23.27 $\frac{\partial^{m+n} f}{\partial x^m \partial y^n}$, $m, n \in N$, $f(x,y) = \ln(1+x+y)$, $x+y > -1$

$$\frac{\partial^m f}{\partial x^m} = (-1)^{m-1} \cdot (m-1)! \cdot (1+x+y)^{-m}$$

$$\frac{\partial^n}{\partial y^n}\left(\frac{\partial^m f}{\partial x^m}\right) = \frac{\partial^{m+n} f}{\partial x^m \partial y^n} = (-1)^{m-1}(m-1)! \cdot (-m) \cdot (-m-1) \cdot \ldots \cdot (-m-n+1) \cdot (1+x+y)^{-m-n} =$$

$$= (-1)^{m+n-1} \cdot (m+n-1)! \cdot (1+x+y)^{-(m+n)} \quad //$$

23.32 $\frac{x}{z} = \ln\frac{z}{y} + 1$

$$z'_x = \frac{z}{x+z}, \quad z'_y = \frac{z^2}{y(x+z)}$$

$$z''_{xx} = \frac{-z^2}{(x+z)^3}, \quad z''_{yy} = \frac{-xz^2}{y^2(x+z)^3}, \quad z''_{xy} = \frac{xz^2}{y(x+z)^3}$$

$$\frac{x}{z} = \ln z - \ln y + 1$$

$$\frac{z\,dx - x\,dz}{z^2} = \frac{y}{z}\,dz - dy$$

$$dz = \frac{z(y\,dx + z\,dy)}{y(x+z)} \quad //$$

$$yz\,dx - xy\,dz - yz\,dz + z^2 dy = 0$$

$$d^2z = -\frac{z^2(y\,dx - x\,dy)^2}{y^2(x+z)^3} \quad //$$

$$y(x+z)d^2z = z\,dx\,dy + (z\,dy - x\,dy)dz - y\,dz^2$$

23.38 $f(x,y) = \frac{x}{y}$, $y \neq 0$, $P_0(1,1)$

$$f(x,y) = \frac{1 + (x-1)}{1 + (y-1)}$$

$$(1+(y-1))^{-1} = 1 - (y-1) + (y-1)^2$$

$$f(x,y) = (1 + (x-1))(1 - (y-1) + (y-1)^2) =$$

$$= 1 + (x-1) - (y-1) - (x-1)(y-1) + (y-1)^2 + o(\|h\|^2) \quad //$$

23.42 $f(x,y) = \sin x \cdot \sin y$, $P_0\left(\frac{\pi}{2}, \frac{\pi}{3}\right)$

$$f(x,y) = \left(f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2!} f''(x_0)(x-x_0)^2 + o((x-x_0)^2)\right) \cdot$$

$$\cdot \left(f(y_0) + f'(y_0)(y-y_0) + \frac{1}{2!} f''(y_0) \cdot (y-y_0)^2 + o((y-y_0)^2)\right) =$$

$$= \left(1 - \frac{1}{2}\left(x - \frac{\pi}{2}\right)^2 + o\left(\left(x-\frac{\pi}{2}\right)^2\right)\right) \cdot \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\left(y - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(y - \frac{\pi}{3}\right)^2 + o\left(\left(y-\frac{\pi}{3}\right)^2\right)\right) =$$

$$= \frac{\sqrt{3}}{2} + \frac{1}{2}\left(y - \frac{\pi}{3}\right) - \frac{\sqrt{3}}{4}\left(y - \frac{\pi}{3}\right)^2 - \frac{\sqrt{3}}{4}\left(x - \frac{\pi}{2}\right)^2 + o(\|h\|^2), \text{ где } h = (x-x_0, y-y_0) \quad //$$

24.6 $f(x,y) = xy + \frac{50}{x} + \frac{20}{y}$

$f'_x = y - \frac{50}{x^2} = 0$, $f'_y = x - \frac{20}{y^2} = 0 \Rightarrow x = 5, y = 2$

$f''_{xx} = \frac{100}{x^3}$, $f''_{xy} = 1$, $f''_{yy} = \frac{40}{y^3}$, $\Delta(x,y) = \frac{4000}{x^3 y^3} - 1$

$\Delta(5,2) = 3 > 0$; $A_{11}(5,2) = \frac{4}{5} > 0 \Rightarrow$ в т. $(5,2)$ - min

$f_{min} = 30$ //

24.12 $f(x,y,z) = xy^2z^3(a - x - 2y - 3z)$, $a > 0$, $x, y, z > 0$

$f'_x = y^2z^3(a - 2x - 2y - 3z) = 0$

$f'_y = 2xyz^3(a - x - 3y - 3z) = 0$

$f'_z = 3xy^2z^2(a - x - 2y - 4z) = 0$

$M_1(0, y, z)$, $2y + 3z = a$

$M_1 \in x = 0$, $2y + 3z = a$

$M_2(x, 0, z) \in y = 0$

$M_3(x, y, 0) \in z = 0$

$$\begin{cases} a - 2x - 2y - 3z = 0 \\ a - x - 3y - 3z = 0 \\ a - x - 2y - 4z = 0 \end{cases} \Rightarrow M_0\left(\frac{a}{7}, \frac{a}{7}, \frac{a}{7}\right)$$

$f''_{xx} = -2y^2z^3$, $f''_{yy} = 2xz^3(a - x - 6y - 3z)$, $f''_{zz} = 6xy^2z(a - 2y - x - 8z)$

$f''_{xy} = 2yz^3(a - 2x - 3y - 3z)$, $f''_{yz} = 6xyz^2(a - x - 3y - 4z)$, $f''_{xz} = 3y^2z^2(a - 2x - 2y - 4z)$

у точці M_0: $f''_{xx} = -\frac{2a^5}{7^5}$, $f''_{yy} = -\frac{6a^5}{7^5}$, $f''_{xy} = -\frac{2a^5}{7^5}$, $f''_{xz} = -\frac{3a^5}{7^5}$

$f''_{yz} = -\frac{24a^5}{7^5}$

$\Delta_1 < 0$, $\Delta_2 > 0$, $\Delta_3 < 0$ $f_{max} = \frac{a^7}{7^7}$ //

$M_1: d^2f = -2y^2z^3dx^2 + 4yz^3(a-3y-3z)dxdy + 6y^2z^2(a-2y-4z)dydz$ —

— не знаковизн. форма від dx, dy, dz, тобто в т. M_1 немає екстремуму

$M_2: d^2f = 2xz^3(a-x-3z)dy^2$ при $a-x-3z \neq 0$, $x \neq 0$, $z \neq 0$

екстремум — 0

$M_3: d^3f \neq 0$ в M_3 немає екстремуму

$d^2f = 0$

24.35 $f(x,y) = x^2 - xy + y^2$, $D = \{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \leq 1\}$

$$\begin{cases} f_x' = 2x - y = 0 \\ f_y' = 2y - x = 0 \end{cases} \Rightarrow (0;0)$$

$f(0,0) = 0$

Макс. значення досягається при $|x| + |y| = 1$, тобто в точках $(1,0), (-1,0), (0,1), (0,-1)$

$\min f = f(0,0) = 0$

$\max f = f(\pm 1, 0), f(0, \pm 1) = 1$ //