1) $S^2 = \dfrac{\sum_{i=1}^{n} (\xi_i - \bar{\xi})^2}{n}$, $MS^2 = \dfrac{n-1}{n}\sigma^2$

$$S^2 = \frac{\sum_{i=1}^{n} (\xi_i - \bar{\xi})^2}{n} = \frac{\sum_{i=1}^{n} (\xi_i - a)^2}{n} - (\bar{\xi} - a)^2 = \frac{(n\bar{\xi} - na)^2}{n} - (\bar{\xi} - a)^2 =$$

$$= (n-1)(\bar{\xi} - a)^2 \Rightarrow MS^2 = \frac{n-1}{n}\sigma^2 \,//$$

2) $P(\xi = k) = \dfrac{\theta^k}{(1+\theta)^{k+1}}$, $\theta > 0$, $k = 0, 1, \dots$, $\hat{\theta} = \dfrac{\sum_{i=1}^{n} \xi_i}{n}$

1. $M\hat{\theta} = M\dfrac{\sum_{i=1}^{n} \xi_i}{n} = \theta$

2. $D\hat{\theta} = D\dfrac{1}{n}\sum_{i=1}^{n} \xi_i = \dfrac{1}{n^2}\sum_{i=1}^{n} D\xi_i = \dfrac{n\theta}{n^2} = \dfrac{\theta}{n}$

3. $L(x, \theta) = \prod_{k=1}^{n} P(\xi = x_k) = \prod_{k=1}^{n} \dfrac{\theta^{x_k}}{(1+\theta)^{x_k+1}} = \dfrac{\theta^{\sum_{k=1}^{n} x_k}}{(1+\theta)^{\sum_{k=1}^{n} x_k + n}}$

$$\ln L(x, \theta) = \sum_{k=1}^{n} x_k \ln\theta - \left(\sum_{k=1}^{n} x_k + n\right)\ln(1+\theta)$$

$$\frac{\partial}{\partial\theta}\ln L(x, \theta) = \frac{\sum_{k=1}^{n} x_k (1+\theta) - \left(\sum_{k=1}^{n} x_k + n\right)\theta}{\theta + \theta^2} = \frac{\sum_{k=1}^{n} x_k - n\theta}{\theta + \theta^2}$$

$$I(\theta) = -M\frac{\partial^2}{\partial\theta^2}\ln L(\xi, \theta) = M\frac{\sum_{k=1}^{n} \xi_k}{(\theta+\theta^2)^2} = \frac{n\theta}{(\theta+\theta^2)^2} = \frac{n}{\theta(1+\theta)^2}$$

4. $D\hat{\theta} > (I(\theta))^{-1} \Rightarrow$ оцінка є ефективною //