18.4 $\int\limits_{x_0}^{x} \frac{dt}{\sin^4 t\cdot\cos^2 t} = \left| \begin{matrix} y=tg\,t & dt=\frac{dy}{1+y^2} \\ \sin t=\frac{y}{\sqrt{1+y^2}} & \cos t=\frac{1}{\sqrt{1+y^2}} \end{matrix}\right| = \int\limits_{tg\,x_0}^{tg\,x} \frac{dy}{(1+y^2)\left(\frac{y}{\sqrt{1+y^2}}\right)^4\left(\frac{1}{\sqrt{1+y^2}}\right)^2} =$

$= \int\limits_{tg\,x_0}^{tg\,x} \frac{dy\,(1+y^2)^2}{y^4} = \int\limits_{tg\,x_0}^{tg\,x} \frac{1+2y^2+y^4}{y^4}\,dy = \int\limits_{tg\,x_0} \frac{1}{y^4}dy + 2\int\limits_{tg\,x_0} \frac{1}{y^2} + \int\limits_{tg\,x_0} 1\,dy =$

$= -\frac{1}{3}ctg^3 x - 2\,ctg\,x + tg\,x$ //

18.5 $\int\limits_{x_0}^{x} \frac{dt}{\sin 2t}$ $\quad R(-\sin t, \cos t) = -R(\sin t, \cos t)$

(непарна по синусу)

$\int\limits_{x_0}^{x} \frac{dt}{\sin 2t} = \int\limits_{x_0}^{x} \frac{\sin 2t\,dt}{1-\cos^2 2t} = \left| \begin{matrix} y=\cos 2t \\ dy=-2\sin 2t\,dt \end{matrix}\right| = \int\limits_{\cos x_0}^{\cos x} -\frac{1}{2}\cdot\frac{dy}{1-y^2} =$

$= \frac{1}{2}\int\limits_{\cos 2x_0}^{\cos 2x} \frac{dy}{y^2-1} = \frac{1}{4}\ln\left|\frac{\cos 2t-1}{\cos 2t+1}\right|\Bigg|_{x_0}^{x} = \frac{1}{4}\ln|tg^2 x| = \frac{1}{2}\ln|tg\,x|$ //

18.7 $\int\limits_{x_0}^{x} \frac{dt}{3-2\cos t} = \left| \begin{matrix} tg\frac{t}{2}=y \\ \cos t=\frac{1-y^2}{1+y^2} \\ dt=\frac{2dy}{1+y^2} \end{matrix}\right| = \int\limits_{tg\frac{x_0}{2}}^{tg\frac{x}{2}} \frac{2dy}{(1+y^2)\left(3-2\frac{1-y^2}{1+y^2}\right)} = \int\limits_{tg\frac{x_0}{2}}^{tg\frac{x}{2}} \frac{2dy}{\cancel{(1+y^2)}\frac{3+3y^2-2+2y^2}{\cancel{1+y^2}}} =$

$= \int\limits_{tg\frac{x_0}{2}}^{tg\frac{x}{2}} \frac{2dy}{1+5y^2} = 2\int\limits_{\sqrt{5}y_0}^{\sqrt{5}tg} \frac{\frac{1}{\sqrt{5}}dz}{1+z^2} = \frac{2}{\sqrt{5}}\,arctg\,\sqrt{5}y = \frac{2}{\sqrt{5}}\,arctg\,\sqrt{5}\,tg\frac{x}{2}$ //

18.11 $\int\limits_{x_0}^{x} \frac{8\cos^2 t\sin t}{\sin t+\cos t}dt = \left| \begin{matrix} y=tg\,t & dt=\frac{dy}{1+y^2} \\ \sin t=\frac{y}{\sqrt{1+y^2}} & \cos t=\frac{1}{\sqrt{1+y^2}} \end{matrix}\right| = 8\int\limits_{tg\,x_0}^{tg\,x} \frac{\frac{1}{(1+y^2)}\frac{y}{\sqrt{1+y^2}}}{\frac{y}{\sqrt{1+y^2}}+\frac{1}{\sqrt{1+y^2}}}\cdot\frac{dy}{1+y^2}$

$R(-\sin t, -\cos t) = R(\sin t, \cos t)$

$= 8\int\limits_{tg\,x_0}^{tg\,x} \frac{y\,dy}{(1+y^2)^2(1+y)} =$

За методом невизначених коефіцієнтів

$\frac{8y}{(1+y^2)^2(1+y)} = \frac{A}{1+y} + \frac{By+C}{1+y^2} + \frac{Dy+E}{(1+y^2)^2}$

$8y = A + 2Ay^2 + Ay^4 + By^4 + By^3 + By^2 + Cy^3 + Cy^2 + Cy + C + Dy^2 + Dy + Ey + E$

$\begin{array}{l|l} y^4 & A+B=0 \\ y^3 & B+C=0 \\ y^2 & 2A+B+C+D=0 \\ y^1 & B+C+D+E=8 \\ y^0 & A+C+E=0 \end{array} \qquad \begin{array}{l} A=-2 \\ B=2 \\ C=-2 \\ D=4 \\ E=4 \end{array}$

$$= -\int_{tg x_0}^{tg x} \frac{2}{1+y}dy + \int_{tg x_0}^{tg x} \frac{2y-2}{(1+y^2)}dy + \int_{tg x_0}^{tg x} \frac{4y+4}{(1+y^2)^2}dy =$$

$$= -2\ln|tg x+1| + \int_{tg x_0}^{tg x} \frac{dy^2}{1+y^2} - \int_{tg x_0}^{tg x} \frac{2}{1+y^2}dy + 4\int_{tg x_0}^{tg x} \frac{1+y}{(1+y^2)^2}dy =$$

$$4\int_{tg x_0}^{tg x} \frac{1+y}{(1+y^2)^2}dy = \left| \begin{matrix} y = tg z \\ dy = \frac{dz}{\cos^2 z} \end{matrix} \right| = 4\int \frac{(tg z+1)dz}{(1+tg^2 z)^2\cos^2 z} = 4\int \cos^2 z(tg z+1) =$$

$$= 4\int (1-\sin^2 z)(1+tg z)dz = 4\int(-\sin^2 z + tg z + \sin^2 z)(1+tg z)dz =$$

$$= -4\int \sin^2 z\, tg z\, dz + 4\int tg z\, dz - 4\sin^2 dz + 4\int 1dz =$$

$$= 4\int \sin z\cos z\, dz - 2\int 1dz + 2\int \cos 2z\, dz + 4\int 1dz = 2\sin^2 z + 2z + \sin 2z$$

$$= -2\ln|tg x+1| + \ln|\cos^2 x| - \cancel{2arctg(tg x)} + 2\sin^2(arctg(tg x)) +$$

$$+ \cancel{2x} + \sin 2x = -2\ln|tg x+1| + \ln|\cos^2 x| + 2\sin^2 x + \sin 2x$$

18.13 $I_n = \int_{x_0}^{x} \cos^n t\, dt$

$n=1 \Rightarrow I_1 = \int_{x_0}^{x} \cos t\, dt = \sin x$

$n=2 \Rightarrow I_2 = \int_{x_0}^{x} \cos^2 t\, dt = \int_{x_0}^{x} \frac{1+\cos 2t}{2}dt = \frac{1}{2}x + \frac{1}{4}\sin 2x$

$n\geq 3 \Rightarrow I_n = \int_{x_0}^{x} \cos^n t\, dt = \left| \begin{matrix} u = \cos^{n-1} t & du = -(n-1)\cos^{n-2} t\sin t\, dt \\ dv = \cos t\, dt & v = \sin t \end{matrix} \right| =$

$= \cos^{n-1} t\sin t + (n-1)\int \sin t \cdot \cos^{n-2} t\sin t\, dt = \cos^{n-1} t\sin t +$

$+ (n-1)\int(1-\cos^2 t)\cos^{n-2} t\, dt = \cos^{n-1} t\sin t + (n-1)\int \cos^{n-2} t\, dt -$

$-(n-1)\int \cos^n t\, dt$

$I_n = \cos^{n-1} t\sin t + (n-1)I_{n-2} - (n-1)I_n \Rightarrow I_n = \frac{1}{n}\cos^{n-1} t\sin t + \frac{n-1}{n}I_{n-2}$

18.14 $I_n = \int_{x_0}^{x} \frac{dt}{\sin^n t}$

$n=1 \Rightarrow I_1 = \int_{x_0}^{x} \frac{dt}{\sin t} = \left| \begin{matrix} tg\frac{t}{2} = y & dt = \frac{2dy}{1+y^2} \\ \sin t = \frac{2y}{1+y^2} & \end{matrix} \right| = \int_{tg\frac{x_0}{2}}^{tg\frac{x}{2}} \frac{2dy}{2y} = \ln|tg\frac{x}{2}|$

$n=2 \Rightarrow I_2 = \int_{x_0}^{x} \frac{1}{\sin^2 t}dt = -ctg\, x$

$$n\geq 3 \Rightarrow J_n = \int_{x_0}^{x} \frac{dt}{\sin^n t} = \left| \begin{matrix} v=\cos t & dv=-\sin t\,dt \\ u=\frac{1}{\sin^n t} & du=-\frac{(n-1)\cos t\,dt}{\sin^n t} \end{matrix} \right| =$$

$$= \frac{\cos t}{\sin^{n-1} t} + \int \frac{(n-1)\cos^2 t\,dt}{\sin^n t} = \frac{\cos t}{\sin^{n-1} t} + (n-1)\int \frac{dt}{\sin^n t} - (n-1)\int \frac{dt}{\sin^{n-2} t}$$

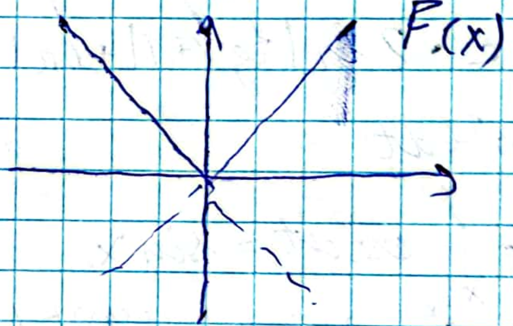
$$J_n = \frac{\cos t}{\sin^{n-1} t} + (n-1) J_n - (n-1) J_{n-2}$$

$$(n-2) J_n = -\frac{\cos t}{(n-2)\sin^{n-1} t} + \frac{n-1}{n-2} J_{n-2} \quad //$$

18.15 $f(x) = \operatorname{sgn} x$

$$f(x) = \begin{cases} 1, & x>0 \\ 0, & x=0 \\ -1, & x<0 \end{cases} \qquad F(x) = \begin{cases} x+C_1, & x>0 \\ 0+C_2, & x=0 \\ -x+C_3, & x<0 \end{cases}$$

$F(0+1) = F(0-1)$.



$x + C_1 = -x + C_3$

$0 + C_1 = 0 + C_3$

$C_1 = C_3 = C_2 \Rightarrow$

$$\Rightarrow F(x) = \begin{cases} x+C, & x>0 \\ C, & x=0 \\ -x+C, & x<0 \end{cases} \qquad F(x) = |x| + C \quad //$$

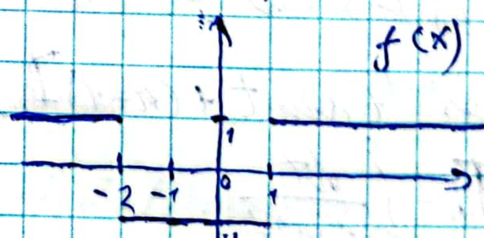
18.16 $f(x) = \operatorname{sgn}(x^2+x-2) = \frac{|x^2+x-2|}{x^2+x-2}$

$$f(x) = \begin{cases} \frac{x^2+x-2}{x^2+x-2} = 1, & x^2+x-2 \geq 0 \\ & x\in(-\infty;-2]\cup[1;+\infty) \\ \frac{-(x^2+x-2)}{x^2+x-2} = -1, & x^2+x-2<0 \\ & x\in(-2;1) \end{cases}$$

$x^2+x-2>0$

$(x+2)(x-1)>0$

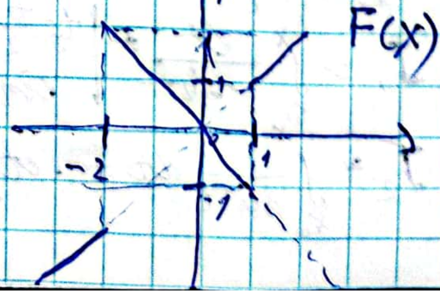
$$F(x) = \begin{cases} x+C_1, & x\in \mathbb{R}\setminus(-2;1) \\ -x+C_2, & x\in(-2;1) \end{cases}$$



$F(-2-0) = F(-2+0)$ $\quad F(1-0) = F(1+0)$

$-2 + C_1 = 2 + C_2$ $\quad -1 + C = 1 + C_1$

$C_1 = 4 + C_2$ $\quad C_1 = C_2 - 2$



$x < -2 : F(x) = x + 4 + C$

$x \in [-2, 1] : F(x) = -x + C$

$x > 1 : F(x) = x - 2 + C \quad //$