Home work 7

Due Date: March 10, 2017, by 8:00p.m. at the homework box.

Reading assignment: Chapter 10 of the text book.

Bounded signals. A signal $x[n]$ is said to be bounded if there exists a finite number M such that $|x[n]| < M$ for $-\infty < n < \infty$.

Stable system. A system is said to be (bounded-input-bounded-output) stable if for *all* bounded input signals the output signal is also bounded.

Exercise 1. Show that an LTI system is stable if, and only if, its impulse response $h[n]$ is *absolutely summable*; that is

$$\sum_{n=-\infty}^{\infty} |h[n]| < \infty.$$

Exercise 2. System A has impulse response $h_A[n] = (-3/7)^n u[n]$, while system B has impulse response $h_B[n] = (-3/7)^n u[-n]$. Which of the systems, if any, is stable?

Exercise 3. The input $x[n]$, and output $y[n]$, of an LTI system satisfy the equation

$$2\,y[n] - 5\,y[n-1] + 2\,y[n-2] = 2\,x[n].$$

i. Find the impulse response if the system is known to be causal.
ii. Find the impulse response if the system is known to be stable.
iii. Can the system be both causal and stable at the same time? Justify your answer.

Exercise 4. The input $x[n]$ and output $y[n]$ of a **causal** system (with system transfer function $H(z)$) satisfy the difference equation

$$y[n] - 5y[n-1] + 6y[n-2] = x[n].$$

Show that this system is unstable. To stabilize it a system with transfer function $G(z)$ is added in the feed-back path as shown in the following figure:

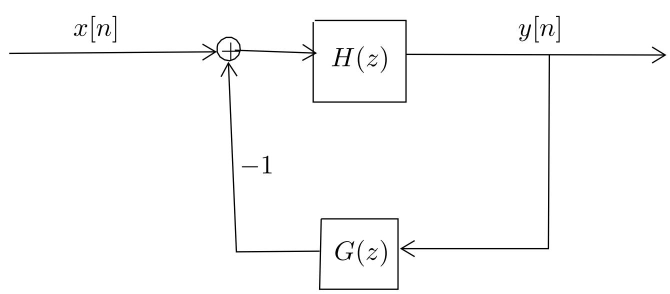

Figure 1.

You will try three choices for $G(z)$:

i. $G(z) = K$
ii. $G(z) = K\,z^{-1}$
iii. $G(z) = K\,z^{-2}$

For each choice try to find a real value of K for which the overall system is stable and causal. Use Matlab (or Octave) if necessary to plot the magnitude of the poles as a function of K, and visually inspect the graph to find an answer, if possible.

What does this tell you about delays in the feedback loop?