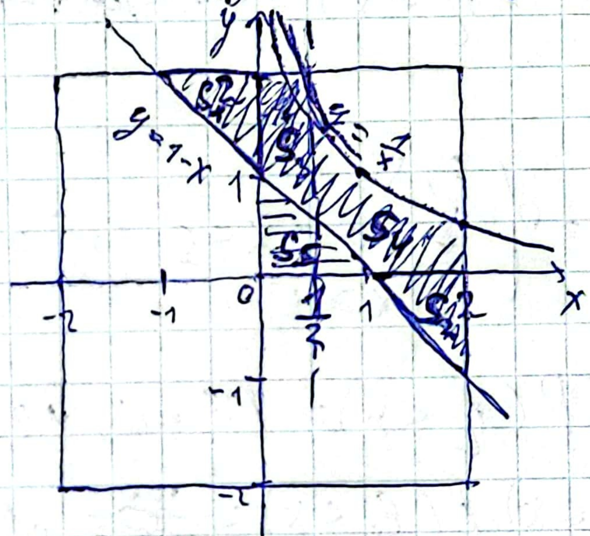
Варіант №9

1. $\Omega = \{(x,y): x,y \in [-2;2]\}$

$A = \{(x,y): x,y \in [-2;2], (x+y>1) \wedge (xy<1)\}$



$P(A) = \frac{S_A}{S}$

$S = 4 \cdot 4 = 16$

$S_A = S_1 + S_2 + S_3 + S_4 - S_5$

1) $S_1 = S_2 = S_5 = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$

2) $S_3 = 2 \cdot \frac{1}{2} = 1$

3) $S_4 = \int_{\frac{1}{2}}^{2} \frac{1}{x}\,dx = \ln x \Big|_{\frac{1}{2}}^{2} = \ln 2 - \ln\frac{1}{2} = \ln 4$

$S_A = \frac{1}{2} + \cancel{\frac{1}{2}} + 1 + \ln 4 - \cancel{\frac{1}{2}} = 1{,}5 + \ln 4$

$P(A) = \frac{S_A}{S} = \frac{1{,}5 + \ln 4}{16}$

2. $F_{\xi}(x) = \begin{cases} 0, & x < 0 \\ \frac{x}{5}, & 0 \le x < 5 \\ 1, & x \ge 5 \end{cases}, \quad i = \overline{1,n}$

$F_{\eta}(x) = P(\eta \le x) = P(\min\{2, \xi\} \le x) =$

$= P(2 \le x)\,P(\xi \le x) = \begin{cases} 0, & x < 0 \\ \frac{3x}{25}, & 0 \le x < 5 \\ 1, & x \ge 5 \end{cases}$

$f_{\eta}(x) = \frac{dF_{\eta}}{dx} = \begin{cases} \frac{3}{25}, & x \in [0;5] \\ 0, & x \notin [0;5] \end{cases}$

$M\eta = \int_{-\infty}^{+\infty} x \cdot f_{\eta}(x)\,dx = \int_{0}^{5} \frac{3x}{25}\,dx = 1{,}5$

$D\eta = \int_{-\infty}^{+\infty} x^2 \cdot f_{\eta}(x)\,dx = \int_{0}^{5} \frac{3x^2}{25}\,dx = 5$

3. $\xi = \begin{cases} -1, & P(\xi = -1) = p = 0{,}3 \\ 1, & P(\xi = 1) = q = 0{,}7 \end{cases}$

$$\eta = \begin{cases} -1, & P(\eta = -1) = \sum_{i=0}^{[\frac{n}{2}]} \left(C_n^{2i+1} (0{,}7)^{2i+1} (0{,}3)^{n-2i-1} \right) \\ 1, & P(\eta = 1) = \sum_{i=1}^{[\frac{n}{2}]} \left(C_n^{2i} (0{,}7)^{2i} (0{,}3)^{n-2i} \right) \end{cases}$$

$$M(\eta) = n \sum_{i=0}^{[\frac{n}{2}]} C_n^{2i+1} 0{,}7^{2i+1} 0{,}3^{n-2i-1}$$

$$D(\eta) = n \sum_{i=0}^{[\frac{n}{2}]} C_n^{2i+1} 0{,}7^{2i+1} 0{,}3^{n-2i-1} \sum_{i=1}^{[\frac{n}{2}]} C_n^{2i} 0{,}7^{2i} 0{,}3^{n-2i}$$