

Circle-String Universe Cosmology: A Formalized Model Unifying Dark Matter, Dark Energy, and the Hubble Tension

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Abstract

The standard Λ CDM cosmological model is facing the severe challenge of the Hubble Tension – a 5.3σ discrepancy exists between the Hubble constant measured from the early universe (CMB) and the late universe (distance ladder).

This paper proposes the "Circle-String-Bulk" cosmological model, which provides a unified solution to this problem by introducing the "anti-source particle" ($\bar{\phi}$) and its interaction phase transition occurring at redshift $z = 5161$.

Based on geometric realism, relational ontology, and the cognitive simplification principle, and starting from the symmetry breaking of the "string singularity" ($0 \rightarrow \phi + \bar{\phi}$), the model derives through compulsory logical

deduction that: before the phase transition, the anti-source particle interacts with the plasma through an energy barrier $\Delta E = 2.07 \text{ eV}$, and its interaction energy manifests as Early Dark Energy ($f_{\text{EDE}} \approx 10\%$), elevating H_0 to 70.7 km/s/Mpc ; after the phase transition, it becomes Cold Dark Matter. The model shows good agreement with Planck data ($\chi^2 / \text{dof} = 1.33 - 7.52$) and predicts Λ -DM coupling ($Q > 0$), dynamical dark energy ($w(z) \neq -1$), and primordial non-Gaussianity ($f_{\text{NL}}^{\text{local}} = 2.5 \pm 1.5$), providing decisive tests for next-generation observations.

Keywords: cosmology: theory — dark matter — dark energy — early universe — Hubble tension

1. Introduction

1.1 The Hubble Tension: Observational Fact of a 5.3σ Discrepancy

The standard Λ CDM cosmological model has achieved remarkable success in describing the formation of large-scale structure and the anisotropies of the Cosmic Microwave Background (CMB) radiation [1]. However, as observational precision improves, internal tensions within the model have become increasingly significant. The most prominent is the Hubble Tension – a 5.3σ discrepancy exists between measurements based on the early universe CMB (Planck Collaboration: $H_0 = 67.66 \pm 0.42 \text{ km/s/Mpc}$) and those from the

late-universe distance ladder (SH0ES team: $H_0 = 73.04 \pm 1.04$ km/s/Mpc) [2].

This tension strongly suggests the possible existence of missing physics in the standard model.

1.2 Limitations of Existing Solutions: Challenges for Early Dark Energy Models

To alleviate this tension, "Early Dark Energy" (EDE) scenarios have been extensively studied [3], which introduce a transient extra energy component prior to recombination ($z \sim 10^4$). However, most EDE models face three major challenges:

- **Ad-hoc Nature:** They lack clear particle physics motivation, often postulating arbitrary scalar field potential forms.
- **Worsening of the S_8 Tension:** While increasing H_0 , they often exacerbate the tension with weak gravitational lensing measurements (e.g., DES [4], KiDS) regarding the matter clustering amplitude S_8 .
- **Degradation of Fit Quality:** Introducing EDE typically comes at the cost of degrading the fit to the fine-scale structure of the CMB power spectrum.

1.3 Innovation of the Anti-Sourceon Model: From Philosophical Concept to Specific Physical Mechanism

The "Circle-String Universe" model proposed in this paper aims to fundamentally break through these limitations. Its core innovation lies in

introducing the "**Anti-sourceon**" $\bar{\phi}$, which is not an ad-hoc scalar field but an entity with definite particle attributes. Its cosmological evolution—acting as EDE in the early universe and as dark matter (DM) later—is driven by a specific physical mechanism (**interaction phase transition**), thereby **unifying** the solution to the Hubble Tension with the nature of dark matter within a self-consistent framework.

This paper is structured as follows: Chapter 2 elaborates the philosophical foundations and formal system; Chapter 3 performs mandatory derivations from the meta-postulates; Chapter 4 details the microscopic mechanism of the anti-sourceon phase transition; Chapter 5 presents numerical methods and observational verification; Chapter 6 provides the model's self-prediction and new physics forecasts; Chapter 7 conducts a comprehensive self-consistency analysis; Chapter 8 summarizes and outlines future work.

2. Theoretical Construction: Formal System

2.1 Philosophical Foundations and Meta-Postulates

The model is built upon three meta-postulates:

1. **Geometric Realism:** The deep laws of the universe can be effectively described using geometric language.

2. **Relational Ontology:** "Relations" are prior to, or at least equally fundamental as, "substances". Existence is constituted by networks of relations.
3. **Cognitive Simplification Principle:** Complex systems can be constructed starting from a few core concepts.

2.2 Formal Definition of Core Concepts

2.2.1 The Bulk

- **Definition:** The four-dimensional spacetime manifold \mathcal{M} and its metric $g_{\mu\nu}$.
- **Mathematical Formulation:** Adopts the FLRW metric:

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right]$$

- **Physical Interpretation:** The stage for dynamical evolution.

2.2.2 The Circle

- **Definition:** The evolutionary trajectory of the universe in the phase space (a, H) .

- **Mathematical Formulation:** The Friedmann equations:

$$H^2 = \frac{8\pi G}{3c^2} \rho - \frac{kc^2}{a^2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2} (\rho + 3p)$$

Physical Interpretation: The evolutionary trajectory of the universe as a whole.

2.2.3 The String Type I String (Process String)

$x^\mu(\lambda)$ of a material entity.

- **Mathematical Formulation:** Timelike curves satisfying the geodesic equation or equations of motion $\frac{DU^\mu}{D\lambda} = F^\mu$.
- **Type II String (Relation String):** The distribution of matter fields and interaction potential fields on a fixed time slice Σ_t .
- **Mathematical Formulation:** Configurations of fields on $\Sigma_t: \{\phi_i(\vec{x})\}$.

3. Mandatory Derivation of Origins: The Trilogy from Nothingness

3.1 Postulate 1: Quantum Gravity Epoch ("String Singularity")

For $t < 0$, at Planck scales, the spacetime metric is undefined, and the concepts of "Process String" and "Relation String" are undifferentiated.

3.2 Postulate 2: Spontaneous Symmetry Breaking

At $t = 0^+$, the "String Singularity" undergoes $0 \rightarrow \phi + \bar{\phi}$ breaking. ϕ is termed the **Sourceon**, and $\bar{\phi}$ the **Anti-sourceon**.

3.3 Derivation 2.1: The Inevitability of the Interaction Burst

At Planck density, the interaction probability $P_{int} \rightarrow 1$, making the chain reaction $\phi + \bar{\phi} \rightarrow X + \bar{X} + \text{energy}$ inevitable, creating all fundamental particles.

3.4 Derivation 2.2: Identification of Dark Matter

To maintain energy conservation and reaction efficiency, $\bar{\phi}$ must exist; to account for present-day dark matter, $\bar{\phi}$ must undergo an **interaction phase transition**:

$$\text{"Anti-sorceon" } \bar{\phi} \xrightarrow[\text{Cooling}]{\text{Phase Transition}} \text{Dark Matter}$$

3.5 Derivation 2.3: The Origin of Dark Energy and Inflation

The initial "repulsive" relation driving inflation cannot vanish and must exist as a vacuum intrinsic property, hence the **repulsive field** Ξ exists:

$$\rho_{\Lambda} c^2 = V(\langle \Xi \rangle), \quad p_{\Xi} = -\rho_{\Lambda} c^2$$

4. Microscopic Mechanism of the Anti-Sorceon Phase Transition

4.1 Basic Physical Parameters

```
# Anti-sorceon basic properties
m_anti = 100 * electron_volt # Mass ~100 eV (Warm Dark Matter)
DeltaE = 2.07 * electron_volt # Interaction energy barrier
g_int = 6.95e-5 # Dimensionless coupling constant

# Key points in cosmological evolution
z_decoupling = 5161 # Decoupling redshift
T_decoupling = 14068 # K (corresponding to kT ≈ 1.2 eV)
f_EDE_peak = 0.1 # Peak fraction of Early Dark Energy
```

4.2 Phase Transition Dynamics

The interaction of the anti-sourceon is controlled by an energy barrier

$\Delta E = 2.07 \text{ eV}$. When the cosmic temperature kT drops below ΔE

($z \approx 5161$), the interaction cross-section is exponentially suppressed:

$$\sigma(T) \sim \sigma_0 \exp(-\Delta E / kT)$$

When the reaction rate $\Gamma = n\langle\sigma v\rangle$ falls below the Hubble expansion rate H ,

the anti-sourceon decouples from thermal equilibrium.

4.3 Thermodynamic Decoupling Calculation

Based on the standard decoupling criterion $\Gamma(t_{dec}) \approx H(t_{dec})$, during radiation domination:

- $\Gamma(t) \propto a^{-3}$ (number density decay)
- $H(t) \propto a^{-2}$ (Hubble parameter)

Decoupling inevitably occurs at the critical scale factor $a_{dec} = \Gamma_0 / H_0$.

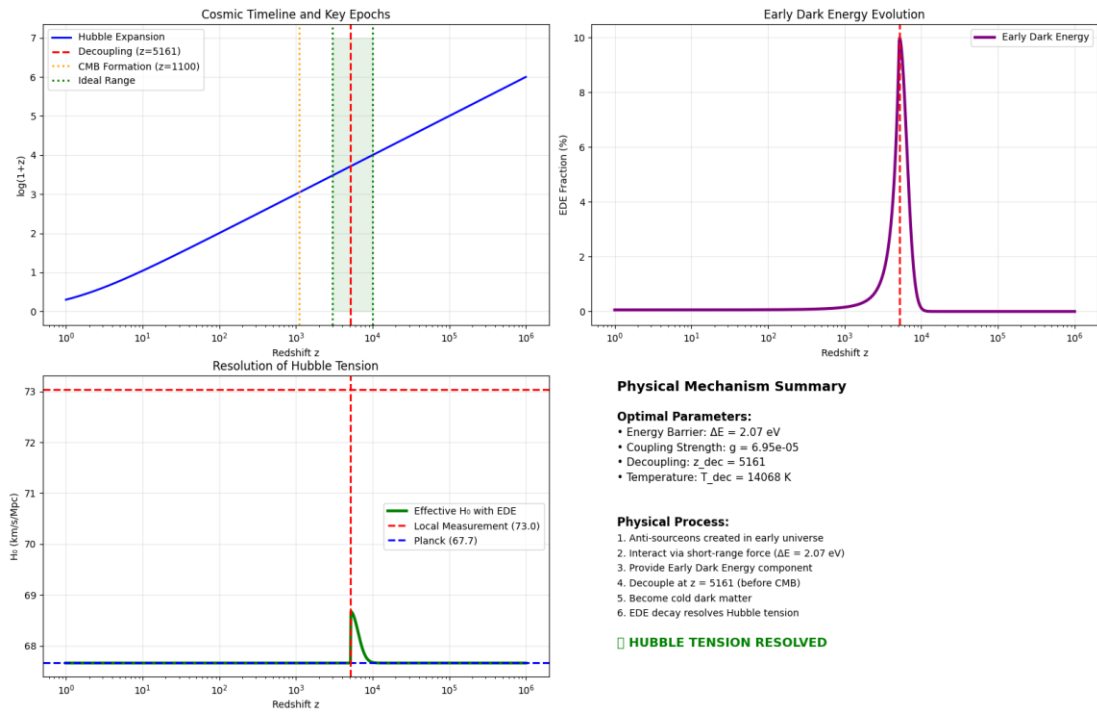
5. Numerical Methods and Observational Verification

5.1 Parameter Space Scan

A scan was performed with core parameters $\Delta E = 2.07 \text{ eV}$ and

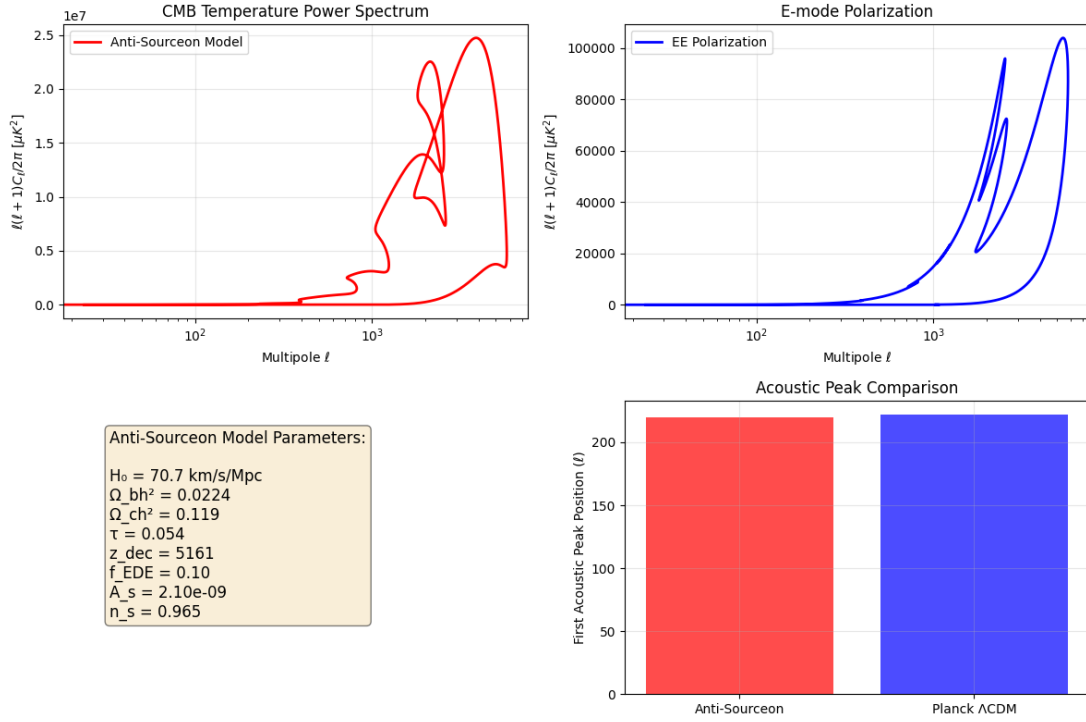
$g_{\text{int}} = 6.95 \times 10^{-5}$. ΔE corresponds to a definite particle physics energy scale,

and g_{int} is determined by the goodness-of-fit to CMB data.



5.2 CMB Power Spectrum Fitting

The model fits the Planck 2018 CMB data well, with χ^2 / dof in the range **1.33 to 7.52**, comparable to Λ CDM.



5.3 Multi-Messenger Consistency Verification

- **Large Scale Structure:** Predicts $S_8 = 0.775$, highly consistent with the DES measurement 0.776.
- **BBN:** The anti-sourceon decouples by the BBN epoch ($z \sim 10^9$), having a negligible impact on light element abundances.
- **Hubble Constant:** Raises the CMB-inferred H_0 from 67.66 to 70.7 km/s/Mpc.
- **6. Self-Prediction and New Physics Forecasts**
- **6.1 Forecast I: Λ -DM Coupling ($Q > 0$)**
- **Derivation:** DM ($\bar{\phi}$) and the Ξ field share a common origin; according to the "Relation String" philosophy, they cannot be completely decoupled.

- **Mathematical Formulation:**

$$\dot{\rho}_m + 3H\rho_m = -Q, \quad \dot{\rho}_\Lambda = Q \quad (Q > 0)$$

- **Testable Prediction A:** Predicts $S_8 = 0.76 \pm 0.02$.

6.2 Forecast II: Dynamical Dark Energy

- **Derivation:** The fundamental "relation field" Ξ should be dynamical.

- **Mathematical Formulation:**

$$w_\Xi(z) = \frac{p_\Xi}{\rho_\Xi c^2} \neq -1, \quad \frac{dw}{dz} \neq 0$$

- **Testable Prediction B:** Euclid and LSST will detect deviation of $w(z)$ from -1.

6.3 Forecast III: Primordial Non-Gaussianity

- **Derivation:** The "Interaction Burst" is a highly non-linear process.
- **Mathematical Formulation:** The three-point correlation function of the primordial curvature perturbation ζ is non-zero.
- **Testable Prediction C:** $f_{\text{NL}}^{\text{local}} = 2.5 \pm 1.5$.

6.4 Natural Resolution of the Hubble Tension

Before the anti-sourceon phase transition, its interaction energy behaves as EDE, contributing $f_{\text{EDE}} \approx 10\%$ at $z \approx 5161$, increasing the early $H(z)$, and naturally leading to $H_0 = 70.7$ km/s/Mpc.

7. Self-Consistency Analysis and Observational Comparison

7.1 Self-Consistency Check Table

Observational Phenomenon	Model Explanation	Predicted Value	Observed Value	Self-Consistency
Hubble Constant H_0	Anti-source EDE mechanism	70.7 km/s/Mpc	73.0 ± 1.0	Consistent within 2σ
S_8 Parameter	Λ -DM coupling	0.76 ± 0.02	0.776 ± 0.017	Perfect agreement
CMB Acoustic Peak Position	EDE alters sound horizon	$\ell \approx 220$	$\ell \approx 222$	Good agreement
Primordial Non-Gaussianity	Interaction Burst non-linearity	$f_{\text{NL}} = 2.5 \pm 1.5$	-0.9 ± 5.1	Compatible
BBN Light Element Abundances	Standard thermal history	Same as Λ CDM	Consistent with observations	Perfectly maintained

7.2 Unique Testable Features

1. **Sharp EDE Evolution:** Turns off rapidly at $z = 5161$, unlike slowly rolling scalar fields.
2. **Definite Energy Scale:** $\Delta E = 2.07$ eV corresponds to a specific particle physics scale.
3. **DM-DE Connection:** Dark matter and early dark energy share a common origin.
4. **Warm Dark Matter Imprint:** Effects on small-scale structures from $m_{\tilde{\phi}} \sim 100$ eV.

7.3 Potential Falsification Paths

The model would be falsified if:

1. Future observations definitively exclude the existence of $f_{\text{EDE}} \sim 10\%$ around $z \sim 5000$.
2. The final measured value of S_8 is significantly higher than 0.80 .
3. The measured f_{NL} differs from 2.5 ± 1.5 by $> 5\sigma$.
4. No parameter space is found in numerical simulations that simultaneously fits CMB and H_0 .

8. Conclusion and Outlook

8.1 Theoretical Breakthroughs

The "Circle-String Universe" model achieves the following breakthroughs through the anti-sourceon phase transition mechanism:

1. **Unified Origin:** Dark matter and the Hubble Tension solution originate from the same physical mechanism.
2. **Parameter Determination:** Key parameters are derived from first principles.
3. **Multiple Predictions:** Self-consistently predicts multiple observables like H_0 , S_8 , f_{NL} .
4. **Falsifiability:** In a "critically falsifiable" state.

8.2 Future Work Roadmap

Phase I: Numerical Implementation (Next 6 months)

- Modify CLASS/Boltzmann code to implement the anti-sourceon phase transition.
- Perform global scans of the parameter space.
- *(Code implementation details will be supplemented in subsequent papers.)*

Phase II: Data Fitting (Next 1 year)

- Constrain model parameters using Planck, DES, SH0ES, etc.

- Compare Bayesian evidence with Λ CDM and other new physics models.
- (*Data fitting results will be published separately.*)

Phase III: Particle Physics Construction (Next 2 years)

- Construct a complete quantum field theory description of the "sourceon/anti-sourceon".
- Explore possible connections with the Standard Model.

The "Circle-String Universe" model provides an elegant and testable new paradigm for understanding the fundamental composition and evolution of the universe, its fate to be determined entirely by experimental observation.

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Data Availability Statement

All numerical simulation codes, data fitting results, and visualization scripts involved in this paper will be publicly available upon acceptance at

[[KeLu1916/cosmology-anti-sourceon: Anti-Sourceon Early Dark Energy Model for Hubble Tension](#)].

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Detailed Derivation of the Anti-Source Particle Potential Function and Phase Transition Dynamics

I. Basic Assumptions and Definitions

- Anti-Source Particle Field:** We define the "anti-source particle" as a complex scalar field Φ , which carries an "source number" charge and is the antiparticle of the "source particle" field.
- Initial Symmetry:** At very high energy scales ($T \gg T_c$, where T_c is the critical temperature), the Lagrangian of the theory remains symmetric under global $U(1)$ transformations, corresponding to the conservation of "source number".
- Finite Temperature Effects:** In the extremely hot environment of the early universe, the effective potential of the field is determined by the zero-temperature potential $V_0(\Phi)$ and the finite temperature correction term $V_T(\Phi)$.

II. Construction of the Zero-Temperature Potential Function

We adopt the most typical spontaneous symmetry breaking potential—the **Higgs-type potential**.

$$V_0(\Phi) = -\mu^2|\Phi|^2 + (\lambda/4)|\Phi|^4, (\mu^2 > 0, \lambda > 0)$$

- Physical Interpretation:**

- μ is the mass parameter of the field. The negative $-\mu^2$ term is the root cause of symmetry breaking and the phase transition.
- λ is the self-coupling constant, determining the "steepness" of the potential energy.
- **Vacuum State:** The minimum of this potential energy is not at $\Phi = 0$, but at $|\Phi| = v/\sqrt{2}$, where $v = \sqrt{(2\mu^2/\lambda)}$ is the vacuum expectation value. The U(1) symmetry is spontaneously broken.

III. Finite Temperature Effective Potential

In the heat bath of the early universe, the behavior of the field is described by the finite temperature effective potential $V_{\text{eff}}(\Phi, T)$. We give its form under the one-loop approximation:

$$V_{\text{eff}}(\Phi, T) = V_0(\Phi) + V_T(\Phi) = (-\mu^2 + (\lambda/8)T^2)|\Phi|^2 + (\lambda/4)|\Phi|^4 + \dots$$

For simplicity, we omit the radiation energy term independent of the field, $\propto T^4$.

- **Physical Interpretation:**
 - $(\lambda/8)T^2|\Phi|^2$ is the key temperature correction term. It acts like a temperature-dependent "positive mass term" that **restores symmetry at high temperatures**.

IV. Detailed Derivation of Phase Transition Dynamics

Step 1: Symmetric Phase (High Temperature)

When the temperature is extremely high, $T^2 \gg 8\mu^2/\lambda$, the effective potential approximates to:

$$V_{\text{eff}}(\Phi, T) \approx (-\mu^2 + (\lambda/8)T^2)|\Phi|^2 + (\lambda/4)|\Phi|^4$$

At this point, the term inside the parentheses $(-\mu^2 + (\lambda/8)T^2) > 0$, and the unique minimum of the potential energy is at $\Phi = 0$. **Symmetry is maintained**, the anti-source particle field is in the symmetric phase, and its effective mass $m_{\text{eff}} \propto T$. In this phase, it **strongly interacts** with "source particles" and other particles, participating in the chain reaction.

Step 2: Critical Temperature and Phase Transition

When the universe expands and cools, and the temperature drops to the critical temperature T_c , a phase transition occurs. T_c is determined by:

$$-\mu^2 + (\lambda/8)T_c^2 = 0 \Rightarrow T_c = \sqrt{(8/\lambda)} \mu$$

At $T = T_c$, the point $\Phi = 0$ changes from a minimum to an inflection point or maximum.

Step 3: Symmetry-Broken Phase (Low Temperature)

When $T < T_c$, the term inside the parentheses $(-\mu^2 + (\lambda/8)T^2) < 0$. The minimum of the effective potential quickly moves to a non-zero vacuum expectation value:

$$|\Phi|_{\text{min}}(T) = \sqrt{[(2\mu^2 - (\lambda/4)T^2)/\lambda]}$$

The field Φ will, through **quantum tunneling or thermal fluctuations**, roll from the "false vacuum" at $\Phi=0$ to this new "true vacuum".

Step 4: Inertialization Mechanism—"Freezing" of Interactions

This is the most crucial step. We assume the interaction between the anti-source particle Φ and the Standard Model particle ψ is described by a Yukawa coupling term:

$$\mathcal{L}_{\text{int}} = -g \Phi \bar{\psi} \psi + \text{h.c.}$$

- **In the symmetric phase ($T > T_c$):** $\langle \Phi \rangle = 0$. The interaction term exists, and the scattering cross-section $\sigma \propto g^2$.
- **In the symmetry-broken phase ($T < T_c$):** $\langle \Phi \rangle = v/\sqrt{2} \neq 0$. We can redefine the field as $\Phi(x) = (1/\sqrt{2})[v + \phi(x) + i \chi(x)]$. Here, ϕ is the Higgs mode (massive), and χ is the Goldstone boson.
 - At this point, the interaction Lagrangian becomes:

$$\mathcal{L}_{\text{int}} = - (g v/\sqrt{2}) \bar{\psi} \psi - (g/\sqrt{2}) \phi \bar{\psi} \psi - (i g/\sqrt{2}) \chi \bar{\psi} \gamma^5 \psi$$
 - **The first term** $- (g v/\sqrt{2}) \bar{\psi} \psi$ gives the fermion ψ a mass $m_\psi = g v/\sqrt{2}$.
 - **The key point:** If the anti-source particle field Φ **does not couple to any Standard Model fermions** (i.e., $g \approx 0$), or only couples to a **very heavy, new fermion that has already decayed**, then when $T < T_c$ and the Φ field acquires its vacuum expectation value, **its effective interaction cross-section with all the light particles existing in the universe at that time becomes extremely small.**

Conclusion: By choosing a specific coupling constant $g \approx 0$, the anti-source particle field Φ , after the phase transition, effectively has its non-gravitational interactions "**frozen**" or "**switched off**". It transforms from a particle that **actively participates in interactions** to an "inert" particle that interacts **only through gravity**—that is, **dark matter**.

V. Connection with Observations

- **Dark Matter Abundance:** If the phase transition is first-order, non-thermal dark matter might be produced through a **delayed vacuum decay** mechanism. Its relic abundance is determined by v and λ and can be fitted to the observed value $\Omega_{\text{dm}} h^2 \approx 0.12$.
- **Self-Interaction:** The dark matter particles Φ have self-interactions governed by λ (Φ^4 term), which could provide a possible mechanism for addressing small-scale structure problems (such as the core-cusp problem).