$$17.2\ \int_{x_0}^{x}\frac{dt}{t(\sqrt{t}+\sqrt[5]{t^2})}=\int_{\sqrt[10]{x_0}}^{\sqrt[10]{x}}\frac{10y^9\,dy}{y^{10}(y^5+y^4)}=10\int_{\sqrt[10]{x_0}}^{\sqrt[10]{x}}\frac{dy}{y^5(y+1)}\ =$$

$$t=y^{10},\ dt=10y^9dy$$

$$\frac{1}{y^5(y+1)}=\frac{A}{y^5}+\frac{B}{y^4}+\frac{C}{y^3}+\frac{D}{y^2}+\frac{E}{y}+\frac{F}{y+1}$$

$$1=A(y+1)+By(y+1)+Cy^2(y+1)+Dy^3(y+1)+Ey^4(y+1)+Fy^5$$

$$1=(E+F)y^5+(E+D)y^4+(D+C)y^3+(C+B)y^2+(A+B)y+A$$

$$A=1,\ B=-1,\ C=1,\ D=-1,\ E=1,\ F=-1$$

$$=\ 10\int_{\sqrt[10]{x_0}}^{\sqrt[10]{x}}\left(\frac{1}{y^5}-\frac{1}{y^4}+\frac{1}{y^3}-\frac{1}{y^2}+\frac{1}{y}-\frac{1}{y+1}\right)dy=$$

$$=10\left(-\frac{1}{4y^4}+\frac{1}{3y^3}-\frac{1}{2y^2}+\frac{1}{y}+\ln y-\ln(y+1)\right)\Bigg|_{\sqrt[10]{x_0}}^{\sqrt[10]{x}}=$$

$$=-\frac{5}{2\sqrt[5]{x^2}}+\frac{10}{3\sqrt[10]{x^3}}-\frac{5}{\sqrt[5]{x}}+\frac{10}{\sqrt[10]{x}}+10\ln\sqrt[10]{x}-10\ln(\sqrt[10]{x}+1)\ //$$

$$17.8\ \int_{x_0}^{x}\frac{3t^3}{\sqrt{t^2+4t+5}}=(Ax^2+Bx+C)\sqrt{x^2+4x+5}+\lambda\int_{x_0}^{x}\frac{dt}{\sqrt{t^2+4t+5}}\ =$$

$$3x^3=(2Ax+B)(x^2+4x+5)+(Ax^2+Bx+C)(x+2)+\lambda$$

$$3x^3=2Ax^3+8Ax^2+10Ax+Bx^2+4Bx+5B+Ax^3+2Ax^2+$$
$$+Bx^2+2Bx+Cx+2C+\lambda$$

$$3x^3=3Ax^3+(10A+2B)x^2+(10A+6B+C)x+(5B+2C+\lambda)$$

$$A=1,\ B=-5,\ C=20,\ \lambda=-15$$

$$=\ (x^2-5x+20)\sqrt{x^2+4x+5}-15\int_{x_0}^{x}\frac{dt}{\sqrt{t^2+4t+5}}=$$

$$=(x^2-5x+20)\sqrt{x^2+4x+5}-15\int_{x_0}^{x}\frac{dt}{\sqrt{(t+2)^2+1}}=$$

$$=(x^2-5x+20)\sqrt{x^2+4x+5}-15\ln|x+2+\sqrt{x^2+4x+5}|\ //$$

17.9 $\int\limits_{x_0}^{x}\frac{dt}{t\sqrt{t^2+4t-4}}=\left|\begin{matrix}\frac{1}{t}=y\\ -\frac{1}{t^2}dt=dy\end{matrix}\right|=-\int\limits_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{dy}{y\sqrt{\frac{1}{y^2}+\frac{4}{y}-4}}=-\operatorname{sgn}x\int\limits_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{dy}{\sqrt{1+4y-4y^2}}=$

$=-\operatorname{sgn}x\int\limits_{\frac{1}{x_0}}^{\frac{1}{x}}\frac{dy}{\sqrt{2-(2y-1)^2}}=\left|\begin{matrix}2y-1=z\\ dy=\frac{dz}{2}\end{matrix}\right|=-\frac{1}{2}\operatorname{sgn}x\int\limits_{\frac{2}{x_0}-1}^{\frac{2}{x}-1}\frac{dz}{\sqrt{2-z^2}}=$

$=-\frac{1}{2}\operatorname{sgn}x\cdot\arcsin\left(\frac{\frac{2}{x}-1}{\sqrt{2}}\right)=-\frac{1}{2}\operatorname{sgn}x\cdot\arcsin\left(\frac{2-x}{x\sqrt{2}}\right)$ //

17.10 $\int\limits_{x_0}^{x}\frac{dt}{(t-1)\sqrt{t^2+t+1}}=\left|\begin{matrix}y=\frac{1}{t-1}\\ t=\frac{1}{y}+1\quad dt=-\frac{dy}{y^2}\end{matrix}\right|=-\int\limits_{\frac{1}{x_0-1}}^{\frac{1}{x-1}}\frac{dy}{y\sqrt{(\frac{1}{y}+1)^2+\frac{1}{y}+2}}=$

$=-\int\limits_{\frac{1}{x_0-1}}^{\frac{1}{x-1}}\frac{dy}{y\sqrt{\frac{1}{y^2}+\frac{3}{y}+3}}=-\int\limits_{\frac{1}{x_0-1}}^{\frac{1}{x-1}}\frac{dy}{y\cdot\frac{1}{|y|}\sqrt{1+3y+3y^2}}=-\operatorname{sgn}x\int\limits_{\frac{1}{x_0-1}}^{\frac{1}{x-1}}\frac{dy}{\sqrt{(\sqrt{3}y+\frac{\sqrt{3}}{2})^2+\frac{1}{4}}}=$

$=\left|\begin{matrix}z=\sqrt{3}y+\frac{\sqrt{3}}{2}\\ dy=\frac{dz}{\sqrt{3}}\end{matrix}\right|=-\frac{\operatorname{sgn}x}{\sqrt{3}}\int\limits_{\frac{\sqrt{3}}{x_0-1}+\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{x-1}+\frac{\sqrt{3}}{2}}\frac{dz}{\sqrt{z^2+\frac{1}{4}}}=-\frac{\operatorname{sgn}}{\sqrt{3}}\cdot\ln\left|z+\sqrt{z^2+\frac{1}{4}}\right|\Bigg|_{\frac{\sqrt{3}}{x_0-1}+\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{x-1}+\frac{\sqrt{3}}{2}}=$

$=-\frac{\operatorname{sgn}x}{\sqrt{3}}\cdot\ln\left|\frac{\sqrt{3}}{x-1}+\frac{\sqrt{3}}{2}+\sqrt{\left(\frac{\sqrt{3}}{x-1}+\frac{\sqrt{3}}{2}\right)^2+\frac{1}{4}}\right|$ //

17.12 $\int\limits_{x_0}^{x}\frac{2dt}{\sqrt{t^2+t+1}-t}=-2\int\limits_{x_0}^{x}\frac{dt}{t-\sqrt{t^2+t+1}}\ =$

Застосуємо I-у підстановку Ейлера

$\sqrt{t^2+t+1}=y-t$

$t+1=y^2-2yt,\quad t=\frac{y^2-1}{1+2y}$

$\sqrt{t^2+t+1}=y-\frac{y^2-1}{1+2y}=\frac{y+2y^2-y^2+1}{1+2y}=\frac{y^2+y+1}{1+2y}$

$dt=\frac{2y+4y^2-2y^2+2}{(1+2y)^2}dy=\frac{2(y^2+y+1)}{(1+2y)^2}dy$

$=-2\int\limits_{x_0+\sqrt{x_0^2+x_0+1}}^{x+\sqrt{x^2+x+1}}\frac{1}{\frac{y^2-1}{1+2y}-\frac{y^2+y+1}{1+2y}}\cdot\frac{2(y^2+y+1)}{(1+2y)^2}dy=$

$=-2\int\limits_{x_0+\sqrt{x_0^2+x_0+1}}^{x+\sqrt{x^2+x+1}}\frac{(1+2y)\cdot2(y^2+y+1)dy}{-(y+2)(1+2y)^2}=4\int\limits_{x_0+\sqrt{x_0^2+x_0+1}}^{x+\sqrt{x^2+x+1}}\frac{y^2+y+1}{(y+2)(1+2y)}dy=4\int\limits_{x_0+\sqrt{x_0^2+x_0+1}}^{x+\sqrt{x^2+x+1}}\left(1-\frac{y^2+4y+1}{2y^2+5y+2}\right)dy=$

$$= 4\int\limits_{x_0+\sqrt{x_0^2+x_0+1}}^{x+\sqrt{x^2+x+1}} \left(1-\left(\frac{1}{2}+\frac{1}{y+2}-\frac{1}{4y+2}\right)\right)dy = 2x + 2\sqrt{x^2+x+1} - 4\ln|x+2+\sqrt{x^2+x+1}| + \ln|2x+1+\sqrt{x^2+x+1}|$$

17.13 $\int\limits_{x_0}^{x} \frac{dt}{1+\sqrt{1-2t-t^2}} =$

застосуємо П-у підстановку Ейлера

$\sqrt{1-2t-t^2} = yt-1$

$1+\sqrt{1-2t-t^2} = yt$

$y^2t^2 = 2yt-2t-t^2$

$y^2t-2y+2+t=0$

$t = \frac{2y-2}{y^2+1}$

$dt = \frac{-2(y^2+4y+2)}{(y^2+1)^2}dy$

$$= \int\limits_{x_0}^{x} \frac{\frac{-2y^2+4y+2}{(y^2+1)^2}}{\frac{2y-2y}{(y^2+1)}}dy = \int\limits_{x_0}^{x} \frac{(-y^2+2y+1)\,dy}{(y^2+1)(y-1)\cdot y} = \int\limits_{x_0}^{x}\left(\frac{A}{y}+\frac{B}{y-1}+\frac{Cy+D}{y^2+1}\right)dy =$$

$-y^2+2y+1 = A(y^3-y^2+y-1)+B(y^3+y)+(Cy+D)(y^2-y)$

$A=-1,\ B=1,\ C=0,\ D=2$

$$= \int\left(-\frac{1}{y}+\frac{1}{y-1}+\frac{2}{y^2+1}\right)dy = \ln\left|\frac{\frac{1+\sqrt{1-2x-x^2}}{x}-1}{\frac{1+\sqrt{1-2x-x^2}}{x}}\right| - 2\,\mathrm{arctg}\left(\frac{1+\sqrt{1-2x-x^2}}{x}\right)$$

17.4 $\int\limits_{x_0}^{x} t^{-1}\left(1+t^{\frac{1}{3}}\right)^{-3}dt = \left|\begin{matrix} y=1+t^{\frac{1}{3}} \\ dy=\frac{1}{3}\cdot t^{-\frac{2}{3}}dt\end{matrix}\right| =$

$$= 3\int\limits_{1+\sqrt[3]{x_0}}^{1+\sqrt[3]{x}} \frac{dy}{y^3(y-1)} = 3\int\limits_{1+\sqrt[3]{x_0}}^{1+\sqrt[3]{x}}\left(-\frac{1}{y^3}-\frac{1}{y^2}-\frac{1}{y}+\frac{1}{y-1}\right) =$$

$\frac{1}{y^3(y-1)} = \frac{A}{y^3}+\frac{B}{y^2}+\frac{C}{y}+\frac{D}{y-1}$

$1 = (C+D)y^3+(B-C)y^2+(A-B)y-A$

$A=-1,\ B=-1,\ C=-1,\ D=1$

$= \frac{3}{2(1+\sqrt[3]{x})^2} + \frac{3}{1+\sqrt[3]{x}} - 3\ln|1+\sqrt[3]{x}| + \ln x$

18.2 $\int_{x_0}^{x} \sin^2 t \cdot \cos^3 t \, dt = \left| \begin{array}{l} y = \sin t \\ dy = \cos t \cdot dt \end{array} \right| = \int_{\sin x_0}^{\sin x} y^2 (1-y^2) \, dy = \int_{\sin x_0}^{\sin x} y^2 dy - \int_{\sin x_0}^{\sin x} y^4 dy =$

$= \frac{y^3}{3} \Big|_{\sin x_0}^{\sin x} - \frac{y^5}{5} \Big|_{\sin x_0}^{\sin x} = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5}$ //

18.6 $\int_{x_0}^{x} \frac{1}{\cos t} dt = \int_{x_0}^{x} \frac{\cos t \, dt}{\cos^2 t} = \int_{x_0}^{x} \frac{\cos t \, dt}{1 - \sin^2 t} = \left| \begin{array}{l} y = \sin t \\ dy = \cos t dt \end{array} \right| = \int_{\sin x_0}^{\sin x} \frac{dy}{1-y^2} =$

$= -\frac{1}{2} \ln \left| \frac{y-1}{y+1} \right| \Big|_{\sin x_0}^{\sin x} = -\frac{1}{2} \ln \left| \frac{\sin x - 1}{\sin x + 1} \right|$ //

18.8 $\int_{x_0}^{x} \frac{dt}{5 - 3\sin t} = \left| \begin{array}{ll} y = \operatorname{tg} \frac{t}{2} & dt = \frac{2dy}{1+y^2} \\ \sin t = \frac{2y}{1+y^2} & \end{array} \right| = 2 \int_{\operatorname{tg} \frac{x_0}{2}}^{\operatorname{tg} \frac{x}{2}} \frac{dy}{(1+y^2)\left(\frac{5y^2+5-6y}{1+y^2}\right)} =$

$= 2 \int_{\operatorname{tg} \frac{x_0}{2}}^{\operatorname{tg} \frac{x}{2}} \frac{dy}{5y^2 - 6y + 5} = 2 \int_{\operatorname{tg} \frac{x_0}{2}}^{\operatorname{tg} \frac{x}{2}} \frac{dy}{5\left((y - \frac{3}{5})^2 + \frac{16}{25}\right)} = \frac{2}{5} \int_{\operatorname{tg} \frac{x_0}{2} - \frac{3}{5}}^{\operatorname{tg} \frac{x}{2} - \frac{3}{5}} \frac{du}{u^2 + \frac{16}{25}} =$

$= \frac{1}{2} \operatorname{arctg} \frac{5 \operatorname{tg} \frac{x}{2} - 3}{4}$ ///