621. $\begin{pmatrix} 24 & 19 & 36 & 72 & -38 \\ 49 & 40 & 73 & 147 & -80 \\ 23 & 59 & 98 & 219 & -118 \\ 42 & 96 & 21 & 141 & -72 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 1 & 3 & -4 \\ 24 & 19 & 36 & 72 & -38 \\ 1 & 2 & -10 & 3 & -4 \\ -1 & -2 & -1 & -3 & 4 \end{pmatrix} \begin{matrix} \\ -24\,I \\ -I \\ +I \end{matrix} \sim$

$\sim \begin{pmatrix} 1 & 2 & 1 & 3 & -4 \\ 0 & -29 & -12 & 0 & 58 \\ 0 & 0 & -11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ $r(A) = 3$ //

644. $a_1 = (1, 0, 0, 2, 5)$
$a_2 = (0, 1, 0, 3, 4)$
$a_3 = (0, 0, 1, 4, 7)$
$a_4 = (2, -3, 4, 11, 12)$

$(a_1 | a_2 | a_3 | a_4) = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 2 & 3 & 4 & 11 \\ 3 & 4 & 7 & 12 \end{pmatrix} \begin{matrix} \\ \\ \\ -2I \\ -3I \end{matrix} \sim \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 3 & 4 & 3 \\ 0 & 4 & 7 & 6 \end{pmatrix} \begin{matrix} \\ \\ \\ -3II \\ -4II \end{matrix} \sim \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 4 & 12 \\ 0 & 0 & 7 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 4 & 12 \\ 0 & 0 & 3 & 0 \end{pmatrix} \begin{matrix} \\ \\ \\ -4III \\ -3III \end{matrix} \sim$

$\sim \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & -4 \\ 0 & 0 & 0 & -12 \end{pmatrix} \begin{matrix} +\frac{1}{2}IV \\ +\frac{3}{4}IV \\ \sim \\ +IV \\ \cdot(-1) \\ +3IV \end{matrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ — лін. незалежні //

~~672. $a_1 = (4, -1, 3, -2)$ $a_3 = (3, -1, 4, -2)$~~
$a_2 = (8, -2, 6, -4)$ $a_4 = (6, -2, 8, -4)$

673. $a_1 = (1, 2, 0, 0)$
$a_2 = (1, 2, 3, 4)$
$a_3 = (3, 6, 0, 0)$

$(a_1 | a_2 | a_3) = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 2 & 6 \\ 0 & 3 & 0 \\ 0 & 4 & 0 \end{pmatrix} \begin{matrix} -2I \\ \\ \\ -4III \end{matrix} \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 4 & 0 \end{pmatrix} \begin{matrix} \\ \\ \\ \cdot 3 \end{matrix} \sim \begin{pmatrix} 1 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow a_1, a_2$ – база //
$r(A) = 2$, a_1 і a_3 – упорядковані
$\Rightarrow a_2, a_3$ – база //

688. $a_1 = (2, -1, 3, 5)$
$a_2 = (4, -3, 1, 3)$
$a_3 = (3, -2, 3, 4)$
$a_4 = (4, -1, 15, 17)$
$a_5 = (7, -6, -9, 0)$

$$(a_1|a_2|a_3|a_4|a_5) = \left(\begin{array}{ccccc|l} 2 & 4 & 3 & 4 & 7 & \\ -1 & -3 & -2 & -1 & -6 & \\ 3 & 1 & 3 & 15 & -2 & \\ 5 & 3 & 4 & 17 & 0 & \end{array}\right) \begin{array}{l} +2\text{II} \\ \times(-1)\text{I} \\ +3\text{II} \\ +5\text{II} \end{array} \sim \left(\begin{array}{ccccc|l} 1 & 3 & 2 & 1 & 6 & \\ 0 & -2 & -1 & 2 & -5 & \\ 0 & -8 & -3 & 12 & -25 & \\ 0 & -12 & -6 & 12 & -30 & \end{array}\right) \begin{array}{l} \times(-1) \\ -4\text{II} \\ -6\text{II} \end{array} \sim$$

$$\sim \left(\begin{array}{cccc} \textcircled{1} & 3 & 2 & 1 \\ 0 & \textcircled{2} & 1 & -2 \\ 0 & 0 & \textcircled{1} & 4 \\ 0 & 0 & 0 & 0 \end{array}\right) \Rightarrow \boxed{a_1, a_2, a_3 - \text{база}}$$

$x_1 a_1 + x_2 a_2 + x_3 a_3 = a_4$

$$(a_1|a_2|a_3||a_4) \sim \left(\begin{array}{ccc|c} \textcircled{1} & 3 & 2 & 1 \\ 0 & \textcircled{2} & 1 & -2 \\ 0 & 0 & \textcircled{1} & 4 \end{array}\right)$$

$x_3 = 4, \quad x_2 = \frac{-2-4}{2} = -3, \quad x_1 = 1 + 9 - 8 = 2$

$\boxed{a_4 = 2a_1 - 3a_2 + 4a_3}$ //

689. $\begin{cases} 2x_1 + 4x_2 + 3x_3 + x_4 = 6 \\ 3x_1 + 5x_2 + 2x_3 + 2x_4 = 4 \\ 9x_1 + 4x_2 + x_3 + 7x_4 = 2 \end{cases}$

$$\left(\begin{array}{cccc|l} 2 & 4 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{array}\right) \begin{array}{l} \\ -1\text{I} \\ -11 \end{array} \sim \left(\begin{array}{cccc|l} 1 & -2 & -1 & 1 & -2 \\ 2 & 4 & 3 & 1 & 6 \\ 9 & -1 & -1 & 5 & -2 \end{array}\right) \begin{array}{l} \\ -2\text{I} \\ \times(-1) \end{array} \sim \left(\begin{array}{cccc|l} 1 & -2 & -1 & 1 & -2 \\ 0 & 11 & 5 & -1 & 10 \\ 0 & 1 & 1 & -5 & 2 \end{array}\right) \begin{array}{l} \\ -11\text{III} \\ \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|l} 1 & -2 & -1 & 1 & -2 \\ 0 & 1 & 1 & -5 & 2 \\ 0 & 0 & -6 & 54 & -12 \end{array}\right) \begin{array}{l} \\ \\ :(6) \end{array} \sim \left(\begin{array}{cccc|l} 1 & -2 & -1 & 1 & -2 \\ 0 & 1 & 1 & -5 & 2 \\ 0 & 0 & 1 & -9 & 2 \end{array}\right)$$

$$\left(\begin{array}{cccc|l} 2 & 4 & 3 & 1 & 6 \\ 3 & 5 & 2 & 2 & 4 \\ 9 & 4 & 1 & 7 & 2 \end{array}\right) \begin{array}{l} \\ -1\text{I} \\ -3\text{II} \end{array} \sim \left(\begin{array}{cccc|l} 1 & -2 & -1 & 1 & -2 \\ 2 & 4 & 3 & 1 & 6 \\ 0 & -11 & -5 & 1 & -10 \end{array}\right) \begin{array}{l} \\ -2\text{I} \\ \end{array} \sim \left(\begin{array}{cccc|l} \textcircled{1} & -2 & -1 & 1 & -2 \\ 0 & \textcircled{11} & 5 & -1 & 10 \\ 0 & -11 & -5 & 1 & -10 \end{array}\right) \begin{array}{l} \Rightarrow x_1, x_2 - \text{базисні} \\ x_3, x_4 - \text{вільні} \\ +\text{II} \end{array}$$

$$\begin{cases} x_1 = \frac{x_3 - 9x_4 - 2}{11} \\ x_2 = -\frac{5x_3 + x_4 + 10}{11} \end{cases}$$

$\begin{cases} x_3 = 0 \\ x_4 = 1 \end{cases} \quad \begin{cases} x_2 = 1 \\ x_1 = -1 \end{cases}$ – част. розв.

691. $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 4x_1 - 4x_2 + x_3 + 3x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$

$\left(\begin{array}{cccc|c} 3 & -5 & 2 & 4 & 2 \\ 4 & -4 & 1 & 3 & 5 \\ 5 & 7 & -4 & -6 & 3 \end{array}\right) \begin{matrix} \\ -2I \\ \\ \end{matrix} \overset{\uparrow I_2}{\sim} \left(\begin{array}{cccc|c} 1 & 6 & -3 & -5 & 1 \\ 3 & -5 & 2 & 4 & 2 \\ 5 & 7 & -4 & -6 & 3 \end{array}\right) \begin{matrix} \\ -3I \\ -5I \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 6 & -3 & -5 & 1 \\ 0 & -23 & 11 & 19 & -1 \\ 0 & -25 & 11 & 19 & -2 \end{array}\right) \sim$

$x_3 = 1 - 3x_1 - 4x_2, \; x_4 = 1$

$x_1 = -1, \; x_2 = 1, \; x_3 = 0, \; x_4 = 1$

715. $\begin{cases} 2x_1 - x_2 + 3x_3 + 4x_4 = 5 \\ 4x_1 - 2x_2 + 5x_3 + 6x_4 = 7 \\ 6x_1 - 3x_2 + 7x_3 + 8x_4 = 9 \\ 8x_1 - 4x_2 + 9x_3 + 10x_4 = 11 \end{cases}$

$\left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 4 & -2 & 5 & 6 & 7 \\ 6 & -3 & 7 & 8 & 9 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right) \begin{matrix} \\ -2I \\ -3I \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & -1 & -2 & -3 \\ 0 & 0 & -2 & -4 & -6 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right) \begin{matrix} \\ \cdot(-1) \\ \\ -4I \end{matrix} \sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -2 & -4 & -6 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right) \sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & -3 & -6 & -9 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right) \begin{matrix} \\ \\ +3II \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|c} 2 & -1 & 3 & 4 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 \\ 8 & -4 & 9 & 10 & 11 \end{array}\right)$

Має розв'язок при $\lambda \in \mathbb{R}$

При $\lambda = 8$: $x_2 = 4 + 2x_1 - 2x_4, \; x_3 = 3 - 2x_4$

При $\lambda \neq 8$: $x_1 = 0, \; x_2 = 4 - 2x_4, \; x_3 = 3 - 2x_4$

725. $\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 = 0 \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 = 0 \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 = 0 \end{cases}$

$\left(\begin{array}{cccc|c} 2 & -4 & 5 & 3 & 0 \\ 3 & -6 & 4 & 2 & 0 \\ 4 & -8 & 17 & 11 & 0 \end{array}\right) \begin{matrix} \\ -1I \\ -2I \end{matrix} \overset{\uparrow I_2}{\sim} \left(\begin{array}{cccc|c} 1 & -2 & -1 & -1 & 0 \\ 2 & -4 & 5 & 3 & 0 \\ 0 & 0 & 7 & 5 & 0 \end{array}\right) \begin{matrix} \\ -2I \\ \\ \end{matrix} \sim \left(\begin{array}{cccc|c} \textcircled{1} & -2 & -1 & -1 & 0 \\ 0 & 0 & \textcircled{7} & 5 & 0 \\ 0 & 0 & 7 & 5 & 0 \end{array}\right)$

$x_3 = -\frac{5}{7}x_1 + 5x_2, \; x_4 = \frac{7}{2}x_1 - 7x_2$

x_1	x_2	x_3	x_4
1	0	$-\frac{5}{7}$	$\frac{7}{2}$
0	1	5	-7

790. $\begin{cases} 6x_1 - 2x_2 + 2x_3 + 5x_4 + 2x_5 = 0 \\ 9x_1 - 3x_2 + 4x_3 + 8x_4 + 9x_5 = 0 \\ 6x_1 - 2x_2 + 6x_3 + 7x_4 + x_5 = 0 \\ 3x_1 - x_2 + 4x_3 + 4x_4 - x_5 = 0 \end{cases}$

$$\left(\begin{array}{ccccc|c} 6 & -2 & 2 & 5 & 2 & 0 \\ 9 & -3 & 4 & 8 & 9 & 0 \\ 6 & -2 & 6 & 7 & 1 & 0 \\ 3 & -1 & 4 & 4 & -1 & 0 \end{array}\right) \begin{matrix} \cdot(-2/3) \\ \cdot 5/4 \\ \cdot(-2/3) \\ \cdot 1 \end{matrix} \sim \left(\begin{array}{ccccc|c} 3 & -1 & 4 & 4 & -1 & 0 \\ 0 & 0 & -8 & -4 & 12 & 0 \\ 0 & 0 & -2 & -1 & 3 & 0 \\ 0 & 0 & -6 & -3 & 9 & 0 \end{array}\right) \sim \left(\begin{array}{ccccc|c} \textcircled{3} & -1 & 4 & 4 & -1 & 0 \\ 0 & 0 & \textcircled{2} & 1 & -3 & 0 \end{array}\right)$$

$x_4 = \dfrac{-9x_1 + 3x_2 - 10x_3}{11}$, $x_5 = \dfrac{-3x_1 + x_2 - 4x_3}{11}$

x_1	x_2	x_3	x_4	x_5
•	0	0	$-\frac{9}{11}$	$-\frac{3}{11}$
0	1	0	$\frac{3}{11}$	$\frac{1}{11}$
0	0	1	$-\frac{10}{11}$	$\frac{4}{11}$

791. $\begin{pmatrix} 5 & 8 & -4 \\ 6 & 9 & -5 \\ 4 & 7 & -3 \end{pmatrix} \cdot \begin{pmatrix} 9 & 2 & 5 \\ 4 & -1 & 3 \\ 9 & 6 & 5 \end{pmatrix} = \begin{pmatrix} 15+32-36 & 10-8-24 & 25+24-20 \\ 18+36-45 & 12-9-30 & 30+27-25 \\ 12+28-27 & 8-7-18 & 20+21-15 \end{pmatrix} =$

$= \begin{pmatrix} 11 & -22 & 29 \\ 9 & -27 & 32 \\ 13 & -17 & 26 \end{pmatrix}$ //

804. $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{n-2} = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^{n-3} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}$

808. $\begin{pmatrix} 17 & -6 \\ 35 & -12 \end{pmatrix}^5 = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}^5 \cdot \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}^5 \cdot \begin{pmatrix} -7 & 3 \\ 5 & -2 \end{pmatrix}^5$

$\begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix}^2 = \begin{pmatrix} 19 & 27 \\ 45 & 64 \end{pmatrix}$, $\begin{pmatrix} 19 & 27 \\ 45 & 64 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} = \begin{pmatrix} 173 & 246 \\ 410 & 583 \end{pmatrix}$

$\begin{pmatrix} 173 & 246 \\ 410 & 583 \end{pmatrix} \cdot \begin{pmatrix} 7 & 3 \\ 5 & 2 \end{pmatrix} = \begin{pmatrix} 1231 & 519 \\ 2915 & 1229 \end{pmatrix}$; $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix} = 6$

$\begin{pmatrix} 17 & -6 \\ 35 & -12 \end{pmatrix}^5 = \begin{pmatrix} 3194 & -1266 \\ 4385 & -922 \end{pmatrix}$