7.50 $\lim\limits_{x\to 0} \dfrac{3x^4 - 5x^3 + o(x^4)}{x^3 + 4x^4 + o(x^3)} = \dfrac{-5x^3 + o(x^3)}{x^3 + o(x^3)} = \dfrac{-5 + o(1)}{1 + o(1)} = -5$ //

8.6 $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{7 + x^3} - \sqrt{3 + x^2}}{1 - x} = \dfrac{\sqrt[3]{8} - \sqrt{4}}{1 - 1} = \dfrac{0}{0}$

$= \left| \begin{matrix} x - 1 = t \\ x = t + 1 \\ t \to 0 \end{matrix} \right| = \lim\limits_{t\to 0} \dfrac{\sqrt[3]{7 + (t+1)^3} - \sqrt{3 + (t+1)^2}}{t} =$

$= \dfrac{\sqrt[3]{8 + t^3 + 3t^2 + 3t} - \sqrt{t^2 + 2t + 4}}{t} = \dfrac{2\left(1 + \frac{3}{8}t + o(t)\right)^{\frac{1}{3}} - 2\left(1 + \frac{t}{2} + o(t)\right)^{\frac{1}{2}}}{t} =$

$= \dfrac{2\left(1 + \frac{1}{8}t + o(t)\right) - 2\left(1 + \frac{1}{4}t + o\,t\right)}{t} =$

$= \dfrac{\frac{1}{4}t - \frac{1}{2}t + o(t)}{t} = -\dfrac{1}{4}$ //

8.12 $\lim\limits_{x\to \pi} \left(\dfrac{1 + \operatorname{tg} x}{1 + \sin x}\right)^{\frac{1}{\sin^3 x}} = \left(\dfrac{1 + 0}{1 + 0}\right)^{\frac{1}{0}} = [1]^{\infty}$

$= e^{\lim\limits_{x\to\pi} \ln \left(\frac{1 + \operatorname{tg} x}{1 + \sin x}\right)^{\frac{1}{\sin^3 x}}}$

$\lim\limits_{x\to \pi} \dfrac{1}{\sin^3 x} \cdot \ln\left(\dfrac{1 + \operatorname{tg} x}{1 + \sin x}\right) = \left| \begin{matrix} t = -\pi + x \\ x = t + \pi \end{matrix} \right| = \lim\limits_{t\to 0} \dfrac{1}{-\sin^3 t} \cdot \ln\left(\dfrac{1 + \operatorname{tg} t}{1 - \sin t}\right) =$

$= \lim\limits_{t\to 0} \left(\dfrac{1}{-\sin^3 t} \cdot \left(\dfrac{1 + \operatorname{tg} t}{1 - \sin t} - 1\right)\right) =$

$= \lim\limits_{t\to 0} \left(\dfrac{1}{-\sin^3 x} \cdot \left(\dfrac{\operatorname{tg} t + \sin t}{1 - \sin t}\right)\right) = \lim\limits_{t\to 0} \dfrac{2t + o(t)}{t^2 t^3 - o(t^3)} = \lim\limits_{t\to 0} \dfrac{2 + o(1)}{-t^2 + o(t^2)} = -\infty$

$\lim\limits_{n\to -\infty} e^{n} = 0$ //

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8,26 $\lim\limits_{x \to 0} \dfrac{\operatorname{th} x}{x} = \lim\limits_{x \to 0} \dfrac{1}{\operatorname{ch} x} \cdot \lim\limits_{x \to 0} \dfrac{\operatorname{sh} x}{x} = \lim\limits_{x \to 0} \dfrac{e^x - \frac{1}{e^x}}{2x} =$

$= \lim\limits_{x \to 0} \dfrac{1 + x + o(x) - \frac{1}{1 + x + o(x)}}{2x} = \dfrac{2x + o(x)}{2x + o(x)} = \dfrac{2 + o(1)}{2 + o(1)} = 1$ //

8.3 $\lim\limits_{x \to 0} \dfrac{\sqrt[3]{27+x} - \sqrt[4]{16-x} - e^x}{\ln(1+e^x - \cos x)} = \dfrac{3\left(1+\frac{1}{27}x\right)^{\frac{1}{3}} - 2\left(1-\frac{1}{16}x\right)^{\frac{1}{4}} - 1 - x + o(x)}{\ln\left(1+1+x-1+\frac{x^2}{2}+o(x)\right)}$

$= \dfrac{3\left(\cancel{1}+\frac{1}{81}x+o(x)\right) - 2\left(\cancel{1}-\frac{1}{64}x+o(x)\right) - \cancel{1} - x + o(x)}{\ln(1+x+o(x))} =$

$= \dfrac{\frac{1}{81}x + \frac{1}{32}x - x + o(x)}{x + o(x)} = \dfrac{\frac{1}{81} + \frac{1}{32} - 1 + o(1)}{1 + o(1)} = -\dfrac{805}{864}$ //

8.48 $f(x) = \cos x - 1$, $g(x) = 1 - \operatorname{ch} x$, $x \to 0$

$\lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} \cos x - 1 = 1 - 1 = 0 \Rightarrow f = o(1)$

$\lim\limits_{x \to 0} g(x) = \lim\limits_{x \to 0} 1 - \operatorname{ch} x = 1 - 1 = 0 \Rightarrow g = o(1)$

$$\lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{\cos x - 1}{1 - \operatorname{ch} x} = \frac{1 - \frac{x^2}{2} + o(x^2) - 1}{1 - \left(1 + \frac{x^2}{2} + o(x^2)\right)} = \frac{-\frac{x^2}{2} + o(x^2)}{-\frac{x^2}{2} + o(x^2)} = 1$$

$\Longrightarrow f(x) \sim g(x)$ //

8.51 $f(x) = x^x$; $g(x) = 1$, $x \to 0$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x^x = \lim_{x\to 0} e^{x \cdot \ln x} = e^{\lim\limits_{x\to 0} x \cdot \ln x} =$$

$$= e^{\lim\limits_{x\to 0} x \cdot \ln(x-1+1)} = e^{\lim\limits_{x\to 0} x \cdot (x-1+o(x-1))} = e^{\lim\limits_{x\to 0} -x+o(x-1)} =$$

$$= e^0 = 1 \Rightarrow$$

$$\Rightarrow \lim_{x\to 0} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to 0} f(x)}{\lim\limits_{x\to 0} g(x)} = 1 \Rightarrow f(x) \sim g(x)$$

8.61 $f(x) = \dfrac{\ln(1+x^{\alpha})}{x^{\beta}}$, $x_0 = 0$, $x \to x_0$

$\lim\limits_{x\to 0} f(x) = \dfrac{x^{\alpha} + o(x)}{x^{\beta}} = o(\cancel{1}) \Rightarrow x^{\alpha} = o(x^{\beta}) \Rightarrow$

$\Rightarrow \alpha > \beta, \begin{cases} \alpha > 0, \beta < \alpha \\ \alpha \leq 0, \beta < 0 \end{cases}$

9.25 $f(x) = \begin{cases} \dfrac{x^2-4}{x^3-8} + \dfrac{\sin x}{2x}, & x \notin \{0, 2\}, \\ \alpha, & x = 0, \\ \beta, & x = 2; \end{cases}$

$\lim\limits_{x\to 0}\left(\dfrac{x^2-4}{x^3-8} + \dfrac{\sin x}{2x}\right) = \lim\limits_{x\to 0}\left(\dfrac{1}{2} + \dfrac{x+o(x)}{2x}\right) = \dfrac{1}{2} + \dfrac{1}{2} = 1 \Rightarrow$

$\Rightarrow \alpha = 1$

$\lim\limits_{x\to 2}\left(\dfrac{x^2-4}{x^3-8} + \dfrac{\sin x}{2x}\right) = \dfrac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x^2+2x+4)} + \dfrac{\sin 2}{4} =$

$= \dfrac{2+2}{2^2+2\cdot 2+4} + \dfrac{\sin 2}{4} = \dfrac{1}{3} + \dfrac{\sin 2}{4} \Rightarrow \beta = \dfrac{1}{3} + \dfrac{\sin 2}{4}$

9.27 $f(x) = [x^2]$

$n = [x^2]$, $n \in N$

$\sqrt{n} - 0 \longrightarrow n-1$ $\qquad$ $-\sqrt{n} - 0 \longrightarrow n$

$\sqrt{n} + 0 \longrightarrow n$ $\qquad$ $-\sqrt{n} + 0 \longrightarrow n-1$

неперервна зліва на $R \setminus \{\sqrt{n} \mid n \in N\}$ та

неперервна справа на $R \setminus \{-\sqrt{n} \mid n \in N\}$