+e^{\frac{1}{x}})^2}$ //

11.18 $f(x) = x \cdot |x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$ $f'(x) = \begin{cases} 2x, & x \geq 0 \\ -2x, & x < 0 \end{cases}$

$f(x) = x^2 \operatorname{sgn} x$

$f'(x) = 2x \cdot \operatorname{sgn} x = 2|x|$

$f'_п(0) = \lim\limits_{\Delta x \to 0} \frac{f(0+\Delta x) - f(0)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{(\Delta x)^2 - 0}{\Delta x} = 0$

$f'_л(0) = \lim\limits_{\Delta x \to 0} \frac{0 - (0-\Delta x)(-\Delta x)}{\Delta x} = \lim\limits_{\Delta x \to 0} \frac{(\Delta x)^2}{\Delta x} = 0$

$f'_п(0) = f'_л(0) \Rightarrow$ диференційовна //

11.21 $f(x) = [x] \sin \pi x$ При $x \in \mathbb{R} \setminus \mathbb{Z}$:

$f(k) = 0$, $k \in \mathbb{Z}$ $f'(x) = \pi [x] \cos \pi x$

$f'_п(k) = \lim\limits_{\Delta x \to 0} \frac{f(k+\Delta x) - f(k)}{\Delta x} = k\pi \cdot (-1)^k$

$f'_л(k) = \lim\limits_{\Delta x \to 0} \frac{f(k) - f(k-\Delta x)}{\Delta x} = (k-1)\pi \cdot (-1)^k$ //

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11.26 $f(x) = \begin{cases} xe^{-\alpha x}, & x<0 \\ x^2-\beta x+2\alpha, & x\ge 0\end{cases}$

$f(0-0)=0,\ f(0+0)=2\alpha \Rightarrow \alpha=0$

Функція неперервна при $\alpha=0$ //

$f'_п(0)=\lim\limits_{\Delta x\to 0}\frac{\Delta x^2+\cancel{2\alpha}-\beta\Delta x-\cancel{2\alpha}}{\Delta x}=-\beta$ $\Longrightarrow$

$f'_л(0)=\lim\limits_{\Delta x\to 0}\frac{2\alpha-(0-\Delta x)e^{-\alpha(0-\Delta x)}}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{0+\Delta x}{\Delta x}=1$

$\Longrightarrow \boxed{\beta=-1,\ \alpha=0}$

11.31 $f(x)=\begin{cases}\alpha e^{x}+x-1, & x\le 0\\ \beta, & x>0\end{cases}$, 1) $x_0=0$, 2) $x_0=1$

$f(0+0)=\beta,\ f(0+0)=\alpha-1 \Rightarrow \boxed{\beta=\alpha-1 \text{ - функція неперервна}}$

1) $f'_п(0)=\lim\limits_{\Delta x\to 0}\frac{\beta+1-\alpha}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\cancel{\alpha-1+1-\alpha}}{\Delta x}=0$

$f'_л(0)=\lim\limits_{\Delta x\to 0}\frac{\alpha-1-(\alpha e^{(-\Delta x)}-\Delta x-1)}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\alpha-1-(\alpha(1-\Delta x)-\Delta x-1)}{\Delta x}=$

$=\lim\limits_{\Delta x\to 0}\frac{\cancel{\alpha-1-\alpha}+\alpha\Delta x+\Delta x\cancel{+1}}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{\alpha\Delta x+\Delta x}{\Delta x}=\alpha+1$

$f'_п(0)=f'_л(0)\Rightarrow\boxed{\alpha+1=0}$ $\alpha=-1,\ \beta=-2$ //

2) $f(1)=\beta$ $f'(x)=\begin{cases}\alpha e^{x}+1, & x\le 0\\ 0, & x>0\end{cases}$

$\alpha,\beta\in\mathbb{R}$ //