545. a) $\underbrace{4x_1x_2 + x_2^2 + 4x_2x_3 + 2x_3^2}_{f(x)} - 4x_1 - 2x_2 - 5 = 0$

$$A = \begin{pmatrix} 0 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} -\lambda & 2 & 0 \\ 2 & 1-\lambda & 2 \\ 0 & 2 & 2-\lambda \end{vmatrix} = -\lambda(1-\lambda)(2-\lambda) + 4\lambda - 4(2-\lambda) =$$

$$= (2+\lambda)(\lambda-4)(1-\lambda) = 0$$

$\lambda_1 = 4$

$\lambda_2 = 1$

$\lambda_3 = -2$

$\underline{\lambda_1 = 4}$

$$A - 4E = \begin{pmatrix} -4 & 2 & 0 \\ 2 & -3 & 2 \\ 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & -2 \end{pmatrix} \sim \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	2	2

$$e_1 = \frac{a_1}{|a_1|} = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$$

$\lambda_2 = 1$

$$A - E = \begin{pmatrix} -1 & 2 & 0 \\ 2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 \\ 0 & 4 & 2 \\ 0 & 2 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 0 \\ 0 & 2 & 1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
2	1	-2

$$e_2 = \frac{a_2}{|a_2|} = \left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$$

$e_3 = \left(\frac{2}{3}, -\frac{2}{3}, \frac{1}{3}\right)$

$\lambda_3 = -2$

$$A + 2E = \begin{pmatrix} 2 & 2 & 0 \\ 2 & 3 & 2 \\ 0 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 2 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
2	-2	1

$$Q = (l_1 | l_2 | l_3) = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix}$$

$$D = Q^{-1} A Q = Q^{T} A Q = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$f(X) = 4y_1^2 + y_2^2 - 2y_3^2$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$$x_1 = \frac{1}{3} y_1 + \frac{2}{3} y_2 + \frac{2}{3} y_3$$
$$x_2 = \frac{2}{3} y_1 + \frac{1}{3} y_2 - \frac{2}{3} y_3$$
$$x_3 = \frac{2}{3} y_1 - \frac{2}{3} y_2 + \frac{1}{3} y_3$$

$$4y_1^2 + y_2^2 - 2y_3^2 - 4\left(\frac{1}{3} y_1 + \frac{2}{3} y_2 + \frac{2}{3} y_3\right) - 2\left(\frac{2}{3} y_1 + \frac{1}{3} y_2 - \frac{2}{3} y_3\right) - 5 = 0;$$

$$4y_1^2 + y_2^2 - 2y_3^2 - \frac{8}{3} y_1 - \frac{10}{3} y_2 - \frac{4}{3} y_3 - 5 = 0;$$

$$4\left(y_1^2 - \frac{2}{3} y_1 + \frac{1}{9}\right) - \frac{4}{9} + \left(y_2^2 - \frac{10}{3} y_2 + \frac{25}{9}\right) - \frac{25}{9} - 2\left(y_3^2 + \frac{2}{3} y_3 + \frac{1}{9}\right) + \frac{2}{9} - 5 = 0;$$

$$4\left(y_1 - \frac{1}{3}\right)^2 + \left(y_2 - \frac{5}{3}\right)^2 - 2\left(y_3 + \frac{1}{3}\right)^2 - 8 = 0$$

$$y_1 - \frac{1}{3} = z_1 \qquad y_1 = z_1 + \frac{1}{3}$$
$$y_2 - \frac{5}{3} = z_2 \qquad y_2 = z_2 + \frac{5}{3}$$
$$y_3 + \frac{1}{3} = z_3 \qquad y_3 = z_3 - \frac{1}{3}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} \frac{1}{3} \\ \frac{5}{3} \\ -\frac{1}{3} \end{pmatrix}$$

$$4z_1^2 + z_2^2 - 2z_3^2 - 8 = 0 \quad //$$

$$\frac{z_1^2}{2} + \frac{z_2^2}{8} - \frac{z_3^2}{4} = 1$$ – однопорожнинний гіперболоїд

Остаточна заміна:

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} 1/3 \\ 5/3 \\ -1/3 \end{pmatrix};$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{pmatrix} \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \quad //$$

д) $\underbrace{x_1^2 + 2x_1x_2 - 2x_1x_3 + x_2^2 - 2x_2x_3 + x_3^2}$ $- 2x_1 = 0$

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} 1-\lambda & 1 & -1 \\ 1 & 1-\lambda & -1 \\ -1 & -1 & 1-\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & -\lambda \\ -1 & -1 & -\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1 & 0 \\ 2 & 2-\lambda & 0 \\ -1 & -1 & -\lambda \end{vmatrix} = -\lambda \begin{vmatrix} 1-\lambda & 1 \\ 2 & 2-\lambda \end{vmatrix} =$$

$$= -\lambda \begin{vmatrix} -\lambda & 1 \\ \lambda & 2-\lambda \end{vmatrix} = -\lambda \begin{vmatrix} -\lambda & 1 \\ 0 & 3-\lambda \end{vmatrix} = \lambda^2(3-\lambda)$$

$\lambda_1 = 3$

$\lambda_2 = 0$ кр. 2

$\lambda_1 = 3$

$$A - 3E = \begin{pmatrix} -2 & 1 & -1 \\ 1 & -2 & -1 \\ -1 & -1 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & -3 & -3 \\ 0 & 3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3	
-1	-1	1	$= a_1$

$$e_1 = \frac{a_1}{|a_1|} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}\right)$$

$\lambda_2 = 0$

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} \sim (1\ 1\ -1)$$

ФСР:

x_1	x_2	x_3	
1	-1	0	$= a_2$
1	0	1	$= a_3$

$b_2 = a_2 = (1, -1, 0)$

$b_3 = a_3 - \alpha b_2 \qquad \alpha = \frac{(a_3, b_2)}{(b_2, b_2)} = \frac{1}{2}$

$$b_3 = a_3 - \frac{1}{2} b_2 = (1, 0, 1) + \left(-\frac{1}{2}, \frac{1}{2}, 0\right) = \left(\frac{1}{2}, \frac{1}{2}, 1\right) = \frac{1}{2}(1, 1, 2)$$

$$e_2 = \frac{b_2}{|b_2|} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$$

$$e_3 = \frac{b_3}{|b_3|} = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}\right)$$

$$Q = (e_1 | e_2 | e_3) = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ -1/\sqrt{3} & 0 & 2/\sqrt{6} \end{pmatrix} \qquad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = Q \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$

$x_1 = \frac{1}{\sqrt{3}} y_1 + \frac{1}{\sqrt{2}} y_2 + \frac{1}{\sqrt{6}} y_3$

$x_2 = \frac{1}{\sqrt{3}} y_1 - \frac{1}{\sqrt{2}} y_2 + \frac{1}{\sqrt{6}} y_3$

$x_3 = -\frac{1}{\sqrt{3}} y_1 + \cancel{\ldots y_2} + \frac{2}{\sqrt{6}} y_3$

$f(X) = 3y_1^2$

$$3y_1^2 - 2\left(\frac{1}{\sqrt{3}}y_1 + \frac{1}{\sqrt{2}}y_2 + \frac{1}{\sqrt{6}}y_3\right) = 0;$$

$$3y_1^2 - \frac{2}{\sqrt{3}}y_1 - \sqrt{2}\,y_2 - \frac{2}{\sqrt{6}}y_3 = 0;$$

$$3\left(y_1^2 - \frac{2}{3\sqrt{3}}y_1 + \frac{1}{27}\right) - \frac{1}{9} - \sqrt{2}\,y_2 - \frac{2}{\sqrt{6}}y_3 = 0$$

$$3\left(y_1 - \frac{1}{3\sqrt{3}}\right)^2 - \sqrt{2}\,y_2 - \frac{2}{\sqrt{6}}y_3 = \frac{1}{9}$$

$$y_1 = z_1$$

$$y_2 = \frac{-\sqrt{2}\,z_2 - \frac{2}{\sqrt{6}}z_3}{-\sqrt{2} -}$$