13.1 $f(x)=\sqrt[3]{1+x}$, $n=5$

$f(x)=\sqrt[3]{1+x}=(1+x)^{\frac{1}{3}}=1+\frac{x}{3}-\frac{x^2}{9}+\frac{5x^3}{81}-\frac{10x^4}{243}+\frac{22x^5}{725}+o(x^5)$ //

13.3 $f(x)=\frac{1}{1+x}$, $n=5$

$f(x)=\frac{1}{1+x}=(1+x)^{-1}=1-x+x^2-x^3+x^4-x^5+o(x^5)$ //

13.5 $f(x)=\operatorname{ch}x$, $n=5$

$f(x)=\operatorname{ch}x=1+\frac{x^2}{2!}+\frac{x^4}{4!}+o(x^5)=1+\frac{x^2}{2}+\frac{x^4}{24}+o(x^5)$ //

13.6 $f(x)=\operatorname{sh}x$, $n=5$

$f(x)=\operatorname{sh}x=x+\frac{x^3}{3!}+\frac{x^5}{5!}+o(x^5)=x+\frac{x^3}{6}+\frac{x^5}{120}+o(x^5)$ //

13.9 $f(x)=e^{2x-x^2}$, $n=5$

$f(x)=e^{2x-x^2}=1+(2x-x^2)+\frac{1}{2}(2x-x^2)^2+\frac{1}{6}(2x-x^2)^3+\frac{1}{4!}(2x-x^2)^4+\frac{1}{5!}(2x-x^2)^5+o(x^5)=$

$=1+2x+x^2-\frac{2}{3}x^3-\frac{5}{6}x^4-\frac{1}{15}x^5+o(x^5)$ //

13.10 $f(x)=\sin(\sin x)$, $n=5$

$f(x)=\sin(\sin x)=\sin\left(x-\frac{x^3}{6}+\frac{x^5}{120}+o(x^5)\right)=$

$=x-\frac{x^3}{6}+\frac{x^5}{120}-\frac{1}{6}\left(x^3-\frac{1}{2}x^5\right)+\frac{x^5}{120}+o(x^5)=$

$=x-\frac{x^3}{3}+\frac{x^5}{10}+o(x^5)$ //

13.12 $f(x) = \sin(\cos x)$, $n = 5$

$\frac{x^2}{2} - \frac{x^4}{24} = \alpha(x)$

$f(x) = \sin(\cos x) = \sin\left(1 - \frac{x^2}{2} + \frac{x^4}{24} + o(x^5)\right) = \sin(1 - \alpha(x)) =$

$= \sin 1 \cos \alpha(x) - \cos 1 \sin \alpha(x) = \sin 1 \left(1 - \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^4}{24}\right)^2\right) - \cos 1 \left(\sin\left(\frac{x^2}{2} - \frac{x^4}{24}\right)\right) =$

$= \sin 1 - \frac{\cos 1}{2} x^2 + \frac{\cos 1 - 3\sin 1}{24} x^4 + o(x^5)$ //

13.19 $\lim\limits_{x\to 0} \frac{e^x \sin x - x(1+x)}{3x^3} = \lim\limits_{x\to 0} \frac{\left(1 + x + \frac{x^2}{2} + o(x^3)\right)\left(x - \frac{x^3}{6} + o(x^3)\right) - x - x^2}{3 \cdot x^3} =$

$= \lim\limits_{x\to 0} \frac{x + x^2 + \frac{x^3}{2} - \frac{x^3}{6} - \frac{x^4}{6} - \frac{x^5}{12} - x - x^2}{3x^3} =$

$= \lim\limits_{x\to 0} \frac{\frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{12}}{3x^3} = \lim\limits_{x\to 0} \frac{\frac{x^3}{3} + o(x^3)}{3x^3} = \frac{1}{9}$ //

13.21 $\lim\limits_{x\to 0} \frac{1}{x}\left(\frac{1}{x} - \operatorname{ctg} x\right) = \lim\limits_{x\to 0} \frac{\sin x - x\cos x}{x^2 \cdot \sin x} =$

$= \lim\limits_{x\to 0} \frac{x - \frac{x^3}{6} + o(x^3) - x\left(1 - \frac{x^2}{2} + o(x^2)\right)}{x^2 \cdot (x + o(x))} =$

$= \lim\limits_{x\to 0} \frac{\cancel{x} - \frac{x^3}{6} - \cancel{x} + \frac{x^3}{2} + o(x^3)}{x^3 + o(x^3)} = \frac{1}{3}$ //