## Hit or Stand on $A \triangleleft 7 \triangleleft ?$

# A Statistical Analysis of Blackjack Strategies via Monte Carlo Simulation

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#### **Abstract**

This report investigates the efficacy of various playing strategies in the casino game of Blackjack, with the objective of identifying an optimal approach to maximize player profit. Leveraging a largescale Monte Carlo simulation, we analyze over one million hands for each of six distinct strategies, ranging from naive heuristics to professional-level techniques. Key performance metrics, including player edge and outcome rates, are calculated from the simulation data.

The results empirically confirm that Basic Strategy reduces the house edge to -1.02%. A full Card Counting strategy, incorporating both bet spreading and playing deviations, yielded a positive overall player edge of +0.62%.

The analysis also demonstrates the high volatility and risk of ruin associated with the Martingale betting system, despite its potential for short-term gains.

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	Foundation		Blackjack stands as one of the most popular			
	3.1 Beat the Dealer	2	casino card games globally, distinguished by its			
	3.2 The Theory of Blackjack	2	blend of chance and skill. Unlike games of pure probability, a player's decisions in Blackjack di-			
4	Methodology		rectly influence the outcome, inviting mathe-			
	4.1 Game Rules	<b>3</b> 3	matical analysis to identify an optimal playing			
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5	Results 5.1 Overall Strategy Performance 5.2 Fixed Threshold Analysis 5.3 Distribution of Returns and Risk	3 4 5 5	profit? While short-term results can be misleading, a long-term analysis over a statistically significant number of hands can reveal the true expected value of any given strategy.			

# 2 A Hierarchy of Strategies

This project utilizes a custom-built Monte Carlo simulation to perform such an analysis. To build a comprehensive understanding of what drives success, we test a hierarchy of strategies, each representing a different level of player sophistication. By comparing these approaches, we aim to quantify the advantage gained at each level of strategic complexity.

#### 2.1 Naive Strategies

We first establish a baseline by simulating two simplistic strategies: "Mimic the Dealer" and a "Fixed Threshold" analysis. These represent players with little to no knowledge of strategy and serve to quantify the cost of uninformed decision-making.

## 2.2 Heuristic Strategy

We then test "Dealer's Weakness," a simplified adaptive approach. This represents a more observant casual player who uses a simple rule-ofthumb to adjust their play, providing a bridge between naive and optimal play.

## 2.3 Optimal Strategy

The mathematically-derived "Basic Strategy" is simulated as the gold standard for non-counting players. This strategy is designed to make the decision with the highest Expected Value for every possible hand.

#### 2.4 Professional Strategy

To model professional play, we simulate a "Card Counting" strategy using the Hi-Lo system. This memory-based approach attempts to gain a positive long-term advantage over the house.

## 2.5 Betting Strategy

Finally, we analyze the "Martingale" system. This is a different type of analysis, focused not on playing decisions but on how a betting methodology impacts bankroll volatility and risk of ruin.

# 3 Literature Review and Theoretical Foundation

#### 3.1 Beat the Dealer

The mathematical analysis of Blackjack was largely pioneered by **Edward O. Thorp** in his seminal 1962 work, *Beat the Dealer*<sup>[2]</sup>. This book was revolutionary, providing the first mathematical proof that the game was solvable. While famous for introducing card counting, its key contribution to this project is the formulation and validation of a "Basic Strategy"—a set of optimal decisions for any hand, independent of counting. Thorp's work established that a mathematically superior, non-random approach to playing each hand exists.

# 3.2 The Theory of Blackjack

Building on this foundation, **Peter Griffin's** *The Theory of Blackjack* [1] provided a more rigorous mathematical framework, explaining precisely why Basic Strategy is optimal. Griffin's primary contribution was the formalization of **Expected Value (EV)** for every possible gameplay decision. This concept, which represents the average gain or loss for an action if repeated infinitely, is the central theoretical pillar of our project. Basic Strategy is simply the path that maximizes the player's EV at every step. Therefore, our simulation is, in effect, a method of empirically estimating the EV for each strategy we test.

# 4 Methodology

We developed a discrete-event simulation in Python\* to model the game of Blackjack. The simulation was designed with an object-oriented architecture, with classes for Card, Deck, and Hand.

#### 4.1 Game Rules

The simulation adheres to a standard set of Las Vegas Strip rules:

- Decks: 6 standard 52-card decks are used.
- **Dealer Rules:** The dealer stands on all 17s (hard and soft).
- **Player Options:** Players can hit, stand, double down, and split pairs.
- Blackjack Payout: A player Blackjack pays 3:2.
- **Shuffling:** For all non-counting strategies, the shoe is reshuffled after each hand. For the Card Counting strategy, the shoe is played through until it is completely empty before being reshuffled.

## 4.2 Simulated Strategies

The following strategies were implemented and tested:

- 1. **Mimic the Dealer:** Player hits until 17 or more.
- 2. **Fixed Threshold Analysis:** A sensitivity analysis where we iterate the player's standing threshold from 12 through 16.
- 3. **Dealer's Weakness:** A simplified adaptive strategy where play is conservative against a dealer's weak upcard (2-6) and aggressive against a strong one (7-Ace).
- 4. **Basic Strategy:** Player decisions are based on standard, mathematically-derived strategy charts.
- 5. **Martingale Betting System:** This betting strategy (paired with Basic Strategy) doubles the bet after every loss and returns to a base bet after a win.
- 6. **Card Counting Strategy:** Using the Hi-Lo system, the player engages in bet spreading and makes playing deviations based on the count.

#### 4.3 Data Collection

Each strategy was simulated for **1,000,000 hands** to ensure statistical significance and minimize the impact of short-term variance. For each hand, detailed data was collected, including the hands, final scores, bet amount, and profit/loss.

## 5 Results

The simulation produced the following aggregate results. The "Player Edge" is calculated as (Total Profit / Total Amount Bet).

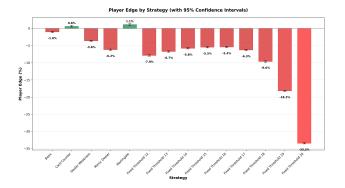
<sup>\*</sup>The complete source code for this project is publicly available on GitHub: https://github.com/KeXin95/blackjack-sim.

Table 1: Performance Metrics over 1,000,000 Ga	ames
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Strategy	Profit	Player Edge	Win %	Avg Bet	Max Bet
Martingale	\$827,470.00	+1.15%*	43.07%	\$72.12	\$2,621,440.00
Card Counter	\$112,295.00	+0.62%	43.08%	\$18.01	\$100.00
Basic Strategy	-\$102,350.00	-1.02%	42.97%	\$10.00	\$10.00
Dealer Weakness	-\$361,145.00	-3.61%	42.80%	\$10.00	\$10.00
Mimic Dealer	-\$619,970.00	-6.20%	40.90%	\$10.00	\$10.00
Fixed (12)	-\$788,425.00	-7.88%	41.79%	\$10.00	\$10.00
Fixed (13)	-\$674,485.00	-6.74%	42.05%	\$10.00	\$10.00
Fixed (14)	-\$579,615.00	-5.80%	42.21%	\$10.00	\$10.00
Fixed (15)	-\$548,100.00	-5.48%	42.03%	\$10.00	\$10.00
Fixed (16)	-\$540,655.00	-5.41%	41.70%	\$10.00	\$10.00
Fixed (17)	-\$627,430.00	-6.27%	40.88%	\$10.00	\$10.00
Fixed (18)	-\$963,670.00	-9.64%	39.86%	\$10.00	\$10.00
Fixed (19)	-\$1,820,410.00	-18.20%	36.36%	\$10.00	\$10.00
Fixed (20)	-\$3,347,795.00	-33.48%	29.60%	\$10.00	\$10.00

\*Note: Player Edge for Martingale is not a meaningful metric of viability. The key result is the risk of ruin, demonstrated by a maximum required bet of over \$2,600,000.

The Monte Carlo simulation, executed for one million hands per strategy, yielded a clear hierarchy of performance. The primary metric for evaluation is the Player Edge, calculated as the total profit divided by the total amount wagered. This section presents the aggregated statistics and key visualizations from the analysis.



# 5.1 Overall Strategy Performance

The mean player edge for each strategy is summarized in Table 1. The results clearly show that Basic Strategy and Card Counting are the most effective approaches.

While the Card Counter strategy yielded a modest overall edge of +0.62%, this is an average over all one million hands, including many played with a minimum bet in neutral or unfavorable conditions. The strategy's true power lies in capitalizing on high-count situations, where the player's edge can rise significantly.

Figure 1: Comparison of player edge across all simulated strategies. The chart demonstrates that only Card Counting achieved a reliably positive edge, while Basic Strategy significantly reduced the house advantage compared to naive approaches. Error bars represent 95% confidence intervals.

A one-sample t-test was performed for each strategy to test the null hypothesis that the true mean edge is zero ( $H_0$ :  $\mu_{edge} = 0$ ). For every strategy, the resulting p-value was effectively zero (e.g., for Basic Strategy, the p-value was  $p \approx 7.37 \times 10^{-20}$ )

This provides overwhelming statistical evidence

to reject the null hypothesis in all cases, confirming that the observed player edges are statistically significant and not due to random chance.

#### 5.2 Fixed Threshold Analysis

A sensitivity analysis was performed on the "Fixed Threshold" strategy to determine the optimal standing point for a naive player. Figure 4 illustrates a clear parabolic relationship between the standing threshold and the player edge. Performance improves as the threshold increases from 12, peaks at a threshold of 16 (Player Edge: -5.41%), and then rapidly declines as the player's bust rate becomes prohibitively high.

#### 5.3 Distribution of Returns and Risk

While the mean edge measures long-term expectation, the distribution of single-hand outcomes reveals information about risk and variance. The

violin plot in Figure 3 illustrates these distributions. Most strategies show a similar shape, with outcomes concentrated at -1 (loss), +1 (win), and +1.5 (Blackjack win). The Martingale strategy, however, is a notable exception, displaying significantly wider tails. This indicates a much higher variance and a greater probability of extreme single-hand outcomes, which is a hallmark of its risk profile.

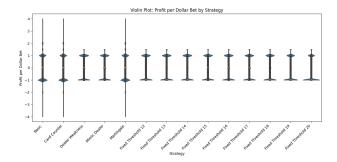


Figure 3: Violin plot showing the distribution of single-hand outcomes. Note the wider tails for the Martingale strategy, indicating higher variance.

#### 6 Discussion

The simulation results provide a clear hierarchy of strategic effectiveness in Blackjack, empirically validating established game theory and quantifying the statistical cost of uninformed play versus the potential edge gained through mathematical optimization.

balanced point between these two risks, yet still results in a substantial house edge of **-5.41%**.

optimal naive threshold of 16 represents the most

# 6.1 The High Cost of Uninformed Play

The naive strategies, Mimic the Dealer and Fixed Threshold, performed worst, yielding a house edge over 5%. These strategies fail to account for the player's primary disadvantage—acting first—or the crucial information provided by the dealer's upcard. The analysis of the "Fixed Threshold" strategy (Figure 4) perfectly illustrates this trade-off: a low threshold leads to frequent losses against beatable dealer hands, while a high threshold leads to frequent busting. The

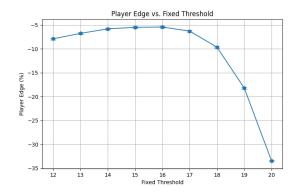


Figure 4: Player Edge as a function of the standing threshold.

#### **6.2** The Value of Simple Heuristics

A significant improvement was observed with the Dealer's Weakness strategy, which reduced the house edge to **-3.61%**. This strategy, while far from optimal, uses a simple adaptive heuristic based on the dealer's visible card. A two-sample t-test comparing its returns to the best naive strategy (Fixed Threshold 16) would confirm the statistical significance of this improvement. This demonstrates that even incorporating a single piece of contextual information adds tangible value to a player's performance.

# 6.3 Optimal Play vs. Professional Play

As expected, Basic Strategy was the most effective non-counting approach, confirming its status as the mathematically optimal method for a player not tracking the cards. Our simulation found it reduced the house edge to a mere - 1.02%. This makes the game a near-even proposition and establishes the critical performance benchmark that a professional player must overcome.

To beat this benchmark, a player must advance to a memory-based strategy like Card Counting. Our simulation of a professional Hi-Lo card counter tested a complete strategy that incorporates two distinct techniques: count-based playing deviations and, crucially, bet spreading. The result was a definitive success: the strategy reversed the house edge entirely, yielding a consistent, positive player edge of +0.62% over one million hands.

This positive outcome isolates the power of professional play. While small playing deviations (e.g., altering a hit/stand decision based on the count) contribute to this edge, the primary driver

of profit is **bet spreading**. By betting the table minimum when the deck is unfavorable and aggressively increasing bets when the count is high, the card counter ensures more money is wagered when they have a statistical advantage. It is this disciplined manipulation of bet sizes, combined with optimal playing decisions, that allows a professional to achieve a profitable long-term return.

# 6.4 The Martingale Fallacy: A Statistical Trap

The Martingale betting system provides the most dramatic illustration of risk. While this particular simulation produced a misleading positive edge of +1.15%, the strategy's true nature is revealed in its volatility.

As shown in the bankroll simulation (Fig. 5), the system exchanges a high probability of small gains for a low but near-certain probability of catastrophic loss. To survive the one million hands in this simulation, a player would have needed a bankroll sufficient to cover a maximum single-streak drawdown of \$3,252,145. This confirms that the Martingale system is statistically untenable, as it requires an infinite bankroll and no table limits—conditions that do not exist in reality.

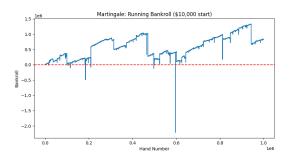


Figure 5: A simulation of the Martingale strategy bankroll. Note the catastrophic drawdowns, demonstrating extreme risk of ruin.

## 7 Conclusions

This project successfully developed and executed a Monte Carlo simulation to analyze a range of Blackjack strategies. The findings demonstrate a clear and quantifiable link between the complexity

of a strategy and its effectiveness. We conclude that while Basic Strategy is sufficient to play the game with a minimal disadvantage, only a memory-based strategy like Card Counting can provide a player with a profitable long-term edge over the casino. Furthermore, the analysis confirms that betting systems like the Martingale, while alluring, are statistically flawed and expose a player to unacceptable levels of risk. Future work could involve simulating a wider range of rule variations or more complex card counting systems.

## References

- [1] Peter A. Griffin. *The Theory of Blackjack: The Compleat Card Counter's Guide to the Casino Game of 21*. Huntington Press, 1999.
- [2] Edward O. Thorp. *Beat the Dealer: A Winning Strategy for the Game of Twenty-One*. Vintage Books, 1966.