A Polynomial Time Algorithm to Minimize Total Travel Time in k-Depot Storage/Retrieval System

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A warehouse

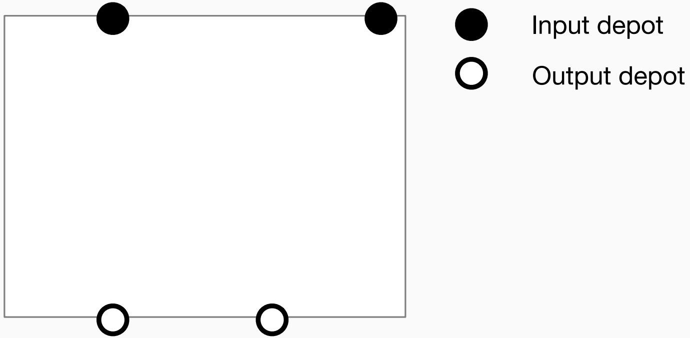
An automated warehouse with input depots and output depots. It has to complete input(storage) and output(retrieval) requests.

Figure 1: Demag V-type crane machine. Source: `demagcranes.com`

The storage and retrial machine

- The machine start at some depot.
- The machine can hold at most one item.
- The machine can pick up an item from any input depot, and drop off the item at a input request location.
- The machine can pick up an item from a output request location, and drop off the item at any output depot.
- The machine must return to the original depot.

Example



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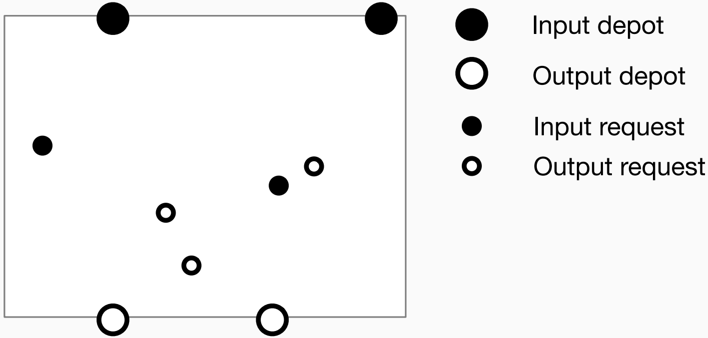

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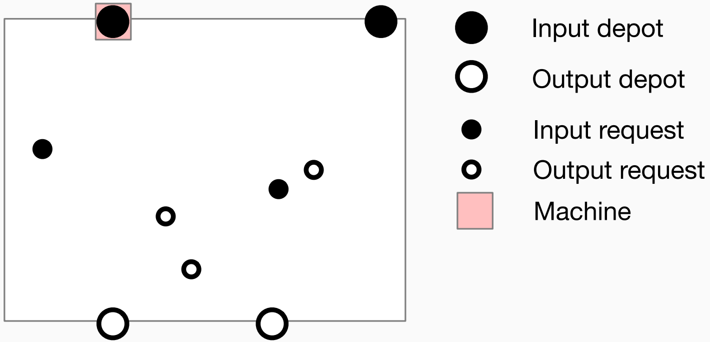
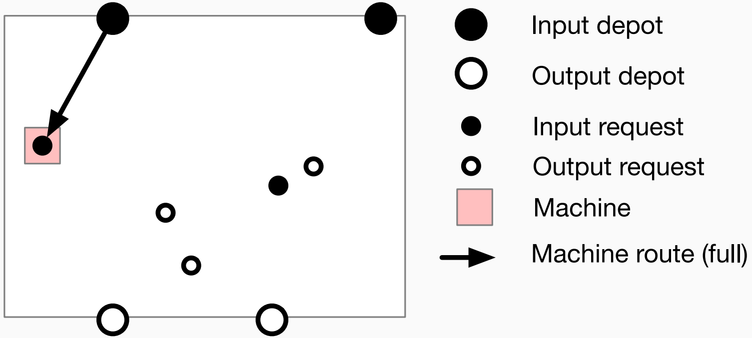


Input depot
Output depot
Input request
Output request
Machine

Example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Machine route (full)

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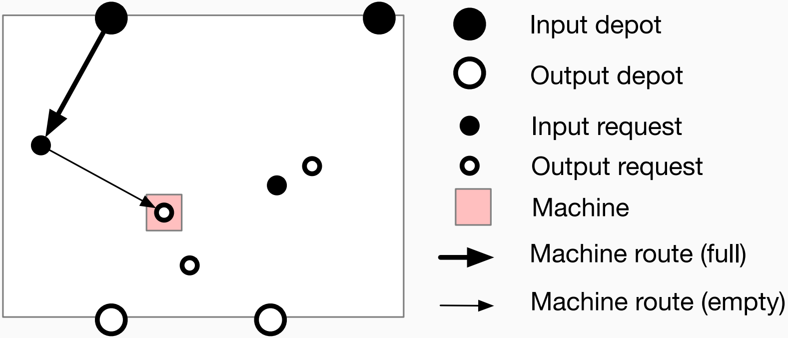
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Machine route (empty)

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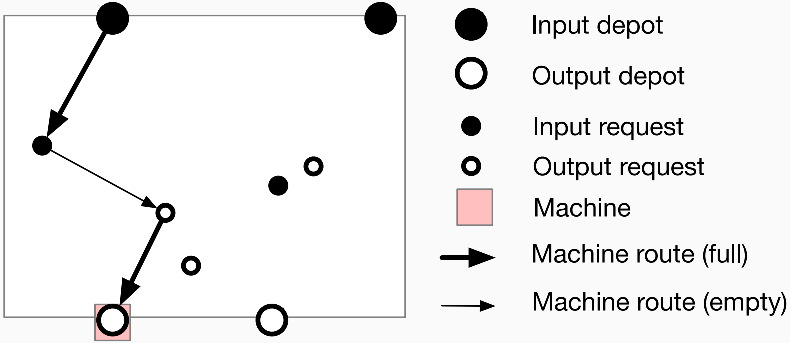
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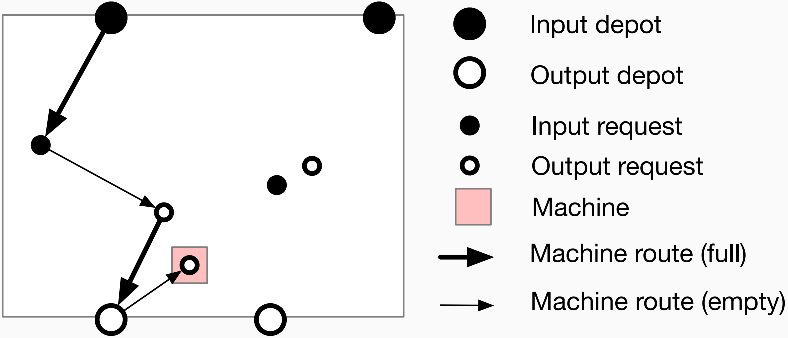
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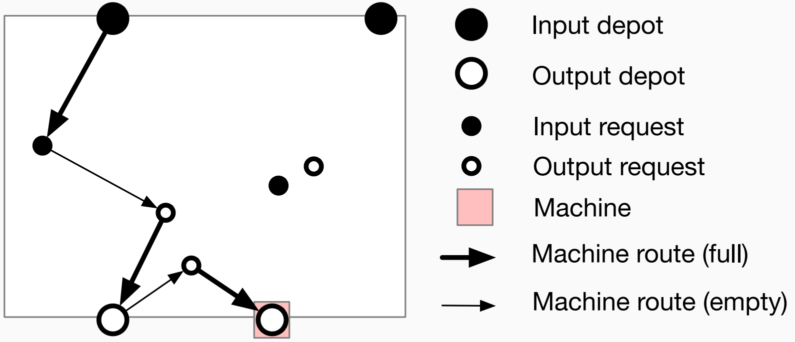

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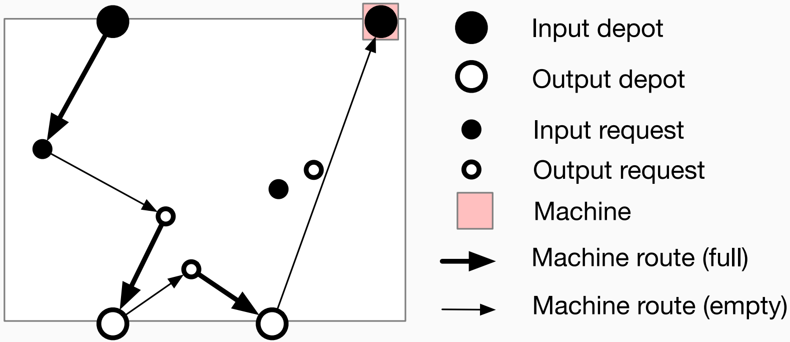
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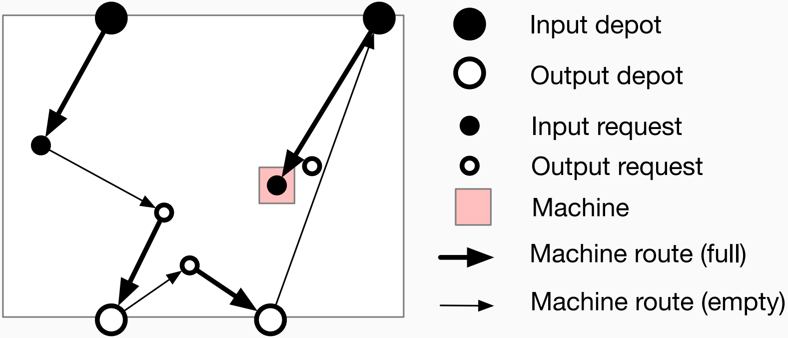
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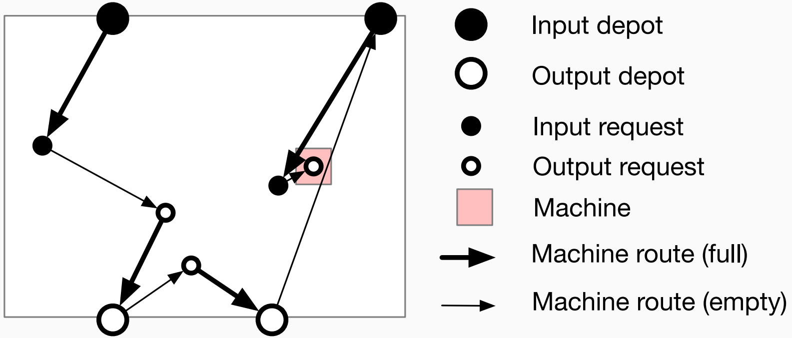

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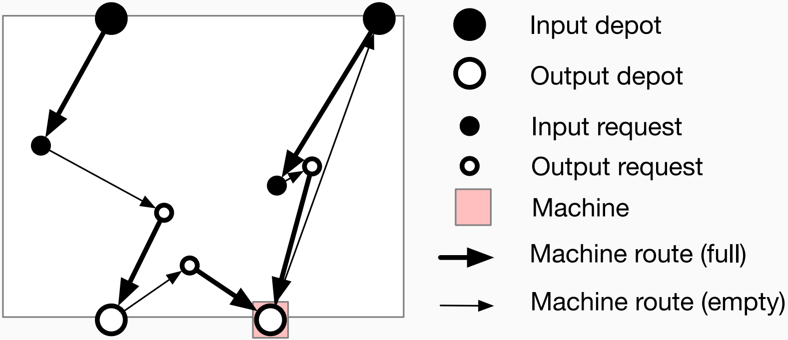
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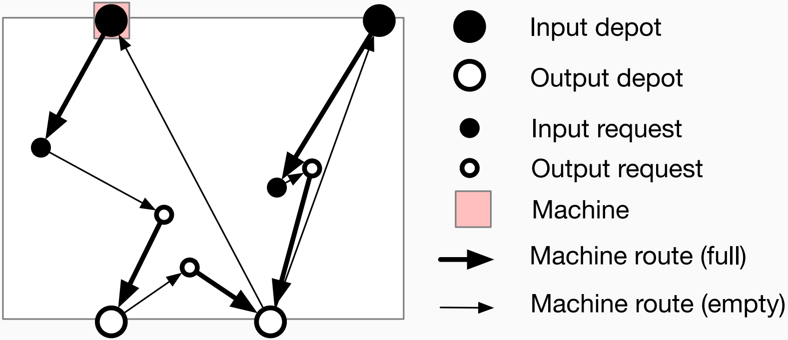

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Input of the problem

- Input depots D_I, output depots D_O, $D = D_I \cup D_O$. $|D| = k$.
- Input request R_I, output request R_O, $R = R_I \cup R_O$. $|R| = n$.
- $V = D \cup R$, the set of vertices.
- dist : $V \times V \to \mathbb{R}_+$ a asymmetric metric.

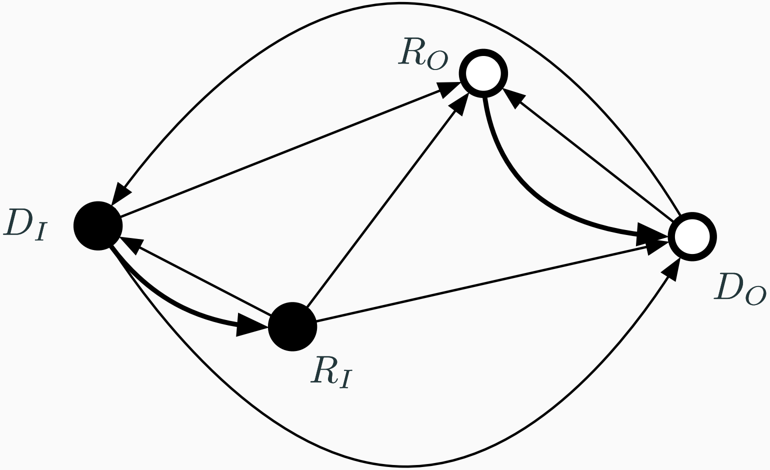
Model as a walk on a graph

For input D_I, D_O, R_I, R_O, c, we construct the following weighted directed graph G.

- For $d, d' \in D$, there is an edge (d, d').
- For $v \in V$, $u \in R_I$, there is an edge (u, v).
- For $v \in V$, $u \in R_O$, there is an edge (v, u).
- For $v \in R_I$, $d \in D_I$, there is an edge (d, v).
- For $v \in R_O$, $d \in D_O$, there is an edge (v, d).

The cost of an edge $c(u, v) = \text{dist}(u, v)$. Such graph G is called a warehouse network.

Warehouse network, high level view



The abstract problem

Problem: k-depot warehouse tour

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Observation: the optimal solution goes through each vertex in R exactly once, because of the metric.

Previous results

A regular depot is a pair of input depot d and output depot d' with $\mathrm{dist}(d, d') = 0$.

[Gharehgozli, Yu, Zhang, de Koster '17] considered special cases of the problem.

- $k = 4$: 2 pairs of regular depots. Running time $O(n^6)$.
- $k = 2$: 2 depots, one input, one output. Running time $O(n^3)$.

Our result

Theorem
The k-depot warehouse tour can be solved in

- $O(n^{k+1} + n^{2.5})$ *time if all depots are input(output) depots.*
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Counterintuitive! Having depots of only one type is harder.

A simple polynomial time algorithm

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A simpler connectivity condition

Theorem
If H is valid, then it is connected if and only if D is connected.

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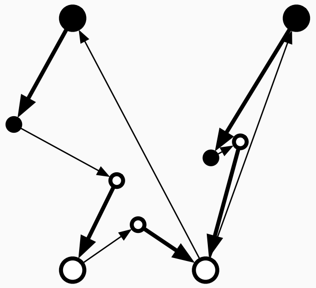
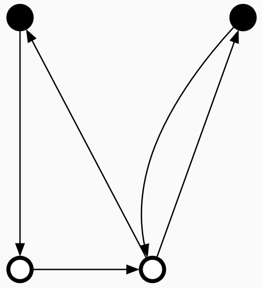
3. If an optimum solution to the original problem contains all edges in F, then H is an optimum solution to the original problem.

Structure graph of a solution

The structure graph of a solution is obtained by the following transformation. For each depot to depot path that does not contain any other depot P. Let P' be the sequence of internal vertices, and P is from d to d'. We create an edge e from d to d', and give it the label P'.





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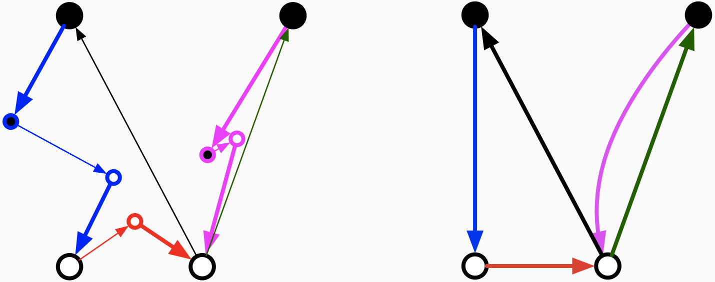

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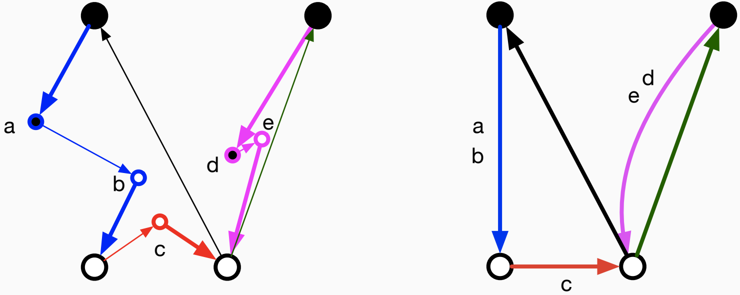

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A candidate set of trees

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There are

$$f(k)\binom{n}{2}\binom{n-2}{2}\dots\binom{n-2(k-1)}{2} = O(n^{2(k-1)})$$

trees.

A slow polynomial time algorithm

Theorem
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Worse than the state of the art for $k \leq 4$.

Faster algorithm: using a better set of trees.

Our analysis:

- $\mathcal{T}$ set of possible spanning trees in structure graphs.
- Bound $|\mathcal{T}|$ by $O(n^w)$, where w is the maximum weight over all trees in $\mathcal{T}$.

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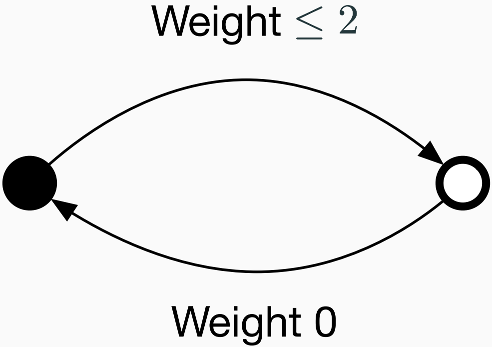
Idea: Let $\mathcal{T}$ be the set of *minimum* spanning trees.

Do we expect improvements?



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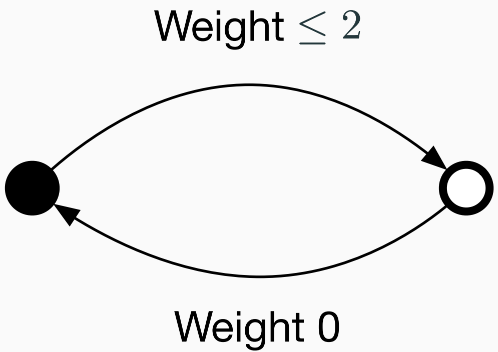

Yes!

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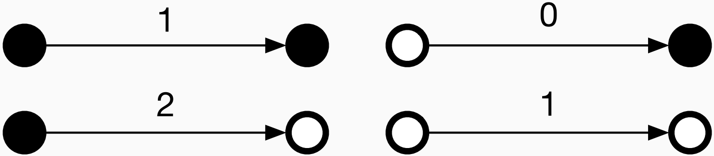
Corollary
There exists an algorithm for k-depot warehouse tour with running time $O(n^k + n^{2.5})$, for the case when there is at least one input and output depot.

Weights of the structure graph

Upper bound on the weights of edges, depending on depot type.



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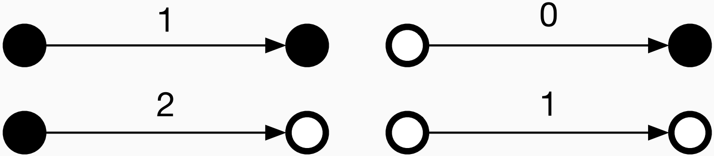

Summarized by having vertex weights.

- $w_0(v) = 0$ if $v \in D_O$, $w_0(v) = 1$ if $v \in D_I$.
- $w_1(v) = 0$ if $v \in D_I$, $w_1(v) = 1$ if $v \in D_O$.
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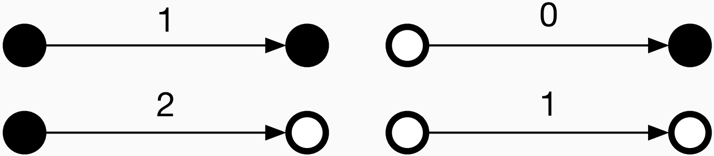

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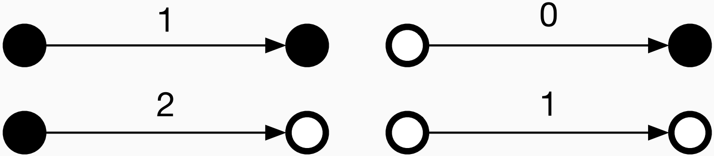

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$w'(e)$ is an upper bound to the edge weight of e. We will abuse the notation and refer $w'(e)$ as the edge weight.

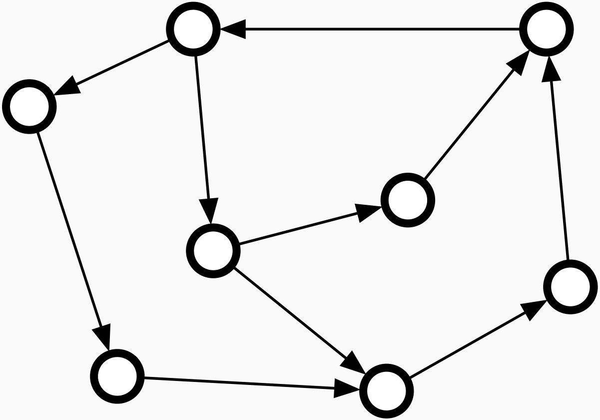
Ear decomposition

Let $G = (V, E)$ be a directed graph. A sequence of set of edges $E_1, \ldots, E_k$ that partitions E is a ear decomposition if:

- E_1 is a cycle, each $E_2, \ldots, E_k$ is a path(including cycles).
- The start and end of the path E_i are vertices in $V(E_1 \cup \ldots \cup E_{i-1})$. No other vertex in $V(E_i)$ is in $V(E_1 \cup \ldots \cup E_{i-1})$.

$E_1, \ldots, E_k$ are called ears.

Example of an ear decomposition



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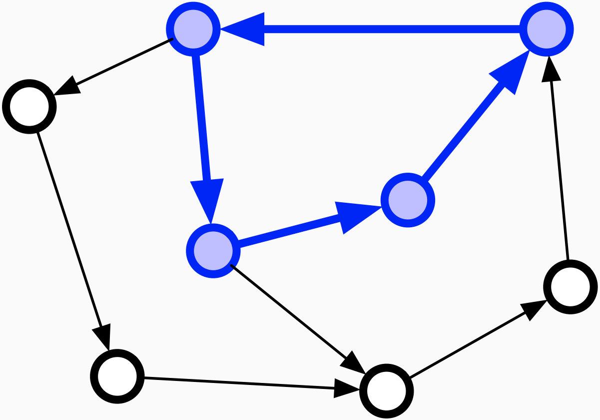

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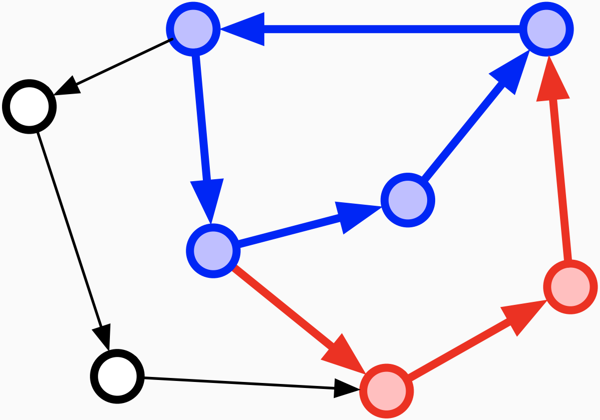

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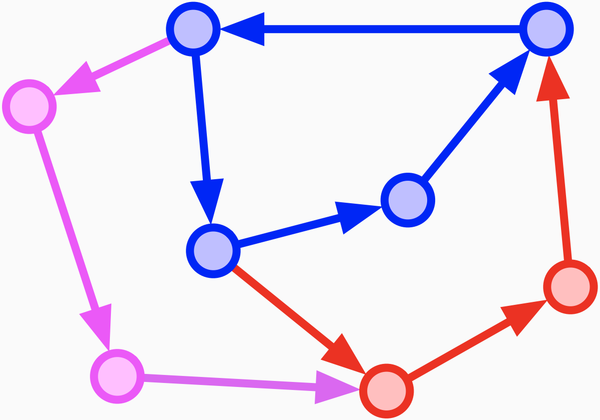

Ear decomposition

Theorem
Let G be a strongly connected directed graph, and C is a cycle in G. There exists a ear decomposition $E_1, \ldots, E_j$ where $E_1 = C$.

Proof of the MST theorem

Proof by induction on the number of ears in the ear decomposition.

Let H have ear decomposition $E_1, \ldots, E_t$. We can chose E_1 to be a cycle with at least one input depot and one output depot.

Base Case

Theorem
Let $P = v_1, \ldots, v_n$ be a path and P start with a input depot, and end with an output depot, then there exists an edge of weight 2.

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C is a cycle of k vertices with at least one input depot and one output depot, then $\mathrm{mst}(C) = k - 2$.

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Theorem
C is a cycle of k vertices with at least one input depot and one output depot, then $\mathrm{mst}(C) = k - 2$.

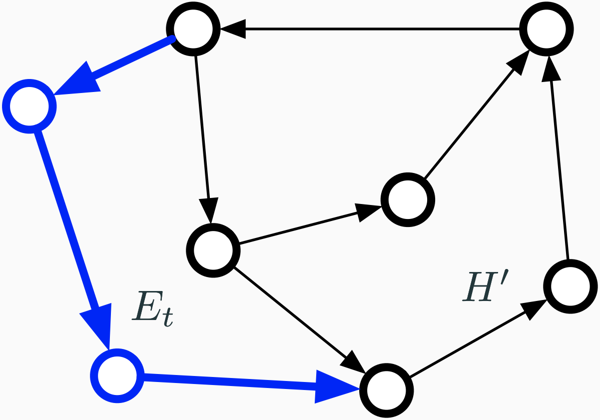
Proof.
The total edge weight is $\sum_{e \in C} w'(e) = \sum_{v \in C} w_0(v) + w_1(v) = k$. Take any path from an input depot to an output depot, and remove the weight 2 edge in the path. $\square$

Inductive step

Path case. Assume E_t is a path and not a cycle.

$H' = (V(E_1 \cup \ldots \cup E_{t-1}), E_1 \cup \ldots \cup E_{t-1})$.

$\mathrm{mst}(H) \leq \mathrm{mst}(H') + w'(E_t) - \max_{e \in E_t} w'(e)$.



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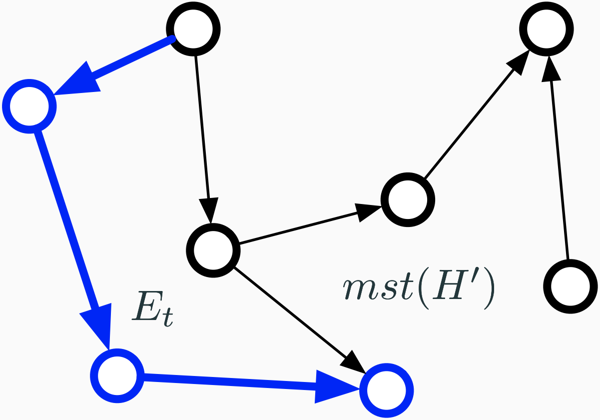

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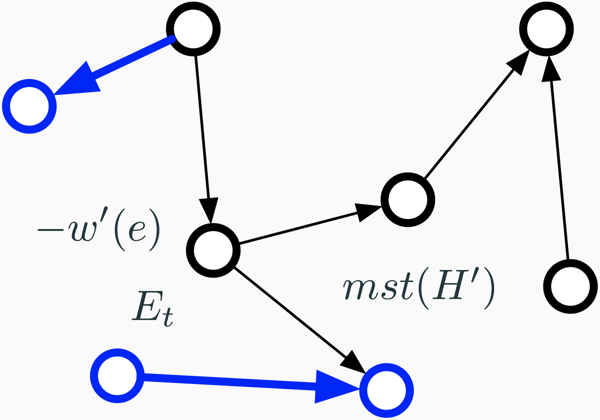

Inductive step. cont.

$$\begin{aligned}
\operatorname{mst}(H) &\leq \operatorname{mst}(H') + \sum_{e \in E_t} w'(e) - \max_{e \in E_t} w'(e) \\
&\leq (|V(H')| - 2) + \sum_{e \in E_t} w'(e) - \max_{e \in E_t} w'(e) \\
&= (|V(H')| - 2) + (|V(E_t)| - w_1(u) - w_0(v)) - \max_{e \in E_t} w'(e) \\
&= (|V(H)| - 2) + 2 - w_1(u) - w_0(v) - \max_{e \in E_t} w'(e).
\end{aligned}$$

We have to show that $w_1(u) + w_0(v) + \max_{e \in E_t} w'(e) \geq 2$.

Inductive step. cont.

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The case where E_t is a cycle is similar. This completes the proof.

What about only input depots?

Theorem
Let H be a k-vertex structure graph with only input depots, then $\mathrm{mst}(H) \leq k - 1$.

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Let H be a k-vertex structure graph with only input depots, then $\mathrm{mst}(H) \leq k - 1$.

Same proof by induction on ear decomposition. The base case is a single cycle C, where $\mathrm{mst}(C) = k - 1$, the rest of the proof follows.

Some ongoing work

- Output requests only, but a machine can hold two item at a time.

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- Each request is a set of locations. Unknown status, preliminary work with Madan and Shen.

Thank you!