445. F_1 і $F_2 \in Oy$, ~~$b=7$, $a=2$~~

1) $b=7$, $a=2$;

$$\frac{x^2}{4}+\frac{y^2}{49}=1;$$

2) $2b=10$, $2c=8$;

$b=5$, $c=4$, $a=\sqrt{b^2-c^2}$;

$a=\sqrt{25-16}=3$,

$$\frac{x^2}{9}+\frac{y^2}{25}=1;$$

3) $2c=24$, $\varepsilon=\frac{12}{13}$,

$$\varepsilon=\frac{c}{b} \Rightarrow b=\frac{c}{\varepsilon}=\frac{12}{\frac{12}{13}}=13;$$

$a^2=169-144=25$;

$$\frac{x^2}{25}+\frac{y^2}{169}=1;$$

4) $2a=16$, $\varepsilon=\frac{3}{5}$;

$c=\frac{3}{5}b$,

$a^2=b^2-c^2$;

$64=b^2-\frac{9}{25}b^2$;

$64=\frac{16}{25}b^2$;

$b^2=100$

$$\frac{x^2}{64}+\frac{y^2}{100}=1;$$

5) $2c=6$, $\frac{2b}{\varepsilon}=\frac{50}{3}$;

$\varepsilon=\frac{3}{b}$;

$\frac{2b}{\frac{3}{b}}=\frac{50}{3}$;

$b^2=25$;

$a^2=25-9=16$;

$\frac{x^2}{16}+\frac{y^2}{25}=1$;

6) $\frac{2b}{\varepsilon}=\frac{32}{3}$, $\varepsilon=\frac{3}{4}$

$\frac{8b}{3}=\frac{32}{3}$;

$b=4$;

$a^2=16-\cancel{\frac{9}{16}}\cdot\cancel{16}=7$;

$\frac{x^2}{7}+\frac{y^2}{16}=1$.

495. $A(4;-1)$, $x+4y-10=0$

$a^2k^2+b^2=m^2$

$y=kx+m$;

$y=-\frac{1}{4}x+\frac{5}{2}$; $\Rightarrow k=-\frac{1}{4}$; $m=\frac{5}{2}$

$\begin{cases}\frac{1}{16}a^2+b^2=\frac{25}{4},\\ \frac{16}{a^2}+\frac{1}{b^2}=1,\end{cases}$ $\begin{cases}a^2+16b^2-100=0,\\ a^2+16b^2-a^2b^2=0;\end{cases}$

Замінимо $a^2=x$, $b^2=y$

$\begin{cases}x+16y-100=0,\\ x+16y-xy=0;\end{cases}$

$xy=100 \Rightarrow x=\frac{100}{y}$

$\frac{100}{y}+16y-\frac{100}{y}y=0;$

$16y^2-100y+100=0;$

$D=3600, \sqrt{D}=60;$

$y_1=\frac{100+60}{32}=5;$

$y_2=\frac{100-60}{32}=\frac{40}{32}=\frac{5}{4};$

$x_1=\frac{100}{y_1}=20; \ x_2=\frac{100}{y_2}=80$

$\boxed{\frac{x^2}{20}+\frac{y^2}{5}=1 \text{ або } \frac{x^2}{80}+\frac{4y^2}{5}=1}$

576. $\frac{x^2}{a_1^2}+\frac{y^2}{b_1^2}$ - еліпс $\quad c_1^2=a_1^2-b_1^2$

$\frac{x^2}{a_2^2}-\frac{y^2}{b_2^2}$ - гіпербола $\quad c_2=a_2^2+b_2^2$

$c_1=c_2 \Rightarrow a_1^2-b_1^2=a_2^2+b_2^2 \Rightarrow a_1^2-a_2^2=b_1^2+b_2^2 \ (1).$

Нехай (x_0, y_0) - точка перетину

Тоді дотичні в цій точці мають бути перпендикулярні

$\frac{xx_0}{a_1^2}+\frac{yy_0}{b_1^2}=1 \qquad \frac{xx_0}{a_2^2}-\frac{yy_0}{b^2}=1$

$\frac{yy_0}{b_1^2}=1-\frac{xx_0}{a_1^2} \qquad \frac{yy_0}{b_2^2}=\frac{xx_0}{a_2^2}-1$

$y=\frac{b_1^2}{y_0}-x\cdot\frac{x_0b^2}{y_0a_1^2} \qquad y=x\cdot\frac{x_0b_2^2}{y_0\cdot a_2^2}-\frac{b_2^2}{y_0}$

$k_1=-\frac{x_0b_1^2}{y_0a_1^2} \qquad k_2=\frac{x_0\cdot b_2^2}{y_0\cdot a_2^2}$

$\begin{cases} x_0^2b_1^2-y_0^2a_1^2=a_1^2b_1^2 \\ x_0^2b_2^2-y_0^2a_2^2=a_2^2b_2^2 \end{cases}$ $\quad \Delta=-a_2^2b_1^2-a_1^2b_2^2$

$\Delta x=-a_1^2a_2^2b_1^2-a_1^2a_2^2b_2^2$

$\Delta y=a_2^2b_1^2b_2^2-a_1^2b_1^2b_2^2$

$$y_0^2 = \frac{a_1^2 a_2^2 b_1^2 + a_1^2 a_2^2 b_2^2}{a_1^2 b_2^2 + a_2^2 b_1^2}; \quad y_0^2 = \frac{a_1^2 b_1^2 b_2^2 - a_2^2 b_1^2 b_2^2}{a_1^2 b_2^2 + a_2^2 b_1^2}$$

$$\frac{x_0^2}{y_0^2} = \frac{a_1^2 a_2^2 b_1^2 + a_1^2 a_2^2 b_2^2}{a_1^2 b_1^2 b_2^2 - a_2^2 b_1^2 b_2^2} = \frac{(a_1 a_2)^2 (b_1^2 - b_2^2)}{(b_1 b_2)^2 (a_1^2 - a_2^2)}$$

$$k_1 k_2 = -\frac{x_0^2}{y_0^2} \cdot \frac{b_1^2 \cdot b_2^2}{a_1^2 a_2^2} = \frac{-(b_1^2 + b_2^2)}{(a_1^2 - a_2^2)} \overset{(1)}{=} 1.$$

Доведено.

610. $y = kx + b, \quad y^2 = 2px$

$(kx+b)^2 = 2px$

$k^2x^2 + 2(bk - p)x + b^2 = 0$ – має єдиний розв'язок $\Rightarrow$

$\Rightarrow D = 0 \Rightarrow 4(bk-p)^2 - 4b^2k^2 = 0$

$\cancel{4b^2k^2} - 8bkp + 4p^2 - \cancel{4b^2k^2} = 0$

$\boxed{p = 2bk}$

612. $y^2 = 2px, \quad M_1(x_1; y_1)$

$\begin{cases} y_1^2 = 2px_1 \\ y_1 = kx_1 + b \\ p = 2kb \end{cases} \quad p = \frac{y_1^2}{2x_1}, \quad \begin{cases} y_1 = kx_1 + b \Rightarrow b = y_1 - kx_1 \\ \frac{y_1^2}{2x_1} = 2kb \Rightarrow \frac{y_1^2}{2x_1} = 2k(y_1 - kx_1) \Rightarrow \end{cases}$

$\Rightarrow k^2 x_1 - ky_1 + \frac{y_1^2}{4x_1} = 0$; – має один розв'язок $\Rightarrow$

$\Rightarrow D = 0 \Rightarrow y_1^2 - 4\frac{y_1^2}{4x_1}x_1 = 0$

$k = \frac{y_1}{2x_1}, \quad b = y_1 - \frac{y_1}{2x_1}x_1 = \frac{y_1}{2} \Rightarrow$

$\Rightarrow \boxed{y = \frac{y_1}{2x_1}x + \frac{y_1}{2}}$

$\boxed{yy_1 = p(x + x_1)}$

627. $\begin{cases} x = Ay^2 + By + C, \\ y = Dx^2 + Ex + F \end{cases}$ $\quad (x-x_0)^2 (y-y_0)^2 = R^2$

$\begin{cases} y^2 = \frac{1}{A} - \frac{B}{A}y - \frac{C}{A} \\ x^2 = \frac{1}{D} - \frac{F}{D}x - \frac{E}{D} \end{cases} \Big| +$ $\quad x^2 + y^2 = \frac{1}{A}x - \frac{B}{A}y - \frac{C}{A} + \frac{1}{D}y - \frac{F}{D}x - \frac{E}{D}$

$x^2 + y^2 - (\frac{1}{A} - \frac{F}{D})x - (\frac{1}{D} - \frac{B}{A})y = -\frac{C}{A} - \frac{E}{D}$

$[x^2 - (\frac{1}{A} - \frac{E}{D})x + \frac{1}{4}(\frac{1}{A} - \frac{F}{D})^2] + [y^2 - (\frac{1}{D} - \frac{B}{A})y + \frac{1}{4}(\frac{1}{D} - \frac{B}{A})^2] =$

$= -\frac{C}{A} - \frac{F}{D} + \frac{1}{4}(\frac{1}{A} - \frac{E}{D})^2 + \frac{1}{4}(\frac{1}{D} - \frac{B}{A})^2$

$[x - \frac{1}{2}(\frac{1}{A} - \frac{F}{D})]^2 + [y - \frac{1}{2}(\frac{1}{D} - \frac{B}{A})]^2 = \frac{1}{4}[(\frac{1}{A} - \frac{F}{D})^2 + (\frac{1}{D} - \frac{B}{A})^2] - \frac{C}{A} - \frac{E}{D}$

677. 2) $11x^2 - 20xy - 4y^2 - 20x - 8y + 1 = 0;$

$\delta = \begin{vmatrix} 11 & -10 \\ -10 & 4 \end{vmatrix} = 44 - 100 < 0$ – гіперболічний тип

$\tilde{x}: \begin{cases} 11x_0 - 10y_0 - 10 = 0 \\ -10x_0 - 4y_0 - 4 = 0 \end{cases}$ $\tilde{y}:$ $\quad \begin{cases} x_0 = 0 \\ y_0 = -1 \end{cases}$

$\begin{cases} x = \tilde{x} \\ y = \tilde{y} - 1 \end{cases}$

$11\tilde{x}^2 - 20\tilde{x}\tilde{y} + 20\tilde{x} - 4(y+1)^2 + 4 + 1 = 0$

$11\tilde{x}^2 - 20\tilde{x}\tilde{y} - 4\tilde{y}^2 + 5 = 0$

$\tilde{x}\tilde{y}: -10 tg^2\alpha - 15 tg\alpha + 10 = 0$

$tg\alpha_1 = -\frac{1}{2} \quad tg\alpha_2 = 2$

$\sin\alpha = \frac{tg\alpha}{\sqrt{1+tg^2\alpha}} = \frac{2}{\sqrt{5}}$

$\cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{1}{\sqrt{5}}$

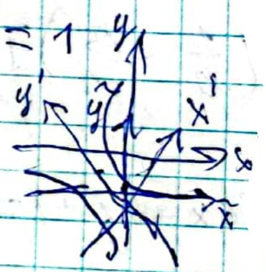
$$\begin{cases} \tilde{x} = \dfrac{x'}{\sqrt{5}} - \dfrac{2y'}{\sqrt{5}} \\ \tilde{y} = \dfrac{2x'}{\sqrt{5}} + \dfrac{y'}{\sqrt{5}} \end{cases}$$

$11 \cdot \frac{1}{5}(x'-2y')^2 - 20 \cdot \frac{1}{5}(x'-2y')(2x'+y') - 4\frac{1}{5}(2x'+y')^2 + 5 = 0$

$\frac{11}{5}x'^2 - \frac{44}{5}x'y' + \frac{44}{5}y'^2 - 8x'^2 + 12x'y' + 8y'^2 - \frac{16}{5}x'^2 - \frac{16}{5}x'y' - \frac{4}{5}y'^2 + 5 = 0$

$\boxed{9x'^2 - 16y'^2 = 5}$

$\frac{9x'^2}{5} - \frac{16y'^2}{5} = 1$



3) $7x^2 + 60xy + 32y^2 - 14x - 60y + 7 = 0$;

$\delta = 224 - 900 < 0$ - гіперболічний тип

$$\begin{cases} 7x_0 + 30y_0 - 7 = 0 \\ 30x_0 + 32y_0 - 30 = 0 \end{cases} \quad \begin{cases} x_0 = 1 \\ y_0 = 0 \end{cases}$$

$$\begin{cases} x = \tilde{x} + 1 \\ y = \tilde{y} \end{cases}$$

$7(\tilde{x}+1)^2 + 60(\tilde{x}+1)\tilde{y} + 32\tilde{y}^2 + 7 = 0 \Rightarrow 7\tilde{x}^2 + 14\tilde{x} + 7 + 60\tilde{x}\tilde{y} +$ $14(\tilde{x}+1)$

$+ 60\tilde{y} + 32\tilde{y}^2 - \cancel{14\tilde{x}} - 14 - 60y + 7 = 0$;

$7\tilde{x}^2 + 60\tilde{x}\tilde{y} + 32\tilde{y}^2 = 0$

$30\,tg^2\alpha - 25\,tg\,\alpha - 30 = 0$;

$tg\,\alpha_1 = \frac{3}{2}$; $tg\,\alpha_2 = -\frac{2}{3}$

$\sin\alpha = \frac{tg\,\alpha}{\sqrt{1+tg^2\alpha}} = \frac{3}{2}\cdot\frac{2}{\sqrt{13}} = \frac{3}{\sqrt{13}}$

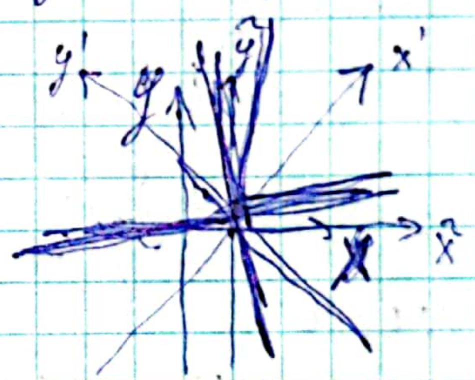
$\cos\alpha = \frac{1}{\sqrt{1+tg^2\alpha}} = \frac{2}{\sqrt{13}}$

$\begin{cases} x' = \frac{2\hat{x}}{\sqrt{13}} - \frac{3\hat{y}}{\sqrt{13}} = \frac{1}{\sqrt{13}}(2\hat{x}-3\hat{y}) \\ y' = \frac{3\hat{x}}{\sqrt{13}} + \frac{2\hat{y}}{\sqrt{13}} = \frac{1}{\sqrt{13}}(3\hat{x}+2\hat{y}) \end{cases}$

$7(2x'-3y')^2+60(2x'-3y')(3x'+2y')+32(3x+2y)^2=0$

$\overset{169}{\cancel{6..}}\,x'^2 - \cancel{476}\,y'^2 = 0$

$\boxed{\cancel{x}'^2 - 4y^2 = 0}$



689. 2) $9x^2+12xy+4y^2-24x-16y+3=0;$

$\delta = 36-36=0$ – параболічний тип

$6\,tg^2\alpha + 5\,tg\,\alpha - 6 = 0$

$tg\,\alpha_1 = \frac{2}{3} \quad tg\,\alpha_2 = -\frac{3}{2}$

$\sin\alpha = \frac{2}{\sqrt{13}},\ \cos = \frac{3}{\sqrt{13}}$

$\begin{cases} x = \frac{3}{\sqrt{13}}x' - \frac{2}{\sqrt{13}}y' = \frac{1}{\sqrt{13}}(3x'-2y') \\ y = \frac{2}{\sqrt{13}}x' + \frac{3}{\sqrt{13}}y' = \frac{1}{\sqrt{13}}(2x'+3y') \end{cases}$

$13x^2 - 8\sqrt{13}\,x + 3 = 0$

$(\sqrt{13}-4)^2 - 13 = 0 \quad \left(x - \frac{4}{\sqrt{13}}\right) = 1 \quad \hat{x} = x - \frac{4}{\sqrt{13}} \quad \boxed{\hat{x}^2 = 1}$

3) $16x^2-24xy+9y^2-160x+120y+425=0$

~~$-12 tg^2\alpha+7 tg\alpha+12=0$~~

$\delta = 144-144=0$ – параболічний тип

$-12\, tg^2\alpha+7\, tg\alpha+12=0$

$tg\alpha_1=-\frac{3}{4}$, $tg\alpha_2=\frac{4}{8}$

$\sin\alpha=-\frac{3}{5}$ $\cos\alpha=\frac{4}{5}$

$x=\frac{4}{5}x'+\frac{3}{5}y'=\frac{1}{5}(4x'+3y')$

$y=-\frac{3}{5}x'+\frac{4}{5}y'=\frac{1}{5}(-3x'+4y')$

$25\tilde{x}^2-200\tilde{x}+425=0$

$(x-4)^2=-1$