Вербицький Артем 1ПZ-Я1 „Диференціальні рівняння першого порядку"

1. $\dfrac{y - xy'}{x + xy'} = 2;$

$y' = \dfrac{y}{3x} - \dfrac{2}{3};$

$\dfrac{dy}{dx} = \dfrac{y}{3x} - \dfrac{2}{3};$

$dy = \left(\dfrac{y}{3x} - \dfrac{2}{3}\right)dx;$

$u = \dfrac{y}{x}, \ dy = u\,dx + x\,du;$

$u\,dx + x\,du = \left(\dfrac{u}{3} - \dfrac{2}{3}\right)dx;$

$3x\,du = (-2u - 2)\,dx;$

$-\dfrac{du}{2u+2} = \dfrac{dx}{3x};$

$\int -\dfrac{1}{2u+2}\,du = \int \dfrac{1}{3x}\,dx$

$-\dfrac{\ln|u+1|}{2} = \dfrac{\ln|x|}{3} + \ln c$

$\dfrac{1}{\sqrt{u+1}} = c\sqrt[3]{x}$

$\dfrac{1}{\sqrt{\frac{y}{x}+1}} = c\sqrt[3]{x} \qquad \sqrt{x} = c\sqrt[3]{x}\sqrt{y+x}$ //

2. $2x^2y' = y^2(2xy' - y);$

$2x^2 = -x'y^3 + 2xy^2; \quad y' = \frac{1}{x'}$

$\dfrac{2x^2}{y^3} = -x' + \dfrac{2x}{y};$

$x' - \dfrac{2x}{y} = -\dfrac{2x^2}{y^3};$

$\dfrac{x'}{x^2} - \dfrac{2}{xy} = -\dfrac{2}{y^3};$

$u = \dfrac{1}{x}, \ u' = -\dfrac{x'}{x^2};$

$-u' - \dfrac{2u}{y} = -\dfrac{2}{y^3};$

$u = \dfrac{1}{z^2}, \ du = -\dfrac{2dz}{z^3};$

$\dfrac{2dz}{z^3} - \dfrac{2dy}{yz^2} = -\dfrac{2dy}{y^3};$

$y^3dz - y^2z\,dy = -z^3dy;$

$y^3dz = (y^2z - z^3)dy;$

$v = \dfrac{z}{y}, \ dz = v\,dy + y\,dv;$

$y^3(v\,dy + y\,dv) = (v - v^3)y^3dy;$

$vy^3dy + y^4dv = vy^3dy - v^3y^3dy;$

$y^4dv = -v^3y^3dy;$

$\dfrac{dv}{v^3} = -\dfrac{dy}{y};$

$\int \dfrac{1}{v^3}\,dv = \int -\dfrac{1}{y}\,dy;$

$-\dfrac{1}{2v^2} = c - \ln|y|$

$-\dfrac{y^2}{2z^2} = c - \ln|y|$

$-\dfrac{y^2}{2x} = c - \ln|y|$

$x = \dfrac{y^2}{2\ln|y| + c}, \ y = 0$ //

3. $2(xe^y + y^4)y' = ye^y;$

$y' = \dfrac{1}{x'};$

$2(xe^y + y^4) = x'ye^y;$

$-\dfrac{2y^3}{e^y} - \dfrac{2x}{y} = -x';$

$x' - \dfrac{2x}{y} = \dfrac{2y^3}{e^y};$

$x' = \dfrac{2x}{y};$

$\dfrac{dx}{dy} = \dfrac{2x}{y};$

$dx = \dfrac{2x\,dy}{y};$

$\dfrac{dx}{x} = \dfrac{2\,dy}{y};$

$\int \dfrac{1}{x}\,dx = \int \dfrac{2}{y}\,dy;$

$\ln|x| = 2\ln|y| + \ln c;$

$x = e^c y^2;$

$x = cy^2;$

$c = u(y);$

$x = uy^2;$

$u'y^2 = \dfrac{2y^3}{e^y} \qquad du = \dfrac{2y\,dy}{e^y};$

$u' = \dfrac{2y}{e^y}; \qquad \int 1\,du = \int \dfrac{2y}{e^y}\,dy;$

$\dfrac{du}{dy} = \dfrac{2y}{e^y}; \qquad u = c - \dfrac{2y+2}{e^y}, \ y = 0;$

$x = \dfrac{y^2(ce^y - 2y - 2)}{e^y}$

$x = cy^2 - \dfrac{2y^3 + 2y^2}{e^y}, \ y = 0$ //

4. $y = y'\sqrt{1+y'^2}$;

$t = y'$; $dy = t\,dx$; $t\,dx = \frac{(2t^2+1)\,dt}{\sqrt{t^2+1}}$;

$y = t\sqrt{t^2+1}$; $dx = \frac{\sqrt{t^2+1}\,(2t^2+1)\,dt}{t^3+t}$;

$dy = \frac{dt + 2t^2\,dt}{\sqrt{t^2+1}}$; $\int 1\,dx = \int \frac{\sqrt{t^2+1}\,(2t^2+1)}{t^3+t}\,dt$;

$t\,dx = \frac{dt + 2t^2\,dt}{\sqrt{t^2+1}}$; $x = \frac{\ln\left|\frac{\sqrt{2t^2+2}-\sqrt{2}}{\sqrt{2t^2+2}+\sqrt{2}}\right|}{2} + \sqrt{2}\sqrt{2t^2+2} + C$

$y = t\sqrt{t^2+1}$, $x = -\frac{\ln|\sqrt{t^2+1}+1|}{2} + \frac{\ln|\sqrt{t^2+1}-1|}{2} + 2\sqrt{t^2+1} + C$ //