146 б) $\cos 8x$

$(\cos x + i\sin x)^n = \cos nx + i\sin nx$

$\sum_{k=0}^{n} i^k C_n^k \cos^{n-k}x \sin^k x$

$(\cos x + i\sin x)^8 = \cos 8x + i\sin 8x$

$\underline{\cos^8 x} + i 8\cos^7 x \sin x \underline{- 28\cos^6 x \sin^2 x} + i 56\cos^5 x \sin^3 x +$

$\underline{+ 70\cos^4 x \sin^4 x} + i 56\cos^3 x \sin^5 x \underline{- 28\cos^2 x \sin^6 x} + i 8\cos x \sin^7 x \underline{+ \sin^8 x}$

$Re: \cos 8x = \cos^8 x - 28\cos^6 x \sin^2 x + 70\cos^4 x \sin^4 x - 28\cos^2 x \sin^6 x + \sin^8 x$ //

149 б) $\sin^4 x = \left(\frac{\alpha - \alpha^{-1}}{2i}\right)^4 = \frac{\alpha^4 - 4\alpha^3\alpha^{-1} + 6\alpha^2\alpha^{-2} - 4\alpha\alpha^{-3} + \alpha^{-4}}{16} =$

$= \frac{\cos 4x - 4\cos 2x + 3}{8}$ //

153 б) $C_n^1 - C_n^3 + C_n^5 - C_n^7 + \dots$

$(1+i)^n = 1 + iC_n^1 - C_n^2 - iC_n^3 + C_n^4 + iC_n^5 - C_n^6 - iC_n^7 + \dots + i^n C_n^n$

$(\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}))^n = 2^{\frac{n}{2}}(\cos\frac{n\pi}{4} + i\sin\frac{n\pi}{4})$

$\overset{Re}{\Rightarrow} C_n^1 - C_n^3 + C_n^5 - C_n^7 + \dots = 2^{\frac{n}{2}}\sin\frac{n\pi}{4}$ //

167. $\sin x + \sin 2x + \dots + \sin nx = \frac{\sin\frac{n+1}{2}x \cdot \sin\frac{nx}{2}}{\sin\frac{x}{2}}$

$\alpha = \cos\frac{x}{2} + i\sin\frac{x}{2}$

$C = \cos x + \cos 2x + \dots + \cos nx$; $S = \sin x + \sin 2x + \dots + \sin nx$

$C + iS = \alpha^2 + \alpha^4 + \alpha^6 + \dots + \alpha^{2n} = \alpha^2 \frac{\alpha^{2n} - 1}{\alpha^2 - 1} =$

$= \alpha^{2-1+n}\frac{\alpha^n - \alpha^{-n}}{\alpha - \alpha^{-1}} = \left(\cos\frac{x(n+1)}{2} + i\sin\frac{x(n+1)}{2}\right)\left(\frac{\cos\frac{nx}{2} + i\sin\frac{nx}{2} - \cos\frac{nx}{2} + i\sin\frac{nx}{2}}{\cos\frac{x}{2} + i\sin\frac{x}{2} - \cos\frac{x}{2} + i\sin\frac{x}{2}}\right)$

$= \frac{\sin\frac{nx}{2}}{\sin\frac{x}{2}}\left(\cos\frac{x(n+1)}{2} + i\sin\frac{x(n+1)}{2}\right)$

$Re\ S = \frac{\sin\frac{n+1}{2}x \cdot \sin\frac{nx}{2}}{\sin\frac{x}{2}}$ //

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544. б) $x^3-3x^2-x-1 \mid 3x^2-2x+1$

$-\; x^3-\frac{2}{3}x^2+\frac{1}{3}x \quad \mid \frac{1}{3}x-\frac{7}{9}$

$-\frac{7}{3}x^2-\frac{2}{3}x-1$

$-\; -\frac{7}{3}x^2+\frac{14}{9}x-\frac{7}{9}$

$-\frac{16}{9}-\frac{2}{9}$

4

549 б) $\cancel{2x^5-5x^3-8x \mid x+3}$ $\cancel{-\;2x^5+6x^4 \mid 2x^4}$

$2x^5-5x^3-8x=f(x)$

| | 2 | 0 | -5 | 0 | -8 | 0 |
|---|---|---|---|---|---|---|
| $\alpha=-3$ | 2 | -6 | 13 | -39 | 109 | -327 |

$f(x)=(x+3)(2x^4-6x^3+13x^2-39x+109)-327$ 4

550 б) $f(x)=x^5+(1+2i)x^4-(1+3i)x^3+7,\ x_0=-2-i$

551. б) $f(x)=x^5,\quad x_0=1$

| | 1 | 0 | 0 | 0 | 0 | 0 | |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | $=a_0$ |
| 1 | 1 | 2 | 3 | 4 | 5 | $=a_1$ | |
| 1 | 1 | 3 | 6 | 10 | $=a_2$ | | |
| 1 | 1 | 4 | 10 | $=a_3$ | | | |
| 1 | 1 | 5 | $=a_4$ | | | | |
| 1 | 1 | $=a_5$ | | | | | |

$f(x)=(x-1)^5+5(x-1)^4+10(x-1)^3+10(x-1)^2+5(x-1)+1$ 4

555. в) $-2,\ f(x)=x^5+7x^4+16x^3+8x^2-16x-16$

ділимо на $x+2$:

| | 1 | 7 | 16 | 8 | -16 | -16 |
|---|---|---|---|---|---|---|
| -2 | 1 | 5 | 6 | -4 | -8 | 0 |
| -2 | 1 | 3 | 0 | -4 | 0 | |
| -2 | 1 | 1 | -2 | 0 | | |
| -2 | 1 | -1 | 0 | | | |
| -2 | 1 | -3 | | | | |
| -2 | 1 | | | | | |

кратність 4 4

547. б) $x^5+x^4-x^3-2x-1, \quad 3x^4+2x^3+x^2+2x-2$

$$\begin{array}{l|l} x^5+x^4-x^3-2x-1 & 3x^4+2x^3+x^2+2x-2 \\ \hline - x^5+\frac{2}{3}x^4+\frac{1}{3}x^3+\frac{2}{3}x^2-\frac{2}{3}x & \frac{1}{3}x+\frac{1}{9} \end{array}$$

$$\frac{1}{3}x^4-\frac{4}{3}x^3-\frac{2}{3}x^2-\frac{4}{3}x-1$$

$$-\ \frac{1}{3}x^4+\frac{2}{9}x^3+\frac{1}{9}x^2+\frac{2}{9}x-\frac{2}{9}$$

$$-\frac{14}{9}x^3-\frac{7}{9}x^2-\frac{14}{9}x-\frac{7}{9} \quad (\times \tfrac{9}{3})$$

$$\begin{array}{l|l} 6x^4+4x^3+2x^2+4x-4 & 2x^3-x^2-2x-1 \\ \hline - 6x^4-3x^3-6x^2-3x & 3x+3,5 \end{array}$$

$$7x^3+8x^2+7x-4$$

$$-\ 7x^3-3,5x^2-7x-3,5$$

$$11,5x^2+14x+7,5$$

598. б) $f_1(x)M_2(x)+f_2(x)M_1(x)=\delta(x)=\text{НОД}(f_1(x), f_2(x))$

$M_1(x), M_2(x)$ - ?

$f_1(x)=x^5+3x^4+x^3+x^2+3x+1$

$f_2(x)=x^4+2x^3+x+2$

$$\begin{array}{l|l} x^5+3x^4+x^3+x^2+3x+1 & x^4+2x^3+x+2 \\ \hline - x^5+2x^4+x^2+2x & x+1 \end{array}$$

$$x^4+x^3+x+1$$

$$-\ x^4+2x^3+x+2$$

$$-x^3-1$$

$$\begin{array}{l|l} x^4+2x^3+x+2 & -x^3-1 \\ \hline - x^4+x & -x-2 \end{array}$$

$$-2x^3+2$$

$$2x^3+2$$

$$0$$

$f_1(x)=f_2(x)(x+1)-x^3-1$

$f_2(x)=(x^3+1)(x+2)$

$\delta(x)=-x^3-1=f_1(x)-f_2(x)(x+1)$ $\quad M_1=-x-1$, $M_2=1$

579. б) $f_1(x)M_2(x)+f_2(x)M_1(x)=1$

$f_1(x)=x^4-x^3-4x^2+4x+1$

$f_2(x)=x^2-x-1$

$$\begin{array}{l|l} x^4-x^3-4x^2+4x+1 & x^2-x-1 \\ \hline -\,x^4-x^3-x^2 & x^2-3 \\ \quad -3x^2+4x+1 & \\ \quad -\,-3x^2+3x+3 & \\ \qquad x-2 & \end{array}$$

$$\begin{array}{l|l} x^2-x-1 & x-2 \\ \hline -\,x^2-2x & x+1 \\ \quad x-1 & \\ \quad -\,x-2 & \\ \qquad 1 & \end{array}$$

$f_1(x)=f_2(x)(x^2-3)+x-2$

$f_2(x)=(x-2)(x+1)+1$

$1=f_2(x)-(x-2)(x+1)=f_2(x)-(f_1(x)-f_2(x)(x^2-3))(x+1)=$

$=f_2(x)-f_1(x)(x+1)+f_2(x)(x^2-3)=f_1(x)(-x-1)+f_2(x)(x^2-2)$

586. б) $x^5-10x^3-20x^2-15x-4=f(x)$

$f'(x)=5x^4-30x^2-40x-15=5(x^4-6x^2-8x-3)$

$d_1(x)=НОД(f(x),f'(x))$

$$\begin{array}{l|l} x^5-10x^3-20x^2-15x-4 & x^4-6x^2-8x-3 \\ \hline -\,x^5-6x^3-8x^2-3x & x \\ \quad -4x^3-12x^2-12x-4 & \end{array}$$

$$\begin{array}{l|l} x^4-6x^2-8x-3 & x^3+3x^2+3x+1 \\ \hline -\,x^4+3x^3+3x^2+x & x-3 \\ \quad -3x^3-9x^2-9x-3 & \\ \quad -\,-3x^3-9x^2-9x-3 & \\ \qquad 0 & \end{array}$$

$d_1(x)=x^3+3x^2+3x+1 \quad d_1'(x)=3x^2+6x+3$

$d_2(x) = НОД(d_1(x), d_1'(x))$

$$\begin{array}{l|l} x^3 + 3x^2 + 3x + 1 & x^2 + 3x + 1 \\ \hline -\,x^3 + 3x^2 + x & x \\ \hline \quad 2x + 1 & \end{array}$$

$$\begin{array}{l|l} x^2 + 3x + 1 & 2x + 1 \\ \hline -\,x^2 + 0{,}5x & 0{,}5x + 1{,}25 \\ \hline \quad 2{,}5x + 1 & \\ -\,2{,}5x + 1{,}25 & \\ \hline \end{array}$$