1) $f(x,\theta)=\begin{cases}\theta(1-\theta|x|), & |x|\le\frac{1}{\theta};\\ 0, & |x|>\frac{1}{\theta}.\end{cases}$

$\alpha_1 = M\xi = \int\limits_{-\frac{1}{\theta}}^{\frac{1}{\theta}} \theta(1-\theta|x|)x\,dx = \int\limits_{0}^{\frac{1}{\theta}} \theta(1-\theta|x|)x\,dx + \int\limits_{-\frac{1}{\theta}}^{0} \theta(1-\theta|x|)x\,dx = 0$

$\alpha_2 = M\xi^2 = \int\limits_{-\frac{1}{\theta}}^{\frac{1}{\theta}} \theta(1-\theta|x|)x^2\,dx = \frac{\frac{1}{\theta^3}\cdot\theta}{3}\cdot\frac{\frac{1}{\theta^4}\cdot\theta^2}{4} - \left(\frac{-\frac{1}{\theta^3}\cdot\theta}{3}+\frac{\frac{1}{\theta^4}\cdot\theta^2}{4}\right) =$

$= \frac{1}{6\theta^2} \Rightarrow \alpha_2 = \alpha_2(\theta)$

$\frac{1}{6\theta^2} = \frac{1}{n}\sum_{i=1}^{n} x_i^2 \Rightarrow \hat{\theta} = \frac{1}{\sqrt{\frac{6}{n}\sum_{i=1}^{n} x_i^2}}$ ~~оцінка конзистентна~~

~~2) $N(\theta, 2\theta)$, $f(x,\theta)=\frac{1}{2\sqrt{\pi\theta}}e^{-\frac{(x-\theta)^2}{4\theta}}$, $\theta>0$, $x\in\mathbb{R}$~~

1. Закон великих чисел

$M\xi^2 = \frac{1}{6\theta^2}$

$\frac{\sum_{i=1}^{n}\xi_i^2}{n} \xrightarrow{P} M\xi^2 = \frac{1}{6\theta^2}, \; n\to\infty$

2. Теорема про суперпозицію

$\hat{\theta} = \frac{1}{\sqrt{\frac{6}{n}\sum_{i=1}^{n}\xi_i^2}} \longrightarrow \frac{1}{\sqrt{6M\xi^2}} = \frac{1}{\sqrt{6\cdot\frac{1}{6\theta^2}}} = \theta, \; n\to\infty$

Отже оцінка є конзистентною.

2) $N(\theta, 2\theta)$

$f(x,\theta) = \frac{1}{2\sqrt{\pi\theta}}e^{-\frac{(x-\theta)^2}{4\theta}}, \theta>0, x\in\mathbb{R}$

$L(x,\theta) = \prod_{k=1}^{n}\frac{1}{2\sqrt{\pi\theta}}e^{-\frac{(x-\theta)^2}{4\theta}} = \frac{1}{(2\sqrt{\pi\theta})^n}e^{-\frac{\sum_{k=1}^{n}x_k^2}{4\theta}+\frac{\sum_{k=1}^{n}x_k}{2}-\frac{n\theta}{4}}$

$\ln L(x,\theta) = -n\ln 2\sqrt{\pi\theta} - \frac{\sum_{k=1}^{n}x_k^2}{4\theta} + \frac{\sum_{k=1}^{n}x_k}{2} - \frac{n\theta}{4}$

$$\frac{\partial}{\partial x} \ln L(x,\theta) = -\frac{n}{2\theta} + \frac{\sum_{k=1}^{n} x_k^2}{4\theta^2} - \frac{n}{4} = 0$$

$$-2n\theta + \sum_{k=1}^{n} x_k^2 - n\theta^2 = 0 \Rightarrow$$

$$\Rightarrow \hat{\theta} = \frac{-\cancel{n} + \sqrt{n^2 + n\sum_{k=1}^{n} x_k^2}}{n}$$

$$\hat{\theta} = \frac{-n + \sqrt{n^2 + n\sum_{k=1}^{n} x_k^2}}{n} \Rightarrow$$

$$\Rightarrow \hat{\theta} = \sqrt{1 + \frac{1}{n}\sum_{k=1}^{n} x_k^2} - 1$$

1. Закон великих чисел

$$M\xi^2 = D\xi + (M\xi)^2 = 2\theta + \theta^2$$

$$\frac{\sum_{i=1}^{n} \xi_i^2}{n} \xrightarrow{P} M\xi^2 = 2\theta + \theta^2, \; n \to \infty$$

2. Теорема про суперпозицію

$$\hat{\theta} = \sqrt{1 + \frac{1}{n}\sum_{i=1}^{n} \xi_i^2} - 1 \xrightarrow{P} \sqrt{1 + M\xi^2} - 1 = \sqrt{1 + 2\theta + \theta^2} - 1 = \sqrt{(1+\theta)^2} - 1 = \theta, \; n \to \infty$$

Отже оцінка є консистентною