1283. $A = \{1, x, x^2, \dots, x^n\}$

$B = \{1, x-\alpha, (x-\alpha)^2, \dots, (x-\alpha)^n\}$

$$(x-\alpha)^k = \sum_{i=0}^{k} C_k^i x^i (-\alpha)^{k-i} \Rightarrow$$

$$\Rightarrow (x-\alpha)^k = (-\alpha)^k + C_k^1 (-\alpha)^{k-1} x + C_k^2 (-\alpha)^{k-2} x^2 + \dots + 1 \cdot x^k + 0 \cdot x^{k+1} + \dots + 0 \cdot x^n$$

$$\Rightarrow T_{A \to B} = \begin{pmatrix} 1 & -\alpha & \alpha^2 & -\alpha^3 & \dots & (-\alpha)^n \\ 0 & 1 & -2\alpha & 3\alpha^2 & \dots & n(-\alpha)^{n-1} \\ - & - & - & - & - & - \\ 0 & 0 & 0 & 0 & \dots & 1 \end{pmatrix} //$$

1284. $A = \{a_1, a_2, \dots, a_n\}$, $B = \{b_1, b_2, \dots, b_n\}$

$$\begin{aligned} b_1 &= \alpha_{11} a_1 + \alpha_{21} a_2 + \dots + \alpha_{n1} a_n \\ b_2 &= \alpha_{12} a_1 + \alpha_{22} a_2 + \dots + \alpha_{n2} a_n \\ &- - - - - - - \\ b_n &= \alpha_{1n} a_1 + \alpha_{2n} a_2 + \dots + \alpha_{nn} a_n \end{aligned} \qquad T = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ - & - & - & - \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{pmatrix}$$

a) $$\begin{aligned} b_1 &= \alpha_{21} a_2 + \alpha_{11} a_1 + \dots + a_{n1} a_n \\ b_2 &= \alpha_{22} a_2 + \alpha_{12} a_1 + \dots + a_{n2} a_n \\ &- - - - - - - \\ b_n &= \alpha_{2n} a_2 + \alpha_{1n} a_1 + \dots + a_{nn} a_n \end{aligned} \qquad T = \begin{pmatrix} \alpha_{21} & \alpha_{22} & \dots & \alpha_{2n} \\ \alpha_{11} & \alpha_{12} & \dots & \alpha_{1n} \\ - & - & - & - \\ \alpha_{n1} & \alpha_{n2} & \dots & \alpha_{nn} \end{pmatrix}$$

- помінялись місцями відповідні рядки

б) $$\begin{aligned} b_2 &= \alpha_{12} a_1 + \alpha_{22} a_2 + \dots + \alpha_{n2} a_n \\ b_1 &= \alpha_{11} a_1 + \alpha_{21} a_2 + \dots + \alpha_{n1} a_n \\ &- - - - - - - \\ b_n &= \alpha_{1n} a_1 + \alpha_{2n} a_2 + \dots + \alpha_{nn} a_n \end{aligned} \qquad T = \begin{pmatrix} \alpha_{12} & \alpha_{11} & \dots & \alpha_{1n} \\ \alpha_{22} & \alpha_{21} & \dots & \alpha_{2n} \\ - & - & - & - \\ \alpha_{n2} & \alpha_{n1} & \dots & \alpha_{nn} \end{pmatrix}$$

- помінялись місцями відповідні стовпці

в) $b_n = \alpha_{nn} a_n + \dots + \alpha_{2n} a_n + \alpha_{1n} a_1$

$b_2 = \alpha_{n2} a_n + \dots + \alpha_{22} a_2 + \alpha_{12} a_2$

$b_1 = \alpha_{n1} a_n + \dots + \alpha_{21} a_1 + \alpha_{11} a_1$

$$T = \begin{pmatrix} \alpha_{nn} & \dots & \alpha_{n2} & \alpha_{n1} \\ \dots & \dots & \dots & \dots \\ \alpha_{2n} & \dots & \alpha_{22} & \alpha_{21} \\ \alpha_{1n} & \dots & \alpha_{12} & \alpha_{11} \end{pmatrix}$$

симетрично відносно центру

1311. $a_1 = (1, 1, 1, 1, 0)$ $B(L) - ?$, $\dim L - ?$

$a_2 = (1, 1, -1, -1, -1)$

$a_3 = (2, 2, 0, 0, -1)$

$a_4 = (1, 1, 5, 5, 2)$

$a_5 = (1, -1, -1, 0, 0)$

$$A = (a_1|a_2|a_3|a_4|a_5) = \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 & -1 \\ 1 & -1 & 0 & 5 & -1 \\ 1 & -1 & 0 & 5 & 0 \\ 0 & -1 & -1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & -2 & -2 & 4 & -2 \\ 0 & -2 & -2 & 4 & -1 \\ 0 & -1 & -1 & 2 & 0 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & -1 & -1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 & 1 \\ 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow a_1, a_2, a_5 \text{ - базис } L$$

$\dim L = 3$

1313. $a_1 = (1, -1, 1, -1, 1)$, $a_2 = (1, 1, 0, 0, 3)$, $a_3 = (3, 1, 1, -1, 7)$

$a_4 = (0, 2, -1, 1, 2)$

$L = \langle a_1, a_2, a_3, a_4 \rangle$

$\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 + \alpha_5 x_5 = 0$

$$\begin{cases} \alpha_1 - \alpha_2 + \alpha_3 - \alpha_4 + \alpha_5 = 0 \\ \alpha_1 + \alpha_2 + 3\alpha_5 = 0 \\ 3\alpha_1 + \alpha_2 + \alpha_3 - \alpha_4 + 7\alpha_5 = 0 \\ 2\alpha_2 - \alpha_3 + \alpha_4 + 2\alpha_5 = 0 \end{cases}$$

$$A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 1 & 1 & 0 & 0 & 3 \\ 3 & 1 & 1 & -1 & 7 \\ 0 & 2 & -1 & 1 & 2 \end{pmatrix} \sim$$

$$\sim \begin{pmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 2 & -1 & 1 & 2 \\ \cancel{0} & \cancel{4} & \cancel{-2} & \cancel{2} & \cancel{4} \\ \cancel{0} & \cancel{2} & \cancel{-1} & \cancel{1} & \cancel{2} \end{pmatrix}$$

ФСР:

α_1	α_2	α_3	α_4	α_5
-1	1	2	0	0
0	0	1	1	0
-3	0	2	0	1

$$\begin{cases} x_1 - x_2 - 2x_3 = 0 \\ x_3 + x_4 = 0 \\ 3x_1 - 2x_3 - x_5 = 0 \end{cases}$$

1319. $a_1 = (1,2,0,1) \qquad b_1 = (1,0,1,0)$

$a_2 = (1,1,1,0) \qquad b_2 = (1,3,0,1)$

$L_1 = \langle a_1, a_2 \rangle \qquad L_2 = \langle b_1, b_2 \rangle$

$L_1 + L_2 = \langle a_1, a_2, b_1, b_2 \rangle$

$$(a_1|a_2|b_1|b_2) = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 1 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & -2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow \dim L_1 + L_2 = 3$$

$$A = (a_1|a_2) = \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \dim L_1 = 2$$

$$B = (b_1|b_2) = \begin{pmatrix} 1 & 1 \\ 0 & 3 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 3 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow \dim L_2 = 2$$

$$\dim L_1 \cap L_2 = \dim L_1 + \dim L_2 - \dim L_1 + L_2 = 2 + 2 - 3 = 1$$

1320. $a_1 = (1,2,1) \qquad b_1 = (2,3,-1)$

$a_2 = (1,1,-1) \qquad b_2 = (1,2,2)$

$a_3 = (1,3,3) \qquad b_3 = (1,1,-3)$

$L_1 = \langle a_1, a_2, a_3 \rangle \qquad L_2 = \langle b_1, b_2, b_3 \rangle$

$$\begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 3 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 \\ 0 & -1 & 1 \\ \cancel{0} & \cancel{-2} & \cancel{2} \end{pmatrix} \quad \dim L_1 = 2 \quad \begin{pmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ -1 & 2 & -3 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 3 \\ 0 & 5 & -8 \\ \cancel{0} & \cancel{8} & \cancel{-8} \end{pmatrix} \quad \dim L_2 = 2$$

$$(a_1|a_2|b_1|b_2) = \begin{pmatrix} 1 & 1 & 2 & 1 \\ 2 & 1 & 3 & 2 \\ 1 & -1 & -1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & -1 & -1 & 0 \\ 0 & -2 & -3 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 2 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

$\Rightarrow \boxed{a_1, a_2, b_1 \text{ — базис } L_1 + L_2}$

Пусть $x \in L_1 \cap L_2 \Rightarrow x \in L_1,\ x \in L_2$

$$x = \alpha_1 a_1 + \alpha_2 a_2 = \beta_1 b_1 + \beta_2 b_2$$

$$\alpha_1 a_1 + \alpha_2 a_2 - \beta_1 b_1 - \beta_2 b_2 = 0$$

$$(a_1 | a_2 | b_1 | b_2) \sim \begin{pmatrix} 1 & 1 & -2 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 1 \end{pmatrix}$$

ФСР:

α_1	α_2	β_1	β_2
2	1	1	1

$$X = 2a_1 + a_2 = b_1 + b_2 = (3, 5, 1)$$