Aufgabe 1

$$\ddot{\varphi} + \frac{d}{m}\dot{\varphi} = -\frac{g}{l}\sin\varphi \quad \rightarrow \quad \ddot{\varphi} = -\frac{d}{m}\dot{\varphi} - \frac{g}{l}\sin\varphi$$

1) $$\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \varphi \\ \dot{\varphi} \end{pmatrix}$$

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -\frac{d}{m}x_2 - \frac{g}{l}\sin(x_1)$$

$$\underset{\sim}{f}(\underset{\sim}{x}) = \begin{pmatrix} x_2 \\ -\frac{d}{m}x_2 - \frac{g}{l}\sin x_1 \end{pmatrix}$$

2) $$\underset{\sim}{\dot{x}} \overset{!}{=} \underset{\sim}{0}$$

$$0 = x_{2,0}$$
$$0 = \cancel{-\frac{d}{m}x_{2,0}} - \frac{g}{l}\sin(x_{1,0})$$
$$x_{1,0} = k \cdot \pi$$

$k=0$ gerade untere RL

$k=1$ ungerade obere RL

3) Linearisierung um $\underset{\sim}{x}_{01} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\underset{\sim}{\dot{x}}(t) = \underset{\sim}{f}\Big|_{\underset{\sim}{x}_{01}} + \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{pmatrix}\Bigg|_{\underset{\sim}{x}_{01}} (\underset{\sim}{x} - \underset{\sim}{x}_{01})$$

$$= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\cos(x_1) & -\frac{d}{m} \end{pmatrix}\Bigg|_{\underset{\sim}{x}_{01}} (\underset{\sim}{x} - \underset{\sim}{x}_{01})$$

$$\underset{\sim}{\dot{x}} = \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{g}{l} & -\frac{d}{m} \end{pmatrix}}_{\underset{=}{A}} \underset{\sim}{x}$$

$$\underset{\sim}{\dot{x}} = \underset{=}{A}\,\underset{\sim}{x}$$

$$\underset{\sim}{\xi} = \underset{\sim}{x} - \underset{\sim}{x}_0$$

$$\underset{\sim}{\dot{\xi}} = \underset{=}{A}\,\underset{\sim}{\xi}$$
$$= \underset{=}{A}\,(\underset{\sim}{x} - \underset{\sim}{x}_0)$$

<u>Ansatz</u>: $\underset{\sim}{x} = \underset{\sim}{r}\, e^{\lambda t}$

$$\underset{\sim}{r}\,\lambda e^{\lambda t} = \underset{=}{A}\,\underset{\sim}{r}\, e^{\lambda t}$$
$$0 = (\underset{=}{A} - \lambda \mathbb{1})\,\underset{\sim}{r}\, e^{\lambda t}$$
$$\det(\underset{=}{A} - \lambda \mathbb{1}) \overset{!}{=} 0$$

Aufgabe 1 (Fortsetzung)

$$\det(\underline{\underline{A}} - \lambda \mathbb{1}) \overset{!}{=} 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -\frac{g}{l} & -\lambda - \frac{d}{m} \end{vmatrix} = (-\lambda)\left(-\lambda - \frac{d}{m}\right) + \frac{g}{l} = 0$$

$$\lambda^2 + \frac{d}{m}\lambda + \frac{g}{l} = 0$$

$$\left(\lambda + \frac{d}{2m}\right)^2 = -\frac{g}{l} + \left(\frac{d}{2m}\right)^2$$

$$\lambda = -\frac{d}{2m} \pm \sqrt{\left(\frac{d}{2m}\right)^2 - \frac{g}{l}}$$

$\frac{d}{m} > 0$ ✓ $\quad \frac{g}{l} > 0$ ✓ $\rightarrow$ asympt. stabil

Linearisierung um $\underset{\sim}{x}_{02} = \begin{pmatrix} \pi \\ 0 \end{pmatrix}$

$$\dot{\underset{\sim}{x}} \approx \cancel{\underset{\sim}{f}\Big|_{\underset{\sim}{x}_{02}}} + \underbrace{\begin{pmatrix} 0 & 1 \\ \frac{g}{l} & -\frac{d}{m} \end{pmatrix}}_{A} \underbrace{\left(\underset{\sim}{x} - \underset{\sim}{x}_{02}\right)}_{\underset{\sim}{\xi}}$$

$$\dot{\underset{\sim}{\xi}} = \underline{\underline{A}}\,\underset{\sim}{\xi}$$

$$\det(\underline{\underline{A}} - \lambda \mathbb{1}) = \begin{vmatrix} -\lambda & 1 \\ \frac{g}{l} & -\lambda - \frac{d}{m} \end{vmatrix} \overset{!}{=} 0$$

$\rightarrow \lambda = -\frac{d}{2m} \pm \sqrt{\left(\frac{d}{2m}\right)^2 + \frac{g}{l}}$

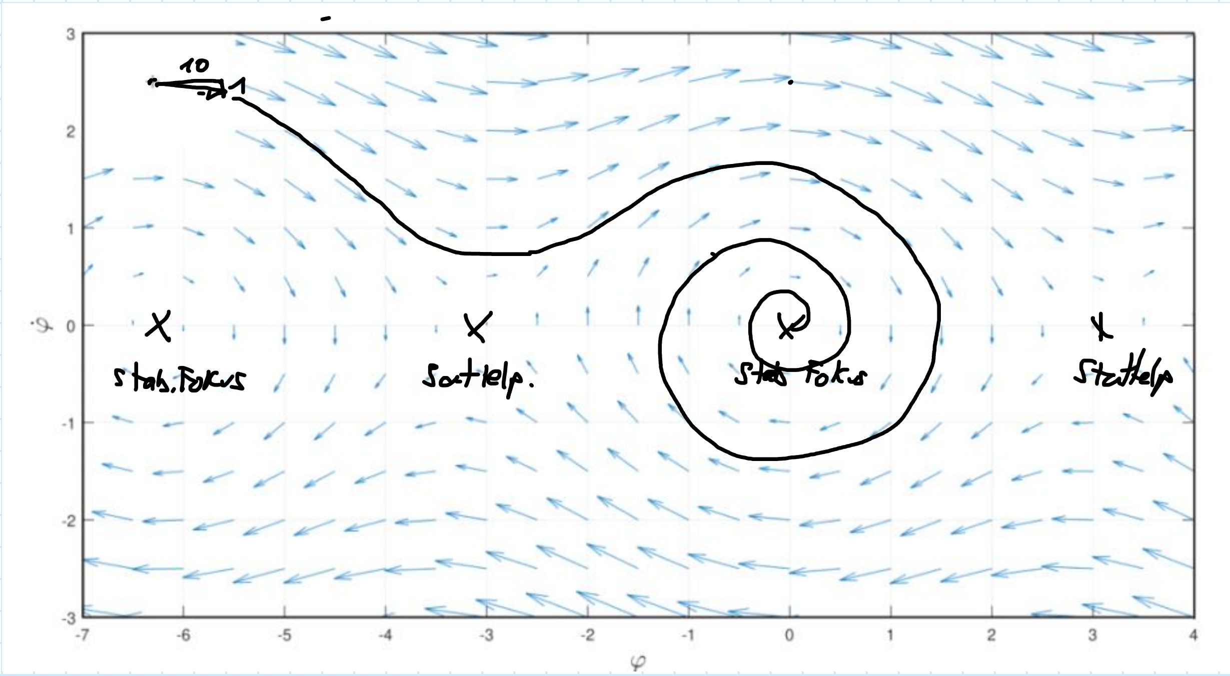
$\rightarrow$ Instabilität, ein positiver Realteil

4) $\boxed{\underset{\sim}{x}_{01}}$

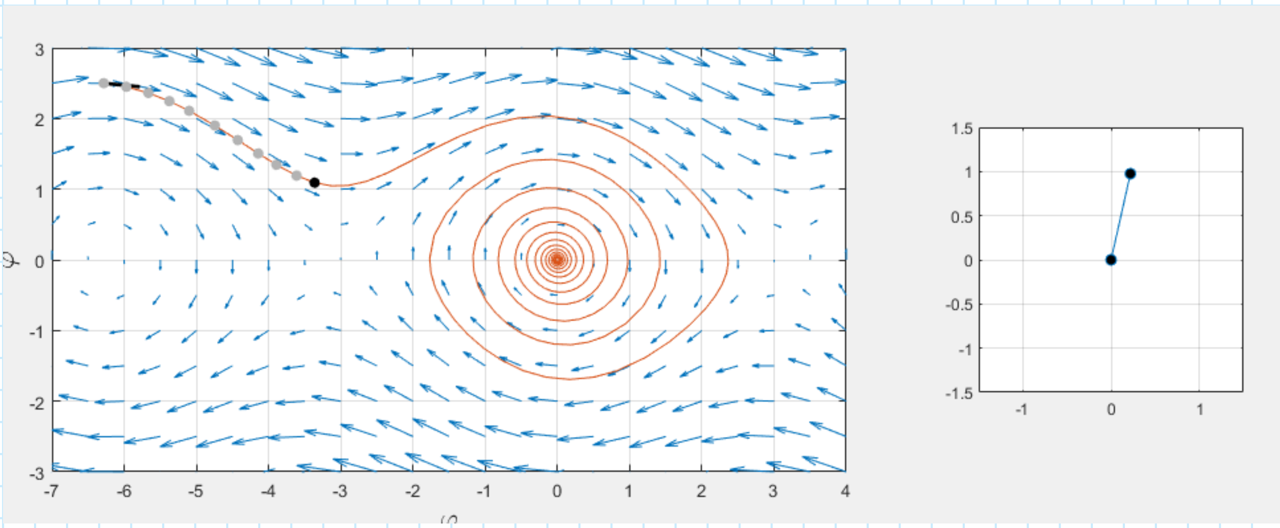
- $\left(\frac{d}{2m}\right)^2 > \frac{g}{l} \rightarrow$ stabiler Knoten
- $\left(\frac{d}{2m}\right)^2 < \frac{g}{l} \rightarrow$ stabiler Strudel / Fokus
- $\left(\frac{d}{2m}\right)^2 = \frac{g}{l} \rightarrow$ stabiler Knoten

$\boxed{\underset{\sim}{x}_{02}}$ Sattelpunkt $(\lambda_1 < 0,\ \lambda_2 > 0)$

Aufgabe 1 (Fortsetzung)



G) $\left.\begin{array}{l} x_1 = -2\pi \\ x_2 = 2{,}5 \end{array}\right\} \quad f\left(\begin{pmatrix} -2\pi \\ 2{,}5 \end{pmatrix}\right) = \begin{pmatrix} 2{,}5 \\ -0{,}25 \end{pmatrix}$



Aufgabe 3

$$\dot{\psi} - \Omega(\gamma_1 + \gamma_2) = c_2\gamma_2$$
$$\dot{\gamma}_1 + \dot{\gamma}_2 + \Omega\psi = -c_1\gamma_2$$
$$\dot{\gamma}_1 + \Omega\psi = -k(\gamma_1 - \kappa), \quad -1.$$

$$\dot{\gamma}_2 = -c_1\gamma_2 + k(\gamma_1 - \kappa)$$

1)
$$\underset{\sim}{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \psi \end{pmatrix}$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} = \begin{pmatrix} -\Omega x_3 - k(x_1 - \kappa) \\ -c_1 x_2 + k(x_1 - \kappa) \\ \Omega(x_1 + x_2) + c_2 x_2 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} -k & 0 & -\Omega \\ k & -c_1 & 0 \\ \Omega & \Omega + c_2 & 0 \end{pmatrix}}_{\underline{\underline{A}}} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + \begin{pmatrix} k\kappa \\ -k\kappa \\ 0 \end{pmatrix}$$

$$\dot{\underset{\sim}{x}} \overset{!}{=} 0 \qquad \underset{\sim}{x}_0 = \begin{pmatrix} x_{10} \\ x_{20} \\ x_{30} \end{pmatrix} = \begin{pmatrix} \dfrac{k\kappa(\Omega + c_2)}{(\Omega + c_2)k + c_1\Omega} \\ / \\ / \end{pmatrix}$$

$$\Rightarrow \quad \dot{\underset{\sim}{x}} = \underline{\underline{A}}\,\underbrace{(\underset{\sim}{x} - \underset{\sim}{x}_0)}$$
$$\dot{\underset{\sim}{\xi}} = \underline{\underline{A}}\,\underset{\sim}{\xi}$$

$$\det(\underline{\underline{A}} - \lambda\underline{\underline{1}}) \overset{!}{=} 0$$

$$\begin{vmatrix} -k-\lambda & 0 & -\Omega \\ k & -c_1-\lambda & 0 \\ \Omega & \Omega + c_2 & -\lambda \end{vmatrix} = (-k-\lambda)\begin{vmatrix} -c_1-\lambda & 0 \\ \Omega + c_2 & -\lambda \end{vmatrix} - \Omega \begin{vmatrix} k & -c_1-\lambda \\ \Omega & \Omega + c_2 \end{vmatrix}$$

$$0 = \underset{a_0 = 1}{\lambda^3} + \underbrace{(k + c_1)}_{a_1}\lambda^2 + \underbrace{(kc_1 + \Omega^2)}_{a_2 > 0\ \checkmark}\lambda + \underbrace{(\Omega^2 k + \Omega k c_2 + \Omega^2 c_1)}_{a_3}$$

Aufgabe 3 (Fortsetzung)

$$a_1 a_2 - a_0 a_3 = \boxed{k c_1 + c_1^2 - 2 c_2 > 0}$$

Richtungsfeld 3D $\rightsquigarrow$ $f\left(\begin{pmatrix}0\\0\\0\end{pmatrix}\right) = k k \begin{pmatrix}1\\-1\\0\end{pmatrix}$