1190. $f = 2x_1^2 + 9x_2^2 + 3x_3^2 + 8x_1x_2 - 4x_1x_3 - 10x_2x_3$

$g = 2y_1^2 + 3y_2^2 + 6y_3^2 - 4y_1y_2 - 4y_1y_3 + 8y_2y_3$

$f = (2x_1^2 + 8x_1x_2 - 4x_1x_3) + 9x_2^2 + 3x_3^2 - 10x_2x_3 =$

$= (\sqrt{2}x_1 + 2\sqrt{2}x_2 - \sqrt{2}x_3)^2 =$

$= (2x_1^2 + 8x_2^2 + 2x_3^2 + 8x_1x_2 - 4x_1x_3 - 8x_2x_3) + x_2^2 + x_3^2 - 2x_2x_3$

$= \left| \sqrt{2}x_1 + 2\sqrt{2}x_2 - \sqrt{2}x_3 = z_1 \right| = z_1^2 + (x_2^2 + x_3^2 - 2x_2x_3) =$

$= z_1^2 + (x_2 - x_3)^2 = \left| \begin{matrix} x_2 - x_3 = z_2 \\ x_3 = z_3 \end{matrix} \right| = z_1^2 + z_2^2$

$g = (2y_1^2 - 4y_1y_2 - 4y_1y_3) + 3y_2^2 + 6y_3^2 + 8y_2y_3 =$

$= (\sqrt{2}y_1 - \sqrt{2}y_2 - \sqrt{2}y_3)^2 =$

$= (2y_1^2 + 2y_2^2 + 2y_3^2 - 4y_1y_2 - 4y_1y_3 + 4y_2y_3) + y_2^2 + 4y_3^2 + 4y_2y_3 =$

$= \left| \sqrt{2}y_1 - \sqrt{2}y_2 - \sqrt{2}y_3 = a_1 \right| = a_1^2 + (y_2^2 + 4y_3^2 + 4y_2y_3) =$

$= a_1^2 + (y_2 + 2y_3)^2 = \left| \begin{matrix} y_2 + 2y_3 = a_2 \\ y_3 = a_3 \end{matrix} \right| = a_1^2 + a_2^2$

$\begin{matrix} z_1 = a_1 \\ z_2 = a_2 \\ z_3 = a_3 \end{matrix} \left| \begin{matrix} x_1 = \frac{1}{\sqrt{2}}(z_1 - 2\sqrt{2}x_2 + \sqrt{2}x_3) = \frac{z_1}{\sqrt{2}} - 2z_2 - z_3 \\ x_2 = z_2 + x_3 = z_2 + z_3 \\ x_3 = z_3 \end{matrix} \right.$

$x_1 = \frac{\sqrt{2}y_1 - \sqrt{2}y_2 - \sqrt{2}y_3}{\sqrt{2}} - 2y_2 - 4y_3 - y_3 = y_1 - 3y_2 - 6y_3$

$x_2 = y_2 + 2y_3 + y_3 = y_2 + 3y_3$

$x_3 = y_3$

$$\begin{cases} x_1 = y_1 - 3y_2 - 6y_3 \\ x_2 = y_2 + 3y_3 \\ x_3 = y_3 \end{cases}$$

1202. $f_1 = x_1^2 + 4x_2^2 + x_3^2 + 4x_1x_2 - 2x_1x_3$

$f_2 = y_1^2 + 2y_2^2 - y_3^2 + 4y_1y_2 - 2y_1y_3 - 4y_2y_3$

$f_3 = -4z_1^2 - z_2^2 - z_3^2 - 4z_1z_2 + 4z_1z_3 + 18z_2z_3$

$$f_1 = (x_1 + 2x_2 - x_3)^2 + 4x_2x_3 = \left| \begin{array}{l} x_1 + 2x_2 - x_3 = a_1 \\ x_2 = a_2 + a_3 \\ x_3 = a_2 - a_3 \end{array} \right| = a_1^2 + a_2^2 - a_3^2$$

$$f_2 = (y_1 + 2y_2 - y_3)^2 - 2y_2^2 - 2y_3^2 = \left| \begin{array}{l} y_1 + 2y_2 - y_3 = b_1 \\ y_2 = \sqrt{2}\, b_2 \\ y_3 = \sqrt{2}\, b_3 \end{array} \right| = b_1^2 - a_2^2 - a_3^2$$

$$f_3 = -(2z_1 + z_2 - z_3)^2 + 16z_2z_3 = \left| \begin{array}{l} 2z_1 + z_2 - z_3 = c_1 \\ z_2 = c_2 + c_3 \\ z_3 = c_2 - c_3 \end{array} \right| = -c_1^2 + c_2^2 - c_3^2$$

$\Rightarrow f_2$ і f_3 – еквівалентні //

1213. $f(x) = 2x_1^2 + x_2^2 + 3x_3^2 + 2\lambda x_1x_2 + 2x_1x_3$

$$A = \begin{vmatrix} 2 & \lambda & 1 \\ \lambda & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix}$$

$\Delta_1 = 2 > 0$

$\Delta_2 = \begin{vmatrix} 2 & \lambda \\ \lambda & 1 \end{vmatrix} = 2 - \lambda^2 > 0 \Rightarrow \lambda^2 < 2$

$\Delta_3 = \begin{vmatrix} 2 & \lambda & 1 \\ \lambda & 1 & 0 \\ 1 & 0 & 3 \end{vmatrix} = 6 - 1 - 3\lambda^2 = 5 - 3\lambda^2 > 0 \Rightarrow \lambda^2 < \frac{5}{3}$

Отже, $|\lambda| < \sqrt{\frac{5}{3}}$ //

1243. $f(x) = 3x_2^2 + 3x_3^2 + 4x_1x_2 + 4x_1x_3 - 2x_2x_3$

$$A = \begin{pmatrix} 0 & 2 & 2 \\ 2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

$$|A - \lambda E| = \begin{vmatrix} -\lambda & 2 & 2 \\ 2 & 3-\lambda & -1 \\ 2 & -1 & 3-\lambda \end{vmatrix} = \begin{vmatrix} -\lambda & 2 & 2 \\ 2 & 3-\lambda & -1 \\ 0 & -4+\lambda & 4-\lambda \end{vmatrix} = \begin{vmatrix} -\lambda & 4 & 2 \\ 2 & 2-\lambda & -1 \\ 0 & 0 & 4-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} -\lambda & 4 \\ 2 & 2-\lambda \end{vmatrix} =$$

$$= (4-\lambda)\begin{vmatrix} 4-\lambda & 4 \\ 4-\lambda & 2-\lambda \end{vmatrix} = (4-\lambda)\begin{vmatrix} 4-\lambda & 4 \\ 0 & -2-\lambda \end{vmatrix} = -(4-\lambda)^2(2+\lambda)$$

$\lambda_1 = 4$ кр. 2

$\lambda_2 = -2$

$f(x) = 4y_1^2 + 4y_2^2 - 2y_3^2$ //

1250. $x_1^2+x_2^2+5x_3^2-6x_1x_2-2x_1x_3+2x_2x_3$

$$A=\begin{pmatrix}1&-3&-1\\-3&1&1\\-1&1&5\end{pmatrix}$$ - симетрична

$$|A-\lambda E|=\begin{vmatrix}1-\lambda&-3&-1\\-3&1-\lambda&1\\-1&1&5-\lambda\end{vmatrix}=\begin{vmatrix}-2-\lambda&-2-\lambda&0\\-3&1-\lambda&1\\-1&1&5-\lambda\end{vmatrix}=\begin{vmatrix}-2-\lambda&0&0\\-3&4-\lambda&1\\-1&2&5-\lambda\end{vmatrix}=-(2+\lambda)\begin{vmatrix}4-\lambda&1\\2&5-\lambda\end{vmatrix}=$$

$$=-(2+\lambda)((4-\lambda)(5-\lambda)-2)=-(2+\lambda)(\lambda-3)(\lambda-6)=0$$

$\lambda_1 = -2$
$\lambda_2 = 3$
$\lambda_3 = 6$

$\underline{\lambda_1 = -2}$

$$A + 2E = \begin{pmatrix} 3 & -3 & -1 \\ -3 & 3 & 1 \\ -1 & 1 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -7 \\ 0 & 0 & 1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	1	0

$$e_1 = \frac{a_1}{|a_1|} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$$

$\underline{\lambda_2 = 3}$

$$A - 3E = \begin{pmatrix} -2 & -3 & -1 \\ -3 & -2 & 1 \\ -1 & 1 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -2 \\ 0 & -5 & -5 \\ \cancel{0} & \cancel{-5} & \cancel{-5} \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & -2 \\ 0 & 1 & 1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	-1	1

$$e_2 = \frac{a_2}{|a_2|} = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$\underline{\lambda_3 = 6}$

$$A - 6E = \begin{pmatrix} -5 & -3 & -1 \\ -3 & -5 & 1 \\ -1 & 1 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & -8 & 4 \\ \cancel{0} & \cancel{-8} & \cancel{4} \end{pmatrix} \sim \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \end{pmatrix}$$

ФСР:

x_1	x_2	x_3
1	-1	-2

$$e_3 = \frac{a_3}{|a_3|} = \left(\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}\right)$$

$$Q = (e_1 | e_2 | e_3) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \end{pmatrix}$$

$$\begin{cases} x_1 = \frac{1}{\sqrt{2}} y_1 + \frac{1}{\sqrt{3}} y_2 + \frac{1}{\sqrt{6}} y_3 \\ x_2 = \frac{1}{\sqrt{2}} y_1 - \frac{1}{\sqrt{3}} y_2 - \frac{1}{\sqrt{6}} y_3 \\ x_3 = \frac{1}{\sqrt{3}} y_2 - \frac{2}{\sqrt{6}} y_3 \end{cases}$$

$$B = Q^T A Q = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{pmatrix}$$

$$f(x) = -2y_1^2 + 3y_2^2 + 6y_3^2$$

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