1.1 $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

$n_1: \sum_{k=1}^{1} k^2 = \frac{1\cdot 2\cdot 3}{6} = 1$ (+)

$n_{k+1}: \sum_{k=1}^{n+1} k^2 = \frac{(n+1)(n+2)(2n+3)}{6}$?

$\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2 = \frac{2n^3+n^2+2n^2+n+6n^2+12n+6}{6} =$

$= \frac{2n^3+9n^2+13n+6}{6} = \frac{(n^2+3n+2)\cdot(2n+3)}{6} = \frac{(n+1)(n+2)(2n+3)}{6}$ //

1.2 $\sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$

$n_1: \sum_{k=1}^{1} k^3 = \left(\sum_{k=1}^{1} 1\right)^2 = 1^2 = 1$ (+)

$n_{n+1}: \sum_{k=1}^{n+1} k^3 = \left(\sum_{k=1}^{n+1} k\right)^2$?

$\sum_{k=1}^{n+1} k^3 = \sum_{k=1}^{n} k^3 + (n+1)^3 = \left(\frac{n(n+1)}{2}\right)^2 + (n+1)^3 = \frac{n^4+2n^3+n^2}{4} + n^3+3n^2+3n+1 =$

$= \frac{n^4+2n^3+n^2+4n^3+12n^2+12n+4}{4} = \frac{n^4+9n^2+4+6n^3+4n^2+12n}{4} =$

$= \frac{(n^2+3n+2)^2}{4} = \left(\frac{n^2+3n+2}{2}\right)^2 = \left(\frac{n^2+2n+n+2}{2}\right)^2 =$

$= \left(\frac{(n+1)\cdot(n+2)}{2}\right)^2$

1.3 $\sum_{k=1}^{n-1} (-1)^{k-1} k^2 = (-1)^n \frac{(n-1)n}{2}, \; n \geq 2$

$n_1: \sum_{k=1}^{1} (-1)^{k-1} k^2 = (-1)^0 \cdot 1^2 = 1 = (-1)^2 \frac{(2-1)\cdot 2}{2} = 1$

$n_{n+1}: \sum_{k=1}^{n} (-1)^{k-1} k^2 = (-1)^{n+1} \frac{n(n+1)}{2}$? $n = m+1$

$\sum_{k=1}^{m} (-1)^{k-1} k^2 = (-1)^m \cdot \frac{(m-1)m}{2} + (-1)^{m-1} \cdot m^2 = (-1)^{m-1} \cdot m \left((-1)\left(\frac{m-1}{2}\right)+m\right) =$

$= (-1)^{m-1} \cdot m \left(\frac{1-m}{2} + m\right) = (-1)^{m-1} \cdot \frac{m(m+1)}{2}$

1.4 $\sum_{k=1}^{n} k\,k! = (n+1)! - 1$

$A_1: 1\cdot 1! = 2! - 1 = 1$ ✓

$A_{n+1}: \sum_{k=1}^{n+1} k\,k! = \sum_{k=1}^{n} + (n+1)(n+1)! = (n+1)! - 1 + (n+1)(n+1!) =$

$= (n+1)!(n+1+1) - 1 = (n+2)! - 1$

1.5 $\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$

$A_1: 1\cdot(1+1) = \frac{1\cdot 2\cdot 3}{3} = 2$

$A_{n+1}: \sum_{k=1}^{n+1} k(k+1) = \frac{(n+1)(n+2)(n+3)}{3}$?

$\sum_{k=1}^{n+1} k(k+1) = \sum_{k=1}^{n} k(k+1) + (n+1)(n+2) = \frac{n(n+1)(n+2)}{3} + (n+1)(n+2) =$

$= \frac{n(n+1)(n+2) + 3(n+1)(n+2)}{3} = \frac{(n+1)(n+2)(n+3)}{3}$

1.6 $\sum_{k=1}^{n} \operatorname{arctg}\frac{1}{2k^2} = \operatorname{arctg}\frac{n}{n+1}$

$A_1: \operatorname{arctg}\frac{1}{2\cdot 1} = \operatorname{arctg}\frac{1}{1+1} = \operatorname{arctg}\frac{1}{2}$

$A_{n+1}: \sum_{k=1}^{n+1} \operatorname{arctg}\frac{1}{2(n+1)^2} = \operatorname{arctg}\frac{n+1}{n+2}$?

$\sum_{k=1}^{n+1} \operatorname{arctg}\frac{1}{2k^2} = \sum_{k=1}^{n} \operatorname{arctg}\frac{1}{2k^2} + \operatorname{arctg}\frac{1}{2(n+1)^2} = \operatorname{arctg}\frac{n}{n+1} + \operatorname{arctg}\frac{1}{2(n+1)^2} =$

$= \operatorname{arctg}\left(\frac{\frac{n}{n+1} + \frac{1}{2(n+1)^2}}{1 - \frac{n}{2(n+1)^3}}\right) = \operatorname{arctg}\left(\frac{\frac{2n(n+1)+1}{2(n+1)^2}}{1 - \frac{n}{2(n+1)^3}}\right) =$

$= \operatorname{arctg}\left(\frac{\frac{2n^2+2n+1}{2(n+1)^2}}{\frac{2(n+1)^3 - n}{2(n+1)^3}}\right) = \operatorname{arctg}\left(\frac{(2n^2+2n+1)(n+1)}{2(n+1)^3 - n}\right) =$

$= \operatorname{arctg}\left(\frac{2n^3+2n^2+2n^2+2n+n+1}{2(n^3+3n^2+3n+1) - n}\right) = \operatorname{arctg}\left(\frac{2n^3+4n^2+3n+1}{2n^3+6n^2+6n+2-n}\right) =$

$= \operatorname{arctg}\left(\frac{2n^3+2n^2+2n^2+2n+n+1}{2n^3+4n^2+2n^2+5n+2}\right) = \operatorname{arctg}\left(\frac{2n^2(n+1)+2n(n+1)+1(n+1)}{2n^2(n+2)+2n(n+2)+1(n+2)}\right) =$

$= \arctan\left(\frac{(n+1)(2n^2+2n+1)}{(n+2)(2n^2+2n+1)}\right) = \arctan\frac{(n+1)}{(n+2)}$

1.10 $\sum_{k=1}^{n} \frac{1}{n+k} > \frac{13}{24}, n \geq 2$

A_1: $\sum_{k=1}^{2} \frac{1}{2+2} > \frac{13}{24} = \frac{\frac{1}{4}\cdot\frac{5}{4}}{2} > \frac{13}{24} = \frac{5}{8} > \frac{13}{24} = \frac{15}{24} > \frac{13}{24}$ (+)

A_n: $\frac{1+2n}{2n^2} > \frac{13}{24}$ - припустимо

A_{n+1}: $\sum_{k=1}^{n+1} \frac{1}{n+k} > \frac{13}{24}$ - ?

$\sum_{k=1}^{n+1} \frac{1}{(n+1)+(n+1)} > \frac{13}{24} = \sum_{k=1}^{n+1} \frac{1}{n+k}$ ~~$> \frac{13}{24}$~~ $=$ ~~$\frac{1}{2(n+1)} \cdot \frac{1+2(n+1)}{2(n+1)} > \frac{13}{24}$~~

$= \sum_{k=1}^{n} \frac{1}{n+k} + \frac{1}{2n+2} = \frac{1+2n}{2n^2} + \frac{1}{2n+2} > \frac{13}{24}$

~~$= \frac{1+2(n+1)}{2(n+1)^2} > \frac{13}{24}$~~

1.11 $\sqrt{n} < \sum_{k=1}^{n} \frac{1}{\sqrt{k}} < 2\sqrt{n}, n \geq 2$

A_1: $\sqrt{2} < \sum_{k=1}^{2} \frac{1}{\sqrt{2}} < 2\sqrt{2}$;

$\sqrt{2} < 1+\sqrt{2} < \sqrt{2}+\sqrt{2}$ (+)

A_n: $\sqrt{2} < \frac{2\cdot(1+\sqrt{n})}{n} < 2\sqrt{2}$ - припустимо

A_{n+1}: $\sqrt{2} < \sum_{k=1}^{n+1} \frac{1}{\sqrt{n+1}} < 2\sqrt{2}$

$\sqrt{2} < \sum_{k=1}^{n} + \frac{1}{\sqrt{n+1}} < 2\sqrt{2}$

$\sqrt{2} < \frac{2\cdot(1+\sqrt{n})}{n} + \frac{1}{\sqrt{n+1}} < 2\sqrt{2}$

1.12 $n^{n+1} > (n+1)^n, n \geq 3$

A_1: $3^4 > 4^3$, $81 > 64$ (+)

A_{n+1}: $(n+1)^{n+2} > (n+2)^{n+1}$

$(n+1)^n\cdot(n+1)^2 > (n+2)^n(n+2)$

$\frac{(n+2)^{n+1}}{(n+1)^{n+2}} < 1$;

$$\frac{n+2}{(n+1)^2}\cdot\left(\frac{n+2}{n+1}\right)^n < 1$$

$$\frac{n+2}{(n+1)^2}\cdot\left(1+\frac{1}{n+1}\right)^n < \frac{n+2}{(n+1)^2}\cdot\left(1+\frac{1}{n}\right)^n$$

$$\frac{(n+2)n}{(n+1)^2}\cdot\boxed{\frac{(n+1)^n}{n^{n+1}}} < 1$$

$$\Rightarrow < 1$$

$$\frac{(n+2)n}{(n+1)^2} = \frac{n^2+2n}{n^2+2n+1} < 1$$

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