1) $f_\xi(x) = a e^{-\lambda|x|}, a>0, \lambda>0$

$\int_{-\infty}^{\infty} f_\xi(x)dx = 1;$

$\int_{-\infty}^{\infty} a e^{-\lambda|x|} dx = a\int_{-\infty}^{\infty} e^{-\lambda|x|}dx = 2a\int_0^{\infty} e^{-\lambda x}dx = -\frac{2a}{\lambda} e^{-\lambda|x|}\Big|_0^{\infty} =$

$= -\frac{2a}{\lambda}(0-1) = \frac{2a}{\lambda} = 1 \Rightarrow a = \frac{\lambda}{2};$ //

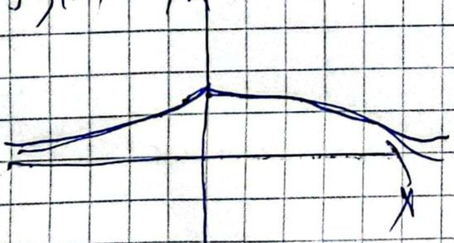
$F_\xi(x) = \int_{-\infty}^{x} f_\xi(u)$

$F_\xi(x) = \int_{-\infty}^{x} \frac{\lambda}{2} e^{-\lambda|x|} du = \frac{\lambda}{2}\int_{-\infty}^{x} e^{-\lambda|x|} du = \frac{1}{2}(1-e^{-\lambda x})$

$$\begin{cases} \frac{1}{2}(1-e^{-\lambda x}), x \geq 0 \\ \frac{1}{2}(1+e^{\lambda x}), x<0 \end{cases}$$ //

$M\xi = \int_{-\infty}^{\infty} x \frac{\lambda}{2} e^{-\lambda|x|} dx = \frac{\lambda}{2}\int_{-\infty}^{\infty} x e^{-\lambda|x|}dx = \frac{\lambda}{2}\left(\int_{-\infty}^{0}(\_) + \int_0^{\infty}(\_)\right) = 0 = \mu$

$D\xi = \int_{-\infty}^{\infty}(x-\mu)^2 \cdot f_\xi(x)dx = \int_{-\infty} x^2 f_\xi(x)dx = \int_{-\infty} x^2 \cdot \frac{\lambda}{2} e^{-\lambda|x|} dx = -x^2$ //



2) $f_\xi(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \\ 0, & x < 0 \end{cases}$

$M\xi = \int_{-\infty}^{\infty} x f_\xi(x)\,dx = \int_0^{\infty} x \lambda e^{-\lambda x}\,dx = \left[\begin{matrix} u = x, \int dv = \int \lambda e^{-\lambda x}dx \\ du = dx, v = -e^{-\lambda x} \end{matrix}\right] =$

$= -x \cdot e^{-\lambda x}\Big|_0^\infty - \frac{1}{\lambda}\int_0^\infty e^{-\lambda x} d(-x\cdot\lambda) = -\frac{1}{\lambda} e^{-\lambda x}\Big|_0^\infty = \frac{1}{\lambda} - \frac{1}{\lambda}\lim_{x\to\infty}\frac{1}{e^{\lambda x}} = \frac{1}{\lambda}$ //

$M\xi = \frac{1}{\lambda}$ //

$D\xi = \int_{-\infty}^{\infty} x^2 f_\xi(x)\,dx - \frac{1}{\lambda^2} = \int_0^\infty x^2 \lambda e^{-\lambda x} dx - \frac{1}{\lambda} = \left[\begin{matrix} u = x^2, dv = \lambda e^{-\lambda x}dx \\ du = 2x\,dx, v = e^{-\lambda x} \end{matrix}\right] =$

$= -x^2 \cdot e^{-\lambda x}\Big|_0^\infty + \int_0^\infty 2x e^{-\lambda x} dx - \frac{1}{\lambda^2} = \left[\begin{matrix} x = u, dv = \frac{1}{\lambda}e^{-\lambda x}dx \\ dx = du, v = -\frac{1}{\lambda}e^{-\lambda x} \end{matrix}\right] =$

$= -2x\frac{1}{\lambda}e^{-\lambda x}\Big|_0^\infty - 2\frac{1}{\lambda^2}\int_0^\infty e^{-\lambda x} d(-x\lambda) - \frac{1}{\lambda^2} = -\frac{2}{\lambda^2}e^{-\lambda x}\Big|_0^\infty = \frac{2}{\lambda^2} - \lim_{x\to\infty}\frac{2}{\lambda^2}\frac{1}{e^{\lambda x}} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$

$D\xi = \frac{1}{\lambda^2}$ //

3) $N(m, \sigma^2)$, $M\xi = m$, $D\xi = \sigma^2$

$M\xi = \int_{-\infty}^{\infty} x f(x)\,dx = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} x e^{-\frac{(x-m)^2}{2\sigma^2}} dx = \left[\frac{x-m}{\sigma} = z;\ x = m + z\sigma;\ dx = \sigma dz\right] =$

$= \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}(m + \sigma z)e^{-\frac{z^2}{2}}dz\,\sigma = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}dz + \frac{\sigma}{\sqrt{2\pi}}\int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}}dz =$

$$\boxed{\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{z^2}{2}}dz = 1,\ \int_{-\infty}^{\infty} z e^{-\frac{z^2}{2}}dz = 0} = m \ \square$$

$D\xi = \int_{-\infty}^{\infty}(x - M\xi)^2 f(x)\,dx = \frac{1}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty}(x-m)^2 e^{-\frac{(x-m)^2}{2\sigma}} dx =$

$= \left[\frac{x-m}{\sqrt{2}\sigma} = t;\ x = m + t\sqrt{2}\sigma;\ dx = dt\sqrt{2}\sigma;\ (x-m)^2 = 2t^2\sigma^2\right] =$

$= \frac{2\sigma^2}{\sqrt{2\pi}\sigma}\int_{-\infty}^{\infty} t^2 e^{-t^2} dt\sqrt{2}\sigma = \frac{\sigma^2}{\sqrt{\pi}}\int_{-\infty}^{\infty} t\, 2t e^{-t^2} dt = \left[\begin{matrix} t = u;\ dt = du; \\ 2te^{-t^2}dt = dv;\ v = -e^{-t^2} \end{matrix}\right] =$

$= \frac{\sigma^2}{\sqrt{\pi}}\left[-te^{-t^2}\Big|_{-\infty}^{+\infty} + \int_{-\infty}^{\infty} e^{-t^2}dt\right] = \sigma^2 \ \square$