1823. б) $\ln : V \to V'$ $\quad e^{f} : V' \to V$

1) $\forall f, g \in V : \ln(f \oplus g) = \ln(f \cdot g) = \ln(f) + \ln(g)$

2) $\forall f \in V, \forall \alpha \in \mathbb{R} : \ln(\alpha \odot f) = \ln(f^{\alpha}) = \alpha \cdot \ln(f)$

1) $\forall f, g \in V' : e^{f \oplus g} = e^{f \cdot g} =$

2) $\forall f \in V', \forall \alpha \in \mathbb{R} : e^{\alpha \odot f} = e^{f^{\alpha}} = e^{f}$

1823. в) $1, x, x^2, x^3, \dots, x^n, \ n \geq 1$

$\underline{n \geq 1}$ $\alpha \cdot 1 = 0 \Rightarrow \alpha = 0 \Rightarrow 1$ - лін. незал.

Припустимо $1, x, x^2, \dots, x^{n-1}$ - лін. незал.

Доведемо, що $1, x, x^2, \dots, x^{n-1}, x^n$ - лін. незал.

$\underline{n \geq 2}$ $\frac{d}{dx}$:

$\alpha_1 \cdot 0 + \alpha_2 \cdot 1 + 2\alpha_3 x + 3\alpha_4 x^2 + \dots + n\alpha_{n+1} x^{n-1}$

За припущенням

$\Rightarrow \alpha_1 = \alpha_2 = 2\alpha_3 = 3\alpha_4 = \dots = n\alpha_{n+1} = 0$

$\Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \dots = \alpha_{n+1} = 0$

$\Rightarrow 1, x, x^2, \dots, x^n$ - лін. незал.

$\Rightarrow V'$ - нескінченновимірний //

1828. а) $\sin x, \cos x$

$$\alpha \sin x + \beta \cos x = 0 \mid \cdot \beta$$

$$\frac{d}{dx}: \alpha \cos x - \beta \sin x = 0 \mid \cdot \alpha$$

$+ \Rightarrow$

$\Rightarrow \alpha^2 \cos x + \beta^2 \cos x = 0 \Rightarrow (\alpha^2+\beta^2)\cos x = 0 \quad x \in \mathbb{R},\ x = 0$

$\Rightarrow \alpha^2 + \beta^2 = 0 \Rightarrow \alpha = \beta = 0$ //

1828. б) $\sin x, \cos x, \sin^2 x, \cos^2 x, \sin^3 x, \cos^3 x, \underline{\sin^4 x}, \underline{\cos^4 x}, \dots, \sin^n x, \cos^n x,$

$n \geq 4$

$\cos^4 x - \sin^4 x - \cos^2 x + \sin^2 x = 0 \Rightarrow$

$\Rightarrow \sin^2 x, \cos^2 x, \sin^4 x, \cos^4 x$ - лін. залежна підсистема

$\Rightarrow$ вся система лін. залежна

1279. $e_1 = (1,2,-1,-2)$, $e_2 = (2,3,0,-1)$, $e_3 = (1,2,1,4)$,
$e_4 = (1,3,-1,0)$, $x = (7,14,-1,2)$

$x_1 e_1 + x_2 e_2 + x_3 e_3 + x_4 e_4 = x \quad (e_1 e_2 e_3 e_4 \mid x)$

$$\left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 2 & 3 & 2 & 3 & 14 \\ -1 & 0 & 1 & -1 & -1 \\ -2 & -1 & 4 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 2 & 2 & 0 & 6 \\ 0 & 3 & 6 & 2 & 16 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 6 & 5 & 16 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 7 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -1 & 2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|c} 1 & 2 & 1 & 0 & 5 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 2 & 0 & 0 & 4 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right)$$

$\Rightarrow \operatorname{rank}(e_1, e_2, e_3, e_4) = 4$

$\Rightarrow e_1, e_2, e_3, e_4$ – лін. нез.

$\Rightarrow e_1, e_2, e_3, e_4$ – базис

$x = (0,2,1,2)_e$

1281. $e_1 = (1,1,1,1)$, $e_2 = (1,2,1,1)$, $e_3 = (1,1,2,1)$, $e_4 = (1,3,2,3)$
$e_1' = (1,0,3,3)$, $e_2' = (-2,-3,-5,-4)$, $e_3' = (2,2,5,4)$, $e_4' = (-2,-3,-4,-4)$

$$(e_1', e_2', e_3', e_4') = \begin{pmatrix} 1 & -2 & 2 & -2 \\ 0 & -3 & 2 & -3 \\ 3 & -5 & 5 & -4 \\ 3 & -4 & 4 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 2 & -2 \\ 0 & -3 & 2 & -3 \\ 0 & 1 & -1 & 2 \\ 0 & 2 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 2 & -2 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \Rightarrow e_1', e_2', e_3', e_4' \text{ – базис}$$

$$\left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & -2 & 2 & -2 \\ 1 & 2 & 1 & 3 & 0 & -3 & 2 & -3 \\ 1 & 1 & 2 & 2 & 3 & -5 & 5 & -4 \\ 1 & 1 & 1 & 3 & 3 & -4 & 4 & -4 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & -2 & 2 & -2 \\ 0 & 1 & 0 & 2 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 & -3 & 3 & -2 \\ 0 & 0 & 0 & 2 & 2 & -2 & 2 & -2 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 1 & 1 & -2 & 2 & -2 \\ 0 & 1 & 0 & 2 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & 1 & 2 & -3 & 3 & -2 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 1 & 1 & 0 & 0 & -1 & 1 & -1 \\ 0 & 1 & 0 & 0 & -3 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & -2 & 2 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 1 & 0 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -3 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & -2 & 2 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right) \sim \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 2 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & -3 & 1 & -2 & 1 \\ 0 & 0 & 1 & 0 & 1 & -2 & 2 & -1 \\ 0 & 0 & 0 & 1 & 1 & -1 & 1 & -1 \end{array}\right) \Rightarrow e_1, e_2, e_3, e_4 \text{ – базис}$$

$$\parallel$$
$$T_{e \to e'}$$

$x = (x_1, x_2, x_3, x_4)_e = (x_1', x_2', x_3', x_4')_{e'}$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 2 & 0 & 1 & -1 \\ -3 & 1 & -2 & 1 \\ 1 & -2 & 2 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix} \begin{pmatrix} x_1' \\ x_2' \\ x_3' \\ x_4' \end{pmatrix}$$

$x_1 = 2x_1' + x_3' - x_4'$

$x_2 = -3x_1' + x_2' - 2x_3' + x_4'$

$x_3 = x_1' - 2x_2' + 2x_3' - x_4'$

$x_4 = x_1' - x_2' + x_3' - x_4'$