1. Consider the expression $A \rightarrow B \rightarrow C \rightarrow D$.

   How many possible different sentences can we obtain from it by inserting parentheses?

   Eleven sentences:

   1. $A \rightarrow B \rightarrow C \rightarrow D$
   2. $A \rightarrow B \rightarrow (C \rightarrow D)$
   3. $A \rightarrow (B \rightarrow C) \rightarrow D$
   4. $(A \rightarrow B) \rightarrow C \rightarrow D$
   5. $(A \rightarrow B \rightarrow C) \rightarrow D$
   6. $A \rightarrow (B \rightarrow C \rightarrow D)$
   7. $A \rightarrow (B \rightarrow (C \rightarrow D))$
   8. $A \rightarrow ((B \rightarrow C) \rightarrow D)$
   9. $(A \rightarrow (B \rightarrow C)) \rightarrow D$
   10. $((A \rightarrow B) \rightarrow C) \rightarrow D$
   11. $(A \rightarrow B) \rightarrow (C \rightarrow D)$

   Sentences one, six, and seven are equivalent by right associativity of implication.
   By the same token, sentences five and nine are also equivalent.

2. Find the "distributive laws" for the following cases.

   1. $A \rightarrow (B \vee C) \equiv \neg A \vee (B \vee C) \equiv (\neg A \vee B) \vee (\neg A \vee C) \equiv (A \rightarrow C) \wedge (A \rightarrow B)$

   2. $(A \vee B) \rightarrow C \equiv \neg (A \vee B) \vee C \equiv (\neg A \wedge \neg B) \vee C \equiv (\neg A \vee C) \wedge (\neg B \vee C) \equiv (A \rightarrow C) \wedge (B \rightarrow C)$

   3. $(A \wedge B) \rightarrow C \equiv \neg (A \wedge B) \vee C \equiv (\neg A \vee \neg B) \vee C \equiv (\neg A \vee C) \vee (\neg B \vee C) \equiv (A \rightarrow C) \vee (B \rightarrow C)$

3. Using the "distributive laws" of the previous question, push $\rightarrow$ all the way in, in the following sentences.

   1. $A \vee B \rightarrow C \vee D \equiv [A \rightarrow (c \vee D)] \wedge [A \rightarrow B] \equiv (A \rightarrow B) \wedge (A \rightarrow C) \wedge (A \rightarrow D)$

   2. $(A \vee B) \rightarrow (C \wedge D) \equiv [A \rightarrow (C \wedge D)] \wedge [B \rightarrow (C \wedge D)] \equiv (A \rightarrow C) \wedge (A \rightarrow D) \wedge (B \rightarrow C) \wedge (B \rightarrow D)$

3. $(A \wedge B) \rightarrow (C \vee D) \equiv [A \rightarrow (C \vee D)] \vee [B \rightarrow (C \vee D)] \equiv [(A \rightarrow C) \wedge (A \rightarrow D)] \vee [(B \rightarrow C) \wedge (B \rightarrow D)]$

4. $(A \wedge B) \rightarrow (C \wedge D) \equiv [A \rightarrow (C \wedge D)] \vee [B \rightarrow (C \wedge D)] \equiv [(A \rightarrow C) \wedge (A \rightarrow D)] \vee [(B \rightarrow C) \wedge (B \rightarrow D)]$

4. Simplify the following sentences.

Sentence 4: $(A \rightarrow B) \vee (B \rightarrow A)$

$$(A \rightarrow B) \vee (B \rightarrow A) \equiv (\neg A \vee B) \vee (A \vee \neg B) \equiv (\neg B \vee A) \vee (\neg B \vee B) \equiv A \vee \neg A$$

Sentence 5: $\neg(A \rightarrow B) \vee \neg(B \rightarrow A)$

$$\neg(A \rightarrow B) \vee \neg(B \rightarrow A) \equiv \neg(\neg A \vee B) \vee \neg(\neg B \vee A) \equiv (A \wedge \neg B) \vee (B \wedge \neg A) \equiv \neg(A \Leftrightarrow B) \equiv A \Leftrightarrow \neg B$$

Sentence 6: $(A \Leftrightarrow B) \Leftrightarrow A$

$$(A \Leftrightarrow B) \Leftrightarrow A \equiv A \Leftrightarrow B$$

Sentence 9: $(A \rightarrow B) \Leftrightarrow (B \rightarrow A)$

$$(A \rightarrow B) \Leftrightarrow (B \rightarrow A) \equiv [\neg(\neg A \vee B) \vee (\neg B \vee A)] \wedge [\neg(\neg B \vee A) \vee (\neg A \vee B)]$$

$$[(A \wedge \neg B) \vee (\neg B \vee A)] \wedge [(B \wedge \neg A) \vee (\neg A \vee B)] \equiv (A \wedge \neg B) \vee (B \wedge \neg A) \equiv \neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$

Sentence 11: $(A \vee B) \Leftrightarrow (A \wedge B)$

$$(A \vee B) \Leftrightarrow (A \wedge B) \equiv [\neg(A \vee B) \vee (A \wedge B)] \wedge [\neg(A \wedge B) \vee (A \vee B)] \equiv [(\neg A \vee \neg B) \vee (A \wedge B)] \wedge [(\neg A \vee \neg B) \vee (A \vee B)]$$

$$[(\neg A \wedge \neg B) \vee (A \wedge B)] \equiv [A \vee (\neg A \wedge \neg B)] \wedge [B \vee (\neg A \wedge \neg B)] \equiv [(A \wedge \neg A) \vee (A \wedge \neg B)] \wedge [(B \wedge \neg B) \vee (B \wedge \neg A)]$$

$$\neg(A \rightarrow B) \vee \neg(B \rightarrow A)$$