### SIAM Conference on Uncertainty Quantification (UQ20)

Thursday - 26.03.2020

08:30

# MS741: Machine learning methods for reliability analysis and risk assessment (Part I of II)

#### Chair(s)

Qifeng Liao (ShanghaiTech University) Jinglai Li (University of Liverpool)

Room: Topic:

MW ZS 2050 Rare events and Risk

Form of presentation: Duration:
Mini-symposium 120 Minutes

Reliability analysis and risk assessment for complex physical and engineering systems governed by partial differential equations (PDEs) are computationally intensive, especially when high-dimensional random parameters are involved. Since standard numerical schemes for solving these complex PDEs are expensive, traditional Monte Carlo methods which require repeatedly solving PDEs are infeasible. Alternative approaches which are typically the surrogate based methods suffer from the so-called ``curse of dimensionality", which limits their application to problems with high-dimensional parameters. The purpose of this mini-symposium is to bring researchers from different fields to discuss the recent machine learning methods for such problems, focusing on both novel machine learning surrogates and alternative Monte Carlo methods.

08:30

#### A modified Multicanonical Monte Carlo method for failure probability estimation

Jinglai Li | University of Liverpool | United Kingdom

#### Author

Jinglai Li | University of Liverpool | United Kingdom

In this talk we shall discuss the implementation of Multicanonical Monte Carlo (MMC) method for estimating rare failure probability. Here we present certain treatment to avoid constructing bins in the output space, an important step that has substantial impact on the estimation accuracy in the original MMC algorithm. The proposed method is based on Gaussian process regression and kernel density estimation. We provide numerical examples to illustrate the performance of the proposed method.

09:00

### Coupling the reduced-order model and the generative model for an importance sampling estimator

Xiaoliang Wan | Louisiana State University | United States

#### Author:

Xiaoliang Wan | Louisiana State University | United States

In this work, we develop an importance sampling estimator by coupling the reduced-order model and the generative model in a problem setting of uncertainty quantification. The target is to estimate the probability that the quantity of interest (QoI) in a complex system is beyond a given threshold. To avoid the prohibitive cost of sampling a large scale system, the reduced-order model is usually considered for a trade-off between efficiency and accuracy. However, the Monte Carlo estimator given by the reduced-order model is biased due to the error from dimension reduction. To correct the bias, we still need to sample the fine model. An effective technique to reduce the variance reduction is importance sampling, where we employ the generative model to estimate the distribution of the data from the reduced-order model and use it for the change of measure in the importance sampling estimator. To compensate the approximation errors of the reduced-order model, more data that induce a slightly smaller QoI than the threshold need to be included into the training set. Although the amount of these data can be controlled by a posterior error estimate, redundant data, which may outnumber the effective data, will be kept due to the epistemic uncertainty. To deal with this issue, we introduce a weighted empirical distribution to process the data from the reduced-order model.

09:30

## - CANCELED - A model reduction method for multiscale elliptic PDEs with random coefficients using an optimization approach

Zhiwen Zhang | University of Hong Kong | Hong Kong

#### Author:

Zhiwen Zhang | University of Hong Kong | Hong Kong

We propose a model reduction method for solving multiscale elliptic PDEs with random coefficients setting using an optimization approach. The optimization approach enables us to construct a set of localized multiscale data-driven stochastic basis functions that give optimal approximation property of the solution operator. Our method consists of offline and online stages. In the offline stage, we construct the localized multiscale data-driven stochastic basis functions by solving an optimization problem. In the online stage, using our basis functions, we can efficiently solve multiscale elliptic PDEs with random coefficients with relatively small computational costs. Therefore, our method is very efficient in solving target problems with many different force functions. The convergence analysis of the proposed method is also presented and has been verified by the numerical simulations.

10:00

#### A Hierarchical Neural Hybrid Method for Failure Probability Estimation

Ke Li | ShanghaiTech University | China

#### Author:

Ke Li | ShanghaiTech University | China

Failure probability evaluation for complex physical and engineering systems governed by partial differential equations (PDEs) are computationally intensive, especially when high-dimensional random parameters are involved. Since standard numerical schemes for solving these complex PDEs are expensive, traditional Monte Carlo methods which require repeatedly solving PDEs are infeasible.

Alternative approaches which are typically the surrogate based methods suffer from the so-called ``curse of dimensionality", which limits their application to problems with high-dimensional parameters. For this purpose, we develop a novel hierarchical neural hybrid (HNH) method to efficiently compute failure probabilities of these challenging high-dimensional problems. Especially, multifidelity surrogates are constructed based on neural networks with different levels of layers, such that expensive highfidelity surrogates are adapted only when the parameters are in the suspicious domain. The efficiency of our new HNH method is theoretically analyzed and is demonstrated with numerical experiments. From numerical results, we show that to achieve an accuracy in estimating the rare failure probability (e.g., 1e-5), the traditional Monte Carlo method needs to solve PDEs more than a million times, while our HNH only requires solving them a few thousand times.

14:00

# MS742: Machine learning methods for reliability analysis and risk assessment (Part II of II)

#### Chair(s)

Qifeng Liao (ShanghaiTech University) Jinglai Li (University of Liverpool)

Room:

MW ZS 2050

Topic:

Rare events and Risk

Form of presentation: Mini-symposium Duration: 120 Minutes

Reliability analysis and risk assessment for complex physical and engineering systems governed by partial differential equations (PDEs) are computationally intensive, especially when high-dimensional random parameters are involved. Since standard numerical schemes for solving these complex PDEs are expensive, traditional Monte Carlo methods which require repeatedly solving PDEs are infeasible. Alternative approaches which are typically the surrogate based methods suffer from the so-called ``curse of dimensionality", which limits their application to problems with high-dimensional parameters. The purpose of this mini-symposium is to bring researchers from different fields to discuss the recent machine learning methods for such problems, focusing on both novel machine learning surrogates and alternative Monte Carlo methods.

14.00

#### Generative Model based on Winslow Mapping for Sampling unnormalised Distribution

Xiang Zhou | City University of Hong Kong | Hong Kong

#### Author:

Xiang Zhou | City University of Hong Kong | Hong Kong

As an alternative to MCMC, the generative model transforms a simple distribution to a complicated target distribution and becomes increasingly popular due to the advance of deep neural network (such as the Knothe-Rosenblatt mapping in NICE and NVP works) to represent the high dim map. The principle is to minimise the discrepancy (such as KL divergence in variational inference or the Wasserstein distance in Monge-Ampere flow) between the target density and the transformed density. Variational structure is critically beneficial in this context for the implementation by machine learning techniques. In this work, our generative model is based on the harmonic map which is the foundation of many traditional moving mesh methods in adaptive numerical method for PDE. Our approach fully enjoys the variational formulation associated with the so-called Winslow's energy. With a simple choice of the monitor function in the Winslow's energy, the induced density is already close to the target density in many cases and thus serves a good initial guess for the further refining in the same framework of variational inference. Furthermore, the functional derivative to iteratively refine the monitor function also enjoys a variational formulation and thus by learning the map and gradient simultaneously, we have a robust and efficient descent algorithm. Numerical examples are included in the talk to demonstrate our ideas.

14:30

#### Deep density estimation for Fokker-Planck equations using flow-based generative model

Kejun Tang | ShanghaiTech University | China

#### Author:

Kejun Tang | ShanghaiTech University | China

In this work, we proposed a general flow-based generative model based onKnothe-Rosenblatt rearrangement, and we study the performance and applications of flow-based generative models subject to an invertible mapping. We apply this generative model to solve the Fokker-Planck equation. In particular, an adaptive deep learning based algorithm is proposed for solving the Fokker-Planck equation. Unlike the standard deep learning based method for solving partial differential equations, this approach can provide an exact sampling instead of uniform sampling on the domain. Numerical experiments demonstrate the efficiency of our method.

#### 15:00

#### Proper orthogonal decomposition method for multiscale elliptic PDEs with random coefficients

Dingjiong Ma | University of Hong Kong | Hong Kong

#### Authors:

Dingjiong Ma | University of Hong Kong | Hong Kong

Zhiwen Zhang | University of Hong Kong | Hong Kong

In this talk, we develop an efficient multiscale reduced basis method to solve elliptic PDEs with multiscale and random coefficients in a multi-query setting. Our method consists of offline and online stages. In the offline stage, a small number of reduced multiscale basis functions are constructed within each coarse grid block using the proper orthogonal decomposition (POD) method. Moreover, local tensor spaces are defined to approximate the solution space of the multiscale random PDEs. In the online stage, a weak formulation is derived and discretized using the Galerkin method to compute the solution. Since the multiscale reduced basis functions can efficiently approximate the high-dimensional solution space, our method is very efficient in solving multiscale elliptic PDEs with random coefficients. Convergence analysis of the proposed method is presented, which shows the dependence of the numerical error on the number of snapshots and the truncation threshold in the POD method. Finally, numerical results are presented to demonstrate the accuracy and efficiency of the proposed method for several multiscale problems with or without scale separation in the physical space.

#### 15:30

#### - CANCELED - Domain decomposed uncertainty analysis based on RealNVP

Qifeng Liao | ShanghaiTech University | China

#### Author:

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during the importance sampling procedure.

The domain decomposition uncertainty quantification method (DDUQ) (SIAM J. SCI. COMPUT (37) pp. A103-A133) provides a decomposed strategy to conduct uncertainty analysis for complex engineering systems governed by PDEs. In DDUQ, uncertainty analysis on each local component is independently conducted in an ``offline" phase, and global uncertainty analysis results are assembled using precomputed local information in an ``online" phase through importance sampling. The performance of DDUQ relies on the coupling surrogates and probability density estimation

Since coupling surrogates can give high-dimensional interface parameters, we in this work develop a RealNVP based interface coupling strategy, which dramatically improve the efficiency of DDUQ.