A Polynomial Time Algorithm to Minimize Total Travel Time in k-Depot Storage/Retrieval System

Amir Gharehgozli, **Chao Xu**, Wenda Zhang

Aug 24, 2018

A warehouse

An automated warehouse with input depots and output depots. It has to complete input(storage) and output(retrieval) requests.
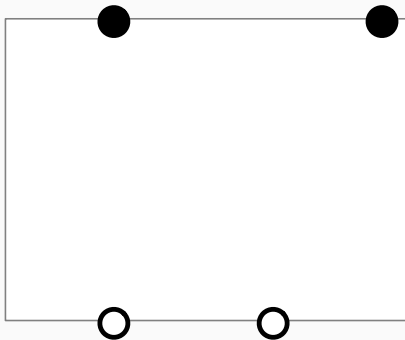

Figure 1: Demag V-type crane machine. Source: `demagcranes.com`

The storage and retrial machine

- The machine start at some depot.
- The machine can hold at most one item.
- The machine can pick up an item from any input depot, and drop off the item at a input request location.
- The machine can pick up an item from a output request location, and drop off the item at any output depot.
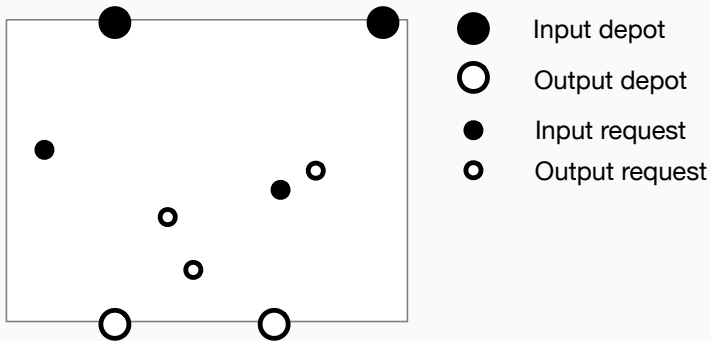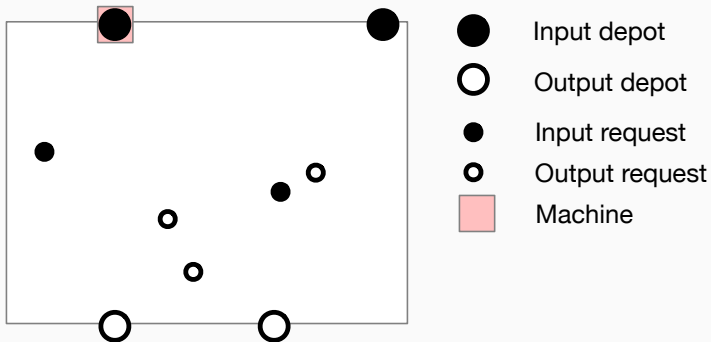- The machine must return to the original depot.

Input depot
Output depot

● Input depot
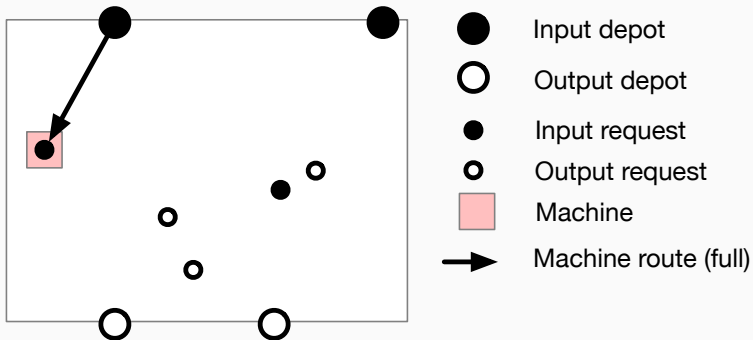○ Output depot
● Input request
○ Output request

● Input depot
○ Output depot
• Input request
∘ Output request
▨ Machine

- ● Input depot
- ○ Output depot
- ● Input request
- ○ Output request
- ▪ Machine
- → Machine route (full)

●	Input depot
○	Output depot
•	Input request
∘	Output request
▨	Machine
→	Machine route (full)
→	Machine route (empty)

- ● Input depot
- ○ Output depot
- • Input request
- ○ Output request
- ▨ Machine
- → Machine route (full)
- → Machine route (empty)

- ● Input depot
- ○ Output depot
- • Input request
- ∘ Output request
- ▪ Machine
- → Machine route (full)
- → Machine route (empty)

- ● Input depot
- ○ Output depot
- ● Input request
- ○ Output request
- ▨ Machine
- → Machine route (full)
- → Machine route (empty)

● Input depot

○ Output depot

• Input request

∘ Output request

▨ Machine

→ Machine route (full)

→ Machine route (empty)

- ● Input depot
- ○ Output depot
- • Input request
- ∘ Output request
- ▨ Machine
- → Machine route (full)
- → Machine route (empty)

- ● Input depot
- ○ Output depot
- ● Input request
- ○ Output request
- ▨ Machine
- → Machine route (full)
- → Machine route (empty)

- ● Input depot
- ○ Output depot
- • Input request
- ∘ Output request
- ■ Machine
- → Machine route (full)
- → Machine route (empty)

- ● Input depot
- ○ Output depot
- • Input request
- ∘ Output request
- ▪ Machine
- → Machine route (full)
- → Machine route (empty)
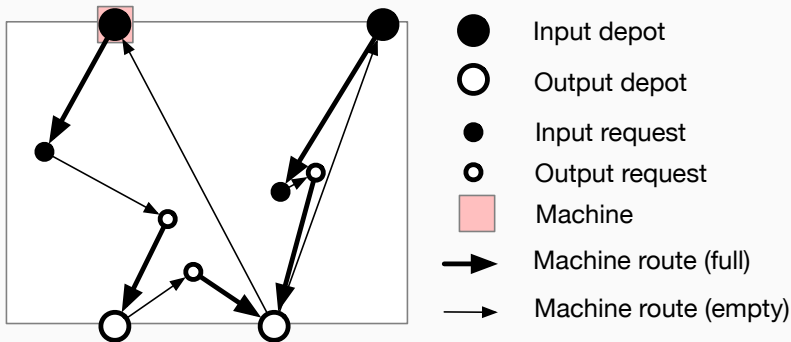
Observations

At most 2 request can be completed per depot to depot trip.

- input depot $\rightarrow$ input request$\rightarrow$ output request$\rightarrow$ output depot
- input depot$\rightarrow$ input request$\rightarrow$ arbitrary depot
- arbitrary depot$\rightarrow$ output request$\rightarrow$ output depot
- output depot $\rightarrow$ input depot

Formalizing the problem: The input

- Input depots D_I, output depots D_O, $D = D_I \cup D_O$. $|D| = k$.
- Input request R_I, output request R_O, $R = R_I \cup R_O$. $|R| = n$.
- $V = D \cup R$, the set of vertices.
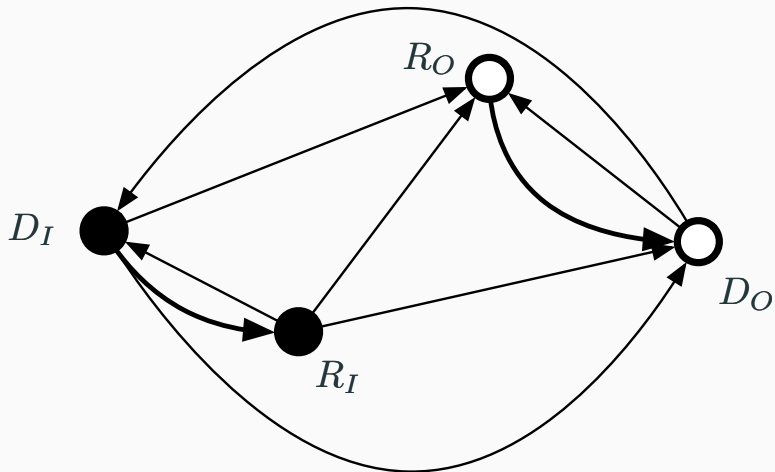- dist : $V \times V \to \mathbb{R}_+$ a asymmetric metric.

Model as a walk on a graph

For input D_I, D_O, R_I, R_O, c, we construct the following weighted directed graph G on vertices V.

- For $d, d' \in D$, there is an edge (d, d').
- For $v \in V$, $u \in R_I$, there is an edge (u, v).
- For $v \in V$, $u \in R_O$, there is an edge (v, u).
- For $v \in R_I$, $d \in D_I$, there is an edge (d, v).
- For $v \in R_O$, $d \in D_O$, there is an edge (v, d).

The cost of an edge $c(u, v) = \text{dist}(u, v)$. Such graph G is called a warehouse network.

Problem: k-depot warehouse tour
Input: A warehouse network with k depot vertices.
Output: A minimum cost closed walk that goes through every vertex at least once.

Problem: k-depot warehouse tour
Input: A warehouse network with k depot vertices.
Output: A minimum cost closed walk that goes through every vertex at least once.

Note: we assume in the optimal solution, all k depot has to be visited.

The abstract problem

Problem: k-depot warehouse tour
Input: A warehouse network with k depot vertices.
Output: A minimum cost closed walk that goes through every vertex at least once.

Note: we assume in the optimal solution, all k depot has to be visited.

Closely related to TSP.

Problem: k-depot warehouse tour
Input: A warehouse network with k depot vertices.
Output: A minimum cost closed walk that goes through every vertex at least once.

Note: we assume in the optimal solution, all k depot has to be visited.

Closely related to TSP.

Observation: there is an optimal solution that goes through each vertex in R exactly once, because dist is a metric.

A regular depot is a pair of input depot d and output depot d' with $\text{dist}(d, d') = 0$.

[Gharehgozli, Yu, Zhang, de Koster '17] considered special cases of the problem.

- $k = 4$: 2 pairs of regular depots. Running time $O(n^6)$.
- $k = 2$: 2 depots, one input, one output. Running time $O(n^3)$.

Our result

Let $MCF(n, m)$ be the running time to solve min-cost flow on a unit capacity graph with m edges and n vertices.

Theorem
The k-depot warehouse tour can be solved in

- $O(n^{k+1} + MCF(n, n^2))$ *time if all depots are input(output) depots.*

- $O(n^k + MCF(n, n^2))$ *time otherwise.*

$MCF(n, m) = \tilde{O}(\sqrt{n}m)$, **[Lee-Sidford '13]**.

Our result

Let $MCF(n, m)$ be the running time to solve min-cost flow on a unit capacity graph with m edges and n vertices.

Theorem
The k-depot warehouse tour can be solved in

- $O(n^{k+1} + MCF(n, n^2))$ *time if all depots are input(output) depots.*
- $O(n^k + MCF(n, n^2))$ *time otherwise.*

$MCF(n, m) = \tilde{O}(\sqrt{n}m)$, **[Lee-Sidford '13]**. Counterintuitive! Having depots of only one type is harder.

A simple polynomial time algorithm

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.
2. Covering property: Each vertex has in-degree at least 1.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.
2. Covering property: Each vertex has in-degree at least 1. Each vertex in R has in-degree exactly 1.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.
2. Covering property: Each vertex has in-degree at least 1. Each vertex in R has in-degree exactly 1.
3. Connectivity property: H is (weakly) connected.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.
2. Covering property: Each vertex has in-degree at least 1. Each vertex in R has in-degree exactly 1.
3. Connectivity property: H is (weakly) connected.

Every graph with the above properties induces a feasible solution: it is a Eulerian graph that contains all vertices.

What does a solution look like?

The feasible solution is a closed walk W. The (disjoint) union of the edges in the solution is a multigraph H with the following properties.

1. Circulation property: The in-degree and out-degree are the same for each vertex.

2. Covering property: Each vertex has in-degree at least 1. Each vertex in R has in-degree exactly 1.

3. Connectivity property: H is (weakly) connected.

Every graph with the above properties induces a feasible solution: it is a Eulerian graph that contains all vertices. H is valid if it has circulation and covering property.

A simpler connectivity condition

Theorem
If H is valid, then it is connected if and only if D is connected.

Idea

1. Connect D by some small graph F.

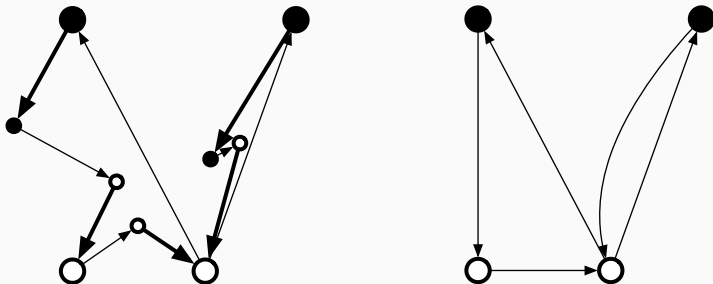1. Connect D by some small graph F.
2. Find a minimum weight valid subgraph H of G containing all the edges of F.

1. Connect D by some small graph F.
2. Find a minimum weight valid subgraph H of G containing all the edges of F.
3. If an optimum solution to the original problem contains all edges in F, then H is an optimum solution to the original problem.
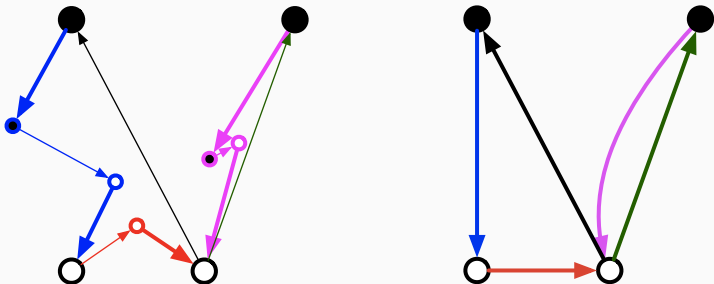
Structure graph of a solution

The structure graph of a solution is obtained by the following transformation. For each depot to depot path that does not contain any other depot P. Let P' be the sequence of internal vertices, and P is from d to d'. We create an edge e from d to d', and give it the label P'.

Structure graph of a solution

The structure graph of a solution is obtained by the following transformation. For each depot to depot path that does not contain any other depot P. Let P' be the sequence of internal vertices, and P is from d to d'. We create an edge e from d to d', and give it the label P'.
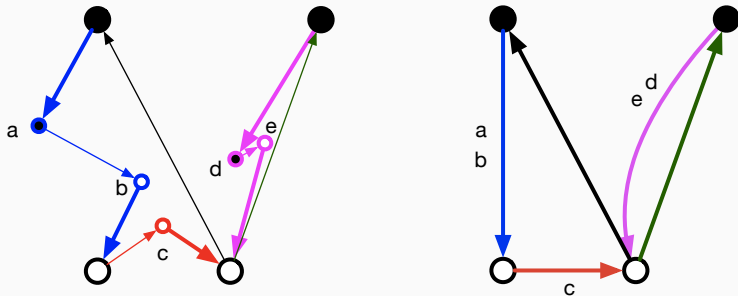
Structure graph of a solution

The structure graph of a solution is obtained by the following
transformation. For each depot to depot path that does not
contain any other depot P. Let P' be the sequence of internal
vertices, and P is from d to d'. We create an edge e from d to d',
and give it the label P'.

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the corresponding edges of the solution.

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the corresponding edges of the solution.

For each $T \in \mathcal{T}$

　　Find an optimal valid subgraph of G that contains all edges in $\phi(T)$.

Return the minimum

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the corresponding edges of the solution.

For each $T \in \mathcal{T}$

Find an optimal valid subgraph of G that contains all edges in $\phi(T)$.

Return the minimum

Running time $O(|\mathcal{T}| \times$ time to find optimal valid subgraph).

$\mathcal{T}$ be a set of trees such that every structure graph must contain
at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the
corresponding edges of the solution.

For each $T \in \mathcal{T}$

    Find an optimal valid subgraph of G that contains all edges in
$\phi(T)$.

Return the minimum

Running time $O(|\mathcal{T}| \times$ time to find optimal valid subgraph$)$.

Time to find optimal valid subgraph:

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the corresponding edges of the solution.

For each $T \in \mathcal{T}$

Find an optimal valid subgraph of G that contains all edges in $\phi(T)$.

Return the minimum

Running time $O(|\mathcal{T}| \times$ time to find optimal valid subgraph).

Time to find optimal valid subgraph: reduces to a min-cost flow computation on a unit capacity graph.

$\mathcal{T}$ be a set of trees such that every structure graph must contain at least one of the tree as a subgraph.

ϕ that takes a subgraph of the structure graph, and return the corresponding edges of the solution.

For each $T \in \mathcal{T}$

    Find an optimal valid subgraph of G that contains all edges in $\phi(T)$.

Return the minimum

Running time $O(|\mathcal{T}| \times$ time to find optimal valid subgraph).

Time to find optimal valid subgraph: reduces to a min-cost flow computation on a unit capacity graph. $\tilde{O}(n^{2.5})$ **[Lee-Sidford '13]**.

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.

The weight of a tree is the number of labels on the edges.

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.

The weight of a tree is the number of labels on the edges.

Claim: $|\mathcal{T}| = O(n^{2(k-1)})$.

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.

The weight of a tree is the number of labels on the edges.

Claim: $|\mathcal{T}| = O(n^{2(k-1)})$.

- There are k nodes, so there can be $f(k)$ trees (ignoring labels).

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.

The weight of a tree is the number of labels on the edges.

Claim: $|\mathcal{T}| = O(n^{2(k-1)})$.

- There are k nodes, so there can be $f(k)$ trees (ignoring labels).
- Each tree has $k - 1$ edges. Each edge can have at most 2 labels.

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.
The weight of a tree is the number of labels on the edges.
Claim: $|\mathcal{T}| = O(n^{2(k-1)})$.

- There are k nodes, so there can be $f(k)$ trees (ignoring labels).
- Each tree has $k - 1$ edges. Each edge can have at most 2 labels.
- Each tree has at most $2(k - 1)$ labels (weight at most $2(k - 1)$).

A candidate set of trees

$\mathcal{T}$ is the set of spanning trees that can appear in a solution subgraph.

The weight of a tree is the number of labels on the edges.

Claim: $|\mathcal{T}| = O(n^{2(k-1)})$.

- There are k nodes, so there can be $f(k)$ trees (ignoring labels).
- Each tree has $k - 1$ edges. Each edge can have at most 2 labels.
- Each tree has at most $2(k - 1)$ labels (weight at most $2(k - 1)$).

There are

$$f(k)\binom{n}{2}\binom{n-2}{2}\ldots\binom{n-2(k-1)}{2} = O(n^{2(k-1)})$$

trees.

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^{2.5}) = O(n^{2k+\frac{1}{2}})$ time.

A slow polynomial time algorithm

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^{2.5}) = O(n^{2k+\frac{1}{2}})$ *time.*

A simple improvement: use dynamic min-cost flow. Update the valid subgraph in $O(n^2)$ time.

A slow polynomial time algorithm

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^{2.5}) = O(n^{2k+\frac{1}{2}})$ time.

A simple improvement: use dynamic min-cost flow. Update the valid subgraph in $O(n^2)$ time.

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^2 + n^{2.5}) = O(n^{2k} + n^{2.5})$ time.

A slow polynomial time algorithm

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^{2.5}) = O(n^{2k+\frac{1}{2}})$ time.

A simple improvement: use dynamic min-cost flow. Update the valid subgraph in $O(n^2)$ time.

Theorem
There exists an algorithm that solves the k-depot warehouse tour problem in $O(n^{2(k-1)} \cdot n^2 + n^{2.5}) = O(n^{2k} + n^{2.5})$ time.

Worse than the state of the art for $k \leq 4$.

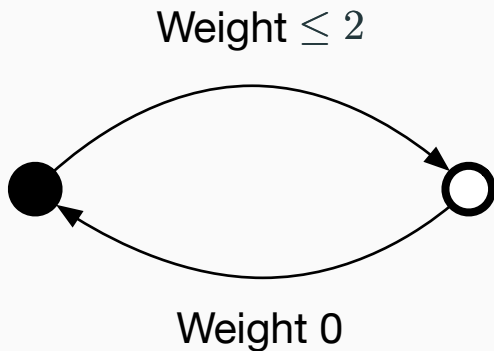**Faster algorithm: using a better set of trees.**

Our analysis:

- $\mathcal{T}$ set of possible spanning trees in structure graphs.
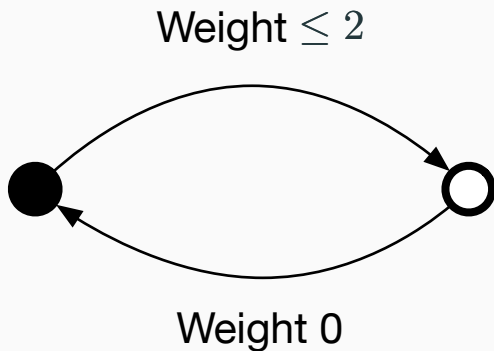- Bound $|\mathcal{T}|$ by $O(n^w)$, where w is the maximum weight over all trees in $\mathcal{T}$.

Our analysis:

- $\mathcal{T}$ set of possible spanning trees in structure graphs.
- Bound $|\mathcal{T}|$ by $O(n^w)$, where w is the maximum weight over all trees in $\mathcal{T}$.

Idea: Let $\mathcal{T}$ be the set of *minimum* spanning trees.

Weight ≤ 2

Weight 0

Do we expect improvements?

Weight ≤ 2

Weight 0

Yes!

The punch line

mst(H): the weight of the minimum spanning tree in H.

mst(H): the weight of the minimum spanning tree in H.

Theorem
Let H be a solution graph on k vertices, and $|D_I|, |D_O| \geq 1$, then
mst(H) $\leq k - 2$.

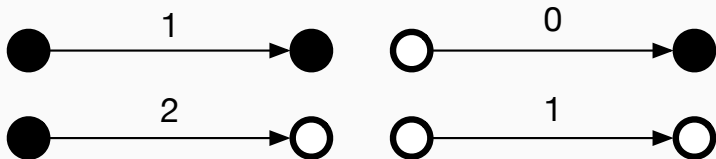mst(H): the weight of the minimum spanning tree in H.

Theorem
Let H be a solution graph on k vertices, and $|D_I|, |D_O| \geq 1$, then mst(H) $\leq k - 2$.

Corollary
There exists an algorithm for k-depot warehouse tour with running time $O(n^k + n^{2.5})$, for the case when there is at least one input and output depot.
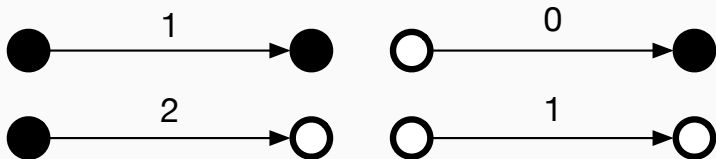
Weights of the structure graph

Upper bound on the weights of edges, depending on depot type.

Weights of the structure graph

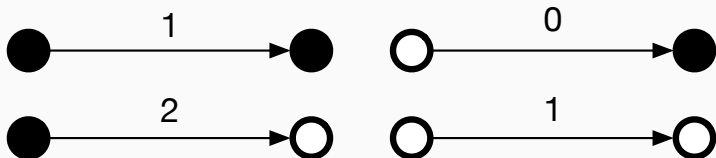Upper bound on the weights of edges, depending on depot type.

Summarized by having vertex weights.

- $w_0(v) = 0$ if $v \in D_O$, $w_0(v) = 1$ if $v \in D_I$.
- $w_1(v) = 0$ if $v \in D_I$, $w_1(v) = 1$ if $v \in D_O$.
- $w'((u, v)) = w_0(u) + w_1(v)$.

Weights of the structure graph

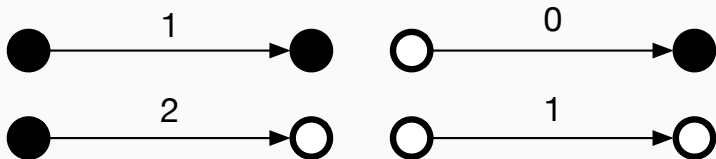Upper bound on the weights of edges, depending on depot type.

Summarized by having vertex weights.

- $w_0(v) = 0$ if $v \in D_O$, $w_0(v) = 1$ if $v \in D_I$.
- $w_1(v) = 0$ if $v \in D_I$, $w_1(v) = 1$ if $v \in D_O$.
- $w'((u, v)) = w_0(u) + w_1(v)$.

$w'(e)$ is an upper bound to the edge weight of e.

Weights of the structure graph

Upper bound on the weights of edges, depending on depot type.

Summarized by having vertex weights.

- $w_0(v) = 0$ if $v \in D_O$, $w_0(v) = 1$ if $v \in D_I$.
- $w_1(v) = 0$ if $v \in D_I$, $w_1(v) = 1$ if $v \in D_O$.
- $w'((u, v)) = w_0(u) + w_1(v)$.

$w'(e)$ is an upper bound to the edge weight of e. We will abuse the notation and refer $w'(e)$ as the edge weight.

Ear decomposition

Let $G = (V, E)$ be a directed graph. A sequence of set of edges $E_1, \ldots, E_k$ that partitions E is a ear decomposition if:
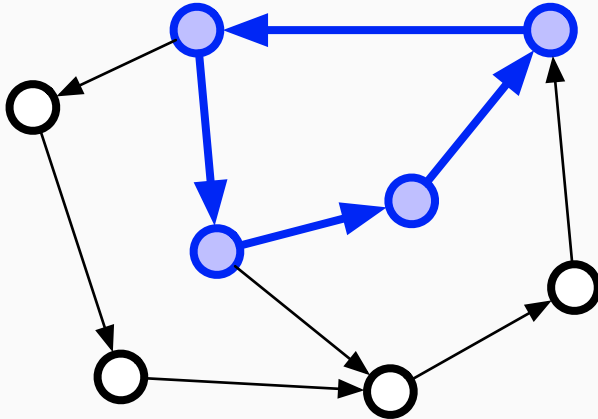
- E_1 is a cycle, each $E_2, \ldots, E_k$ is a path(including cycles).
- The start and end of the path E_i are vertices in $V(E_1 \cup \ldots \cup E_{i-1})$. No other vertex in $V(E_i)$ is in $V(E_1 \cup \ldots \cup E_{i-1})$.

$E_1, \ldots, E_k$ are called ears.
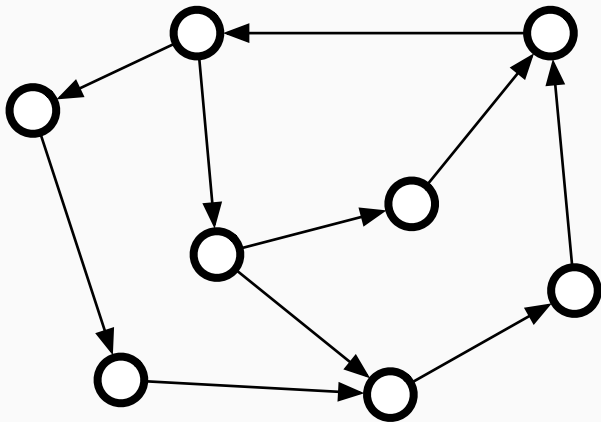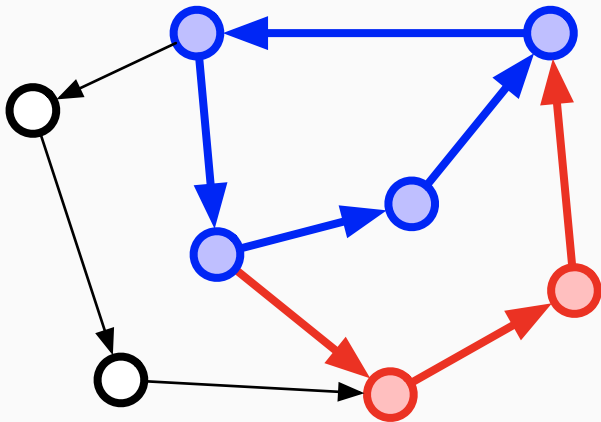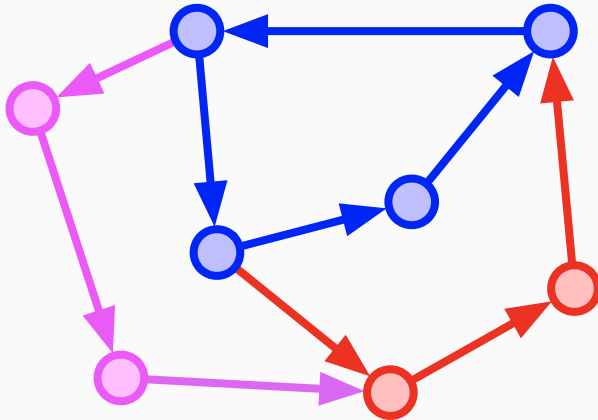
Theorem
Let G be a strongly connected directed graph, and C is a cycle in G. There exists a ear decomposition $E_1, \ldots, E_j$ where $E_1 = C$.

Proof of the MST theorem

Proof by induction on the number of ears in the ear decomposition.

Let H have ear decomposition $E_1, \ldots, E_t$. We can chose E_1 to be a cycle with at least one input depot and one output depot.

Theorem
Let $P = v_1, \ldots, v_n$ be a path and P start with a input depot, and end with an output depot, then there exists an edge of weight 2.

Base Case

Theorem
Let $P = v_1, \ldots, v_n$ be a path and P start with a input depot, and end with an output depot, then there exists an edge of weight 2.

Proof.
Since v_1 is an input depot, v_n is an output depot. For some i, v_i is an input depot and v_{i+1} is an output depot. The edge $v_i v_{i+1}$ has weight 2. $\qquad\square$

Base Case

Theorem
Let $P = v_1, \ldots, v_n$ be a path and P start with a input depot, and end with an output depot, then there exists an edge of weight 2.

Proof.
Since v_1 is an input depot, v_n is an output depot. For some i, v_i is an input depot and v_{i+1} is an output depot. The edge $v_i v_{i+1}$ has weight 2. $\qquad\square$

Theorem
C is a cycle of k vertices with at least one input depot and one output depot, then $\text{mst}(C) = k - 2$.

Base Case

Theorem
Let $P = v_1, \ldots, v_n$ be a path and P start with a input depot, and end with an output depot, then there exists an edge of weight 2.

Proof.
Since v_1 is an input depot, v_n is an output depot. For some i, v_i is an input depot and v_{i+1} is an output depot. The edge $v_i v_{i+1}$ has weight 2. $\qquad\square$

Theorem
C is a cycle of k vertices with at least one input depot and one output depot, then $\mathrm{mst}(C) = k - 2$.

Proof.
The total edge weight is $\sum_{e \in C} w'(e) = \sum_{v \in C} w_0(v) + w_1(v) = k$. Take any path from an input depot to an output depot, and remove the weight 2 edge in the path. $\qquad\square$

Inductive step

Path case. Assume E_t is a path and not a cycle.
$H' = (V(E_1 \cup \ldots \cup E_{t-1}), E_1 \cup \ldots \cup E_{t-1})$.
$\mathsf{mst}(H) \leq \mathsf{mst}(H') + w'(E_t) - \max_{e \in E_t} w'(e)$.

Inductive step

Path case. Assume E_t is a path and not a cycle.
$H' = (V(E_1 \cup \ldots \cup E_{t-1}), E_1 \cup \ldots \cup E_{t-1})$.
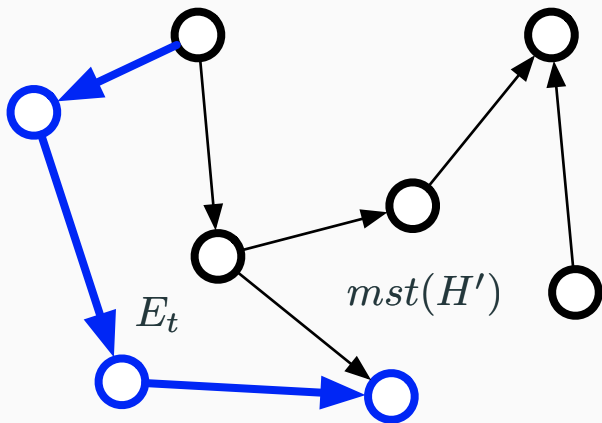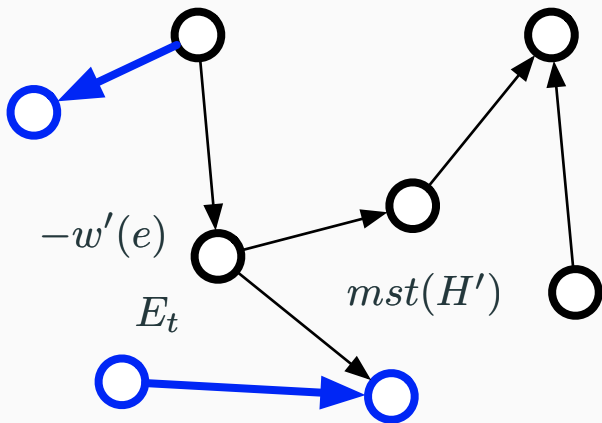$\mathsf{mst}(H) \leq \mathsf{mst}(H') + w'(E_t) - \max_{e \in E_t} w'(e)$.

Inductive step

Path case. Assume E_t is a path and not a cycle.
$H' = (V(E_1 \cup \ldots \cup E_{t-1}), E_1 \cup \ldots \cup E_{t-1})$.
$\text{mst}(H) \leq \text{mst}(H') + w'(E_t) - \max_{e \in E_t} w'(e)$.

Inductive step. cont.

$$
\begin{aligned}
\mathsf{mst}(H) &\leq \mathsf{mst}(H') + \sum_{e \in E_t} w'(e) - \max_{e \in E_t} w'(e) \\
&\leq (|V(H')| - 2) + \sum_{e \in E_t} w'(e) - \max_{e \in E_t} w'(e) \\
&= (|V(H')| - 2) + (|V(E_t)| - w_1(u) - w_0(v)) - \max_{e \in E_t} w'(e) \\
&= (|V(H)| - 2) + 2 - w_1(u) - w_0(v) - \max_{e \in E_t} w'(e).
\end{aligned}
$$

We have to show that $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

Inductive step. cont.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

- $w_1(u) = 1$, one of the edges containing u has weight at least 1.

Inductive step. cont.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

- $w_1(u) = 1$, one of the edges containing u has weight at least 1.
- Similarly for $w_0(v) = 1$.

Inductive step. cont.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

- $w_1(u) = 1$, one of the edges containing u has weight at least 1.

- Similarly for $w_0(v) = 1$.

- $w_1(u) + w_0(v) = 0$, then E_t is a path from a input depot to a output depot, hence there exists an edge of weight 2 in E_t.

Inductive step. cont.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

- $w_1(u) = 1$, one of the edges containing u has weight at least 1.

- Similarly for $w_0(v) = 1$.

- $w_1(u) + w_0(v) = 0$, then E_t is a path from a input depot to a output depot, hence there exists an edge of weight 2 in E_t.

The case where E_t is a cycle is similar.

Inductive step. cont.

Prove that: $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

- $w_1(u) = 1$, one of the edges containing u has weight at least 1.

- Similarly for $w_0(v) = 1$.

- $w_1(u) + w_0(v) = 0$, then E_t is a path from a input depot to a output depot, hence there exists an edge of weight 2 in E_t.

The case where E_t is a cycle is similar. This completes the proof.

Theorem
Let H be a k-vertex structure graph with only input depots, then
$\mathsf{mst}(H) \leq k - 1$.

What about only input depots?

Theorem
Let H be a k-vertex structure graph with only input depots, then
$mst(H) \leq k - 1$.

Same proof by induction on ear decomposition. The base case is a
single cycle C, where $mst(C) = k - 1$, the rest of the proof follows.

- What if the dist is not a metric?

- What if the dist is not a metric? Use a shortest path metric dist' instead.

Variations

- What if the dist is not a metric? Use a shortest path metric dist$'$ instead.
- What if the machine can start only in locations L_{start}, and end in a set of locations L_{end}?

Variations

- What if the dist is not a metric? Use a shortest path metric dist$'$ instead.
- What if the machine can start only in locations L_{start}, and end in a set of locations L_{end}? Simple transformation to the case where machine start and end at same position.

Variations

- What if the dist is not a metric? Use a shortest path metric dist' instead.
- What if the machine can start only in locations L_{start}, and end in a set of locations L_{end}? Simple transformation to the case where machine start and end at same position.
- What if each input request can only be completed by a particular input depot?

Variations

- What if the dist is not a metric? Use a shortest path metric dist' instead.

- What if the machine can start only in locations L_{start}, and end in a set of locations L_{end}? Simple transformation to the case where machine start and end at same position.

- What if each input request can only be completed by a particular input depot? Remove edges from depots to the request in the warehouse network, compute a new metric, and use the new metric to construct the warehouse network.

- Output requests only, but a machine can hold two item at a time.

- Output requests only, but a machine can hold two item at a time. Solvable in polynomial time if the metric is symmetric. [Xu, Yang, Zhang Unpublished]

Some ongoing work

- Output requests only, but a machine can hold two item at a time. Solvable in polynomial time if the metric is symmetric. [Xu, Yang, Zhang Unpublished]
- Multiple machines.

- Output requests only, but a machine can hold two item at a time. Solvable in polynomial time if the metric is symmetric. [Xu, Yang, Zhang Unpublished]
- Multiple machines. Solvable in polynomial time if number of empty depot is constant. [Unpublished]

Some ongoing work

- Output requests only, but a machine can hold two item at a time. Solvable in polynomial time if the metric is symmetric. [Xu, Yang, Zhang Unpublished]
- Multiple machines. Solvable in polynomial time if number of empty depot is constant. [Unpublished]
- Each request is a set of locations.

Some ongoing work

- Output requests only, but a machine can hold two item at a time. Solvable in polynomial time if the metric is symmetric. [Xu, Yang, Zhang Unpublished]

- Multiple machines. Solvable in polynomial time if number of empty depot is constant. [Unpublished]

- Each request is a set of locations. Unknown status, preliminary work with Madan and Shen.

Thank you!