1. $\exists x (A(x) \vee B(x)) \to \exists x A(x) \vee \exists x B(x)$

$\vdash \exists x (A(x) \vee B(x))$, $\vdash \exists x A(x) \vee \exists x B(x)$

$\vdash A(y) \vee B(y)$, $\vdash \exists x A(x)$, $\vdash \exists x B(x)$

$\vdash A(y), \vdash \exists x A(x), \vdash \exists x B(x)$ | $\vdash B(y), \vdash \exists A(x), \vdash \exists A(x)$

$\vdash A(y), \vdash A(y)$ ... | $\vdash B(y), \vdash B(y)$ ...

X X

2. $\exists x (A(x) \& B(x)) \to \exists x A(x) \& \exists x B(x)$

$\vdash \exists x (A(x) \& B(x))$, $\vdash \exists x A(x) \& \exists x B(x)$

$\vdash A(y), \vdash B(y)$, $\vdash \exists x A(x) \& \exists x B(x)$

$\vdash \exists x A(x), \vdash A(y) \vdash B(y)$ | $\vdash \exists x B(x), \vdash A(y), \vdash B(y)$

$\vdash A(y), \vdash A(y)$ ... | $\vdash B(y), \vdash B(y)$ ...

X X

3. $\forall x (A(x) \vee B(x)) \to \forall x A(x) \vee \forall x B(x)$

$\vdash \forall x (A(x) \vee B(x))$, $\vdash \forall x A(x) \vee \forall x B(x)$

$\vdash \forall x (A(x) \vee B(x))$, $\vdash A(y)$, $\vdash B(y)$

$\vdash A(y) \vee B(y), \vdash A(y), \vdash B(y), \vdash \forall x (A(x) \vee B(x))$

$\vdash A(y), \vdash A(y)$ ... | $\vdash B(y), \vdash B(y)$ ...

X X

4. $\forall x(A(x) \& B(x)) \to \forall x A(x) \& \forall x B(x)$

$\vdash \forall x(A(x) \& B(x)), \neg \forall x A(x) \& \forall x B(x)$

$\vdash \forall x A(x), \vdash \forall x B(x), \neg A(y) \& \neg B(y)$

$\vdash \forall x A(x), \vdash \forall x B(x), \neg A(y) \quad | \quad \vdash \forall x A(x), \vdash \forall x B(x), \neg B(y)$

$\vdash A(y), \neg A(y) \ldots \quad | \quad \vdash B(y), \neg B(y) \ldots$

(x, d) (d, x, d)