Aufgabe 1

1)
$$l\ddot{\varphi} = -k\dot{\varphi} - g\sin\varphi + l\omega^2\sin(\varphi)\cos(\varphi)$$
$$\ddot{\varphi} = -\frac{k}{l}\dot{\varphi} - \frac{g}{l}\sin\varphi + \omega^2\sin(\varphi)\cos(\varphi)$$

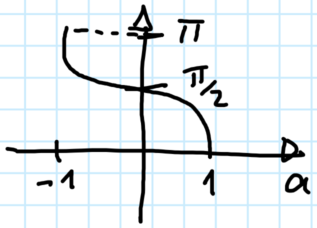
$$\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \varphi \\ \dot{\varphi} \end{pmatrix}$$

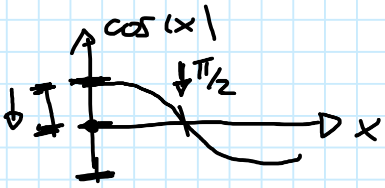
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{k}{l}x_2 - \frac{g}{l}\sin(x_1) + \omega^2\sin(x_1)\cos(x_1)$$
$$\dot{\underset{\sim}{x}} = \underset{\sim}{f}(\underset{\sim}{x}) \neq \underline{\underline{A}}\,\underset{\sim}{x}$$

2)
$$\dot{\underset{\sim}{x}} \overset{!}{=} \underset{\sim}{0} \qquad \dot{x}_1 \overset{!}{=} 0 \qquad \dot{x}_2 \overset{!}{=} 0$$

$$x_2 = 0$$
$$\cancel{-\frac{k}{l}x_2} - \frac{g}{l}\sin(x_1) + \omega^2\sin(x_1)\cos(x_1) = 0$$
$$\sin(x_1)\Big[\underbrace{-\frac{g}{l} + \omega^2\cos(x_1)}_{\cos(x_1) = \frac{g}{l\omega^2}}\Big] = 0$$

$$x_{10,1} = 0$$
$$x_{10,2} = \pi$$
$$x_{10,3} = \arccos\left(\frac{g}{l\omega^2}\right)$$
$$\hookrightarrow \frac{g}{l\omega^2} \leq 1$$
$$\underline{\underline{\frac{g}{l} \leq \omega^2}}$$





3)
$$\underset{\sim}{x}_{01} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 \approx -\frac{k}{l}x_2 + \frac{\partial}{\partial x_1}\left(-\frac{g}{l}\sin(x_1) + \omega^2\sin(x_1)\cos(x_1)\right)\Bigg|_{x_{10}} \cdot (x_1 - x_{10})$$
$$= -\frac{k}{l}x_2 + \left[-\frac{g}{l}\cos(x_1) + \omega^2(-\sin^2(x_1) + \cos^2(x_1))\right]_{x_{10}}(x_1 - x_{10})$$
$$= -\frac{k}{l}x_2 + \left(-\frac{g}{l} + \omega^2\right)x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -\frac{g}{l} + \omega^2 & -\frac{k}{l} \end{bmatrix}}_{\underline{\underline{A}}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad 0 \overset{!}{=} \det(\underline{\underline{A}} - \lambda\underline{\underline{1}})$$

$$\begin{vmatrix} -\lambda & 1 \\ -\frac{g}{l} + \omega^2 & -\lambda - \frac{k}{l} \end{vmatrix} \overset{!}{=} 0 \iff 0 = -\lambda\left(-\lambda - \frac{k}{l}\right) - \left(-\frac{g}{l} + \omega^2\right)$$

Aufgabe 1 (Fortsetzung)

$$0 = -\lambda\left(-\lambda - \frac{k}{e}\right) - \left(-\frac{g}{e} + \omega^2\right)$$

$$\lambda = -\frac{k}{2e} \pm \sqrt{\left(\frac{k}{2e}\right)^2 + \left(\omega^2 - \frac{g}{e}\right)}$$

$\omega^2 - \frac{g}{e} > 0 \rightarrow$ Instabilität

$\omega^2 - \frac{g}{e} < 0 \rightarrow$ as Stabilität

$\omega^2 - \frac{g}{e} = 0 \rightarrow Re(\lambda_1) = 0$ keine Aussage möglich

$\boxed{\underset{\sim}{x}_{02}} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$

$$\lambda = -\frac{k}{2e} \pm \sqrt{\left(\frac{k}{2e}\right)^2 + \underbrace{\left(\omega^2 + \frac{g}{e}\right)}_{>0}}$$

größter EW $Re(\lambda_1) > 0 \rightarrow$ Instabilität

$\boxed{\underset{\sim}{x}_{03}}$

$$\lambda = -\frac{k}{2e} \pm \sqrt{\left(\frac{k}{2e}\right)^2 + \left(\frac{g}{\omega e}\right)^2 - \omega^2}$$

$\frac{g^2}{\cancel{\omega^2} e^2} > \omega^{\cancel{2}4} \leftarrow \left(\frac{g}{\omega e}\right)^2 - \omega^2 > 0$ Instab.

$\frac{g}{e} > \omega^2$

$\cdot = \cdot \rightarrow$ keine Aussage

$\frac{g}{e} < \omega^2 \leftarrow \left(\frac{g}{\omega e}\right)^2 - \omega^2 < 0$ as. Stab