992. $5x-2y-z-3=0$, $x+3y-2z+5=0$, $\Pi \parallel \bar{l}=\{7;9;17\}$

$7(5\alpha+\beta)+9(-2\alpha+3\beta)+17(-\alpha-2\beta)=0$

$35\alpha+7\beta-18\alpha+27\beta-17\alpha-34\beta=0$

$0=0$

$\Rightarrow \Pi \in \alpha(5x-2y-z-3=0)+\beta(x+3y-2z+5)=0.$ //

1020. 1) $2x+3y-z-4=0$, $3x-5y+2z+1=0$;

$$\begin{cases} 2x+3y-z-4=0, \\ 3x-5y+2z+1=0 \end{cases}$$

$x=1$

$$\begin{cases} 3y-z-2=0, \\ -5y+2z+4=0 \end{cases}$$

$y=0$, $z=-2$ $\quad (1;0;-2)$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 3 & -1 \\ 3 & -5 & 2 \end{vmatrix} = 6\bar{i}-3\bar{j}-10\bar{k}-9\bar{k}-4\bar{j}-5\bar{i}=\bar{i}-7\bar{j}-19\bar{k}$$

$\bar{l}=\{1;-7;-19\}$

$$\frac{x-1}{1}=\frac{y}{-7}=\frac{z+2}{-19}=t;$$

$$\begin{cases} x=t+1 \\ y=-7t \\ z=-19t-2 \end{cases} //$$

2) $x+2y-z-6=0,\ 2x-y+z+1=0$

$$\begin{cases} x+2y-z-6=0, \\ 2x-y+z+1=0 \end{cases}$$

$x=0$

$$\begin{cases} 2y-z-6=0 \\ -y+z+1=0 \end{cases} \Big| +$$

$y=5,\ z=4 \quad (0;5;4)$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -1 \\ 2 & -1 & 1 \end{vmatrix} = 2\bar{i}-2\bar{j}-\bar{k}-4\bar{k}-\bar{i}-\bar{j}=\bar{i}-3\bar{j}-5\bar{k}$$

$\bar{l}=\{1;-3;-5\}$

$$\frac{x}{1}=\frac{y-5}{-3}=\frac{z-4}{-5}=t$$

$$\begin{cases} x=t \\ y=-3t+5 \\ z=-5t+4 \end{cases} //$$

1022. 1) $\frac{x}{1}=\frac{y-1}{-2}=\frac{z}{3},\ \begin{cases} 3x+y-5z+1=0, \\ 2x+3y-8z+3=0 \end{cases}$

$\bar{l}_1=\{1;-2;3\}$

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & 1 & -5 \\ 2 & 3 & -8 \end{vmatrix} = -8\bar{i}-10\bar{j}+9\bar{k}-2\bar{k}+15\bar{i}+24\bar{j}=7\bar{i}+14\bar{j}+7\bar{k}$$

$\bar{l}_2=\{7;14;7\}$

$(\bar{l}_1,\bar{l}_2)=7-28+21=0 \Rightarrow \bar{l}_1 \perp \bar{l}_2 //$

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2) $\begin{cases} x=2t+1 \\ y=3t-2 \\ z=-6t+1 \end{cases}$, $\begin{cases} 2x+y-4z+2=0 \\ 4x-y-5z+4=0 \end{cases}$

$\overline{l_1}=\{2;3;-6\}$

$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 2 & 1 & -4 \\ 4 & -1 & -5 \end{vmatrix} = -5\overline{i}-16\overline{j}-2\overline{k}-4\overline{k}-4\overline{i}+10\overline{j} = -9\overline{i}-6\overline{j}-6\overline{k}$

$\overline{l_2}=\{-9;-6;-6\}$

$(\overline{l_1};\overline{l_2}) = -18-18+36=0 \Rightarrow \overline{l_1}\perp\overline{l_2}$ //

3) $\begin{cases} x+y-3z-1=0 \\ 2x-y-9z-2=0 \end{cases}$, $\begin{cases} 2x+y+2z+5=0 \\ 2x-2y-z+2=0 \end{cases}$

$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 1 & 1 & -3 \\ 2 & -1 & -9 \end{vmatrix} = -9\overline{i}-6\overline{j}-\cancel{k}-2\overline{k}-3\overline{i}+9\overline{j} = -12\overline{i}+3\overline{j}-3\overline{k}$

$\overline{l_1}=\{-12;3;-3\}$

$\begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 2 & 1 & 2 \\ 2 & -2 & -1 \end{vmatrix} = -\overline{i}+4\overline{j}-4\overline{k}-2\overline{k}+4\overline{i}+2\overline{j} = 3\overline{i}+6\overline{j}-6\overline{k}$

$\overline{l_2}=\{3;6;-6\}$

$(\overline{l_1},\overline{l_2}) = -36+18+18=0 \Rightarrow \overline{l_1}\perp\overline{l_2}$ //

1027. $\frac{x+2}{2}=\frac{y}{-3}=\frac{z-1}{4}$, $\frac{x-3}{l}=\frac{y-1}{4}=\frac{z-7}{2}$;

$\overline{p}=\{-3-2;-1-0;-7+1\}=\{-5;-1;-6\}$

$\begin{vmatrix} -5 & -1 & -6 \\ 2 & 3 & 4 \\ l & 4 & 2 \end{vmatrix} = -30-4l-48-18l+4+180 = 66-22l \Rightarrow$

$\Rightarrow \boxed{l=3}$

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1031. $l_1: \begin{cases} x = 3t-7 \\ y = -2t+4 \\ z = 3t+4 \end{cases}$ $l_2: x = t+1$, $y = 2t-6$, $z = -t-12$

$\bar{p}_1 = \{3; -2; 3\}$ $\bar{p}_2 = \{1; 2; -1\}$

$\bar{p}_1, \bar{p}_2$ – напрямлені вектори

$$\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -2 & 3 \\ 1 & 2 & -1 \end{vmatrix} = 2\bar{i} + 3\bar{j} + 6\bar{k} + 2\bar{k} - 6\bar{i} + 3\bar{j} = -4\bar{i} + 6\bar{j} + 8\bar{k}$$

$\bar{n} = \{-2; 3; 4\}$ або $\bar{n} = \{2; -3; -4\}$

р-ння площини α: $l_1 \subset \alpha$, $\bar{p}_1 \parallel \alpha$

$$\begin{vmatrix} x+7 & y-4 & z-4 \\ 3 & -2 & 3 \\ 2 & -3 & -4 \end{vmatrix} = 0 \Rightarrow 17x + 18y - 5z + 67 = 0$$

р-ння площини β: $l_2 \subset \beta$, $\bar{p}_2 \parallel \beta$

$$\begin{vmatrix} x-1 & y+9 & z+12 \\ 1 & 2 & -1 \\ 2 & -3 & -4 \end{vmatrix} = 0 \Rightarrow 11x - 2y + 7z + 55 = 0$$

$l = \alpha \cap \beta$

$$l: \begin{cases} 17x + 18y - 5z + 67 = 0 \\ 11x - 2y + 7z + 55 = 0 \end{cases}$$

$z_0 = 0$

$$\begin{cases} 17x + 18y + 67 = 0 \\ 11x - 2y + 55 = 0 \end{cases}$$

$x = -\frac{281}{58}$, $y = \frac{99}{116}$

$$\boxed{x = 2t - \frac{281}{58}, \ y = -3t + \frac{99}{116}, \ z = -4t}$$

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1046. $\begin{cases} 3x-2y+z+3=0 \\ 4x-3y+4z+1=0 \end{cases}$, $2x-y+lz-2=0$, $l \parallel \Pi$

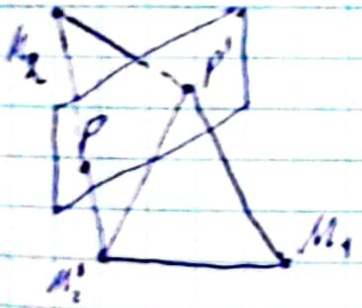
$$\bar{n}_1 \cdot \bar{n}_2 = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 3 & -2 & 1 \\ 4 & -3 & 4 \end{vmatrix} = -8\bar{i}+4\bar{j}-9\bar{k}+8\bar{k}+3\bar{i}-12\bar{j} = -5\bar{i}-8\bar{j}-\bar{k}$$

$$\bar{l} = \{5;8;1\}$$

$$2\cdot 5 - 8 + l = 0 \Rightarrow \boxed{l=-2}$$

1056. $P \in Oxz$, $M_1(3;2;-5)$, $M_2(8;-4;-13)$

M_1 і M_2 по різні сторони Oxz



M_2' симетр. M_2 відн. Oxz, $M_2'(8;4;-13)$

$\forall P' \in Oxz: M_2P' = M_2'P'$

$\forall P' \in Oxz \setminus \{P\}: M_2'P' < M_1M_2' + M_1P'$

$M_1P' < M_1M_2' + M_2'P'$

$M_2'P' - M_1P' < M_1M_2'$

$M_1P' - M_2'P' < M_1M_2'$

$|M_2'P' - M_1P'| < M_1M_2'$

Для $P \in Oxz$, $P = Oxz \cap M_2'M_1$: $|M_2'P - M_1P| = M_1M_2'$

Отже, $P \in Oxz \cap M_1M_2'$ – шукана, $\overline{M_1M_2'} = \{5;2;-8\}$

$$\begin{cases} x = 5t+3 \\ y = 2t+2 \\ z = -8t-5 \end{cases} \quad y=0 \Rightarrow t=-1, \quad \begin{matrix} x=-2 \\ y=0 \\ z=3 \end{matrix}$$

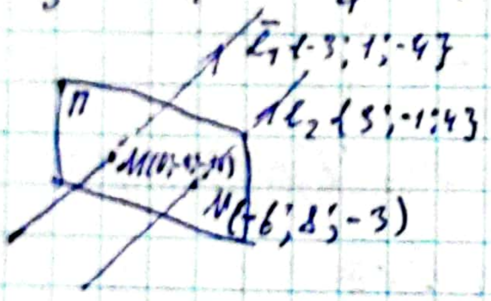
$$\boxed{P(-2;0;3)}$$

1064. $\begin{cases} 2x+2y-z-10=0 \\ x-y-z-22=0 \end{cases}$, $\frac{x+7}{3}=\frac{y-5}{-1}=\frac{z-9}{4}$

$$\bar{n}_1\cdot\bar{n}_2=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & 2 & -1 \\ 1 & -1 & -1 \end{vmatrix}=-3\bar{i}+\bar{j}-4\bar{k}$$

$\Rightarrow \bar{l}_1=\{-3;1;-4\}$ $\quad \bar{l}_2=\{3;-1;4\}$

$\frac{3}{-3}=\frac{-1}{1}=\frac{-4}{4}=-1 \Rightarrow \bar{l}_1 \parallel \bar{l}_2$



$M:\ x=0$

$\begin{cases} 2y-z-10=0 \\ -y-z-22=0 \end{cases} \Rightarrow$

$\Rightarrow y=-4$
$z=-18$

$M\in П\cap\bar{l}_1,\ \bar{l}_1\perp П$

$П:\ -3(x-0)+(y+4)-4(z+18)=0$

$3x-y+4z+68=0$

$\bar{l}_2\cap П=N:$

$\begin{cases} x=3t-7 \\ y=-t+5 \\ z=4t+9 \\ 3x-y+4z+68=0 \end{cases}$ $\quad 3(3t-7)-(-t+5)+4(4t+9)+68=0;$

$9t-21+t-5+16t+36+68=0;$

$26t+78=0;\ t=-3.$

$\begin{cases} x=-16 \\ y=8 \\ z=-3 \end{cases}$

$|\overline{MN}|=\sqrt{256+144+225}=\sqrt{625}=25.$

Відп. 25.//

№75. $P(3;-4;-6)$, $M_1(-6;1;-5)$, $M_2(7;-2;-1)$, $M_3(10;-7;1)$

$$\Pi: \begin{vmatrix} x+6 & y-1 & z+5 \\ 13 & -3 & 4 \\ 16 & -8 & 6 \end{vmatrix} = 0;$$

$$-18x-108+64y-64-104z-520+48z+240+$$
$$+32x+192-78y+78=0$$

$$14x-14y-56z-182=0$$

$\boxed{x-y-4z-13=0}$ – р-ння площини

$$\frac{x-3}{1}=\frac{y+4}{-1}=\frac{z+6}{-4}$$ – р-ння прямої PQ

$$\begin{cases} x=t+3 \\ y=-t-4 \\ z=-4t-6 \end{cases}$$

$PQ \cap \Pi = E$:

$$\begin{cases} x=t+3 \\ y=-t-4 \\ z=-4t-6 \\ x-y-4z-13=0 \end{cases}$$

$t+3+t+4+16t+24-13=0$

$18t+18=0 \Rightarrow t=-1$

$E(2;-3;-2) \Rightarrow \boxed{Q(1;-2;2)}$

1083. 2) $\begin{cases} x=2t-4 \\ y=-t+4 \\ z=-2t-1 \end{cases}$ $\begin{cases} x=4t-5 \\ y=-3t+5 \\ z=-5t+5 \end{cases}$

$$\bar{n}=\bar{l}_1\cdot\bar{l}_2=\begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 2 & -1 & -2 \\ 4 & -3 & -5 \end{vmatrix} = -i+2j-2k$$

$\bar{n}=\{-1;2;-2\}$

$M(-4;4;-1)$

$-(x+4)+2(y-4)-2(z+1)=0;$

$\boxed{x-2y+2z+14=0}$ – р-ння площини

$M(-5;5;5)$

$n=3;\quad d=\frac{1}{3}[-5-10+10+14]=\frac{9}{3}=3$

Від. 3. //