1572. $A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & 0 \end{pmatrix}$

$$|A-\lambda E| = \begin{vmatrix} \frac{1}{2}-\lambda & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & \frac{1}{2}-\lambda & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\lambda \end{vmatrix} = \begin{vmatrix} 1-\lambda & 1-\lambda & 0 \\ \frac{1}{2} & \frac{1}{2}-\lambda & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} -\lambda & \frac{\sqrt{2}}{2} \\ -\sqrt{2} & -\lambda \end{vmatrix} = (1-\lambda)(\lambda^2+1)$$

$\lambda_1 = 1$ кр. 1
$\lambda_2 = i$ кр. 1
$\lambda_3 = -i$ кр. 1

$\lambda_1 = 1$

$$A-E = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & -\frac{\sqrt{2}}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -\sqrt{2} \\ 1 & -1 & \sqrt{2} \\ \sqrt{2} & -\sqrt{2} & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & -\sqrt{2} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

1525. $A = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4} & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{2} \end{pmatrix}$.

$$|A-\lambda E| = \begin{vmatrix} \frac{3}{4}-\lambda & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4}-\lambda & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{2}-\lambda \end{vmatrix} = \begin{vmatrix} 3-4\lambda & 1 & \sqrt{6} \\ 1 & 3-4\lambda & \sqrt{6} \\ \sqrt{6} & -\sqrt{6} & 2-4\lambda \end{vmatrix} = \begin{vmatrix} 3-4\lambda & 4-4\lambda & -\sqrt{6} \\ 1 & 4-4\lambda & \sqrt{6} \\ \sqrt{6} & 0 & 2-4\lambda \end{vmatrix} =$$

$$= (4-4\lambda)\begin{vmatrix} 2-4\lambda & -\sqrt{6} \\ \sqrt{6} & 2-4\lambda \end{vmatrix} = (1-\lambda)(\lambda^2-\lambda+1)$$

$\lambda_1 = 1$ кр. 1

$\lambda_2 = \frac{1+i\sqrt{3}}{2}$ кр. 1

$\lambda_3 = \frac{1-i\sqrt{3}}{2}$ кр. 1

$\lambda_1 = 1$:

$$\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & -\frac{1}{4} & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & -\frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} -1 & 1 & -\sqrt{6} \\ 1 & -1 & \sqrt{6} \\ \sqrt{6} & -\sqrt{6} & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & \sqrt{6} \\ \sqrt{6} & -\sqrt{6} & -2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -1 & \sqrt{6} \\ 0 & 0 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & \sqrt{6} \\ 0 & 0 & 1 \end{pmatrix}$$

ФРС:

x_1	x_2	x_3
$-\sqrt{6}$	1	0

$e_1 = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$

$\lambda_2 = \frac{1+i\sqrt{3}}{2}$:

$$\begin{pmatrix} \frac{3}{4}-\frac{1+i\sqrt{3}}{2} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4}-\frac{1+i\sqrt{3}}{2} & -\frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{2}-\frac{1+i\sqrt{3}}{2} \end{pmatrix} \sim \begin{pmatrix} \frac{1}{4}-\frac{i\sqrt{3}}{4} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{1}{4}-\frac{i\sqrt{3}}{4} & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & -\frac{i\sqrt{3}}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1-i\sqrt{3} & \sqrt{6} \\ \sqrt{6} & -\sqrt{6} & -2i\sqrt{3} \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -i\sqrt{3} & \sqrt{6} \\ 0 & -2\sqrt{6} & -2i\sqrt{3} \end{pmatrix} \cdot 2i\sqrt{2} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & \sqrt{6} & 2i\sqrt{3} \\ 0 & 0 & 1 \end{pmatrix} \quad (0,0,0)$$

$\lambda_3 = \frac{1-i\sqrt{3}}{2}$:

$$\begin{pmatrix} \frac{3}{4}-\frac{1-i\sqrt{3}}{2} & \frac{1}{4} & -\frac{\sqrt{6}}{4} \\ \frac{1}{4} & \frac{3}{4}-\frac{1-i\sqrt{3}}{2} & \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} & -\frac{\sqrt{6}}{4} & \frac{1}{2}-\frac{1-i\sqrt{3}}{2} \end{pmatrix} \sim \begin{pmatrix} 3-2-2i\sqrt{3} & 1 & -\sqrt{6} \\ 1 & 3-2-2i\sqrt{3} & \sqrt{6} \\ \sqrt{6} & -\sqrt{6} & 2-1-i\sqrt{3} \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & -2i\sqrt{3} & \sqrt{6} \\ 0 & -2\sqrt{6} & 1-i\sqrt{3} \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2\sqrt{6} & 2i\sqrt{3} \\ 0 & -2\sqrt{6} & 1-i\sqrt{3} \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 \\ 0 & 2\sqrt{6} & 2i\sqrt{3} \\ 0 & 0 & 1 \end{pmatrix} \quad (0,0,0)$$

1576.

$$A=\begin{pmatrix} \frac12 & \frac12 & \frac12 & \frac12 \\ \frac12 & \frac12 & -\frac12 & -\frac12 \\ \frac12 & -\frac12 & \frac12 & -\frac12 \\ \frac12 & -\frac12 & -\frac12 & \frac12 \end{pmatrix}$$

$$|A-\lambda E| = \begin{vmatrix} \frac12-\lambda & \frac12 & \frac12 & \frac12 \\ \frac12 & \frac12-\lambda & -\frac12 & -\frac12 \\ \frac12 & -\frac12 & \frac12-\lambda & -\frac12 \\ \frac12 & -\frac12 & -\frac12 & \frac12-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & \frac12 & \frac12 & \frac12 \\ -\lambda & \frac12-\lambda & -\frac12 & -\frac12 \\ -\lambda & -\frac12 & \frac12-\lambda & -\frac12 \\ -\lambda & -\frac12 & -\frac12 & \frac12-\lambda \end{vmatrix} = \begin{vmatrix} 2-\lambda & 1 & \frac12 & \frac12 \\ \lambda & \lambda & \frac12 & \frac12 \\ 0 & -1+\lambda & 1-\lambda & 0 \\ 0 & 0 & 0 & 1-\lambda \end{vmatrix} = (1-\lambda)\begin{vmatrix} 2-\lambda & 1 & \frac12 \\ \lambda & \lambda & \frac12+\lambda \\ 0 & -1+\lambda & 0 \end{vmatrix} =$$

$$= (1-\lambda)(-\lambda+1)\begin{vmatrix} 2-\lambda & \frac32 \\ \lambda & \frac12+\lambda \end{vmatrix} = (1-\lambda)^3(1+\lambda)$$

$\lambda_1 = 1$ кр. 3

$\lambda_2 = -1$ кр. 1

$\lambda_1 = 1$:

$$\begin{pmatrix} -\frac12 & \frac12 & \frac12 & \frac12 \\ \frac12 & -\frac12 & -\frac12 & -\frac12 \\ \frac12 & -\frac12 & -\frac12 & -\frac12 \\ \frac12 & -\frac12 & -\frac12 & -\frac12 \end{pmatrix} \sim (1\ -1\ -1\ -1)$$

прог:

x_1	x_2	x_3	x_4
1	0	0	1
1	0	1	0
1	1	0	0

$\lambda_2 = -1$:

$$\begin{pmatrix} 3 & 1 & 1 & 1 \\ 1 & 3 & -1 & -1 \\ 1 & -1 & 3 & -1 \\ 1 & -1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & 3 \\ 0 & 0 & 4 & -4 \\ 0 & 4 & 0 & -4 \\ 0 & 4 & 4 & -8 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

прог:

x_1	x_2	x_3	x_4
-1	1	1	1

$$e_1 = \left(\frac{1}{\sqrt2}, 0, 0, \frac{1}{\sqrt2}\right)$$

$$e_2 = \left(\frac{1}{\sqrt2}, 0, \frac{1}{\sqrt2}, 0\right)$$

$$e_3 = \left(\frac{1}{\sqrt2}, \frac{1}{\sqrt2}, 0, 0\right)$$

$$e_4 = \left(-\frac12, \frac12, \frac12, \frac12\right)$$

$$Q = (e_1|e_2|e_3|e_4) = \begin{pmatrix} \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & \frac{1}{\sqrt2} & -\frac12 \\ 0 & 0 & \frac{1}{\sqrt2} & \frac12 \\ 0 & \frac{1}{\sqrt2} & 0 & \frac12 \\ \frac{1}{\sqrt2} & 0 & 0 & \frac12 \end{pmatrix}$$

$$B = Q^{-1}AQ = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$