Aufgabe 1

$$L\ddot{\varphi} = -k\dot{\varphi} - g\sin\varphi + L\omega^2\sin\varphi\cos\varphi$$

$$\ddot{\varphi} = -\frac{k}{L}\dot{\varphi} - \frac{g}{L}\sin\varphi + \omega^2\sin\varphi\cos\varphi$$

$$\underset{\sim}{x} = \begin{pmatrix}x_1\\ x_2\end{pmatrix} = \begin{pmatrix}\varphi\\ \dot{\varphi}\end{pmatrix} \qquad \begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= -\frac{k}{L}x_2 - \frac{g}{L}\sin x_1 + \omega^2\sin x_1\cos x_1\end{aligned}$$

$$\dot{\underset{\sim}{x}} = \underset{\sim}{f}(\underset{\sim}{x})$$

Aufgabe 2

$$\dot{\underset{\sim}{x}} \overset{!}{=} 0 \quad \Rightarrow \quad x_2 = 0$$

$$\cancel{-\frac{k}{L}x_2} - \frac{g}{L}\sin x_1 + \omega^2\sin x_1\cos x_1 = 0$$

$$\sin x_1\left(\omega^2\cos x_1 - \frac{g}{L}\right) = 0$$

$$x_{10}\in[0,\pi] \qquad \begin{aligned}x_{01,1} &= 0\\ x_{02,1} &= \pi\\ \cos(x_{03,1}) &= \frac{g}{L\omega^2} \leadsto x_{03,1} = \arccos\left(\frac{g}{L\omega^2}\right)\end{aligned}$$

$$\underset{\sim}{x}_{01} = \begin{pmatrix}0\\0\end{pmatrix}, \quad \underset{\sim}{x}_{02} = \begin{pmatrix}\pi\\0\end{pmatrix}, \quad \underset{\sim}{x}_{03} = \begin{pmatrix}\arccos\left(\frac{g}{L\omega^2}\right)\\ 0\end{pmatrix}$$

Aufgabe 3

$\boxed{\underset{\sim}{x}_{01}}$

$$\dot{\underset{\sim}{x}} \approx \underbrace{\underset{\sim}{f}\Big|_{\underset{\sim}{x}_{01}}}_{\to 0} + \begin{pmatrix}\frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2}\\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2}\end{pmatrix}\Bigg|_{\underset{\sim}{x}_{01}}(\underset{\sim}{x} - \underset{\sim}{x}_{01})$$

$$= \underbrace{\begin{pmatrix}0 & 1\\ -\frac{g}{L}\cos x_1 + \omega^2(-\sin^2 x_1 + \cos^2 x_1) & -\frac{k}{L}\end{pmatrix}\Bigg|_{x_{01}}}_{\underset{=}{A}}(\underset{\sim}{x} - \underset{\sim}{x}_{01})$$

$$\underset{=}{A} = \begin{pmatrix}0 & 1\\ -\frac{g}{L} + \omega^2 & -\frac{k}{L}\end{pmatrix} \leadsto \det(\underset{=}{A} - \lambda\underset{=}{1}) \overset{!}{=} 0$$

$$\begin{vmatrix}-\lambda & 1\\ -\frac{g}{L} + \omega^2 & -\frac{k}{L} - \lambda\end{vmatrix} = -\lambda\left(-\frac{k}{L} - \lambda\right) - \left(\omega^2 - \frac{g}{L}\right) \overset{!}{=} 0$$

$$\lambda = -\frac{k}{2L} \pm \sqrt{\left(\frac{k}{2L}\right)^2 + \left(\omega^2 - \frac{g}{L}\right)}$$

- $\triangleright\ \omega^2 - \frac{g}{L} < 0 \longrightarrow$ asympt. Stabilität (alle $Re(\lambda) < 0$)
- $\triangleright\ \omega^2 - \frac{g}{L} > 0 \longrightarrow$ instabil (mind. ein $Re(\lambda) > 0$)
- $\triangleright\ \omega^2 - \frac{g}{L} = 0 \longrightarrow$ keine Aussage möglich, System nicht linear und größter $Re(\lambda) = 0$

$\boxed{\underset{\sim}{x}_{02}}$

$$\dot{\underset{\sim}{x}} \approx \underbrace{\begin{pmatrix} 0 & 1 \\ \frac{g}{L} + \omega^2 & -\frac{k}{L} \end{pmatrix}}_{\underline{\underline{A}}} (\underset{\sim}{x} - \underset{\sim}{x}_{02})$$

$$0 \overset{!}{=} \det(\underline{\underline{A}} - \lambda \underline{\underline{I}})$$

$$= \lambda^2 + \frac{k}{L}\lambda - \left(\frac{g}{L} + \omega^2\right) \Longrightarrow \lambda = -\frac{k}{2L} \pm \sqrt{\left(\frac{k}{2L}\right)^2 + \underbrace{\left(\frac{g}{L} + \omega^2\right)}_{>0}}$$

$\longrightarrow$ immer instabil, da größter $Re(\lambda) > 0$

$\boxed{\underset{\sim}{x}_{03}}$

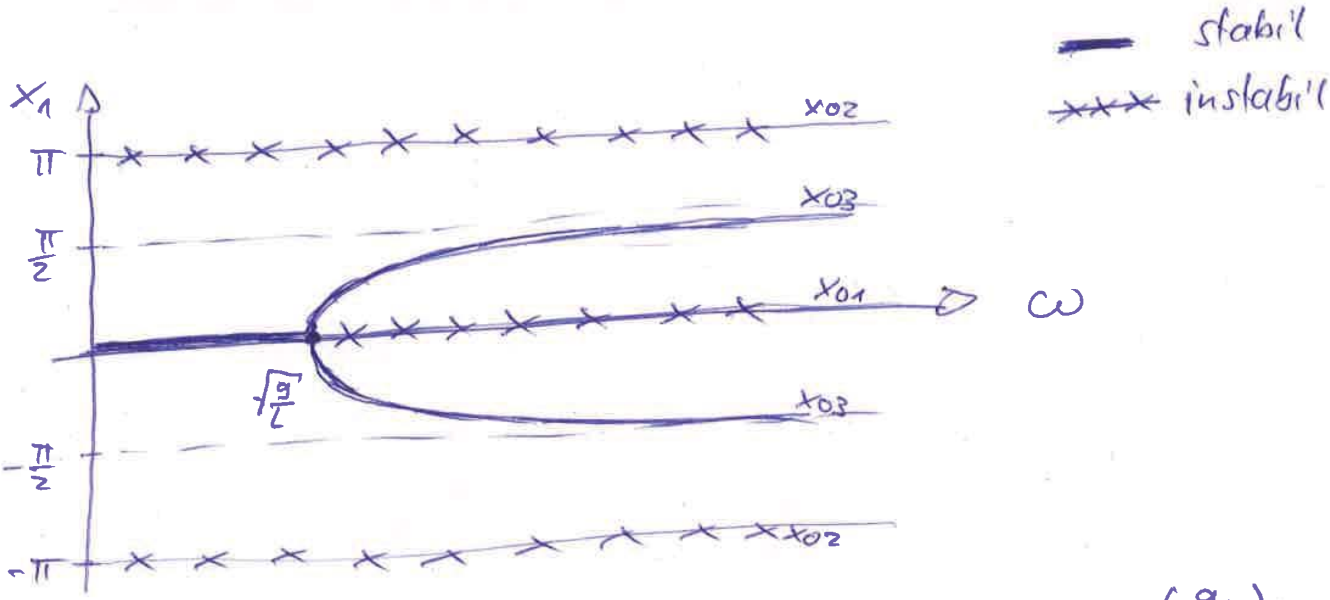
$$\dot{\underset{\sim}{x}} \approx \underbrace{\begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\cos x_1 + \omega^2(2\cos^2 x_1 - 1) & -\frac{k}{L} \end{pmatrix}\Bigg|_{\underset{\sim}{x}_{03}}}_{\underline{\underline{A}}} (\underset{\sim}{x} - \underset{\sim}{x}_{03}) \qquad \Bigg| \ \underset{\sim}{x}_{03} = \begin{pmatrix} \arccos\left(\frac{g}{L\omega^2}\right) \\ 0 \end{pmatrix}$$

$$\underline{\underline{A}} = \begin{pmatrix} 0 & 1 \\ -\frac{g}{L}\frac{g}{L\omega^2} + \omega^2\left(2\left(\frac{g}{L\omega^2}\right)^2 - 1\right) & -\frac{k}{L} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ \left(\frac{g}{L\omega}\right)^2 - \omega^2 & -\frac{k}{L} \end{pmatrix} \qquad \det(\underline{\underline{A}} - \lambda \underline{\underline{1}}) \overset{!}{=} 0$$

$$\rightsquigarrow \lambda = -\frac{k}{2L} \pm \sqrt{\left(\frac{k}{2L}\right)^2 + \left(\frac{g}{\omega L}\right)^2 - \omega^2}$$

- $\triangleright\ \left(\frac{g}{L\omega}\right)^2 - \omega^2 < 0 \Longleftrightarrow \left(\frac{g}{L}\right)^2 < \omega^4 \Longleftrightarrow \frac{g}{L} < \omega^2 \rightarrow$ asympt. stabil
  (alle Param. > 0)
- $\triangleright\ \left(\frac{g}{\omega L}\right)^2 - \omega^2 > 0 \Longleftrightarrow \frac{g}{L} > \omega^2 \rightarrow$ instabil
- $\triangleright\ \left(\frac{g}{\omega L}\right)^2 - \omega^2 = 0 \Longleftrightarrow \frac{g}{L} = \omega^2 \rightarrow$ keine Aussage für größter $Re(\lambda) = 0$ möglich, da System nicht linear.



$$x_{03} = \arccos\Big(\underbrace{\frac{g}{l\omega^2}}_{a}\Big)$$