Modellbildung und Simulation
Kapitel 7: Systeme mit verteilten Parametern

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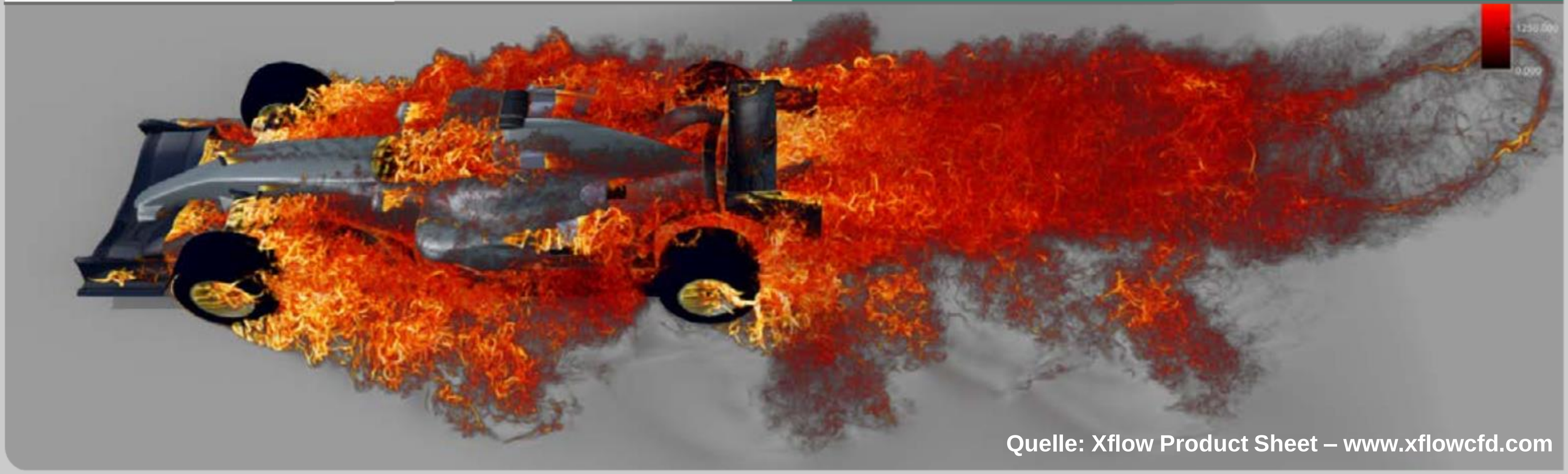


Quelle: Xflow Product Sheet – www.xflowcfd.com

Saalübung

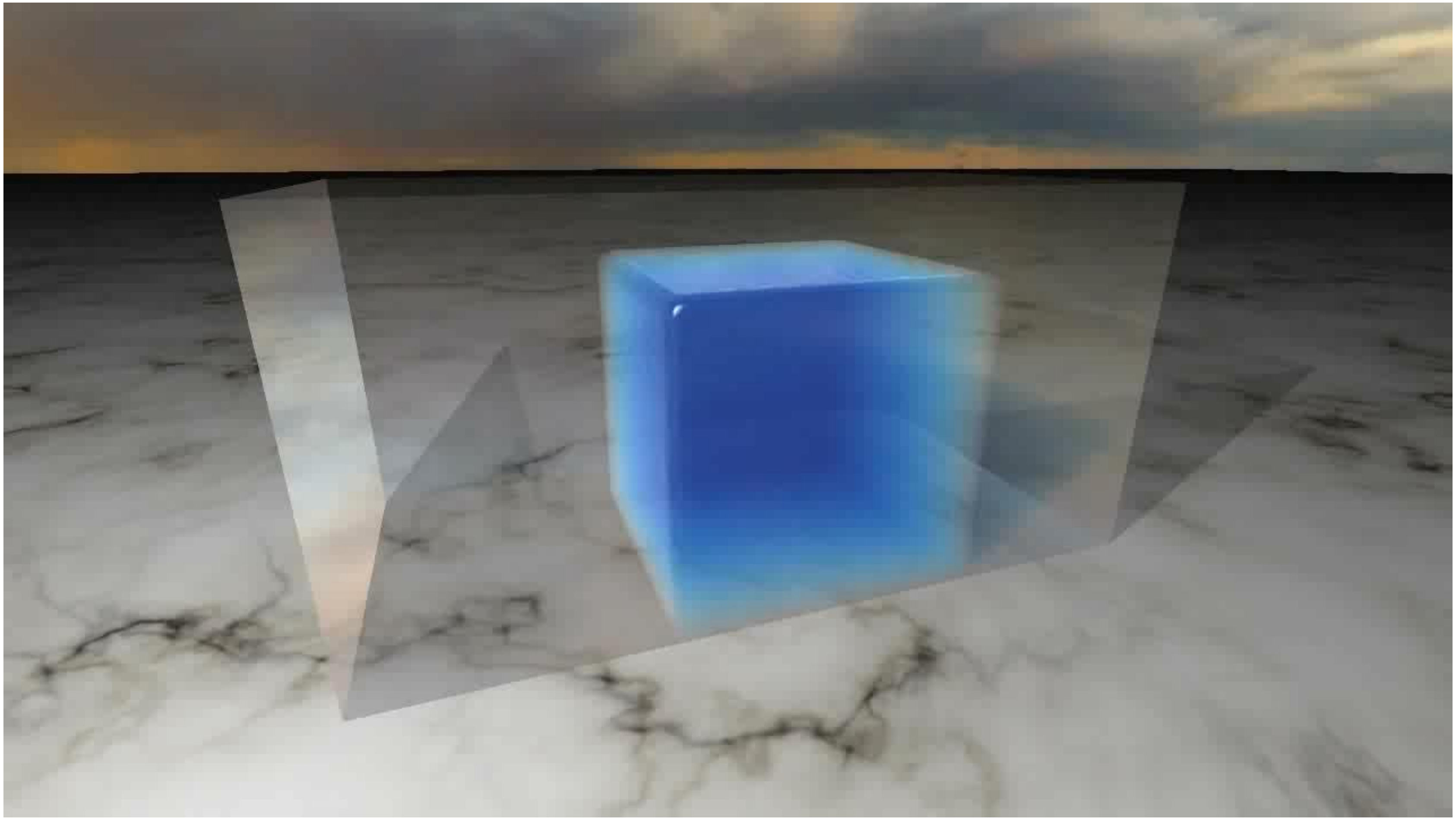
7. Systeme mit verteilten Parametern

7.2 Modellreduktion

Modellreduktion – Mathematische Reduktion

- NVIDIA PhysX - SPH (Smoothed Particle Hydrodynamics)

NVIDIA GF100 Fluids Demo https://www.youtube.com/watch?v=UYIPg8TEMmU



Couette-Poiseuille-Strömung (Kanalströmung)

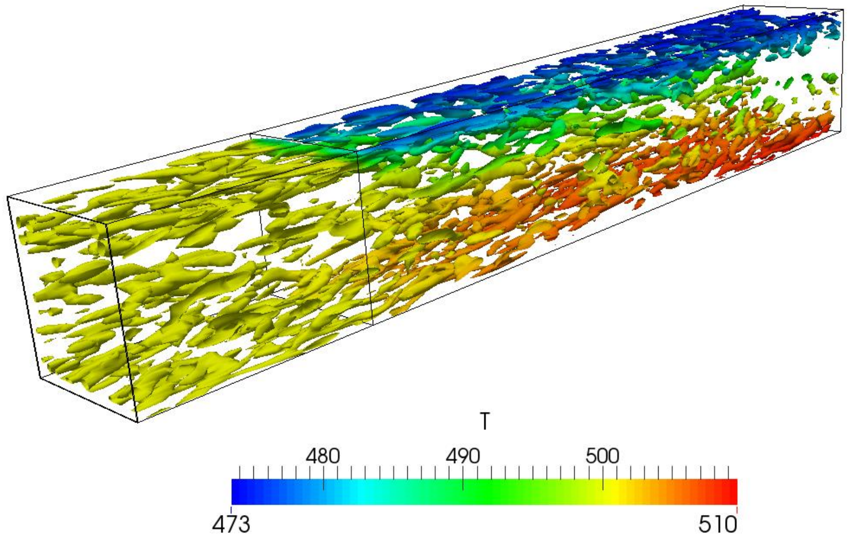
Einsatzgebiet:

Kanalströmung kommt in vielen technischen Bereichen vor, aber die ist für Modelluntersuchungen (vor allem für Turbulenzmodelle) auch ganz gut geeignet.

Als Ausgangspunkt dienen die dreidimensionale kompressible Navier-Stokes Gleichungen.

Die Vereinfachung des Modells durch den folgenden Annahmen wird ausführlich diskutiert:

- 3D → 2D
- Kompressibel → inkompressibel
- Instationär → stationär
- Turbulent → laminar
- Konstante Materialeigenschaften

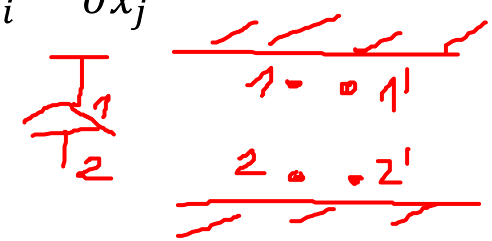


Couette-Poiseuille-Strömung (Kanalströmung)

- Ausgang: 3D, instationär, kompressibel

K $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u_i}{\partial x_i} = 0$$

I $$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial T_{ij}}{\partial x_j} + g_i \qquad T_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\frac{\partial u_k}{\partial x_k}\right)$$



$$\frac{dp}{dx_1} \Rightarrow 1'$$

$$\frac{dp}{dx_2} \Rightarrow 2'$$

$=$

$$\delta_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

E $$\frac{\partial \rho E}{\partial t} + \frac{\partial \rho u_i E}{\partial x_i} = \frac{\partial (q_i - p u_i)}{\partial x_i} + \frac{\partial u_i T_{ij}}{\partial x_j} + u_i g_i$$

$$q_i = \lambda \frac{\partial T}{\partial x_i} \qquad E = \frac{1}{2}u_i u_i + e \qquad e = \frac{p}{\rho(\gamma - 1)} = c_v T \qquad p = \rho R T$$

Couette-Poiseuille-Strömung (Kanalströmung)

- Ausgeschrieben

$$\vec{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

K $$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

I x $$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho u v}{\partial y} + \frac{\partial \rho u w}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} + g_x$$

$$\frac{\partial T_{xx}}{\partial x} = \frac{\partial}{\partial x} \mu \left(2 \frac{\partial u}{\partial x} - \frac{2}{3} \left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) \right)$$

$$\frac{\partial T_{xy}}{\partial y} = \frac{\partial}{\partial y} \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)$$

$$\frac{\partial T_{xz}}{\partial z} = \frac{\partial}{\partial z} \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

Couette-Poiseuille-Strömung (Kanalströmung)

- Ausgeschrieben

$$\vec{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

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I x $$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho u v}{\partial y} + \frac{\partial \rho u w}{\partial z} = -\frac{\partial p}{\partial x} + \frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z} + g_x$$

$$\frac{\partial T_{xx}}{\partial x} = \frac{\partial}{\partial x}\mu\left(2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right)$$

$$\frac{\partial T_{xy}}{\partial y} = \frac{\partial}{\partial y}\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\frac{\partial T_{xz}}{\partial z} = \frac{\partial}{\partial z}\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

$$\frac{\partial \phi}{\partial x} = \frac{\phi_i - \phi_{i-1}}{x_i - x_{i-1}}$$

3

Anzahl der mathematischen Operationen

Couette-Poiseuille-Strömung (Kanalströmung)

- Ausgeschrieben

$$\vec{u} = \begin{pmatrix} u \\ v \\ w \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

K

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

3 1 3 1 3

Σ 11

I x

$$\frac{\partial \rho u}{\partial t} + \frac{\partial \rho u u}{\partial x} + \frac{\partial \rho u v}{\partial y} + \frac{\partial \rho u w}{\partial z} = -\frac{\partial p}{\partial x} + \underbrace{\frac{\partial T_{xx}}{\partial x} + \frac{\partial T_{xy}}{\partial y} + \frac{\partial T_{xz}}{\partial z}}_{46} + g_x$$

1 1 1 1 1 1 1

2+3 2+3 2+3 3 21 11 11

Σ 67

$$\frac{\partial T_{xx}}{\partial x} = \frac{\partial}{\partial x}\mu\left(2\frac{\partial u}{\partial x} - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\right)$$

3 1 1 3 1 1 3 1 3 1 3

Σ 21

$$\frac{\partial T_{xy}}{\partial y} = \frac{\partial}{\partial y}\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

3 1 3 1 3

Σ 11

$$\frac{\partial T_{xz}}{\partial z} = \frac{\partial}{\partial z}\mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

3 1 3 1 3

Σ 11

$$\frac{\partial \phi}{\partial x} = \frac{\phi_i - \phi_{i-1}}{x_i - x_{i-1}}$$

3

$$\frac{(\rho * u * u)_i - (\rho \cdot u \cdot u)_{i-1}}{x_i - x_{i-1}}$$

Couette-Poiseuille-Strömung (Kanalströmung)

- Vereinfachungen
  - Inkompressibel $\rho = \text{konst.} \qquad \Rightarrow \partial\rho = 0$
  - Isotherm $\mu = \text{konst.} \qquad (\mu \neq f(T), \rho \neq f(T))$
  - 2D $\dfrac{\partial}{\partial z} = 0$
  - Stationär $\dfrac{\partial}{\partial t} = 0$
  - Voll entwickelt $\dfrac{\partial}{\partial x} = 0$

Couette-Poiseuille-Strömung (Kanalströmung)

- K

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$$

inkompressibel

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

2D

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

voll entwickelt

$$\boxed{\frac{\partial v}{\partial y} = 0}$$

RB an Wänden

$$\boxed{v = 0}$$

Couette-Poiseuille-Strömung (Kanalströmung)

- E
  - inkomp. $\Rightarrow$ wird entkopelt
  - p muss mit Hilfe einer Poisson-Gleichung iterativ gesucht werden…

Couette-Poiseuille-Strömung (Kanalströmung)

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- I, 3D

$$\frac{\partial \rho u_i}{\partial t} + \frac{\partial \rho u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial T_{ij}}{\partial x_j} + g_i$$

stationär, inkomp.

$$\rho \frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial T_{ij}}{\partial x_j}$$

$$\frac{\partial u_i u_j}{\partial x_j} = u_i \frac{\partial u_j}{\partial x_j} + u_j \frac{\partial u_i}{\partial x_j} = u_j \frac{\partial u_i}{\partial x_j}$$

Kont.

$$\frac{\partial T_{ij}}{\partial x_j} = \frac{\partial}{\partial x_j} \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_k}{\partial x_k} \right)$$

isotherm, inkomp.

$$\begin{aligned}
\frac{\partial T_{ij}}{\partial x_j} &= \mu \frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \\
&= \mu \left(\frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left(\frac{\partial u_j}{\partial x_i} \right) \right) \\
&= \mu \left(\frac{\partial}{\partial x_j} \left(\frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_i} \left(\frac{\partial u_j}{\partial x_j} \right) \right)
\end{aligned}$$

Kont.

$$\frac{\partial T_{ij}}{\partial x_j} = \mu \frac{\partial^2 u_i}{\partial x_j^2}$$

Couette-Poiseuille-Strömung (Kanalströmung)

- I
  - x

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial x_j{}^2}$$

voll entwickelt Kont.

$$0=-\frac{\partial p}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

voll entwickelt

$$\boxed{\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu}\frac{\partial p}{\partial x}}$$

  - y

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v}{\partial x_j{}^2}$$

Kont.

$$0=-\frac{\partial p}{\partial y}+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

Kont.

$$\boxed{\frac{\partial p}{\partial y}=0}$$

Couette-Poiseuille-Strömung (Kanalströmung)

Integration:

$$\frac{\partial u}{\partial y} = \frac{1}{\mu}\frac{\partial p}{\partial x} y + C_1$$

$$u(y) = \frac{1}{\mu}\frac{\partial p}{\partial x}\frac{y^2}{2} + C_1 y + C_2$$

Randbedingungen:

$$u(0) = 0 \qquad \Rightarrow C_2 = 0$$

$$u(h) = U$$

$$U = \frac{1}{2\mu}\frac{\partial p}{\partial x} h^2 + C_1 h$$

$$C_1 = \frac{U}{h} - \frac{1}{2\mu}\frac{\partial p}{\partial x} h$$

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x} y^2 + \left(\frac{U}{h} - \frac{1}{2\mu}\frac{\partial p}{\partial x} h\right) y$$

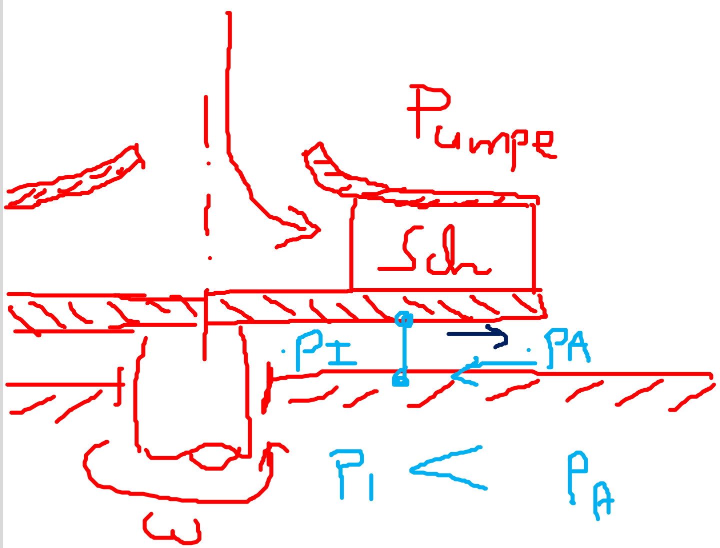
$$\boxed{u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}[y^2 - hy] + \frac{U}{h} y}$$

Couette-Poiseuille-Strömung (Kanalströmung)

$$u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[y^2 - hy\right] + \frac{U}{h}y$$

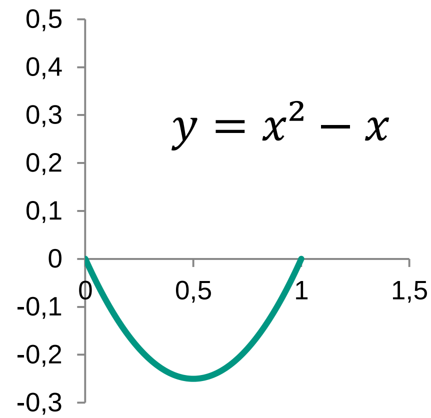
- Couette

$$\frac{\partial p}{\partial x} = 0 \rightarrow \quad u(y) = \frac{U}{h}y$$

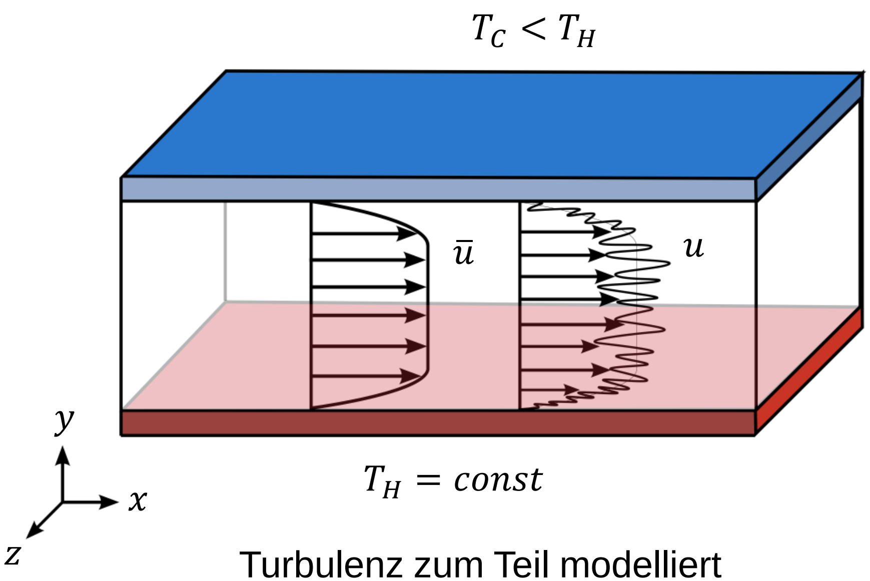


- Poiseuille

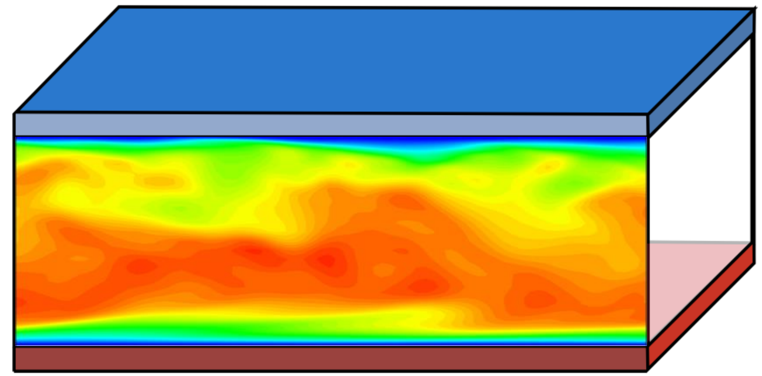
$$U = 0 \rightarrow \quad u(y) = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left[y^2 - hy\right]$$



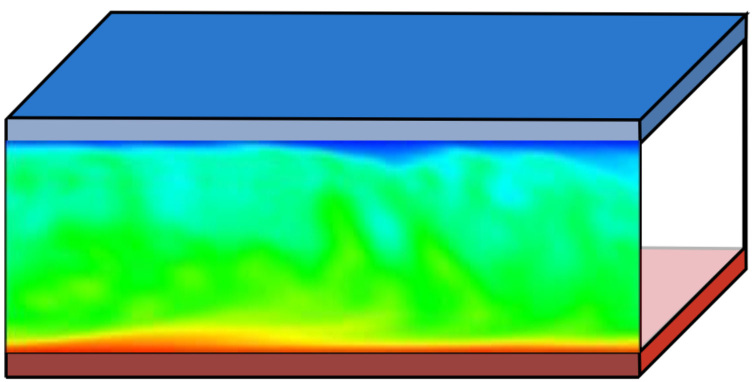
Turbulente Kanalströmung



Instantanes Geschwindigkeitsfeld
Turbulenz zum Teil modelliert

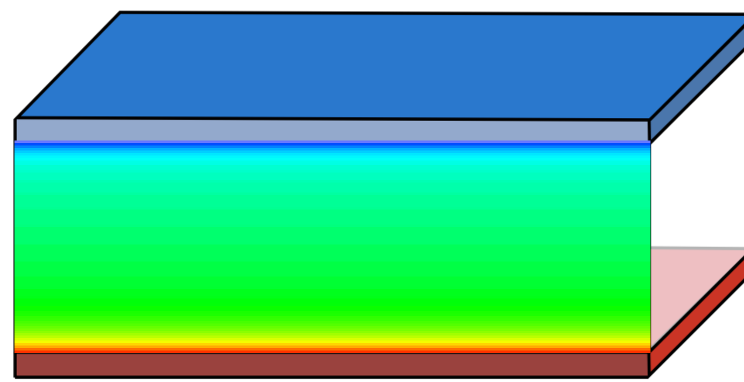


Turbulenz zum Teil modelliert



Instantanes Temperaturfeld

Turbulenz vollständig modelliert



RANS – Temperaturfeld
(**R**eynolds-**a**veraged **N**avier-**S**tokes)

Turbulente Kanalströmung

- Impulsgleichungen (inkompressibel):

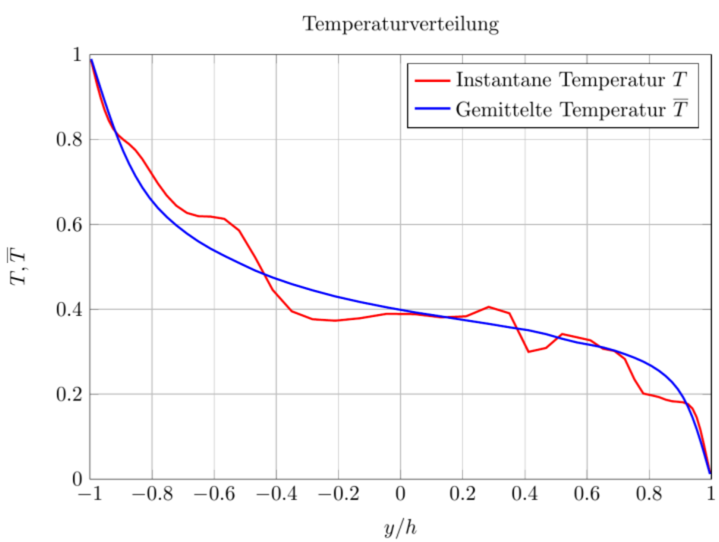
$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho}\frac{\partial p}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial d x_j{}^2}$$

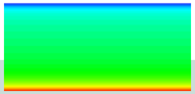
- Reynolds-Zerlegung:

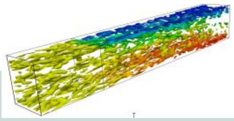
$$u_i = \overline{u_i} + u_i{}'$$

- Reynoldsgemittelte Impulsgleichung (inkompressibel):

$$\frac{\partial \overline{u_i}}{\partial t} + \overline{u_j} \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial\left(\overline{u_i{}' u_j{}'}\right)}{\partial x_j} = -\frac{1}{\rho}\frac{\partial \overline{p}}{\partial x_i} + \nu \frac{\partial^2 \overline{u_i}}{\partial d x_j{}^2}$$



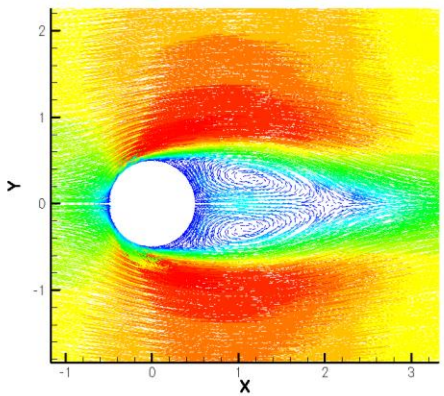




Rechenzeit	
10^{-1} h	10^{3} h
„Genauigkeit“	
+	+++

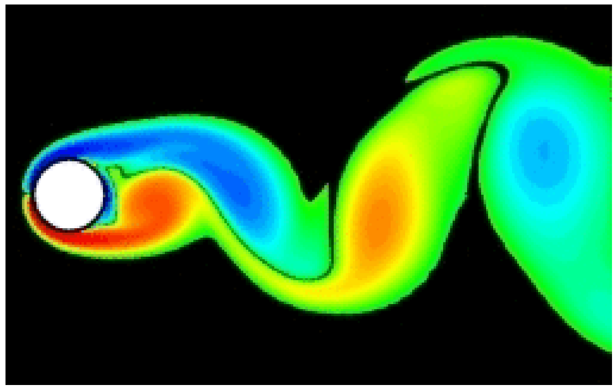
Zylinderumströmung

2D, stationär

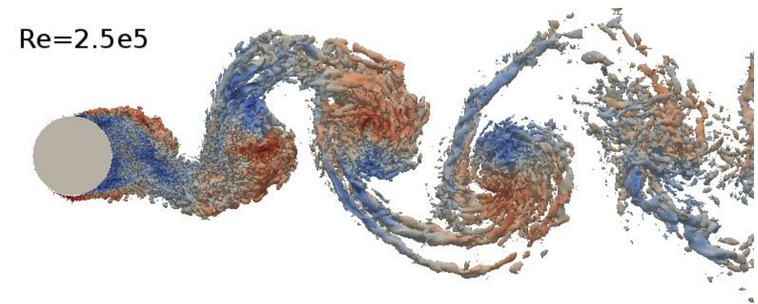


Quelle: cfd.solvcon.net

2D, instationär



3D, instationär



Quelle: www.cttc.upc.edu

Rechenzeit		
10^{-1} h	10^{0} h	10^{6} h
„Genauigkeit“		
+	++	+++

Profilumströmung

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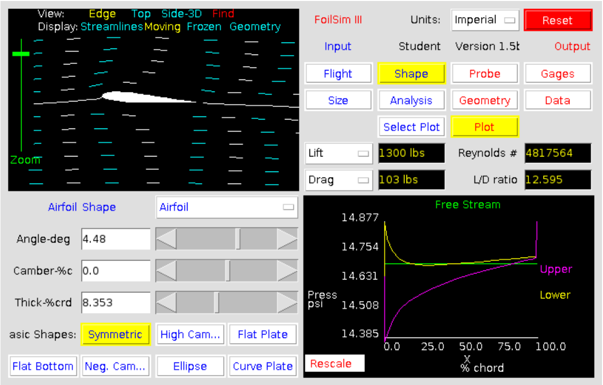
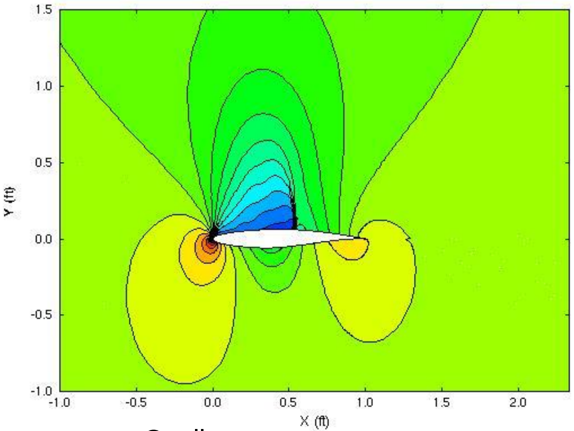
FoilSim
(Euler, inkomp., 2D, stationär)



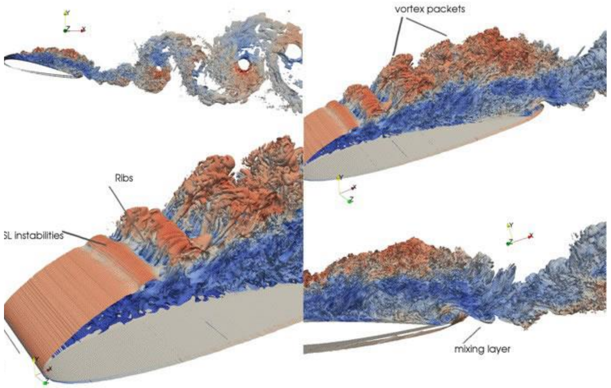
Quelle: www.grc.nasa.gov

NS, 2D, stationär



Quelle: www.grc.nasa.gov

NS, 3D, instationär



Quelle: www.cttc.upc.edu

Rechenzeit		
10^{-5} h	10^{0} h	10^{6} h
„Genauigkeit“		
+	++	+++

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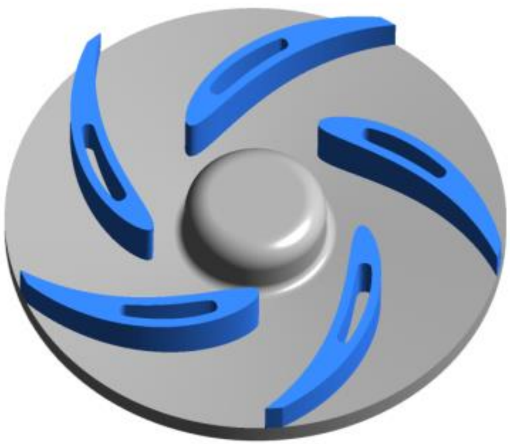
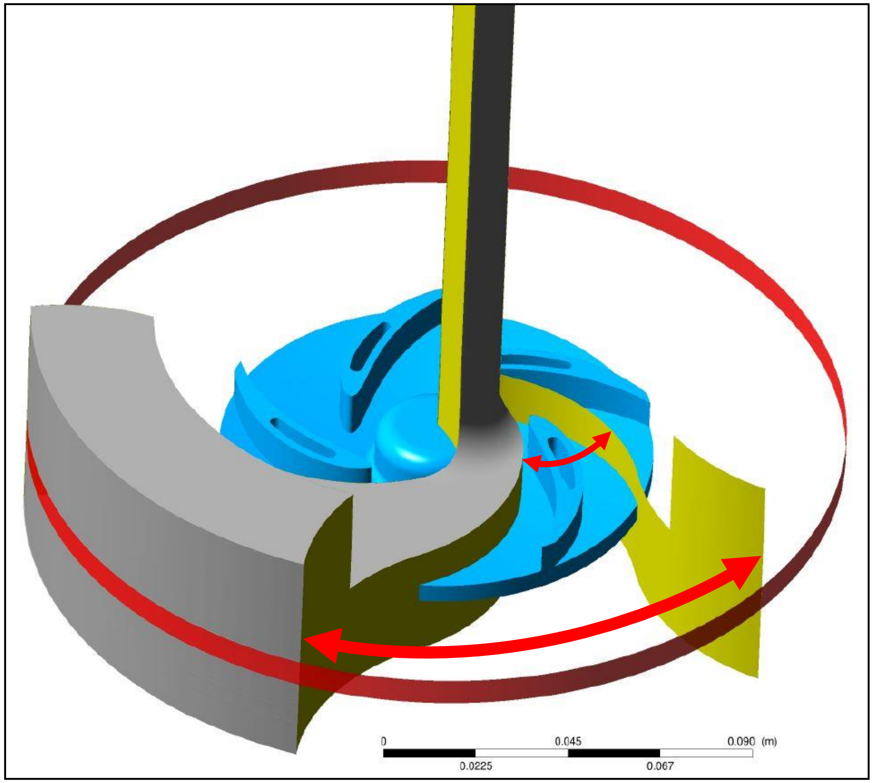
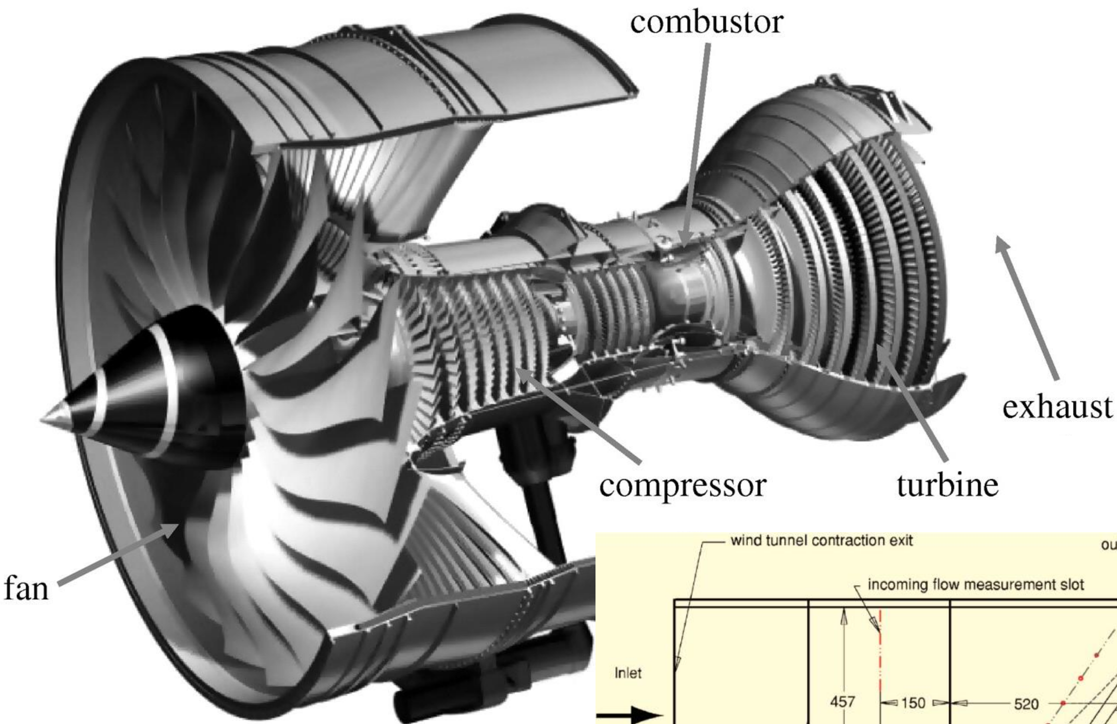
Geometrische Vereinfachung - Periodizität



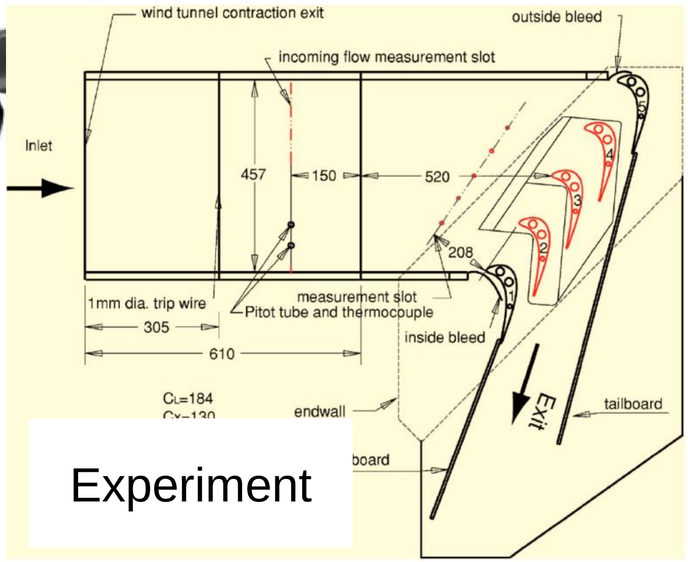


Institut für Thermische Strömungsmaschinen

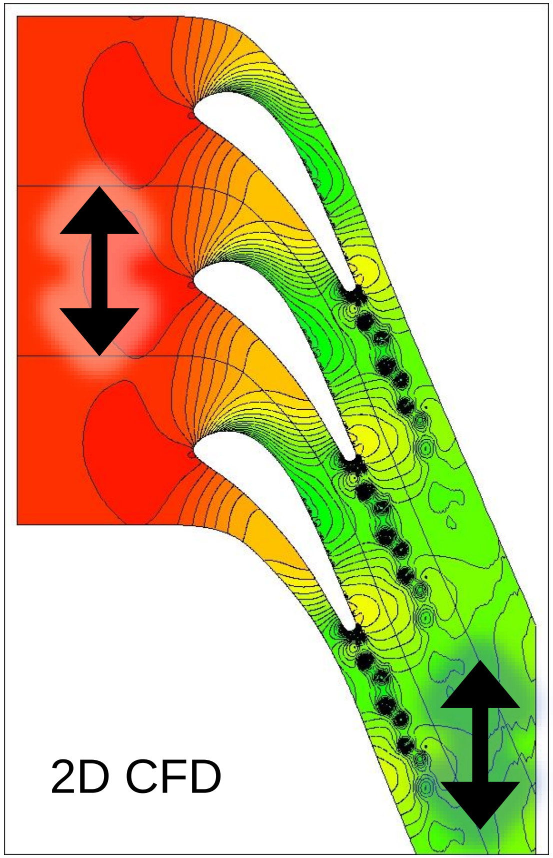
Geometrische Vereinfachung - Periodizität



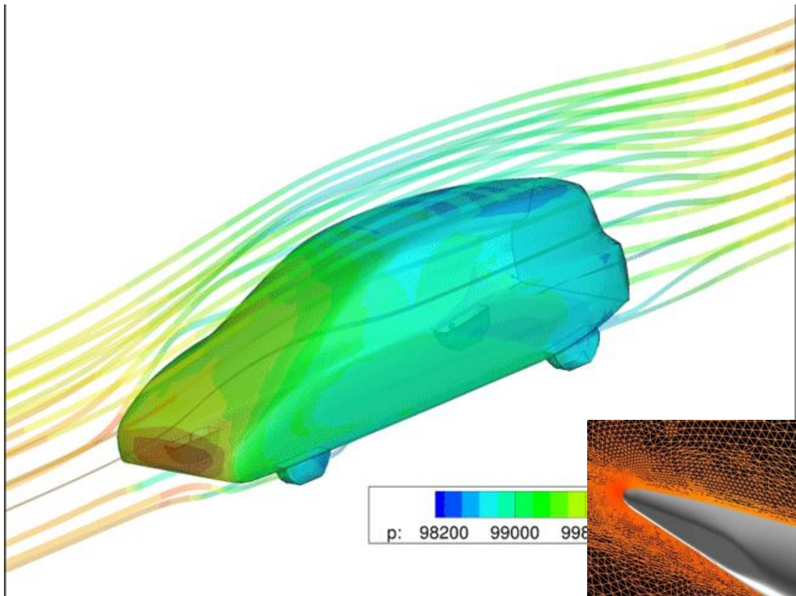
Quelle: DOI: 10.1098/rsta.2009.0064



Quelle: J. Heat Transfer 129(10), 1384-1394 (Mar 01, 2007)

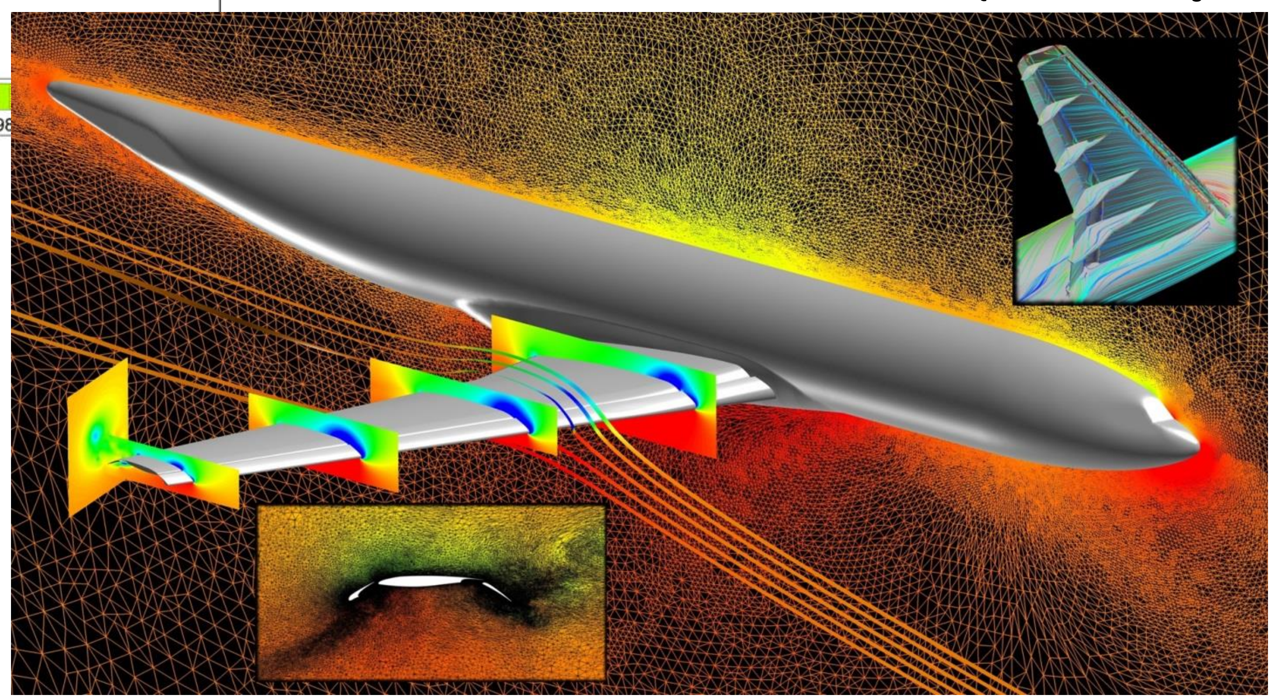


Geometrische Vereinfachung – Symmetrie, …



Quelle: www.karalit.com

Quelle: www.nasa.gov



Modellreduktion am Beispiel eines Radialventilators

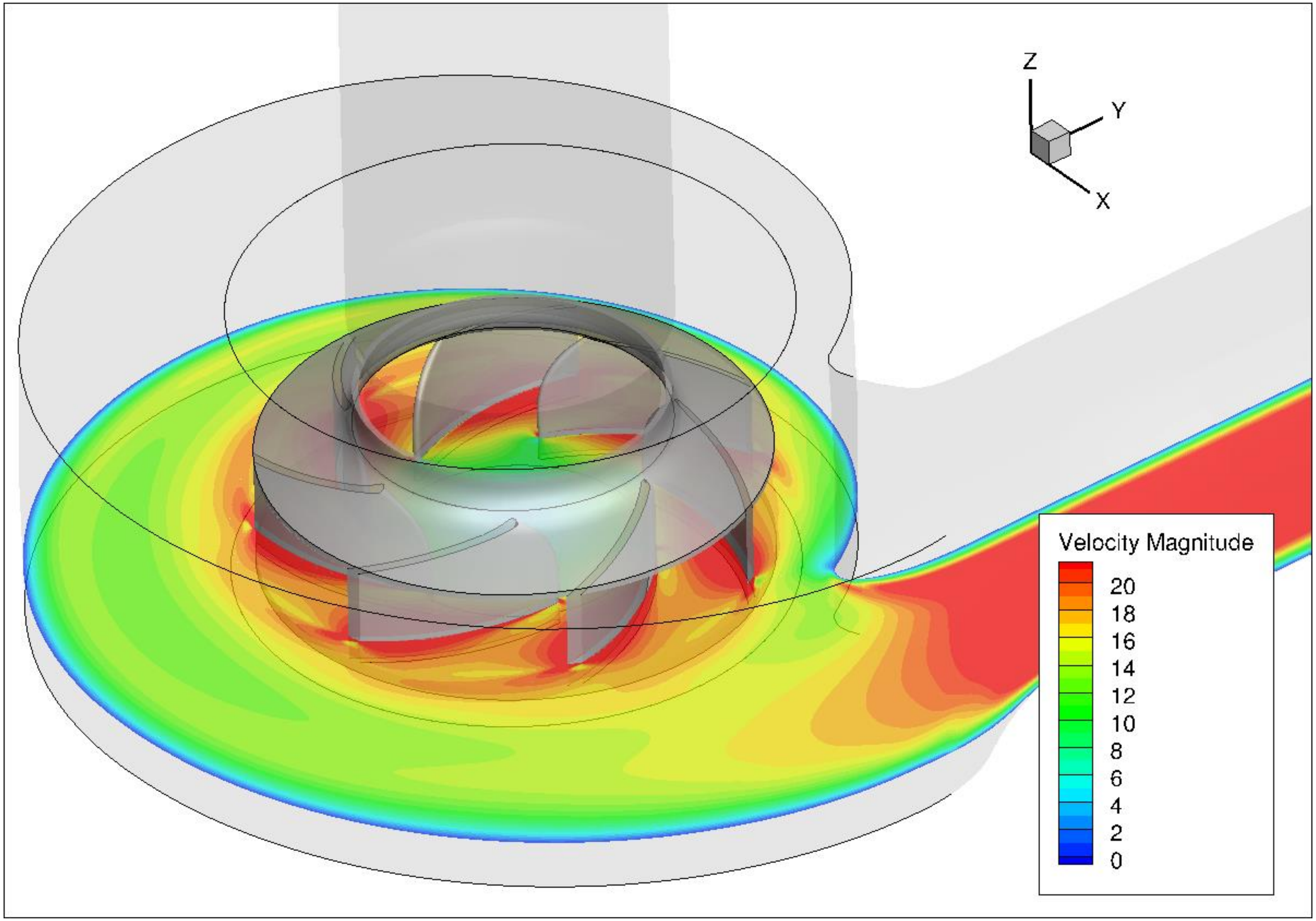
- Was sieht man in der Animation?
  - Größe des Strömungsgebietes
  - Auflösung
  - Was passiert an der Wand?
    - Zusätzliche Modelle für die Strömung.
  - Zeitliche Abläufe

Xflow - Radialventilator

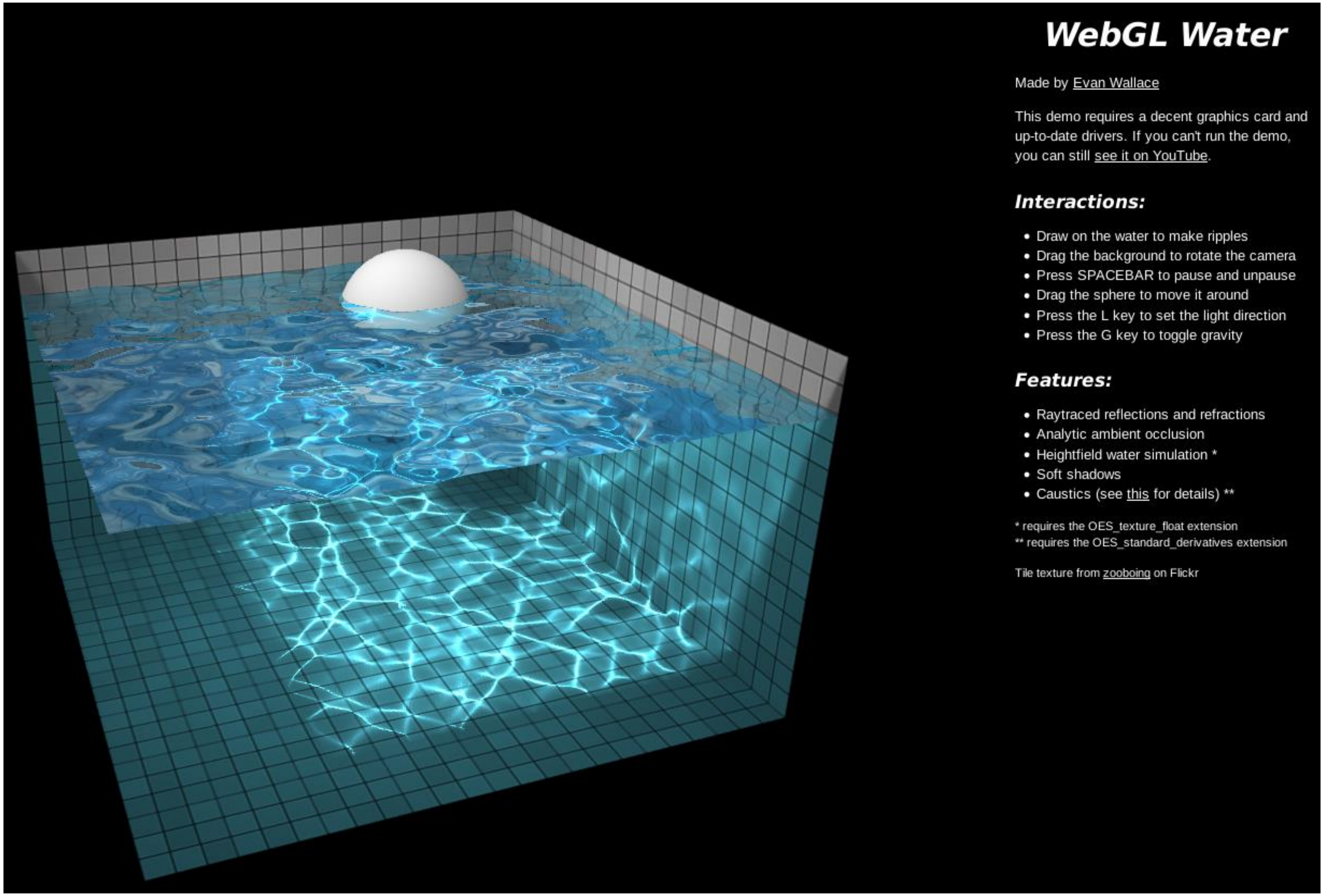
- https://www.youtube.com/watch?v=BnShaho5eQY&list=UUYMvQ8kW8WoKjtl99jUNa1Q
- Veröffentlicht am 16.09.2014
- In this CFD-project of an industrial fan the rotation of the impeller geometry was easily modeled in detail and with real motion laws. The XFlow's particle-based approach avoids any compromises in the modeling of moving parts considering the highest fidelity Wall-Modeled Large Eddy Simulation (WMLES) approach to the turbulence modeling. The accuracy and the pressure increase was predicted with only 0.9% error by XFlow compared to measurements from the test bench.

Xflow - Radialventilator

Radialventilator – Wand aufgelöst



Was wird hier modelliert?



Quelle: madebyevan.com/webgl-water/