ECE130B February 13, 2017

Homework 5

Due date: February 21, 2017, by 11:59p.m. at the homework box.
Reading assignment: Chapter 2 of the text book.

Exercise 1. Find *all* zero-input responses and all impulse-responses of the following difference equations:

1. $\alpha\, y[n] + \beta\, y[n-1] = x[n]$, where $\alpha \neq 0$.
2. $y[n] - (\alpha + \beta) y[n-1] + \alpha\,\beta\, y[n-2] = x[n]$, where $\alpha \neq \beta$.
3. $y[n] - 2\,\alpha\, y[n-1] + \alpha^2\, y[n-2] = x[n]$.

You might want to add some of these solutions to your cheat sheet.

Exercise 2. Find the output $y[n]$ that satisfy the following difference equations:

1. $10\, y[n+1] + 50\, y[n] + 60\, y[n-1] = \left(\frac{7}{10}\right)^{n+1} u[n]$, with $y[-2] = y[-1] = 0$.
2. $\frac{y[n-1] + y[n] + y[n+1]}{3} = u[n]$, with $y[-2] = y[-1] = 0$.
3. $y[n] + y[n-2] = \sin(6\pi n/10)\, u[n]$, with $y[-2] = y[-1] = 0$.

Exercise 3. Find a $y[n]$ that satisfies the difference equation

$$y[n] - \lambda\, y[n-1] = \lambda^n,$$

with $y[0] = 0$. (Finding the particular solution is a bit tricky.)

Exercise 4. Let ω_0 be a given real number. Design a discrete LTI system with impulse response $h[n] = e^{j\omega_0 n} u[n]$. Present a block-diagram of the system and justify your answer. You can **assume** that the multipliers, adders and memory units in your system can handle complex numbers. How many complex memory units did you need?

Exercise 5. Let z and w denote two complex numbers. Let z_R and z_I denote the real and imaginary parts of z; that is, $z = z_R + j\, z_I$. Similarly let $w = w_R + j\, w_I$. Design a device (see Figure 1) that takes the two real inputs z_R and z_I, and outputs the real and imaginary parts of $y = z\, w$ (the product of the two complex numbers z and w). Your design is only allowed to use real multipliers and real adders. Present a block diagram of your system.

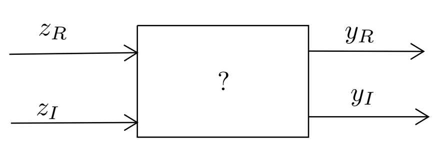

Figure 1.

Exercise 6. Show how to modify your design in Exercise 4 to obtain a discrete LTI system with impulse response $h[n] = \cos(\omega_0 n) u[n]$. This time your system is only allowed to use real multipliers, adders and memory units. Present a block-diagram of your system. How many real memory units did you need? *Hint*: You will find the results of Exercise 5 useful.

Exercise 7. Design a discrete LTI system with impulse response $h[n] = e^{j(\omega_0 n + \varphi_0)} u[n]$, where ω_0 and φ_0 are given real numbers. You can use complex multipliers, adders and memory units. Present a block-diagram of your system.

Exercise 8. Design a discrete LTI system with impulse response $h[n] = \cos(\omega_0 n + \varphi_0)\, u[n]$, where ω_0 and φ_0 are given real numbers. Use only real multipliers, adders and memory units in your system. Present a block diagram of your system.