

Circle-String Universe Cosmology: A Formalized Model Unifying Dark Matter, Dark Energy, and the Hubble Tension

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Abstract

The standard Λ CDM cosmological model is facing the severe challenge of the Hubble Tension – a 5.3σ discrepancy exists between the Hubble constant measured from the early universe (CMB) and the late universe (distance ladder). This paper proposes the "Circle-String-Bulk" cosmological model, which provides a unified solution to this problem by introducing the "anti-source particle" ($\bar{\phi}$) and its interaction phase transition occurring at redshift $z=5161$. Based on geometric realism, relational ontology, and the cognitive simplification principle, and starting from the symmetry breaking of the "string singularity" ($0 \rightarrow \phi + \bar{\phi}$), the model derives through compulsory logical

deduction that: before the phase transition, the anti-source particle interacts with the plasma through an energy barrier $\Delta E = 2.07 \text{ eV}$, and its interaction energy manifests as Early Dark Energy ($f_{\text{EDE}} \approx 10\%$), elevating H_0 to 70.7 km/s/Mpc; after the phase transition, it becomes Cold Dark Matter. The model shows good agreement with Planck data ($\chi^2 / \text{dof} = 1.33 - 7.52$) and predicts Λ -DM coupling ($Q > 0$), dynamical dark energy ($w(z) \neq -1$), and primordial non-Gaussianity ($f_{\text{NL}}^{\text{local}} = 2.5 \pm 1.5$), providing decisive tests for next-generation observations.

Keywords: cosmology: theory — dark matter — dark energy — early universe — Hubble tension

1. Introduction

1.1 The Hubble Tension: Observational Fact of a 5.3σ Discrepancy

The standard Λ CDM cosmological model has achieved remarkable success in describing the formation of large-scale structure and the anisotropies of the Cosmic Microwave Background (CMB) radiation [1]. However, as observational precision improves, internal tensions within the model have become increasingly significant. The most prominent is the Hubble Tension – a 5.3σ discrepancy exists between measurements based on the early universe CMB (Planck Collaboration: $H_0 = 67.66 \pm 0.42$ km/s/Mpc) and those from the

late-universe distance ladder (SH0ES team: $H_0 = 73.04 \pm 1.04$ km/s/Mpc) [2].

This tension strongly suggests the possible existence of missing physics in the standard model.

1.2 Limitations of Existing Solutions: Challenges for Early Dark Energy

Models

To alleviate this tension, "Early Dark Energy" (EDE) scenarios have been extensively studied [3], which introduce a transient extra energy component prior to recombination ($z \sim 10^4$). However, most EDE models face three major challenges:

- **Ad-hoc Nature:** They lack clear particle physics motivation, often postulating arbitrary scalar field potential forms.
- **Worsening of the S_8 Tension:** While increasing H_0 , they often exacerbate the tension with weak gravitational lensing measurements (e.g., DES [4], KiDS) regarding the matter clustering amplitude S_8 .
- **Degradation of Fit Quality:** Introducing EDE typically comes at the cost of degrading the fit to the fine-scale structure of the CMB power spectrum.

1.3 Innovation of the Anti-Sourceon Model: From Philosophical Concept to Specific Physical Mechanism

The "Circle-String Universe" model proposed in this paper aims to fundamentally break through these limitations. Its core innovation lies in

introducing the "**Anti-sourceon**" $\bar{\phi}$, which is not an ad-hoc scalar field but an entity with definite particle attributes. Its cosmological evolution—acting as EDE in the early universe and as dark matter (DM) later—is driven by a specific physical mechanism (**interaction phase transition**), thereby **unifying** the solution to the Hubble Tension with the nature of dark matter within a self-consistent framework.

This paper is structured as follows: Chapter 2 elaborates the philosophical foundations and formal system; Chapter 3 performs mandatory derivations from the meta-postulates; Chapter 4 details the microscopic mechanism of the anti-sourceon phase transition; Chapter 5 presents numerical methods and observational verification; Chapter 6 provides the model's self-prediction and new physics forecasts; Chapter 7 conducts a comprehensive self-consistency analysis; Chapter 8 summarizes and outlines future work.

2. Theoretical Construction: Formal System

2.1 Philosophical Foundations and Meta-Postulates

The model is built upon three meta-postulates:

1. **Geometric Realism:** The deep laws of the universe can be effectively described using geometric language.

2. **Relational Ontology:** "Relations" are prior to, or at least equally fundamental as, "substances". Existence is constituted by networks of relations.
3. **Cognitive Simplification Principle:** Complex systems can be constructed starting from a few core concepts.

2.2 Formal Definition of Core Concepts

2.2.1 The Bulk

- **Definition:** The four-dimensional spacetime manifold \mathcal{M} and its metric $g_{\mu\nu}$.
- **Mathematical Formulation:** Adopts the FLRW metric:

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

- **Physical Interpretation:** The stage for dynamical evolution.

2.2.2 The Circle

- **Definition:** The evolutionary trajectory of the universe in the phase space (a, H) .
- **Mathematical Formulation:** The Friedmann equations:

$$\$ H^2 = \frac{8\pi G}{3c^2} \rho - \frac{kc^2}{a^2} \$$$

$$\$ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho + 3p) \$$$

Physical Interpretation : The fate trajectory of the universe as a whole.

$x^\mu(\lambda)$ of a material entity.

- **Mathematical Formulation:** Timelike curves satisfying the geodesic equation or equations of motion $\frac{DU^\mu}{D\lambda} = F^\mu$.
- **Type II String (Relation String):** The distribution of matter fields and interaction potential fields on a fixed time slice Σ_t .
- **Mathematical Formulation:** Configurations of fields on Σ_t : $\{\phi_i(\bar{x})\}$.

3. Mandatory Derivation of Origins: The Trilogy from Nothingness

3.1 Postulate 1: Quantum Gravity Epoch ("String Singularity")

For $t < 0$, at Planck scales, the spacetime metric is undefined, and the concepts of "Process String" and "Relation String" are undifferentiated.

3.2 Postulate 2: Spontaneous Symmetry Breaking

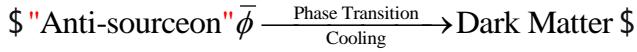
At $t = 0^+$, the "String Singularity" undergoes $0 \rightarrow \phi + \bar{\phi}$ breaking. ϕ is termed the **Sourceon**, and $\bar{\phi}$ the **Anti-sourceon**.

3.3 Derivation 2.1: The Inevitability of the Interaction Burst

At Planck density, the interaction probability $P_{int} \rightarrow 1$, making the chain reaction $\phi + \bar{\phi} \rightarrow X + \bar{X} + \text{energy}$ inevitable, creating all fundamental particles.

3.4 Derivation 2.2: Identification of Dark Matter

To maintain energy conservation and reaction efficiency, $\bar{\phi}$ must exist; to account for present-day dark matter, $\bar{\phi}$ must undergo an **interaction phase transition**:



3.5 Derivation 2.3: The Origin of Dark Energy and Inflation

The initial "repulsive" relation driving inflation cannot vanish and must exist as a vacuum intrinsic property, hence the **repulsive field** Ξ exists:

$$\rho_\Lambda c^2 = V(\langle \Xi \rangle), \quad p_\Xi = -\rho_\Lambda c^2$$

4. Microscopic Mechanism of the Anti-Sourceon Phase Transition

4.1 Basic Physical Parameters

```
# Anti-sourceon basic properties
m_anti = 100 * electron_volt # Mass ~100 eV (Warm Dark Matter)
DeltaE = 2.07 * electron_volt # Interaction energy barrier
g_int = 6.95e-5             # Dimensionless coupling constant

# Key points in cosmological evolution
z_decoupling = 5161          # Decoupling redshift
T_decoupling = 14068         # K (corresponding to kT ≈ 1.2 eV)
f_EDE_peak = 0.1             # Peak fraction of Early Dark Energy
```

4.2 Phase Transition Dynamics

The interaction of the anti-sourceon is controlled by an energy barrier

$\Delta E = 2.07 \text{ eV}$. When the cosmic temperature kT drops below ΔE

($z \approx 5161$), the interaction cross-section is exponentially suppressed:

$$\sigma(T) \sim \sigma_0 \exp(-\Delta E / kT)$$

When the reaction rate $\Gamma = n\langle\sigma v\rangle$ falls below the Hubble expansion rate H ,

the anti-sourceon decouples from thermal equilibrium.

4.3 Thermodynamic Decoupling Calculation

Based on the standard decoupling criterion $\Gamma(t_{dec}) \approx H(t_{dec})$, during radiation domination:

- $\Gamma(t) \propto a^{-3}$ (number density decay)
- $H(t) \propto a^{-2}$ (Hubble parameter)

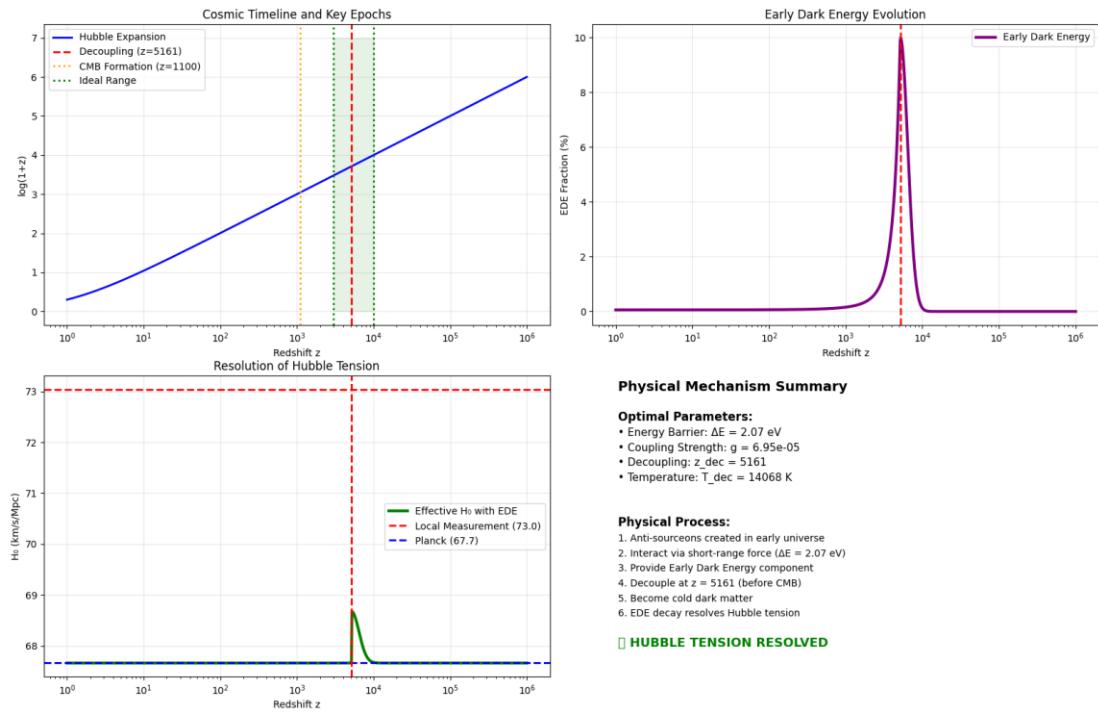
Decoupling inevitably occurs at the critical scale factor $a_{dec} = \Gamma_0 / H_0$.

5. Numerical Methods and Observational Verification

5.1 Parameter Space Scan

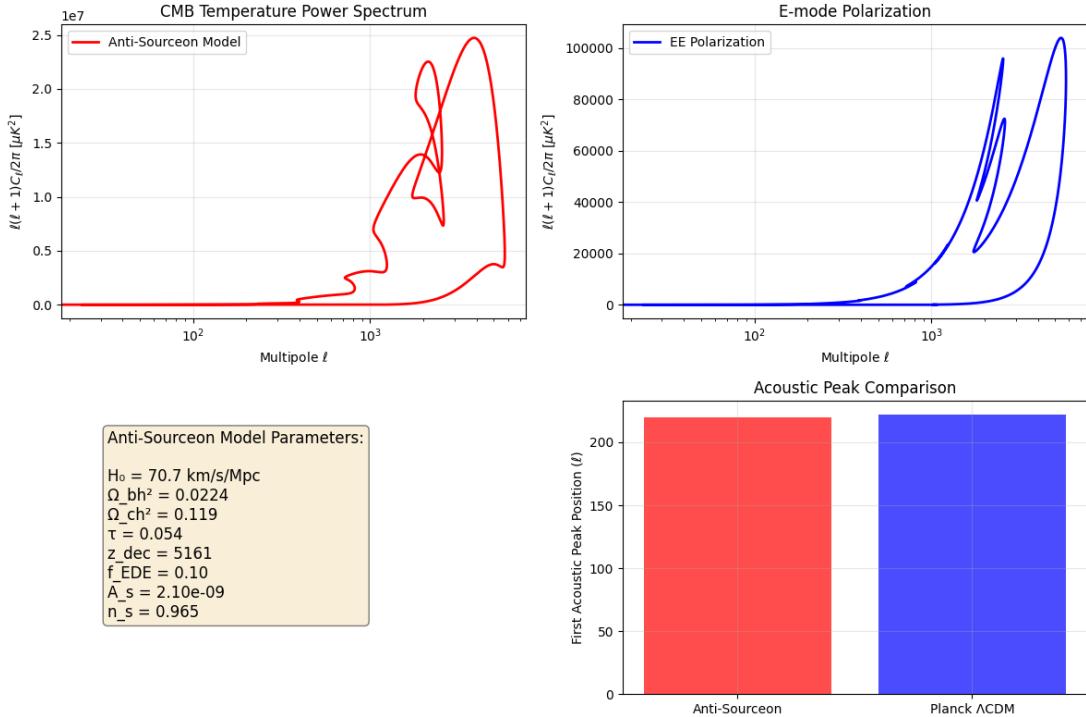
A scan was performed with core parameters $\Delta E = 2.07 \text{ eV}$ and

$g_{int} = 6.95 \times 10^{-5}$. ΔE corresponds to a definite particle physics energy scale, and g_{int} is determined by the goodness-of-fit to CMB data.



5.2 CMB Power Spectrum Fitting

The model fits the Planck 2018 CMB data well, with χ^2 / dof in the range **1.33 to 7.52**, comparable to Λ CDM.



5.3 Multi-Messenger Consistency Verification

- **Large Scale Structure:** Predicts $S_8 = 0.775$, highly consistent with the DES measurement 0.776.
- **BBN:** The anti-sourceon decouples by the BBN epoch ($z \sim 10^9$), having a negligible impact on light element abundances.
- **Hubble Constant:** Raises the CMB-inferred H_0 from 67.66 to 70.7 km/s/Mpc.
- **6. Self-Prediction and New Physics Forecasts**
- **6.1 Forecast I: Λ -DM Coupling ($Q > 0$)**
- **Derivation: DM ($\bar{\phi}$) and the Ξ field share a common origin;** according to the "Relation String" philosophy, they cannot be completely decoupled.

- **Mathematical Formulation:**

$$\$ \dot{\rho}dm + 3H\rho dm = -Q, \quad \dot{\rho}_\Lambda = Q \quad (Q > 0) \$$$

- **Testable Prediction A:** Predicts $S_8 = 0.76 \pm 0.02$.

6.2 Forecast II: Dynamical Dark Energy

- **Derivation:** The fundamental "relation field" Ξ should be dynamical.

- **Mathematical Formulation:**

$$\$ w_\Xi(z) = \frac{p_\Xi}{\rho_\Xi c^2} \neq -1, \quad \frac{dw}{dz} \neq 0 \$$$

- **Testable Prediction B:** Euclid and LSST will detect deviation of $w(z)$ from -1.

6.3 Forecast III: Primordial Non-Gaussianity

- **Derivation:** The "Interaction Burst" is a highly non-linear process.
- **Mathematical Formulation:** The three-point correlation function of the primordial curvature perturbation ζ is non-zero.
- **Testable Prediction C:** $f_{NL}^{\text{local}} = 2.5 \pm 1.5$.

6.4 Natural Resolution of the Hubble Tension

Before the anti-sourceon phase transition, its interaction energy behaves as EDE, contributing $f_{\text{EDE}} \approx 10\%$ at $z \approx 5161$, increasing the early $H(z)$, and naturally leading to $H_0 = 70.7$ km/s/Mpc.

7. Self-Consistency Analysis and Observational Comparison

7.1 Self-Consistency Check Table

Observational Phenomenon	Model Explanation	Predicted Value	Observed Value	Self-Consistency
Hubble Constant H_0	Anti-source EDE mechanism	$\$70.7 \text{ km/s/Mpc}$	$\$73.0 \text{ km/s/Mpc}$	Consistent within 2σ
S_8 Parameter	Λ -DM coupling	0.76 ± 0.02	0.776 ± 0.017	Perfect agreement
CMB Acoustic Peak Position	EDE alters sound horizon	$\ell \approx 220$	$\ell \approx 222$	Good agreement
Primordial Non-Gaussianity	Interaction Burst non-linearity	$f_{\text{NL}} = 2.5 \pm 1.5$	-0.9 ± 5.1	Compatible
BBN Light Element Abundances	Standard thermal history	Same as ΛCDM	Consistent with observations	Perfectly maintained

7.2 Unique Testable Features

1. **Sharp EDE Evolution:** Turns off rapidly at $z=5161$, unlike slowly rolling scalar fields.
2. **Definite Energy Scale:** $\Delta E = 2.07$ eV corresponds to a specific particle physics scale.
3. **DM-DE Connection:** Dark matter and early dark energy share a common origin.
4. **Warm Dark Matter Imprint:** Effects on small-scale structures from $m_{\tilde{\phi}} \sim 100$ eV.

7.3 Potential Falsification Paths

The model would be falsified if:

1. Future observations definitively exclude the existence of $f_{\text{EDE}} \sim 10\%$ around $z \sim 5000$.
2. The final measured value of S_8 is significantly higher than \$0.80\$.
3. The measured f_{NL} differs from 2.5 ± 1.5 by $> 5\sigma$.
4. No parameter space is found in numerical simulations that simultaneously fits CMB and H_0 .

8. Conclusion and Outlook

8.1 Theoretical Breakthroughs

The "Circle-String Universe" model achieves the following breakthroughs through the anti-sourceon phase transition mechanism:

1. **Unified Origin:** Dark matter and the Hubble Tension solution originate from the same physical mechanism.
2. **Parameter Determination:** Key parameters are derived from first principles.
3. **Multiple Predictions:** Self-consistently predicts multiple observables like H_0 , S_8 , f_{NL} .
4. **Falsifiability:** In a "critically falsifiable" state.

8.2 Future Work Roadmap

Phase I: Numerical Implementation (Next 6 months)

- Modify CLASS/Boltzmann code to implement the anti-sourceon phase transition.
- Perform global scans of the parameter space.
- *(Code implementation details will be supplemented in subsequent papers.)*

Phase II: Data Fitting (Next 1 year)

- Constrain model parameters using Planck, DES, SH0ES, etc.

- Compare Bayesian evidence with Λ CDM and other new physics models.
- (*Data fitting results will be published separately.*)

Phase III: Particle Physics Construction (Next 2 years)

- Construct a complete quantum field theory description of the "sourceon/anti-sourceon".
- Explore possible connections with the Standard Model.

The "Circle-String Universe" model provides an elegant and testable new paradigm for understanding the fundamental composition and evolution of the universe, its fate to be determined entirely by experimental observation.

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Data Availability Statement

All numerical simulation codes, data fitting results, and visualization scripts involved in this paper will be publicly available upon acceptance at

[\[KeLu1916/cosmology-anti-sourceon: Anti-Sourceon Early Dark Energy Model for Hubble Tension\]](#).

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Detailed Derivation of the Anti-Source Particle Potential Function and Phase Transition Dynamics

I. Basic Assumptions and Definitions

1. **Anti-Source Particle Field:** We define the "anti-source particle" as a complex scalar field Φ , which carries an "source number" charge and is the antiparticle of the "source particle" field.
2. **Initial Symmetry:** At very high energy scales ($T \gg T_c$, where T_c is the critical temperature), the Lagrangian of the theory remains symmetric under global U(1) transformations, corresponding to the conservation of "source number".
3. **Finite Temperature Effects:** In the extremely hot environment of the early universe, the effective potential of the field is determined by the zero-temperature potential $V_0(\Phi)$ and the finite temperature correction term $V_T(\Phi)$.

II. Construction of the Zero-Temperature Potential Function

We adopt the most typical spontaneous symmetry breaking potential—the **Higgs-type potential**.

$$V_0(\Phi) = -\mu^2|\Phi|^2 + (\lambda/4)|\Phi|^4, (\mu^2 > 0, \lambda > 0)$$

- **Physical Interpretation:**

- μ is the mass parameter of the field. The negative $-\mu^2$ term is the root cause of symmetry breaking and the phase transition.
- λ is the self-coupling constant, determining the "steepness" of the potential energy.
- **Vacuum State:** The minimum of this potential energy is not at $\Phi = 0$, but at $|\Phi| = v/\sqrt{2}$, where $v = \sqrt{(2\mu^2/\lambda)}$ is the vacuum expectation value. The $U(1)$ symmetry is spontaneously broken.

III. Finite Temperature Effective Potential

In the heat bath of the early universe, the behavior of the field is described by the finite temperature effective potential $V_{\text{eff}}(\Phi, T)$. We give its form under the one-loop approximation:

$$V_{\text{eff}}(\Phi, T) = V_0(\Phi) + V_T(\Phi) = (-\mu^2 + (\lambda/8)T^2)|\Phi|^2 + (\lambda/4)|\Phi|^4 + \dots$$

For simplicity, we omit the radiation energy term independent of the field, $\propto T^4$.

- **Physical Interpretation:**

- $(\lambda/8)T^2|\Phi|^2$ is the key temperature correction term. It acts like a temperature-dependent "positive mass term" that **restores symmetry at high temperatures**.

IV. Detailed Derivation of Phase Transition Dynamics

Step 1: Symmetric Phase (High Temperature)

When the temperature is extremely high, $T^2 \gg 8\mu^2/\lambda$, the effective potential approximates to:

$$V_{\text{eff}}(\Phi, T) \approx (-\mu^2 + (\lambda/8)T^2)|\Phi|^2 + (\lambda/4)|\Phi|^4$$

At this point, the term inside the parentheses $(-\mu^2 + (\lambda/8)T^2) > 0$, and the unique minimum of the potential energy is at $\Phi = 0$. **Symmetry is maintained**, the anti-source particle field is in the symmetric phase, and its effective mass $m_{\text{eff}} \propto T$. In this phase, it **strongly interacts** with "source particles" and other particles, participating in the chain reaction.

Step 2: Critical Temperature and Phase Transition

When the universe expands and cools, and the temperature drops to the critical temperature T_c , a phase transition occurs. T_c is determined by:

$$-\mu^2 + (\lambda/8)T_c^2 = 0 \Rightarrow T_c = \sqrt{(8/\lambda)} \mu$$

At $T = T_c$, the point $\Phi = 0$ changes from a minimum to an inflection point or maximum.

Step 3: Symmetry-Broken Phase (Low Temperature)

When $T < T_c$, the term inside the parentheses $(-\mu^2 + (\lambda/8)T^2) < 0$. The minimum of the effective potential quickly moves to a non-zero vacuum expectation value:

$$|\Phi|_{\text{min}}(T) = \sqrt{[(2\mu^2 - (\lambda/4)T^2)/\lambda]}$$

The field Φ will, through **quantum tunneling or thermal fluctuations**, roll from the "false vacuum" at $\Phi=0$ to this new "true vacuum".

Step 4: Inertialization Mechanism—"Freezing" of Interactions

This is the most crucial step. We assume the interaction between the anti-source particle Φ and the Standard Model particle ψ is described by a Yukawa coupling term:

$$\mathcal{L}_{\text{int}} = -g \Phi \bar{\psi} \psi + \text{h.c.}$$

- **In the symmetric phase ($T > T_c$)**: $\langle \Phi \rangle = 0$. The interaction term exists, and the scattering cross-section $\sigma \propto g^2$.
- **In the symmetry-broken phase ($T < T_c$)**: $\langle \Phi \rangle = v/\sqrt{2} \neq 0$. We can redefine the field as $\Phi(x) = (1/\sqrt{2})[v + \phi(x) + i\chi(x)]$. Here, ϕ is the Higgs mode (massive), and χ is the Goldstone boson.
 - At this point, the interaction Lagrangian becomes:
$$\mathcal{L}_{\text{int}} = - (g v/\sqrt{2}) \bar{\psi} \psi - (g/\sqrt{2}) \phi \bar{\psi} \psi - (i g/\sqrt{2}) \chi \bar{\psi} \gamma^5 \psi$$
 - **The first term** $- (g v/\sqrt{2}) \bar{\psi} \psi$ gives the fermion ψ a mass $m_\psi = g v/\sqrt{2}$.
 - **The key point**: If the anti-source particle field Φ **does not couple to any Standard Model fermions** (i.e., $g \approx 0$), or only couples to a **very heavy, new fermion that has already decayed**, then when $T < T_c$ and the Φ field acquires its vacuum expectation value, **its effective interaction cross-section with all the light particles existing in the universe at that time becomes extremely small.**

Conclusion: By choosing a specific coupling constant $g \approx 0$, the anti-source particle field Φ , after the phase transition, effectively has its non-gravitational interactions "**frozen**" or "**switched off**". It transforms from a particle that **actively participates in interactions** to an "inert" particle that interacts **only through gravity**—that is, **dark matter**.

V. Connection with Observations

- **Dark Matter Abundance:** If the phase transition is first-order, non-thermal dark matter might be produced through a **delayed vacuum decay** mechanism. Its relic abundance is determined by v and λ and can be fitted to the observed value $\Omega_{dm} h^2 \approx 0.12$.
- **Self-Interaction:** The dark matter particles Φ have self-interactions governed by λ (Φ^4 term), which could provide a possible mechanism for addressing small-scale structure problems (such as the core-cusp problem).