Білет №7

1. $H_0: m_1 - m_2 = 0$

$H_1: m_1 - m_2 \neq 0$

$\hat{\mu}_1 = 167$

$\hat{S}_1^2 = \frac{1}{4}(38^2 + 17^2 + 33^2 + 57^2 + 7^2) = 1530$

$\hat{\mu}_2 = 176$

$\hat{S}_2^2 = \frac{1}{4}(24^2 + 61^2 + 44^2 + 1^2 + 6^2) = 1567,5$

$\Delta = 9$

$\hat{S} = \sqrt{\frac{4}{8}(\hat{S}_1^2 + \hat{S}_2^2)} = 39,35$

$\frac{\Delta - c}{\hat{S}\sqrt{\frac{10}{25}}} = \frac{9}{39,35 \cdot 0,63} = 0,36 < t_{0,9;8} \Rightarrow$ приймаємо H_0

з рів. 90%

$t_{0,9;8} = 1,397$

2. $F_\xi(x) = \begin{cases} 0, & x < -\frac{\pi}{2} \\ \frac{x+\frac{\pi}{2}}{\pi} & \in [-\frac{\pi}{2}; \frac{\pi}{2}] \\ 1, & x > \frac{\pi}{2} \end{cases}$ $\quad f_\xi(x) = \begin{cases} \frac{1}{\pi}, & x \in [-\frac{\pi}{2}; \frac{\pi}{2}] \\ 0, & x \notin [-\frac{\pi}{2}; \frac{\pi}{2}] \end{cases}$

$F_{|\sin\xi|}(x) = P(|\sin\xi| \le x) = P((\arcsin(-x) \le \xi) \wedge (\xi \le \arcsin x)) =$

$= (1 - P(\xi \le \arcsin(-x)))P(\xi \le \arcsin x)$

$F_{|\sin\xi|}(x) = \begin{cases} 0, & x \in -\frac{\pi}{2} \\ \frac{\pi^2 + 4\pi \arcsin x + 4(\arcsin x)^2}{4\pi^2}, & x \in [-\frac{\pi}{2}; \frac{\pi}{2}] \\ 1, & x > \frac{\pi}{2} \end{cases}$

$f_{|\sin\xi|}(x) = \begin{cases} 0, & x \notin [-\frac{\pi}{2}; \frac{\pi}{2}] \\ \frac{\pi + 2\arcsin x}{\pi^2\sqrt{1-x^2}}, & x \in [-\frac{\pi}{2}; \frac{\pi}{2}] \end{cases}$

$$M\xi = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} x \, \frac{\pi + 2\arcsin x}{\pi^2\sqrt{1-x^2}} \, dx = \frac{-\pi\sqrt{1-x^2} - 2\arcsin x \sqrt{1-x^2} + 2x}{\pi^2}$$

3. $\hat{\theta} = \frac{1}{n-1}\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2$

$$M(\hat{\theta}) = M\left(\frac{1}{n-1}\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2\right) = \frac{n}{n-1} M\left(\frac{1}{n}\sum_{i=1}^{n}(\xi_i - \bar{\xi})^2\right) =$$

$$= \frac{n}{n-1} M\left(\frac{1}{n}\sum_{i=1}^{n}((\xi_i - \mu) - (\bar{\xi} - \mu))^2\right) =$$

$$= \frac{n}{n-1} M\left(\frac{1}{n}\sum_{i=1}^{n}(\xi_i - \mu)^2 - 2(\bar{\xi} - \mu)\frac{1}{n}\sum_{i=1}^{n}(\xi_i - \mu) + (\bar{\xi} - \mu)^2\right) =$$

$$= \frac{n}{n-1} M\left(\frac{1}{n}\sum_{i=1}^{n}(\xi_i - \mu)^2 - (\bar{\xi} - \mu)^2\right) = \frac{n}{n-1}\left(\sigma^2 - \frac{1}{n}\sigma^2\right) = \sigma^2$$

Отже, оцінка $\hat{\theta}$ є незміщеною