16.1 $\int_{x_0}^{x} \frac{4t}{2t+1}\,dt = 2x - \ln|2x+1|$ //

$$\int \frac{4t}{2t+1}\,dt = \int\left(2-\frac{2}{2t+1}\right)dt = 2t - 2\cdot\frac{1}{2}\ln|2t+1| + C$$

16.3 $\int_{x_0}^{x} \frac{3t^4}{t^2+t-2}\,dt = \left( \begin{array}{l|l} 3t^4 & t^2+t-2 \\ \hline -3t^4+3t^3-6t^2 & 3t^2-3t+9 \\ \quad -3t^3+6t^2 & \\ \quad -3t^3-3t^2+6t & \\ \qquad 9t^2-6t & \\ \qquad 9t^2+9t-18 & \\ \qquad\quad -15t+18 & \end{array} \right) =$

$$= \int_{x_0}^{x} (3t^2-3t+9)\,dt + \int_{x_0}^{x} \frac{18-15t}{t^2+t-2}\,dt =$$

$$= x^3 - \frac{3}{2}x^2 + 9x + \int_{x_0}^{x}\left(-\frac{16}{t+2}+\frac{1}{t-1}\right)dt =$$

$$= x^3 - \frac{3}{2}x^2 + 9x - \int_{x_0}^{x}\frac{16\,dt}{t+2} + \int_{x_0}^{x}\frac{1}{x-1}\,dt =$$

$$= x^3 - \frac{3}{2}x^2 + 9x - 16\ln|x+2| + \ln|x-1|$$ //

16.5 $\int_{x_0}^{x}\frac{dt}{t^4-1} = \int_{x_0}^{x}\frac{dt}{(t-1)(t+1)(t^2+1)} = \int_{x_0}^{x}\left(\frac{A}{t-1}+\frac{B}{t+1}+\frac{Ct+D}{t^2+1}\right)dt$

$$\frac{1}{t^4-1} = \frac{A(t+1)(t^2+1)+B(t-1)(t^2+1)+(Ct+D)(t^2-1)}{t^4-1}$$

| | | | |
|---|---|---|---|
| $t^3$ | $0 = A+B+C$ (1) | $A+B=0$ (1)+(3) | $\Rightarrow \frac{1}{2} = A-B,\ A=-B$ |
| $t^2$ | $0 = A-B+D$ (2) $\Rightarrow$ | $2A-2B=1$ (2)+(4) | $-2B=\frac{1}{2},\ B=-\frac{1}{4}$ |
| $t^1$ | $0 = A+B-C$ (3) | $A=\frac{1}{4}$ | |
| $t^0$ | $1 = A-B-D$ (4) | $B=-\frac{1}{4}$ | $1=\frac{1}{4}+\frac{1}{4}-D=\frac{1}{2}-D$ |
| | | $C=0$ | $D=-\frac{1}{2}$ |
| | | $D=-\frac{1}{2}$ | |

$$\int_{x_0}^{x}\frac{dt}{t^4-1} = \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| - \frac{1}{2}\operatorname{arctg} x$$ //

16. $I = \int_{x_0}^{x} \frac{t^4 - 7t^3 - 8t^2 - 23t - 11}{(t^2+4t+5)(t-3)^2(t+2)} = \int_{x_0}^{x} \left( \frac{A}{t-3} + \frac{B}{(t-3)^2} + \frac{C}{t+2} + \frac{Dt+E}{t^2+4t+5} \right) =$

$$t^4 - 7t^3 - 8t^2 - 23t - 11 = A(t-3)(t+2)(t^2+4t+5) + B(t+2)(t^2+4t+5) +$$

$$+ C(t-3)^2(t^2+4t+5) + (Dt+E)(t-3)^2(t+2)$$

$$\begin{array}{l|l} t^4 & 1 = A + C + D \\ t^3 & -7 = 3A + B - 2C + 4D + E \\ t^2 & -8 = -5A + 6B - 10C - 3D - 4E \\ t^1 & -23 = -29A + 13B + 6C + 18D - 3E \\ t^0 & -11 = -30A + 10B + 45C + 18E \end{array}$$

$A = 0,\ B = -2,\ C = 3,\ D = -2,\ E = -7$

$$= \int_{x_0}^{x} \frac{-2}{(t-3)^2} + \int_{x_0}^{x} \frac{3}{t+2} + \int_{x_0}^{x} \frac{-2t-7}{t^2+4t+5} =$$

$$= \frac{2}{x-3} + 3\ln|x+2| - 3\,\mathrm{arctg}(x+2) - \ln(x^2+4x+5) \,//$$

16.9 $\int_{x_0}^{x} t\cos t\,dt = \left|\begin{matrix} t=u, du=dt \\ \cos t\,dt=dv, v=\sin t \end{matrix}\right| = \sin t\cdot t\Big|_{x_0}^{x} -$

$- \int_{x_0}^{x} \sin t\,dt = x\sin x + \cos x$ //

16.11 $\int_{x_0}^{x} e^t \sin t\,dt = \left|\begin{matrix} u=\sin t, du=\cos t\,dt \\ dv=e^t dt, v=e^t \end{matrix}\right| =$

$= e^t\sin t\Big|_{x_0}^{x} - \int_{x_0}^{x} e^t\cos t\,dt = \left|\begin{matrix} u=\cos t, du=-\sin t\,dt \\ dv=e^t dt, v=e^t \end{matrix}\right| =$

$= e^t\cdot\sin t\Big|_{x_0}^{x} - \left(\cos(t)\cdot e^t\Big|_{x_0}^{x} - \int_{x_0}^{x} e^t(-\sin t)\,dt\right) =$

$= e^t\sin t\Big|_{x_0}^{x} - \left(\cos t\cdot e^t\Big|_{x_0}^{x} + \int_{x_0}^{x} e^t \sin t\,dt\right) =$

$= \frac{e^t\sin t\Big|_{x_0}^{x} - e^t\cos t\Big|_{x_0}^{x}}{2} = \frac{\sin t\, e^t - \cos t\, e^t}{2}\Bigg|_{x_0}^{x} =$

$= \frac{e^x(\sin x - \cos x)}{2}$ //

16.13 $\int_{x_0}^{x} \arccos t\,dt = \left|\begin{matrix} \arccos t=u, du=-\frac{1}{\sqrt{1-x^2}}dx \\ dv=dt, v=t \end{matrix}\right| =$

$= \arccos t\cdot t\Big|_{x_0}^{x} - \int_{x_0}^{x} t\cdot\left(-\frac{1}{\sqrt{1-x^2}}\right)dt = \left|y=\sqrt{1-t^2}\right| =$

$= \arccos t\cdot t\Big|_{x_0}^{x} + \int_{x_0}^{x}\left(\frac{\cancel{t}}{\cancel{\sqrt{1-t^2}}}\cdot\frac{1}{\frac{1}{2\cancel{\sqrt{1-t^2}}}\cdot(-2\cancel{t})}\right)dy =$

$= \arccos t\cdot t\Big|_{x_0}^{x} - y = \arccos x\cdot x - \sqrt{1-x^2}$ //

16.16 $\int_{x_0}^{x} t\,tg^2 t\,dt = \left|\begin{matrix} u=t, du=dt \\ dv=tg^2 t\,dt, v=tg\,x-x \end{matrix}\right| =$

$= x\cdot(tg\,x - x) - \int_{x_0}^{x}(tg\,t - t)\,dt =$

$= x\cdot(tg\,x - x) - \left(\int_{x_0}^{x} tg\,t\,dt - \int_{x_0}^{x} t\,dt\right) =$

$= x\cdot(tg\,x - x) - \left(-\ln|\cos x| - \frac{x^2}{2}\right) =$

$= x\cdot tg\,x - \frac{x^2}{2} + \ln|\cos x|$ //

16.17 $\int_{x_0}^{x} \sin \ln t \, dt = \left| y = \ln t \right| = \int_{x_0}^{x} \sin y \cdot e^y dy =$

$= \left| \begin{matrix} u = \sin y, & du = \cos y dy \\ dv = e^y dy, & v = e^y \end{matrix} \right| = \sin y \cdot e^y \Big|_{\ln x_0}^{\ln x} - \int_{\ln x_0}^{\ln x} e^y \cdot \cos y \, dy =$

$= \left| \begin{matrix} u = \cos y, & du = -\sin y dy \\ dv = e^y dy, & v = e^y \end{matrix} \right| = \sin y \cdot e^y \Big|_{\ln x_0}^{\ln x} - \left( \cos y \cdot e^y \Big|_{\ln x_0}^{\ln x} + \int_{\ln x_0}^{\ln x} e^y \cdot \sin y dy \right)$

$= \dfrac{\sin y \cdot e^y \Big|_{\ln x_0}^{\ln x} - \cos y \cdot e^y \Big|_{\ln x_0}^{\ln x}}{2} = \dfrac{(\sin \ln x - \cos \ln x) \cdot x}{2}$ //