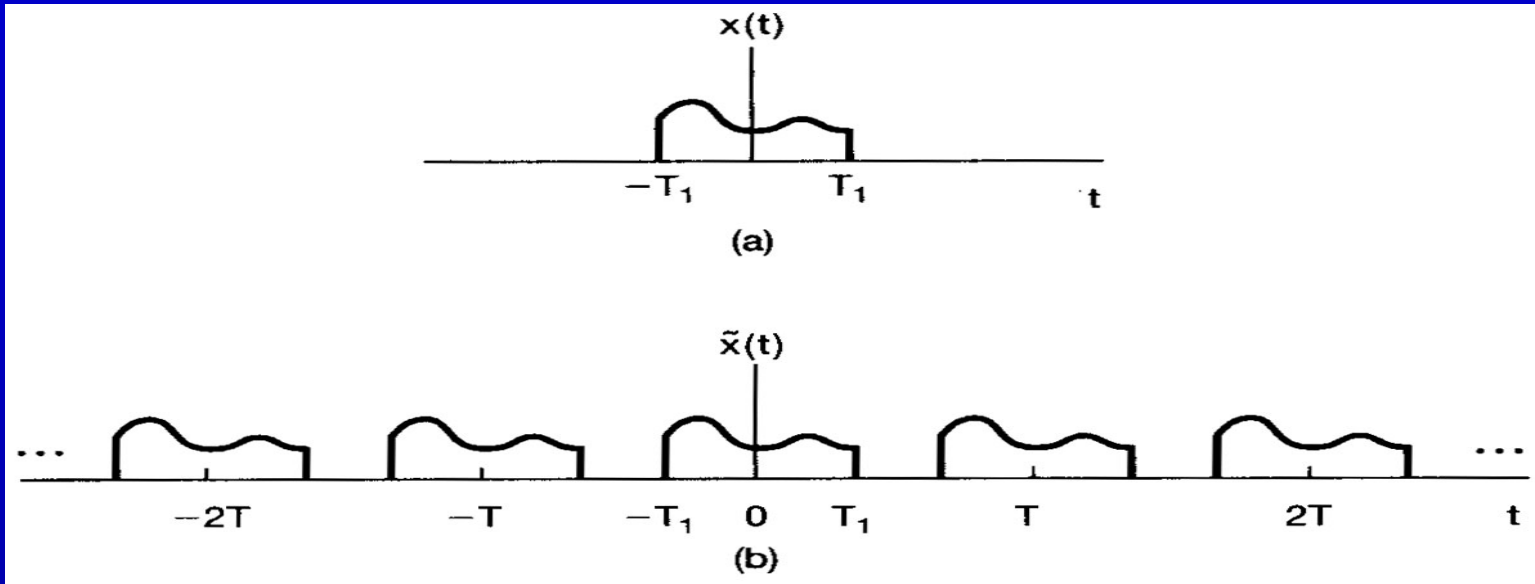
Chapter 5

The Discrete-Time Fourier Transform



$$\tilde{x}(t)=\sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$$

$$a_k=\frac{1}{T}\int_T \tilde{x}(t)e^{-jkw_0 t}dt$$

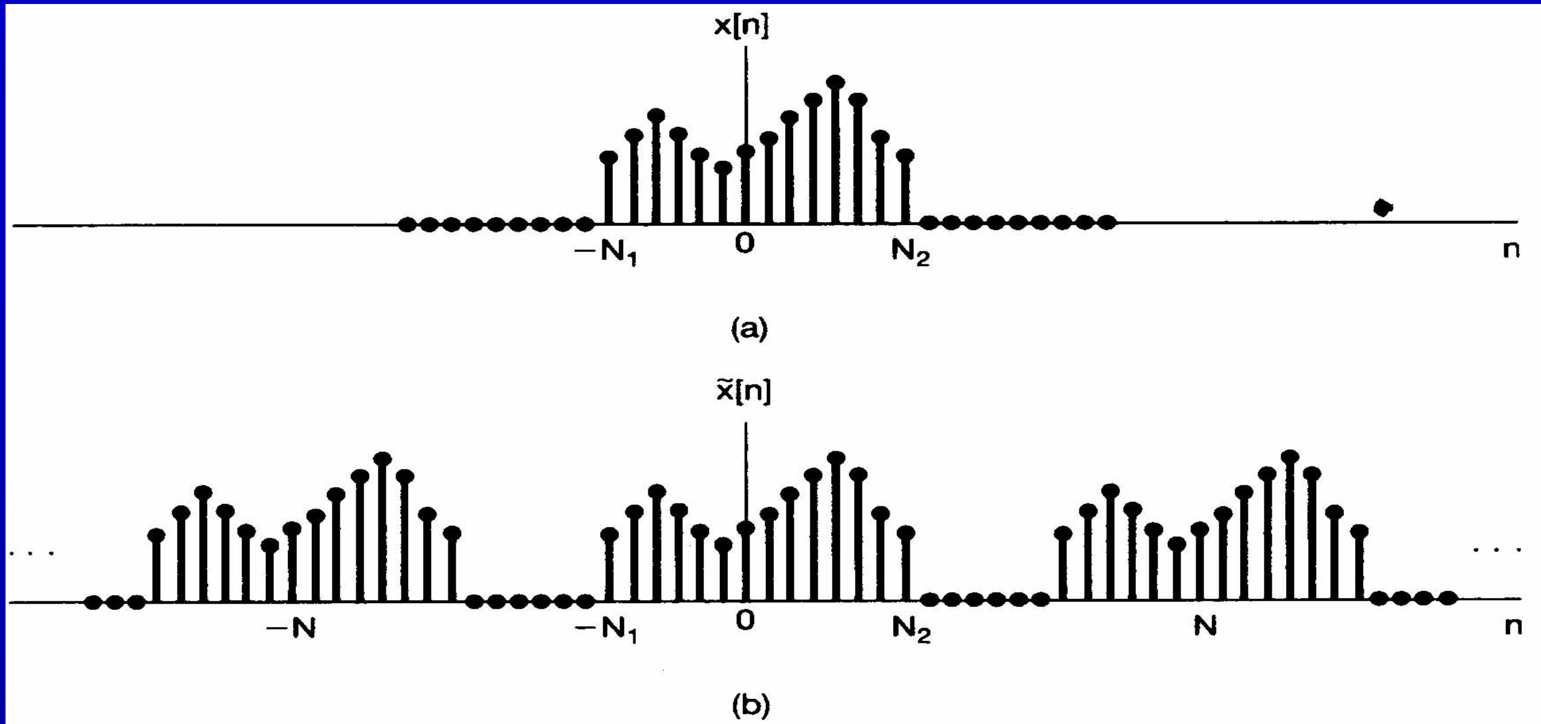
$$T\rightarrow\infty$$

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega)=\int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

Signals & Systems

5.1 Representation of aperiodic signals: the discrete-time Fourier transform



$$\tilde{x}[n] = \sum_{k=\langle N \rangle} a_k e^{jkw_0 n}$$

$$N \to \infty \Rightarrow \quad \tilde{x}[n] \to x[n]$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} \tilde{x}[n] e^{-jkw_0 n}$$

Signals & Systems

Fourier transform

$$X(e^{jw}) = \sum_{n=-\infty}^{+\infty} x[n]e^{-jwn}$$

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X\left(e^{jw}\right)e^{jwn}dw$$

The difference of CFT and DFT

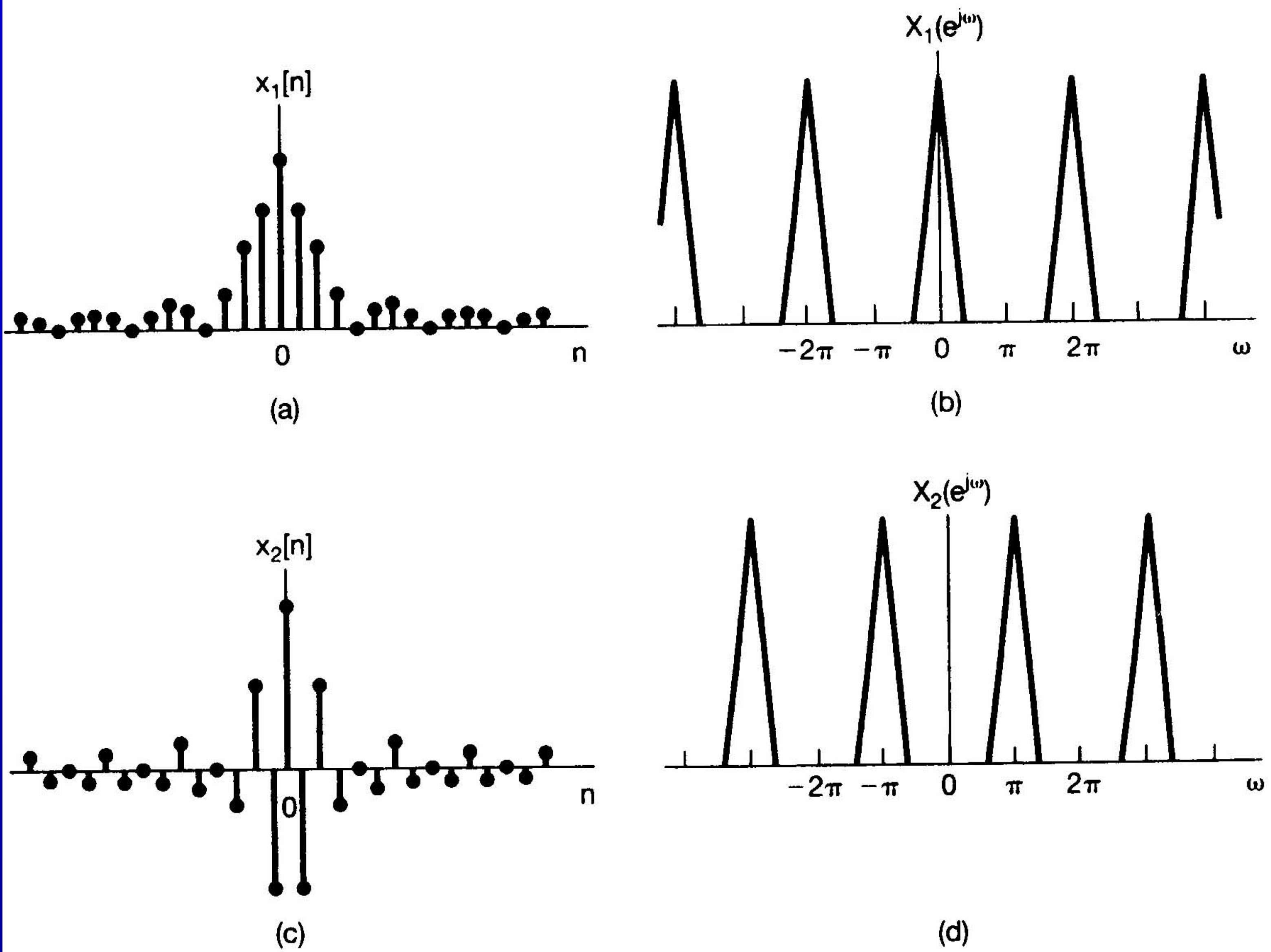
Signals & Systems

CFT	DFT
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$	$x[n]=\frac{1}{2\pi}\int_{2\pi}X(e^{j\omega})e^{j\omega n}d\omega$
$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$	$X(e^{j\omega})=\sum_{n=-\infty}^{+\infty}x[n]e^{-j\omega n}$

$$X(e^{jw})\ \ is\ \ periodic, period\ \ is\ \ 2\pi$$

$$w\rightarrow 0, low\ \ frequency$$

$$w\rightarrow \pi, high\ \ frequency$$

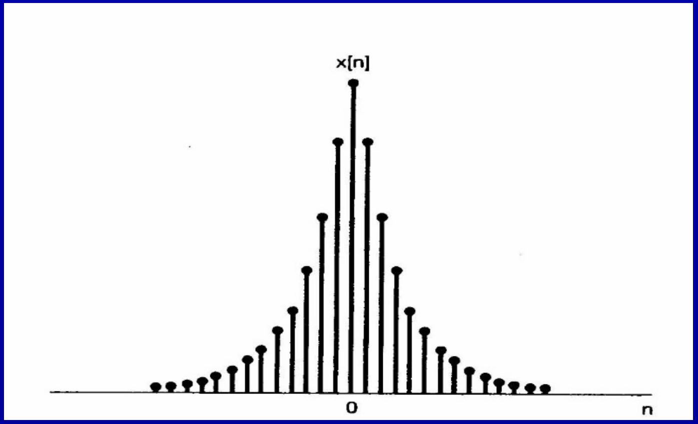

x1[n]
0
n
(a)
X1(e^jω)
−2π
−π
0
π
2π
ω
(b)
x2[n]
0
n
(c)
X2(e^jω)
−2π
−π
0
π
2π
ω
(d)

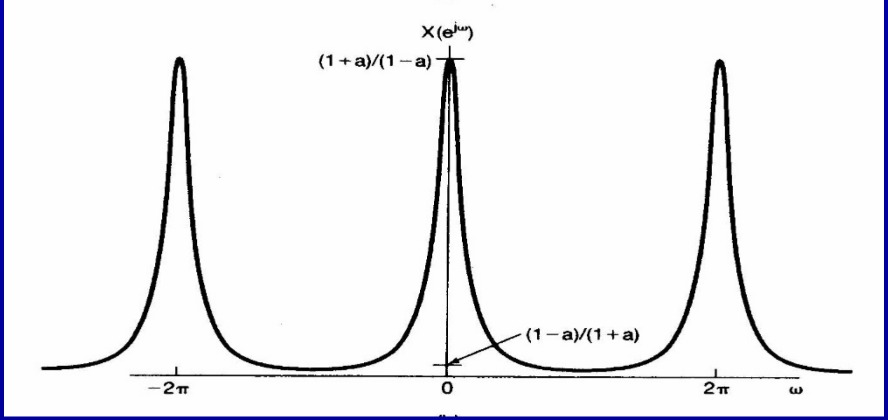
Example 5.1

$$x[n]=a^n u[n],\quad |a|<1, \text{determine } X\left(e^{jw}\right)$$

Example 5.2

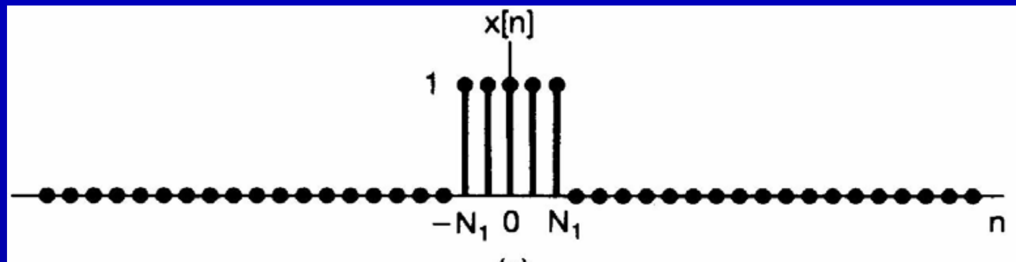
$$x[n]=a^{|n|},\quad |a|<1, \text{determine } X\left(e^{jw}\right)$$



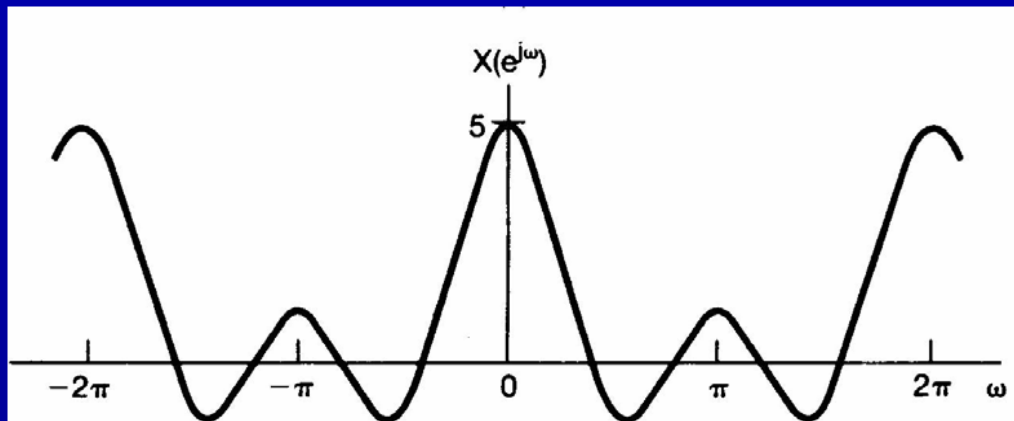


Example 5.3

$$x[n]=\begin{cases}1, & |n|\le N_1\\ 0, & |n|>N_1\end{cases}, \text{determine } X\left(e^{jw}\right)$$



$$X(e^{j\omega})=\frac{\sin\omega\left(N_1+\frac{1}{2}\right)}{\sin(\omega/2)}$$



5.1.3 Convergence Issues Associated with the Discrete-Time Fourier Transform

$$X(j\omega)=\sum_{n=-\infty}^{+\infty} x[n]e^{-j\omega n}$$

$$\sum_{n=-\infty}^{+\infty}\left|x[n]\right|<\infty \quad or \quad \sum_{n=-\infty}^{+\infty}\left|x[n]\right|^{2}<\infty$$

$$x[n]=\frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$