1358. $a_1 = (1, 1, 1, -2)$

$a_2 = (1, 2, 3, 3)$

$(a_1, a_2) = 1 + 2 + 3 - 6 = 0 \Rightarrow a_1 \perp a_2$

$\begin{cases} (x, a_1) = 0 \\ (x, a_2) = 0 \end{cases}$ $x = (x_1, x_2, x_3, x_4)$ $\begin{cases} x_1 + x_2 + x_3 - 2x_4 = 0 \\ x_1 + 2x_2 + 3x_3 + 3x_4 = 0 \end{cases}$

$\begin{pmatrix} 1 & 1 & 1 & -2 \\ 1 & 2 & 3 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & -2 \\ 0 & 1 & 2 & 5 \end{pmatrix}$ ФСР

x_1	x_2	x_3	x_4	
1	-2	1	0	$= b_1$
7	-5	0	1	$= b_2$

$(b_1, b_2) \neq 0$

Процес ортогоналізації

$c_1 = b_1 = (1, -2, 1, 0)$

$c_2 = b_2 - \alpha c_1$ $\quad (c_2, c_1) = 0, \ (b_2, c_1) - \alpha (c_1, c_1) = 0$

$\alpha = \frac{(b_2, c_1)}{(c_1, c_1)} = \frac{17}{6}$

$c_2 = b_2 - \frac{17}{6} c_1 = (7, -5, 0, 1) + \left(-\frac{17}{6}, \frac{34}{6}, -\frac{17}{6}, 0\right) =$

$= (42, -30, 0, 6) + (-17, 34, -17, 0) = (25, 4, -17, 6)$

$\boxed{\begin{aligned} a_3 &= c_1 = (1, -2, 1, 0) \\ a_4 &= c_2 = (25, 4, -17, 6) \end{aligned}}$

1360. $a_1 = \left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ $\quad (a_1, a_2) = \frac{1}{4} + \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = 0$

$a_2 = \left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}\right)$ $\quad (a_1, a_1) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1 \Rightarrow a_1, a_2$ о.н.

$(a_2, a_2) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = 1$

$\begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

ФСР:

x_1	x_2	x_3	x_4	
1	-1	1	-1	$= b_1$
1	-1	-1	1	$= b_2$

$a_3 = c_1 = \frac{b_1}{|b_1|} = \frac{(1, -1, 1, -1)}{\sqrt{1+1+1+1}} = \boxed{\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}\right)}$

$a_4 = c_2 = \frac{b_2}{|b_2|} = \frac{(1, -1, -1, 1)}{\sqrt{1+1+1+1}} = \boxed{\left(\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)}$

1362. $a_1=(1,1,-1,-2)$ $\quad L=\langle a_1,a_2,a_3\rangle$

$a_2=(5,8,-2,-3)$

$a_3=(3,9,3,8)$

$b_1=a_1=(1,1,-1,-2)$

$b_2=a_2-\alpha_{21}b_1$ $\qquad \alpha_{21}=\dfrac{(a_2,b_1)}{(b_1,b_1)}=\dfrac{5+8+2+6}{1+1+1+4}=\dfrac{21}{7}=3$

$\underline{b_2}=a_2-3b_1=\underline{(2,5,1,3)}$

$b_3=a_3-\alpha_{31}b_1-\alpha_{32}b_2$ $\qquad \alpha_{31}=\dfrac{(a_3,b_1)}{(b_1,b_1)}=\dfrac{3+9-3-16}{7}=\dfrac{-7}{7}=-1$

$\alpha_{32}=\dfrac{(a_3,b_2)}{(b_2,b_2)}=\dfrac{6+45+3+24}{4+25+1+9}=\dfrac{78}{39}=2$

$\underline{b_3}=(3,9,3,8)+(1,1,-1,-2)+(-4,-10,-2,-6)=\underline{(0,0,0,0)}$

B: $b_1=(1,1,-1,-2)$, $b_2=(2,5,1,3)$ — о.б. L //

1365. $a_1=(2,1,3,-1)$ $\quad L=\langle a_1,a_2,a_3,a_4\rangle$

$a_2=(7,4,3,-3)$

$a_3=(1,1,-6,0)$

$a_4=(5,7,7,8)$

$b_1=a_1=(2,1,3,-1)$

$b_2=a_2-\alpha_{21}b_1$ $\qquad \alpha_{21}=\dfrac{(a_2,b_1)}{(b_1,b_1)}=\dfrac{14+4+9+3}{4+1+9+1}=\dfrac{30}{15}=2$

$\underline{b_2}=a_2-2b_1=\underline{(3,2,-3,-1)}$

$b_3=a_3-\alpha_{31}b_1-\alpha_{32}b_2$ $\qquad \alpha_{31}=\dfrac{(a_3,b_1)}{(b_1,b_1)}=\dfrac{2+1-18+0}{15}=\dfrac{-15}{15}=-1$

$\alpha_{32}=\dfrac{(a_3,b_2)}{(b_2,b_2)}=\dfrac{3+2+18+0}{9+4+9+1}=\dfrac{23}{23}=1$

$\underline{b_3}=a_3+b_1-b_2=\underline{(0,0,0,0)}$

$b_4=a_4-\alpha_{41}b_1-\alpha_{42}b_2$ $\qquad \alpha_{31}=\dfrac{(a_4,b_1)}{(b_1,b_1)}=\dfrac{10+7+21-8}{15}=\dfrac{30}{15}=2$

$\underline{b_4}=a_4-2b_1=\underline{(1,5,1,10)}$ $\qquad \alpha_{32}=\dfrac{(a_4,b_2)}{(b_2,b_2)}=\dfrac{15+14-21-8}{23}=0$

B: $b_1=(2,1,3,-1)$, $b_2=(3,2,-3,-1)$, $b_3=(1,5,1,10)$ — о.б. L //

1371. $x=(5,2,-2,2)$, $L=\langle a_1,a_2,a_3\rangle$

$a_1=(2,1,1,-1)$, $a_2=(1,1,3,0)$, $a_3=(1,2,8,1)$

$x=y+z$, $y\in L$, $z\in L^{\perp}$

$$(a_1|a_2|a_3)=\begin{pmatrix}2&1&1\\1&1&2\\1&3&8\\-1&0&1\end{pmatrix}\sim\begin{pmatrix}1&0&1\\0&1&3\\0&1&3\\0&3&8\end{pmatrix}\sim\begin{pmatrix}1&0&1\\0&1&3\\0&0&-1\end{pmatrix}\Rightarrow a_1,a_2,a_3 - Б.\ L$$

$$\boxed{y=\alpha_1a_1+\alpha_2a_2+\alpha_3a_3}$$

$$\begin{cases}(x,a_1)=(y,a_1)+\underset{0}{(z,a_1)}=(y,a_1)=\alpha_1(a_1,a_1)+\alpha_2(a_2,a_1)+\alpha_3(a_3,a_1)\\(x,a_2)=(y,a_2)=\alpha_1(a_1,a_2)+\alpha_2(a_2,a_2)+\alpha_3(a_3,a_2)\\(x,a_3)=(y,a_3)=\alpha_1(a_1,a_3)+\alpha_2(a_2,a_3)+\alpha_3(a_3,a_3)\end{cases}$$

$(x,a_1)=10+2-2-2=8$ $\quad(a_1,a_1)=4+1+1+1=7$ $\quad(a_1,a_2)=2+1+3=6$

$(x,a_2)=5+2-6+0=1$ $\quad(a_2,a_2)=1+1+9+0=11$ $\quad(a_1,a_3)=2+2+8-1=11$

$(x,a_3)=5+4-16+2=-5$ $\quad(a_3,a_3)=1+4+64+1=70$ $\quad(a_2,a_3)=1+2+24=27$

$$\begin{cases}7\alpha_1+6\alpha_2+11\alpha_3=8\\6\alpha_1+11\alpha_2+27\alpha_3=1\\11\alpha_1+27\alpha_2+70\alpha_3=-5\end{cases}$$

$$\begin{cases}7\alpha_1+6\alpha_2+11\alpha_3=8 & |\cdot 11\\6\alpha_1+11\alpha_2+27\alpha_3=1 & |\cdot(-6)\end{cases}\qquad\begin{cases}7\alpha_1+6\alpha_2+11\alpha_3=8 & |\cdot 9\\11\alpha_1+27\alpha_2+70\alpha_3=-5 & |\cdot(-2)\end{cases}$$

$41\alpha_1-41\alpha_3=82$ $\qquad 41\alpha_1-41\alpha_3=82$

$\alpha_1=2+\alpha_3$, $\alpha_2=-1-3\alpha_3$

При $\alpha_3=0$: $\alpha_1=2$, $\alpha_2=-1$

$y=2a_1-a_2=(3,1,-1,-2)$

$z=x-y=(5,2,-2,2)+(-3,-1,1,2)=(2,1,-1,4)$

1374. б) $x = (2, 4, -4, 2)$ $\quad d(x, P) - ? = |z|$

$P: \begin{cases} x_1 + 2x_2 + x_3 - x_4 = 1 \\ x_1 + 3x_2 + x_3 - 3x_4 = 2 \end{cases}$ $\quad P = L + x_0$

$L: \begin{cases} x_1 + 2x_2 + x_3 - x_4 = 0 \\ x_1 + 3x_2 + x_3 - 3x_4 = 0 \end{cases}$

x_0 – ч. р. неодн. системы: $\begin{matrix} x_1 = 0 \\ x_2 = 0 \end{matrix}$ $\begin{cases} x_3 - x_4 = 1 \\ x_3 - 3x_4 = 2 \end{cases}$ $\quad 2x_4 = -1$, $x_4 = -\frac{1}{2}$, $x_3 = \frac{1}{2}$

$x_0 = (0, 0, \frac{1}{2}, -\frac{1}{2})$

$x - x_0 = (2, 4, -\frac{9}{2}, \frac{5}{2}) = y + z$, $y \in L$, $z \in L^{\perp}$

$a_1 = (1, 2, 1, -1)$, $a_2 = (1, 3, 1, -3)$ – б. $L^{\perp}$

$z = \alpha_1 a_1 + \alpha_2 a_2$

$(x - x_0, a_1) = 2 + 8 - \frac{9}{2} - \frac{5}{2} = 4$

$(x - x_0, a_2) = 2 + 12 - \frac{9}{2} - \frac{9}{2} = 5$

$(x - x_0, a_1) = \alpha_1 (a_1, a_1) + \alpha_2 (a_2, a_1)$

$(x - x_0, a_2) = \alpha_1 (a_1, a_2) + \alpha_2 (a_2, a_2)$

$(a_1, a_1) = 1 + 4 + 1 + 1 = 7$

$(a_2, a_2) = 1 + 9 + 1 + 9 = 20$

$(a_1, a_2) = 1 + 6 + 1 + 3 = 11$

$\begin{cases} 7\alpha_1 + 11\alpha_2 = 4 \\ 11\alpha_1 + 20\alpha_2 = 5 \end{cases}$ $\quad \boxed{\begin{matrix} \alpha_1 = \frac{15}{19} \\ \alpha_2 = -\frac{9}{19} \end{matrix}}$

$z = \frac{15}{19} a_1 - \frac{9}{19} a_2 = (\frac{15}{19}, \frac{30}{19}, \frac{15}{19}, -\frac{15}{19}) + (-\frac{9}{19}, -\frac{27}{19}, -\frac{9}{19}, \frac{27}{19}) =$

$= (\frac{6}{19}, \frac{3}{19}, \frac{6}{19}, \frac{12}{19})$

$d(x, P) = |z| = \frac{5}{19}$

1385. $A(2, 4, 2, 4, 2)$, $B(6, 4, 4, 4, 6)$, $C(5, 7, 5, 7, 2)$

$AB = (4, 0, 2, 0, 4)$ $\quad |AB| = \sqrt{16 + 4 + 16} = 6$

$BC = (-1, 3, 1, 3, -4)$ $\quad |BC| = \sqrt{1 + 9 + 1 + 9 + 16} = 6$

$AC = (3, 3, 3, 3, 0)$ $\quad |AC| = \sqrt{9 + 9 + 9 + 9} = 6$

$AB = BC = AC = 6 \Rightarrow \angle A = \angle B = \angle C = 60°$

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