837. $\begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix} = A$

$$\left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 5 & 7 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 3 & 4 & 1 & 0 \\ 2 & 3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 2 & -1 \\ 2 & 3 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 2 & -1 \\ 0 & 1 & -5 & 3 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & 7 & -4 \\ 0 & 1 & -5 & 3 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 7 & -4 \\ -5 & 3 \end{pmatrix}$$

842. $A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -3 & -3 & 1 & 0 & -2 \\ 0 & -4 & -5 & 0 & 1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & -1 & 1 \\ 0 & -4 & -5 & 0 & 1 & -3 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 & -1 & 1 \\ 0 & 0 & 3 & 4 & -3 & 1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{3} & 2 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) = A^{-1}$$

844. $A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & -1 & -1 & 1 & 0 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -2 & -1 & 0 & 1 & 0 \\ 0 & -2 & -2 & 0 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & -2 & -2 & 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \end{array}\right)$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -2 & \frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & -2 & -2 & -1 & 1 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & 0 & \frac{1}{4} & -\frac{1}{2} \\ 0 & 0 & 0 & -\frac{5}{2} & -\frac{1}{2} & 1 & \frac{1}{2} & -1 \end{array}\right)$$

$$A^{-1} = \frac{1}{4}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

864. $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 5 \\ 5 & 9 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 3 & 4 & 5 & 9 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 0 & -2 & -4 & -6 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 2 & 3 & 5 \\ 0 & 1 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 0 & -1 & -1 \\ 0 & 1 & 2 & 3 \end{array}\right)$$

$X = \begin{pmatrix} -1 & -1 \\ 2 & 3 \end{pmatrix}$ //

865. $X \cdot \begin{pmatrix} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \end{pmatrix} = \begin{pmatrix} -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{pmatrix}$

$$\left(\begin{array}{ccc} 5 & 3 & 1 \\ 1 & -3 & -2 \\ -5 & 2 & 1 \\ \hline -8 & 3 & 0 \\ -5 & 9 & 0 \\ -2 & 15 & 0 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ -2 & 11 & 3 \\ 1 & -10 & -1 \\ \hline 0 & -8 & 3 \\ 0 & -5 & 9 \\ 0 & -2 & 15 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & -19 & 3 \\ 0 & 0 & -1 \\ \hline 3 & -38 & 3 \\ 9 & -95 & 9 \\ 15 & -152 & 15 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{array}\right)$$

$X = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ //

866. $\begin{pmatrix} 2 & -3 & 1 \\ 4 & -5 & 2 \\ 5 & -7 & 3 \end{pmatrix} \cdot X \cdot \begin{pmatrix} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 & -2 \\ 18 & 12 & 9 \\ 23 & 15 & 11 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 2 & -3 & 1 & 2 & 0 & -2 \\ 4 & -5 & 2 & 18 & 12 & 9 \\ 5 & -7 & 3 & 23 & 15 & 11 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 19 & 15 & 15 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 2 & -3 & 1 & 2 & 0 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 19 & 15 & 15 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 0 & -1 & -1 & -36 & -30 & -32 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & 1 & 19 & 15 & 15 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 0 & 0 & 1 & 12 & 18 & 19 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 33 & 27 & 28 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 0 & 0 & 1 & 12 & 18 & 19 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 21 & 9 & 9 \\ 0 & 1 & 0 & 14 & 12 & 13 \\ 0 & 0 & 1 & 12 & 18 & 19 \end{array}\right)$$

$$\left(\begin{array}{ccc} 9 & 7 & 6 \\ 1 & 1 & 2 \\ 1 & 1 & 1 \\ \hline 21 & 9 & 9 \\ 14 & 12 & 13 \\ 12 & 18 & 19 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 9 & 6 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \\ \hline 0 & 21 & 9 \\ -1 & 14 & 13 \\ -1 & 12 & 19 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 10 & 8 \\ 0 & 1 & 1 \\ \hline 0 & 21 & 9 \\ -1 & 23 & 19 \\ -1 & 21 & 25 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 2 & 8 \\ 0 & 0 & 1 \\ \hline 0 & 12 & 9 \\ -1 & 4 & 19 \\ -1 & -4 & 25 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ -1 & 1 & 8 \\ 0 & 0 & 1 \\ \hline 0 & 6 & 9 \\ -1 & 2 & 19 \\ -1 & -2 & 25 \end{array}\right) \sim \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ \hline 6 & 6 & 6 \\ 1 & 2 & 3 \\ -2 & -3 & -11 \end{array}\right)$$

$X = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$

868. $X \cdot \begin{pmatrix} 3 & 6 \\ 4 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 9 & 18 \end{pmatrix}$

$$\left(\begin{array}{cc} 3 & 6 \\ 4 & 8 \\ \hline 2 & 4 \\ 9 & 18 \end{array}\right) \sim \left(\begin{array}{cc} 3 & 0 \\ 4 & 0 \\ \hline 2 & 0 \\ 9 & 0 \end{array}\right)$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 4 & 0 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 9 & 0 \end{pmatrix}$$

$$\begin{cases} 3a + 4b = 2 \\ 3c + 4d = 9 \end{cases} \qquad a = \frac{2-4b}{3}, \quad c = \frac{9-4d}{3}, \quad b, d \in \mathbb{R}.$$

$$X = \begin{pmatrix} \frac{2-4b}{3} & b \\ \frac{9-4d}{3} & d \end{pmatrix}, \; b, d \in \mathbb{R} \; //$$

870. $\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix} \cdot X = \begin{pmatrix} 3 & 9 & 7 \\ 1 & 11 & 7 \\ 7 & 5 & 7 \end{pmatrix}$

$$\left(\begin{array}{ccc|ccc} 3 & -1 & 2 & 3 & 9 & 7 \\ 4 & -3 & 3 & 1 & 11 & 7 \\ 1 & 3 & 0 & 7 & 5 & 7 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 3 & 0 & 7 & 5 & 7 \\ 0 & -10 & 2 & -18 & -6 & -14 \\ \cancel{0} & \cancel{-15} & \cancel{3} & \cancel{-27} & \cancel{-9} & \cancel{-21} \end{array}\right)$$

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & -10 & 2 \\ 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} a & b & c \\ d & e & f \\ m & n & l \end{pmatrix} = \begin{pmatrix} 7 & 5 & 7 \\ -18 & -6 & -14 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned} a + 3d &= 7 & \qquad a &= 7 - 3d \\ b + 3e &= 5 & b &= 5 - 3e \\ c + 3f &= 7 & c &= 7 - 3f \\ -10d + 2m &= -18 & m &= -9 + 5d \\ -10e + 2n &= -6 & n &= -3 + 5e \\ -10f + 2l &= -14 & l &= -7 + 5f \end{aligned}$$

$$X = \begin{pmatrix} 7-3d & 5-3e & 7-3f \\ d & e & f \\ -9+5d & -3+5e & -7+5f \end{pmatrix}, \; d, e, f \in \mathbb{R} \; //$$

469.
$$\begin{vmatrix} a & b & c & d & e & f & g & h \\ -b & a & d & -c & f & -e & -h & g \\ -c & -d & a & b & g & h & -e & -f \\ -d & c & -b & a & h & -g & f & -e \\ e & -f & -g & -h & a & b & c & d \\ -f & e & -h & g & -b & a & -d & c \\ -g & h & e & -f & -c & d & a & -b \\ -h & -g & f & e & -d & -c & b & a \end{vmatrix} = (a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2)^4$$

472.
$$\begin{vmatrix} 1 & \cos(\alpha_1-\alpha_2) & \cos(\alpha_1-\alpha_3) & \dots & \cos(\alpha_1-\alpha_n) \\ \cos(\alpha_1-\alpha_2) & 1 & \cos(\alpha_2-\alpha_3) & \dots & \cos(\alpha_2-\alpha_n) \\ \cos(\alpha_1-\alpha_n) & \cos(\alpha_2-\alpha_n) & \cos(\alpha_3-\alpha_n) & \dots & 1 \end{vmatrix} =$$

$$= \begin{vmatrix} \cos\alpha_1 & \sin\alpha_1 & 0 & \dots & 0 \\ \cos\alpha_2 & \sin\alpha_2 & 0 & \dots & 0 \\ \cos\alpha_n & \sin\alpha_n & 0 & \dots & 0 \end{vmatrix} \cdot \begin{vmatrix} \cos\alpha_1 & \cos\alpha_2 & \dots & \cos\alpha_n \\ \sin\alpha_1 & \sin\alpha_2 & \dots & \sin\alpha_n \\ 0 & 0 & \dots & 0 \end{vmatrix} =$$

$$= \begin{cases} 0, & n \geq 3 \\ \sin^2(\alpha_1-\alpha_2), & n = 2 \end{cases}$$ //

474.
$$\begin{vmatrix} \frac{1-a_1^n b_1^n}{1-a_1 b_1} & \frac{1-a_1^n b_2^n}{1-a_1 b_2} & \dots & \frac{1-a_1^n b_n^n}{1-a_1 b_n} \\ \frac{1-a_2^n b_1^n}{1-a_2 b_1} & \frac{1-a_2^n b_2^n}{1-a_2 b_2} & \dots & \frac{1-a_2^n b_n^n}{1-a_2 b_n} \\ \dots & \dots & \dots & \dots \\ \frac{1-a_n^n b_1^n}{1-a_n b_1} & \frac{1-a_n^n b_2^n}{1-a_n b_2} & \dots & \frac{1-a_n^n b_n^n}{1-a_n b_n} \end{vmatrix} = \begin{vmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \dots & a_2^{n-1} \\ \dots & & & & \\ 1 & a_n & a_n^2 & \dots & a_n^{n-1} \end{vmatrix} \cdot \begin{vmatrix} 1 & 1 & \dots & 1 \\ b_1 & b_2 & \dots & b_n \\ b_1^2 & b_2^2 & \dots & b_n^2 \\ \dots & & & \\ b_1^{n-1} & b_2^{n-1} & \dots & b_n^{n-1} \end{vmatrix} =$$

$$= \prod_{1 \leq j < i \leq n} (a_i - a_j)(b_i - b_j)$$ //

102. $(x-1-i)(x-1+i)(x+1+i)(x+1-i) =$

$= (x^2 - x + \cancel{xi} - x + 1 - \cancel{i} - \cancel{xi} + \cancel{i} - i^2)(x^2 + x - xi + x + 1 - i + xi + i - i^2) =$

$= (x^2 - 2x + 2)(x^2 + 2x + 2) = x^4 + \cancel{2x^3} + 2x^2 - \cancel{2x^3} - 4x^2 - \cancel{4x} + 2x^2 + \cancel{4x} + 4 =$

$= x^4 + 4$ //

Scanned with CamScanner

107. а) $\frac{1+i\,tg\alpha}{1-i\,tg\alpha}=\frac{1-tg^2\alpha}{1+tg^2\alpha}+\frac{2\,tg\alpha}{1+tg^2\alpha}i;$

108. б) $\begin{cases}(2+i)x+(2-i)y=6,\\(3+2i)x+(3-2i)y=8\end{cases}$

$\cancel{y=\frac{6-(2+i)x}{2-i}}$

$\cancel{(3+2i)x+\frac{(3-2i)(6-(2+i)x)}{2-i}=8;}$

$\cancel{3x+2xi+\frac{18-6x-3xi-12i+4xi+2i^2x}{2-i}=8;}$

$3x+2xi+$

$\begin{cases}(3+2i)x+(3-2i)y=8\\0x+\left(\frac{4}{13}+\frac{6i}{13}\right)y=\frac{14}{13}+\frac{8i}{13}\end{cases}$

$\begin{cases}(3+2i)x+(3-2i)y=8\\y=2-i\end{cases}$

$\begin{cases}(3+2i)x=4+7i\\y=2-i\end{cases}$

$\begin{cases}x=2+i\\y=2-i\end{cases}$ //

109. б) $\left(-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)^3=\left(1\left(\cos\left(\frac{2\pi}{3}\right)+i\cdot\sin\left(\frac{2\pi}{3}\right)\right)\right)^3=$

$=1^3\left(\cos\left(3\cdot\frac{2\pi}{3}\right)+i\cdot\sin\left(3\cdot\frac{2\pi}{3}\right)\right)=$

$=\cos(2\pi)+i\cdot\sin(2\pi)=\cos 0+i\cdot\sin 0=1$ //

119. б) $-1=\cos\pi+i\cdot\sin\pi;$

д) $-i=\cos\left(\frac{3\pi}{2}\right)+i\cdot\left(\frac{3\pi}{2}\right);$

ж) $-1+i=\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right)+i\cdot\sin\left(\frac{3\pi}{4}\right)\right)$

и) $1-i=\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right)+i\cdot\sin\left(\frac{7\pi}{4}\right)\right);$

л) $\sqrt{3}-i=2\left(\cos\left(\frac{11\pi}{6}\right)+i\cdot\sin\left(\frac{11\pi}{6}\right)\right)$ //

124. а) $|z| < 2, \quad z \in (-2, 2)$

б) $|z - i| \leq 1, \quad z \in [i - 1, 1 + i]$

в) $|z - 1 - i| < 1, \quad z \in (i, 2 + i)$

143. б) $\sqrt[3]{2 - 2i} = \sqrt[3]{2\sqrt{2}\left(\cos\left(\frac{7\pi}{4}\right) + i \cdot \sin\left(\frac{7\pi}{4}\right)\right)} =$

$= \sqrt[3]{2\sqrt{2}}\left(\cos\left(\frac{\frac{7\pi}{4} + 2k\pi}{3}\right) + i \cdot \sin\left(\frac{\frac{7\pi}{4} + 2k\pi}{3}\right)\right) =$

$= \sqrt{2}\left(\cos\left(\frac{7\pi}{12}\right) + i \cdot \sin\left(\frac{7\pi}{12}\right)\right) = \frac{1 - \sqrt{3}}{2} + \frac{1 + \sqrt{3}}{2} i$

$= \sqrt{2}\left(\cos\left(\frac{5\pi}{4}\right) + i \cdot \sin\left(\frac{5\pi}{4}\right)\right) = -1 - i$

$= \sqrt{2}\left(\cos\left(\frac{23\pi}{12}\right) + i \cdot \sin\left(\frac{23\pi}{12}\right)\right) = \frac{\sqrt{3} + 1}{2} + \frac{1 - \sqrt{3}}{2} i$

145. б) $\sqrt[8]{\frac{1 + i}{\sqrt{3} - i}} = \sqrt[8]{\frac{(1 + i) \cdot (\sqrt{3} + i)}{4}} = \sqrt[8]{\frac{\sqrt{3} + i + \sqrt{3}i - 1}{4}} =$

$= \sqrt[8]{\frac{64\sqrt{3} + 64i + 64\sqrt{3}i - 64}{2}}$