ECE130B Jan. 16, 2017

Home work 2

Due date: Jan. 23, 2017 by 5 p.m. at the home work box.

Reading assignment. Chapter 3 of the text book, including both discrete and continuous Fourier series.

Square wave. A square wave of period T is denoted by $s(t)$ and is given by the formula,

$$s(t) = \begin{cases} 1, & \text{if } 0 \leqslant t \bmod T < T/2, \\ -1, & \text{if } T/2 \leqslant t \bmod T < T, \end{cases}$$

where $t \bmod T$ is the unique number r between 0 and T that makes $t - r$ exactly divisible by T; that is, there exists some integer N, such that $t = NT + r$. Note that N can be a negative integer or even 0.

Triangular wave. A triangular wave of period T is denoted by $\tau(t)$, and is given by the formula,

$$\tau(t) = \begin{cases} \frac{4}{T} t' - 1, & \text{if } 0 \leqslant t' < T/2, \\ 3 - \frac{4}{T} t', & \text{if } T/2 \leqslant t' < T, \end{cases}$$

where $t' = t \bmod T$.

Parabolic wave. A parabolic wave of period T is denoted by $q(t)$, and is given by the formula,

$$q(t) = \begin{cases} \frac{8}{T^2} t' (T - 2t'), & \text{if } 0 \leqslant t' < T/2, \\ \frac{8}{T^2} (T - 2t')(T - t'), & \text{if } T/2 \leqslant t' < T, \end{cases}$$

where $t' = t \bmod T$.

Cosine wave. A cosine wave of period T is given by $\cos(2\pi t/T)$.

Frequency and period. If T is the period of a signal in seconds we will refer to $1/T$ as the frequency (usually denoted by f) in Hertz. We will also refer to $2\pi/T = 2\pi f$, as the frequency in radians per second.

Exercise 1. Using your favorite programming language generate a synthetic audio signal of a square wave, a triangular wave, a parabolic wave and a cosine wave at 262Hz with a sample rate of 8000Hz. Listen to all four recordings and write down your observations; could you distinguish between them? Calculate how many samples should be there in 1 period of the wave. Submit your code fragments and a plot of the first period of each wave that you generated.

Match filtering. Let $x[n]$ be a discrete signal with non-zero values only in the time interval $0 \leqslant n \leqslant N-1$. Let $y[n]$ be another causal ($n \geqslant 0$) discrete signal. The matched filter output for $x[n]$ and $y[n]$ is the causal discrete signal $z[n]$, computed according to the formula

$$z[n] = \sum_{k=0}^{N-1} x[k]\, y[n+k], \qquad n \geqslant 0.$$

Exercise 2. If $y[n]$ is non-zero only in the time-interval $0 \leqslant n < K$, find the time interval in which $z[n]$ *can* be non-zero.

Exercise 3. Write code that takes $x[n]$ and $y[n]$ as inputs and computes $z[n]$. Submit the code fragment.

Exercise 4. Make a recording of yourself saying "Hello world". Call this signal $y[n]$. Make a second recording of yourself saying just "hello". Call this signal $x[n]$. Compute the matched filter output of $x[n]$ and $y[n]$ using the code from Exercise 3. Plot all three signals and write down your observations. Repeat the experiment by just copying out the "Hello" part of $y[n]$ to form $x[n]$.

Exercise 5. Suppose you sample the continuous signal $e^{j\omega_0 t}$ at the rate of f_s samples per second (with the first sample at $t=0$).

i. Find all values of ω_0 for which the resulting discrete signal will be periodic.

ii. Find all values of ω_0 for which the resulting discrete signal will have the fixed period N.

Exercise 6. Suppose $e^{j\frac{2\pi}{N}kn}$ is a discrete time signal with fixed values of N and k (so n is the discrete time variable) that was obtained by sampling a continuous signal of the form $e^{j\omega_0 t}$ at the rate f_s samples per second. Find all possible values of ω_0 in terms of N, k and f_s.

Discrete time Fourier series. Let $x[n]$ be a discrete periodic signal of period N. That is $x[n+N]=x[n]$ for all values of n. The signal $x[n]$ can be written as the sum of discrete harmonic oscillators of period N via the formula

$$x[n]=\sum_{k=0}^{N-1} a_k\, e^{j\frac{2\pi}{N}kn}, \tag{1}$$

where the a_k are called the Fourier coefficients of $x[n]$ and can be found via the formula

$$a_k=\frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}. \tag{2}$$

Exercise 7. Prove that the above formula is correct by plugging back the expression for a_k in equation (2) into equation (1).

Programming. A periodic discrete signal of period N can be conveniently represented as an array of length N. Notice that a period N discrete signal has N Fourier series coefficients which can also be stored in an array of length N.

Exercise 8. Write a program in your favorite programming language that can take as input a discrete time signal of period N and return its Fourier series coefficients. Your program must *explicitly* code the formula in equation (2). You cannot use a library routine to do it. Please submit a printed copy of the program.

Exercise 9. Write a program in your favorite programming language that can take as input the Fourier series coefficients of a discrete time signal of period N and return the discrete time signal itself. Your program must *explicitly* code the formula in equation (1). You cannot use a library routine to do it. Please submit a printed copy of the program.

Exercise 10. Given a length N signal $x[n]$, find the number of arithmetic operations that the algorithm you implemented in Exercise 8, requires. *Note*: complex exponentiation, multiplication, addition, division, subtraction, cosine and sine, can all be viewed as "single" arithmetic operations for this exercise.

Exercise 11. Report the time taken by your code from Exercise 8, for signals of length 2^{12}, 2^{13} and 2^{14}. Make sure you only time the code that computes the Fourier series coefficients.