1599. $A = \begin{pmatrix} 1 & -4 \\ 1 & 4 \end{pmatrix}$

$A = BQ$, $B = B^T$, $Q^T = Q^{-1}$

$AA^T = \begin{pmatrix} 1 & -4 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -4 & 4 \end{pmatrix} = \begin{pmatrix} 17 & -15 \\ -15 & 17 \end{pmatrix}$

$|AA^T - \lambda E| = \begin{vmatrix} 17-\lambda & -15 \\ -15 & 17-\lambda \end{vmatrix} = \lambda^2 - 34\lambda + 289 - 225 = \lambda^2 - 34\lambda + 64 =$

$= (\lambda - 2)(\lambda - 32) = 0$

$\lambda_1 = 2$, $\lambda_2 = 32$

$\lambda_1 = 2$ $\quad AA^T - 2E = \begin{pmatrix} 15 & -15 \\ -15 & 15 \end{pmatrix} \sim (1 \; -1)$

ФСР: $\dfrac{x_1}{1} \Big| \dfrac{x_2}{1} = a_1$ $\qquad p_1 = \dfrac{a_1}{|a_1|} = \left( \dfrac{1}{\sqrt{2}} ; \dfrac{1}{\sqrt{2}} \right)$

$\lambda_2 = 32$

$$AA^T - 32E = \begin{pmatrix} -15 & -15 \\ -15 & -15 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 \end{pmatrix}$$

ФСР: $\begin{array}{c|c} x_1 & x_2 \\ \hline 1 & -1 \end{array} = a_2$ $\qquad p_2 = \dfrac{a_2}{|a_2|} = \left(\dfrac{1}{\sqrt{2}}, -\dfrac{1}{\sqrt{2}}\right)$

$$P = (p_1 | p_2) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \qquad C = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 32 \end{pmatrix}$$

$$C = P^{-1} AA^T P \quad \left( PC = AA^T P \quad PC = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 2 & 0 \\ 0 & 32 \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{2}} & \frac{32}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & -\frac{32}{\sqrt{2}} \end{pmatrix} \right.$$

$$\left. AA^T P = \begin{pmatrix} 17 & -15 \\ -15 & 17 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{2}{\sqrt{2}} & \frac{32}{\sqrt{2}} \\ \frac{2}{\sqrt{2}} & -\frac{32}{\sqrt{2}} \end{pmatrix} \right)$$

$$D = \begin{pmatrix} \sqrt{\lambda_1} & 0 \\ 0 & \sqrt{\lambda_2} \end{pmatrix} = \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 4\sqrt{2} \end{pmatrix}$$

$$B = PDP^T = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{2} & 0 \\ 0 & 4\sqrt{2} \end{pmatrix} P^T = \begin{pmatrix} 1 & 4 \\ 1 & -4 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} \frac{5}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} & \frac{5}{\sqrt{2}} \end{pmatrix}$$ – симм.

$$Q = B^{-1} A \qquad (B | A) \sim (E | B^{-1} A)$$

$$\left(\begin{array}{cc|cc} \frac{5}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 1 & -4 \\ -\frac{3}{\sqrt{2}} & \frac{5}{\sqrt{2}} & 1 & 4 \end{array}\right) \sim \left(\begin{array}{cc|cc} 5 & -3 & \sqrt{2} & -4\sqrt{2} \\ -3 & 5 & \sqrt{2} & 4\sqrt{2} \end{array}\right) \sim \left(\begin{array}{cc|cc} -1 & 7 & 3\sqrt{2} & 4\sqrt{2} \\ -3 & 5 & \sqrt{2} & 4\sqrt{2} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|cc} 1 & -7 & -3\sqrt{2} & -4\sqrt{2} \\ 0 & -16 & -8\sqrt{2} & -8\sqrt{2} \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & -7 & -3\sqrt{2} & -4\sqrt{2} \\ 0 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array}\right) \sim$$

$$\sim \left(\begin{array}{cc|cc} 1 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & 1 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{array}\right)$$

$$A = BQ = \begin{pmatrix} \frac{5}{\sqrt{2}} & -\frac{3}{\sqrt{2}} \\ -\frac{3}{\sqrt{2}} & \frac{5}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} //$$

1178. $x_1^2 - 2x_2^2 + x_3^2 + 2x_1x_2 + 4x_1x_3 + 2x_2x_3 =$

$= (x_1^2 + 2x_1x_2 + 4x_1x_3 + x_2^2 + 2x_3^2 + 4x_2x_3) - 3x_2^2 - x_3^2 - 2x_2x_3 =$

$= \left| \begin{matrix} x_1 + x_2 + 2x_3 = y_1 \\ x_2 = y_2 \\ x_3 = y_3 \end{matrix} \right| = y_1^2 - 3y_2^2 - y_3 - 2y_2y_3 = y_1^2 - (y_3^2 + 2y_2y_3 + y_2^2) - y_2^2 =$

$= \left| \begin{matrix} y_1 = z_1 \\ y_3 + y_2 = z_2 \\ \sqrt{2}y_2 = z_3 \end{matrix} \right| = z_1^2 - z_2^2 - z_3^2$ //

1177. $x_1^2 - 3x_3^2 - 2x_1x_4 + 2x_1x_3 - 6x_2x_3 =$

$= (x_1^2 - 2x_1x_2 + 2x_1x_3 + x_2^2 + x_3^2 - 2x_2x_3) - x_2^2 - 4x_3^2 - 4x_2x_3 =$

$= \left| \begin{matrix} x_1 - x_2 + x_3 = y_1 \\ x_2 = y_2 \\ x_3 = y_3 \end{matrix} \right| = \underbrace{y_1^2}_{= z_1} \underbrace{- y_2^2 - 4y_3 - 4y_2y_3}_{= z_2} = z_1^2 - z_2^2$

1181. $4x_1^2 + x_2^2 + x_3^2 - 4x_1x_2 + 4x_1x_3 - 3x_2x_3 =$

$= (4x_1^2 - 4x_1x_2 + 4x_1x_3 + x_2^2 + x_3^2 - 2x_2x_3) - x_2x_3 = (2x_1 - x_2 + x_3)^2 - x_2x_3 =$

$= \left| \begin{matrix} 2x_1 - x_2 + x_3 = y_1 \\ x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{matrix} \right| = y_1^2 + y_2^2 - y_3^2$ $\quad \begin{matrix} x_1 = \frac{1}{2}y_1 + y_3 \\ x_2 = y_2 - y_3 \\ x_3 = y_2 + y_3 \end{matrix}$

1185. $x_1x_2 + x_2x_3 + x_3x_4 + x_4x_1 = \left| \begin{matrix} x_1 = y_1 - y_2 \\ x_2 = y_1 + y_2 \\ x_3 = y_3 \\ x_4 = y_4 \end{matrix} \right| =$

$= y_1^2 - y_2^2 + y_1y_3 + y_2y_3 + y_3y_4 + y_1y_4 - y_2y_4 =$

$= (y_1^2 + y_1y_3 + y_1y_4 + (\frac{y_3}{2})^2 + (\frac{y_4}{2})^2 + \frac{y_3y_4}{2}) - y_2^2 + y_2y_3 + \frac{y_3y_4}{2} - y_2y_4 =$

$= \left| \begin{matrix} y_1 + \frac{y_3}{2} + \frac{y_4}{2} = z_1 \\ y_2 = z_2 \\ y_3 = z_3 \\ y_4 = z_4 \end{matrix} \right| = z_1^2 - (z_2^2 - z_2z_3 + z_2z_4 + \frac{z_3^2}{4} + \frac{z_4^2}{4} - z_3z_4)$