MOMENTUM OF GAUSSIAN WAVEPACKET

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Compare between directional momentum and spread momentum

Directional momentum

Take 1D for example, if we start a Gaussian wavepacket,

$$\Psi(x) = A \times e^{-x^2/2\sigma_x^2} \times e^{ik_x x}$$

First, with probability density normalization.

$$1 = \int_{-\infty}^{\infty} |\psi|^2 dx \quad => \quad 1 = \int A^2 e^{-x^2/\sigma_x^2} dx \quad => \quad A = \sqrt{\frac{1}{\pi}\frac{1}{\sigma_x}}$$

Thus,

$$\Psi(x) = \sqrt{\frac{1}{\pi}\frac{1}{\sigma_x}} \times e^{-x^2/2\sigma_x^2} \times e^{ik_x x}$$

Moment operator, $\hat{p} = i\hbar\nabla$.

$$<\hat{p}>= \int_{-\infty}^{\infty} \psi^\dagger \hat{p}\psi dx = \int dx \sqrt{\frac{1}{\pi}\frac{1}{\sigma_x}} \times e^{-x^2/2\sigma_x^2} \times e^{-ik_x x} i\nabla(\sqrt{\frac{1}{\pi}\frac{1}{\sigma_x}} \times e^{-x^2/2\sigma_x^2} \times e^{ik_x x})$$

$$<\hat{p}>= \int_{-\infty}^{\infty} dx \frac{1}{\pi}\frac{1}{\sigma_x^2} i\hbar(\frac{1}{2\sigma_x^2}2x + ik)e^{-\frac{x^2}{\sigma_x^2}} = \hbar k$$

The role of uncertainty principle

$$\Delta x \Delta p > \frac{h}{4\pi}$$

Spread Momentum

If we start with

$$\Psi(x) = \sqrt{\frac{1}{\pi}\frac{1}{\sigma_x}} \times e^{-x^2/2\sigma_x^2} \times e^{ik_x x}$$

Then, "Spread momentum" $\vec{p} = \frac{\hbar}{2} * \frac{1}{\sigma_x}$
If we want p to be much smaller than directional momentum,

then

$$\frac{\hbar}{2}\frac{1}{\sigma_x} << \hbar k \qquad => \qquad 2\sigma_x k >> 1$$

The same derivation in 2D

$$\Psi(x,y) = \frac{1}{\pi}\frac{1}{\sqrt{\sigma_x \sigma_y}} \times e^{-x^2/2\sigma_x^2 - y^2/2\sigma_y^2} \times e^{ik_y y}$$

In this expression, electron has a directional momentum and a spread momentum.

Directional Momentum,

$$p_d = \hbar k_y$$

Corresponding Kinetic Energy,

$$E_d = \frac{\hbar^2 k_y^2}{2m}$$

Spread Momentum in Orthogonal direction,

$$p_s = \frac{\hbar}{2}\frac{1}{\sigma_x}$$

Corresponding Kinetic Energy,

$$E_s = \frac{\hbar^2}{8m\sigma_x^2}$$

In our simulation, we choose A=200∼ 400, KK=40, $2\sigma_x^2 = 0.05 \quad => \quad \sigma_x = 0.1581$

Thus, $E_d = 800, E_s = 5, A = 200$. Disorder has a strength of 0.1*A=20.

Diffraction will have response to disorder, but superwire with a large momentum in y direction will ignore weak disorder.