



Topic2 - Circling Magnet

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上海交通大学
SHANGHAI JIAO TONG UNIVERSITY

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正式实验

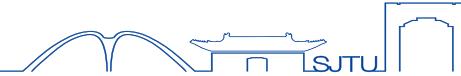
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数据分析和结论



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题目回顾——Circling Magnet



- **Button magnets** with **different diameters** are attached to each end of a cylindrical battery. When placed on **an aluminium foil** the object starts to **circle**. Investigate how the motion depends on relevant parameters.
- 将**不同直径的纽扣磁铁**贴附到**圆柱形电池**的两端。将其放置到**铝箔**上后，物体会开始**旋转**。探究相关参数如何影响该运动。
- 关键词： **不同直径的纽扣磁铁**， **圆柱形电池**， **铝箔**， 物体**旋转**

1 题目回顾和初步分析

2 预实验

3 理论分析

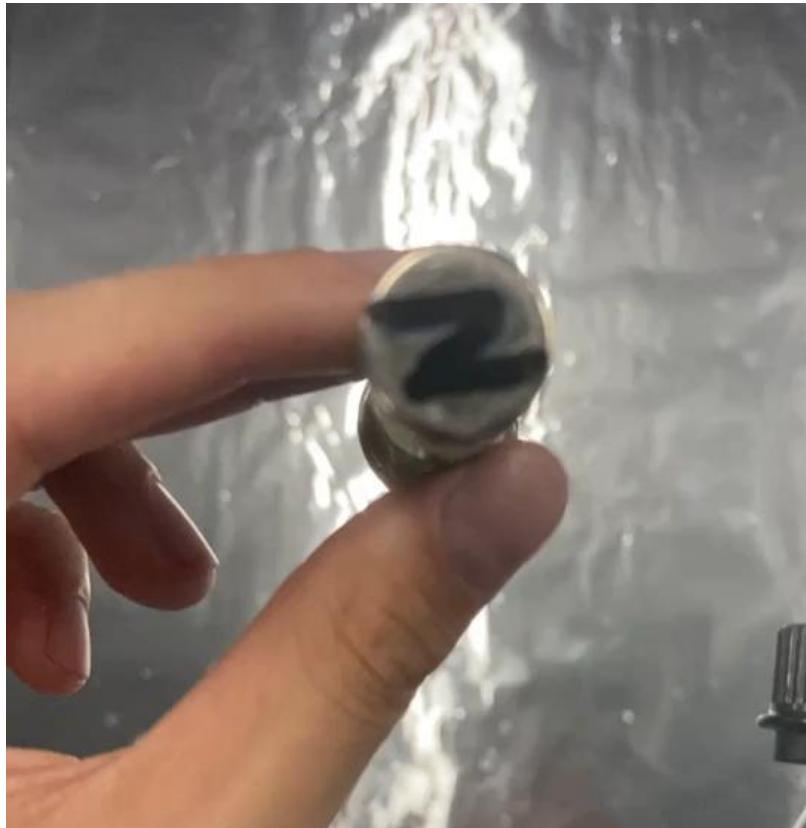
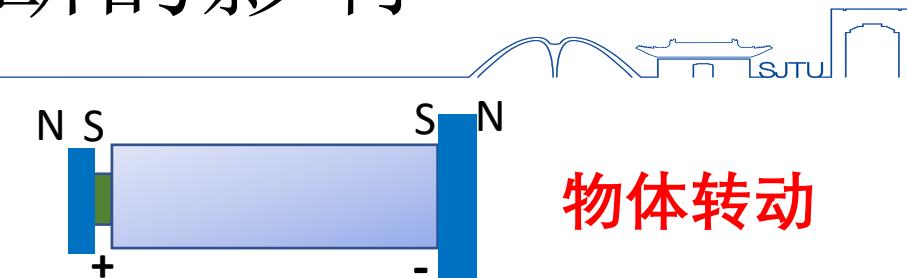
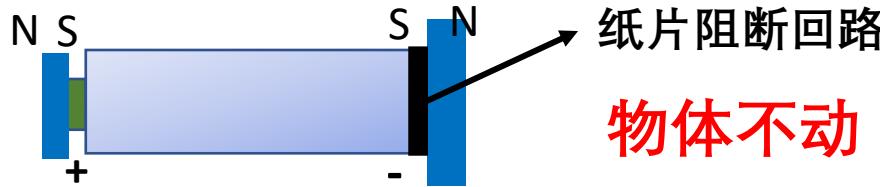
4 正式实验

5 数据分析和结论





预实验——研究电流回路通断的影响



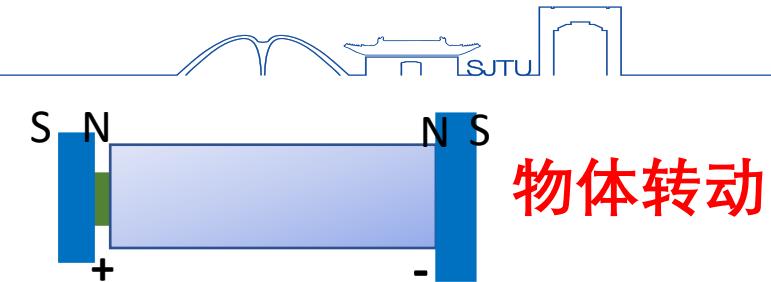
预实验——研究两侧磁铁极性的影响



物体不动



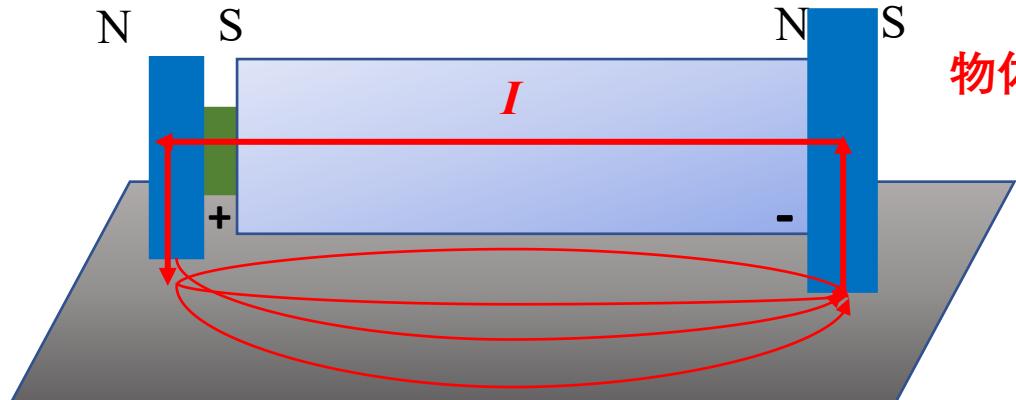
物体转动



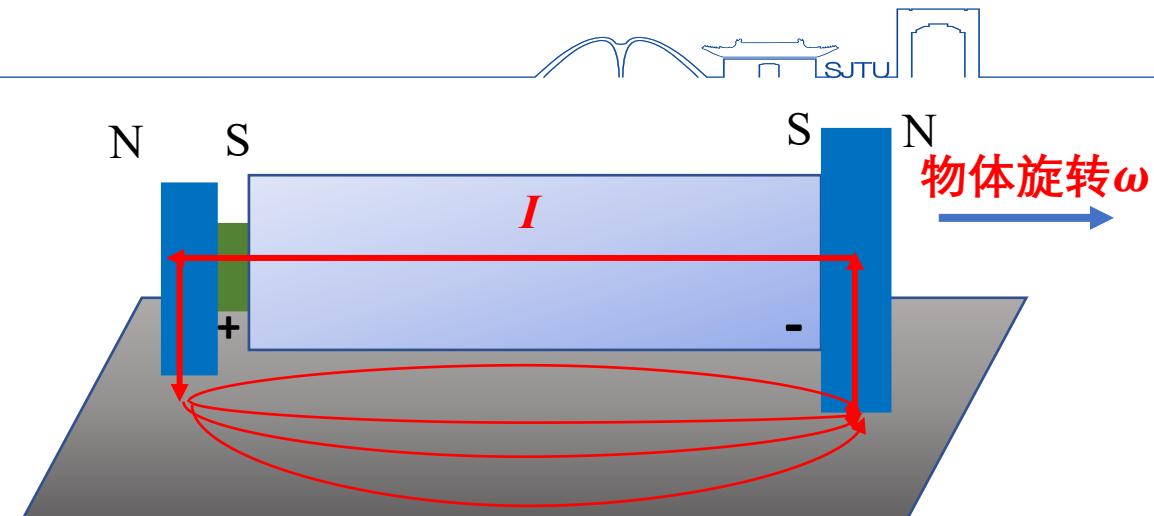
物体转动



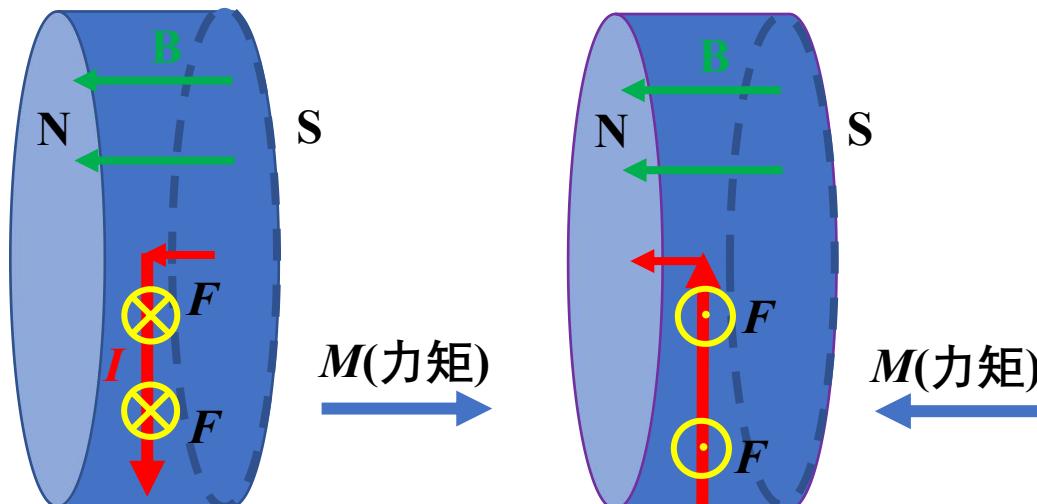
预实验原理分析



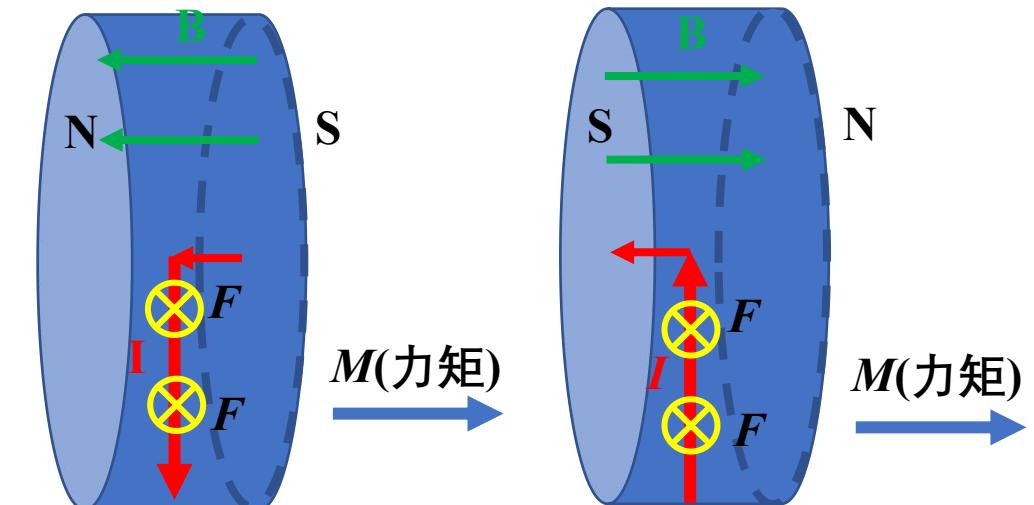
物体不转



物体旋转 ω

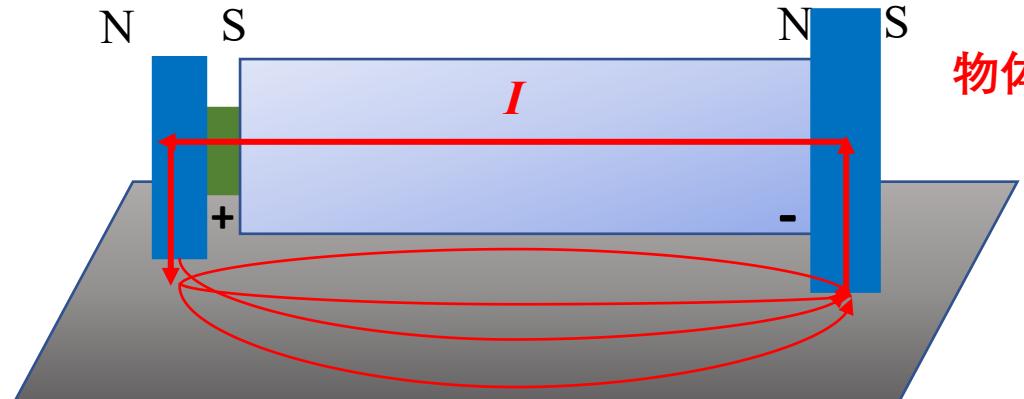


安培力力矩方向相反！

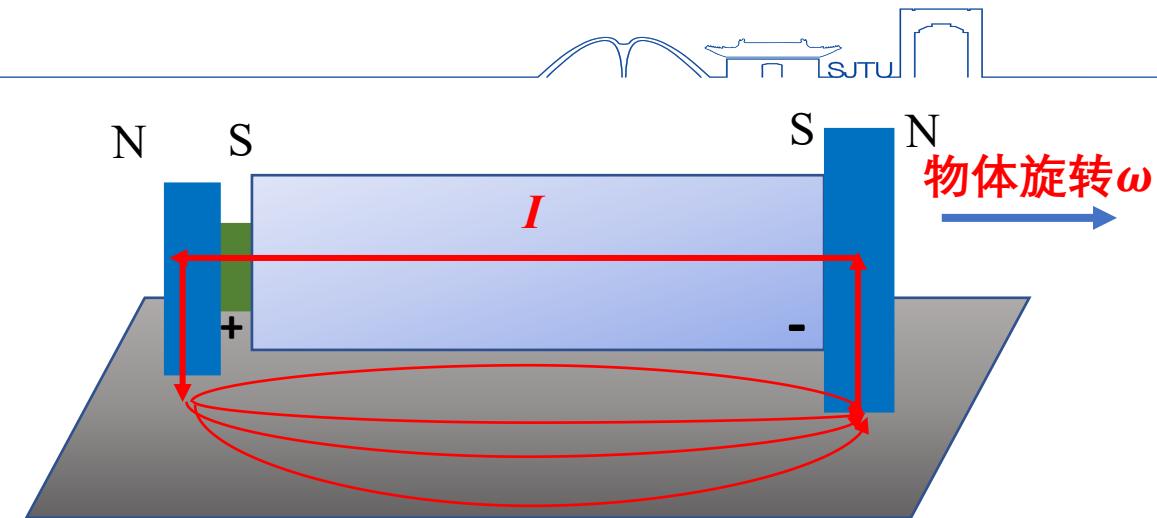


安培力力矩方向相同！

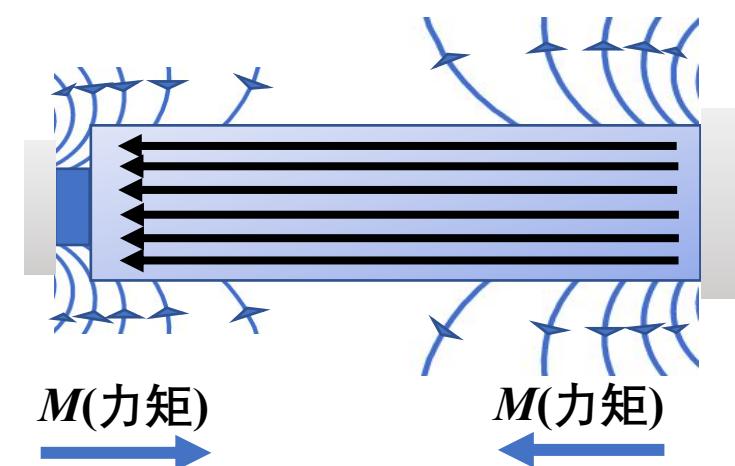
预实验原理分析



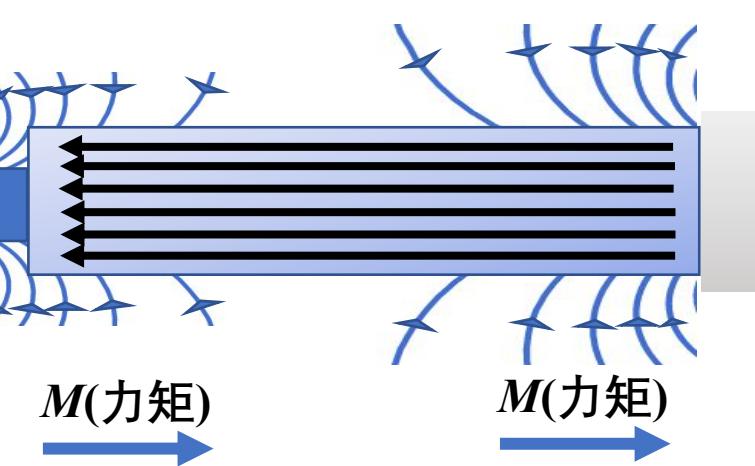
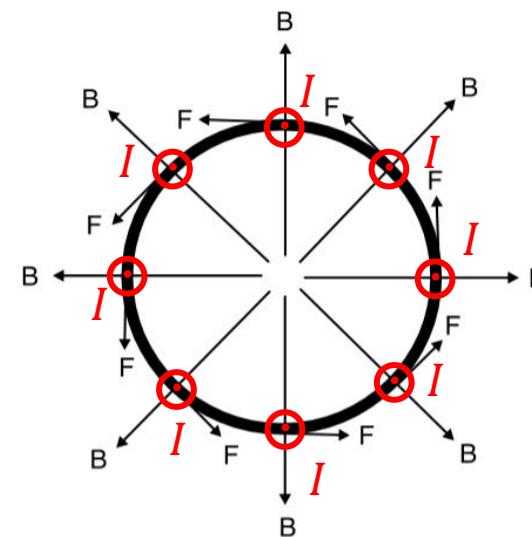
物体不转



物体旋转 ω



安培力力矩方向相反！



安培力力矩方向相同！

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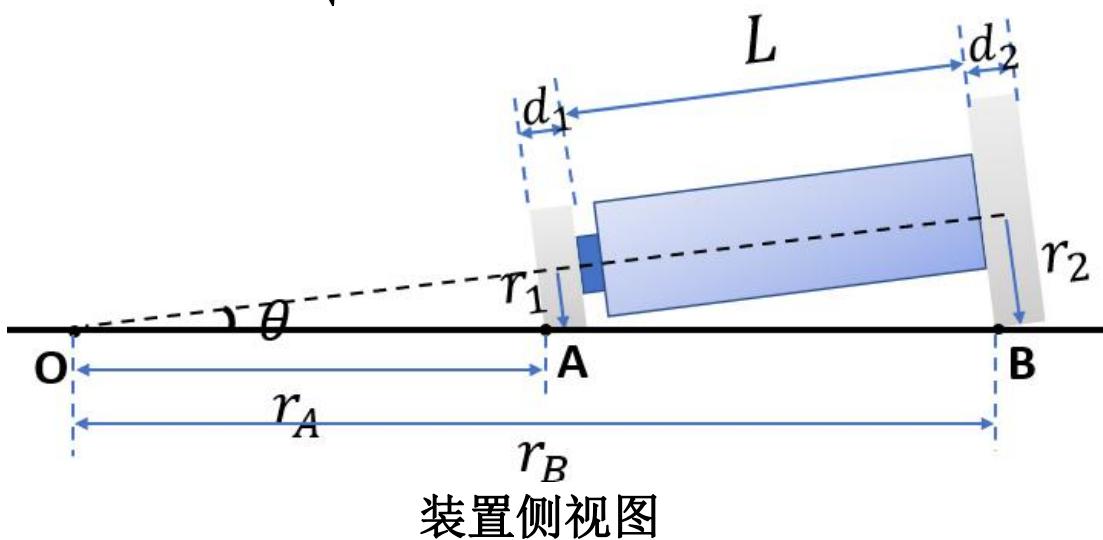
数据分析和结论



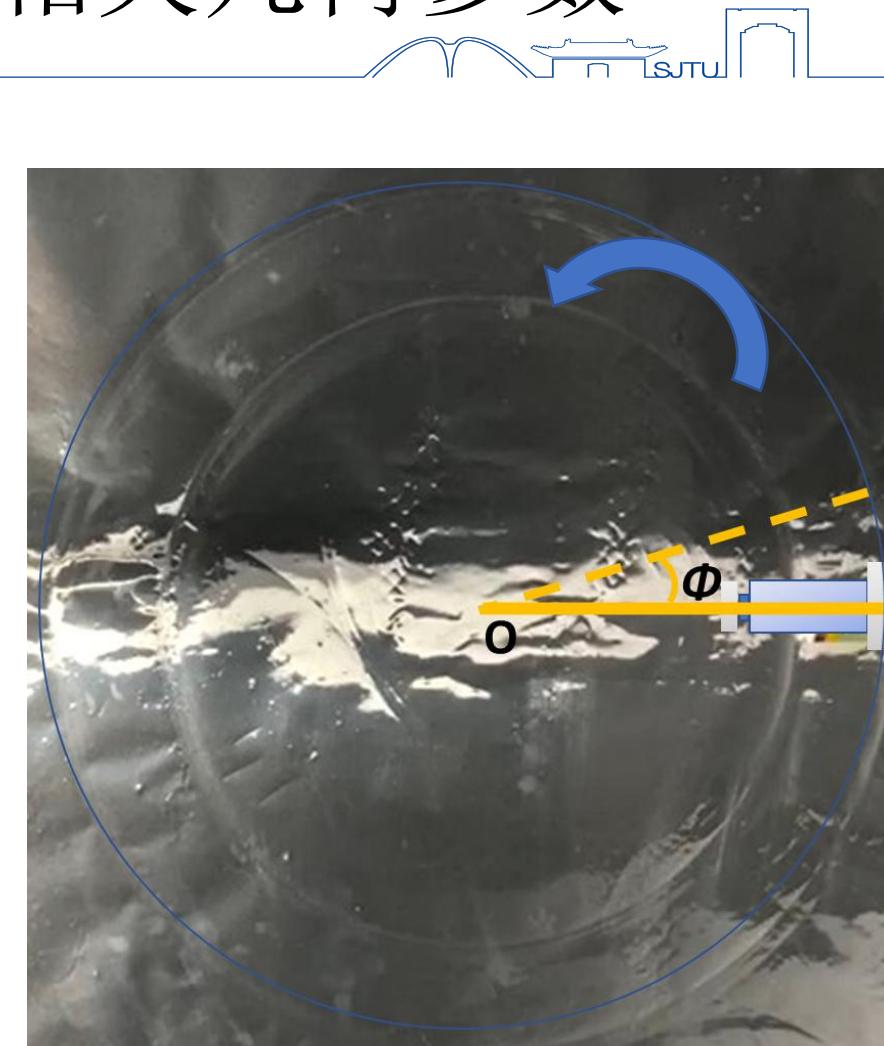
理论分析——物体做圆周运动相关几何参数

$$\left\{ \begin{array}{l} r_A = r_1 \sqrt{\left(\frac{L + d_1}{r_2 - r_1}\right)^2 + 1} \\ r_B = r_2 \sqrt{\left(\frac{L + d_1}{r_2 - r_1}\right)^2 + 1} \\ \Omega = \frac{d\phi}{dt} = \frac{r_1}{r_A} \omega = \frac{1}{\sqrt{\left(\frac{L+d_1}{r_2-r_1}\right)^2+1}} \omega \end{array} \right.$$

Ω 为物体公转角速度
 ω 为物体自转角速度
 $r_1, d_1; r_2, d_2$ 为两侧磁铁的半径, 厚度
 r_A, r_B 为接触点到0点的距离



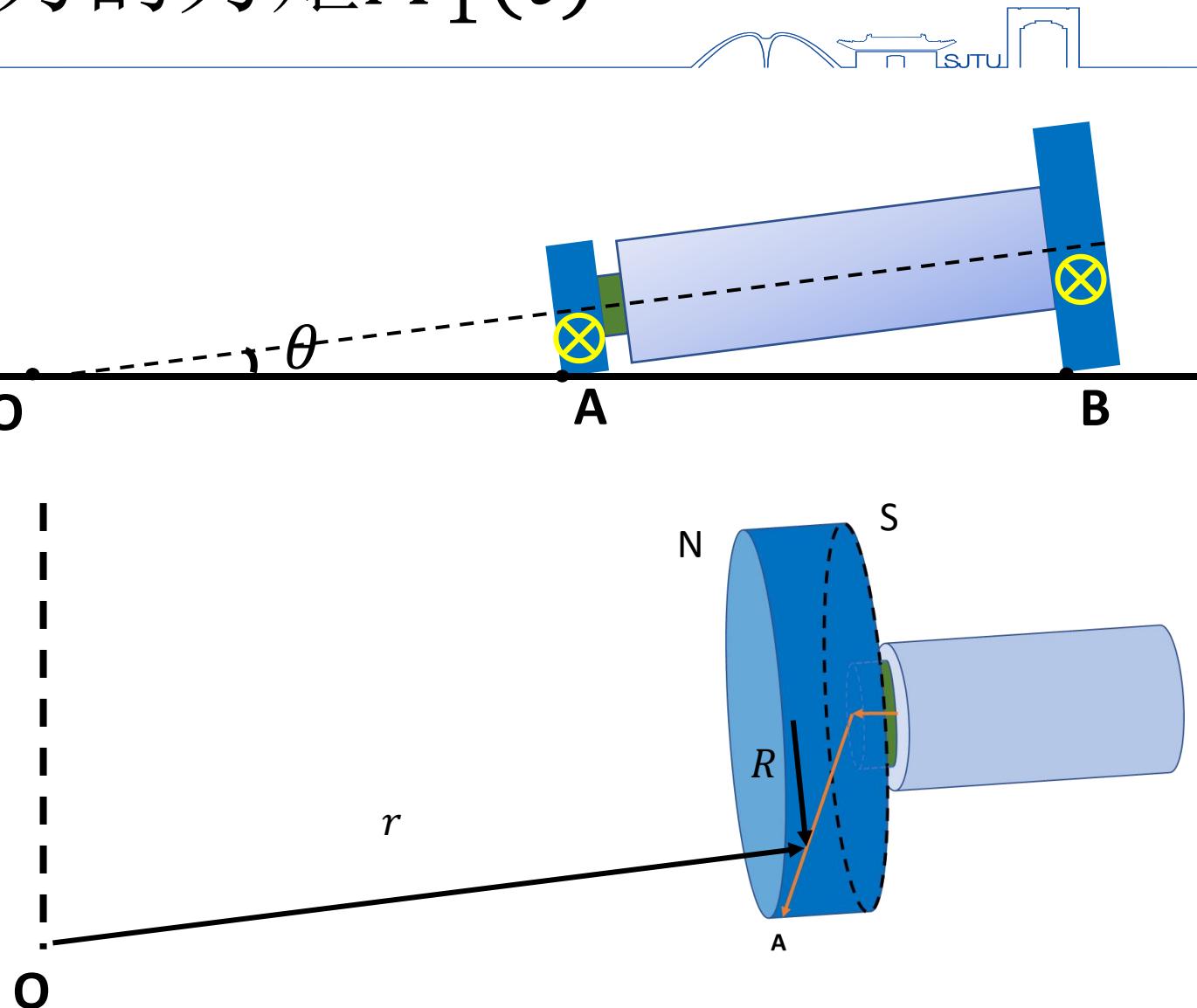
装置侧视图



装置运动俯视图

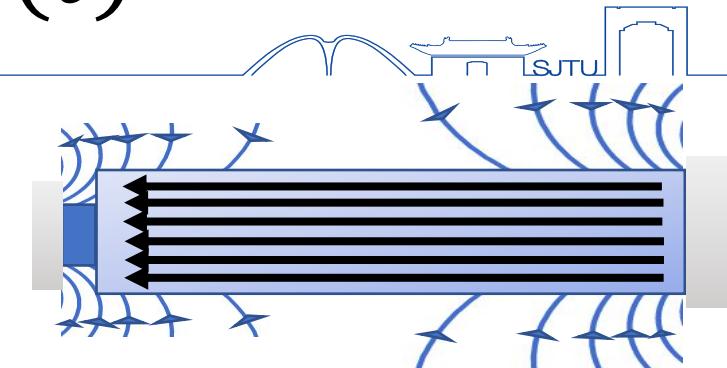
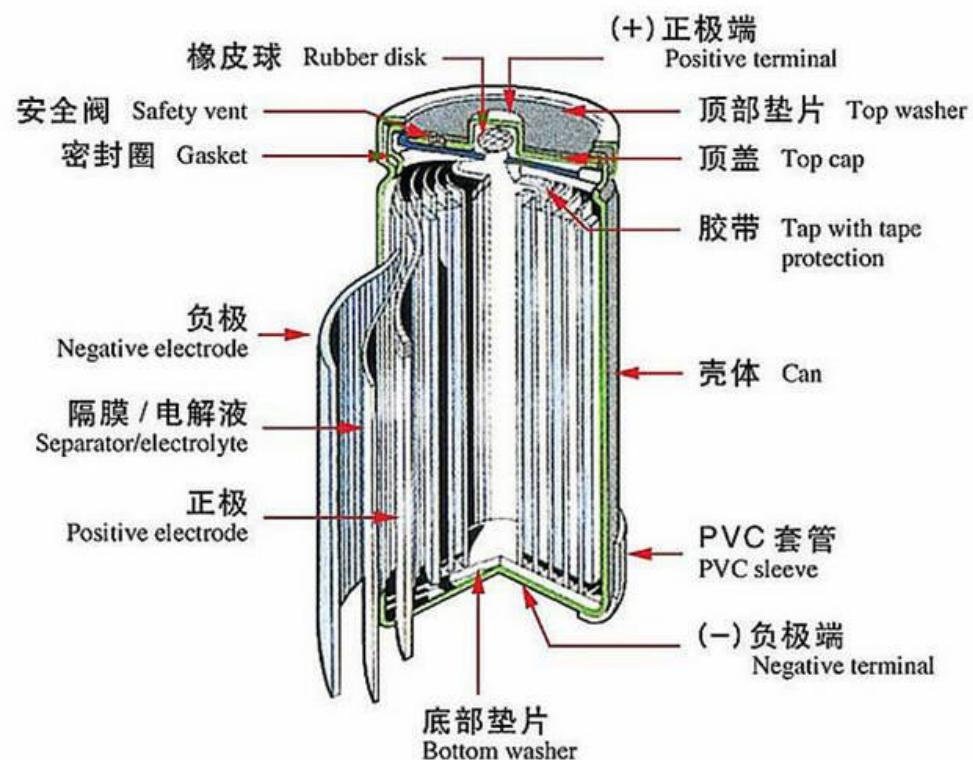
磁铁上电流受到安培力的力矩 $M_1(t)$

- 近似认为 磁铁和铝箔 点接触。电流从磁铁中心流到铝箔上。
- 电流元受到的安培力 $d\vec{F}_R = B_R I dR \sin \theta$,
- 对应的力矩
- $M_1 = \int \vec{r} \times d\vec{F}_R$
- 利用Matlab数值方法计算磁场分布进而对力矩进行积分。



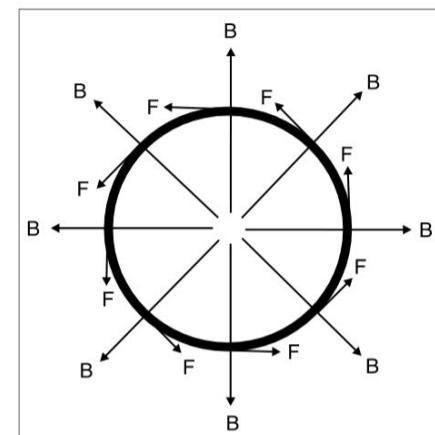
电池上电流受到安培力的力矩 $M_2(t)$

- 镍氢充电电池内部结构如下图



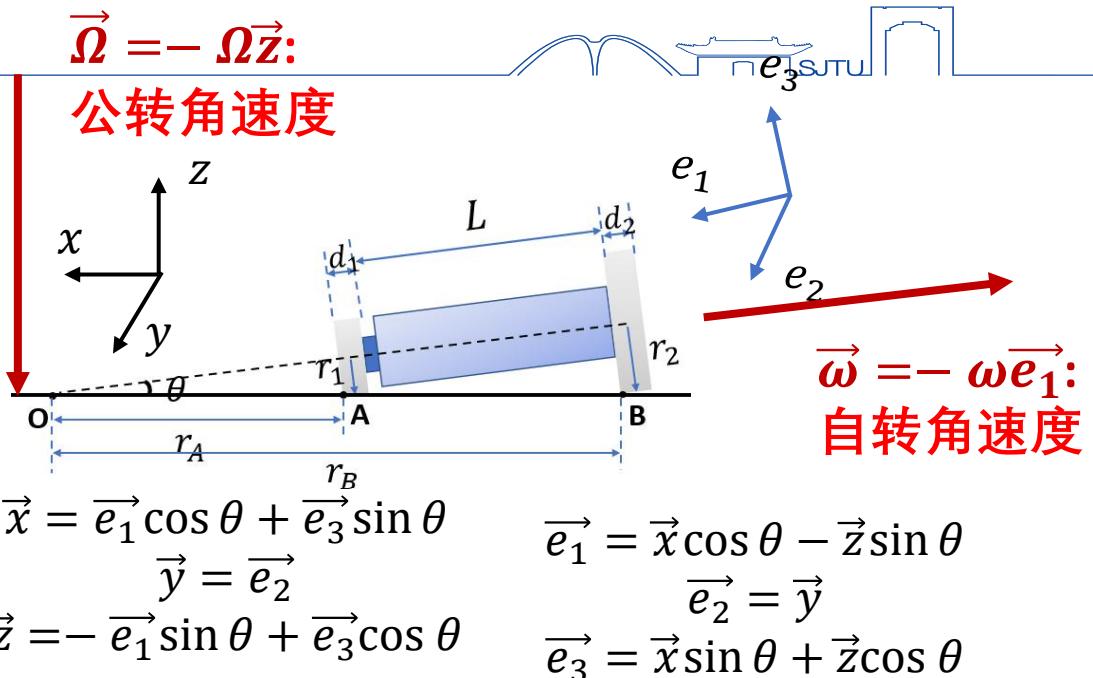
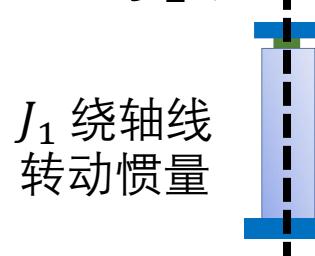
$$M_2 = \iint (B_{r,l} dl \cdot j \cdot 2\pi r dr) r \\ = \int_0^L dl \int_0^R 2\pi r^2 j B_{r,l} dr$$

其中: $j = \frac{I}{\pi R^2}$



理论分析——受力分析

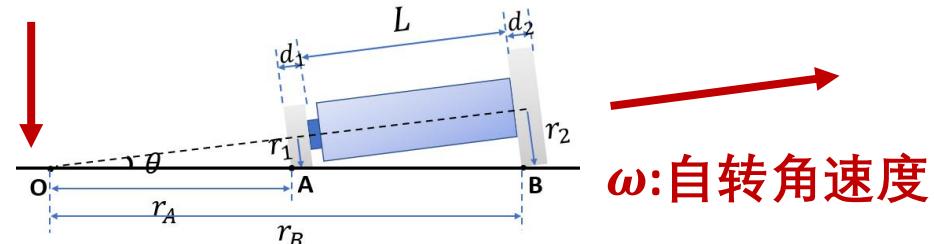
- 绕0点角动量:
- $\vec{L} = J_1(-\omega + \Omega \sin \theta) \vec{e}_1 - J_2 \Omega \cos \theta \vec{e}_3$



- 对0点力矩: 磁铁电流力矩+电池电流力矩+滚动摩擦力矩
 - $\vec{M} = -M_1 e_1(t) \vec{e}_1 + M_1 e_3 \vec{e}_3 - M_2(t) \vec{e}_1 + M_f(t) \vec{x} + \vec{r}_A \times (f_{1x} \vec{x} + f_{1y} \vec{y} + N_1 \vec{z}) + \vec{r}_B \times (f_{2x} \vec{x} + f_{2y} \vec{y} + N_2 \vec{z}) + \vec{r}_c \times (m \vec{g})$
- $$= [-M_1(t) \sin \theta - M_2(t) + M_f(t) \cos \theta + (f_{1y} r_A + f_{2y} r_B) \sin \theta] \vec{e}_1$$
- $$+ [(N_1 r_A + N_2 r_B) - m g r_c] \vec{e}_2 + [+M_1(t) \cos \theta + M_f \sin \theta - (f_{1y} r_A + f_{2y} r_B) \cos \theta] \vec{e}_3$$
- 其中 f_{1x} 、 f_{1y} 、 f_{2x} 、 f_{2y} 为两侧静摩擦， N_1 、 N_2 是两侧的支持力， $M(t)$ 为 动力力矩 + 滚动摩擦力矩

理论分析——受力分析

Ω :公转角速度



ω :自转角速度

$$\vec{M} = \frac{d\vec{L}}{dt} = J_1(-\omega' + \Omega' \sin \theta) \vec{e}_1 + [J_1(-\omega + \Omega \sin \theta) \Omega \cos \theta - J_2 \Omega^2 \sin \theta \cos \theta] \vec{e}_2 - J_2 \Omega' \cos \theta \vec{e}_3$$

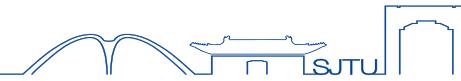
- 上述等式 $\vec{M} = \frac{d\vec{L}}{dt}$ 在 \vec{e}_1 、 \vec{e}_2 、 \vec{e}_3 三个方向分别相等

$$\begin{cases} [-M_{1e_1}(t) - M_2(t) + M_f(t) \cos \theta + (f_{1y}r_A + f_{2y}r_B) \sin \theta] = J_1(-\omega' + \Omega' \sin(\theta)) \\ [(N_1r_A + N_2r_B) - mgr_c] = [J_1(-\omega + \Omega \sin \theta) \Omega \cos \theta - J_2 \Omega^2 \sin \theta \cos \theta] \\ [+M_{1e_3}(t) r_A^2 \Omega' \sin \theta - (f_{1y}r_A + f_{2y}r_B) \cos \theta] = -J_2 \Omega' \cos \theta \end{cases}$$

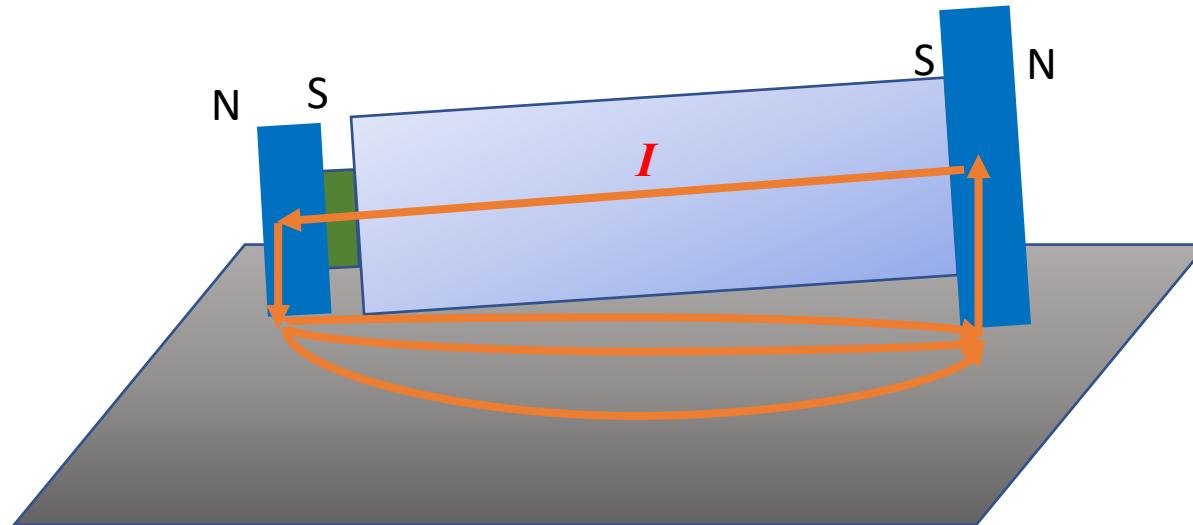
纯滚条件: $r_A^2 \Omega' \sin \theta - (f_{1y}r_A + f_{2y}r_B) \cos \theta = -J_2 \Omega' \cos \theta$

$$\rightarrow M_{1e_3} \tan \theta - M_{1e_1} + \frac{M_f(t)}{\cos \theta} - M_2(t) = \left[J_1 \left(1 - \frac{r_1}{r_A} \sin \theta \right) + J_2 \frac{r_1}{r_A} \sin \theta \right] \omega'(t)$$

物体转动物理分析——动力和阻力

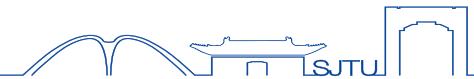


$$\bullet M_{1e_1}(i(t)) - M_{1e_3}(i(t))\tan\theta + M_2(i(t)) - \frac{M_f(t)}{\cos\theta} = \left(J_1\left(1 - \frac{r_1^2}{r_A^2}\right) + J_2 \frac{r_1^2}{r_A^2}\right) \cdot \varphi''(t)$$



其它因素：铝箔表面电流产生磁场对物体产生的二级效应，涡流产生的等效阻力，磁铁升温对磁场的影响，磁铁上的反向电动势，可近似忽略。

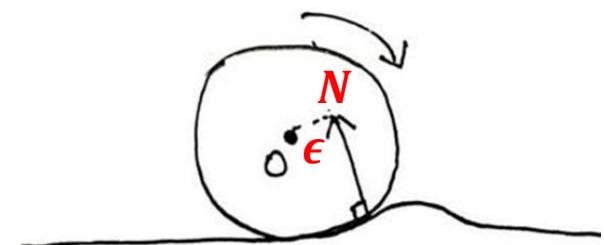
三、阻力项——摩擦阻力 $M_f(t)$



- **摩擦:** 磁铁和铝箔的 滚动摩擦力矩
- 结合受力分析中公式，再利用 $N_1 + N_2 = mg$

$$N_1 = \frac{mg(r_B - r_C) - J_1(\omega - \Omega \sin\theta)\Omega \cos\theta + J_2\Omega^2 \sin\theta \cos\theta}{r_B - r_A} = C_1 - k\omega^2$$

$$N_2 = \frac{mg(r_C - r_A) + J_1(\omega - \Omega \sin\theta)\Omega \cos\theta + J_2\Omega^2 \sin\theta \cos\theta}{r_B - r_A} = C_2 + k\omega^2$$



如右图，滚动摩擦力矩： $M_f = N_1\epsilon_1 + N_2\epsilon_2 = (C_1 - k\omega^2)\epsilon_1 + (C_2 + k\omega^2)\epsilon_2$

考虑到形变程度和速度有关， $\epsilon_1 = \epsilon_{10} + \delta_1\omega$, $\epsilon_2 = \epsilon_{20} + \delta_2\omega$

$$M_f = (C_1\epsilon_{10} + C_2\epsilon_{20}) + C_1\delta_1\omega + C_2\delta_2\omega + k\omega^2(\epsilon_{20} + \delta_2\omega - \epsilon_{10} - \delta_1\omega) \approx \frac{(C_1\epsilon_{10} + C_2\epsilon_{20})}{M_0} + \frac{C_1\delta_1\omega + C_2\delta_2\omega}{\alpha\omega}$$

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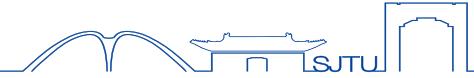
正式实验

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数据分析和结论



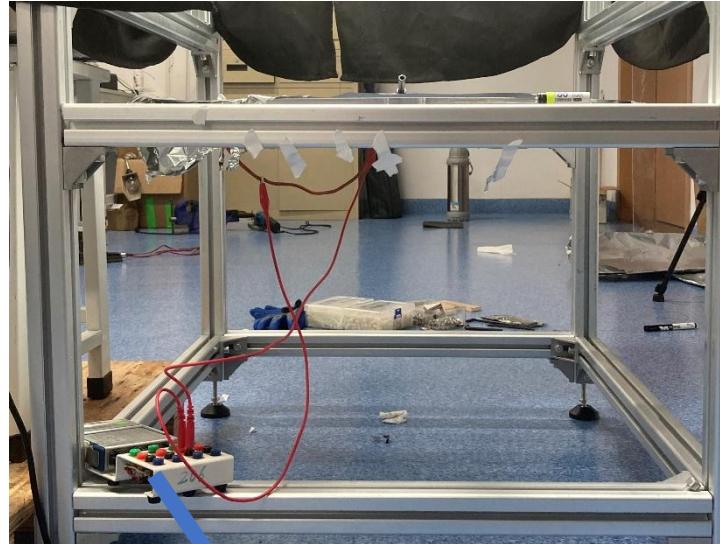
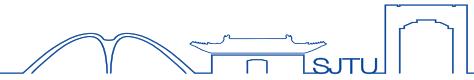
正式实验——实验准备



- 采用磁铁和电池组合而成的物体（如图）
- 贴上黄色贴纸，便于后期跟踪轨迹
- 防止表面因素对运动时接触电阻影响，每次实验前用酒精擦拭磁铁和铝箔表面



正式实验——实验平台搭建



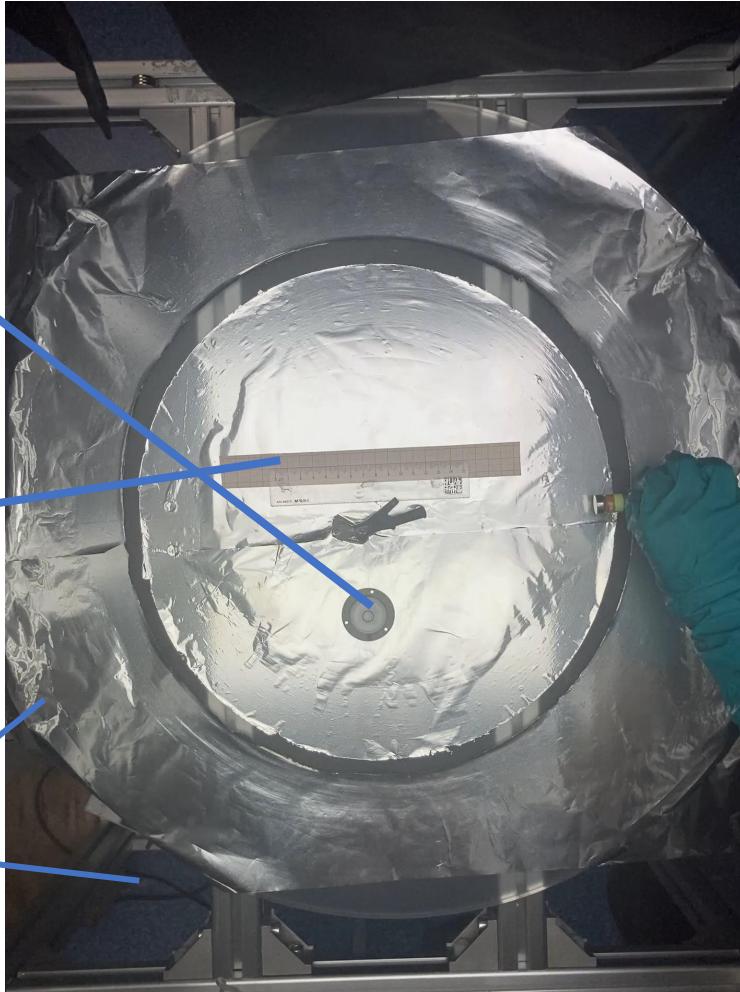
数据采集卡

侧视图

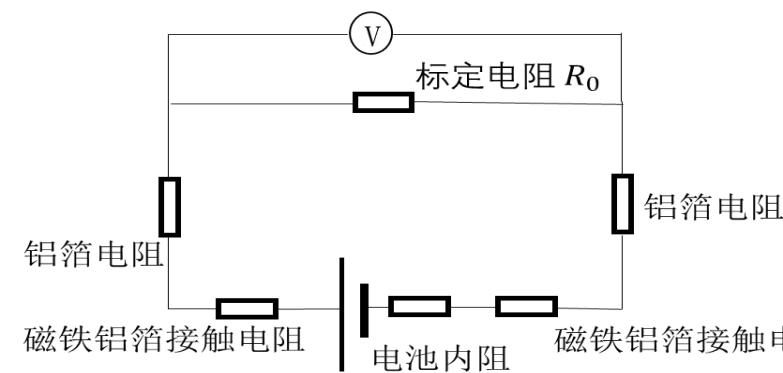
水平仪

定标刻度尺

铝箔



俯视图



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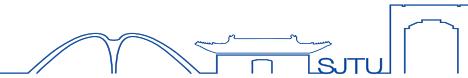
正式实验

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数据分析和结论



实验基本参数



- 电池参数

	开路电压[V]	短路电流[A]	电池长度[m]	电池质量[kg]	电池半径[m]	电池转动惯量[kg·m^2]
	1.3227	8.76	0.0490	0.02905	0.00720	7.53E-07

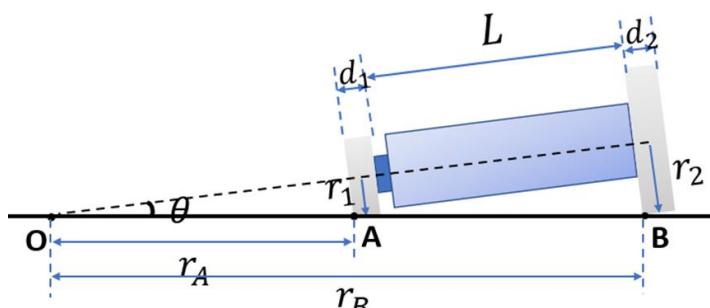
- 磁铁参数

编号	磁铁直径[mm]	磁铁厚度 [mm]	表磁[Gauss]	磁铁型号	磁铁质量 [kg]	剩磁	转动惯量 [kg·m^2]
#1	25	5	2068	N35	0.01870	剩磁12200G	1.46E-06
#2	20	10	3861	N35	0.02390	剩磁12200G	1.20E-06
#3	18	5	2666	N35	0.00970	剩磁12200G	3.93E-07
#4	15	6	3381	N35	0.00810	剩磁12200G	2.28E-07

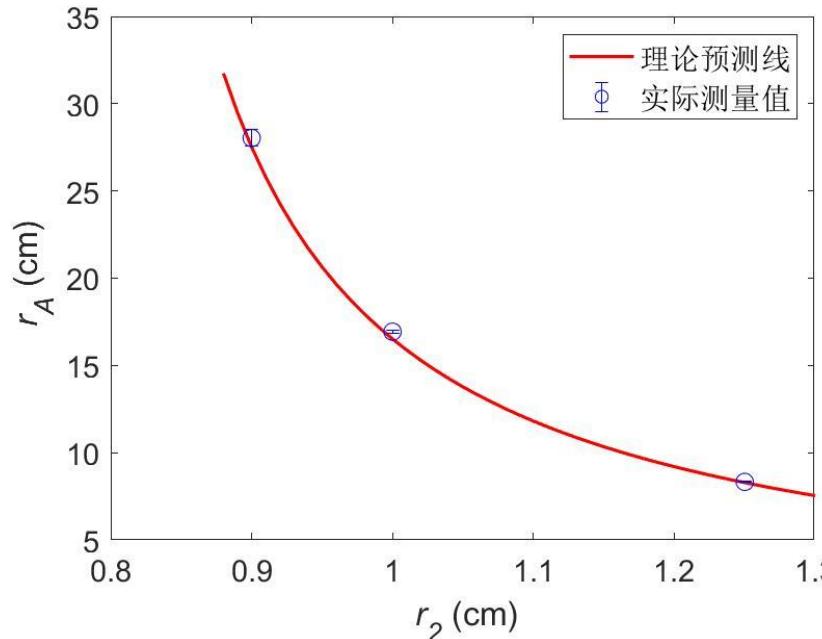
旋转半径比较

$$\left\{ \begin{array}{l} r_A = r_1 \sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1} \\ r_B = r_2 \sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1} \\ \Omega = \frac{d\phi}{dt} = \frac{r_1}{r_A} \omega = \frac{1}{\sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1}} \omega \end{array} \right. \quad \begin{array}{l} ① \\ ② \\ ③ \end{array}$$

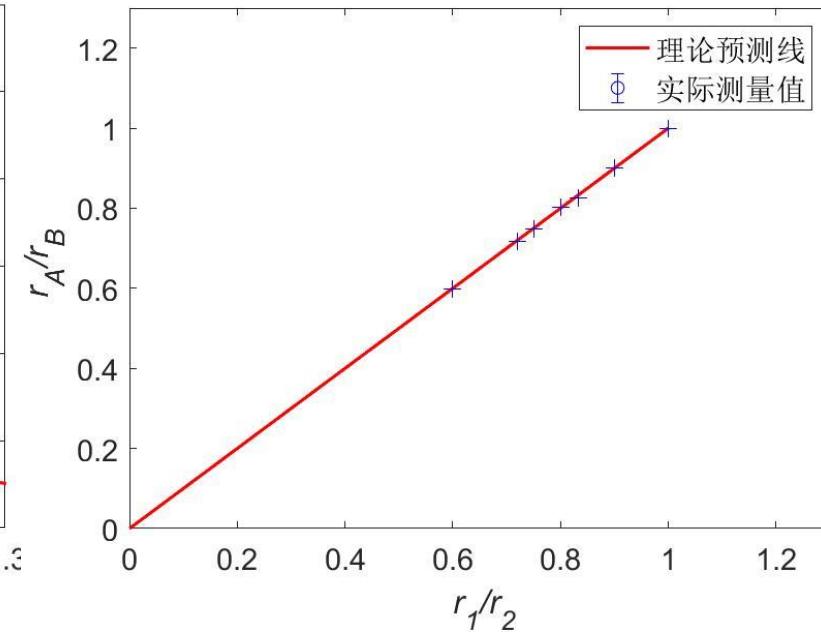
由①/② $\rightarrow \frac{r_A}{r_B} = \frac{r_1}{r_2}$ ④



固定 r_1 , 改变 r_2
 r_A 值的理论与实验的比较

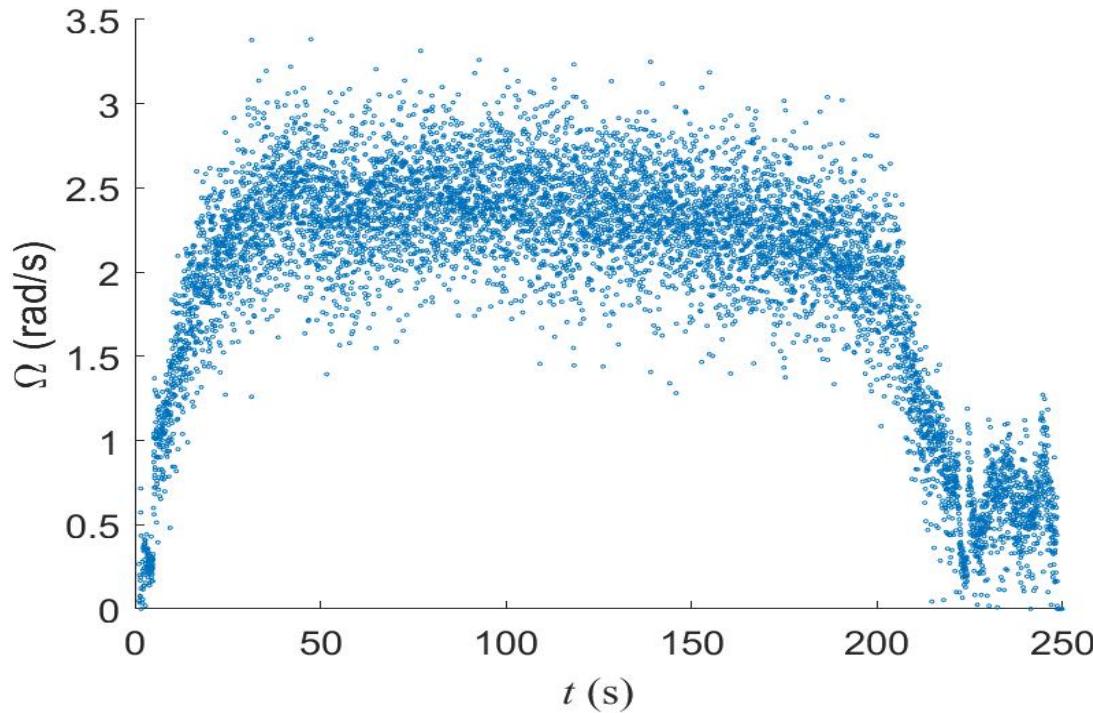


不同 r_1 、 r_2 组合
④式比例关系理论与实验的比较

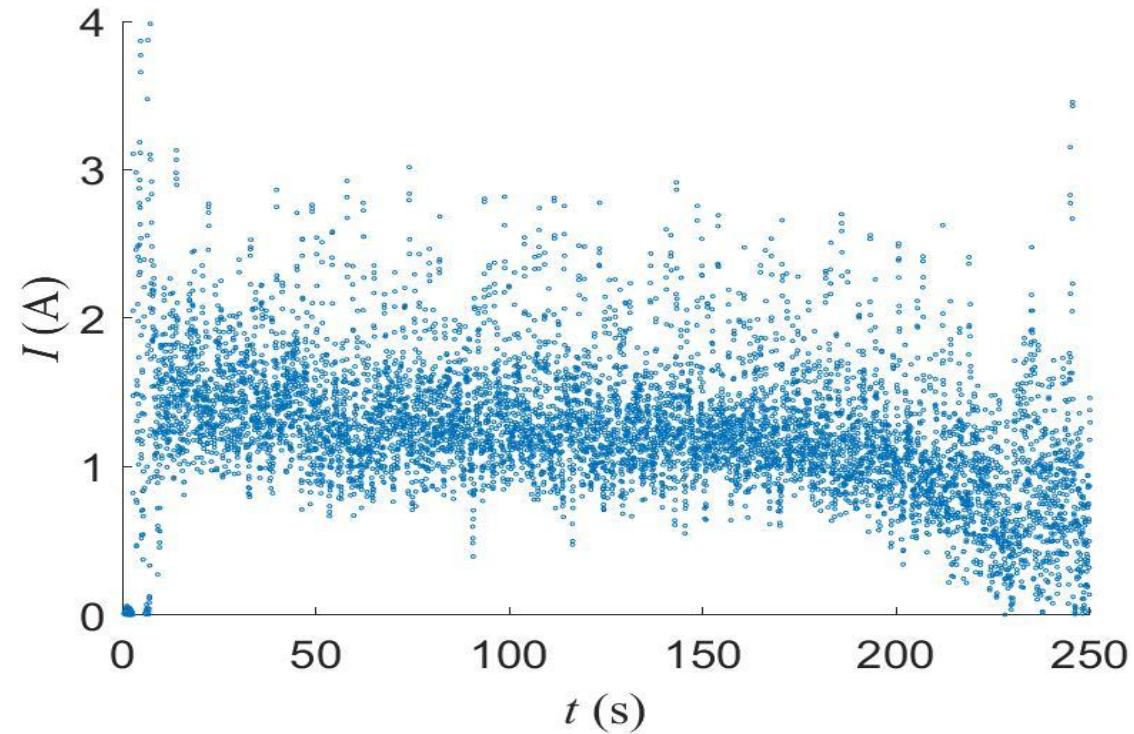


实验与理论一致!

实验结果验证



公转角速度随时间变化关系

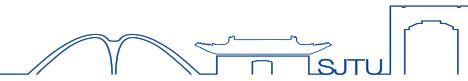


电流随时间变化关系

不同段的电流变化:

上升段和匀速段: 电流基本保持不变
迅速下降段: 电池耗尽, 电流减小

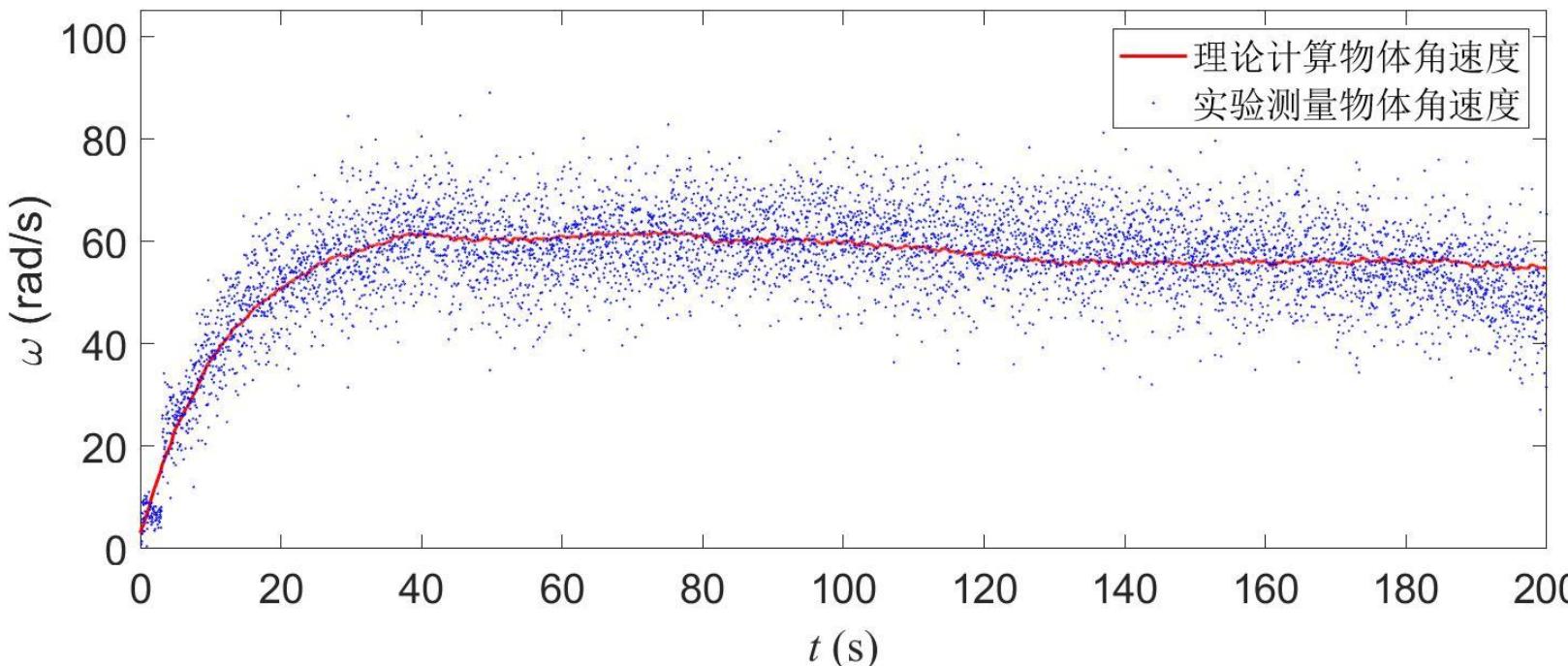
自转角速度随时间变化关系



$$M_{1e_1}(i(t)) - M_{1e_3}(i(t))\tan\theta + M_2(i(t)) - \frac{M_f(t)}{\cos\theta} = [J_1\left(1 - \frac{r_1}{r_A}\sin\theta\right) + J_2\frac{r_1}{r_A}\sin\theta]\cdot\varphi''(t)$$

$$M_{1e_1}(i(t)) - M_{1e_3}(i(t))\tan\theta + M_2(i(t)) - (M_0 + \alpha\omega(t))\sec\theta = [J_1\left(1 - \frac{r_1}{r_A}\sin\theta\right) + J_2\frac{r_1}{r_A}\sin\theta]\cdot\varphi''(t)$$

$$M_0 = 1.548 \times 10^{-6} \text{ N}\cdot\text{m}, \quad \alpha = -0.166 \times 10^{-6} \text{ N}\cdot\text{m}\cdot\text{s}$$

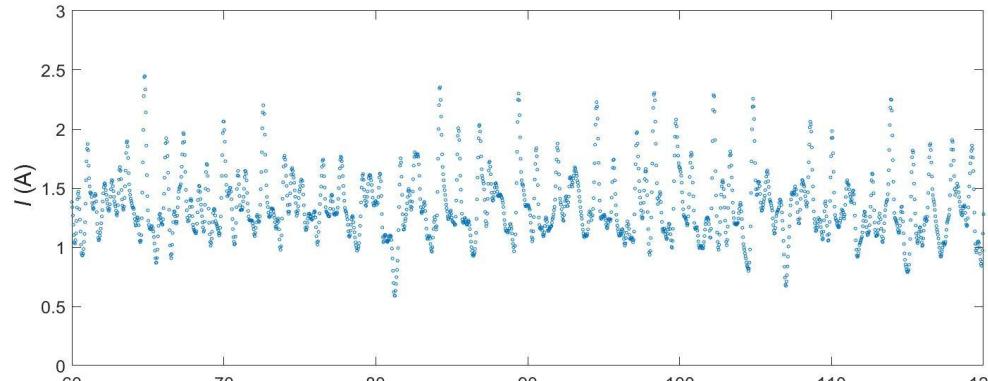


根据理论计算得到角速度和实验测量相近，趋势相同。实验和理论符合很好！

表面性质周期性变化的影响

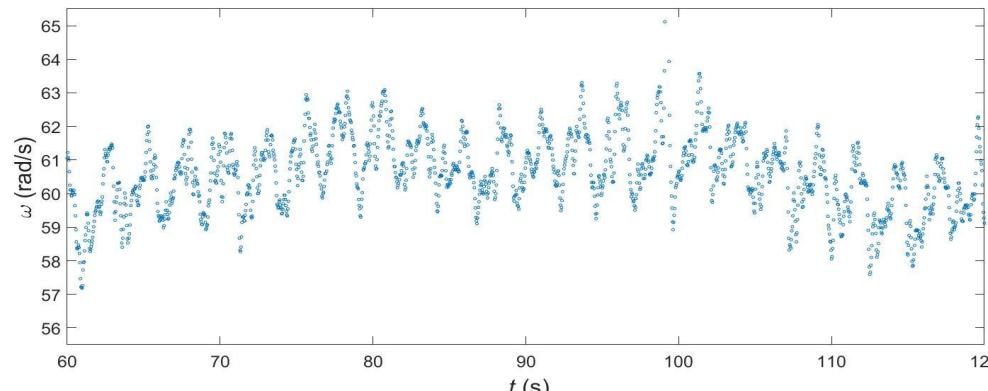
接触电阻周期性变化→电流周期性变化→角速度周期性变化

电流-时间关系



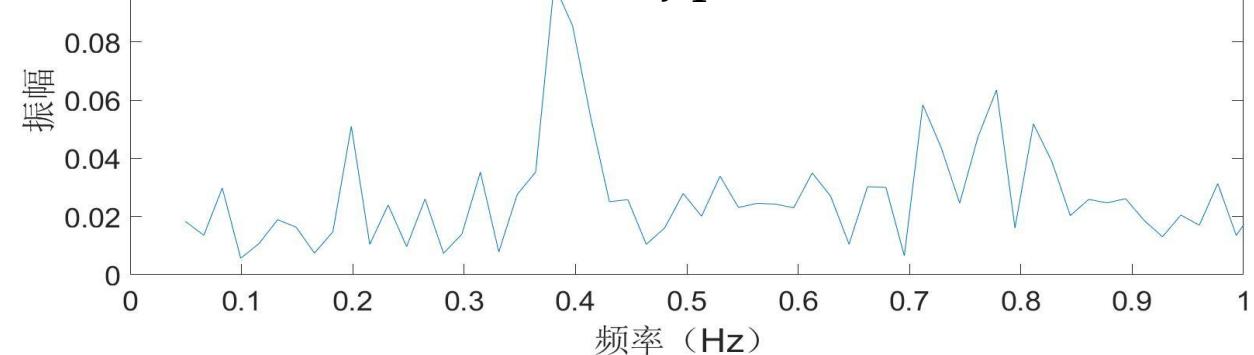
FFT

角速度-时间关系

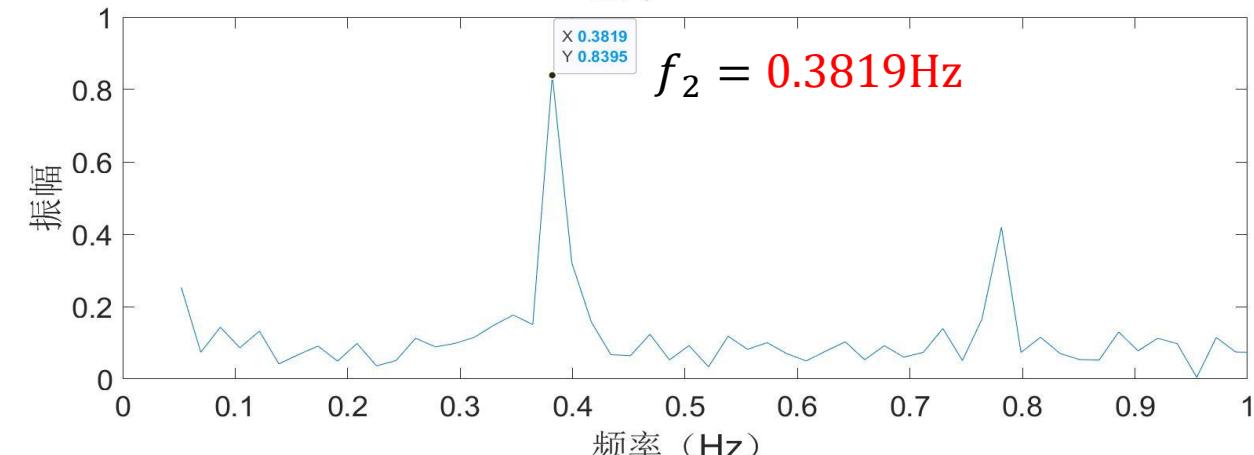


FFT

$f_1 = 0.3808\text{Hz}$



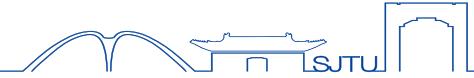
$f_2 = 0.3819\text{Hz}$



匀速运动段公转角速度 $\Omega = \omega \times \frac{r_1}{r_A} \approx 2.40 \text{ rad/s}$, 对应频率 $f = \frac{\Omega}{2\pi} = 0.3819\text{Hz}$

符合很好！

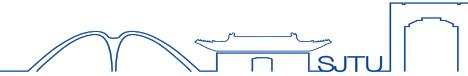
结论



- 研究了回路电阻、磁场大小、磁铁参数、电池参数等对物体运动的影响
- 回路电阻分析：接触电阻+铝箔电阻+电池内阻+磁铁电阻+测压导线电阻
- 动力来源：
 - 磁铁上电流受到安培力力矩 $M_1(t)$
 - 电池内部电流受到的安培力力矩 $M_2(t)$
- 阻力因素：
 - 磁铁和铝箔接触面滚动摩擦 $M_f(t)$
- 表面性质的周期性变化导致电流和角速度周期性变化
- 减速段由于电池电量耗尽



参考文献



- 磁铁电流受力分析: [1] Brauer, H., Ziolkowski, M., Porzig, K., & Toepfer, H. (2011). Modelling and simulation of a simple homopolar motor of Faraday's type. *Facta Universitatis – Series: Electronics and Energetics*, 24(2), 221–242.
- 磁铁磁场分布: [2] 周恩权, 郑仲桥, 张燕红 & 王奇瑞. (2017). 圆柱形永磁体磁场建模及仿真研究. *河南科技*(21), 139–143
- 涡流的影响: [3] González, M. I. (2004). Experiments with eddy currents: the eddy current brake. *European journal of physics*, 25(4), 463.

谢谢！





漂移现象

$$\begin{aligned} -M_1 e_1(t) - M_2(t) + M_f(t) \cos \theta + (f_{1y} r_A + f_{2y} r_B) \sin \theta &= J_1(-\omega' + \Omega' \sin(\theta)) \quad ① \\ [(N_1 r_A + N_2 r_B) - m g r_c] &= [J_1(-\omega + \Omega \sin \theta) \Omega \cos \theta - J_2 \Omega^2 \sin \theta \cos \theta] \quad ② \\ [+ M_1 e_3(t) + M_f \sin \theta - (f_{1y} r_A + f_{2y} r_B) \cos \theta] &= -J_2 \Omega' \cos \theta \quad ③ \\ N_1 + N_2 &= m g \quad ④ \end{aligned}$$

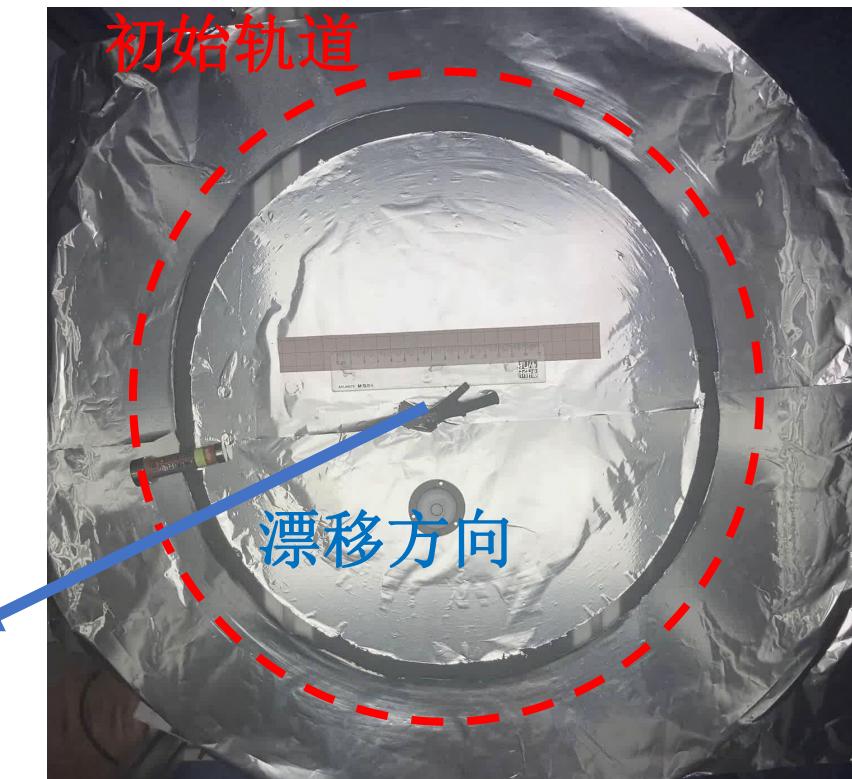
静不定问题！

角速度大时，静摩擦力不足以满足上述方程。

两侧同时达到滑动条件：

$$\sqrt{f_{1x}^2 + f_{1y}^2} = \mu N_1 \quad \sqrt{f_{2x}^2 + f_{2y}^2} = \mu N_2$$

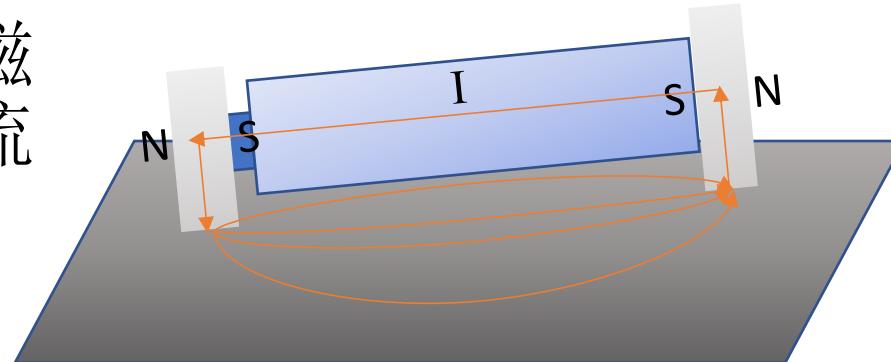
发生漂移！



理论补充A——铝箔上电流产生磁场的二级影响



- 倾斜物体 → 近似于点接触
- 铝箔视作无穷大平面，内侧磁铁处流入电流，外侧磁铁处流出电流。
- 电流分布：



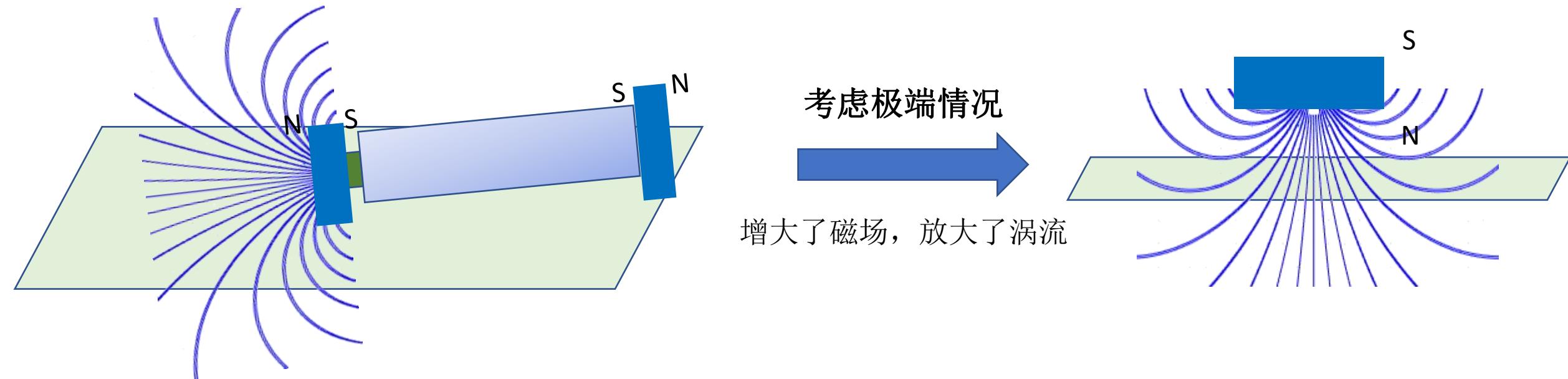
$$j_x = j_{1x} + j_{2x} = \frac{I}{2\pi\sqrt{(x+a)^2 + y^2}} \frac{x+a}{\sqrt{(x+a)^2 + y^2}} + \frac{I}{2\pi\sqrt{(a-x)^2 + y^2}} \frac{a-x}{\sqrt{(a-x)^2 + y^2}}$$

$$j_y = j_{1y} + j_{2y} = \frac{I}{2\pi\sqrt{(x+a)^2 + y^2}} \frac{y}{\sqrt{(x+a)^2 + y^2}} + \frac{I}{2\pi\sqrt{(a-x)^2 + y^2}} \frac{y}{\sqrt{(a-x)^2 + y^2}}$$

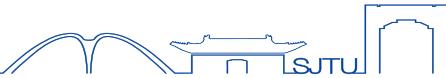
- 根据电流分布，计算得到铝箔上电流产生磁场 $B \approx 0.003T$, 可以近似忽略

理论补充B——涡流的影响

- 涡流：铝箔上磁通量的变化产生涡流，进而产生能量损耗



理论补充B——涡流的影响



1. 电涡流的贯穿深度 h 和径向范围

求贯穿深度可以用平面电磁波入射到半无限大导体来进行处理，根据麦克斯韦方程及边界条件，可以写出方程：

$$\begin{cases} \nabla \times H = J \\ \nabla \times J = -\frac{\mu_0 \mu_r}{\rho} \frac{\partial H}{\partial t} \\ H_y|_{x=0} = H_0 e^{j\omega t} \\ H_x|_{y=0} = H_z|_{x=0} = 0 \end{cases} \quad (1)$$

得：

$$\begin{cases} \frac{\partial H_y}{\partial x} = J_y \\ \frac{\partial J_z}{\partial x} = \frac{\mu_0 \mu_r}{\rho} \frac{\partial H_y}{\partial t} \\ H_y(0, t) = H_0 e^{j\omega t} \\ H_z|_{y=0} = H_x|_{x=0} \end{cases} \quad (2)$$

式中 H —— 磁场强度，

J —— 电流；

μ_0, μ_r —— 真空和被测体的导磁系数。

解 (2) 式得：

$$H_y(x, t) = H_0 e^{-bx} e^{j(\omega t - bx)} \quad (3)$$

$$B_y(x, t) = \mu_0 \mu_r H_0 e^{-bx} e^{j(\omega t - bx)} \quad (4)$$

式中 $b = \sqrt{\frac{\mu_0 \mu_r \omega}{2\rho}}$ ，当 $x = \frac{1}{b}$ 时，幅衰减到 e 分之一，因此得贯穿深度：

$$h = \frac{1}{b} = \sqrt{\frac{2\rho}{\mu_0 \mu_r \omega}} = 5000 \sqrt{\frac{\rho}{\mu_r f}} \quad (5)$$

式中 ρ 为导体电阻率 (欧姆·毫米²/米)。

贯穿深度即为磁场强度的趋肤深度，即电涡流贯穿深度。

2. 电涡流的损耗功率

设有一如图 3 所示的回路元，其厚度为 h （由涡流贯穿深度所决定），回路元的电阻为 $R = \rho \cdot l / s$ ，则所计算范围内电涡流的损耗功率为

$$P = \int dP = \int d \frac{E^2}{R} \quad (7)$$

式中 E 为感应电动势的有效值； R 值为：

$$R = \rho \frac{l}{s} = \rho \cdot \frac{2\pi r}{h \cdot dr} \quad (8)$$

因为感应电动势

$$e = -\frac{d\Phi}{dt} = -\pi r^2 \frac{dB}{dt}$$

如设

$$B = B_m \sin \omega t$$

则

$$e = -\pi r^2 \omega B_m \cos \omega t$$

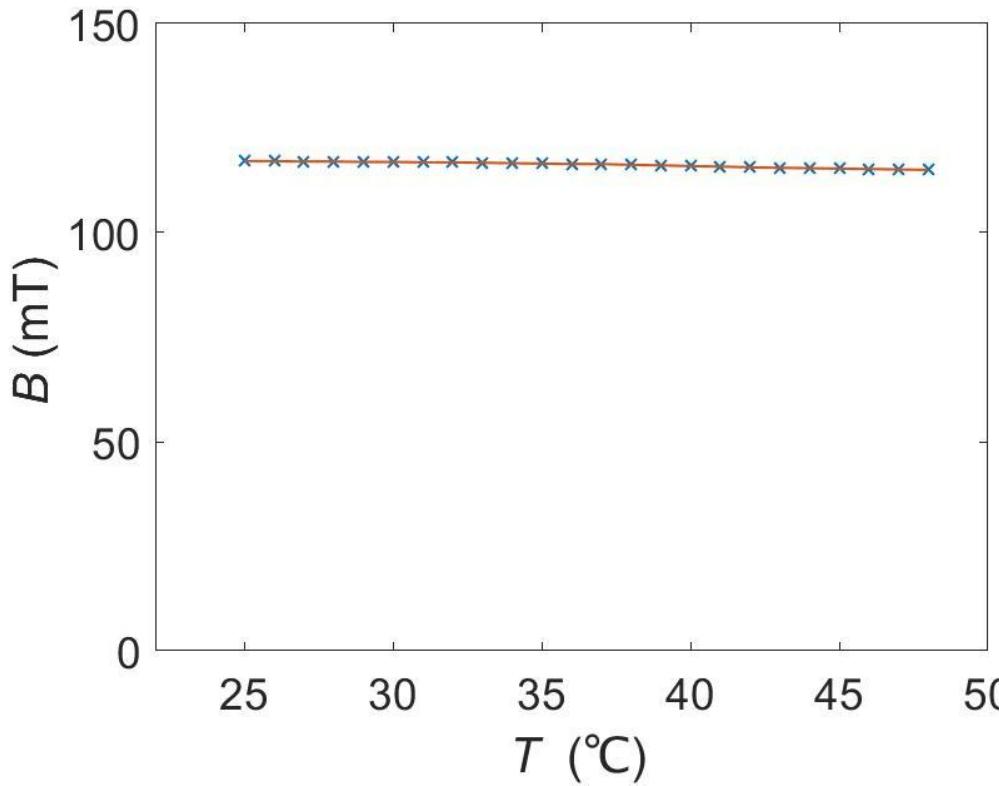
所以在回路元处感应电动势的有效值为：

$$E = \frac{\pi r^2 \omega B_m}{\sqrt{2}}$$

涡流损耗占比 $\frac{P_{eddy}}{UI} \sim 10^{-7}$

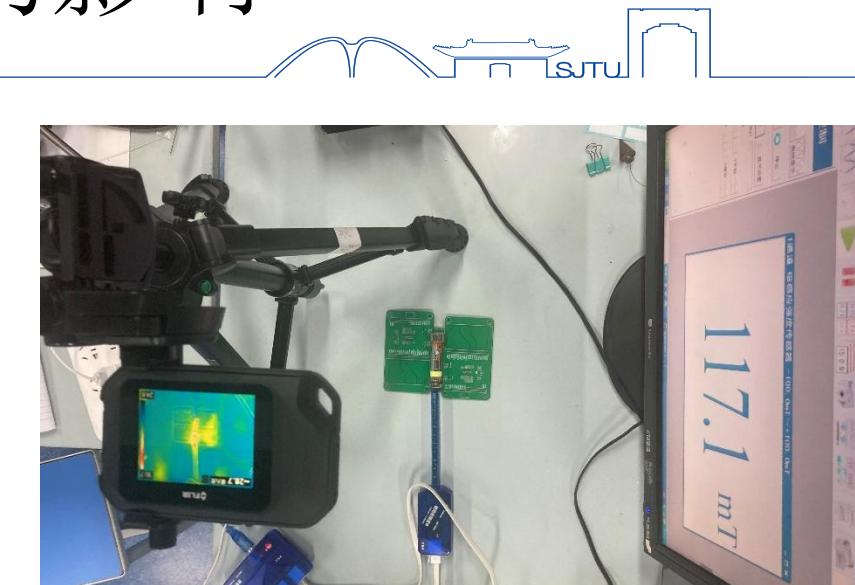
忽略涡流的影响！

理论补充C——磁铁升温对磁场的影响



- 一定距离处磁场强度和温度的关系图

实验测量最大温度40°C，该范围内磁场变化小于0.94%！



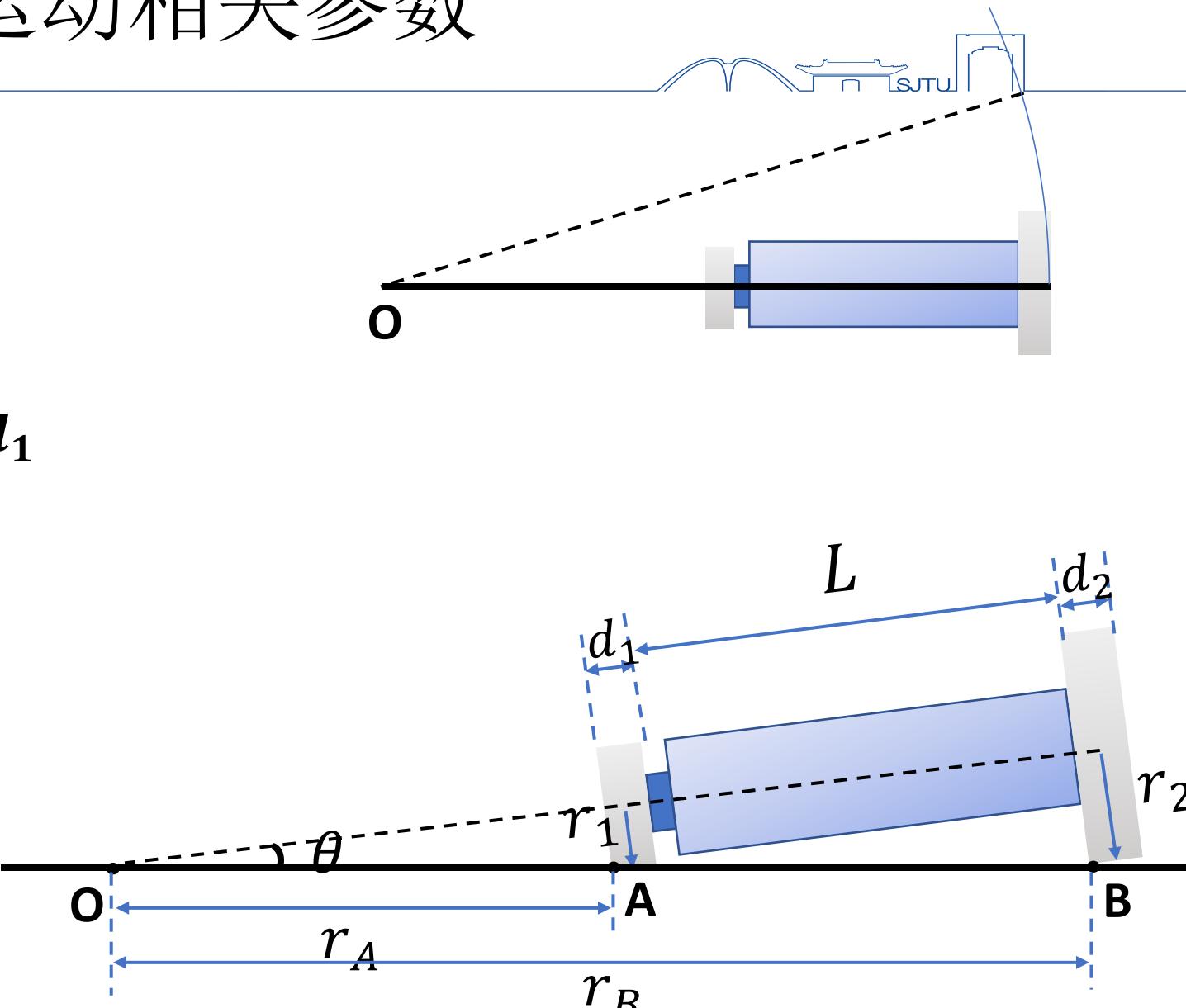
Grade 规格	Br		Temperature 工作温度 /(L/D=0.7)
	kGs	T	
N52	14.9~14.2	1.49~1.42	≤80
N50	14.5~14.0	1.45~1.40	≤80
N48	14.1~13.6	1.41~1.36	≤80
N45	13.8~13.2	1.38~1.32	≤80
N42	13.3~12.8	1.33~1.28	≤80
N40	13.1~12.5	1.31~1.25	≤80
N38	12.7~12.3	1.27~1.23	≤80

附录A——物体圆周运动相关参数

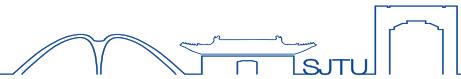
利用几何关系+纯滚条件：

$$\left\{ \begin{array}{l} r_1 = r_A \sin(\theta) \\ r_2 = r_B \sin(\theta) \\ r_B \cos(\theta) - r_A \cos(\theta) = L + d_1 \end{array} \right.$$

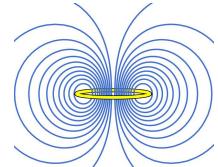
$$\rightarrow \left\{ \begin{array}{l} r_A = r_1 \sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1} \\ r_B = r_2 \sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1} \\ \Omega = \frac{d\Phi}{dt} = \frac{r_1}{r_A} \omega = \frac{1}{\sqrt{\left(\frac{L + d_1}{r_2 - r_1} \right)^2 + 1}} \omega \end{array} \right.$$



附录B——纽扣电池产生的磁场模型



- 分子电流假设:



- 纽扣磁铁→有一定宽度的环形电流→对无厚度环形电流积分
- 侧面环形电流的面电流密度: $J_m = \frac{B_r}{\mu_0}$

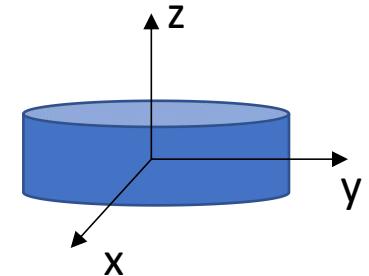
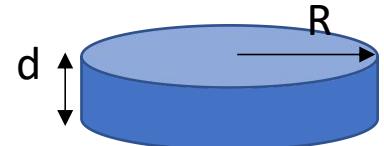
电流环产生磁感应强度 \vec{B} 在 x,y,z 的分量分别为:

$$\left\{ \begin{array}{l} dB_x = \frac{\mu_0 J_m dz_0}{4\pi} \int_0^{2\pi} \frac{(z - z_0)r_0 \cos \theta}{K} d\theta \\ dB_y = \frac{\mu_0 J_m dz_0}{4\pi} \int_0^{2\pi} \frac{(z - z_0)r_0 \sin \theta}{K} d\theta \\ dB_z = \frac{\mu_0 J_m dz_0}{4\pi} \left\{ \int_0^{2\pi} \frac{-r_0(x - r_0 \cos \theta) \cos \theta}{K} d\theta \right. \\ \left. - \int_0^{2\pi} \frac{r_0(y - r_0 \sin \theta) \sin \theta}{K} d\theta \right\} \end{array} \right. \quad (5)$$

$$K = \left[(x - r_0 \cos \theta)^2 + (y - r_0 \sin \theta)^2 + (z - z_0)^2 \right]^{3/2}$$

圆柱永磁体空间磁感应强度为:

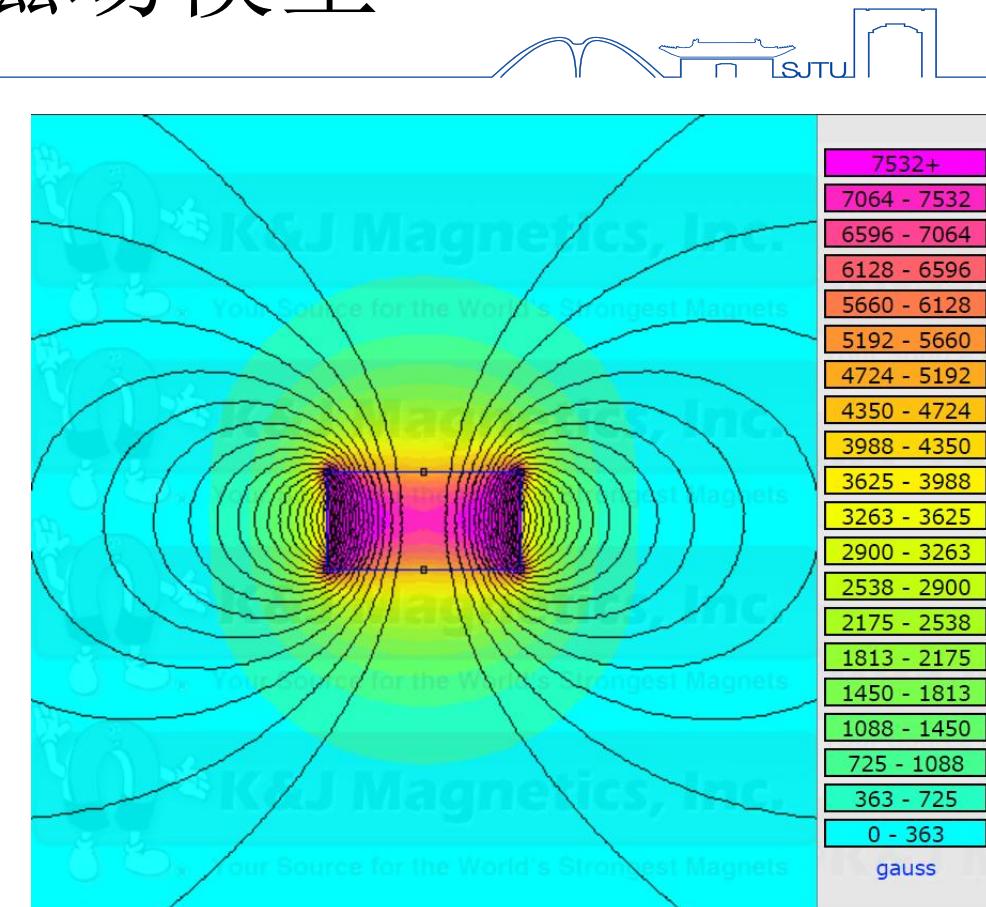
$$\left\{ \begin{array}{l} B_x = \frac{\mu_0 J_m dz_0}{4\pi} \int_0^h \int_0^{2\pi} \frac{r_0(z - z_0) \cos \theta}{K} d\theta \\ B_y = \frac{\mu_0 J_m dz_0}{4\pi} \int_0^h \int_0^{2\pi} \frac{r_0(z - z_0) \sin \theta}{K} d\theta \\ B_z = \frac{\mu_0 J_m dz_0}{4\pi} \left\{ \int_0^h \int_0^{2\pi} \frac{-r_0(x - r_0 \cos \theta) \cos \theta}{K} d\theta \right. \\ \left. - \int_0^h \int_0^{2\pi} \frac{r_0(y - r_0 \sin \theta) \sin \theta}{K} d\theta \right\} \end{array} \right. \quad (6)$$



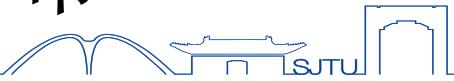


附录B——纽扣电池产生的磁场模型

- 考虑到对称性， X , Y 方向等价，综合成径向 ρ 向磁感应强度
- 根据上述方程，利用Matlab数值方法，得到空间中各个点的磁感应强度。（Matlab程序见附页）

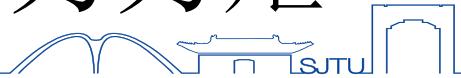


附录C——计算圆柱形磁铁空间磁场分布



```
• function B=FieldSolenoid(a,L,Br,rho,z)
• zp=z+L./2; zm=z-L./2;
• kp=sqrt(4.*a.*rho./((a+rho).^2+zp.^2)); km=sqrt(4.*a.*rho./((a+rho).^2+zm.^2));
• if(rho==0) % Field on the solenoid axis
•     B(1)=0;
• else
•     B(1)=Br./pi.*sqrt(a./rho).*(((kp.^2-2)./(2.*kp).*EllipticK(kp)+...
•         EllipticE(kp)./kp)-((km.^2-2)./(2.*km).*EllipticK(km)+EllipticE(km)./km));
• end
• Bo=Br./pi;
• kp=sqrt((zp.^2+(a-rho).^2)./(zp.^2+(a+rho).^2));
• km=sqrt((zm.^2+(a-rho).^2)./(zm.^2+(a+rho).^2));
• betap=zp./sqrt(zp.^2+(rho+a).^2); betam=zm./sqrt(zm.^2+(rho+a).^2);
• g=(a-rho)./(a+rho);
•
• if(kp<1)
•     B(2)=Bo.*a./(a+rho).*1./(g+1).* (betap.* (EllipticK(sqrt(1-kp.^2))+g.*EllipticPi(1-g.^2,sqrt(1-kp.^2)))...
•         -betam.* (EllipticK(sqrt(1-km.^2))+g.*EllipticPi(1-g.^2,sqrt(1-km.^2)))); 
• else
•     B(2)=Bo.*a./(a+rho).*1./(g.*(g+1)).*(betap./kp.* (EllipticK(sqrt((-1+kp).*(kp+1))./kp).*g+...
•         EllipticPi((-1+g).* (g+1)./g.^2, sqrt((-1+kp).* (kp+1))./kp))...
•         -betam./km.* (EllipticK(sqrt((-1+km).* (km+1))./km).*g+EllipticPi((-1+g).* (g+1)./g.^2, sqrt((-1+km).* (km+1))./km)));
• end
```

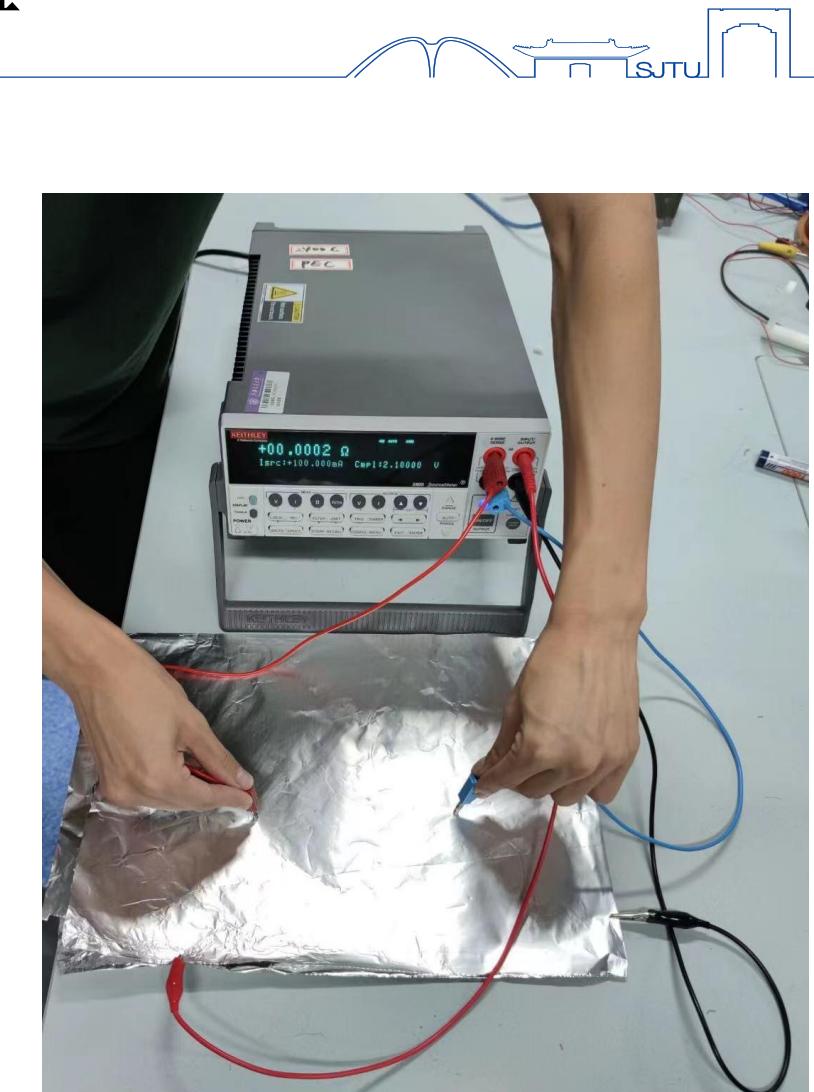
附录D——计算电池上电流受到的安培力力矩



```
• % 长度0.049m, 半径0.0072m
• r1=0.075; d1=0.006 % 磁铁半径和厚度
• r2=0.01; d2=0.005 % 磁铁半径和厚度
• L=0.049; R=0.0072; % 电池长度和半径
• I0=5.6; % 电流
• n=100; % 数值方法, 分成n份求数值积分
• dL=L/n; dR=R/n;
• M_process=0;
• % FieldR=zeros(1,2)
• for jj=1:n
•     Rjj=jj*dR;
•     for ii=1:n
•         Lii=ii*dL;
•         Field=FieldSolenoid(r1,d1,1.22,Rjj,d1/2+Lii)+FieldSolenoid(r2,d2,1.47,Rjj,d2/2+L-Lii);
•         M_process=M_process+dL*dR*Field(1)*Rjj^2;
•     end
• end
• Moment=M_process*I0/R^2
```

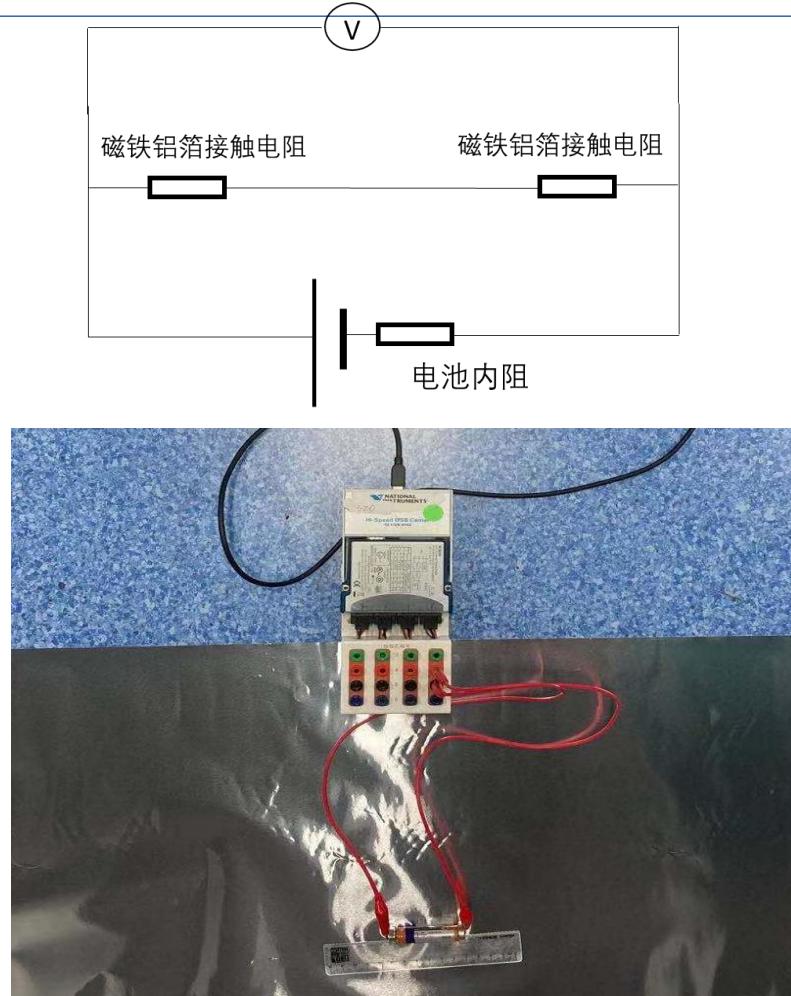
附录E——四线法测量铝箔电阻

- 对铝箔电阻进行测量与估计：
- 采用四线接法测铝箔电流
 - $R < 0.0002 \Omega$
- 对比电池内阻 R 约等于 0.5Ω , 可以忽略不计

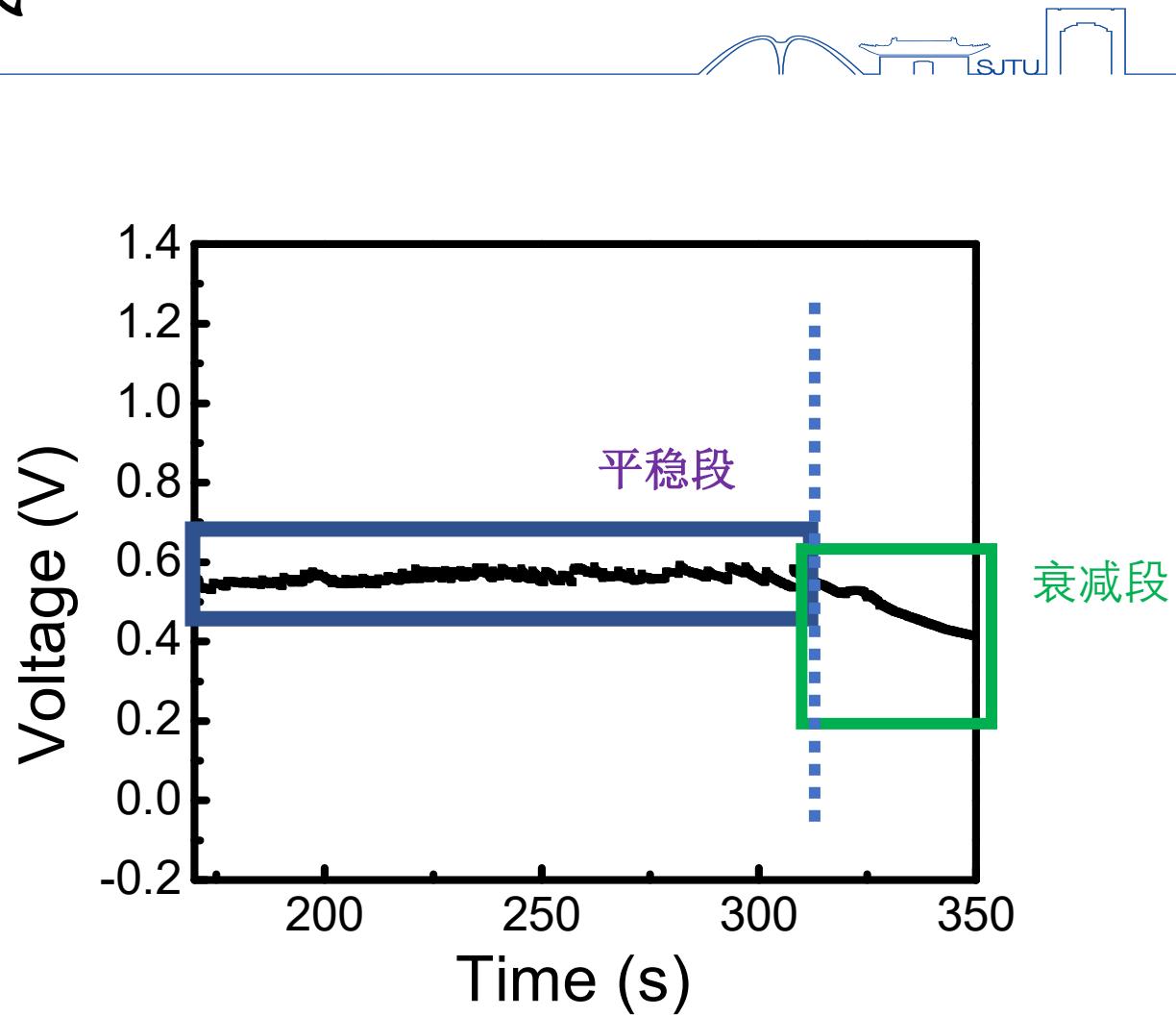




实验测定电池放电曲线



电流曲线测定示意图



电流放电曲线