Karlsruher Institut für Technologie

Modellbildung und Simulation
Kapitel 7: Systeme mit verteilten Parametern

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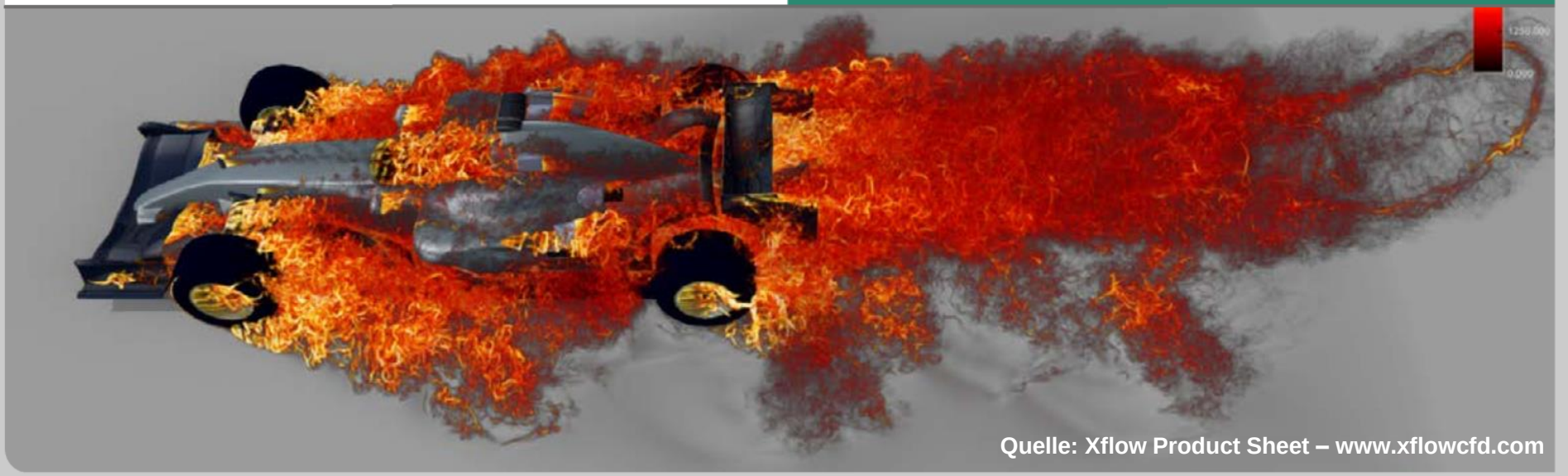
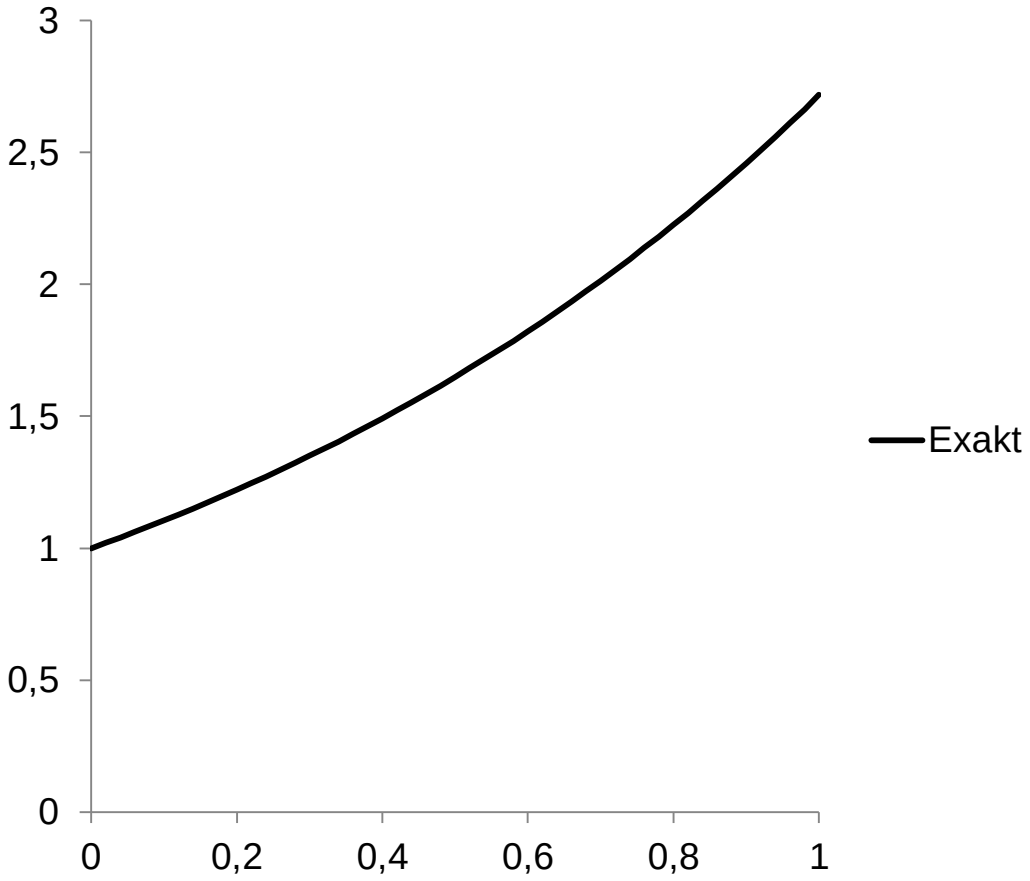
ITS



Quelle: Xflow Product Sheet – www.xflowcfd.com

Saalübung: gew. Residuen und FDM

Aufgabe 1: Methode der gewichteten Residuen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2
1,5
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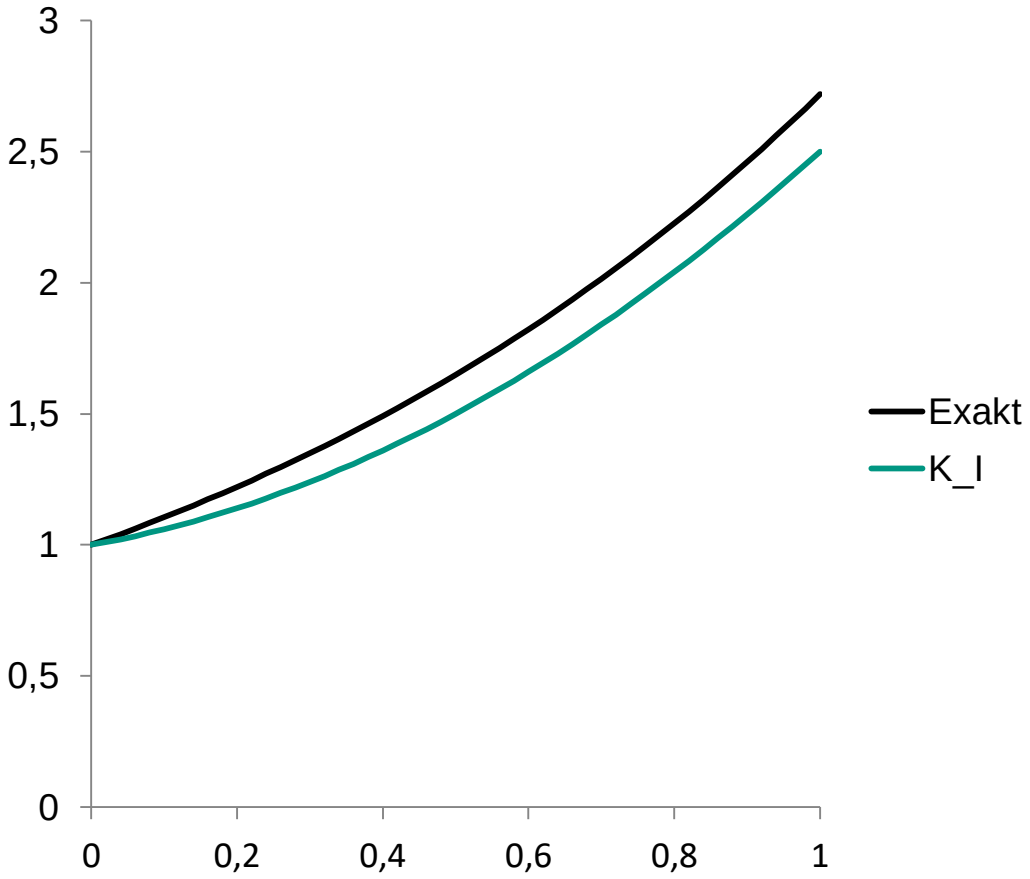
0,8
1
Exakt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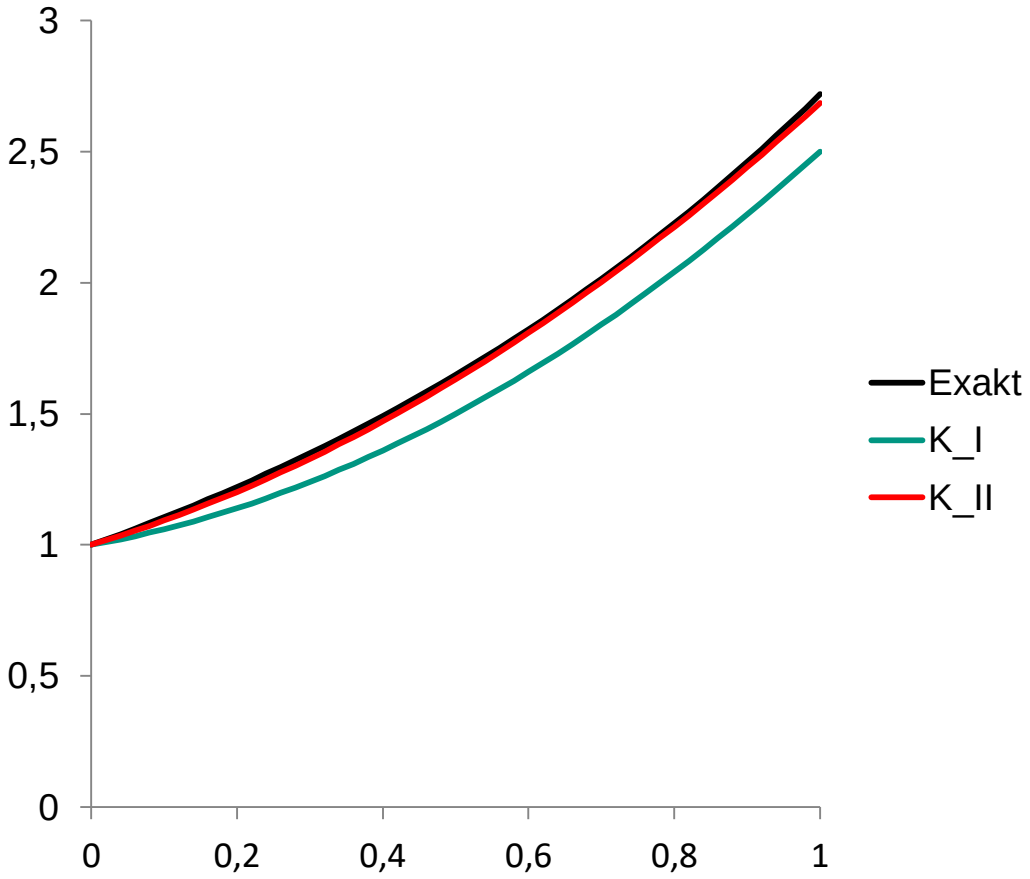
K_I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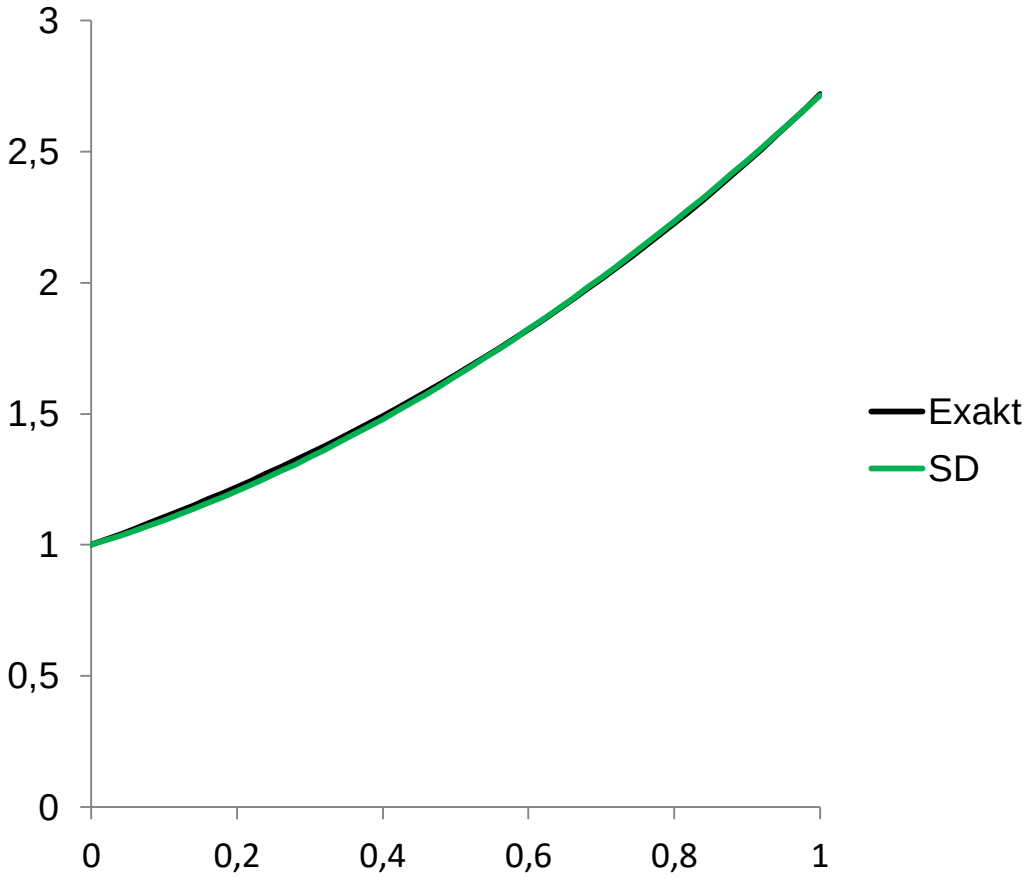
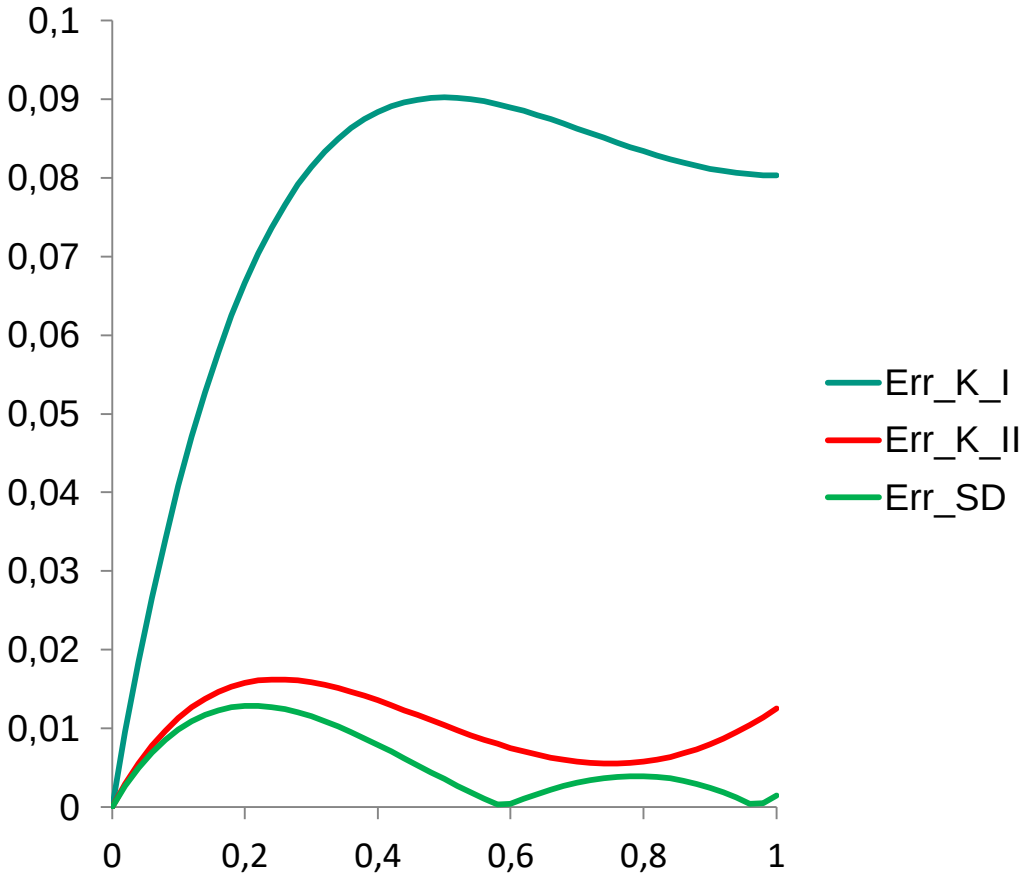
Exakt
K_I
K_II


3
2,5
2
1,5
1
0,5
0
0
0,2
0,4
0,6
0,8
1
Exakt
SD

	$x = \frac{1}{4} = 0,25$	$x = \frac{1}{2} = 0,5$	$x = \frac{3}{4} = 0,75$	$x = 1$
Exakt e^x	**1,2840**	**1,6487**	**2,1170**	**2,7183**
K I	$\frac{19}{16} = 1,1875$	$\frac{3}{2} = 1,5$	$\frac{31}{16} = 1,9375$	$\frac{5}{2} = 2,5$
ε	$0,0965$	$0,1487$	$0,1795$	$0,2183$
K II	$\frac{24}{19} = 1,2632$	$\frac{31}{19} = 1,6316$	$\frac{40}{19} = 2,1053$	$\frac{51}{19} = 2,6842$
ε	$0,0208$	$0,0171$	$0,0177$	$0,0341$
SD	$\frac{71}{56} = 1,2679$	$\frac{23}{14} = 1,6429$	$\frac{119}{56} = 2,125$	$\frac{19}{7} = 2,7143$
ε	$0,0161$	$0,0058$	$0,008$	$0,004$

Relativer Fehler



Aufgabe 2: FDM

$$\frac{\partial(\rho\phi)}{\partial t}+\frac{\partial(\rho v_j\phi)}{\partial x_j}=\frac{\partial}{\partial x_j}\left(\Gamma\frac{\partial\phi}{\partial x_j}\right)+q_\phi$$

$$\phi(x)=\phi(x_i)+(x-x_i)\left(\frac{\partial\phi}{\partial x}\right)_i+\frac{(x-x_i)^2}{2!}\left(\frac{\partial^2\phi}{\partial x^2}\right)_i+$$

$$\frac{(x-x_i)^3}{3!}\left(\frac{\partial^3\phi}{\partial x^3}\right)_i+\cdots+\frac{(x-x_i)^n}{n!}\left(\frac{\partial^n\phi}{\partial x^n}\right)_i+H$$

FDM - Beispielformeln

Beispiel für einfache Differenzenformeln:

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_i - \phi_{i-1}}{x_i - x_{i-1}} + \varepsilon$$

Rückwärtsdifferenz

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_i}{x_{i+1} - x_i} + \varepsilon$$

Vorwärtsdifferenz

$$\left(\frac{\partial \phi}{\partial x}\right)_i = \frac{\phi_{i+1} - \phi_{i-1}}{x_{i+1} - x_{i-1}} + \varepsilon$$

Zentraldifferenz

ε stellt den Abbruchfehler dar