842. $|\bar{a}| = 3, \; |\bar{b}| = 4$

1) $|[(\bar{a}+\bar{b})(\bar{a}-\bar{b})]| = |[\bar{a},\bar{a}] - 2[\bar{a},\bar{b}] - [\bar{b},\bar{b}]| =$

$= |-2[\bar{a},\bar{b}]| = 2 \cdot 3 \cdot 4 = 24;$

2) $|[(3\bar{a}-\bar{b})(\bar{a}-2\bar{b})]| = |3[\bar{a},\bar{a}] - 6[\bar{a},\bar{b}] - [\bar{b},\bar{a}] + 2[\bar{b},\bar{b}]| =$

$= |-6[\bar{a},\bar{b}] + [\bar{a},\bar{b}]| = |-5[\bar{a},\bar{b}]| = -5 \cdot 3 \cdot 4 = 60.$

846. $[\bar{a},\bar{b}]^2 \leq \bar{a}^2\bar{b}^2$

$|\bar{a}|^2|\bar{b}|^2 \sin^2\varphi \leq |\bar{a}|^2|\bar{b}|^2$

$\sin^2\varphi \leq 1 \Rightarrow [\bar{a},\bar{b}]^2 \leq \bar{a}^2\bar{b}^2$

$[\bar{a},\bar{b}]^2 = \bar{a}^2\bar{b}^2$ при $\varphi = \frac{\pi}{2}$

851. $A(2;-1;2), \; B(1;2;-1), \; C(3;2;1)$

$\overline{AB} = \{-1;3;-3\} \quad \overline{BC} = \{2;0;2\}$

$\overline{CB} = \{-2;0;-2\} \quad \overline{AC} = \{1;3;-1\}$

1) $[\overline{AB}, \overline{BC}] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ -1 & 3 & -3 \\ 2 & 0 & 2 \end{vmatrix} = \{6; -4; -6\};$

2) $[(\overline{BC} - 2\overline{CA}), \overline{CB}] = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 0 & -6 & 4 \\ -2 & 0 & -2 \end{vmatrix} = \{12; -8; -12\}.$

861. $\bar{m} \perp Oz$, $\bar{m} \perp \bar{a} = \{8; -15; 3\}$, $(\bar{m}, \hat{Ox}) < \frac{\pi}{2}$, $|\bar{m}| = 51$

$\bar{m} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 8 & -15 & 3 \\ 0 & 0 & 1 \end{vmatrix} = -15\bar{i} - 8\bar{j}$

$k\sqrt{15^2+8^2} = k\sqrt{289} = 17k = 51 \Rightarrow k = 3$

$(\bar{m}, \hat{Ox}) < \frac{\pi}{2} \Rightarrow$ змінюємо знак

$\bar{m} = \{45; 24; 0\}$ //

877. $A(2;3;1)$, $B(4;1;-2)$, $C(6;3;7)$, $D(-5;-4;8)$

$\overline{AB} = \{2; -2; -3\}$, $\overline{AC} = \{4; 0; 6\}$, $\overline{AD} = \{-7; -7; 7\}$

$V = \frac{1}{6} \overline{AB} \cdot \overline{AC} \cdot \overline{AD} = \frac{1}{6} \begin{vmatrix} 2 & -2 & -3 \\ 4 & 0 & 6 \\ -7 & -7 & 7 \end{vmatrix} = \frac{1}{6}(84 + 84 + 84 + 56) =$

$= \frac{1}{6} \cdot 308 = \frac{154}{3};$

$\overline{AB} \cdot \overline{AC} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -2 & -3 \\ 4 & 0 & 6 \end{vmatrix} = -12\bar{i} - 12\bar{j} + 8\bar{k} - 12\bar{j} = -12\bar{i} - 24\bar{j} + 8\bar{k}$

$S = \frac{1}{2}\sqrt{12^2 + 24^2 + 8^2} = \frac{1}{2}\sqrt{784} = \frac{1}{2} \cdot 28 = 14$

$V = \frac{1}{3} S \cdot H \Rightarrow H = \frac{3V}{S} = \frac{154}{14} = 11$

Від. 11. //

919. $M_1(2;-1;3)$, $M_2(3;1;2)$, $\Pi \parallel \bar{a}=\{3;-1;4\}$, $M_1, M_2 \in \Pi$

$\overline{M_1M_2} = \{1;2;-1\}$

$$\bar{a}\cdot\overline{M_1M_2} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -1 & 4 \\ 1 & 2 & -1 \end{vmatrix} = \bar{i}+4j+6\bar{k}+k-8\bar{i}+3\bar{j} =$$
$$= -7i+7j+7k$$

$\bar{n} = \{-7;7;7\}$

$-\cancel{7}(x-2)+\cancel{7}(y+1)+\cancel{7}(z-3)=0$

$-x+\cancel{2}+y\cancel{+1}+z\cancel{-3}=0$

$\boxed{x-y-z=0.}$

926. 1) $2x+ly+3z-5=0$, $mx-6y-6z+2=0$; $l=3$, $m=4$;

2) $3x-y+lz-9=0$, $2x+my+2z-3=0$; $l=3$; $m=-\frac{1}{3}$;

3) $mx+3y-2z-1=0$, $2x-5y-lz=0$; $m=-\frac{6}{5}$; $l=-\frac{10}{3}$.

935. $M_1(x_1;y_1;z_1)$, $M_2(x_2;y_2;z_2)$, $\Pi \perp Ax+By+Cz+D=0$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ A & B & C \end{vmatrix} = 0$$

$\overline{M_1M_2} = \{x_2-x_1; y_2-y_1; z_2-z_1\}$, $M\in\Pi$, $M=\{x;y;z\}$

~~$\bar{n} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \end{vmatrix}$~~ $\overline{M_1M} = \{x-x_1; y-y_1; z-z_1\}$

$\bar{n} = \{A;B;C\}$

$\bar{n} = [\overline{M_1M}, \overline{M_1M_2}] \Rightarrow$

$\Rightarrow [[\overline{M_1M}, \overline{M_1M_2}], \bar{n}] = 0$

$$\begin{vmatrix} x-x_1 & y-y_1 & z-z_1 \\ x_2-x_1 & y_2-y_1 & z_2-z_1 \\ A & B & C \end{vmatrix} = 0$$

977. $M(3;2;-1)$, $5x-y+z+3=0$, $4x-3y+2z+5=0$

$\overline{m_1} = \{5;-1;1\} \uparrow\uparrow \overline{n_1}$

$\overline{m_2} = \{4;-3;2\} \uparrow\uparrow \overline{n_2}$

$(\overline{m_1}, \overline{m_2}) = 20+3+2 > 0$

$\Rightarrow$ т. O лежить в тупому куті

т. O:

$3 > 0$

$5 > 0$

т. M:

$15-2-1+3 > 0$ – по одну сторону

$12-6+2+5>0$ – по одну сторону

Отже, т. M лежить в тупому куті.

979. $2x-y+2z-3=0$, $3x+2y-6z-1=0$, $M(1;2;-3)$

$\overline{m_1} = \{2;-1;2\} \uparrow\uparrow \overline{n_1}$

$\overline{n_2} = \{3;2;-6\} \uparrow\uparrow \overline{m_2}$

$(\overline{m_1}, \overline{m_2}) = 6-2-12 < 0$

$\Rightarrow$ т. O лежить в гострому куті

т. O:

$-3 < 0$

$-1 < 0$

т. M:

$2-2-6-3 < 0$ – по один бік

$3+4+18-1>0$ – по різні боки

$\Rightarrow$ т. M лежить в тупому куті

р-ння бісектриси

$$\frac{2x-y+2z-3}{3} = \frac{3x+2y-6z-1}{7}$$

$$14x-7y+14z-21 = 9x+6y-18z-3$$

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$+$ $\qquad\qquad\qquad\qquad$ $-$

$23x - y - 4z - 24 = 0$ $\qquad\qquad$ $5x - 13y + 32z - 18 = 0$

$$\left[\begin{array}{l} 23x - y - 4z - 24 = 0 \\ 5x - 13y + 32z - 18 = 0 \end{array}\right.$$

$P(0; -3; 0) \in \Pi$

$$\frac{|3 - 24|}{\sqrt{546}} = \frac{21}{\sqrt{546}} \Rightarrow \text{р-ння тупого кута}$$

$$\frac{|39 - 18|}{\sqrt{1218}} = \frac{21}{\sqrt{1218}}$$

Відп. $23x - y - 4z - 24 = 0$. //