$$m \cdot l^2 \cdot \ddot{\varphi} + \underbrace{k \cdot \dot{\varphi}}_{=0} + m \cdot g \cdot l \cdot \sin(\varphi) - m \cdot l^2 \omega^2 \sin(\varphi) \cos(\varphi) = 0$$

$k = 0$

Ruhelage $\varphi = \dot{\varphi} = 0$

$$\ddot{\varphi} = \omega^2 \sin(\varphi) \cos(\varphi) - \frac{g}{l} \cdot \sin(\varphi)$$

$$x = \begin{pmatrix} \varphi \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \dot{x} = \begin{pmatrix} \dot{\varphi} \\ \ddot{\varphi} \end{pmatrix} = \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} \dot{\varphi} \\ \omega^2 \sin(\varphi) \cos(\varphi) - \frac{g}{l} \sin(\varphi) \end{pmatrix}$$

$$\dot{x}(t) = \begin{pmatrix} \varphi_0 \\ \dot{\varphi}_0 \end{pmatrix} + \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix} \begin{pmatrix} x_1 - \varphi_0 \\ x_2 - \dot{\varphi}_0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -\frac{g}{l} \cos(\varphi_0) + \omega^2 (\cos^2(\varphi_0) + \sin^2(\varphi_0)) & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

$$\dot{x}(t) = \begin{pmatrix} 0 & 1 \\ \omega^2 - \frac{g}{l} & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \qquad \text{Systemmatrix } A = \begin{pmatrix} 0 & 1 \\ \omega^2 - \frac{g}{l} & 0 \end{pmatrix}$$

Eigenwerte

$$\det \begin{pmatrix} -\lambda & 1 \\ \omega^2 - \frac{g}{l} & -\lambda \end{pmatrix} = 0 ; \quad \lambda^2 - \left(\omega^2 - \frac{g}{l}\right) = 0$$

$$\lambda^2 = \omega^2 - \frac{g}{l}$$

$$\lambda_1 = \sqrt{\omega^2 - \frac{g}{l}}$$

$$\lambda_2 = -\sqrt{\omega^2 - \frac{g}{l}}$$

Eigenvektoren

$$A \cdot \vec{v_i} = \lambda_i \cdot \vec{v_i}$$

$$(A - \lambda_i) \vec{v_i} = 0$$

für λ_1

$$\begin{pmatrix} -\lambda_1 & 1 \\ \omega^2 - \frac{g}{l} & -\lambda_1 \end{pmatrix} \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = 0 \rightarrow \begin{aligned} &-\lambda_1 \cdot v_{11} + v_{12} = 0 \\ &\left(\omega^2 - \frac{g}{l}\right) \cdot v_{11} + (-\lambda_1 \cdot v_{12}) = 0 \end{aligned} \rightarrow v_{12} = +\lambda_1 v_{11}$$

$$\rightarrow \vec{v}_1 = \begin{pmatrix} v_{11} \\ \lambda_1 v_{11} \end{pmatrix} = v_{11} \begin{pmatrix} 1 \\ \lambda_1 \end{pmatrix}$$

für λ_2:

$$\begin{pmatrix} -\lambda_2 & 1 \\ \omega^2 - \frac{g}{l} & -\lambda_2 \end{pmatrix} \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = 0 \rightarrow \begin{aligned} &-\lambda_2 \cdot v_{21} + v_{22} = 0 \\ &\left(\omega^2 - \frac{g}{l}\right) v_{21} + (-\lambda_2 \cdot v_{22}) = 0 \end{aligned} \rightarrow v_{22} = \lambda_2 \cdot v_{21}$$

$$\rightarrow \vec{v}_2 = \begin{pmatrix} v_{21} \\ \lambda_2 \cdot v_{21} \end{pmatrix} = v_{21} \begin{pmatrix} 1 \\ \lambda_2 \end{pmatrix}$$

$$\begin{pmatrix} v_{11} & v_{21} \\ v_{11} \cdot \lambda_1 & v_{21} \cdot \lambda_2 \end{pmatrix} \begin{pmatrix} e^{\lambda_1 \cdot t} \\ e^{\lambda_2 \cdot t} \end{pmatrix} = \begin{pmatrix} \varphi \\ \dot{\varphi} \end{pmatrix}$$

für $\varphi(t=0) = \frac{\pi}{6}$ und $\dot{\varphi}(t=0) = 0$

$$\begin{pmatrix} v_{11} & v_{21} \\ v_{11} \cdot \lambda_1 & \cdot v_{21} \cdot \lambda_2 \end{pmatrix} \cdot \begin{pmatrix} e^{\lambda_1 \cdot 0} \\ e^{\lambda_2 \cdot 0} \end{pmatrix} = \begin{pmatrix} \frac{\pi}{6} \\ 0 \end{pmatrix}$$

$$\text{I} \quad v_{11} + v_{21} = \frac{\pi}{6} \qquad\qquad \rightarrow v_{11} = v_{21} = \frac{\pi}{12}$$

$$v_{11} \cdot \lambda_1 + v_{21} \cdot \lambda_2 = 0 \quad \lambda_2 = -\lambda_1 \quad \rightarrow v_{11} \cdot \lambda_1 - v_{21} \cdot \lambda_1 = 0 \quad \rightarrow v_{11} = v_{21}$$

$$x(t) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{21} \\ v_{11}\lambda_1 & v_{21} \cdot \lambda_2 \end{pmatrix} \begin{pmatrix} e^{\lambda_1 \cdot t} \\ e^{\lambda_2 \cdot t} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\pi}{12} & \frac{\pi}{12} \\ \frac{\pi}{12} \cdot \sqrt{\omega^2 - \frac{g}{l}} & -\frac{\pi}{12} \sqrt{\omega^2 - \frac{g}{l}} \end{pmatrix} \begin{pmatrix} e^{\sqrt{\omega^2 - \frac{g}{l}} \cdot t} \\ e^{-\sqrt{\omega - \frac{g}{l}} \cdot t} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\pi}{12} \cdot e^{\sqrt{\omega^2 - \frac{g}{l}} \cdot t} + \frac{\pi}{12} \cdot e^{-\sqrt{\omega^2 - \frac{g}{l}} \cdot t} \\ \frac{\pi}{12} \cdot \sqrt{\omega^2 - \frac{g}{l}} \cdot e^{\sqrt{\omega^2 - \frac{g}{l}} \cdot t} - \frac{\pi}{12} \sqrt{\omega^2 - \frac{g}{l}} \cdot e^{-\sqrt{\omega^2 - \frac{g}{l}} \cdot t} \end{pmatrix}$$