

$$15.1 \int \frac{x^5 + 2x^3 - 4x^2 - x + 11}{x^2} dx = \int x^3 dx + \int 2x dx - \int 4 dx - \int x^{-1} dx + \int \frac{11}{x^2} dx$$

$$= \frac{x^4}{4} + x^2 - 4x - \ln|x| - 11x^{-1} + C, \quad x \neq 0 //$$

$$15.2 \int \frac{(1-x)^2}{x\sqrt{x}} dx = \int x^{-\frac{3}{2}} dx - \int \frac{2dx}{\sqrt{x}} + \int \sqrt{x} dx =$$

$$= -\frac{2}{\sqrt{x}} - 4\sqrt{x} + \frac{2x\sqrt{x}}{3} + C = \frac{2x^2 - 12x - 6}{3\sqrt{x}} + C //$$

$$15.3 \int \frac{(1+x)^2}{x(1+x^2)} dx = \int \frac{1}{x} + \frac{2}{1+x^2} dx = \int \frac{1}{x} dx + \int \frac{2}{1+x^2} dx =$$

$$= \ln|x| + 2 \arctan x + C, \quad x \neq 0 //$$

$$15.4 \int 2 \cos^2 \frac{x}{2} dx = 2 \cdot \int \cos^2 \frac{x}{2} dx \stackrel{\frac{x}{2} = t}{=} 2 \cdot \int 2 \cos^2 t dx =$$

$$= 4 \int \frac{1 + \cos 2t}{2} dt = 2 \left(\int 1 dt + \int \cos 2t dt \right) = 2 \left(t + \frac{\sin 2t}{2} \right) =$$

$$= x + \sin(x) + C //$$

$$15.5 \int (\cos 2x \cdot \sin x - \sin 2x \cos x) dx = \int \sin(-x) dx =$$

$$= \int -\sin x dx = -\int \sin x dx = -(-\cos x) + C = \cos x + C //$$

$$15.6 \int \tan^2 x dx = \int \frac{\sin^2 x}{\cos^2 x} dx = \int \frac{1 - \cos^2 x}{\cos^2 x} dx = \int \frac{1}{\cos^2 x} dx - \int 1 dx =$$

$$= \tan x - x + C //$$

$$15.7 \int \sinh x \cdot \cosh x dx = \frac{1}{4} \int \sinh 2x dx = \frac{1}{4} \cosh x + C //$$

$$15.8 \int \frac{3^{x+1} + e^{3x} - e^{x-1}}{e^x} dx = 3 \int \left(\frac{3}{e}\right)^x dx + \int e^{2x} dx - \int \frac{1}{e} dx =$$

$$= 3 \cdot \left(\frac{3}{e}\right)^x \cdot \left(\ln \frac{3}{e}\right)^{-1} + \frac{1}{2} e^{2x} - \frac{x}{e} + C //$$

$$15.9 \int \frac{dx}{(2x-3)^5} = -\frac{1}{8(2x-3)^4} + C //$$

$$15.10 \int \sqrt[3]{(5-8x)^4} dx = -\frac{3}{56} \sqrt[3]{(5-8x)^4} + C //$$

$$15.11 \int e^{-3x+1} dx = -\frac{1}{3} e^{-3x+1} + C //$$

$$15.12 \int \sin(2x-3) dx = -\frac{1}{2} \cos(2x-3) + C //$$

$$15.13 \int \frac{dx}{2x^2+9} = \frac{1}{9} \int \frac{dx}{1+\frac{2}{9}x^2} = \frac{1}{3\sqrt{2}} \int \frac{dt}{1+t^2} = \frac{1}{3\sqrt{2}} \arctg \frac{\sqrt{2}}{3} x + C //$$

$$15.14 \int \frac{dx}{\sqrt{4-9x^2}} = \frac{1}{2} \int \frac{dx}{\sqrt{1-\frac{9}{4}x^2}} = \frac{1}{3} \int \frac{dx}{\sqrt{1-t^2}} = \frac{1}{3} \arcsin \frac{3x}{2} + C //$$

$$15.15 \int_{x_0}^x \frac{t^3}{t^2-4} dt = \frac{1}{2} \int_{x_0}^x \frac{t^2}{t^2-4} dt^2 = |t^2=y| \frac{1}{2} \int_{x_0}^x \frac{y dy}{y-4} =$$

$$= \frac{1}{2} \int_{x_0}^x \frac{y-4+4}{y-4} dy = \frac{1}{2} \int_{x_0}^x \frac{dy}{y-4} + \frac{4}{2} \int_{x_0}^x \frac{dy}{y-4} = \frac{1}{2} y + 2 \ln |y-4| \Big|_{x_0}^x$$

$$= \frac{1}{2} x^2 + 2 \ln |x^2-4| + C //$$

$$15.16 \int_{x_0}^x \frac{t}{\sqrt{b^2 t^2 + a^2}} dt = \frac{1}{a} \int_{x_0}^x \frac{t}{\sqrt{\frac{b^2}{a^2} t^2 + 1}} dt = |y = \frac{b}{a} t| =$$

$$= \frac{1}{b} \int \frac{\frac{a}{b} y}{\sqrt{y^2+1}} dy = \frac{a}{b^2} \cdot \sqrt{y^2+1} \Big|_{x_0}^x = \frac{a}{b^2} \cdot \sqrt{\frac{b^2}{a^2} x^2 + 1} = \frac{1}{b^2} \cdot \sqrt{b^2 x^2 + a^2} //$$

$$15.12 \int_{x_0}^x t^4 \sin(t^5 + 3) dt = \left| \begin{array}{l} \sin(t^5 + 3) = \sin y \\ dy = 5t^4 dt \end{array} \right| =$$

$$= \frac{1}{5} \int \sin y dy = -\frac{\cos(x^5 + 3)}{5}$$

$$15.18 \int_{x_0}^x t \sqrt{b^2 t^2 + a^2} dt = \left| \begin{array}{l} \sqrt{b^2 t^2 + a^2} = \sqrt{y} \\ dy = 2bt dt \end{array} \right| =$$

$$= \frac{1}{2b} \int_{x_0}^x \sqrt{y} dy = \frac{t \sqrt{y}}{\frac{3}{2} b^2} = \frac{(b^2 x^2 + a^2)^{\frac{3}{2}}}{\frac{3}{2} b^2}$$

$$15.19 \int_{x_0}^x \frac{1}{t^3} e^{-\frac{1}{t^2}} dt = \left| \begin{array}{l} -\frac{1}{t^2} = y \\ 2t^{-3} dt = dy \end{array} \right| = \frac{1}{2} \int e^y dy =$$

$$= \frac{1}{2} e^y = \frac{1}{2} e^{-\frac{1}{x^2}}$$

$$15.20 \int_{x_0}^x e^{\sin t} \cos t dt = \left| \begin{array}{l} \sin t = y \\ \cos t dt = dy \end{array} \right| = \int_{x_0}^x e^y dy =$$

$$= e^y = e^{\sin x}$$

$$15.21 \int_{x_0}^x \frac{dt}{t \ln t} = \left| \begin{array}{l} \ln t = y \\ \frac{1}{t} dt = dy \end{array} \right| = \int_{x_0}^x \frac{dy}{y} = \ln|y| = \ln|\ln x|$$

$$15.22 \int_{x_0}^x \frac{dt}{t \ln t \ln \ln t} = \left| \begin{array}{l} \ln t = y \\ \frac{1}{t} dt = dy \end{array} \right| = \int_{x_0}^x \frac{dy}{y \ln y} = \ln|\ln y| = \ln|\ln \ln x|$$