287. $a_{ij} = \max(i, j)$

$$\Delta = \begin{vmatrix} n & n & n & \dots & n \\ n & n-1 & n-1 & \dots & n-1 \\ n & n-1 & n-2 & \dots & n-2 \\ \dots & \dots & \dots & \dots & \dots \\ n & n-1 & n-2 & \dots & 1 \end{vmatrix} = \begin{vmatrix} n & n & n & \dots & n \\ 0 & -1 & -1 & \dots & -1 \\ 0 & 0 & -1 & \dots & -1 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & -1 \end{vmatrix} = n \cdot (-1)^{n-1}$$

288. $a_{ij} = |i-j|$

$$\Delta = \begin{vmatrix} 0 & 1 & 2 & \dots & n-1 \\ 1 & 0 & 1 & \dots & n-2 \\ 2 & 1 & 0 & \dots & n-3 \\ n-1 & n-2 & n-3 & \dots & 0 \end{vmatrix} = \begin{vmatrix} 0 & 1 & 2 & \dots & n-1 \\ 1 & -1 & -1 & \dots & -1 \\ 1 & 1 & -1 & \dots & -1 \\ 1 & 1 & 1 & \dots & -1 \end{vmatrix} = \begin{vmatrix} n-1 & n & n+1 & \dots & n-1 \\ 0 & -2 & -2 & \dots & -1 \\ 0 & 0 & -2 & \dots & -1 \\ 0 & 0 & 0 & \dots & -1 \end{vmatrix} =$$

$$= (n-1)\cdot(-2)^{n-2}\cdot(-1) = (1-n)\cdot(-2)^{n-2} \; //$$

291.

$$\Delta = \begin{vmatrix} a_0 & a_1 & a_2 & \dots & a_n \\ a_0 & x & a_2 & \dots & a_n \\ a_0 & a_1 & x & \dots & a_n \\ a_0 & a_1 & a_2 & \dots & x \end{vmatrix} = a_0 \cdot a_1 \cdot a_2 \cdot \ldots \cdot a_n \cdot \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & \frac{x}{a_1} & 1 & \dots & 1 \\ 1 & 1 & \frac{x}{a_2} & \dots & 1 \\ 1 & 1 & 1 & \dots & \frac{x}{a_n} \end{vmatrix} =$$

$$= a_0 \cdot a_1 \cdot a_2 \cdot \ldots \cdot a_n \cdot \begin{vmatrix} 1 & 1 & 1 & \dots & 1 \\ 0 & \frac{x-a_1}{a_1} & \frac{a_2-x}{a_2} & \dots & 0 \\ 0 & 0 & \frac{x-a_2}{a_2} & \dots & 0 \\ 0 & 0 & 0 & \dots & \frac{x-a_n}{a_n} \end{vmatrix} = a_0 \cdot a_1 \cdot a_2 \cdot \ldots \cdot a_n \left(\frac{x-a_1}{a_1}\right)\cdot\left(\frac{x-a_2}{a_2}\right)\cdot\ldots\cdot\left(\frac{x-a_n}{a_n}\right) =$$

$$= a_0 (x-a_1)(x-a_2)\ldots(x-a_n) \; //$$

294.

$$\begin{vmatrix} 1+x & 1 & 1 & 1 \\ 1 & 1-x & 1 & 1 \\ 1 & 1 & 1+z & 1 \\ 1 & 1 & 1 & 1-z \end{vmatrix} = (1+x)\begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1+z & 1 \\ 1 & 1 & 1-z \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+z & 1 \\ 1 & 1 & 1-z \end{vmatrix} + \begin{vmatrix} 1 & 1-x & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1-z \end{vmatrix} + \begin{vmatrix} 1 & 1-x & 1 \\ 1 & 1 & 1+z \\ 1 & 1 & 1 \end{vmatrix}$$

$$= (1+x)\begin{vmatrix} 1-x & 1 & 1 \\ 1 & 1+z & 1 \\ 1 & 1 & 1-z \end{vmatrix} + \begin{vmatrix} 1 & 1 & 1 \\ 0 & z & 0 \\ 0 & 0 & -z \end{vmatrix} + \begin{vmatrix} 1 & 1-x & 1 \\ 0 & x & 0 \\ 0 & 0 & -z \end{vmatrix} + \begin{vmatrix} 1 & 1-x & 1 \\ 0 & x & z \\ 0 & 0 & -z \end{vmatrix} = x^2 z^2 \; //$$

297.

$$\begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ 1 & a_1 & 0 & \dots & 0 \\ 1 & 0 & a_2 & \dots & 0 \\ 1 & 0 & 0 & \dots & a_n \end{vmatrix} = a_1 a_2 \ldots a_n \begin{vmatrix} 0 & 1 & 1 & \dots & 1 \\ \frac{1}{a_1} & 1 & 0 & \dots & 0 \\ \frac{1}{a_2} & 0 & 1 & \dots & 0 \\ \frac{1}{a_n} & 0 & 0 & \dots & 1 \end{vmatrix} = -a_1 a_2 \ldots a_n \left(\frac{1}{a_1} + \frac{1}{a_2} + \ldots + \frac{1}{a_n}\right) \; //$$

298.

$$\begin{vmatrix} 1 & 0 & 0 & 0 & \dots & 0 & 1 \\ 1 & a_1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 1 & a_2 & 0 & \dots & 0 & 0 \\ 1 & 0 & 1 & a_3 & \dots & 0 & 0 \\ 1 & 0 & 0 & 0 & \dots & 1 & a_n \end{vmatrix} = \begin{vmatrix} a_1 & 0 & 0 & \dots & 0 & 0 \\ 1 & a_2 & 0 & \dots & 0 & 0 \\ 0 & 1 & a_3 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & a_n \end{vmatrix} - \begin{vmatrix} 1 & a_1 & 0 & 0 & \dots & 0 \\ 1 & 1 & a_2 & 0 & \dots & 0 \\ 1 & 0 & 1 & a_3 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 1 \end{vmatrix} =$$

$$=\begin{vmatrix} a_1 & 0 & 0 & \dots & 0 & 0 \\ 1 & a_2 & 0 & \dots & 0 & 0 \\ 0 & 1 & a_3 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 1 & a_n \end{vmatrix} - \left(\begin{vmatrix} a_1 & 0 & 0 & \dots & 0 \\ 1 & a_2 & 0 & \dots & 0 \\ 0 & 1 & a_3 & \dots & 0 \\ 0 & 0 & 0 & \dots & a_{n-1} \end{vmatrix} - \begin{vmatrix} 1 & a_1 & 0 & 0 & \dots & 0 \\ 1 & 1 & a_2 & 0 & \dots & 0 \\ 1 & 0 & 1 & a_3 & \dots & 0 \\ 1 & 0 & 0 & 0 & \dots & 1 \end{vmatrix} \right) =$$

$$= a_1 a_2 \dots a_n - a_1 a_2 \dots a_{n-1} + a_1 a_2 \dots a_{n-2} - \dots + (-1)^{n-1} a_1 + (-1)^n \; //$$

300.

$$\begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 0 & \dots & 3 \end{vmatrix} = \begin{vmatrix} 3 & 2 & 0 & \dots & 0 & 0 \\ 1 & 3 & 2 & \dots & 0 & 0 \\ 0 & 1 & 3 & \dots & 0 & 0 \\ 0 & 0 & 0 & \dots & 3 & 2 \\ 0 & 0 & 0 & \dots & 1 & 3 \end{vmatrix} = 3D_{n-1} - \begin{vmatrix} 3 & 2 & 0 & \dots & 0 \\ 1 & 3 & 2 & \dots & 0 \\ 0 & 1 & 3 & \dots & 0 \\ 0 & 0 & 0 & \dots & 2 \end{vmatrix}$$

$$D_n = 3D_{n-1} - 2D_{n-2}$$

$$x^2 - 3x + 2 = 0;$$

$$\alpha = 1, \beta = 2$$

$$\begin{matrix} n=1 \\ n=2 \end{matrix} \begin{cases} c_1 + 2c_2 = 3 \\ c_1 + 4c_2 = 7 \end{cases} \quad \begin{cases} c_1 = -1, \\ c_2 = 2 \end{cases}$$

$$D_n = -1 \cdot 1^n + 2 \cdot 2^n = 2^{n+1} - 1 \; //$$

302.

$$\begin{vmatrix} 5 & 6 & 0 & 0 & 0 & \dots & 0 & 0 \\ 4 & 5 & 2 & 0 & 0 & \dots & 0 & 0 \\ 0 & 1 & 3 & 2 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 3 & 2 & \dots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 3 & 2 \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 3 \end{vmatrix} = 3D_{n-1} - \begin{vmatrix} 5 & 6 & 0 & 0 & 0 & \dots & 0 \\ 4 & 5 & 2 & 0 & 0 & \dots & 0 \\ 0 & 1 & 3 & 2 & 0 & \dots & 0 \\ 0 & 0 & 1 & 3 & 2 & \dots & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 2 \end{vmatrix}$$

$$D_n = 3D_{n-1} - 2D_{n-2}$$

$$x^2 - 3x + 2 = 0;$$

$$\alpha = 1, \beta = 2$$

$$\begin{matrix} n=1 \\ n=2 \end{matrix} \begin{cases} c_1 + 2c_2 = 5 \\ c_1 + 4c_2 = 1 \end{cases} \quad \begin{cases} c_1 = 9 \\ c_2 = -2 \end{cases}$$

$$D_n = 9 \cdot 1^n - 2 \cdot 2^n = 9 - 2^{n+1} \; //$$