11.26 $f(x)=\begin{cases} x-1, & x<1 \\ x^2-1, & x\geq 1 \end{cases}$

$f'(x)=1, x<1$

$f'(x)=2x, x\geq 1$

$f'_л(1)=1, f'_п(1)=2, f'_л(1)\neq f'_п(1) \Rightarrow$ не диференційовна //

11.27 $f(x)=\begin{cases} \operatorname{arctg} \alpha x, & x\leq 1 \\ \beta \operatorname{sgn}(x-3), & x>1 \end{cases}$

$f(1-0)=\operatorname{arctg}\alpha, \ f(1+0)=-\beta \Rightarrow \operatorname{arctg}\alpha=-\beta$

$f'_{0}(x)=\frac{1}{1+\alpha^2x^2}, \ x\leq 1$ $\Rightarrow$

$f'(x)=0, \ x>1$

$\Rightarrow f'(1)=\frac{1}{1+\alpha^2}=0 \Rightarrow 1=1+\alpha^2 \Rightarrow \alpha=0$

$\Rightarrow -\beta=0 \Rightarrow \boxed{\alpha=\beta=0}$

11.35 $f(x)=y=\arcsin\sqrt{x}, \ x_0=\frac{1}{2}$

$\sqrt{x}=\sin y \quad x=\sin^2 y$

$f^{-1}(y)=\sin^2 y$

$f'(x)=\frac{1}{2\sin y\cos y}=\cancel{\dots}=\frac{1}{2\cdot\sqrt{x}\cdot\sqrt{1-x}}=\frac{1}{2\sqrt{x-x^2}}$ //

$f'\left(\frac{1}{2}\right)=\frac{1}{2\sqrt{\frac{1}{2}-\frac{1}{4}}}=\frac{1}{2\sqrt{\frac{1}{4}}}=\frac{1}{2\cdot\frac{1}{2}}=1$ //

11.37 $f(x) = y = \log_2 x$, $x_0 = 2$

$x = 2^y$, $f'(x) = \cancel{\frac{1}{\ln 2 \cdot y}}\ \frac{1}{\ln 2 \cdot 2^y} = \frac{1}{\ln 2 \cdot x}$

$f'(2) = \frac{1}{2\ln 2}$ //

11.41 $e^y + y = \ln x + x$, $P(1, 0)$

$e^y \cdot y' + y' = \frac{1}{x} + 1$

$y'(e^y + 1) = \frac{1+x}{x}$

$y' = \frac{1+x}{x(e^y+1)}$

$dy = \frac{1+x}{x(e^y+1)} dx$

$P(1,0)$: $dy = \frac{1+1}{1(e^0+1)} dx = \frac{2}{2} dx = dx$ //

11.49 $x(t) = 2^{\sin^2 t}$, $y(t) = 2^{\cos^2 t}$, $t_0 = \frac{\pi}{2}$, $P(1, 2)$

$f'(x) = \frac{y'(t)}{x'(t)} = \frac{\cancel{\ln 2} \cdot 2^{\cos^2 t} \cdot \cancel{2\cos t \cdot (-\sin t)}}{\cancel{\ln 2} \cdot 2^{\sin^2 t} \cdot \cancel{2\sin t \cdot \cos t}} = -\frac{2^{\cos^2 t}}{2^{\sin^2 t}} =$

$= -2^{\cos(2t)}$ //

$f'\left(\frac{\pi}{2}\right) = -2^{\cos \pi} = -2^{-1} = -\frac{1}{2}$ //

11.53 $f = u^v$

$df = u^v \cdot (\ln u\, dv + \frac{v}{u} du)$

11.56 $f = \ln tg \frac{u}{v}$

$df = \frac{1}{tg \frac{u}{v}} \cdot d(tg \frac{u}{v}) = \frac{1}{tg \frac{u}{v}} \cdot \frac{1}{\cos^2 \frac{u}{v}} \cdot d(\frac{u}{v}) = \frac{1}{tg \frac{u}{v}} \cdot \frac{1}{\cos^2 \frac{u}{v}} \cdot \frac{v\,du - u\,dv}{v^2}$,

$v \neq 0 \vee \frac{u}{v} \in (\pi k; \frac{\pi}{2} + \pi k)$, $k \in \mathbb{Z}$

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12.5 $f(x) = \frac{e^x}{x}$, $n = 10$

$$\left(\frac{e^x}{x}\right)^{(10)} = e^x \cdot \sum_{k=0}^{10} C_{10}^k \frac{(-1)^k k!}{x^{k+1}}$$

12.6 $f(x) = \ln \frac{x^2-1}{x^2-4x+4} = \ln \frac{(x-1)(x+1)}{(x-2)^2} =$

$= \ln(x-1) + \ln(x+1) - 2\ln(x-2)$

$$f^{(n)}(x) = (-1)^{n-1} \cdot (n-1)! \cdot \left(\frac{1}{(x-1)^n} + \frac{1}{(x+1)^n} - \frac{2}{(x-2)^n}\right)$$

12.15 $f = u^3$

$df = 3u^2 \cdot du$, $d^2 f = 6u\,du^2 + 3u^2 \cdot d^2 u$

12.20 $f = u \ln v$

$df = \ln v\, du + \frac{u}{v} dv$, $d^2 f = \ln v\, d^2 u + \frac{2}{v} du\, dv + u \cdot \frac{v d^2 v - dv^2}{v^2}$