754. $\bar{a} = \{12; -15; -16\}$

$|\bar{a}| = \sqrt{12^2 + (-15)^2 + (-16)^2} = \sqrt{625} = 25$

$\cos\alpha = \frac{x}{|a|} = \frac{12}{25}$ //

$\cos\beta = \frac{y}{|a|} = -\frac{15}{25} = -\frac{3}{5}$ //

$\cos\gamma = \frac{z}{|a|} = -\frac{16}{25}$ //

759. $\alpha = 60°, \beta = 120°, |\bar{a}| = 2$

$x = |\bar{a}| \cdot \cos\alpha = 2 \cdot 0,5 = 1$

$y = |\bar{a}| \cdot \cos\beta = 2 \cdot (-0,5) = -1$

$\sqrt{1 + 1 + z^2} = 2;$

$2 + z^2 = 4;$

$z^2 = 2; \Rightarrow z = \pm\sqrt{2}$

Від. $\bar{a} = (1; -1; \sqrt{2})$ або $\bar{a} = (1; -1; -\sqrt{2})$.

778. $A(3; -1; 2)$, $B(1; 2; -1)$, $C(-1; 1; -3)$, $D(3; -5; 3)$

$\overline{AB} = \{-2; 3; -3\}$, $\overline{BC} = \{-2; -1; -2\}$,

$\overline{CD} = \{4; -6; 6\}$, $\overline{AD} = \{0; -4; 1\}$

$\overline{AB} \parallel \overline{CD}$: $\frac{-2}{4} = \frac{3}{-6} = \frac{-3}{6} = -\frac{1}{2}$

$\overline{BC} \nparallel \overline{AB}$: $\frac{-2}{-2} \neq \frac{-1}{3} \neq \frac{-2}{3}$

$\overline{AD} \nparallel \overline{AB}$: $\frac{0}{-2} \neq \frac{-4}{3} \neq \frac{1}{-3}$

$\overline{AD} \nparallel \overline{BC}$: $\frac{0}{-2} \neq \frac{-4}{-1} \neq \frac{1}{-2}$

A, B, C, D є вершинами трапеції

Scanned with CamScanner

788. $\bar{a} = \{3; -5; 8\}$, $\bar{b} = \{-1; 1; -4\}$

$\bar{a} + \bar{b} = \{2; -4; 4\}$

$|\bar{a} + \bar{b}| = \sqrt{4 + 16 + 16} = \sqrt{36} = 6$

$\bar{a} - \bar{b} = \{4; -6; 12\}$

$|\bar{a} - \bar{b}| = \sqrt{16 + 36 + 144} = \sqrt{196} = 14$

Відп. $|\bar{a} + \bar{b}| = 6$; $|\bar{a} - \bar{b}| = 14$

789. $\bar{a} = \{3; -1\}$, $\bar{b} = \{1; -2\}$, $\bar{c} = \{-1; 7\}$, $\bar{p} = \bar{a} + \bar{b} + \bar{c}$

$\bar{p} = \{3 + 1 - 1; -1 - 2 + 7\} = \{3; 4\}$

$$\begin{cases} 3\alpha + \beta = 3 \quad | \cdot 2 \\ -\alpha - 2\beta = 4 \end{cases} \quad +$$

$5\alpha = 10$ $\alpha = 2$, $\beta = -3$ $\boxed{\bar{p} = 2\bar{a} - 3\bar{b}}$

791. $A(1; -2)$, $B(2; 1)$, $C(3; 2)$, $D(-2; 3)$

$\overline{AB} = \{1; 3\}$ $\overline{AC}\ \{2; 4\}$

$\overline{AD} = \{-3; 5\}$

$$\begin{cases} \alpha + 2\beta = -3 \quad | \cdot 2 \\ 3\alpha + 4\beta = 5 \end{cases} \quad -$$

$-\alpha = -11$ $\alpha = 11$, $\beta = -7$

$\boxed{\overline{AD} = 11\overline{AB} - 7\overline{AC}}$

$\overline{BD} = \{-4; 2\}$

$$\begin{cases} \alpha + 2\beta = -4 \quad | \cdot 2 \\ 3\alpha + 4\beta = 2 \end{cases} \quad -$$

$-\alpha = -10$ $\alpha = 10$, $\beta = -7$

$\boxed{\overline{BD} = 10\overline{AB} - 7\overline{AC}}$

$\overline{CD} = \{-5; 1\}$

$$\begin{cases} \alpha + 2\beta = -5 \quad | \cdot 2 \\ 3\alpha + 4\beta = 1 \end{cases} \quad -$$

$-\alpha = -11$ $\alpha = 11$, $\beta = -8$

$\boxed{\overline{CD} = 11\overline{AB} - 8\overline{AC}}$

$\overline{AD} + \overline{BD} + \overline{CD} = \{-12; 8\}$

$\begin{cases} \alpha + 2\beta = -12 \;|\cdot 2 \\ -2\alpha + 4\beta = 8 \end{cases} \Big|-$

$-\alpha = -32 \quad \alpha = 32, \beta = -22$

$\boxed{\overline{AD} + \overline{BD} + \overline{CD} = 32\overline{AB} - 22\overline{AC}}$

796. $\varphi = \frac{2\pi}{3}$, $|\bar a| = 3$, $|\bar b| = 4$

1) $\bar a \cdot \bar b = 3 \cdot 4 \cdot (-\frac{1}{2}) = -6$;

2) $\bar a^2 = |\bar a|^2 = 9$;

3) $\bar b^2 = |\bar b|^2 = 16$;

4) $(\bar a + \bar b)^2 = \bar a^2 + 2\bar a\bar b + \bar b^2 = 9 - 12 + 16 = 13$;

5) $(3\bar a - 2\bar b)(\bar a + 2\bar b) = 3\bar a^2 + 6\bar a\bar b - 2\bar a\bar b - 4\bar b^2 =$

$= 3\bar a^2 + 4\bar a\bar b - 4\bar b^2 = 27 - 24 - 64 = -61$;

6) $(\bar a - \bar b)^2 = (\bar a - \bar b)(\bar a - \bar b) = \bar a^2 - 2\bar a\bar b + \bar b^2 =$

$= 9 + 12 + 16 = 37$;

7) $(3\bar a + 2\bar b)^2 = 9\bar a^2 + 12\bar a\bar b + 4\bar b = 81 - 72 + 64 = 73$.

801. $\varphi = 60°$, $|\bar a| = 4$, $|\bar b| = 2$, $|\bar c| = 6$

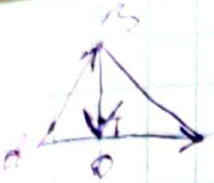
$\bar a^2 = 16$, $\bar b^2 = 4$, $\bar c^2 = 36$

$\bar a\bar b = 4$, $\bar b\bar c = 6$, $\bar a\bar c = 12$

$|\bar p| = \sqrt{(\bar a + \bar b + \bar c)^2} = \sqrt{\bar a^2 + \bar b^2 + \bar c^2 + 2\bar a\bar b + 2\bar b\bar c + 2\bar a\bar c} =$

$= \sqrt{16 + 4 + 36 + 8 + 12 + 24} = 10$;

Від. $|\bar p| = 10$.

804.  $\overline{AB} = \overline{b},\ \overline{AC} = \overline{c},\ \overline{BD} \perp \overline{AC}$

$\overline{BD} = k\overline{c} - \overline{b}$

$(k\overline{c} - \overline{b}) \cdot \overline{c} = 0 \Rightarrow k = \dfrac{\overline{b}\overline{c}}{\overline{c}^2}$

$$\boxed{\overline{BD} = \frac{\overline{b}\overline{c}}{\overline{c}^2} \cdot \overline{c} - \overline{b}}$$

814. $A(-1;3;-7),\ B(2;-1;5),\ C(0;1;-5)$

$\overline{AB} = \{3;-4;12\}$ $\overline{BA} = \{-3;4;-12\}$

$\overline{BC} = \{-2;2;-10\}$ $\overline{CB} = \{2;-2;10\}$

$\overline{AC} = \{1;-2;2\}$

1) $(2\overline{AB} - \overline{CB})(2\overline{BC} + \overline{BA}) = \{6-2;-8+2;24-10\} \times$

$\times \{-4-3;4+4;-20+12\} = \{4;-6;14\} \cdot \{-7;8;-32\} =$

$= -28 - 48 - 448 = -524;$

2) $\sqrt{\overline{AB}^2} = \sqrt{9+16+144} = 13;$

3) $\sqrt{\overline{AC}^2} = \sqrt{1+4+4} = 3;$

4) $(\overline{AB} \cdot \overline{AC})\overline{BC} = (3+8+24) \cdot \{-2;2;-10\} = \{-70;70;-350\};$

$\overline{AB}(\overline{AC} \cdot \overline{BC}) = \{3;-4;12\} \cdot (-2-4-20) = \{-78;104;-312\}.$

828. $\overline{a} = 2\overline{i} - \overline{j} + 3\overline{k},\ \overline{b} = \overline{i} - 3\overline{j} + 2\overline{k},\ \overline{c} = 3\overline{i} + 2\overline{j} - 4\overline{k}$

$\overline{x}\overline{a} = -5,\ \overline{x}\overline{b} = -11,\ \overline{x}\overline{c} = 20$

$\overline{x} = \{x;y;z\}$

$$\begin{cases} 2x - y + 3z = -5 \\ x - 3y + 2z = -11 \\ 3x + 2y - 4z = 20 \end{cases} \Rightarrow \begin{cases} 5y - z = 17 \Rightarrow z = 5y - 17 \\ -7y + 17z = -55 \end{cases}$$

$-7y+85y-289=-55$

$78y=234$

$y=3,\quad z=15-17=-2,\quad x=3\cdot3-2\cdot(-2)-11=2$

$\boxed{\text{Від. } \bar{x}=2\bar{i}+3\bar{j}-2\bar{k}.}$

838. $A(-2;3;-4)$, $B(3;2;5)$, $C(1;-1;2)$, $D(3;2;-4)$.

$\overline{AB}=\{5;-1;9\}$, $\overline{CD}=\{2;3;-6\}$

$\text{пр.}_{\overline{CD}}\overline{AB}=\dfrac{(\overline{AB},\overline{CD})}{|\overline{CD}|}=\dfrac{10-3-54}{\sqrt{4+9+36}}=\dfrac{-47}{7}$.

$\boxed{\text{Від. } -\frac{47}{7}.}$

1205. 1) $\begin{vmatrix}2 & x-4\\1 & 4\end{vmatrix}=0$;

$8-x+4=0$;

$x=12$;

2) $\begin{vmatrix}1 & 4\\3x & x+22\end{vmatrix}=0$;

$x+22-12x=0$;

$-11x=-22$;

$x=2$;

3) $\begin{vmatrix}x & x+1\\-4 & x+1\end{vmatrix}=0$;

$x^2+x+4x+4=0$;

$x^2+5x+4=0$;

$x_1=-1,\ x_2=-4$;

4) $\begin{vmatrix}3x & -1\\x & 2x-3\end{vmatrix}=\frac{3}{2}$;

$6x^2-9x+x=\frac{3}{2}$;

$12x^2-16x-3=0$;

$x_1=-\frac{1}{6}$; $x_2=\frac{3}{2}$;

5) $\begin{vmatrix}x+1 & -5\\1 & x-1\end{vmatrix}=0$;

$x^2-1+5=0$;

$x^2=-4$;

$x_{1,2}=\pm 2i$;

6) $\begin{vmatrix}x^2-4 & -1\\x-4 & x+2\end{vmatrix}=0$;

$x^3+2x^2-4x-8+x-4=0$;

$x^3+2x^2-3x-12=0$;

$x=2$;

7) $\begin{vmatrix}4\sin x & 1\\1 & \cos x\end{vmatrix}=0$;

$4\sin x\cdot\cos x-1=0$;

$2\sin(2x)=1$;

$\sin(2x)=\frac{1}{2}$;

$x=\frac{\pi}{12}+k\pi,\ k\in Z$

$x=\frac{5\pi}{12}+k\pi,\ k\in Z$

8) $\begin{vmatrix}\cos 8x & -\sin 5x\\ \sin 8x & \cos 5x\end{vmatrix}=0$;

$\cos 8x\cdot\cos 5x+\sin 5x\cdot\sin 8x=0$;

$\cos 3x=0$;

$x=\frac{\pi}{6}+\frac{k\pi}{3},\ k\in Z$

1212. $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 5 & 0 & -1 \end{vmatrix} = -1+0+30-0-0-0 = 29.$

1235. 1) $\begin{vmatrix} 3 & -2 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{vmatrix} < 1;$

$-3x \cancel{+2} - 4 + \cancel{x} \cancel{-2} + 12 < 1;$

$-2x + 8 < 1;$

$-2x < -7;$

$x > 3{,}5;$

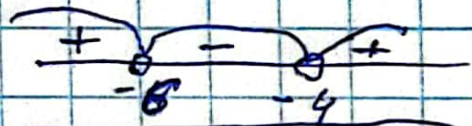
$\boxed{x \in (3{,}5; +\infty)}$

2) $\begin{vmatrix} 2 & x+2 & -1 \\ 1 & 1 & -2 \\ 5 & -3 & x \end{vmatrix} > 0;$

$\cancel{2x} - 10x - 20 + 3 + 5 - x^2 \cancel{-2x} - 12 > 0;$

$-x^2 - 10x - 24 > 0;$

$x^2 + 10x + 24 < 0;$



$\boxed{x \in (-6; -4)}$