22.1 $f(x,y) = \dfrac{x^y}{1+x^y}$, $x>0$; $a=\infty$, $b=0+$

$\lim\limits_{x\to\infty}\left(\lim\limits_{y\to 0+} \dfrac{x^y \to 1}{1+x^y \to 1}\right) = \lim\limits_{x\to\infty} \dfrac{1}{2} = \dfrac{1}{2}$ //

$\lim\limits_{y\to 0+}\left(\lim\limits_{x\to\infty} \dfrac{x^y}{1+x^y}\right) = \lim\limits_{y\to 0+}\left(\lim\limits_{x\to\infty} \dfrac{1}{\underbrace{\frac{1}{x^y\to\infty}}_{0}+1}\right) = \lim\limits_{y\to 0+} 1 = 1$ //

22.3 $f(x,y) = \sin\dfrac{\pi x}{2x+y}$, $y \neq -2x$, $a=b=+\infty$

$\lim\limits_{x\to+\infty}\left(\lim\limits_{y\to+\infty} \sin\dfrac{\pi x}{2x+y}\right) = \lim\limits_{x\to+\infty} 0 = 0$ //

$\lim\limits_{y\to+\infty}\left(\lim\limits_{x\to+\infty} \sin\dfrac{\pi x}{2x+y}\right) = \lim\limits_{y\to+\infty}\left(\lim\limits_{x\to+\infty} \dfrac{\pi}{2+\frac{y}{x}\to 0}\right) = \lim\limits_{y\to+\infty} 1 = 1$ //

22.5 $f(x,y) = \dfrac{x-y}{x+y}$, $x \neq -y$

$\lim\limits_{x\to 0}\left(\lim\limits_{y\to 0} \dfrac{x-y}{x+y}\right) = \lim\limits_{x\to 0} 1 = 1$

$\lim\limits_{y\to 0}\left(\lim\limits_{x\to 0} \dfrac{x-y}{x+y}\right) = \lim\limits_{y\to 0} (-1) = -1$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{x-y}{x+y} = \lim\limits_{\substack{y=kx\\ x\to 0}} \dfrac{x-kx}{x+kx} = \lim\limits_{x\to 0} \dfrac{\cancel{x}(1-k)}{\cancel{x}(1+k)} = \dfrac{1-k}{1+k} \Rightarrow \nexists$

$x_n = \frac{1}{n}$, $y_n = \frac{2}{n}$; $\lim\limits_{n\to\infty} f\left(\frac{1}{n}, \frac{2}{n}\right) = \lim\limits_{n\to\infty} \dfrac{\frac{1}{n}-\frac{2}{n}}{\frac{1}{n}+\frac{2}{n}} = \dfrac{-\frac{1}{n}}{\frac{3}{n}} = -\dfrac{1}{3}$

$x_n^* = \frac{1}{n}$, $y_n^* = \frac{5}{n}$; $\lim\limits_{n\to\infty} \dfrac{\frac{1}{n}-\frac{5}{n}}{\frac{6}{n}} = -\dfrac{4}{6} = -\dfrac{2}{3} \Rightarrow \nexists$

22.15 $f(x,y) = \begin{cases} \frac{1}{xy}, & xy \neq 0 \\ 1, & xy = 0 \end{cases}$

$\lim\limits_{\substack{x\to x_0\\ y\to 0}} f(x,y) = \infty \Rightarrow (x,y)$ при $xy=0$ – точки розриву

22.16 $f(x,y) = \begin{cases} x\sin\frac{1}{y}, & y \neq 0 \\ 0, & y = 0 \end{cases}$

$\lim\limits_{\substack{x\to x_0\\ y\to 0}} x\sin\dfrac{1}{y} = \lim\limits_{x\to x_0} x \cdot \lim\limits_{y\to 0} \sin\dfrac{1}{y} = x_0 \cdot \lim\limits_{y\to 0} \sin\dfrac{1}{y} \to \nexists$

$\lim\limits_{\substack{x\to 0\\ y\to 0}} x\sin\dfrac{1}{y}$ $\quad 0 \le |f(x,y)| = |x\sin\frac{1}{y}| \le |x| \to 0$

$f(x,y)$ – неперервна в т. $(0,0)$

$(x_0, 0)$ – т. розриву

22.22 $f(x,y) = \frac{\sin(x^3+y^3)}{x^2+y^2}$

$\lim\limits_{\substack{x\to0\\y\to0}} \frac{\sin(x^3+y^3)}{x^2+y^2} \le \lim\limits_{\substack{x\to0\\y\to0}} \frac{\sin(x^3+y^3)(x+y)}{(x^2+y^2-xy)(x+y)} = 1\cdot0=0$

$0 \le \left|\frac{\sin(x^3+y^3)}{x^2+y^2}\right| \le \left|\frac{x^3+y^3}{x^2+y^2}\right| \le \frac{(x+y)\cancel{(x^2+y^2-xy)}}{\cancel{x^2+y^2-xy}} = 0$

$\le \frac{x^3}{x^2+y^2} + \frac{y^3}{x^2+y^2} \le \frac{x^3}{x^2} + \frac{y^3}{y^2} = x+y \to 0$ //

23.4 $f(x,y) = \operatorname{arctg}(x^2+2y^2)$

$f'_x = \frac{1}{1+(x^2+2y^2)^2}\cdot 2x$, $f'_y = \frac{4x}{1+(x^2+2y^2)^2}$

$f'_x(0,0) = \lim\limits_{x\to0} \frac{f(x,0)-f(0,0)}{x} = \frac{0}{0}$

$f'_y(0,0) = \frac{0}{0}$

f – диф-на на $\mathbb{R}^2 \setminus \{(0;0)\}$ //

23.6 $f(x,y) = \begin{cases} e^{-\frac{1}{x^2+y^2}}, & x^2+y^2 \ne 0 \\ 0, & x^2+y^2=0 \end{cases}$

$f'_x = e^{-\frac{1}{x^2+y^2}}\cdot\left(\frac{2x}{(x^2+y^2)^2}\right)$, $f'_y = e^{-\frac{1}{x^2+y^2}}\left(\frac{2y}{(x^2+y^2)^2}\right)$

$f'_x(0,0) = \lim\limits_{x\to0} \frac{f(x,0)-f(0,0)}{x} = \lim\limits_{x\to0} \frac{e^{-\frac{1}{x^2}}}{x}$ заміна $= 0$

$f'_y(0,0) = \lim\limits_{y\to0} \frac{f(0,y)-f(0,0)}{y} = \lim\limits_{y\to0} \frac{e^{-\frac{1}{y^2}}}{y} = 0$

$\lim\limits_{\substack{x\to0\\y\to0}} \frac{e^{-\frac{1}{x^2+y^2}}}{\sqrt{x^2+y^2}} = \lim\limits_{\rho\to0} \frac{e^{-\frac{1}{\rho^2}}}{\rho} = 0$, $\rho = \sqrt{x^2+y^2}$

f – диф-на на $\mathbb{R}^2$ //