ImageWarping

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1 目的

实现 IDW 和 RBF 方法。

2 算法原理

我们的问题是：给定 n 对控制点 $(\mathbf{p}_i, \mathbf{q_i})$，$\mathbf{p}_i, \mathbf{q}_i \in \mathbb{R}^2$，$i = 1, \ldots, n$
得到一个函数 $\mathbf{f} : \mathbb{R}^2 \to \mathbb{R}^2$，满足插值条件，即 $\mathbf{f}(\mathbf{p}_i) = \mathbf{q}_i, i = 1, \ldots, n$

2.1 Inverse distance-weighted interpolation methods(IDW)

我们设局部插值函数 $\mathbf{f}_i(\mathbf{p}) : \mathbb{R}^2 \to \mathbb{R}^2$ 满足 $f_i(\mathbf{p}_i) = \mathbf{q}_i$，具体为

$$\mathbf{f}_i(\mathbf{p}) = \mathbf{q}_i + \mathbf{D}_i(\mathbf{p} - \mathbf{p}_i)$$

其中 $\mathbf{D}_i : \mathbb{R}^2 \to \mathbb{R}^2$，满足 $\mathbf{D}_i(\mathbf{0}) = \mathbf{0}$
总的插值函数为

$$\mathbf{f}(\mathbf{x}) = \sum_{i=1}^{n} w_i(\mathbf{x}) \mathbf{f}_i(\mathbf{x})$$

其中 $w_i : \mathbb{R}^2 \to \mathbb{R}$，为

$$w_i(\mathbf{x}) = \frac{\sigma_i(\mathbf{x})}{\sum_{j=1}^{n} \sigma_j(\mathbf{x})}$$

$$\sigma_i(\mathbf{x}) = \frac{1}{\|\mathbf{x} - \mathbf{p}_i\|^{\mu}}$$

其中 $\mu > 1$, 一般我们取 $\mu = 2$。
定义能量

$$\begin{aligned} E_i(\mathbf{D}_i) &= \sum_{j=1, j\neq i}^{n} w_i(\mathbf{p}_j) \left\| \mathbf{q}_i + \begin{pmatrix} d_{i,11} & d_{i,12} \\ d_{i,21} & d_{i,22} \end{pmatrix} (\mathbf{p}_j - \mathbf{p}_i) - \mathbf{q}_j \right\|^2 \\ &= \sum_{j=1, j\neq i}^{n} w_i(\mathbf{p}_j)((d_{i,11}(p_{j,0} - p_{i,0}) + d_{i,12}(p_{j,1} - p_{i,1}) + q_{i,0} - q_{j,0})^2 \\ &+ (d_{i,21}(p_{j,0} - p_{i,0}) + d_{i,22}(p_{j,1} - p_{i,1}) + q_{i,1} - q_{j,1})^2) \end{aligned} \tag{1}$$

最小化该能量可求得 $\mathbf{D}_i$, 即

$$\frac{\partial E_i}{\partial d_{i,11}} = 2\sum_{j=1,j\neq i}^{n} w_i(\mathbf{p}_j)(d_{i,11}(p_{j,0}-p_{i,0}) + d_{i,12}(p_{j,1}-p_{i,1}) + q_{i,0} - q_{j,0})(p_{j,0}-p_{i,0}) = 0$$

$$\frac{\partial E_i}{\partial d_{i,12}} = 2\sum_{j=1,j\neq i}^{n} w_i(\mathbf{p}_j)(d_{i,11}(p_{j,0}-p_{i,0}) + d_{i,12}(p_{j,1}-p_{i,1}) + q_{i,0} - q_{j,0})(p_{j,1}-p_{i,1}) = 0$$

$$\frac{\partial E_i}{\partial d_{i,21}} = 2\sum_{j=1,j\neq i}^{n} w_i(\mathbf{p}_j)(d_{i,21}(p_{j,0}-p_{i,0}) + d_{i,22}(p_{j,1}-p_{i,1}) + q_{i,1} - q_{j,1})(p_{j,0}-p_{i,0}) = 0$$

$$\frac{\partial E_i}{\partial d_{i,22}} = 2\sum_{j=1,j\neq i}^{n} w_i(\mathbf{p}_j)(d_{i,21}(p_{j,0}-p_{i,0}) + d_{i,22}(p_{j,1}-p_{i,1}) + q_{i,1} - q_{j,1})(p_{j,1}-p_{i,1}) = 0$$

2.2 Radial basis functions interpolation method(RBF)

对于给定 n 对控制点 $(\mathbf{p}_i, \mathbf{q_i})$，$\mathbf{p}_i, \mathbf{q}_i \in \mathbb{R}^2$，$i = 1, \ldots, n$，我们要求的插值函数为

$$\boldsymbol{f}(\boldsymbol{p}) = \sum_{i=1}^{n} \boldsymbol{\alpha}_i R(\|\mathbf{p} - \mathbf{p}_i\|) + A\mathbf{p} + \mathbf{b}$$

其中权重系数 $\boldsymbol{\alpha}_i \in \mathbb{R}^2$，$A \in \mathbb{R}^{2\times 2}$，$\mathbf{b} \in \mathbb{R}^2$，径向基函数 $R(d) = (d^2 + r^2)^{\mu/2}$，一般 μ 取 ± 1 .

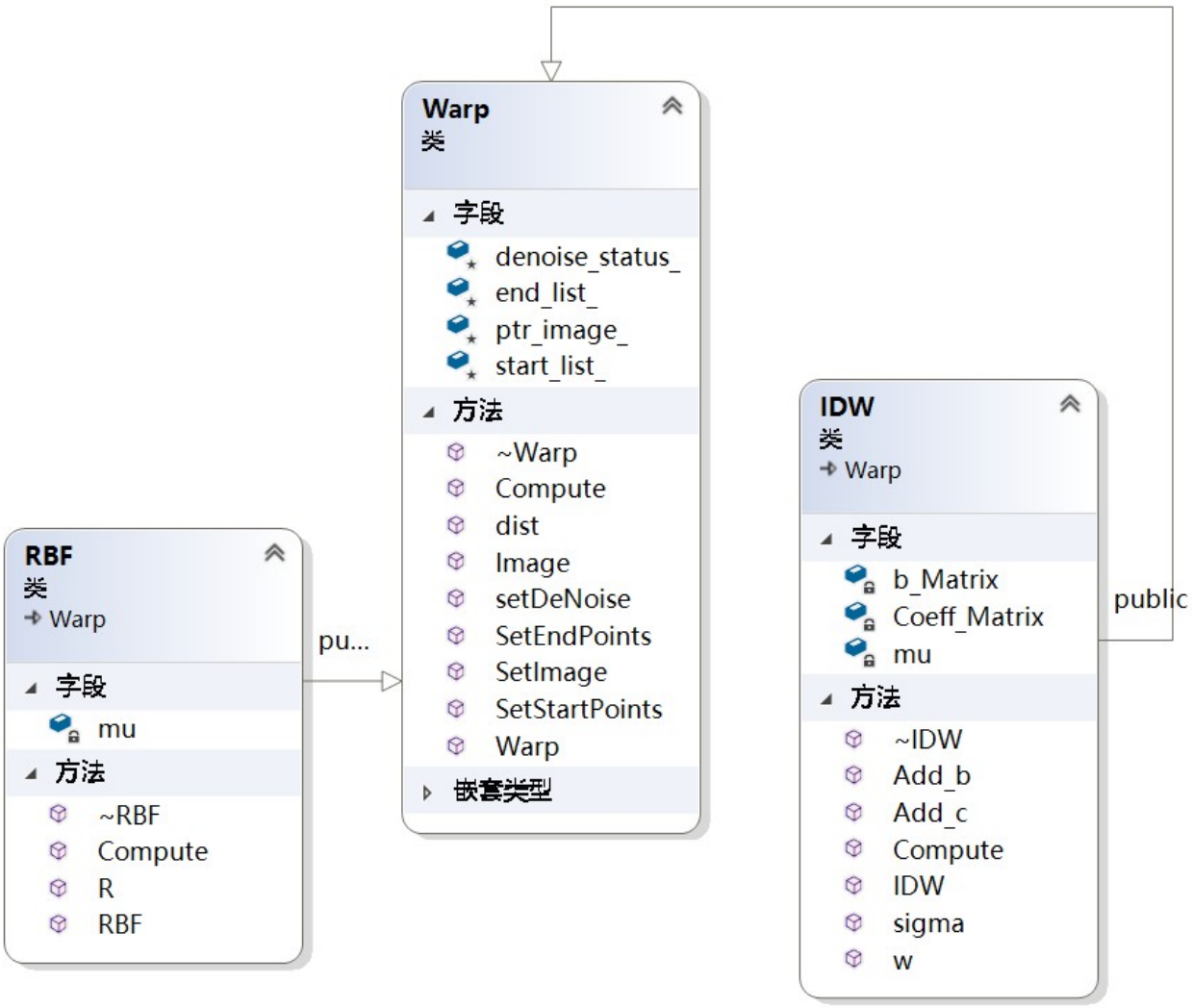
我们要求满足插值条件

$$\mathbf{f}(\mathbf{p}_j) = \sum_{i=1}^{n} \boldsymbol{\alpha}_i R(\|\mathbf{p}_j - \mathbf{p}_i\|) + A\mathbf{p}_j + \mathbf{b} = \mathbf{q}_j, \quad j = 1, \ldots, n$$

由于自由度每维有 $n+3$ 个，我们选取补充约束为

$$\begin{bmatrix} \mathbf{p}_1 \cdots \mathbf{p}_n \\ 1 \cdots 1 \end{bmatrix}_{3\times n} \begin{bmatrix} \boldsymbol{\alpha}_1 \\ \vdots \\ \boldsymbol{\alpha}_n \end{bmatrix}_{n\times 2} = \mathbf{0}_{3\times 2}$$

3 类的设计


Warp
类
字段
denoise_status_
end_list_
ptr_image_
start_list_
方法
~Warp
Compute
dist
Image
setDeNoise
SetEndPoints
SetImage
SetStartPoints
Warp
嵌套类型
RBF
类
Warp
字段
mu
方法
~RBF
Compute
R
RBF
pu...
IDW
类
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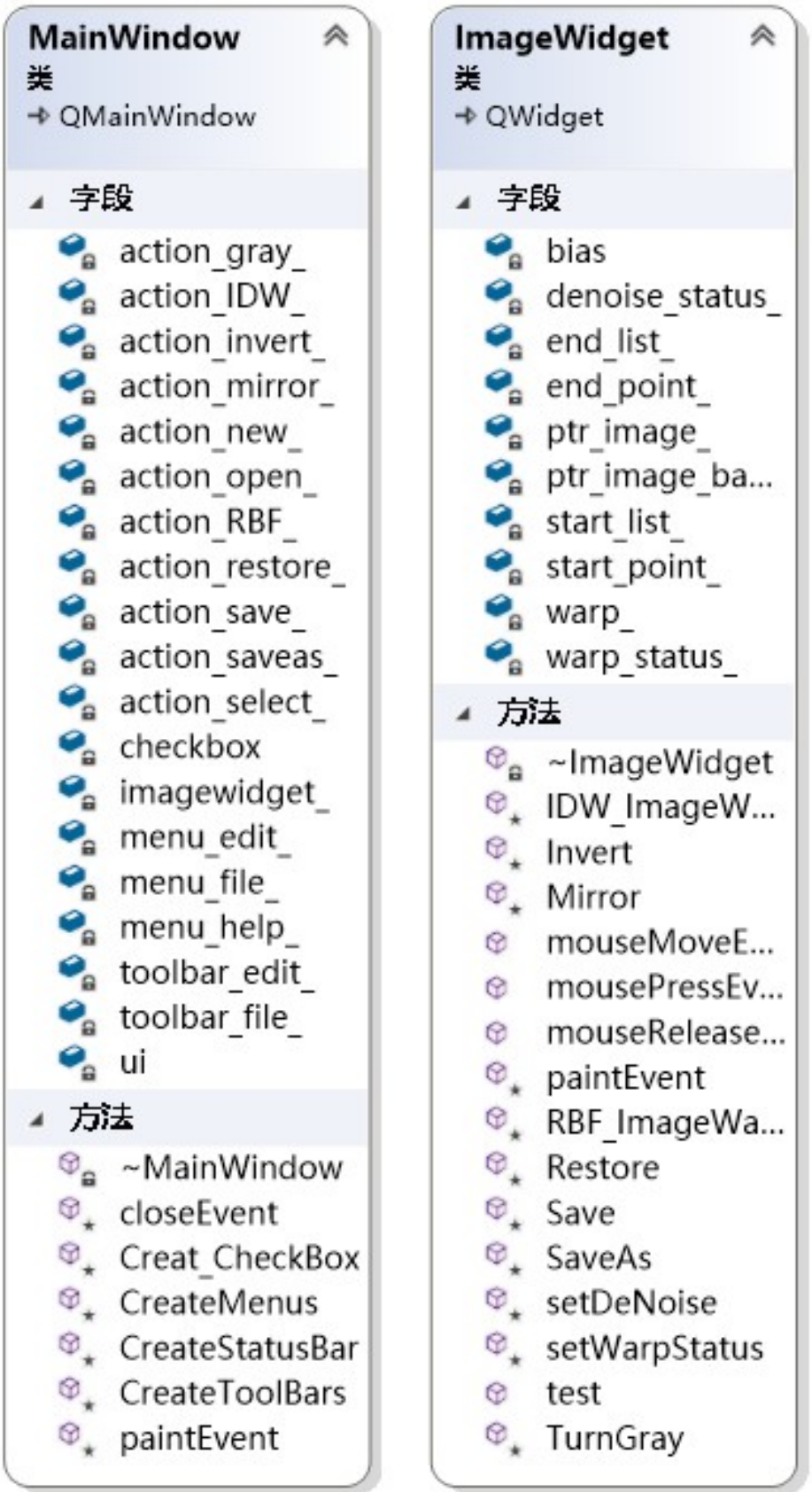
Compute
IDW
sigma
w
public


MainWindow
类
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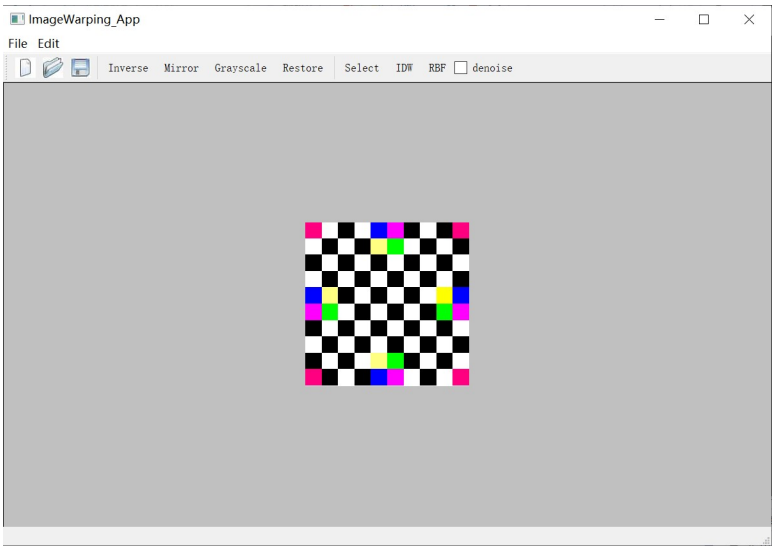
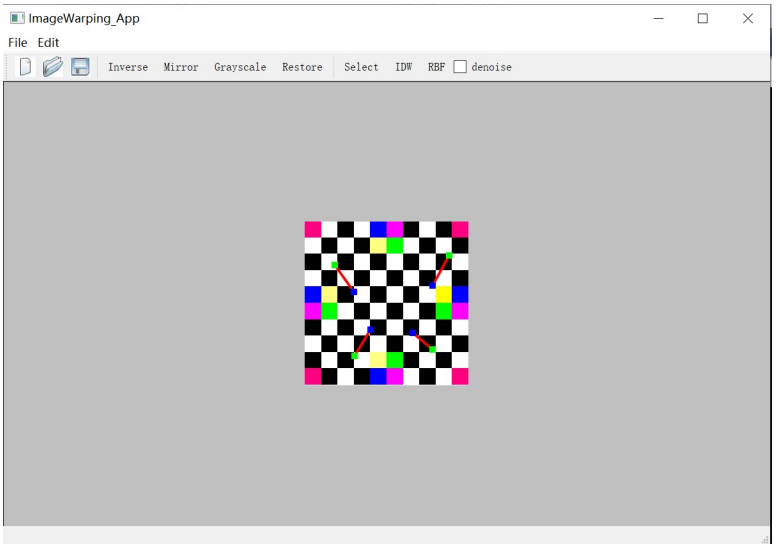
action_IDW_
action_invert_
action_mirror_
action_new_
action_open_
action_RBF_
action_restore_
action_save_
action_saveas_
action_select_
checkbox
imagewidget_
menu_edit_
menu_file_
menu_help_
toolbar_edit_
toolbar_file_
ui
方法
~MainWindow
closeEvent
Creat_CheckBox
CreateMenus
CreateStatusBar
CreateToolBars
paintEvent
ImageWidget
类
QWidget
字段
bias
denoise_status_
end_list_
end_point_
ptr_image_
ptr_image_ba...
start_list_
start_point_
warp_
warp_status_
方法
~ImageWidget
IDW_ImageW...
Invert
Mirror
mouseMoveE...
mousePressEv...
mouseRelease...
paintEvent
RBF_ImageWa...
Restore
Save
SaveAs
setDeNoise
setWarpStatus
test
TurnGray

4 实现结果

4.1 UI 设计



如上图。点击 Select 键后可以开始选择控制点 (蓝色) 和目标点 (绿色)，鼠标左键点击会画出控制点，再释放就可以画出目标点 (如下图)。之后可点击”IDW” 键或”RBF” 来进行图像变形。复选框用于选择是否去噪。



4.2 变形效果

图 1: IDW

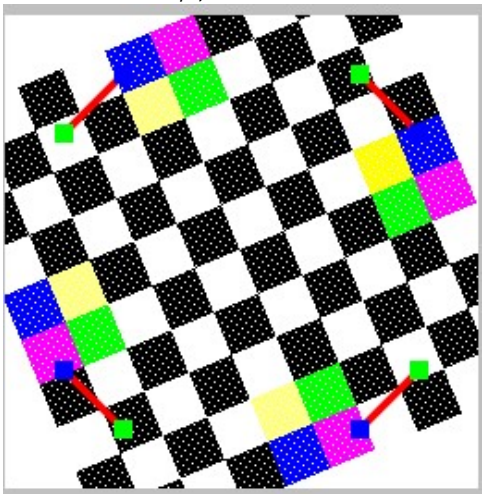


图 2: RBF

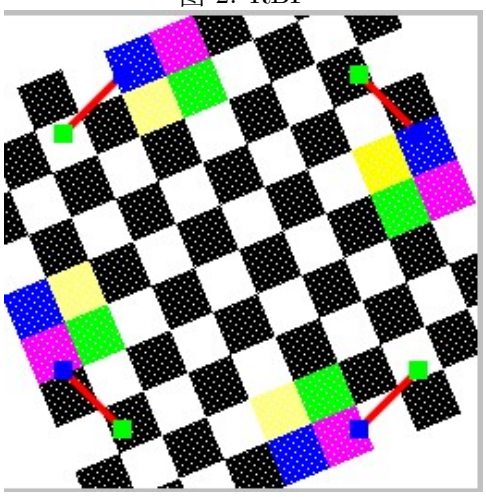


图 3: IDW

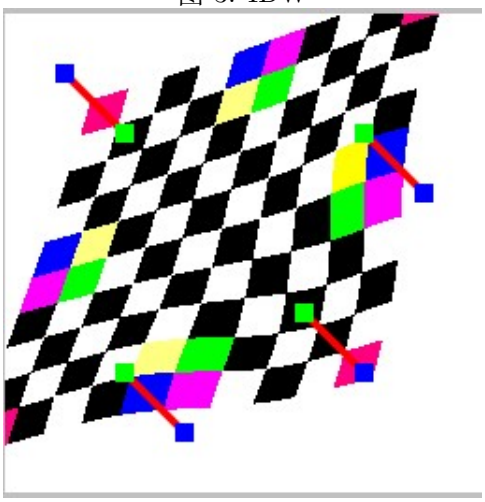


图 4: RBF

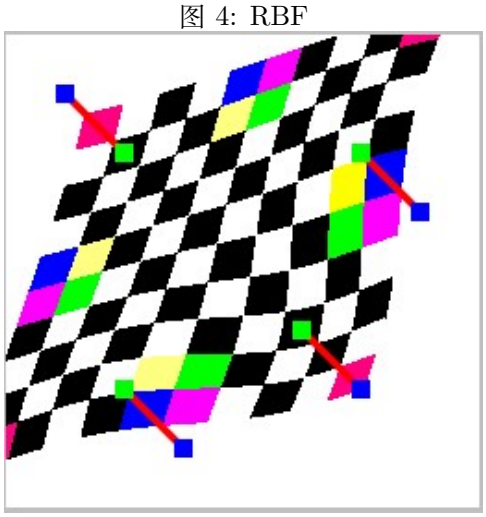


图 5: IDW

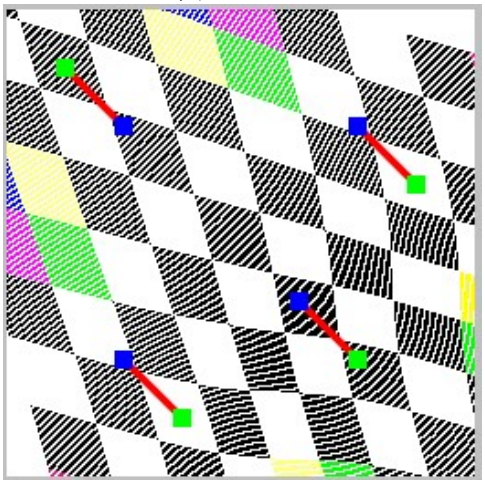
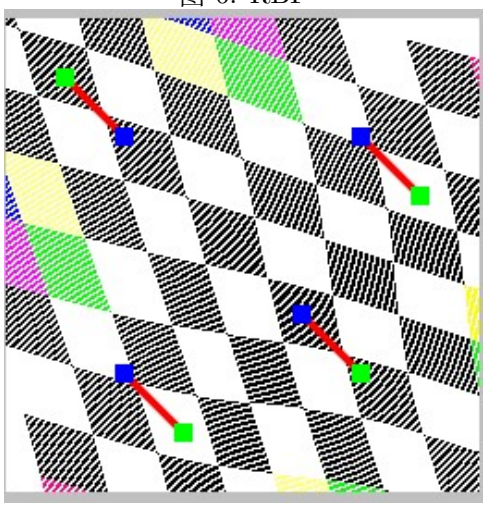


图 6: RBF



我们可以看到 2 种方法效果接近，但是都有噪声的出现，或者说裂缝。所以我们这里采用求插值函数的逆函数的方法来解决这一问题。但是求插值函数的逆函数相当复杂，所以这里考虑将控制点和目标点对调后的插值函数可以近似地作为插值函数的逆函数。实现效果如下：

图 7: IDW

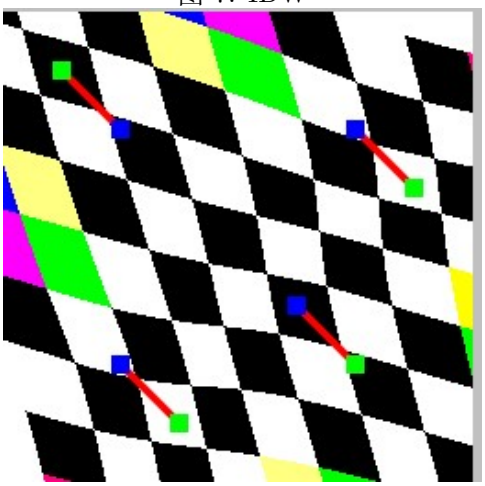
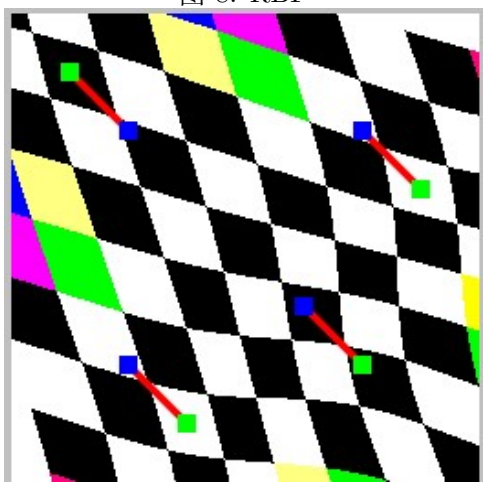


图 8: RBF



5 不足

没有采用插值的方法来补缝。

6 参考文献

[1]Ruprecht D, Muller H. [**Image warping with scattered data interpolation**]IEEE Computer Graphics and Applications, 1995, 15(2): 37-43.

[2]Arad N, Reisfeld D. [**Image warping using few anchor points and radial functions**][C]//Computer graphics forum. Edinburgh, UK: Blackwell Science Ltd, 1995, 14(1): 35-46.