2·2

$x=0$
$2-x+2=4-x$

1. ~~$\hat{\mu}^1 = 62{,}5$~~
~~$\hat{S}^2 = 65{,}01$~~

2. ξ, η - н.в.в.р.р. на $[0;2]$ $\quad f_\eta(x) = f_\xi(x) = \frac{1}{2-0}$

$F, f \quad \xi+\eta$ - ?

$$f_{\xi+\eta}(x) = \int_{-\infty}^{+\infty} f_\xi(x+y) f_\eta(y)\,dy = \int_0^2 f_\xi(x-y)\cdot f_\eta(y)\,dy =$$

$$= \int_{MAX(0;x-2)}^{MIN(2;x)} \frac{1}{4}\,dy = \frac{1}{4}y \Big|_{MAX(0;x-2)}^{MIN(2;x)} = \frac{1}{4}\left(MIN(2;x) - MAX(0;x-2)\right) = \begin{cases} \frac{1}{4}x, & x\in[0;2] \\ 1-\frac{1}{4}x & x\in[2;4] \\ 0, & x\notin[0;4] \end{cases}$$

$x-y\in[0;2]$
$y\in[0;2]$

$$\begin{cases} 0\le x-y\le 2 \\ 0\le y\le 2\end{cases} \Rightarrow \begin{cases} -2+x\le y\le x \\ 0\le y\le 2\end{cases} \qquad MAX(0;x-2)\le y\le \min(2;x)$$

$$~~F_{\xi+\eta}(x) = ~~\int_0^4 f_{\xi+\eta}(x)\,dx = \int_0^2 \frac{1}{4}x\,dx + \int_2^4 1-\frac{1}{4}x\,dx =$$

$$F_{\xi+\eta}(x) = \int f_{\xi+\eta}(x)\,dx = \begin{cases} \frac{1}{4}\int x\,dx, & x\in[0;2] \\ \int 1-\frac{1}{4}x\,dx, & x\in(2;4] \\ 0, & x\notin[0;4]\end{cases} = \begin{cases} \frac{x^2}{8}, & x\in[0;2] \\ x-\frac{x^2}{8}, & x\in(2;4] \\ 0, & x\notin[0;4]\end{cases}$$

3. $$f(x;\theta) = \begin{cases} \frac{1}{\theta}e^{-\frac{x}{\theta}}, & x>0 \\ 0, & x\le 0\end{cases}$$

$$\hat{\theta} = \frac{1}{n}\sum_{k=1}^{n}\xi_k;$$

$$M\xi_k = \int_{-\infty}^{+\infty} x\cdot f(x;\theta)\,dx = \int_0^{+\infty} x\cdot\frac{1}{\theta}\cdot e^{-\frac{x}{\theta}}\,dx = \frac{1}{\theta}\int_0^{+\infty} x\cdot e^{-\frac{x}{\theta}}\,dx = \left| \begin{matrix} x=u & dv=e^{-\frac{x}{\theta}} \\ dx=du & v=-\theta\cdot e^{-\frac{x}{\theta}}\end{matrix}\right| =$$

$$= \left(-\left(x\cdot e^{-\frac{x}{\theta}} - \int e^{-\frac{x}{\theta}}\,dx\right)\right)\Big|_0^{+\infty} = \left(-\left(xe^{-\frac{x}{\theta}} + \theta\cdot e^{-\frac{x}{\theta}}\right)\right)\Big|_0^{+\infty} = \theta$$

$$M\xi_k^2 = \int_0^{+\infty} x^2\cdot\frac{1}{\theta}\cdot e^{-\frac{x}{\theta}}\,dx = \left|\begin{matrix} u=x^2 & dv=e^{-\frac{x}{\theta}} \\ du=2x\,dx & v=-\theta e^{-\frac{x}{\theta}}\end{matrix}\right| = \left(-\left(x^2e^{-\frac{x}{\theta}} - 2\int xe^{-\frac{x}{\theta}}\,dx\right)\right)\Big|_0^{+\infty} =$$

$$= \left(-\left(x^2e^{-\frac{x}{\theta}} + 2\theta xe^{-\frac{x}{\theta}} + 2\theta^2 e^{-\frac{x}{\theta}}\right)\right)\Big|_0^{+\infty} = 2\theta^2$$

$$D\xi_k = \theta^2$$

$$M\hat{\theta} = M\left(\frac{1}{n}\sum_{k=1}^{n}\xi_k\right) = \frac{1}{n}M\left(\sum_{k=1}^{n}\xi_k\right) = \frac{1}{n}\sum_{k=1}^{n}M\xi_k = \frac{1}{n}\cdot n\theta = \theta \Rightarrow \hat{\theta}\text{ - незсунута}$$

$$D\hat{\Theta} = \frac{1}{n^2}\sum_{k=1}^{n} D\xi_k = \frac{1}{n^2}\cdot n \cdot \Theta^2 = \frac{\Theta^2}{n} \xrightarrow{n\to\infty} 0 \Rightarrow \hat{\Theta}$$ конзистентна