

Given $\forall x ((P(x) \wedge \neg Q(x)) \rightarrow \exists y (R(x,y) \wedge G(y)))$
 Deduce $P(x) \wedge \neg Q(x) \vdash \exists y (R(x,y) \wedge G(y))$
 k-29

$$\begin{aligned}
 & \forall x ((\overline{P(x)} \vee Q(x)) \vee \exists y (R(x,y) \wedge G(y))) \\
 & \forall x \exists y ((\overline{P(x)} \vee Q(x)) \vee (R(x,y) \wedge G(y))) \\
 & \forall x \exists y ((\overline{P(x)} \vee Q(x)) \vee R(x,y)) \wedge (\overline{P(x)} \vee Q(x) \vee G(y)) \quad (1)
 \end{aligned}$$

$$\begin{aligned}
 & 2. \quad \forall x (S(x) \rightarrow \neg Q(x)) \\
 & \quad \forall x (S(x) \vee Q(x))
 \end{aligned}$$

$$\begin{aligned}
 & 3. \quad \exists z (S(z) \wedge (P(z) \wedge \forall x (R(x,z) \rightarrow S(x)))) \\
 & \exists z (S(z) \wedge P(z) \wedge \forall x (\neg R(x,z) \vee S(x))) \\
 & \exists z (S(z) \wedge P(z) \wedge (\neg R(z,c) \vee S(c)))
 \end{aligned}$$

$$\begin{aligned}
 & \exists u \forall z (S(u) \wedge P(u) \wedge (\neg R(z,c) \vee S(c))) \\
 & 4. \quad \exists y \exists x \exists u \forall z ((1) \wedge (2) \wedge (3))
 \end{aligned}$$

$$\begin{aligned}
 & y = f(x), \quad u = g(x, k) \\
 & S = \{ P(x) \vee Q(x) \vee R(x, f(x)), \overline{P(x)} \vee Q(x) \vee G(f(x)), S(k) \vee Q(k) \vee Q(x) \vee G(f(x)), S(k) \vee Q(k) \vee S(g(x, k)), P(g(x, k)), R(z, c) \vee S(c) \}
 \end{aligned}$$

$E = \{c, f(x), g(x, k)\}$
 (2) $\neg P, \neg Q \vdash \neg R$
 1. $\neg P, \neg Q$

$E \rightarrow \{c, f(c), f(f(c)), \dots, f^3(c, d)\}$

(2) $\neg P, \neg Q \vdash P \rightarrow Q$

1. $\neg P, \neg Q \vdash P \rightarrow Q$ za *assumptions*