信号与系统
Signals & Systems

何其锐

School of Opto-Electronic Information

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课程介绍

- 课程类型：学科基础课
  - 电子信息类专业基础课
  - 研究生考试课程
- 学时：64学时（教学60学时 实验4）
- 学分：4学分
- 预备知识：高等数学 电路基础 线性代数

教材及参考书

- 《 **Signals & Systems** 》By Oppenheim
  2ed， MIT，1997
- **《信号与系统》**刘树棠 译
  西安交通大学出版 1998
- **《信号与系统计算机练习-利用 Matlab》**刘树棠译，
  西安交大出版社
- **麻省理工学院公开课**-网易公开课

Signals & Systems

成绩考核

- 成绩 = 40%平时 + 10%实验 + 50%期末
- 平时成绩包括：课本作业、出勤率、课程设计、测验等
- 期末考试方式：一页开卷
- 作业按时提交，实时计入总成绩
- 未按时提交，该次作业计零分
- 每人三次延时提交机会

诚信管理

- 《电子科技大学学生纪律处分规定（试行）》
- 第二十二条
- 已提交的平时作业、小论文、实验报告，任课教师发现存在抄袭或伪造数据事实的，给予警告和教育，本次作业或报告成绩以零分记；无视警告再犯的，视情节给予严重警告及以上处分；

- 《电子科技大学学生申诉处理实施办法（试行）》
- 普通本专科学生和研究生对学校给予本人作出取消入学资格、退学处理或处分决定有异议的，向电子科技大学学生申诉处理委员会（以下简称申诉处理委员会）提出申诉的，适用本办法

诚信管理

- 《电子科技大学全日制本科学生考试及学术规范管理规定》
- 第七条考试资格认定。学生有下列情形之一者，**取消考试资格**:
- （一）**平时作业**次数中，有**20%以上未按时完成**。
- （二）平时测验次数中，缺考20%以上。
- （三）实验次数缺少20%以上。
- （四）**缺课学时**数达到课程**总课时的20%以上**。
- （五）经教务处认定的其他行为。

- 被取消考试资格的学生，其相应课程**正考成绩**以“**零分**”记，**取消补考资格**。

Signals & Systems

诚信管理

Signals & Systems

课程学习建议

- 学习目的、方法
- 理解物理含义，掌握内容框架
- **MATLAB Multisim**

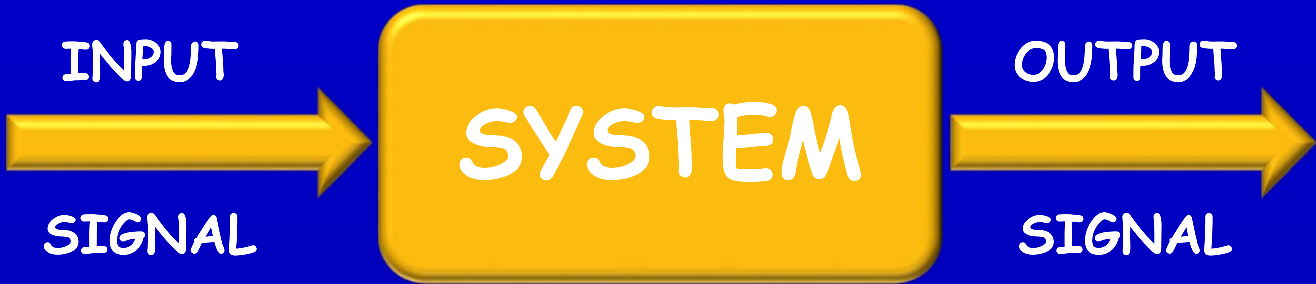
课程收获

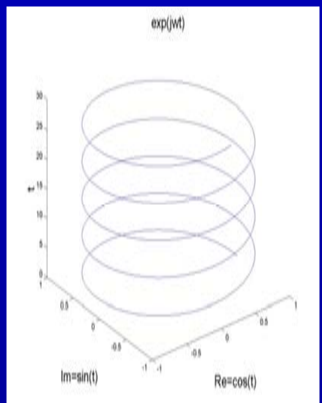
- 课程基本内容、专业英语能力
- 文献查阅能力
- 基本工程设计能力
- MATLAB Multisim
- 文档撰写能力
- 信号与系统的哲学观念

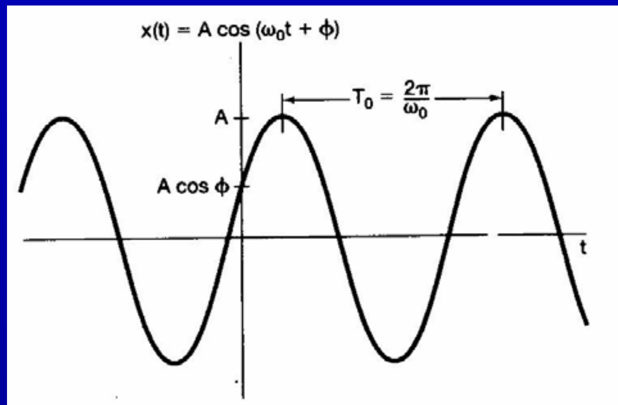
Signals & Systems

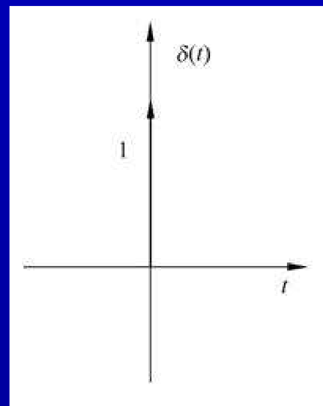
The main content of course

Chapter 1





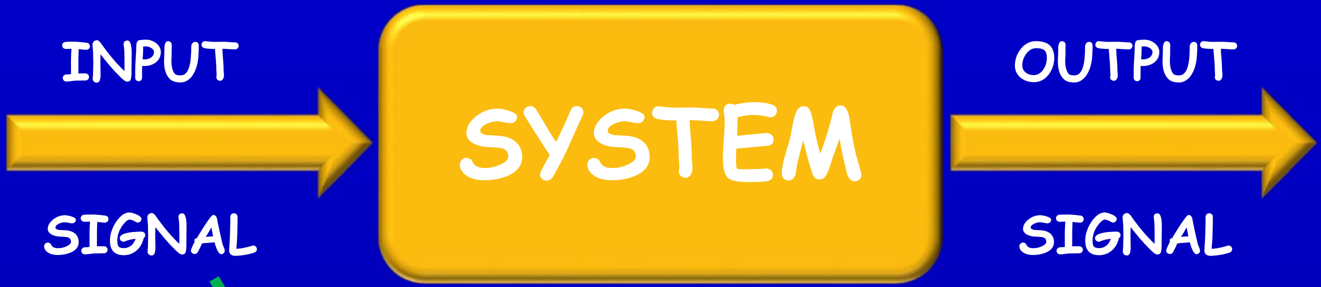


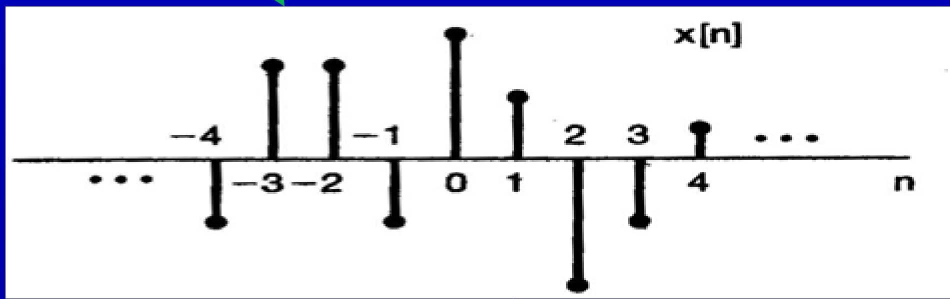


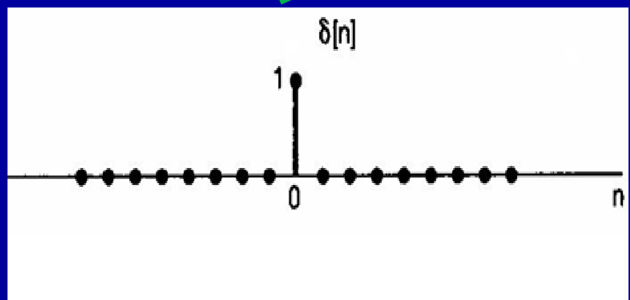
System and their main properties

The main content of course

Chapter 2





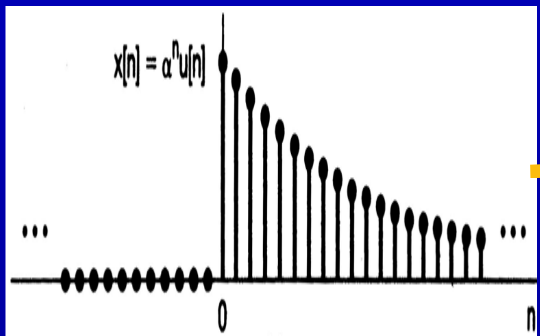


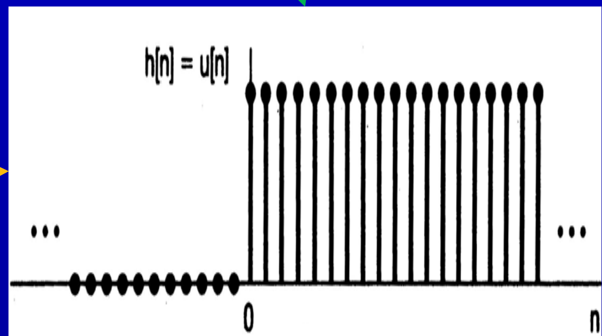
Signals & Systems

The main content of course

Chapter 2

INPUT SIGNAL → SYSTEM → OUTPUT SIGNAL





OUTPUT SIGNAL

Convolution

The main content of course

Chapter 3

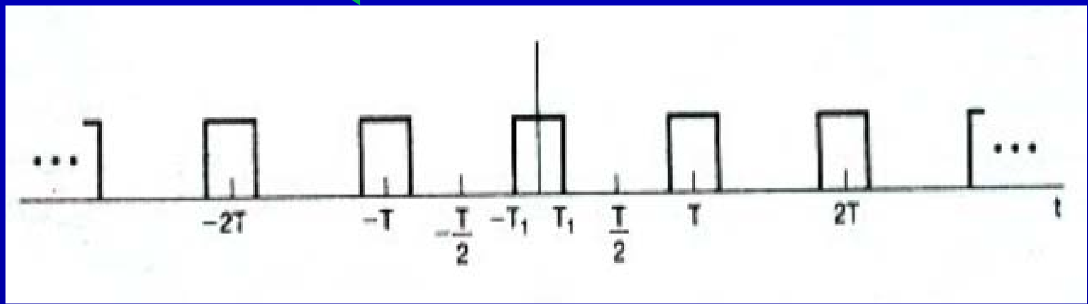
INPUT

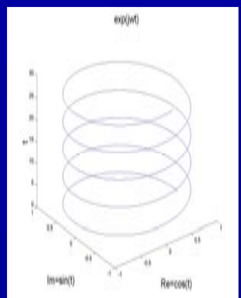
SIGNAL

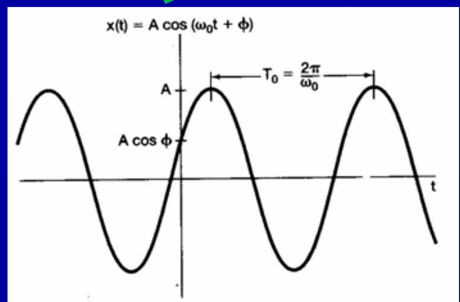
SYSTEM

OUTPUT

SIGNAL



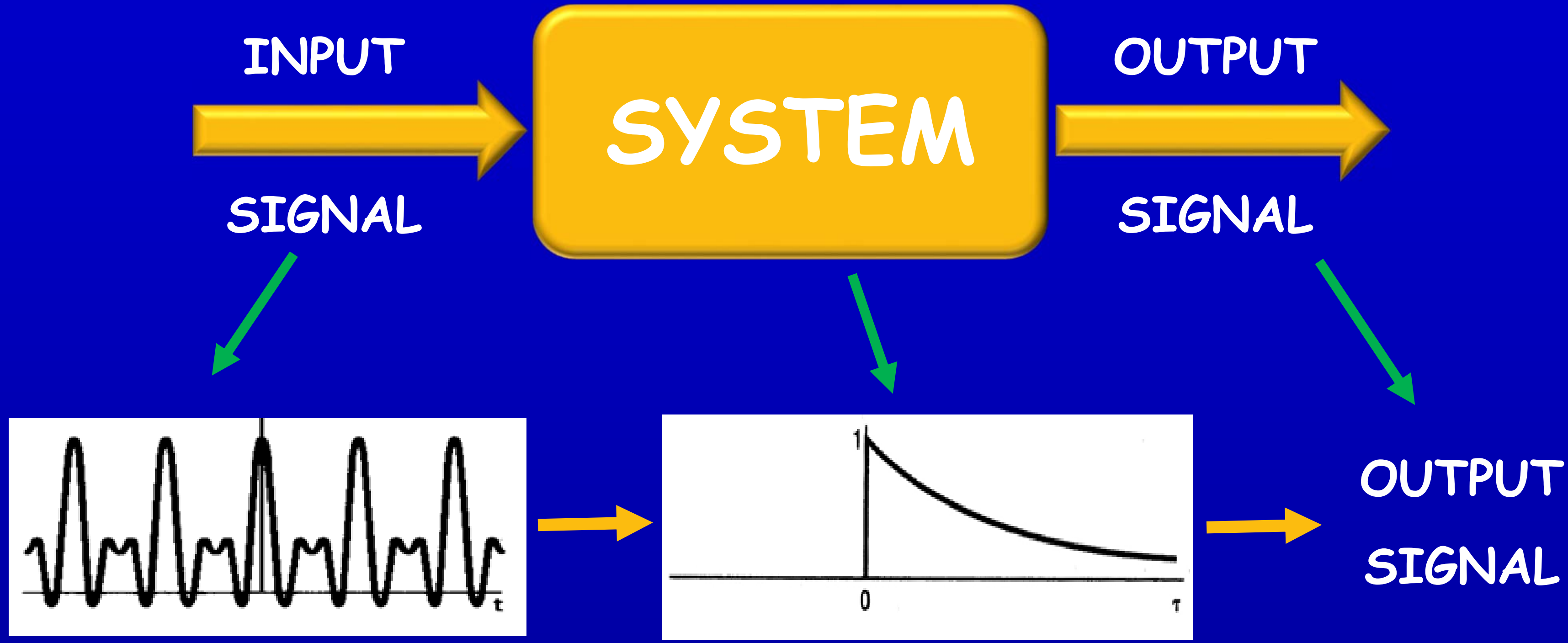




Signals & Systems

The main content of course

Chapter 3

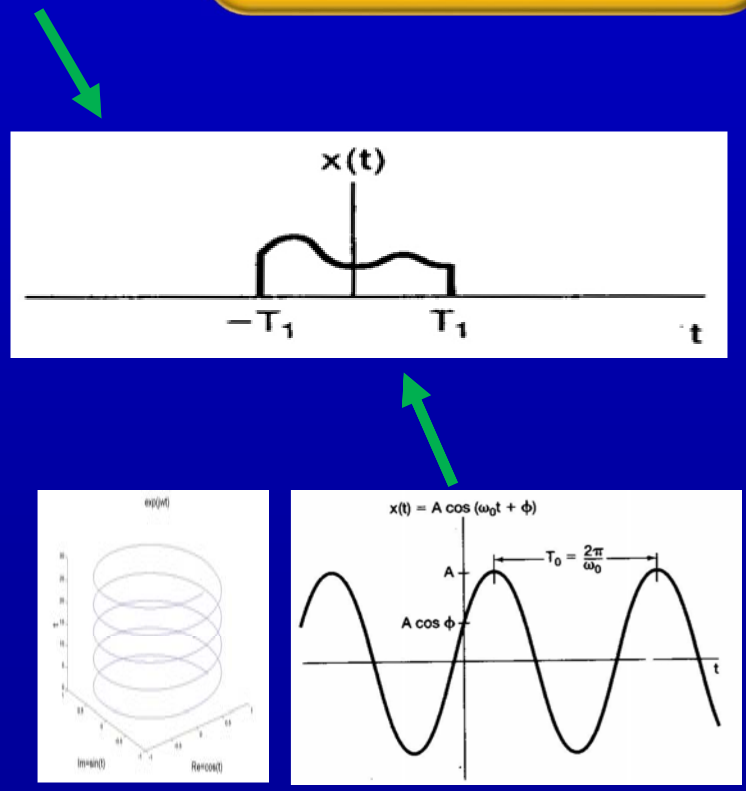


The Response of Periodic Signals

Signals & Systems

The main content of course

Chapter 4



Fourier Transform

The main content of course

Chapter 4

Properties of the Fourier Transform

The main content of course

Chapter 4

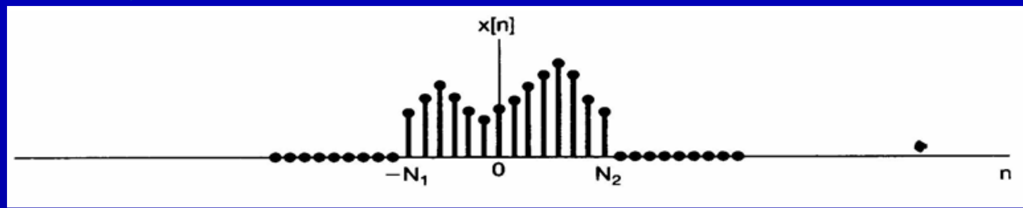
Frequency-Domain Analysis of LTI Systems

Signals & Systems

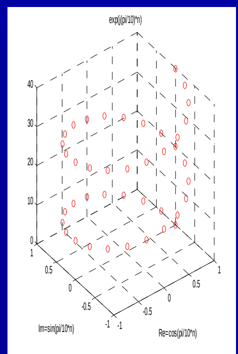
The main content of course

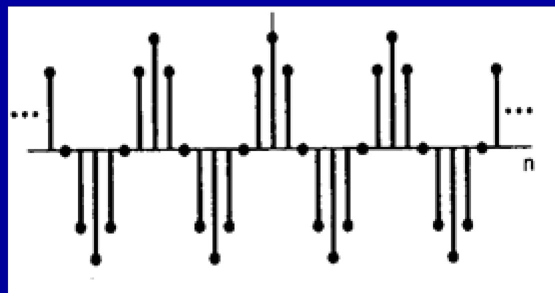
Chapter 5

INPUT SIGNAL → SYSTEM → OUTPUT SIGNAL



Fourier Transform

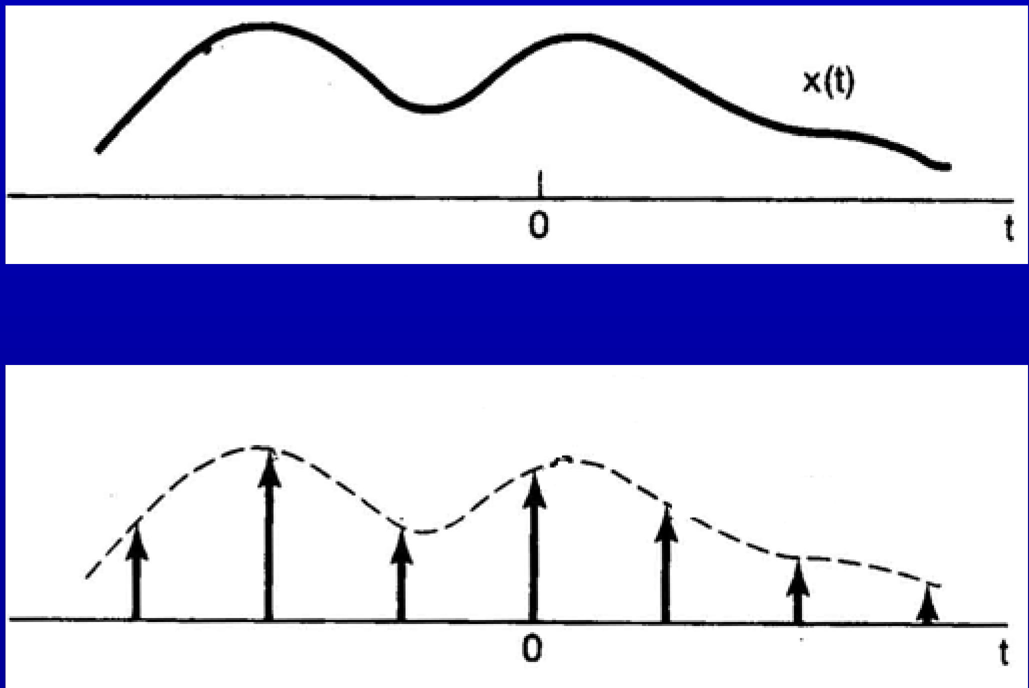




Signals & Systems

The main content of course

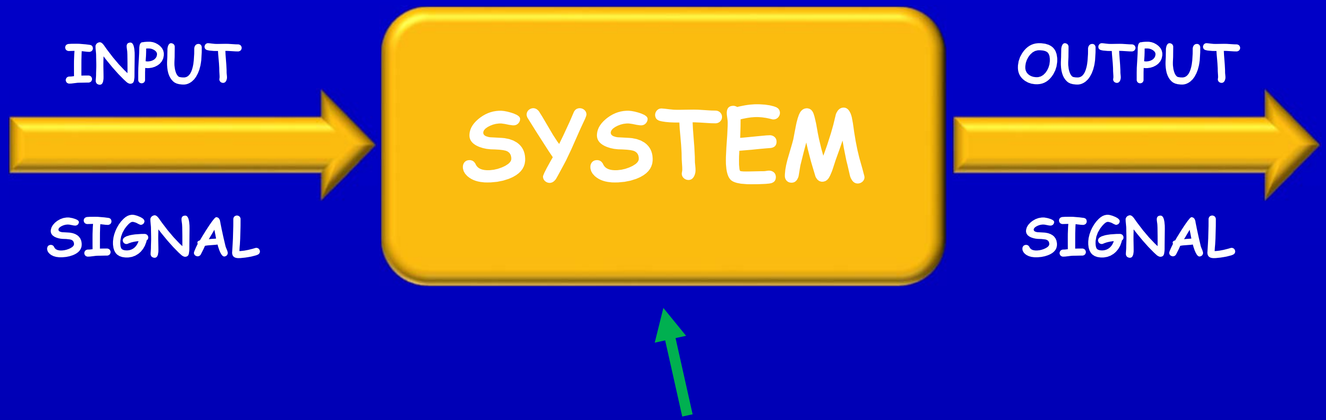
Chapter 7



Sampling Theorem

The main content of course

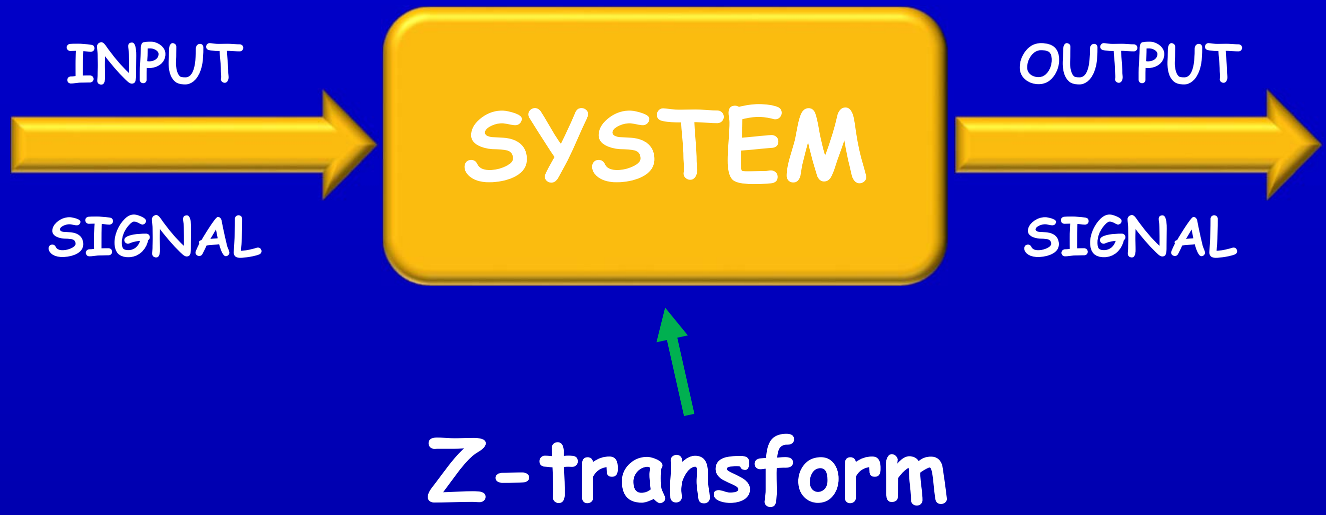
Chapter 9



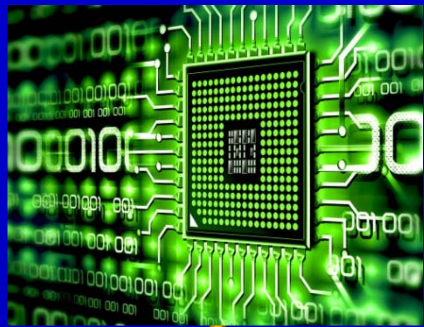
Laplace transform

The main content of course

Chapter 10



FOREWORD



Signals and Systems

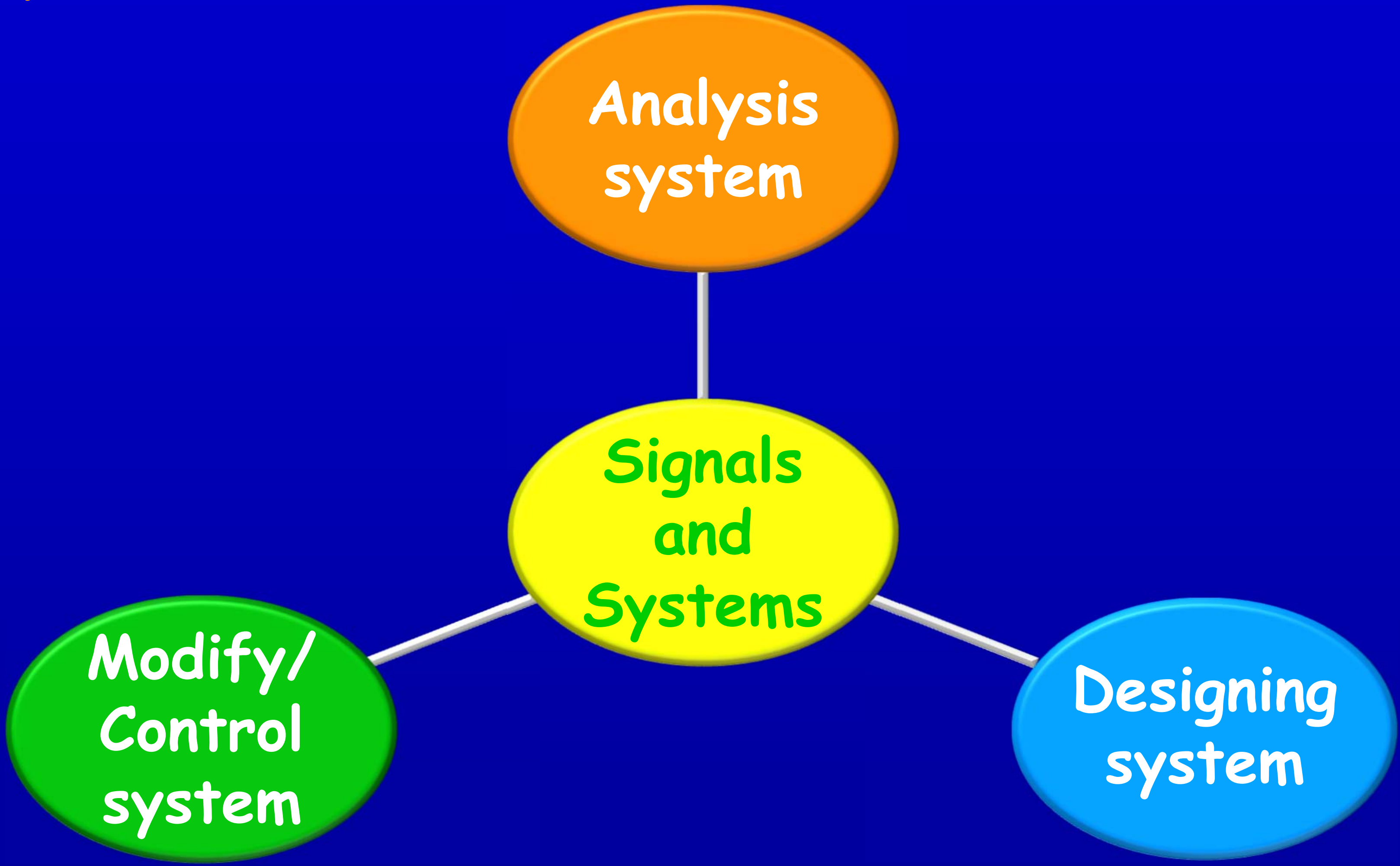
The concept of signal

Signal: **medium** of **information** and **message** contain information about the behavior or nature of some phenomenon

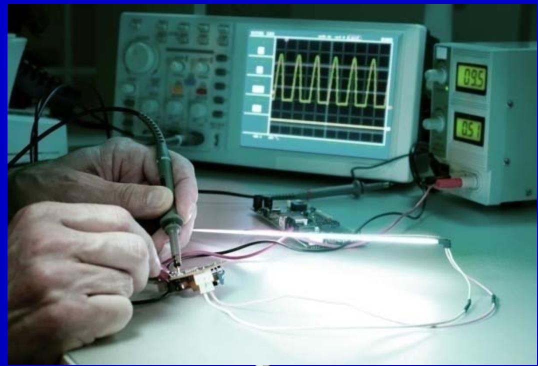
The concept of system

System: objective entity that **produce** 、 **transform** or **process** signal

Application

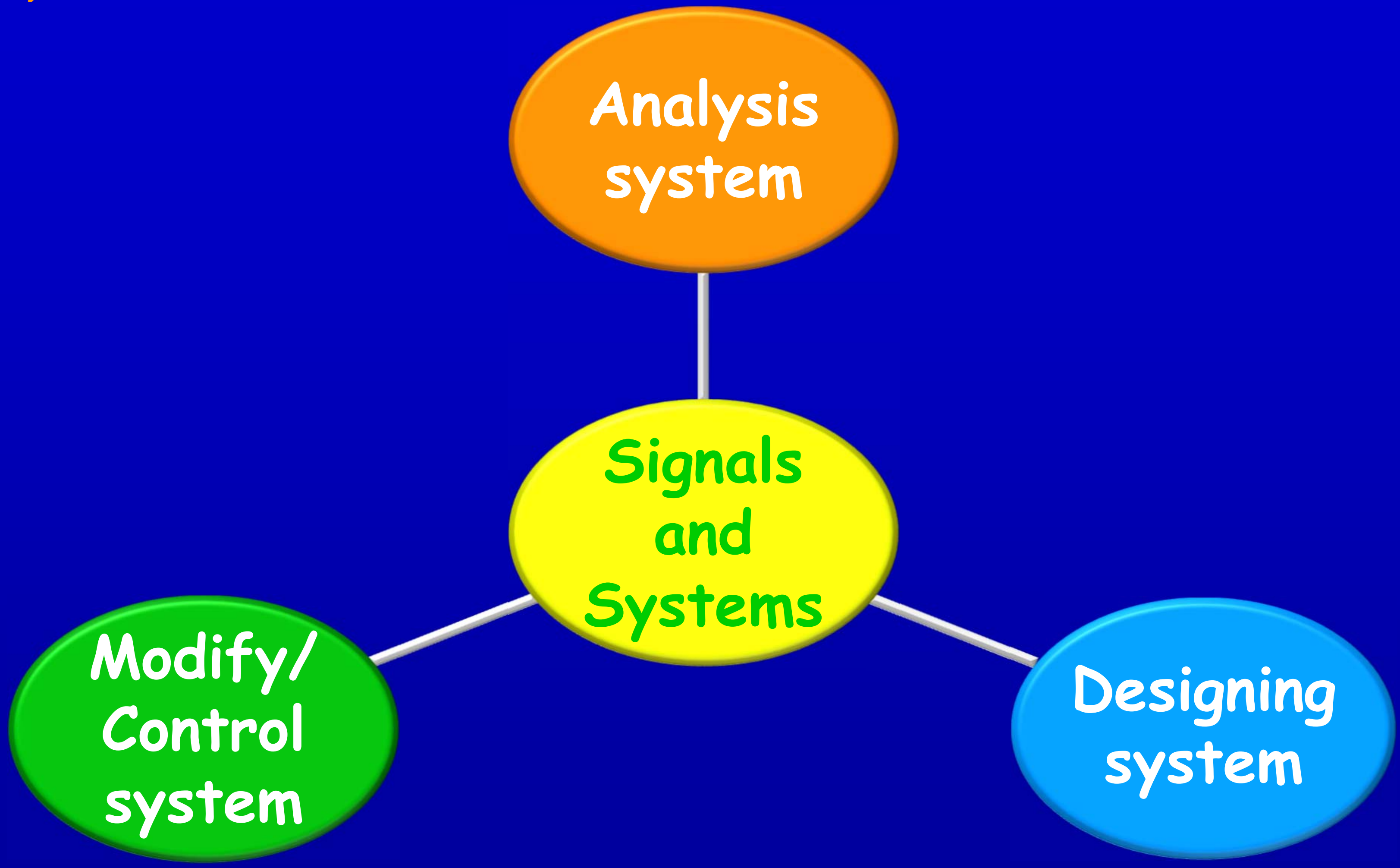


Analysis system



Analysis system

Application

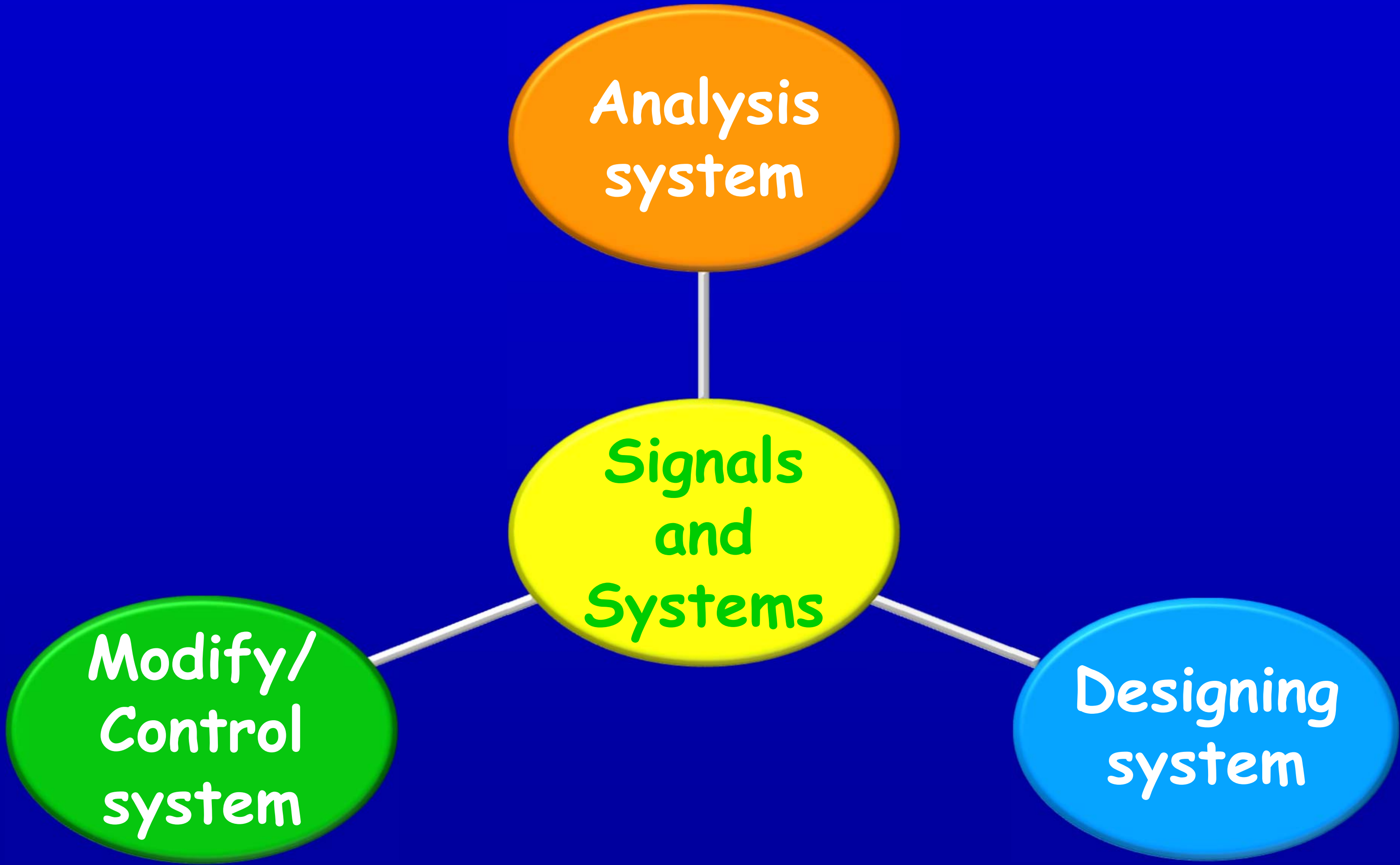


Signals & Systems

Designing system

Designing system

Application



Signals & Systems

Modify/Control system

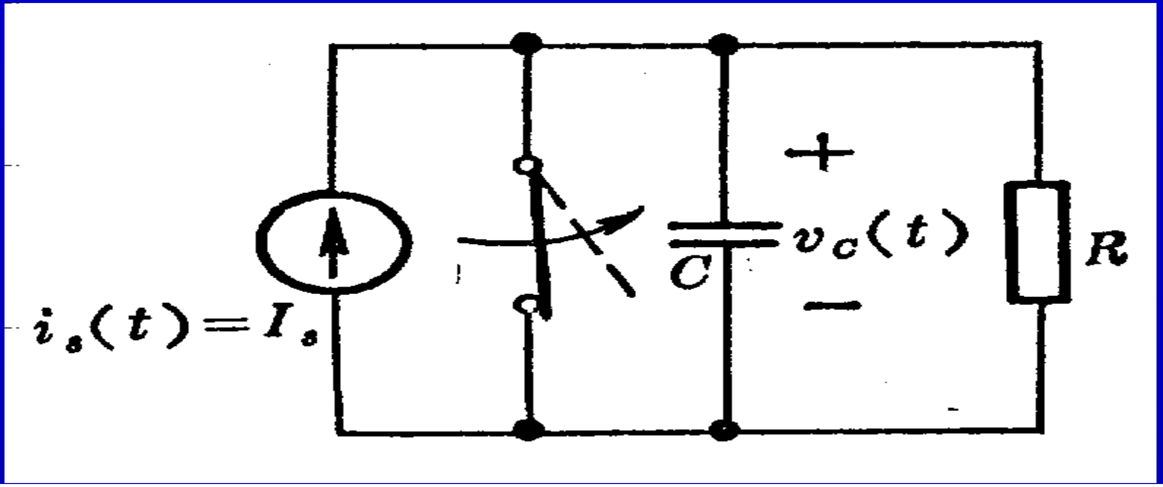
Modify/
Control
system

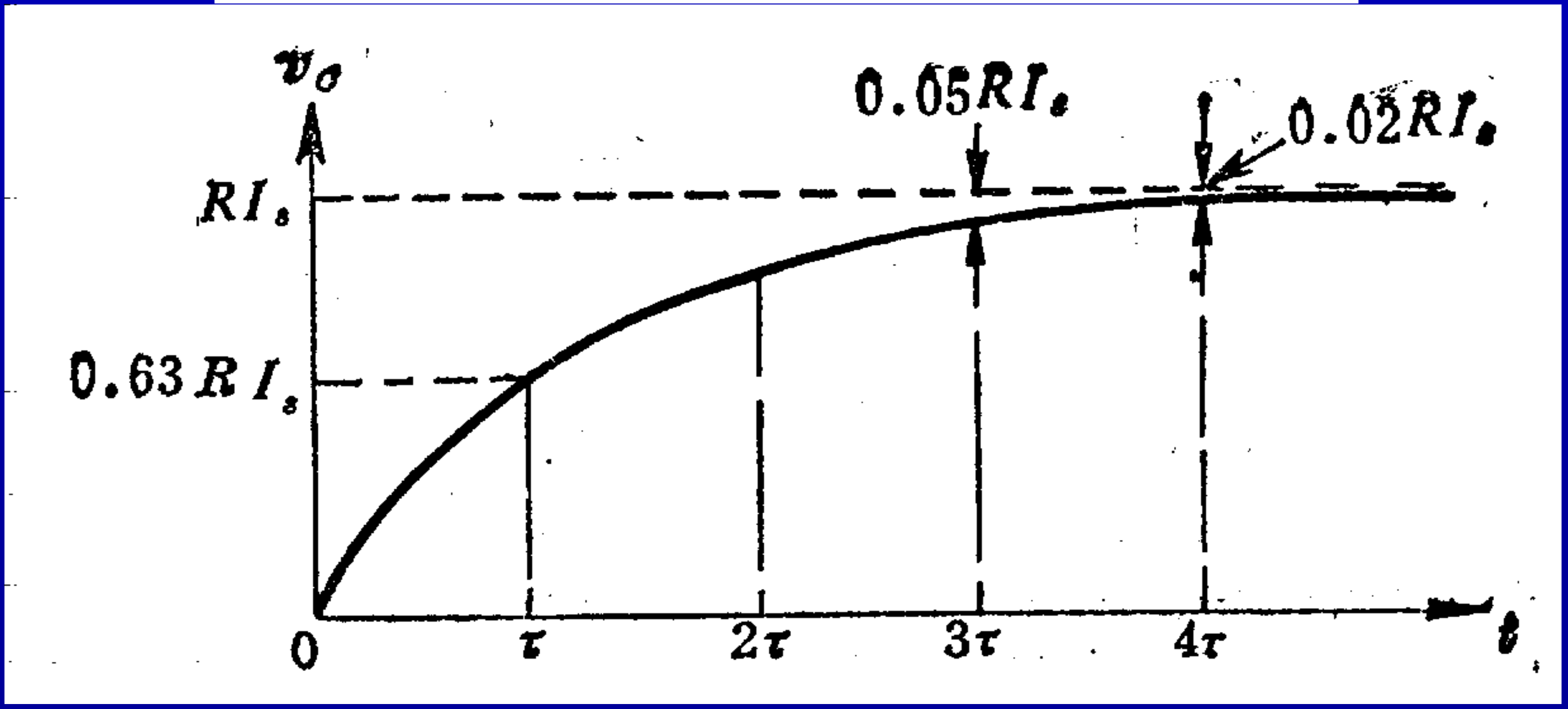
Assignments of this course

Chapter 1

Signals and Systems

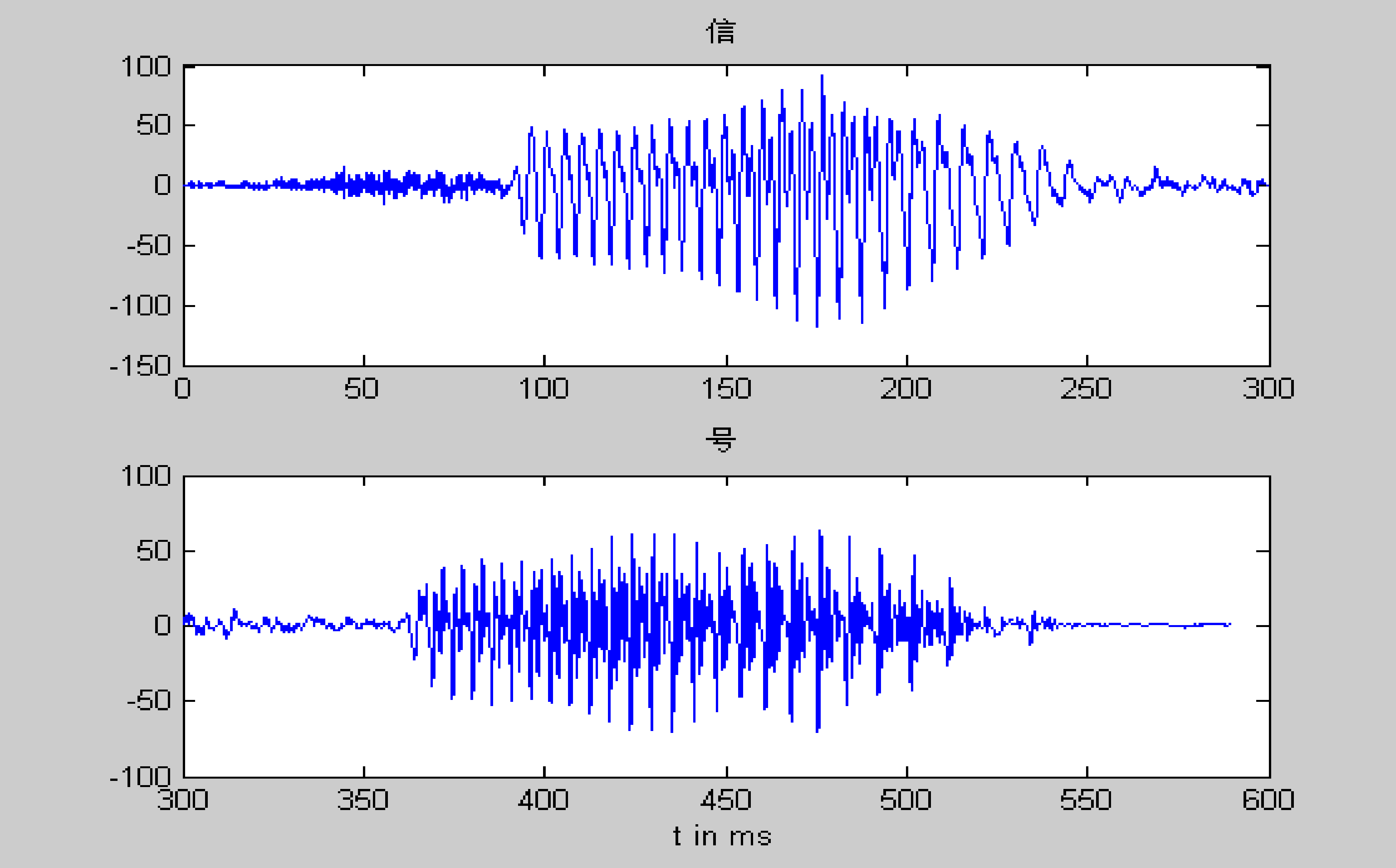
1.1 Continuous-Time and Discrete-Time Signals





Signals & Systems

EXAMPLES



EXAMPLES

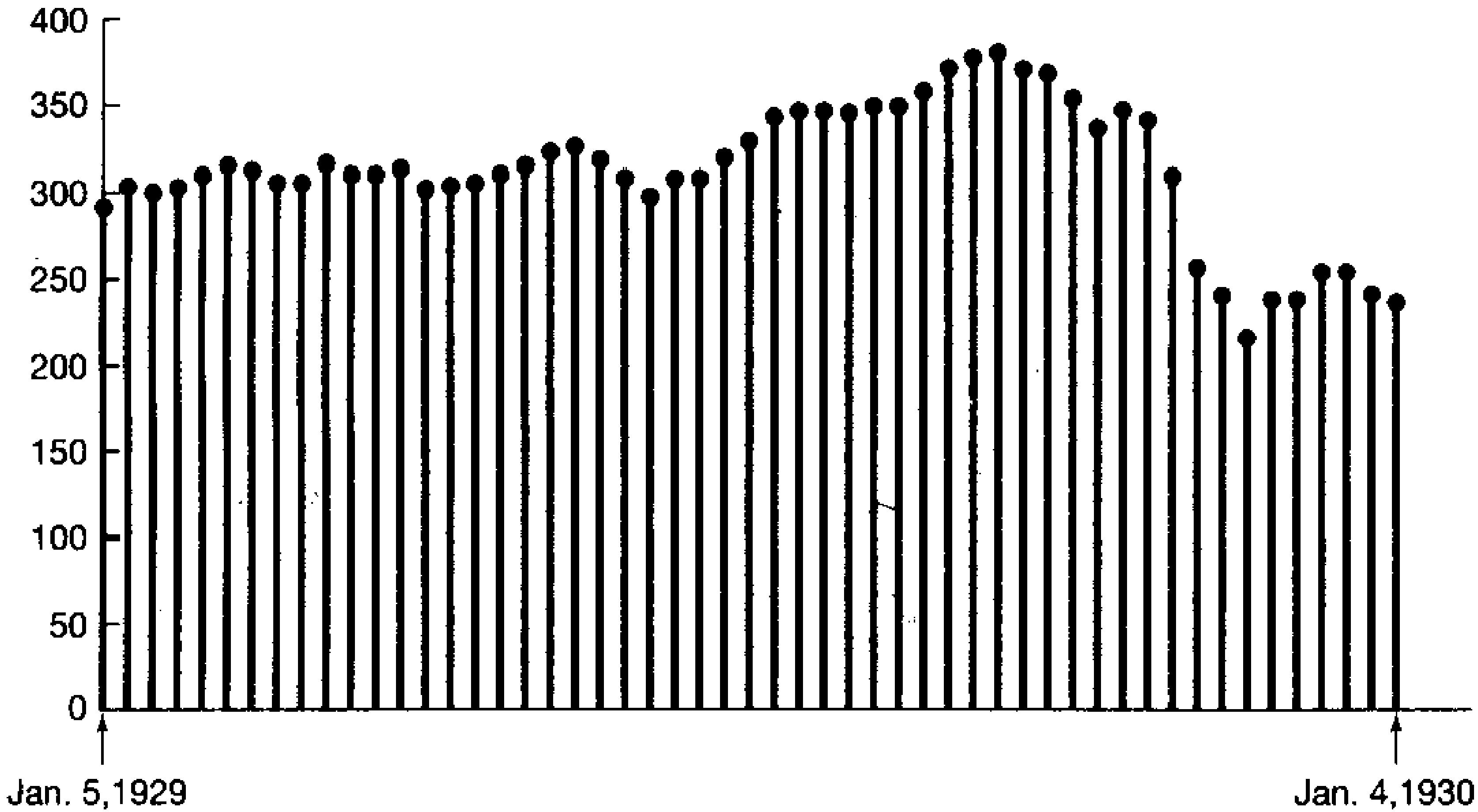


Figure 1.6 An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

Signals & Systems

EXAMPLES

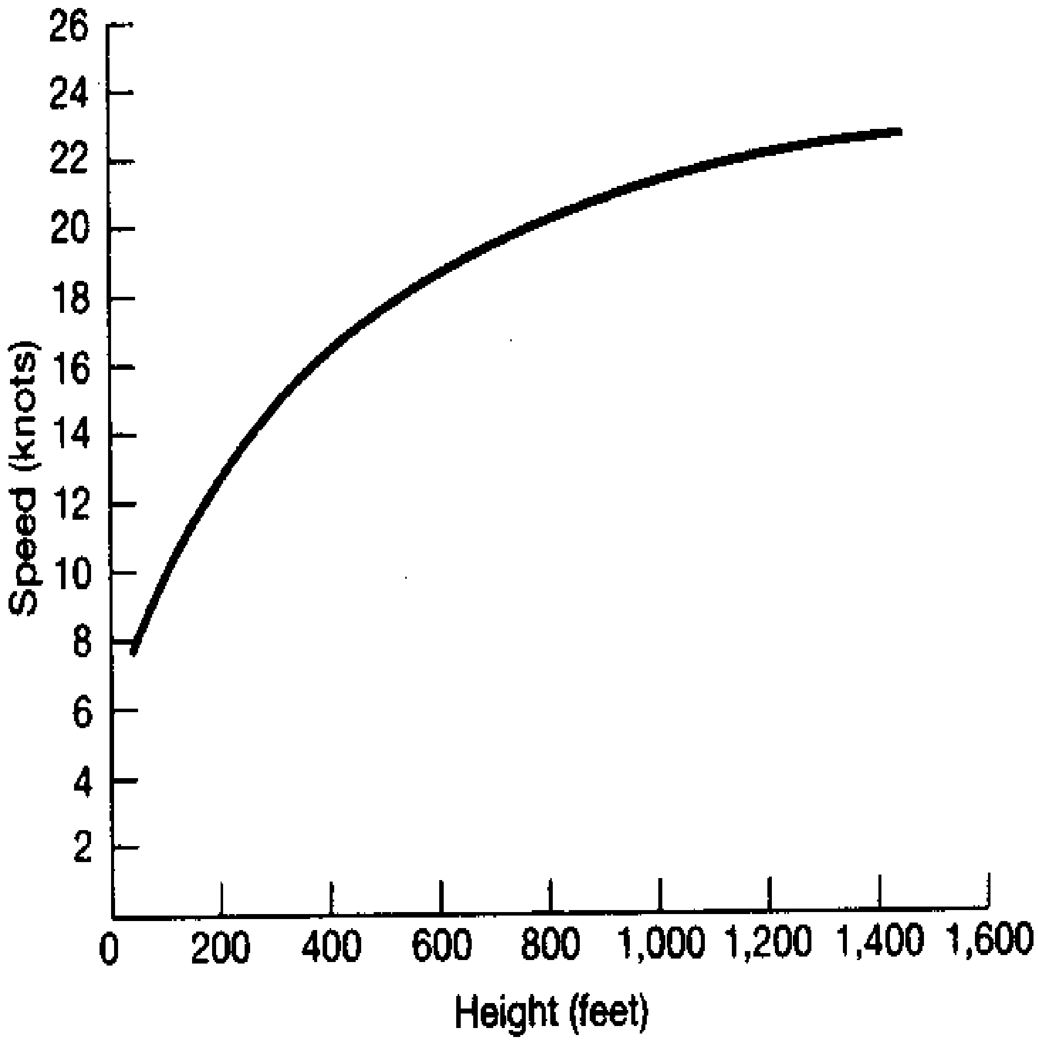
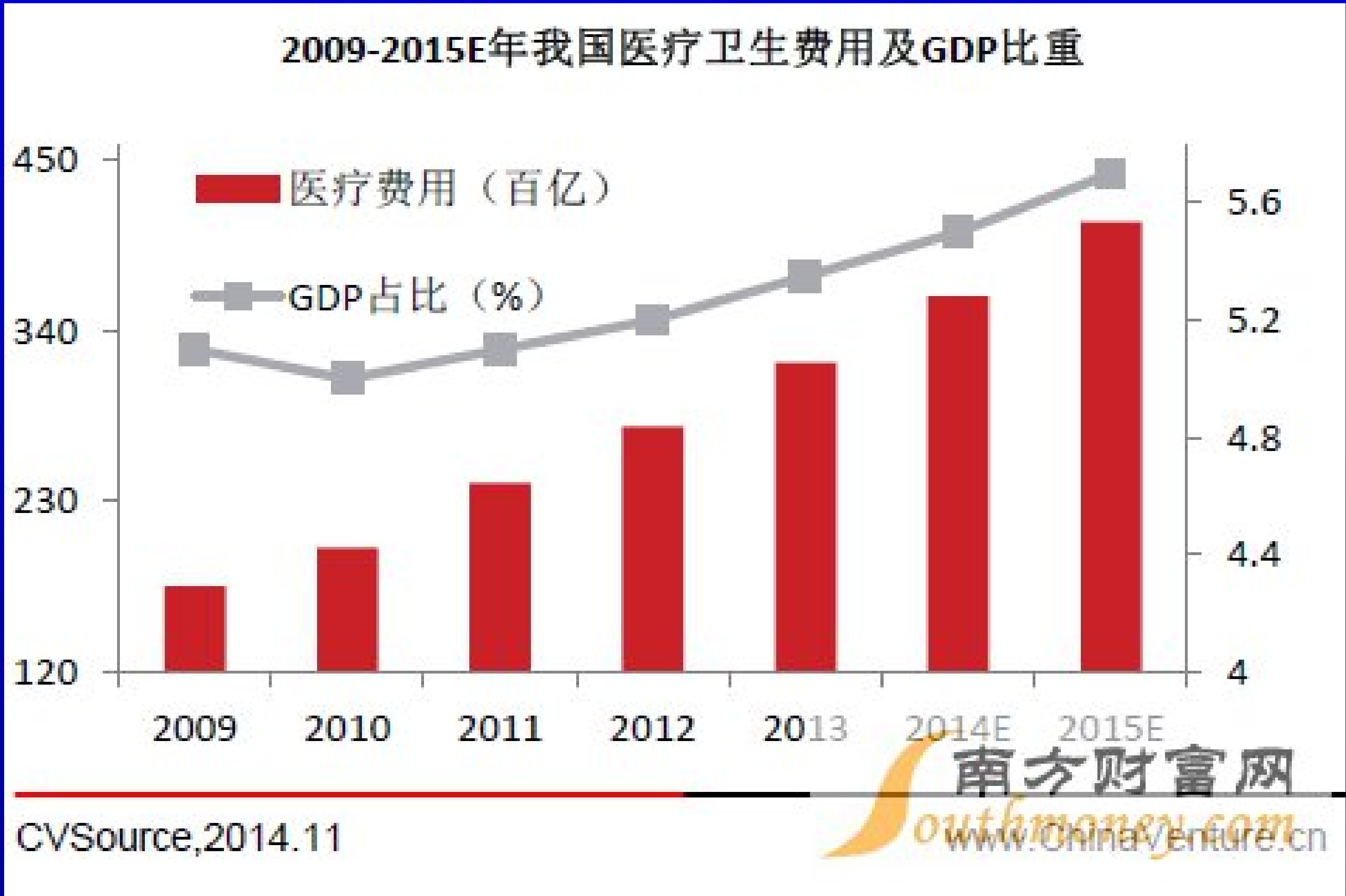


Figure 1.5 Typical annual vertical wind profile. (Adapted from Crawford and Hudson, National Severe Storms Laboratory Report, ESSA ERLTM-NSSL 48, August 1970.)

Signals & Systems

EXAMPLES

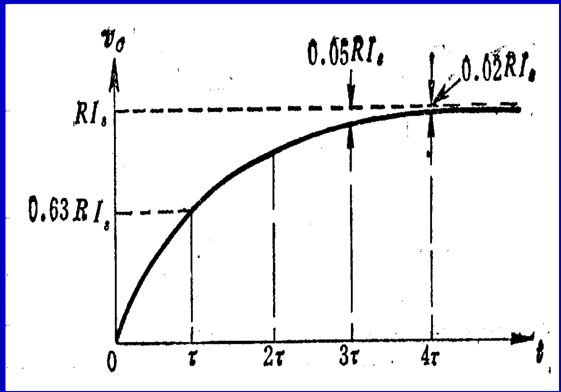


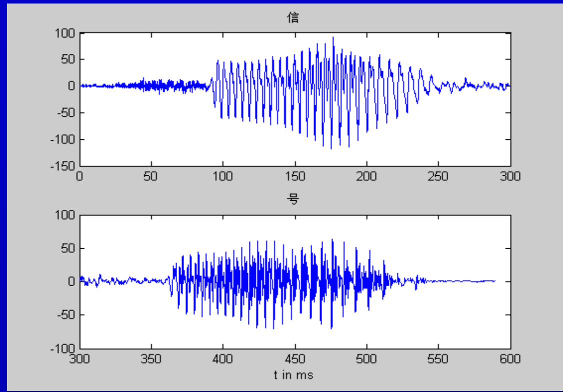
Signals & Systems

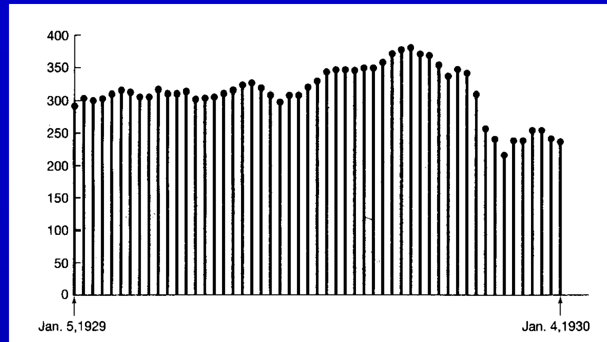
EXAMPLES

Signals & Systems

EXAMPLES







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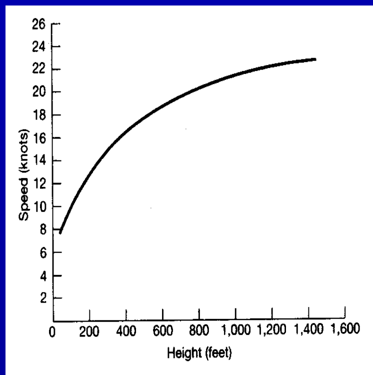

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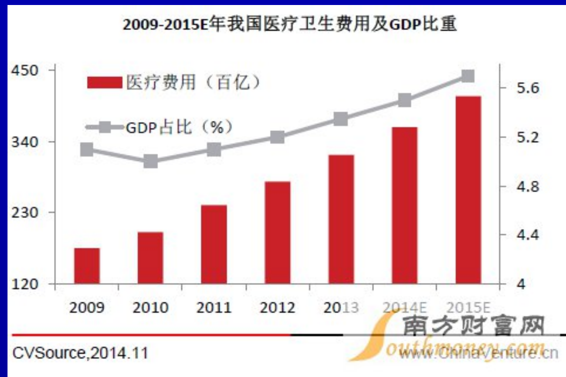
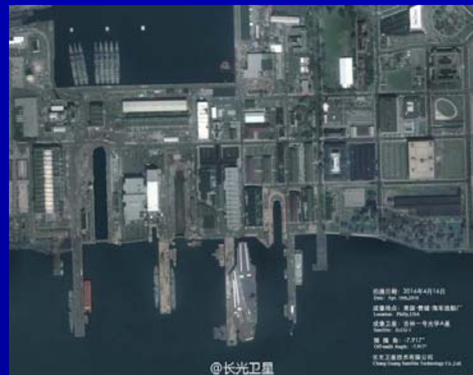
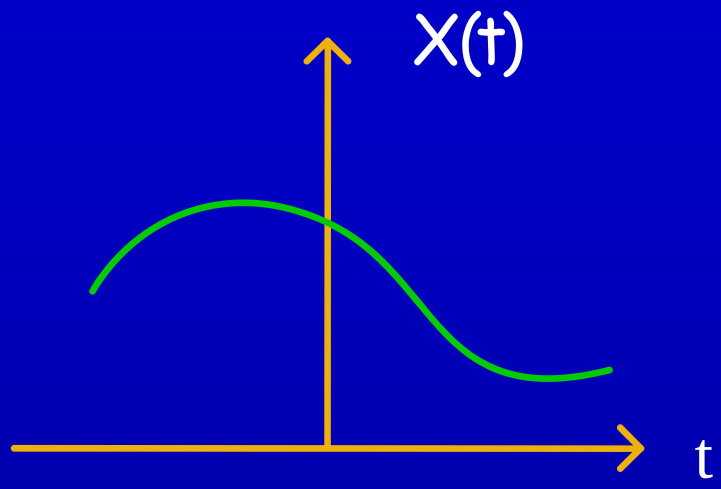




SIGNALS

- SIGNALS are functions of independent variables that carry information.
- The independent variables
  - Can be continuous
  - Can be discrete
  - Can be 1-D, 2-D, ··· N-D
- For this course: Focus on a single (1-D) independent variable which we call "time".
- Continuous-Time (CT) signals:
  - $x(t)$, t —continuous values
- Discrete-Time (DT) signals:
  - $x[n]$, n —integer values only

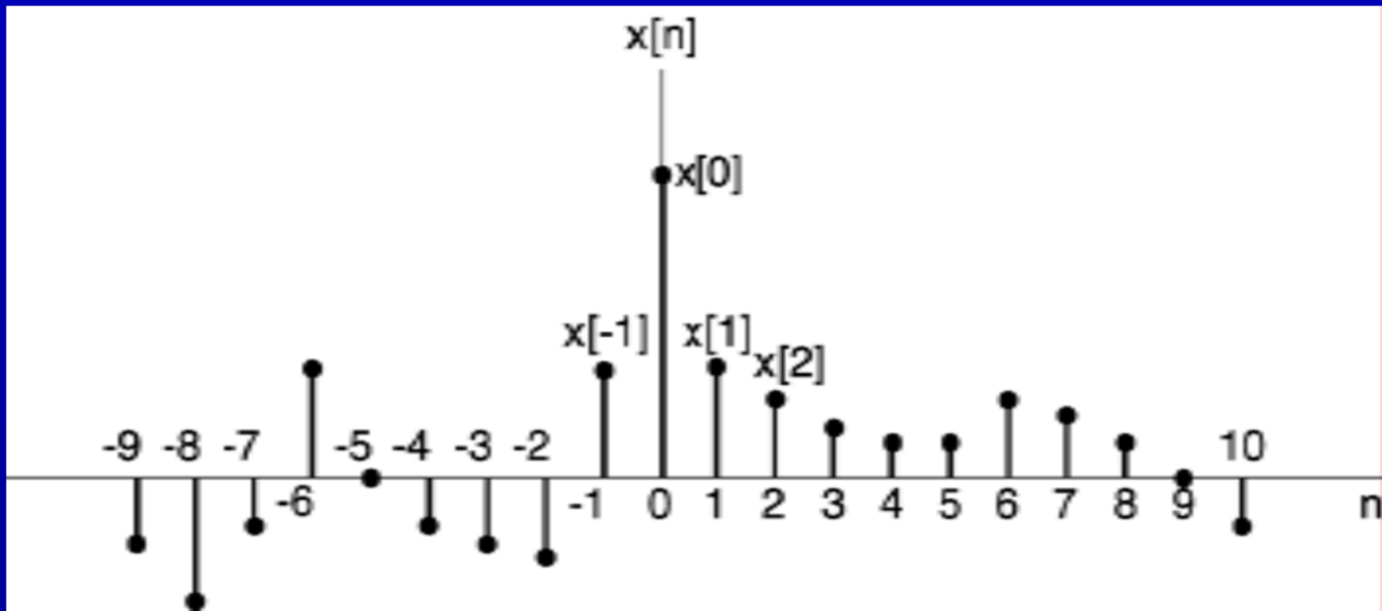
CT Signals



- Most of the signals in the physical world are CT signals
  - E.g. voltage & current, pressure, temperature, velocity, etc.

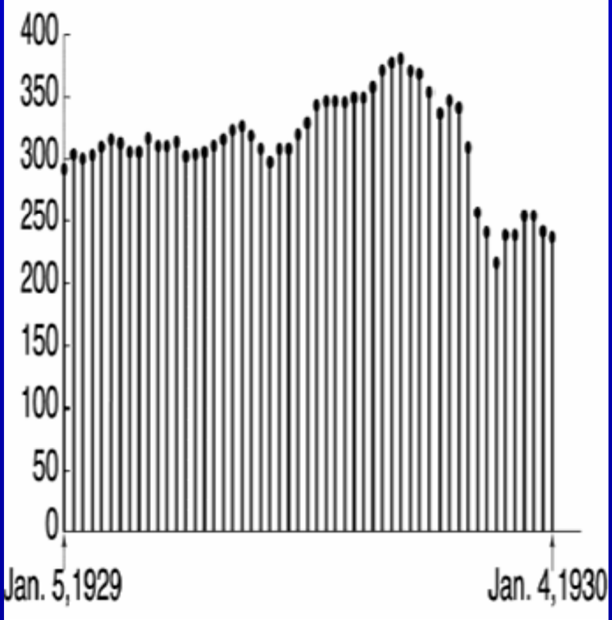
DT Signals

- x[n], n—integer, time varies discretely
- Examples of DT signals in nature:
  - DNA base sequence
  - Population of the nth generation of certain species



Many human-made DT Signals

Ex.#1Weekly Dow-Jones industrial average



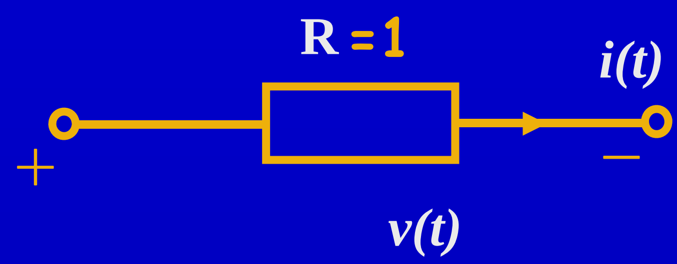
Ex.#2digital image

Why DT? —Can be processed by modern digital computers and digital signal processors (DSPs).

Signals & Systems

1.1.2 Signal Energy and Power

- Example:



Power: $$p(t)=v(t)i(t)=\frac{v^2(t)}{R}=v^2(t)$$

Energy: $$E_{12}=\int_{t_1}^{t_2}p(t)dt=\int_{t_1}^{t_2}\frac{v^2(t)}{R}dt=\int_{t_1}^{t_2}v^2(t)dt$$

Average Power: $$p_{12}=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}p(t)dt=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}\frac{v^2(t)}{R}dt$$

$$=\frac{1}{t_2-t_1}\int_{t_1}^{t_2}v^2(t)dt$$

The energy and power of signal

For Continuous-time signals

$$E = \int_{t_1}^{t_2} |x(t)|^2 \, dt$$

$$P = \frac{1}{t2 - t1} \int_{t1}^{t2} |x(t)|^2 \, dt$$

For Discrete-time signals

$$E = \sum_{n=n1}^{n2} |x[n]|^2$$

$$P = \frac{1}{n2 - n1 + 1} \sum_{n=n1}^{n2} |x[n]|^2$$

Power and energy over an infinite time interval

For Continuous-time signals

$$E_\infty = \lim_{T\to\infty}\int_{-T}^{T}|x(t)|^2\,dt$$

$$P_\infty = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}|x(t)|^2\,dt$$

For Discrete-time signals

$$E_\infty = \lim_{N\to\infty}\sum_{n=-N}^{N}|x[n]|^2$$

$$P_\infty = \lim_{N\to\infty}\frac{1}{2N+1}\sum_{n=-N}^{N}|x[n]|^2$$

Three important classes of signals

- $E_\infty < \infty \quad P_\infty = 0$ finite-energy signal

$$x(t) = \begin{cases} 1 & 0 \le t \le 1 \\ 0 & otherwise \end{cases}$$

- $E_\infty \to \infty \quad P_\infty < \infty$ finite-Power signal

$$x(t) = 4$$

- $E_\infty \to \infty \quad P_\infty \to \infty$ Infinite-energy-and-power signal

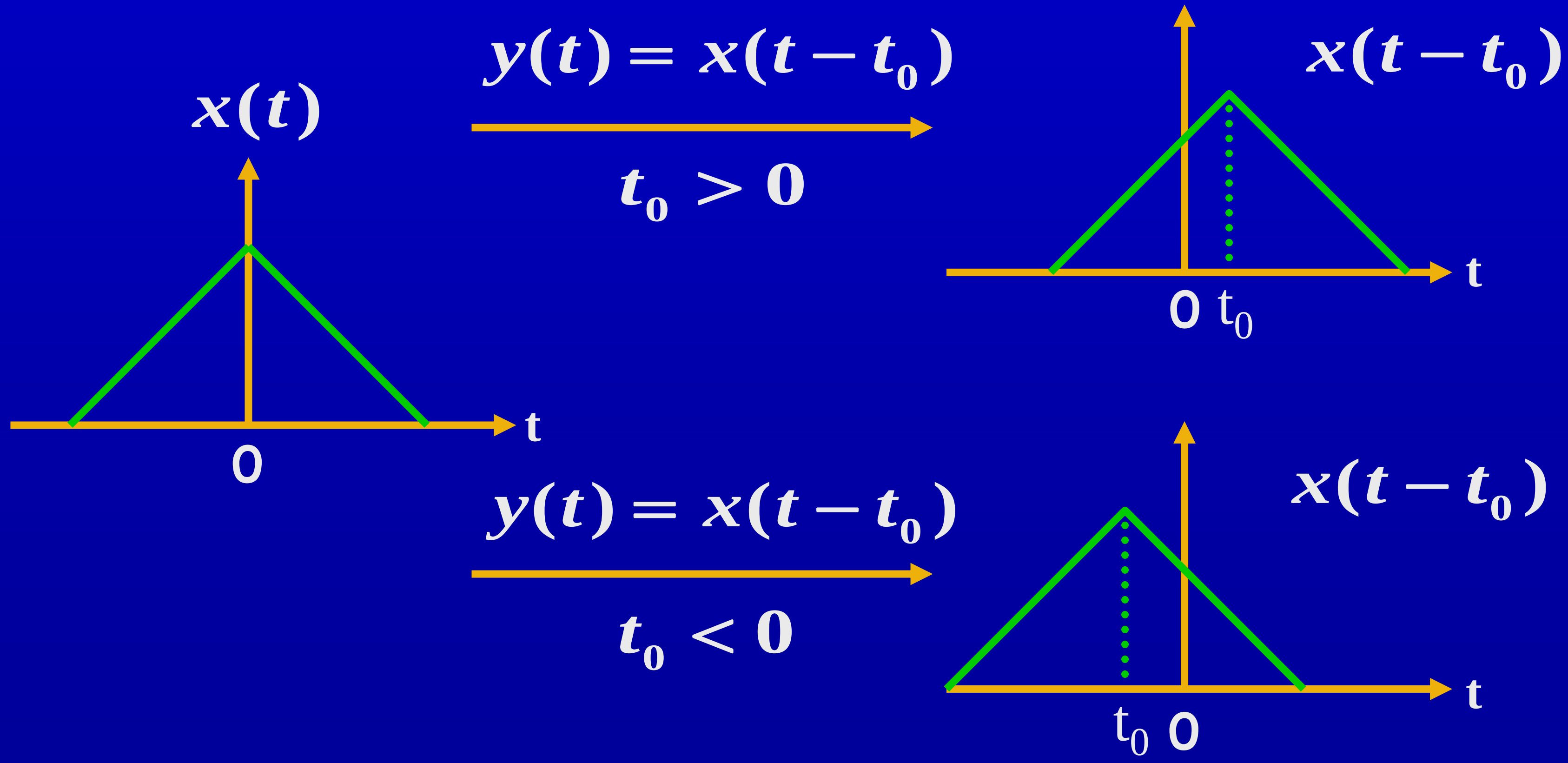
$$x(t) = t$$

1.2 Transformation of the Independent Variable

Signals & Systems

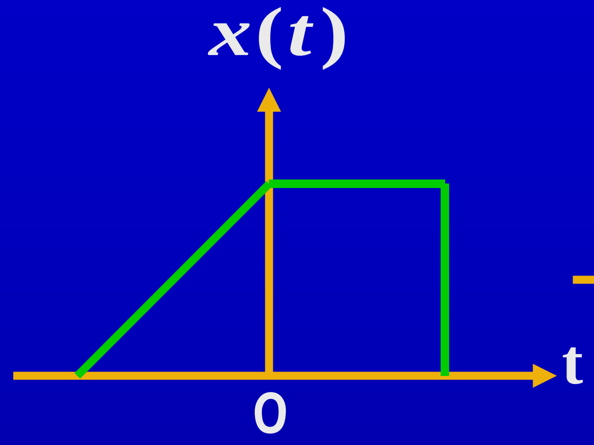
- 1.2.1 Time shift, reversal and scaling

Time shift

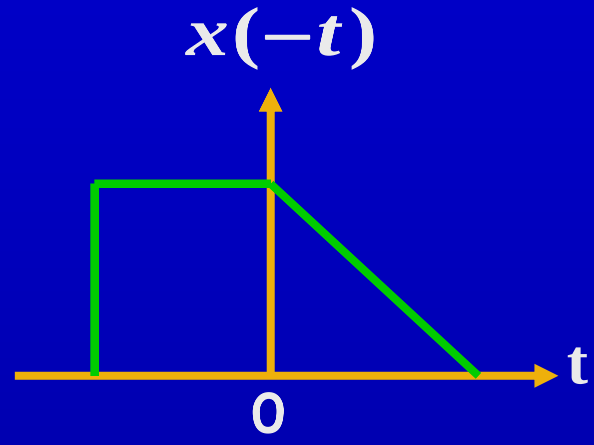


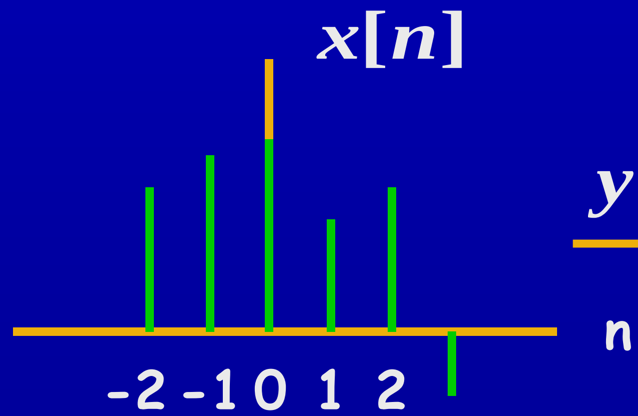
Signals & Systems

Time reversal

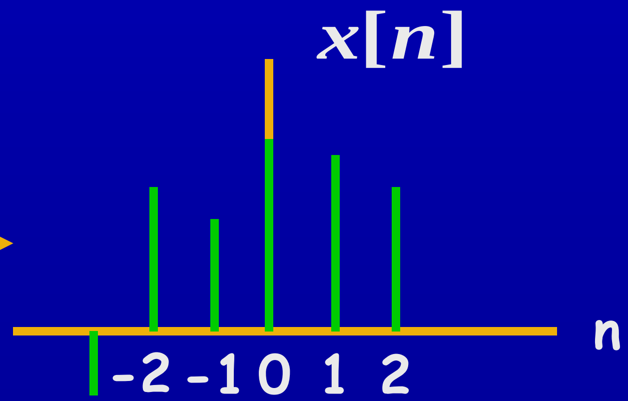


$$y(t) = x(-t)$$

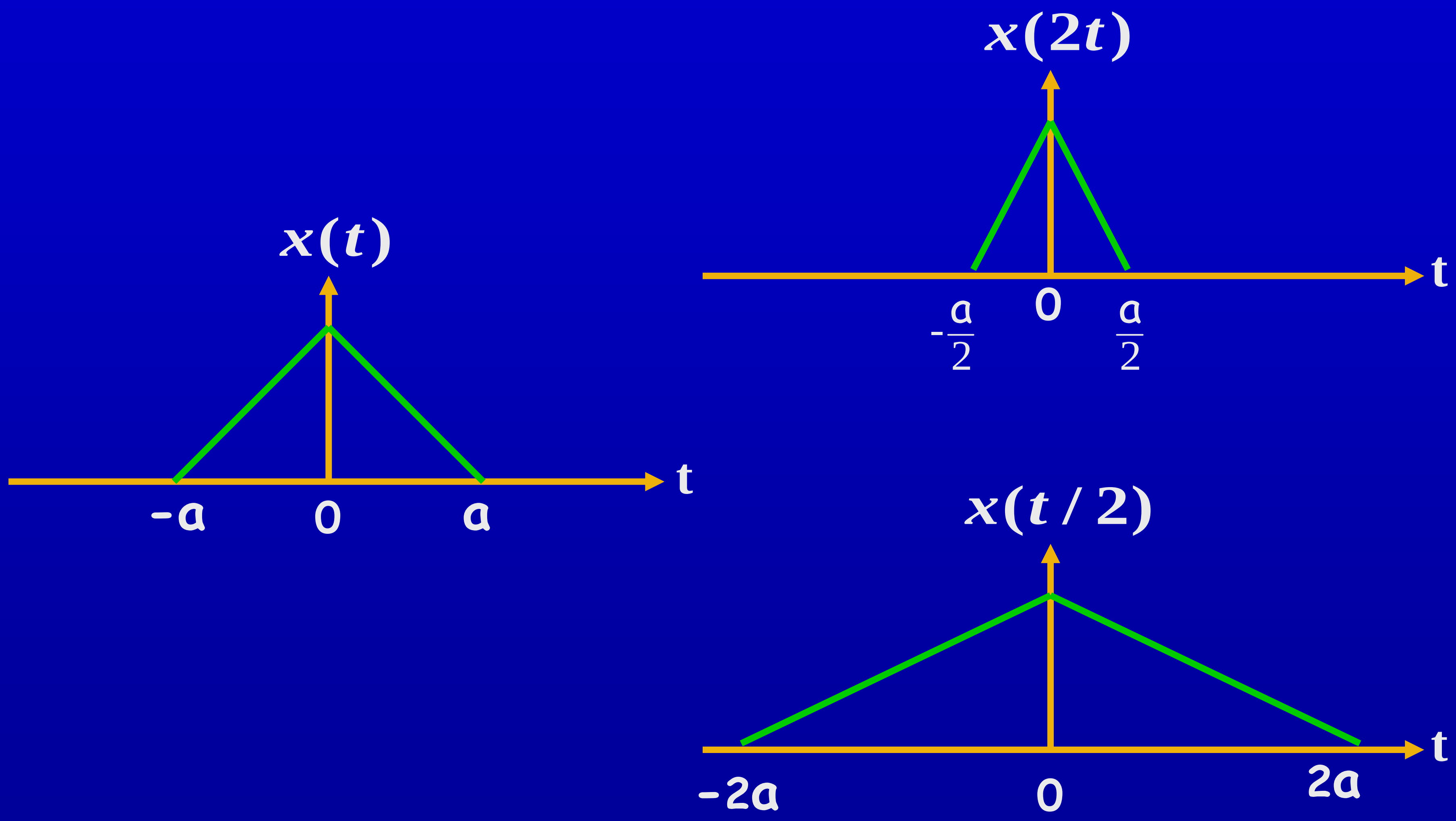




$$y[n] = x[-n]$$



Time scaling



Summary

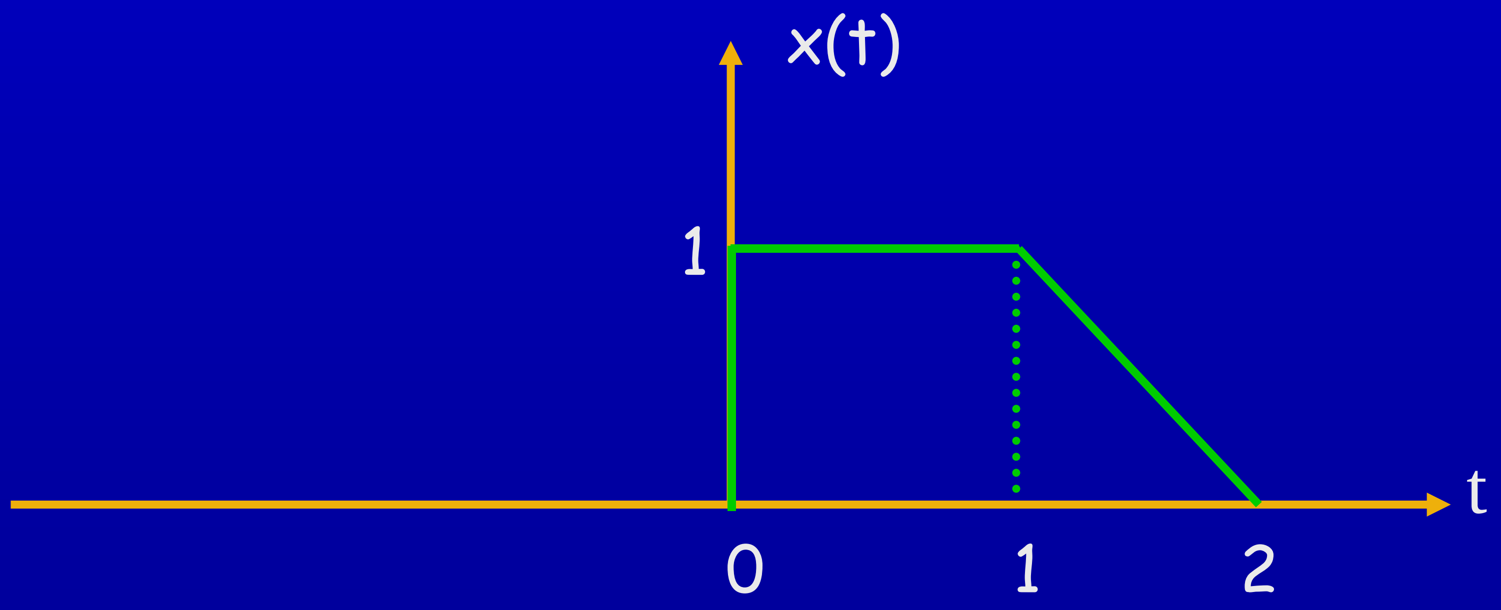
$$x(t) \rightarrow x(\alpha t + \beta)$$

$$\begin{tikzcd}[column sep=huge, row sep=huge]
x(t) \arrow[r, "{|\alpha|<1 \ \text{Stretched}}", "{|\alpha|>1 \ \text{Compressed}}"'] \arrow[d, "{\beta<0 \ \text{Delay}}", "{\beta>0 \ \text{Advanced}}"'] & x(\alpha t) \arrow[d, "{x[\alpha(t+\frac{\beta}{\alpha})]}"] \\
x(t+\beta) \arrow[r, "{|\alpha|>1}", "{|\alpha|<1}"'] & x(\alpha t+\beta)
\end{tikzcd}$$

Signals & Systems

Example 1.1

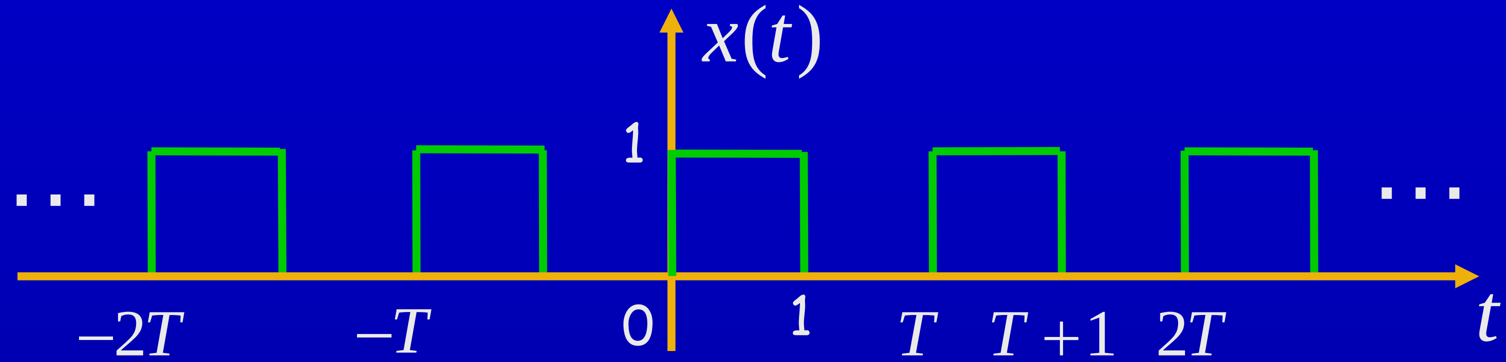
Given the signal $x(t)$ shown as below, please determine the signal $x(-t+1)$, $x(-t+\tau)$ and $x(\frac{3}{2}t+1)$



1.2.2 Periodic signals

Signals & Systems

- The periodic of Continuous-time signal



- The definition of CT periodic signal

$$x(t)=x(t+\underline{T})$$

period

$$x(t)=x(t+T)=x(t+2T)=...=x(t+mT)$$

Signals & Systems

The periodic of CT signal

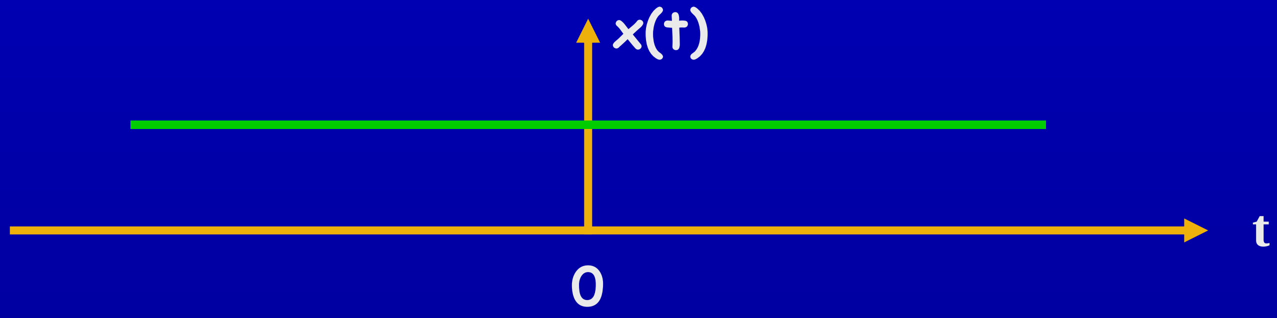
- Fundamental period

$$x(t)=x(t+mT) \qquad m \text{ is integer}$$

fundamental period:

the smallest positive value of period

Exception:



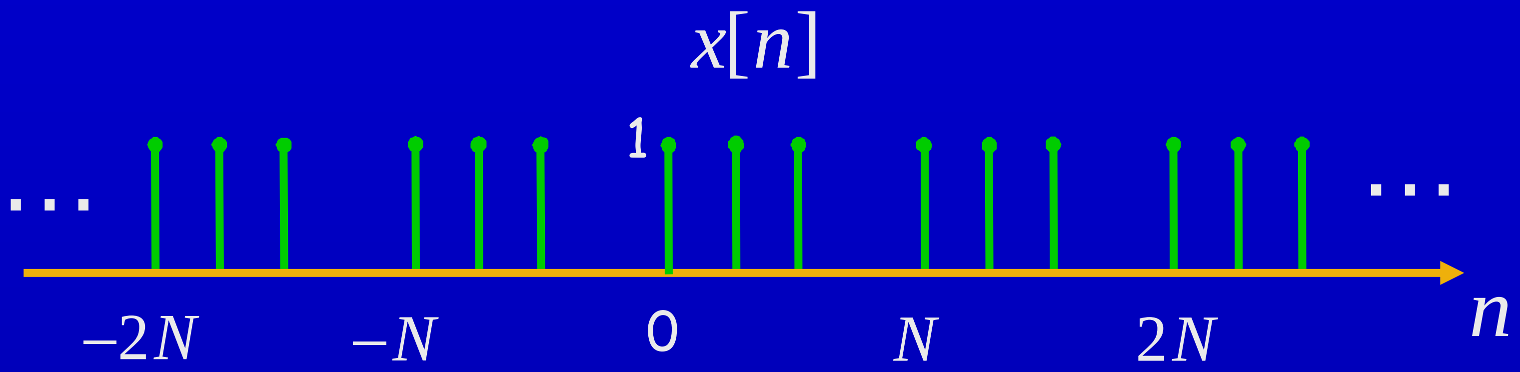
Period? any value ($T \neq 0$)

Fundamental period?

no smallest positive value

Signals & Systems

The periodic of DT signal



- **The definition of periodic signal**

  x[n]=x[n+N] n,N are integer

  period

x[n]=x[n+N]=x[n+2N]=... =x[n+mN]

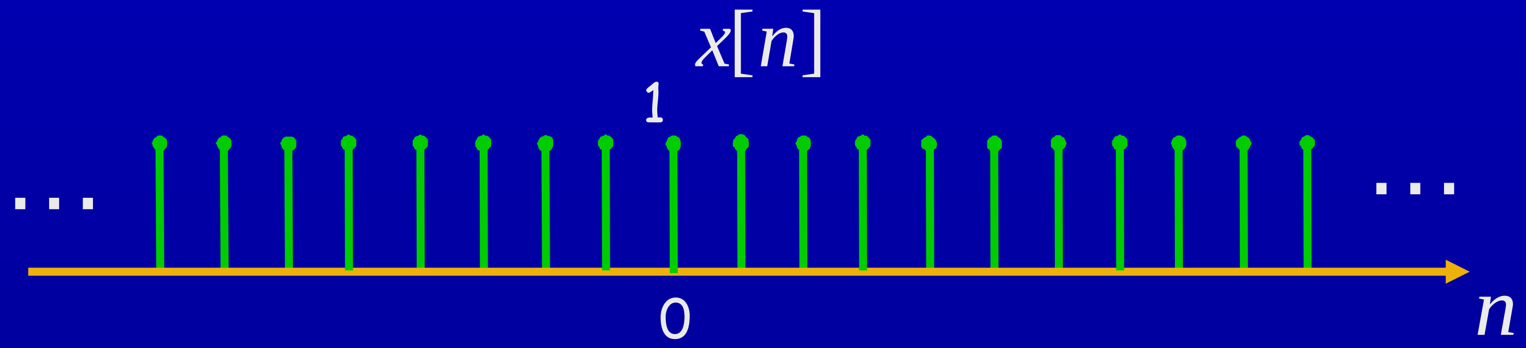
The periodic of DT signal

- Fundamental period

$x[n]=x[n+mN_0]$ n m N_0 are integer

fundamental period:

the smallest positive value of period



period is any integer

the fundamental period is 1

Signals & Systems

Example 1.4

$$x(t)=\begin{cases}\cos(t)\ ,\ \mathrm{t}<0\\ \sin(t)\ ,\ \mathrm{t}>0\end{cases}\qquad \textit{is the signal periodic?}$$

x(t) is discontinuity in t=0,

x(t) is aperiodic

Discontinuity → aperiodic?

Signals & Systems

1.2.3 Even and Odd Signals

Even signal

$$x(-t) = x(t)$$

$$x[-n] = x[n]$$

Odd signal

$$x(-t) = -x(t)$$

$$x[-n] = -x[n]$$

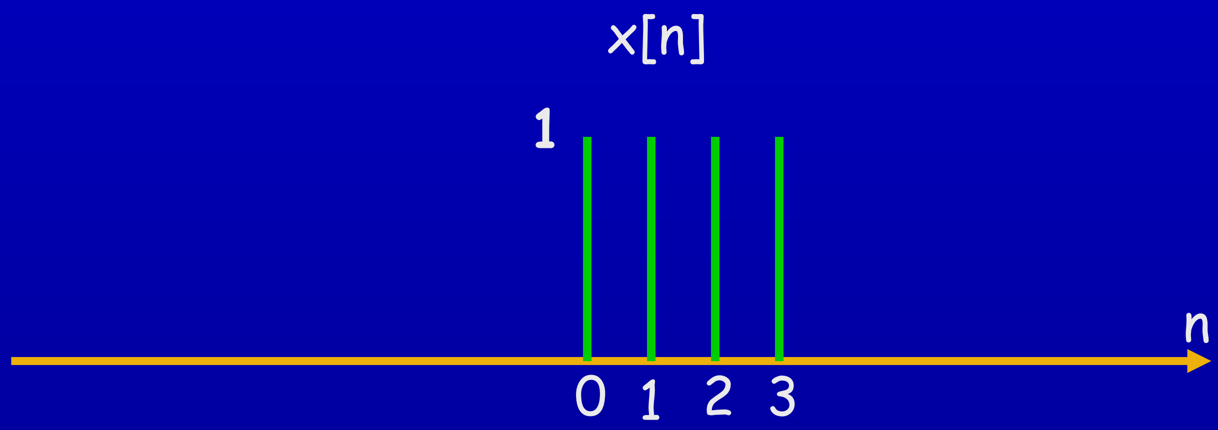
$$x(t) = Ev\{x(t)\} + Od\{x(t)\}$$

$$Ev\{x(t)\} = \frac{x(t) + x(-t)}{2}$$

$$Od\{x(t)\} = \frac{x(t) - x(-t)}{2}$$

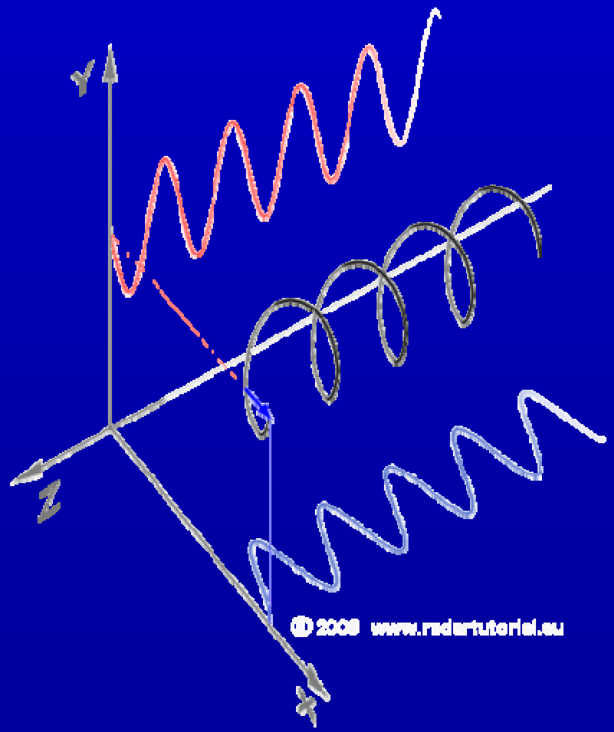
Example

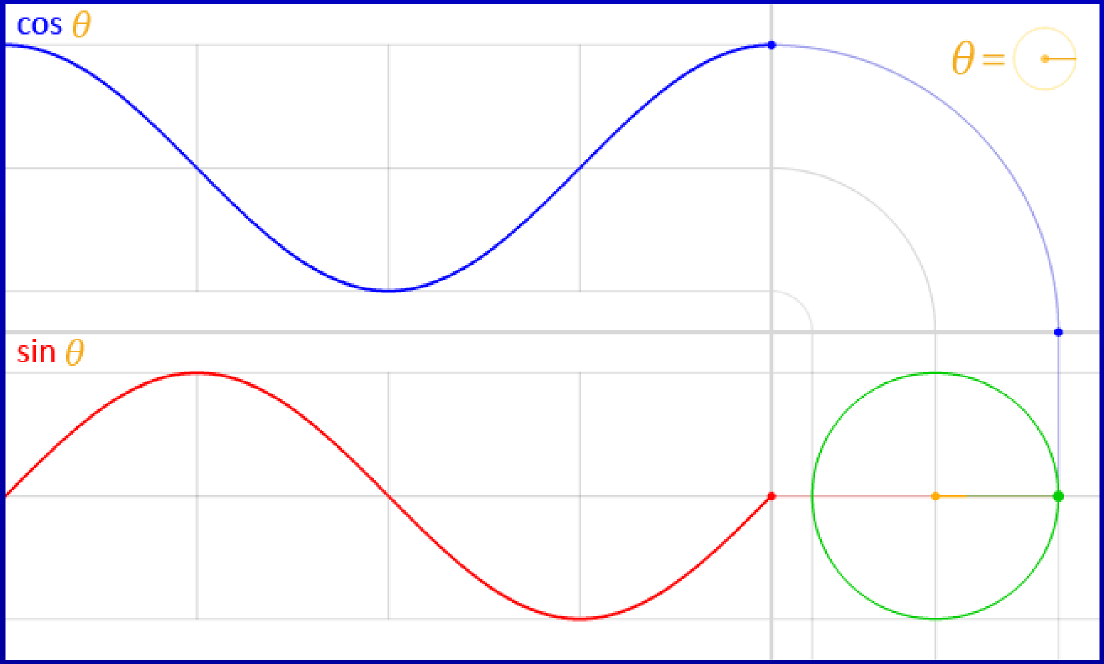
x[n] as follow, determine the even part and the odd part of it



1.3 Exponential and Sinusoidal Signal

1.3.1 Continuous-Time Complex Exponential and Sinusoidal signal





Continuous-Time Complex Exponential

$x(t)=Ce^{st}$ C and s are complex

$e = 2.71828182845...$

C and s are real

x(t) is real exponential signal

C is real and s is pure imaginary

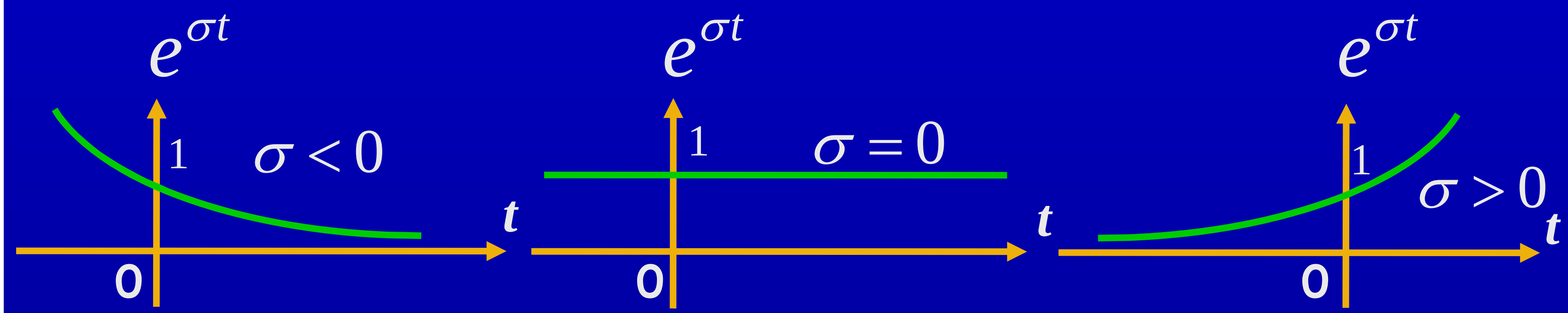
x(t) is periodic complex exponential and sinusoidal signals

C and s are complex

x(t) is general complex exponential signals

Real exponential signals

$$x(t)=Ce^{st} \overset{s=\sigma+j\omega}{\underset{C=1}{=\!=\!=\!=}} e^{(\sigma+j\omega)t} \overset{\omega=0}{=\!=\!=\!=} e^{\sigma t}$$



Decaying　　Constant　　Growing

Radioactive decay

the response of RC circuit

Atomic explosion

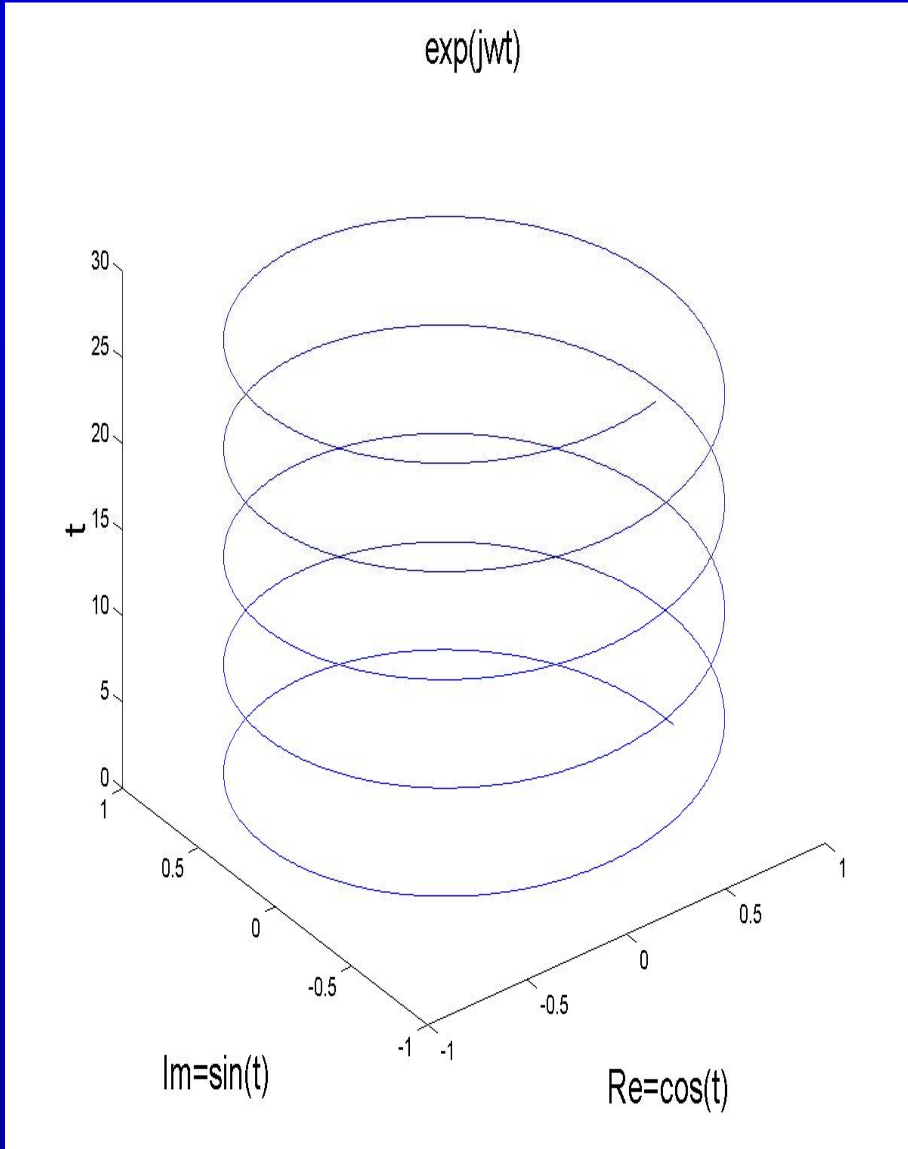
complex chemical reaction

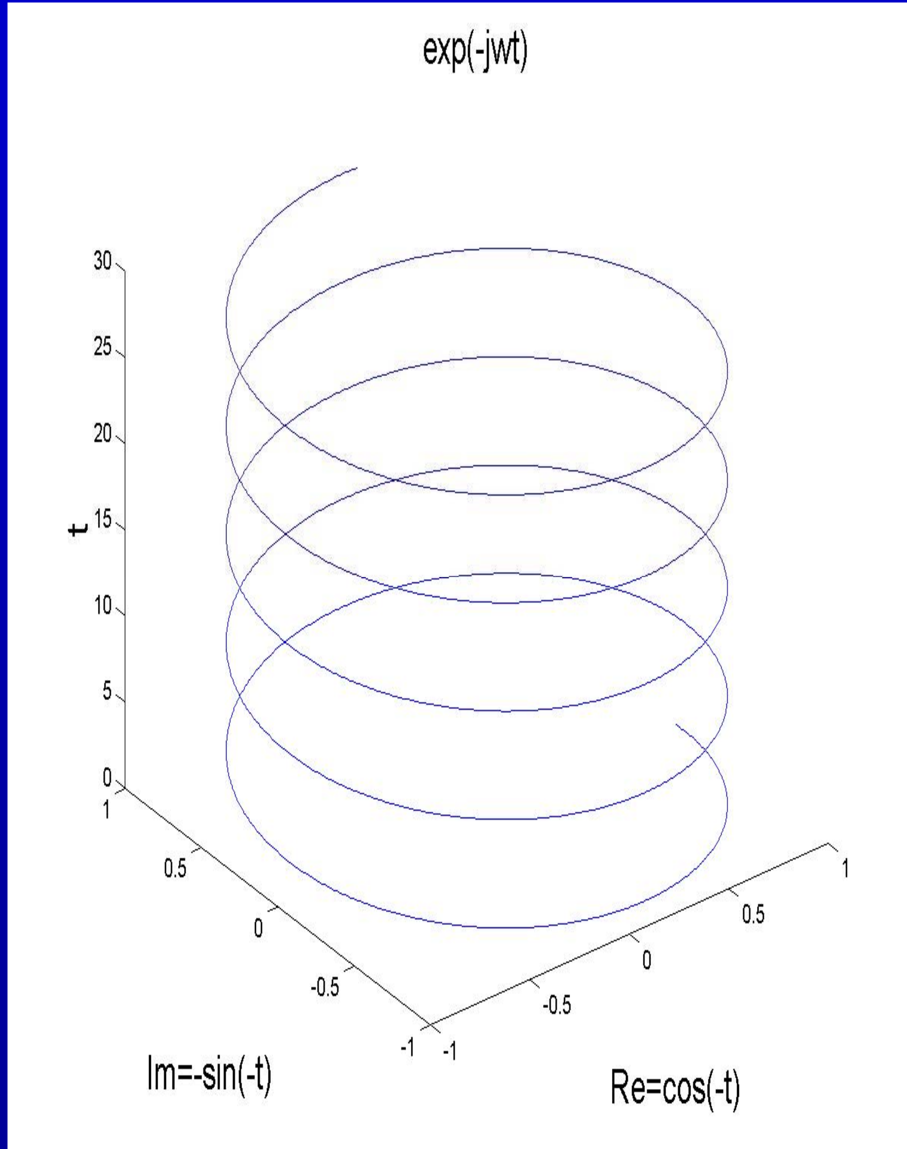
Periodic complex exponential and sinusoidal signal

- Complex exponential signal

$$x(t) = Ce^{(\sigma + j\omega)t} \xrightarrow[\sigma = 0 \quad \omega = \omega_0]{C=1} e^{j\omega_0 t}$$

Periodic complex exponential signal





Periodic complex exponential signal

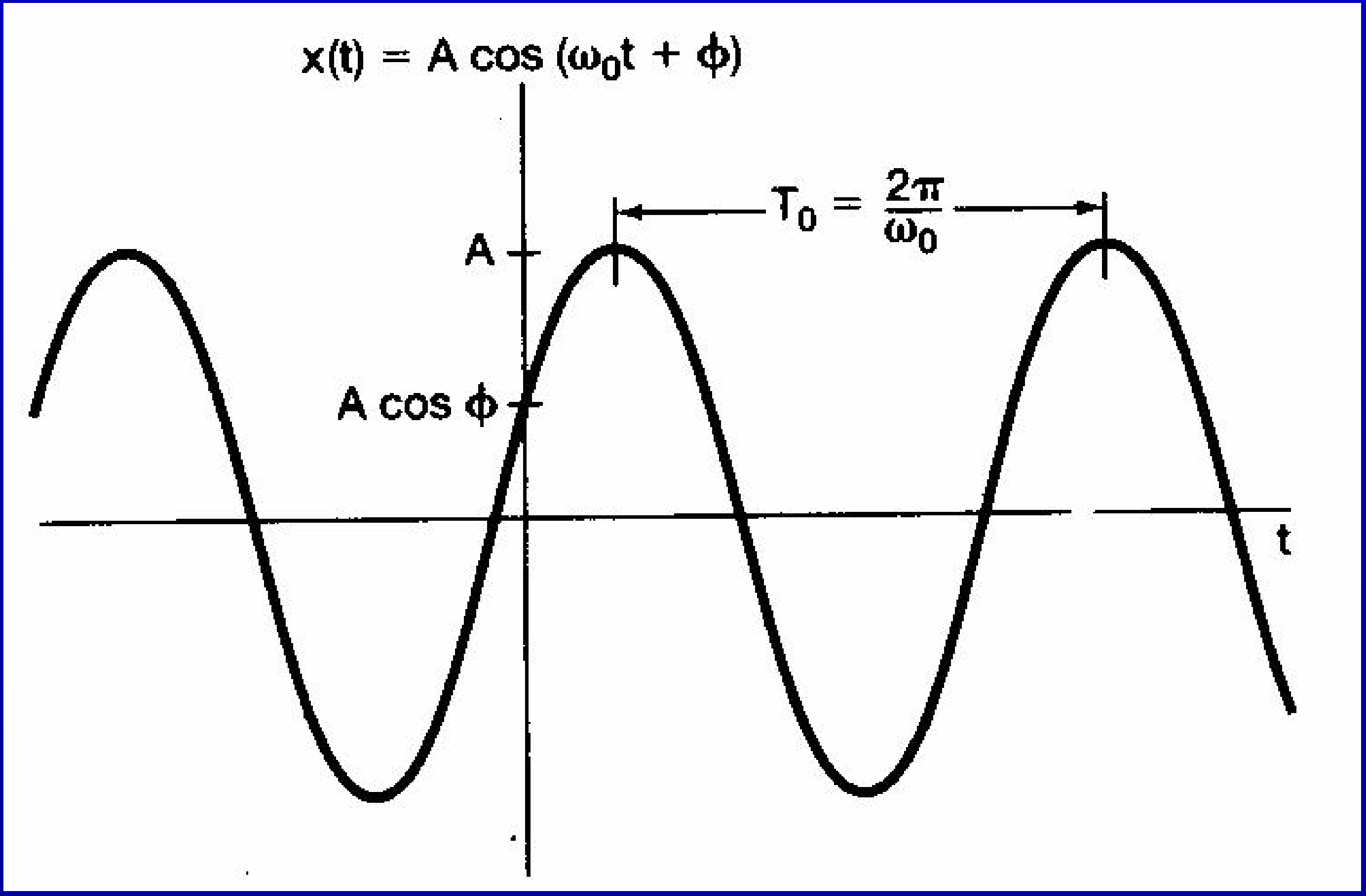
$$x(t) = e^{j\omega_0 t}$$

Fundamental frequency: ω_0

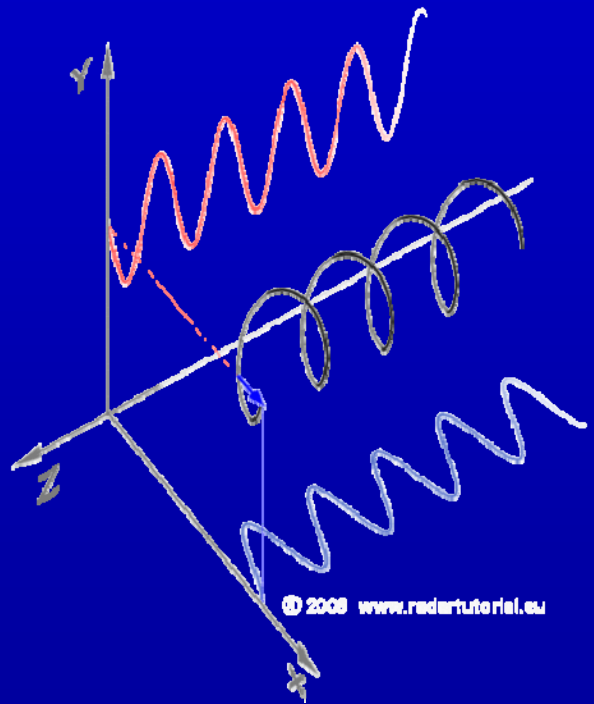
Fundamental period: $T_0 = \dfrac{2\pi}{|\omega_0|}$

Sinusoidal signal

$$x(t) = A\cos(\omega_0 t + \varphi)$$



Euler relation



$$\begin{cases} e^{jw_0t} = \cos w_0t + j\sin w_0t \\ e^{-jw_0t} = \cos w_0t - j\sin w_0t \end{cases}$$

$$\begin{cases} \cos w_0t = \dfrac{e^{jw_0t} + e^{-jw_0t}}{2} \\ \sin w_0t = \dfrac{e^{jw_0t} - e^{-jw_0t}}{2j} \end{cases}$$

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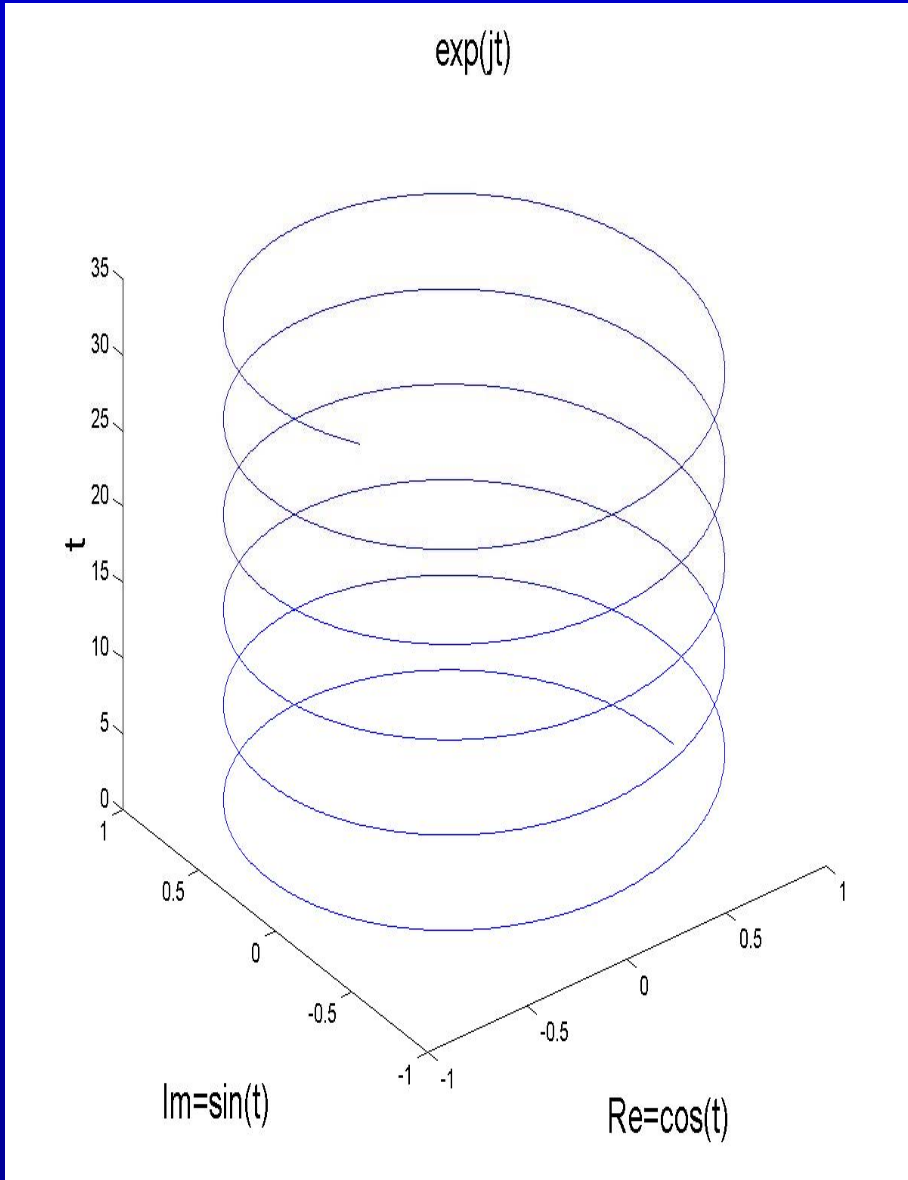
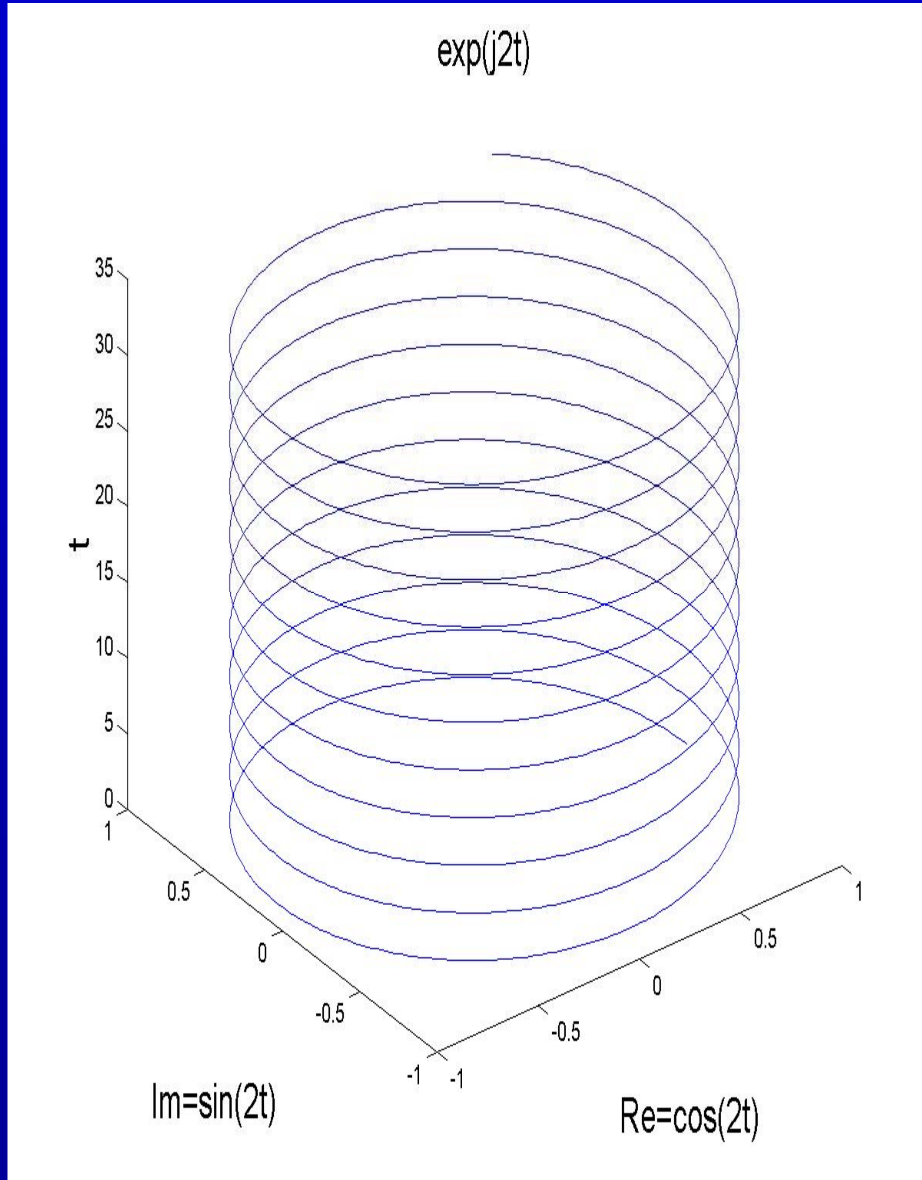
Periodic Complex exponential signal





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Periodic Complex exponential signal

$$\ldots \; e^{-j3w_0t} \; e^{j-2w_0t} \; e^{j-w_0t} \; e^{j0w_0t} \; e^{jw_0t} \; e^{j2w_0t} \; e^{j3w_0t} \; \ldots$$

Harmonically related complex exponential signal

$$\varphi_k(t) = e^{jkw_0t} \qquad k = 0, \pm 1, \pm 2 \ldots$$

Fundamental period: $\dfrac{2\pi}{|k|w_0} = \dfrac{T_0}{|k|}$

Fundamental frequency: $|k|w_0$

General complex exponential signal

$$x(t) = ce^{st}$$

let $c = |c|e^{j\theta}$ and $s = \sigma + jw_0$

$$x(t) = ce^{st}$$

$$= |c|e^{j\theta}e^{(\sigma + jw_0)t}$$

$$= |c|e^{\sigma t}e^{j(w_0 t + \theta)}$$

$$= |c|e^{\sigma t}\cos(w_0 t + \theta) + j|c|e^{\sigma t}\sin(w_0 t + \theta)$$

General complex exponential signal

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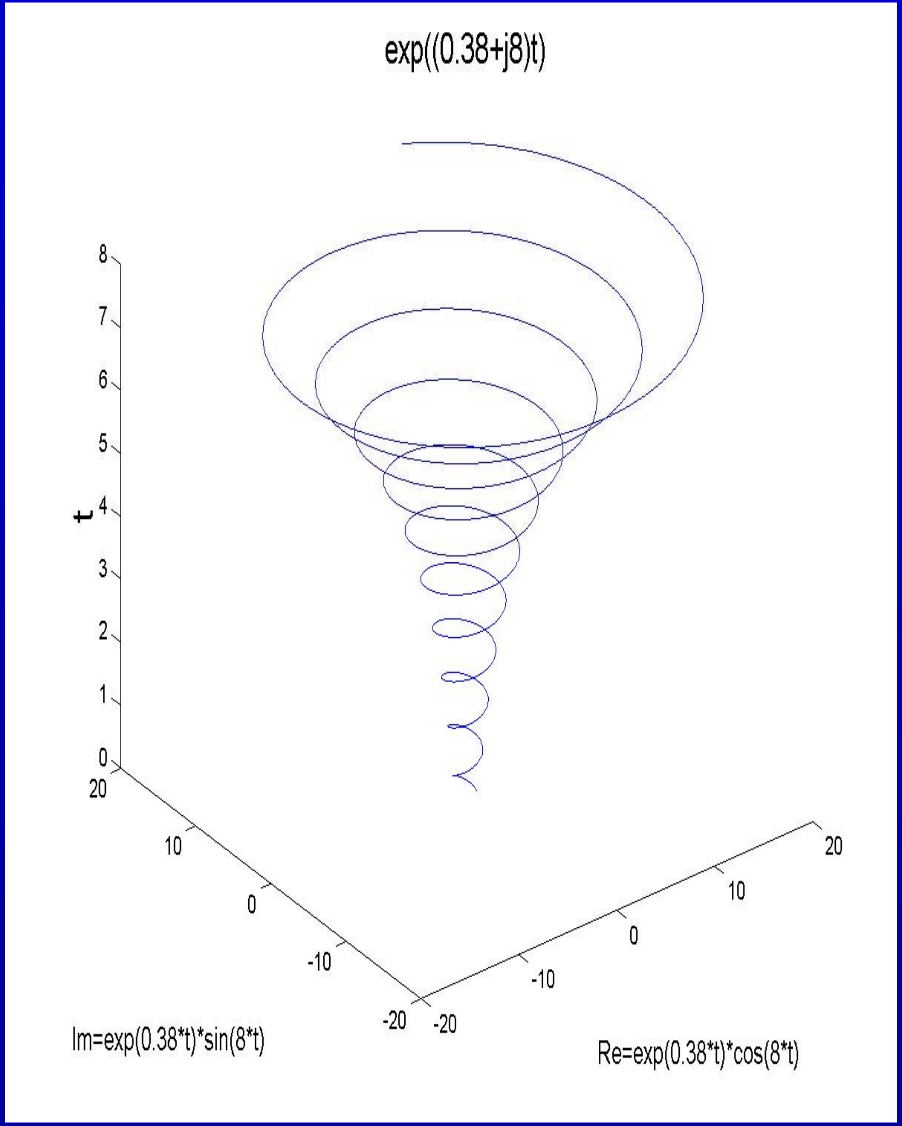

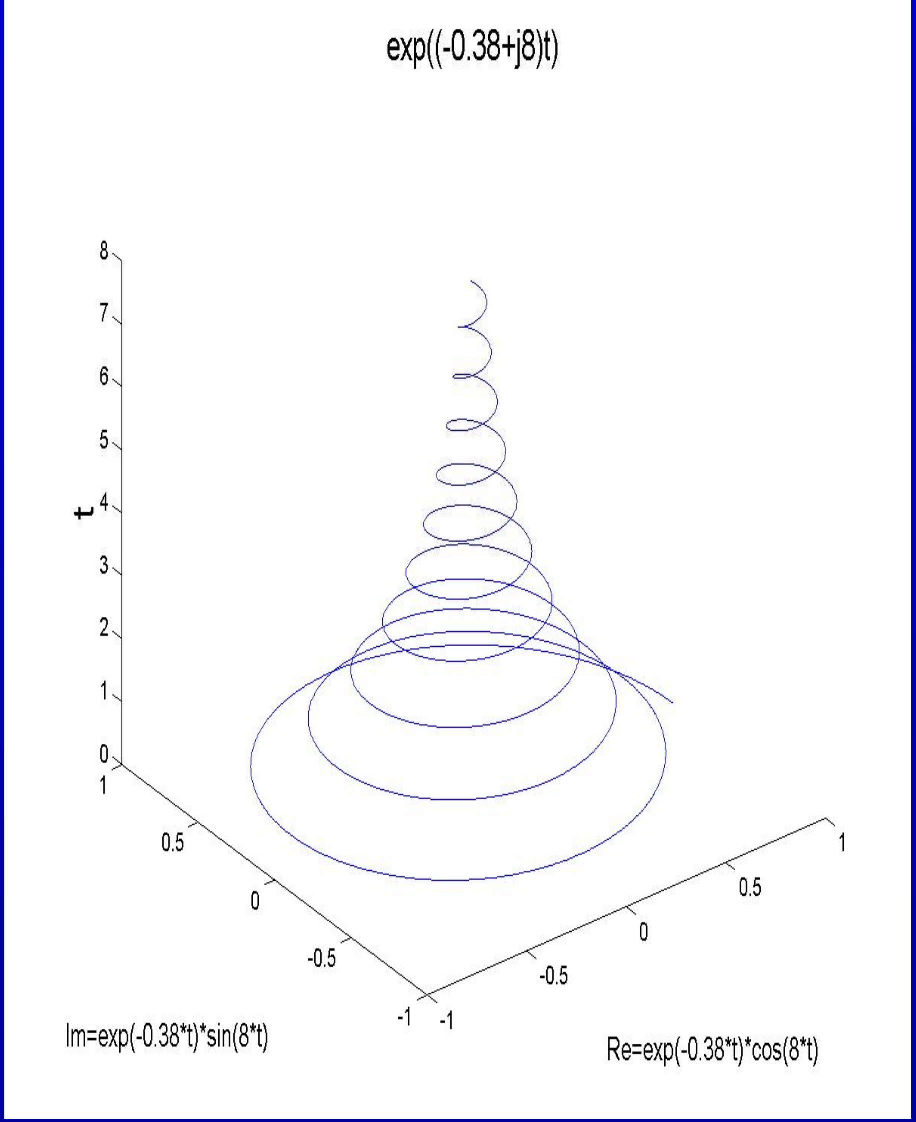

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1.3.2 Discrete-Time Complex Exponential and Sinusoidal signal

$$x[n] = C\alpha^n \xrightarrow[C=|C|e^{j\theta}]{\alpha=|\alpha|e^{j\omega_0}} |C|\cdot|\alpha|e^{j(\omega_0 n+\theta)}$$

C and a are real

x[n] is real exponential signal

C is real and a $=e^{jw_0}$

x[n] is periodic complex exponential and sinusoidal signal

C and a are complex

x[n] is general complex exponential signal

1.3.3 Periodicity properties of DT Complex Exponential signal

- Complex exponential signal

$$x[n] = Ca^n \xrightarrow[a = e^{j\omega_0}]{C=1} e^{j\omega_0 n}$$

Complex exponential signal

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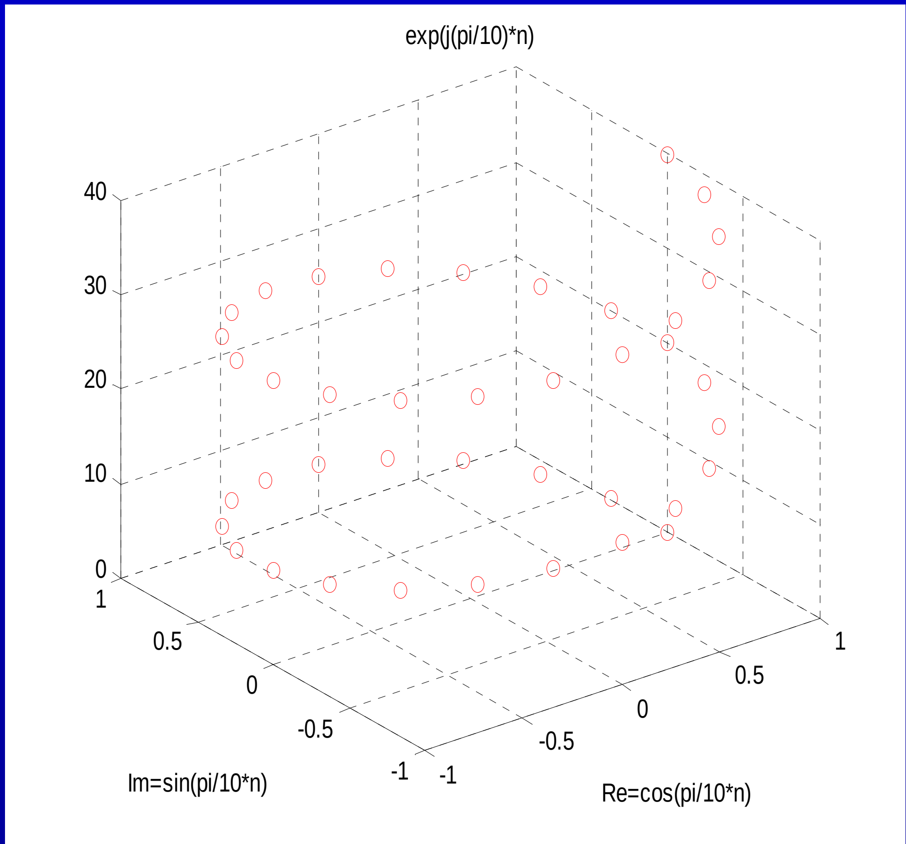

Complex exponential signal

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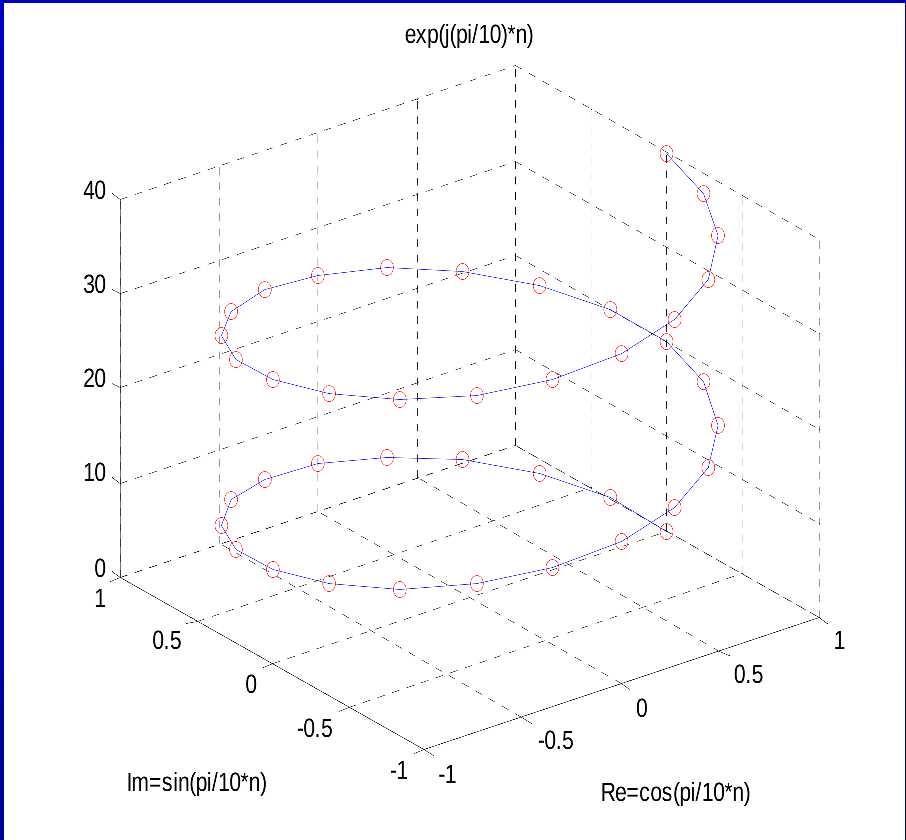

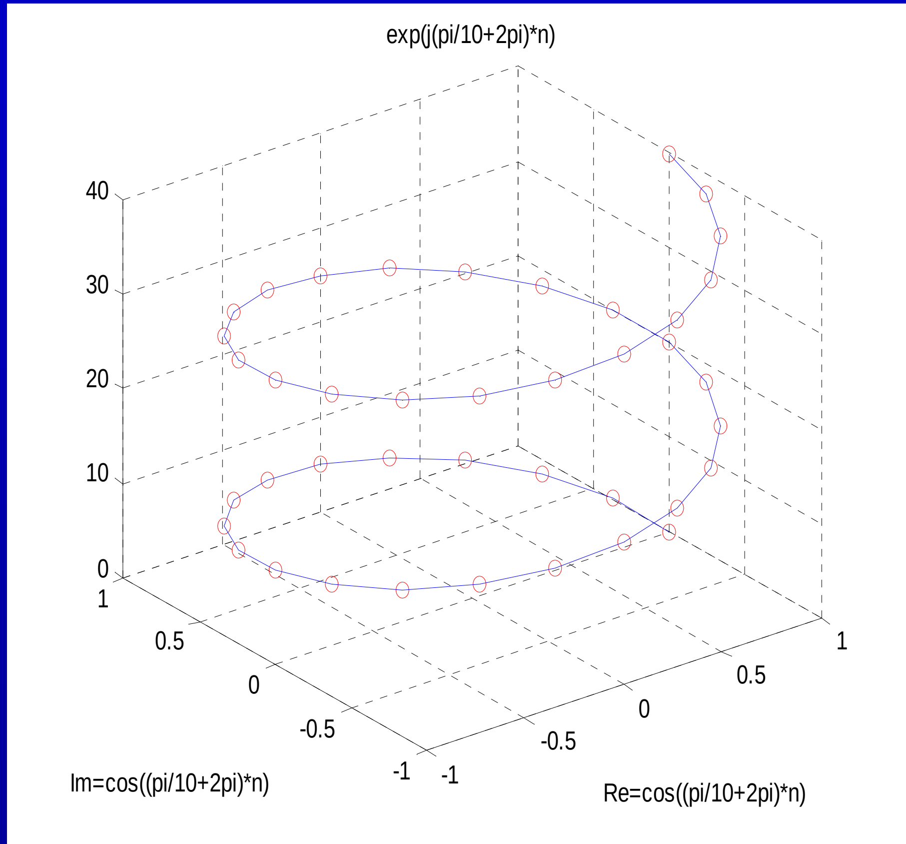

Sinusoidal signal

- Sinusoidal signal

$$x[n] = A\cos(\omega_0 n + \varphi)$$

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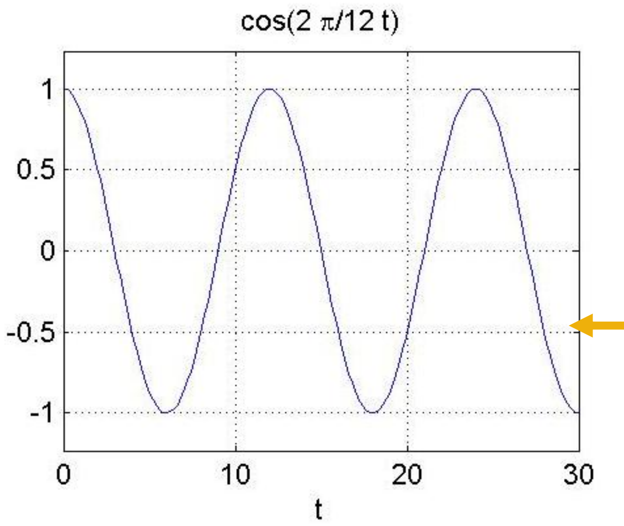
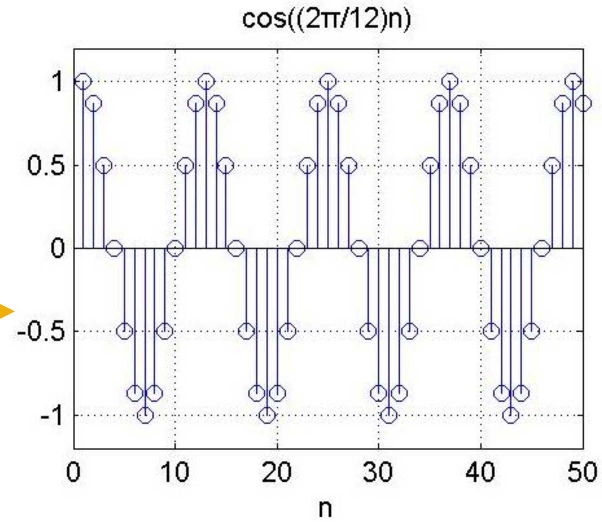
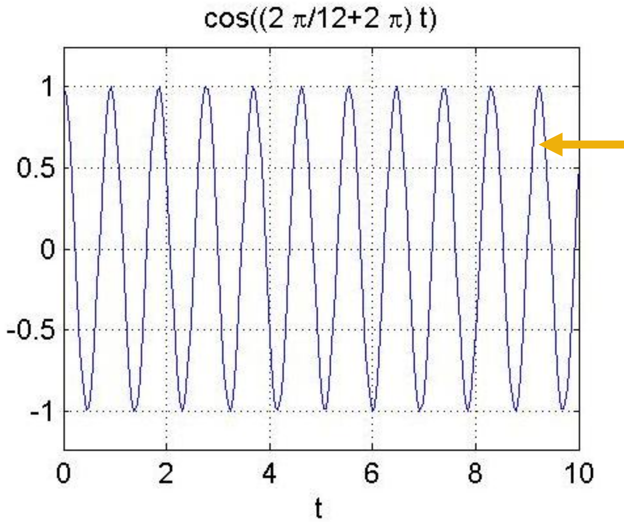
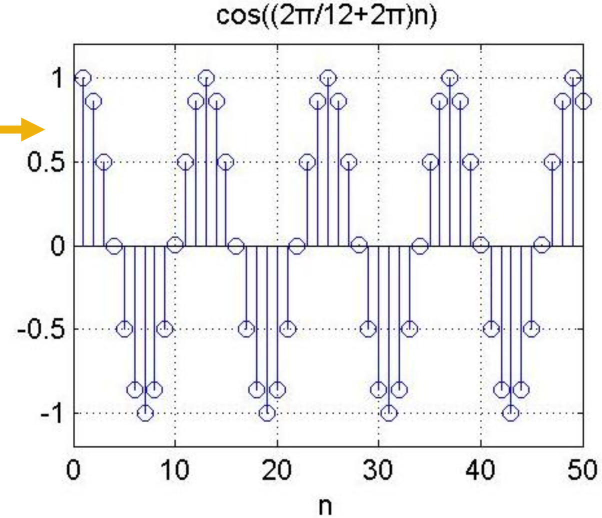
Sinusoidal signal





$$\omega_1 = \frac{2\pi}{12}$$





$$\omega_2 = \frac{2\pi}{12} + 2\pi$$

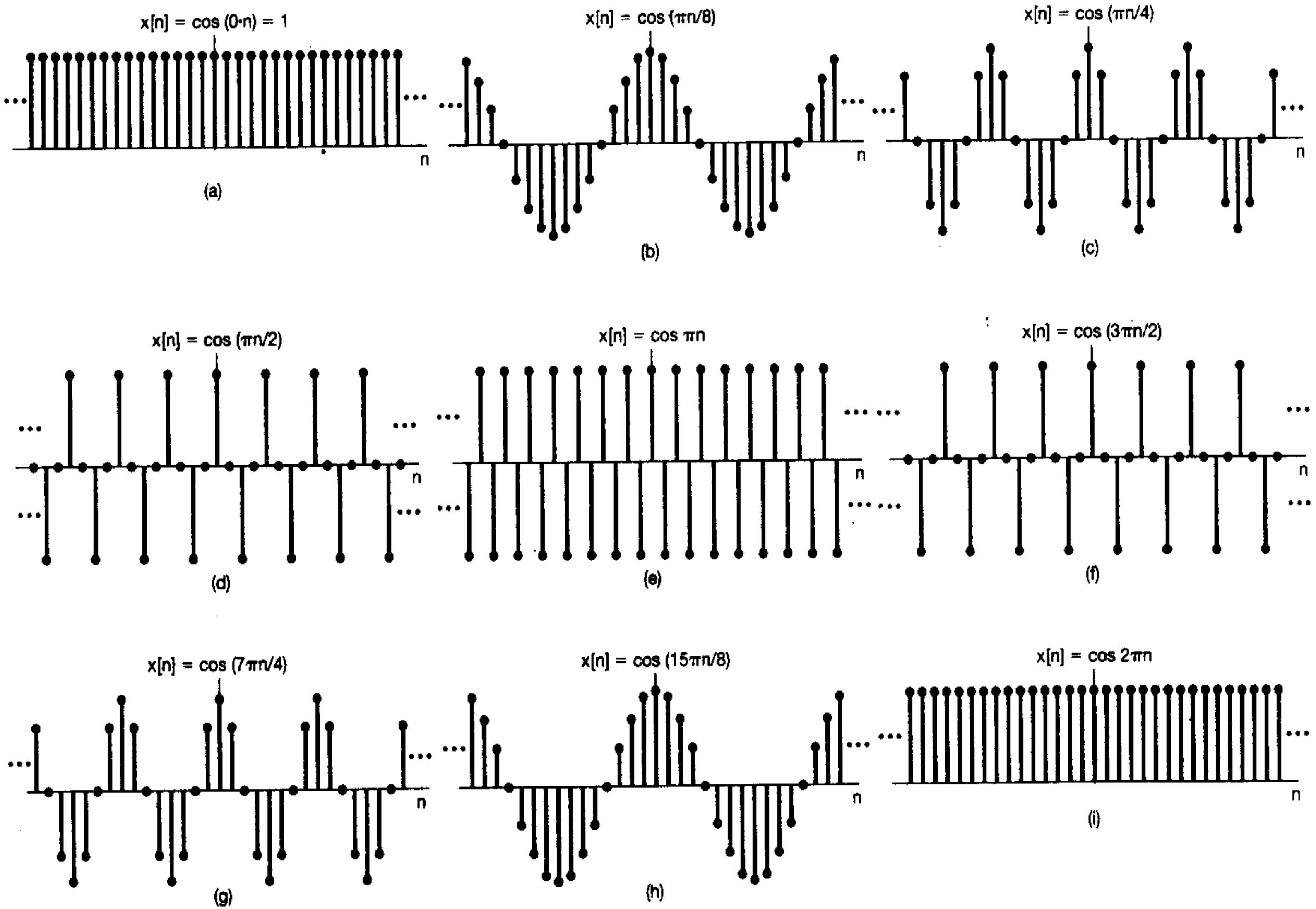


Figure 1.27 Discrete-time sinusoidal sequences for several different frequencies.

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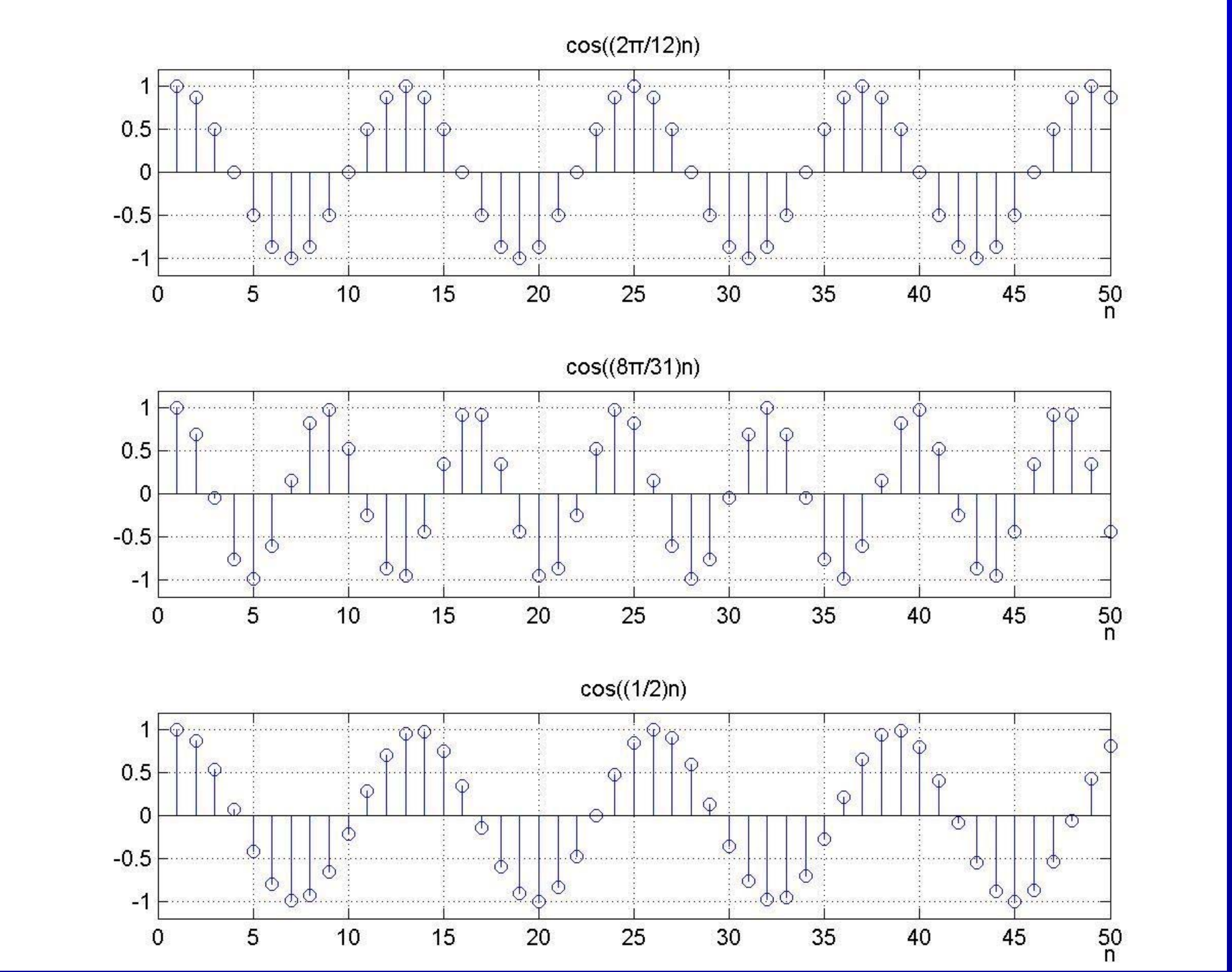

The periodic of the continuous-time complex exponential signals

$$x(t) = e^{jw_0 t} \begin{cases} w_o \Rightarrow rate\ of\ \ oscillation \\ x(t)\ is\ periodic\ for\ any\ value\ of\ \ w_o \end{cases}$$

The periodic of the discrete-time complex exponential signals

$$e^{jw_0 n} = e^{jw_0 (n+N)}$$

$$e^{jw_0 N} = 1 \Rightarrow w_0 N = 2\pi m$$

$$w_0 = \frac{m}{N} 2\pi$$

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Conclusion

- If a discrete-time complex exponential signal is periodic ,then w_0 must be $2\pi \times a$ and a is rational number

Fundamental period: $$N = \frac{2\pi}{|w_0|} m$$

Fundamental frequency: $$\frac{2\pi}{N} = \frac{|w_0|}{m}$$

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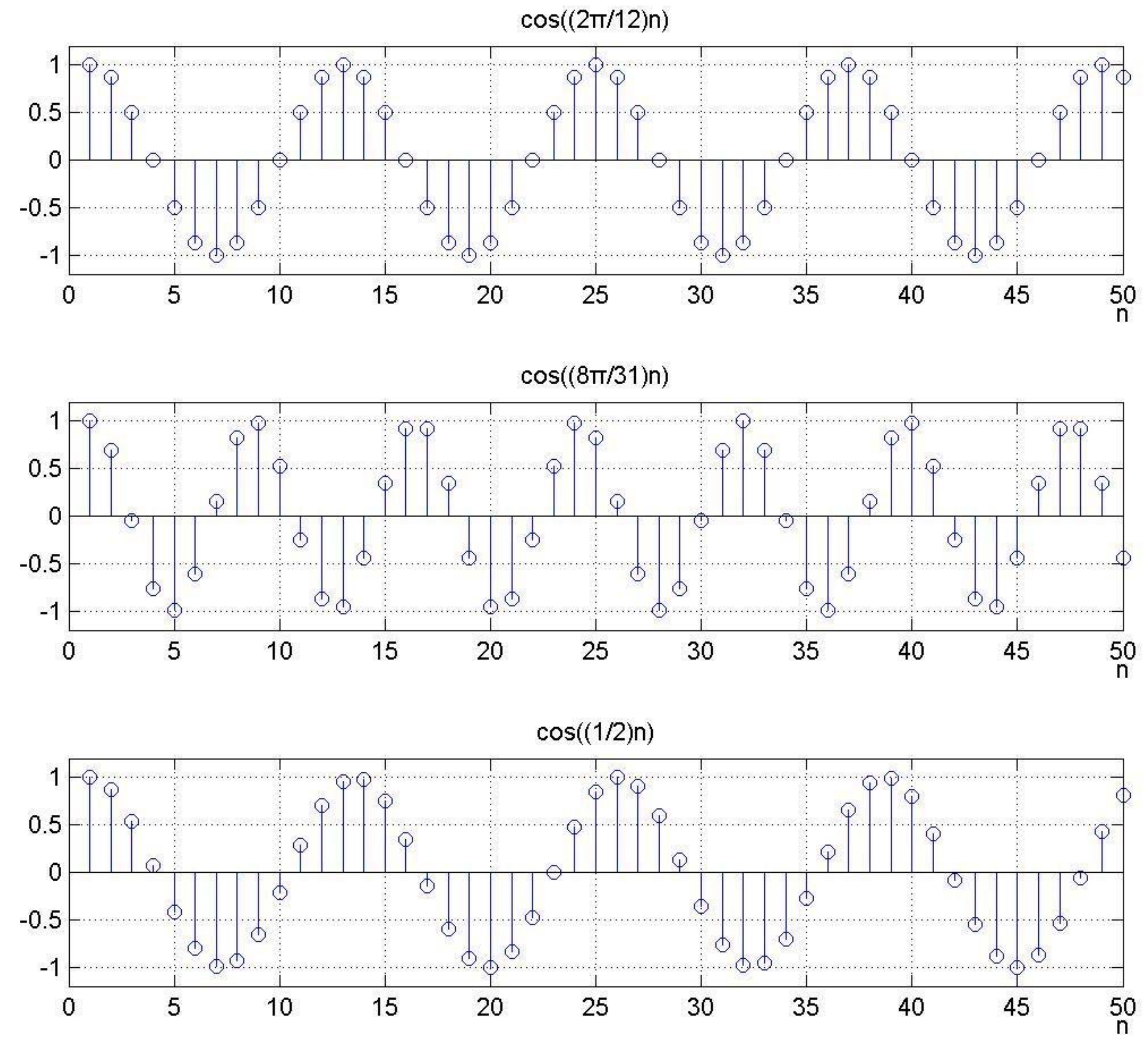

Comparison of $e^{j\omega_0 t}$ and $e^{j\omega_0 n}$

$e^{j\omega_0 t}$	$e^{j\omega_0 n}$
Distinct signal for distinct value of ω_0	Identical signal for ω_0 separated by multiples of 2π
Periodic for any ω_0	Periodic only $N = 2\pi m/\omega_0$ is integers
Fundamental freq ω_0	ω_0 / m
Fundamental period $\omega_0 = 0$, undefined $\omega_0 \neq 0$, $2\pi / \omega_0$	$\omega_0 = 0$, 1 $\omega_0 \neq 0$, $2\pi m / \omega_0$

Example 1.6

If x[n] $= e^{j(\frac{2\pi}{3})n} + e^{j(\frac{3\pi}{4})n}$,

please determine the fundamental period of x[n]

Harmonically related complex exponential signals

Signals & Systems

$$\cdots \; e^{-j2w_0n} \; e^{-jw_0n} \; e^{j0w_0n} \; e^{jw_0n} \; e^{j2w_0n} \; \cdots$$

$$e^{jw_0n} = e^{j(w_0+2\pi)n}$$

$$e^{jkw_0n} = e^{jkw_0n} \cdot e^{j2\pi n}$$

k is integer

$$= e^{j(kw_0+2\pi)n}$$

$$N = \frac{2\pi}{\omega_0} m \quad m = 1$$

$$= e^{j(k\frac{2\pi}{N} + N\frac{2\pi}{N})n}$$

$$= e^{j(k+N)\frac{2\pi}{N}n}$$

$$= e^{j(k+N)w_0n}$$

Example

$$\cdots e^{j\text{-}\frac{4\pi}{5}n} \quad e^{j\text{-}\frac{2\pi}{5}n} \quad e^{j0n} \quad e^{j\frac{2\pi}{5}n} \quad e^{j\frac{4\pi}{5}n} \cdots$$

$$\omega_0 = \frac{2\pi}{5} \qquad N = \frac{2\pi}{\omega_0} = 5 \qquad e^{jkw_0 n} = e^{j(k+N)w_0 n}$$

$$e^{-j\frac{2\pi}{5}n} \quad e^{j0n} \quad e^{j\frac{2\pi}{5}n} \quad e^{j\frac{4\pi}{5}n} \quad e^{j\frac{6\pi}{5}n} \quad e^{j\frac{8\pi}{5}n} \quad e^{j\frac{10\pi}{5}n}$$

$$e^{j\frac{10\pi}{5}n} = e^{j5\frac{2\pi}{5}n} = e^{j(0+5)\frac{2\pi}{5}n} = e^{j0n}$$

$$e^{-j\frac{2\pi}{5}n} = e^{j(-1+5)\frac{2\pi}{5}n} = e^{j\frac{8\pi}{5}n}$$

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Harmonically related complex exponential signals

$$\cdots\ e^{-j2w_0 n}\ e^{-jw_0 n}\ e^{j0w_0 n}\ e^{jw_0 n}\ e^{j2w_0 n}\ \cdots$$

$$e^{jkw_0 n} = e^{j(kw_0 + 2\pi)n} = e^{j(k+N)w_0 n}$$

$$\cdots\ \underbrace{e^{j0w_0 n}\ \overbrace{e^{jw_0 n} \cdots e^{j(N-1)w_0 n}}}_{N}\overbrace{\ e^{jNw_0 n}}^{N}\ \cdots$$

- N distinct periodic exponentials in the set.

But, in the continuous-time case, all signals are distinct !

Example

1、The *period of* $x[n]=\cos(\pi n/8)+\sin(2n)$ is (   )

(a) N=16 (b) N=8 (c) N=32 (d) x[n] is not periodic

2、The *period of* $x(t)=\cos(3\pi t)+\sin(4\pi t)$ is (   )

(a) T=2 (b) T=3 (c) T=4 (d) x(t) is not periodic

1.4 The Unit Impulse And Unit Step Functions

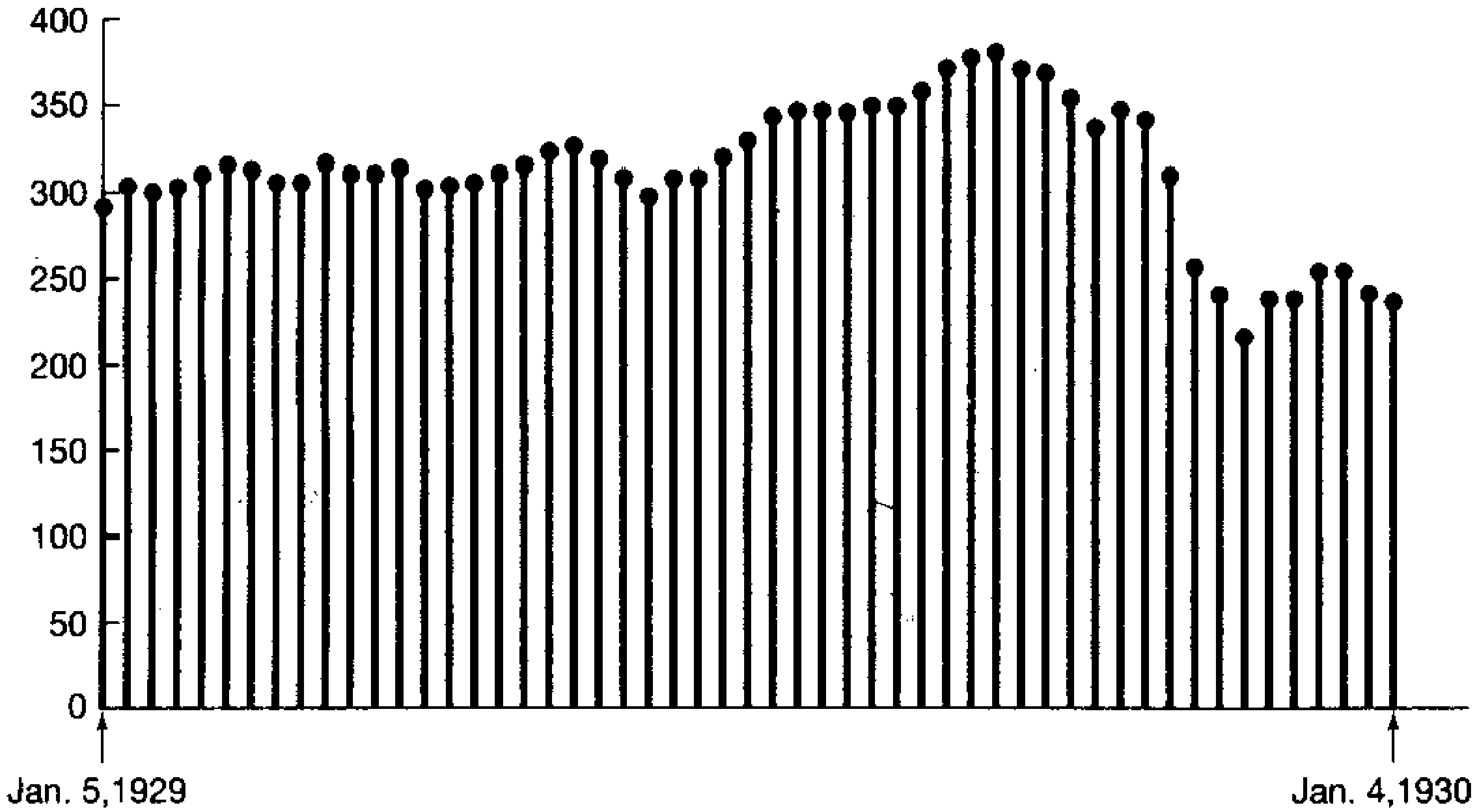
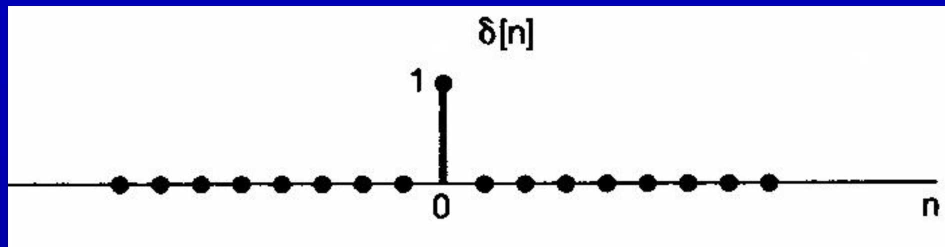


Figure 1.6 An example of a discrete-time signal: The weekly Dow-Jones stock market index from January 5, 1929, to January 4, 1930.

♦ 1.4.1 The Discrete-Time Unit Impulse And Unit Step Sequences

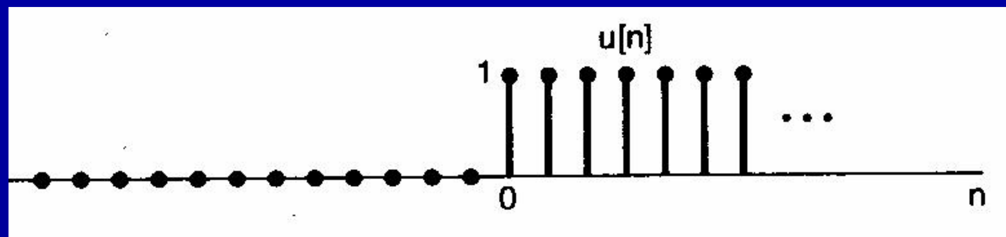
Unit impulse(Unit sample)

$$\delta[n] = \begin{cases} 0 , & n \neq 0 \\ 1 , & n = 0 \end{cases}$$



Unit Step

$$u[n] = \begin{cases} 0 , & n < 0 \\ 1 , & n \geq 0 \end{cases}$$



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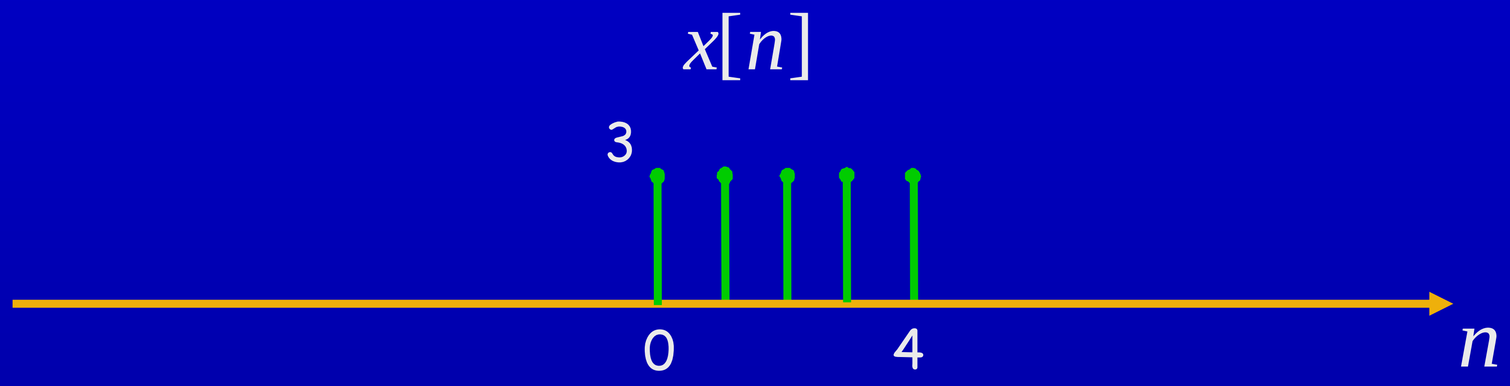
Example

Determine the closed-form expression of x[n]



Relationship between the unit impulse and unit step

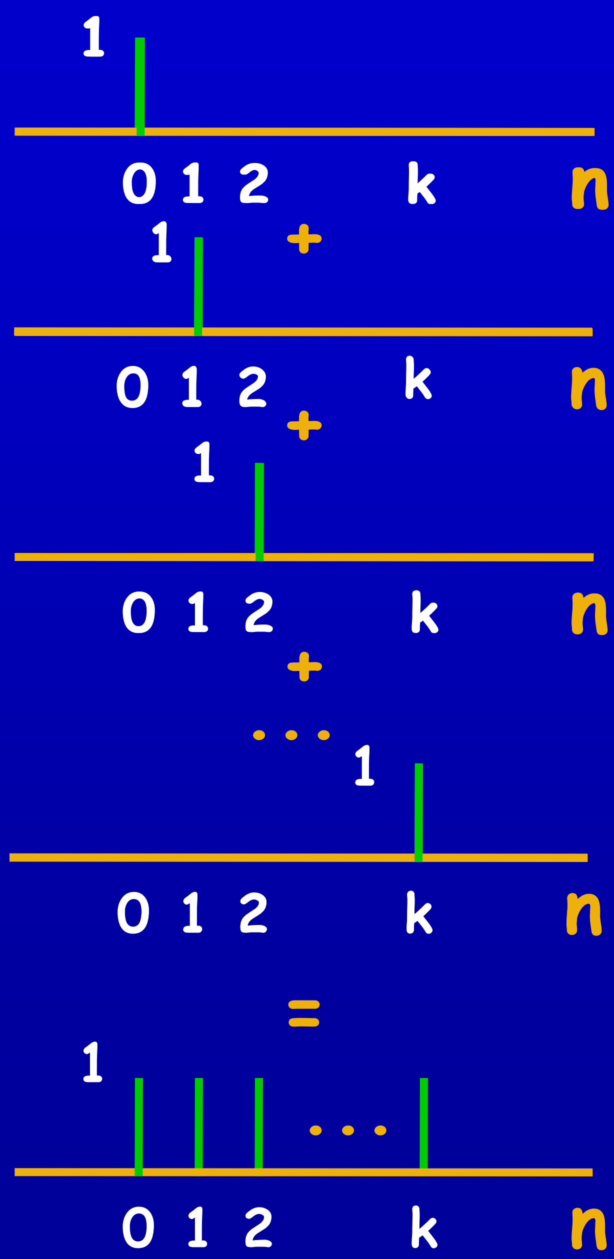
First difference

$$\delta[n] = u[n] - u[n-1]$$

Running sum

$$u[n] = \sum_{m=-\infty}^{n} \delta[m] \qquad u[n] = \sum_{k=0}^{+\infty} \delta[n-k]$$

$$u[n]=\sum_{k=0}^{+\infty}\delta[n-k]$$



$\delta[n]$

$\delta[n-1]$

$\delta[n-2]$

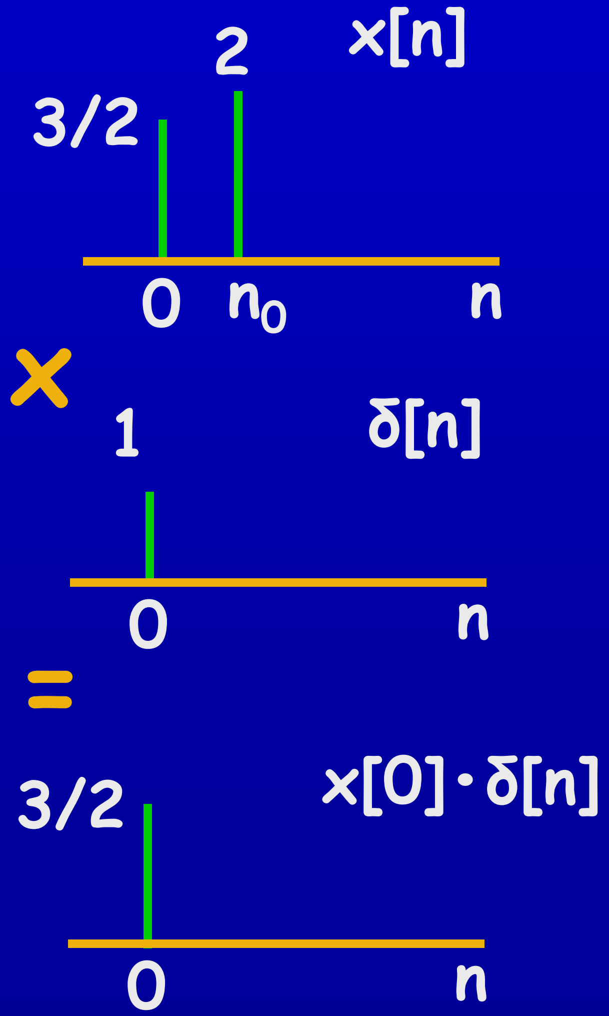
$\delta[n-k]$

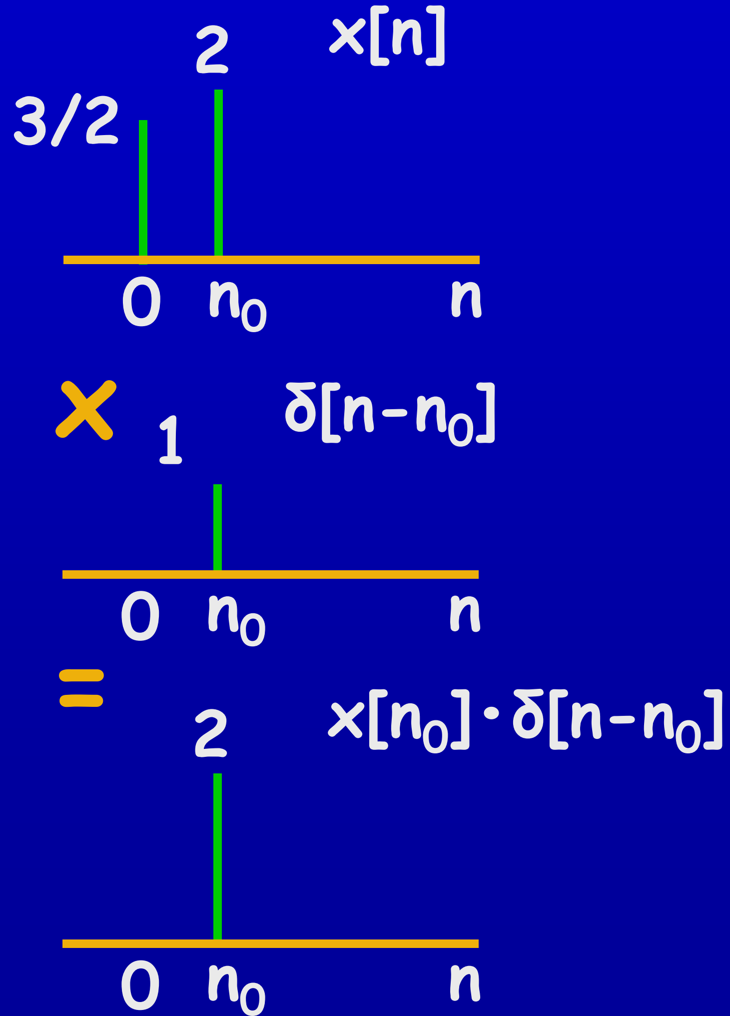
$$u[n]=\sum_{k=0}^{+\infty}\delta[n-k]$$

Sampling Property

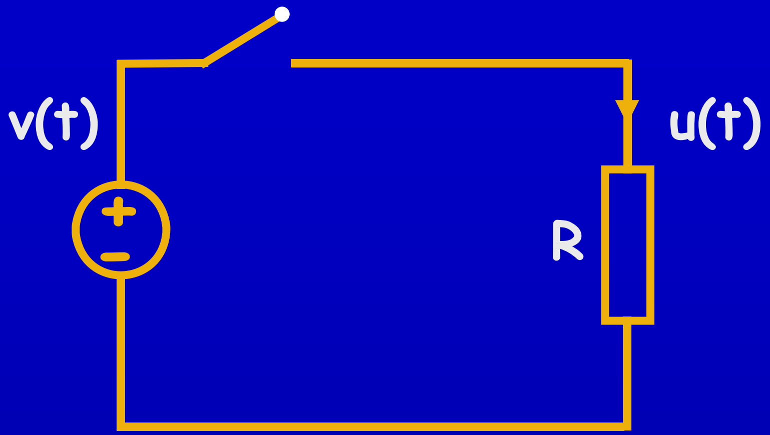
$$x[n]\cdot\delta[n]=x[0]\cdot\delta[n]$$

$$x[n]\cdot\delta[n-n_0]=x[n_0]\cdot\delta[n-n_0]$$



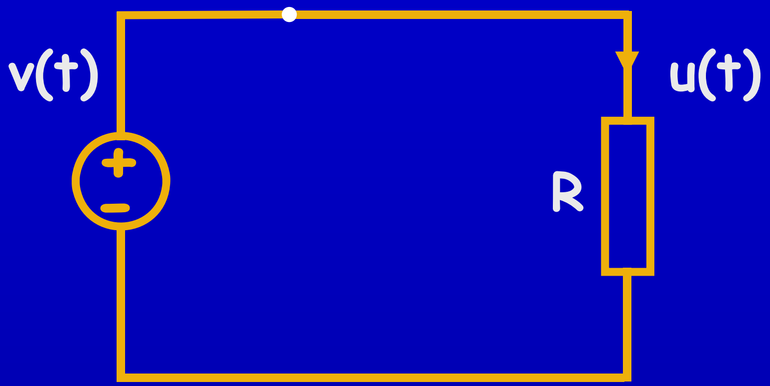


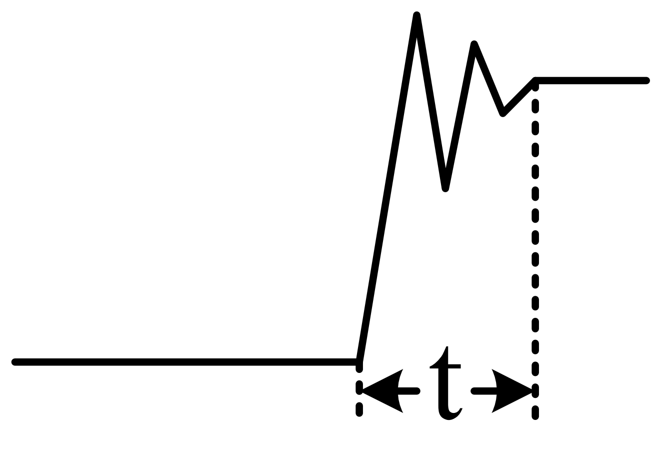
1.4.2 The Continuous-Time Unit Step And Unit Impulse Functions



Signals & Systems

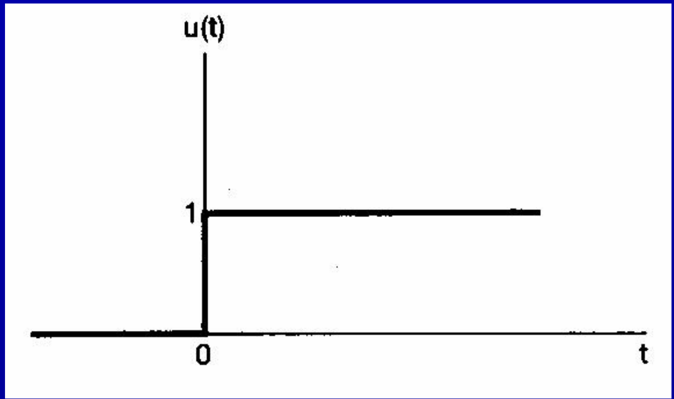
Unit Step





Unit Step

$$u(t) = \begin{cases} 0 \ , \ t < 0 \\ 1 \ , \ t > 0 \end{cases}$$

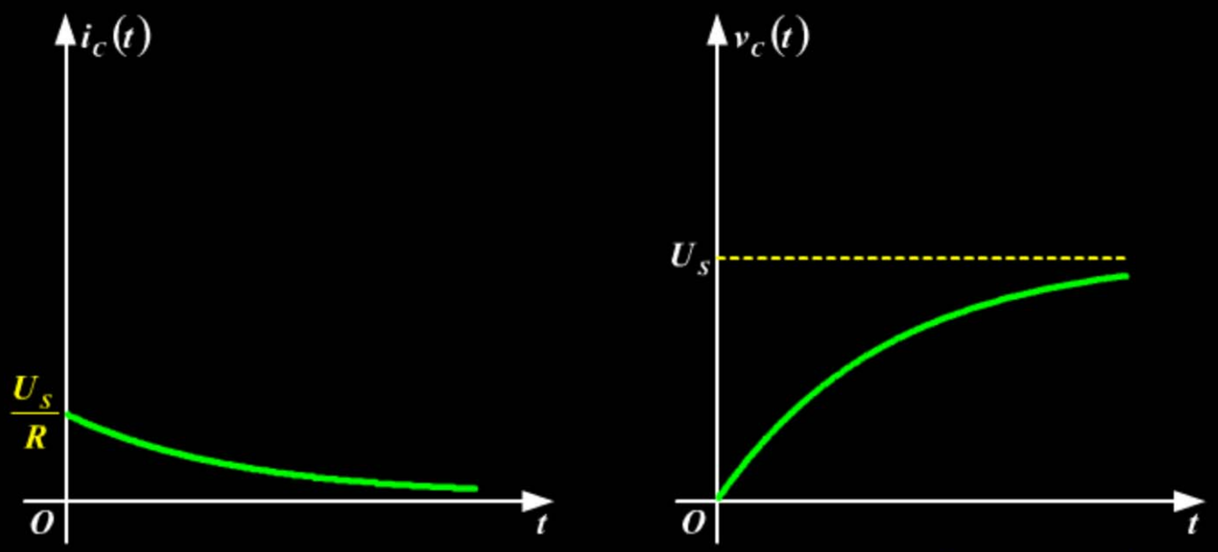


Signals & Systems

Unit impulse

单位冲激信号的引出

R



$$i_C(t) = \frac{U_S}{R} e^{-\frac{t}{RC}}$$

$$v_c(t) = U_s\left(1 - e^{-\frac{t}{RC}}\right)$$

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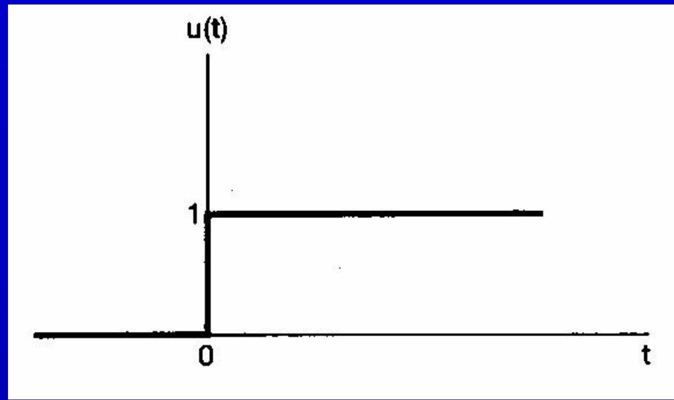
The Continuous-Time Unit Step And Unit Impulse Functions

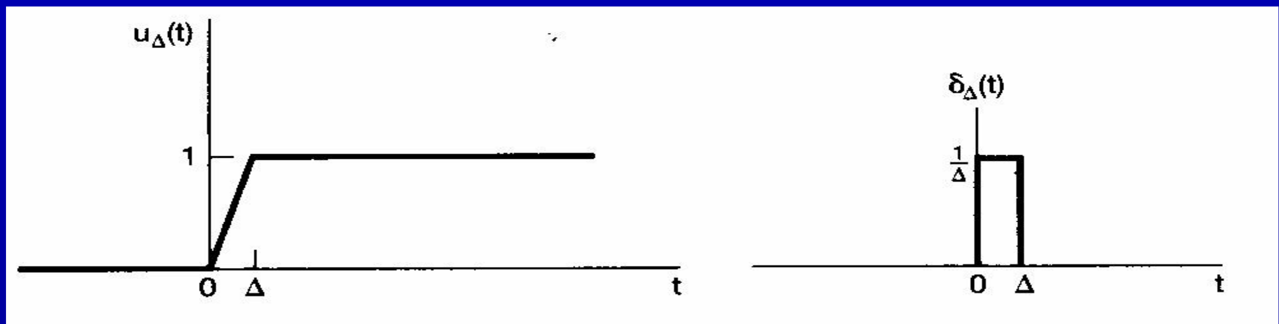
Signals & Systems

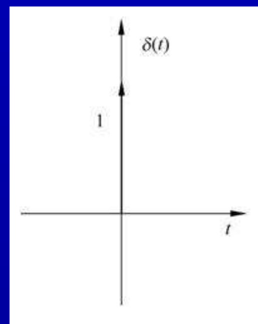
Unit Step

$$u(t) = \begin{cases} 0 \; , \; t < 0 \\ 1 \; , \; t > 0 \end{cases}$$



Unit impulse





$$u(t) = \lim_{\Delta \to 0} u_\Delta(t)$$

$$\delta(t) = \lim_{\Delta \to 0} \delta_\Delta(t)$$ *no duration but unit area!*

$$\boldsymbol{\delta(t) = \frac{du(t)}{dt}}$$

Relationship between the unit impulse and unit step

$$u(t) = \int_{-\infty}^{t} \delta(\tau) d\tau \qquad u(t) = \int_{0}^{+\infty} \delta(t-\tau) d\tau$$

Sampling Property

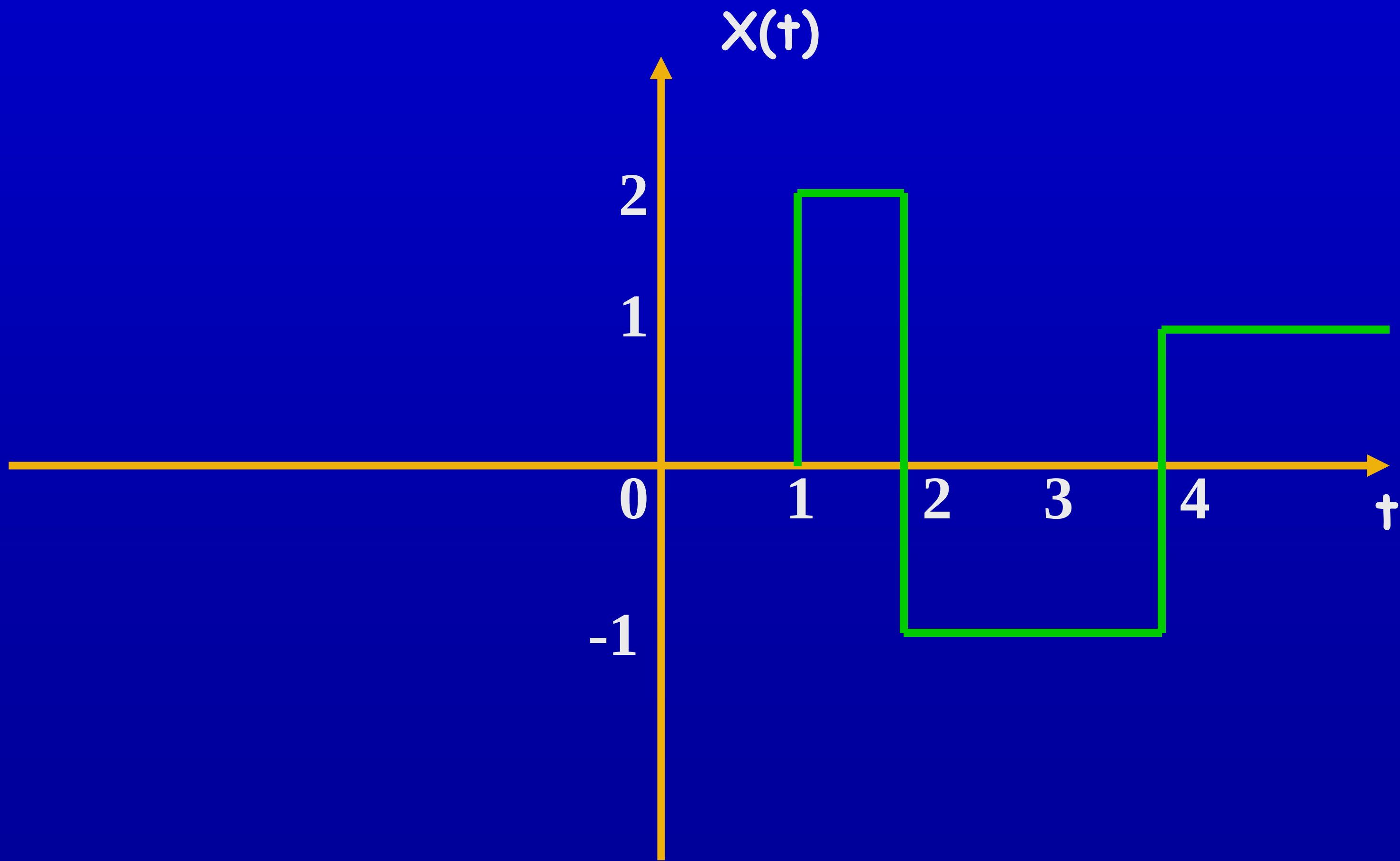
$$x(t) \cdot \delta(t) = x(0) \cdot \delta(t)$$

$$x(t) \cdot \delta(t-t_0) = x(t_0) \cdot \delta(t-t_0)$$

Signals & Systems

Example 1.7

Determine the differential of x(t)



Signals & Systems

Example

1、 $\int_{-\infty}^{+\infty}\left(t^2+4\right)\delta\left(1-t\right)dt = ????$

2、 $\int_{0}^{+\infty}\cos(2\pi t)\delta\left(2t-2\right)dt = ????$

3、 $\int_{0}^{+\infty}(t^2+1)\delta\left(t+1\right)dt = ????$

1.5 Continuous-Time And Discrete-Time System

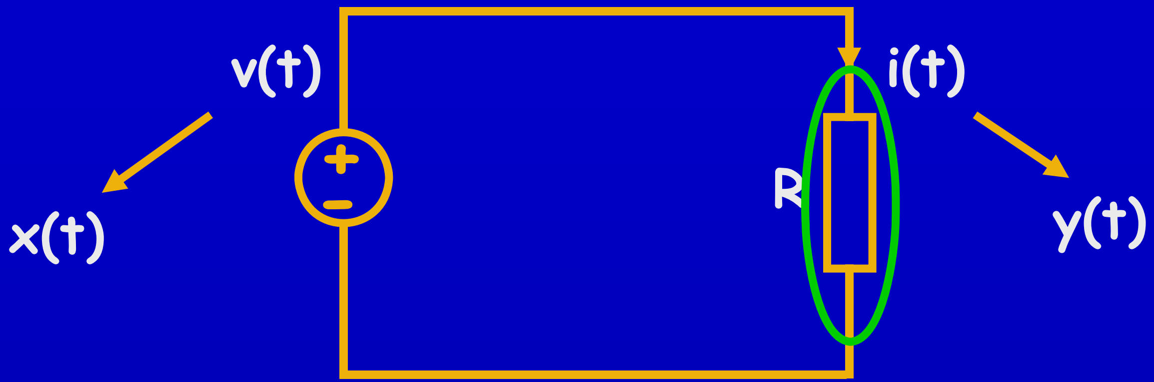
- SYSTEMS
  - For the most part, our view of systems will be from an input-output perspective
  - A system responds to applied input signals, and its response is described in terms of one or more output signals

$$x(t) \longrightarrow \boxed{\text{CT System}} \longrightarrow y(t)$$

$$x[n] \longrightarrow \boxed{\text{DT System}} \longrightarrow y[n]$$

Signals & Systems

CT System



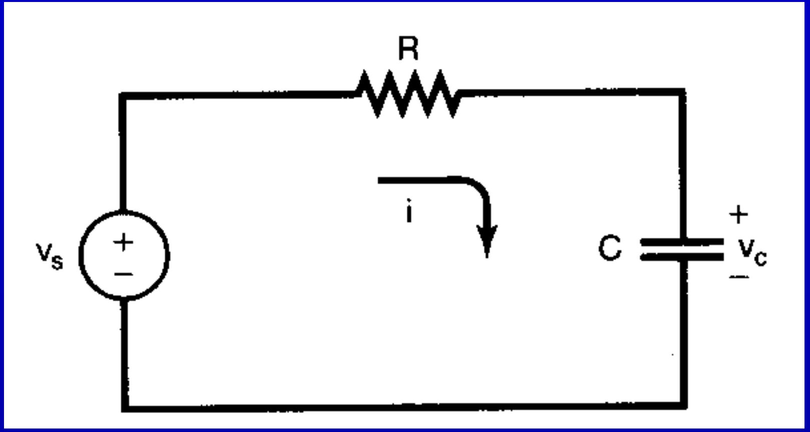
Signals & Systems

DT System

x[n]

y[n]

Example 1.8



Example 1.9

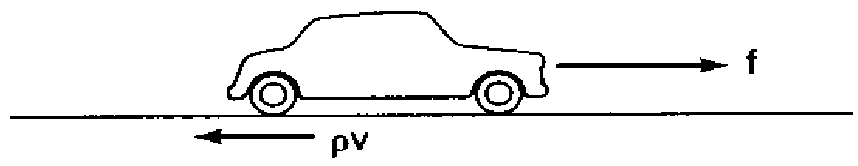


Figure 1.2 An automobile responding to an applied force *f* from the engine and to a retarding frictional force ρv proportional to the automobile's velocity *v*.

First-order linear differential equation

$$\frac{dy(t)}{dt} + ay(t) = bx(t)$$

Example 1.10 ; 1.11

First-order linear difference equation

$$y[n] + ay[n-1] = bx[n]$$

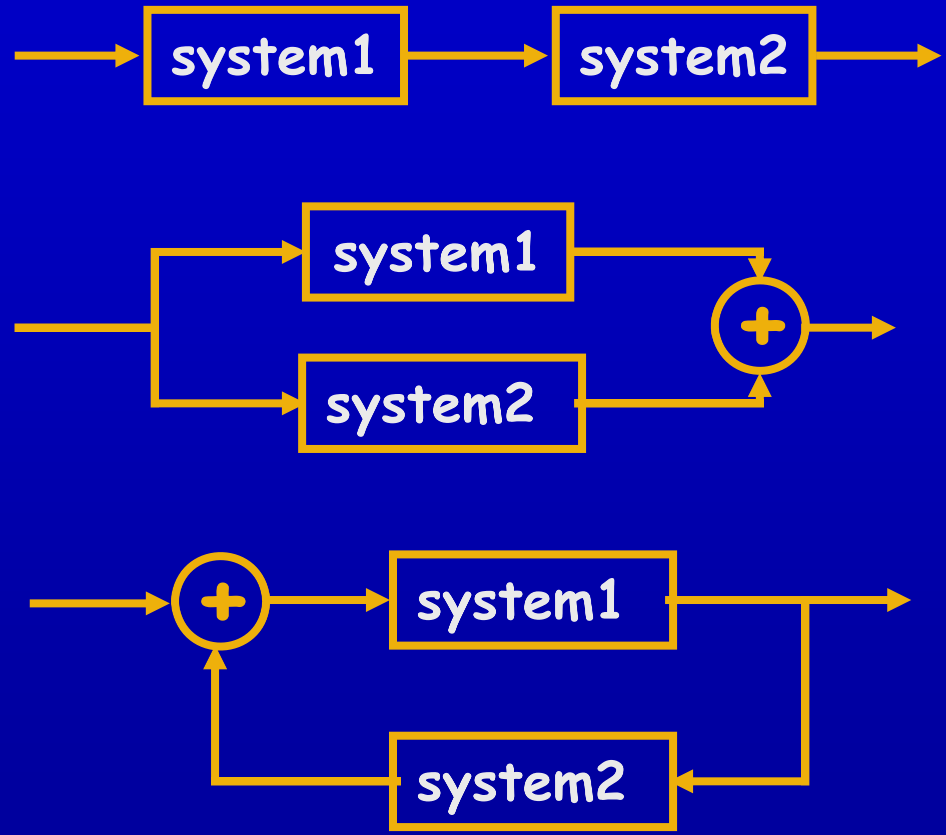
Signals & Systems

System Interconection

Cascade

Parallel

Feedback

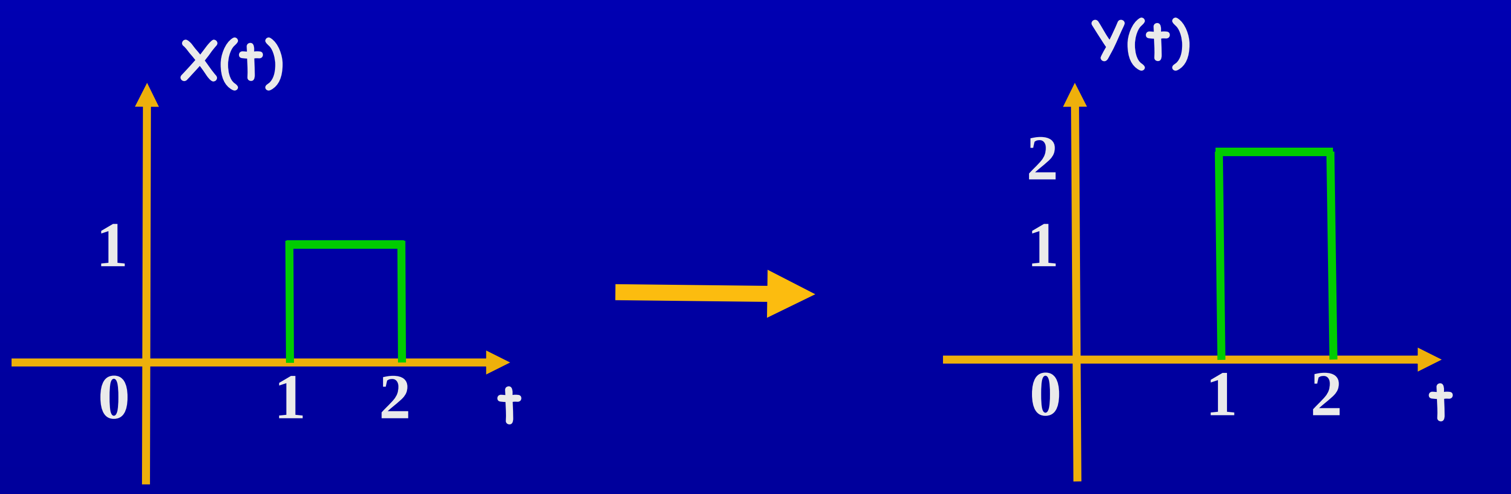


1.6 Basic System Properties

1.6.1 Systems with and without Memory

- Memory less system

It's output is dependent only on the input at the same time.



Example

$$y[n] = (2x[n] - x^2[n])^2$$

$$y(t) = R \cdot x(t)$$

$$y(t) = x(t) \qquad y[n] = x[n]$$ Identity System

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$ Accumulator / Adder

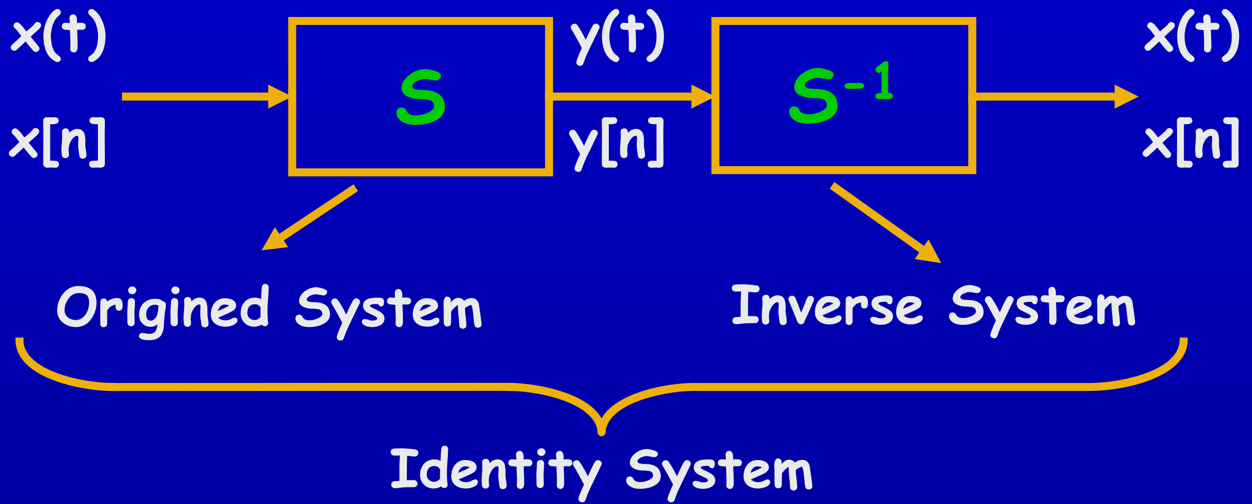
$$y[n] = x[n-1]$$ Delay

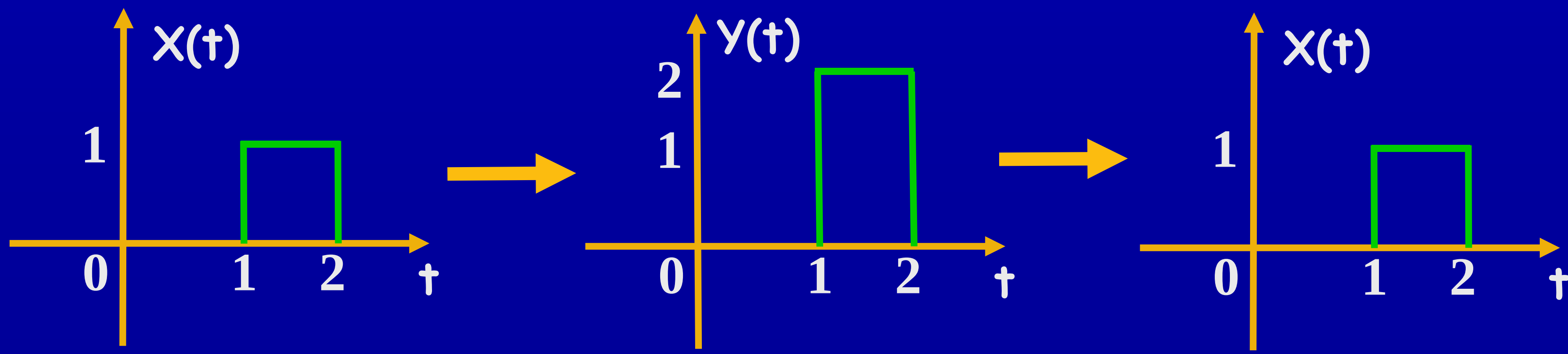
$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(\tau)d\tau$$ Capacitor

Signals & Systems

1.6.2 Invertibility and Inverse Systems

- Invertible
- Distinct input lead to distinct output





Example

$$y(t) = 2 \cdot x(t)$$

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

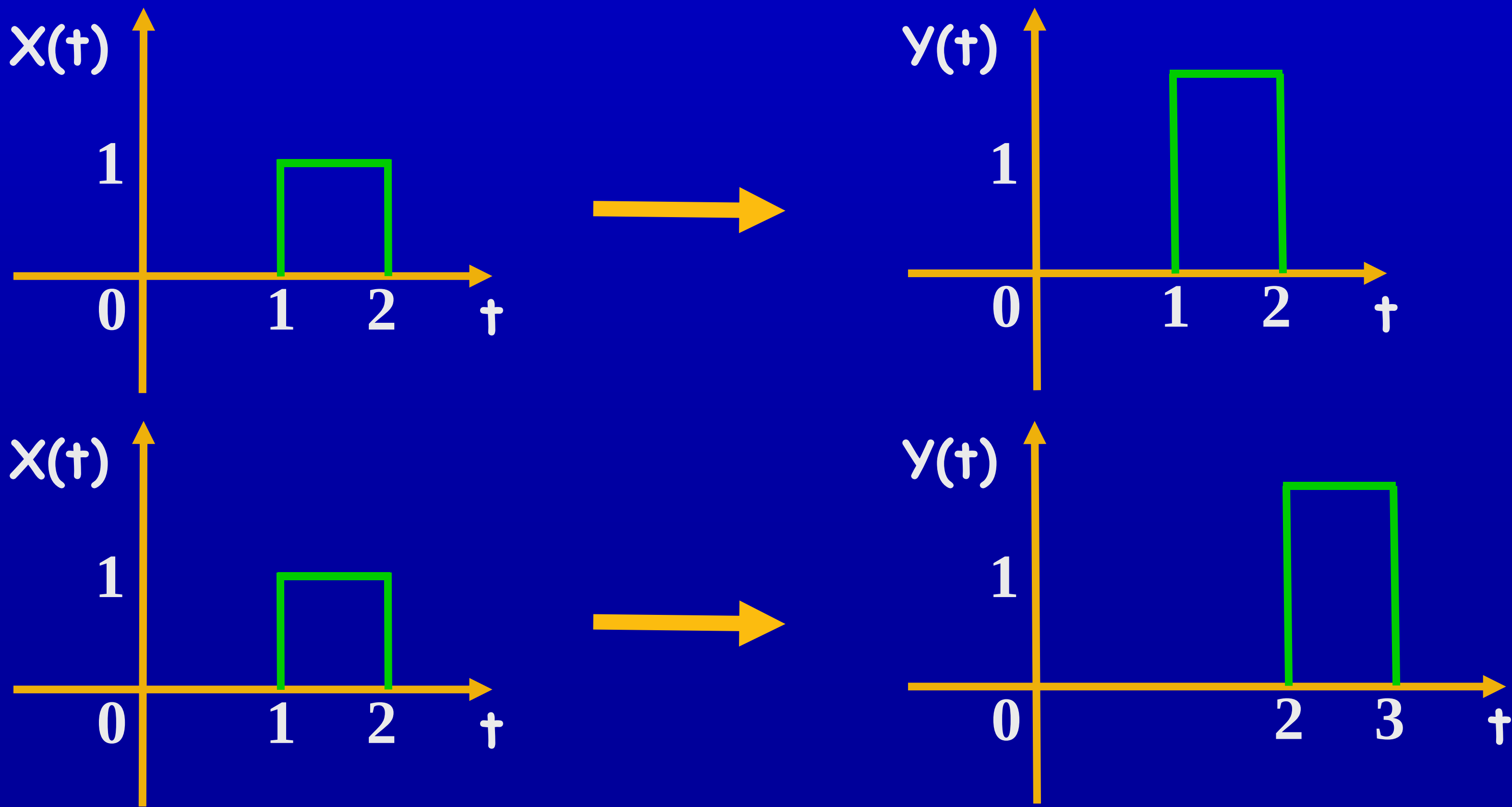
$$y[n] = 0$$

$$y(t) = x^2(t)$$

Signals & Systems

1.6.3 Causality

- Causal System
- The output depend only on the input at present time and in the past.



Example

$$y(t) = x(t+1)$$

$$y[n] = x[n] + x[n+1]$$

$$y[n] = x[-n] \quad \textbf{and} \quad y(t) = x(t) \cdot \cos(t+1)$$

causal ?

1.6.4 Stability

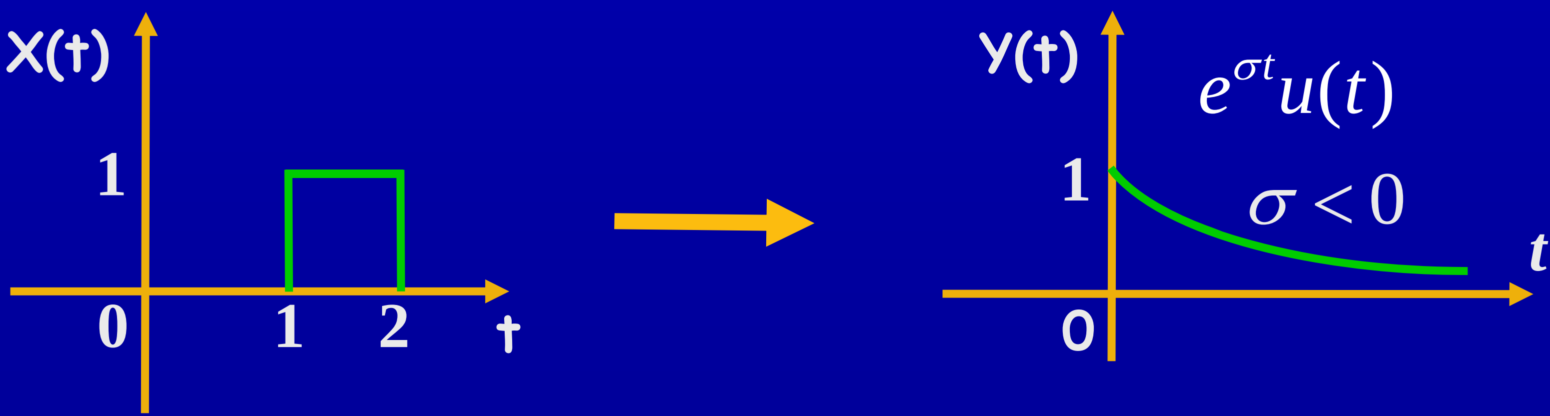
- Stable system

Small input don't lead to divergence of output

if the input bounded, then the output bounded

$x(t) < \infty$ → **System** → $y(t) < \infty$

$x[n] < \infty$ → **System** → $y[n] < \infty$



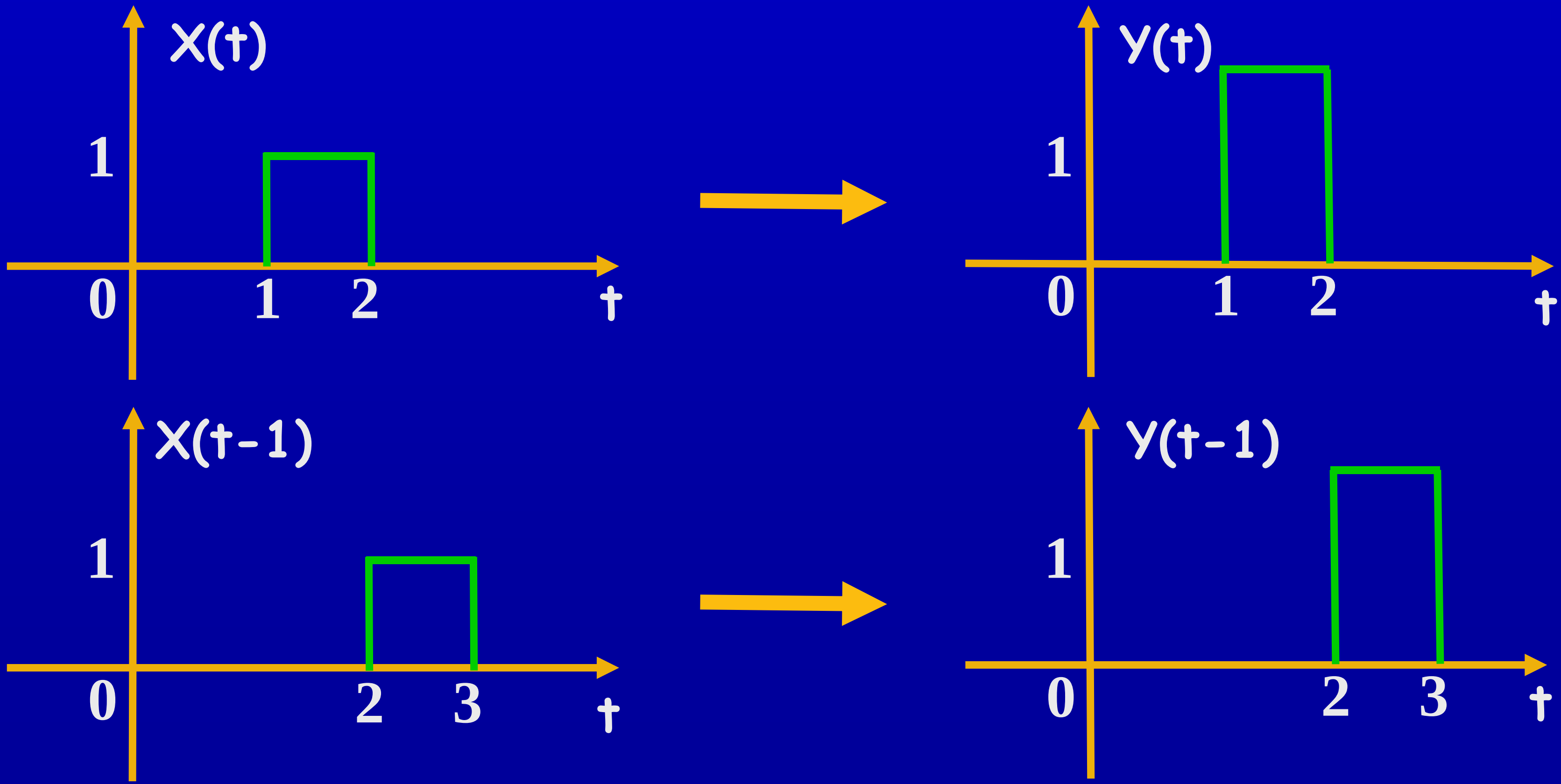
Example

$$y(t) = t \cdot x(t) \qquad y(t) = e^{x(t)}$$

1.6.5 Time Invariance

- Time invariance system

If a time shift in the input results in an identical time shift in the output.



Time Invariance

$$if\ \ x[n] \to y[n], then\ \ x[n-n_0] \to y[n-n_0]$$

$$if\ \ x(t) \to y(t), then\ \ x(t-t_0) \to y(t-t_0)$$

Example

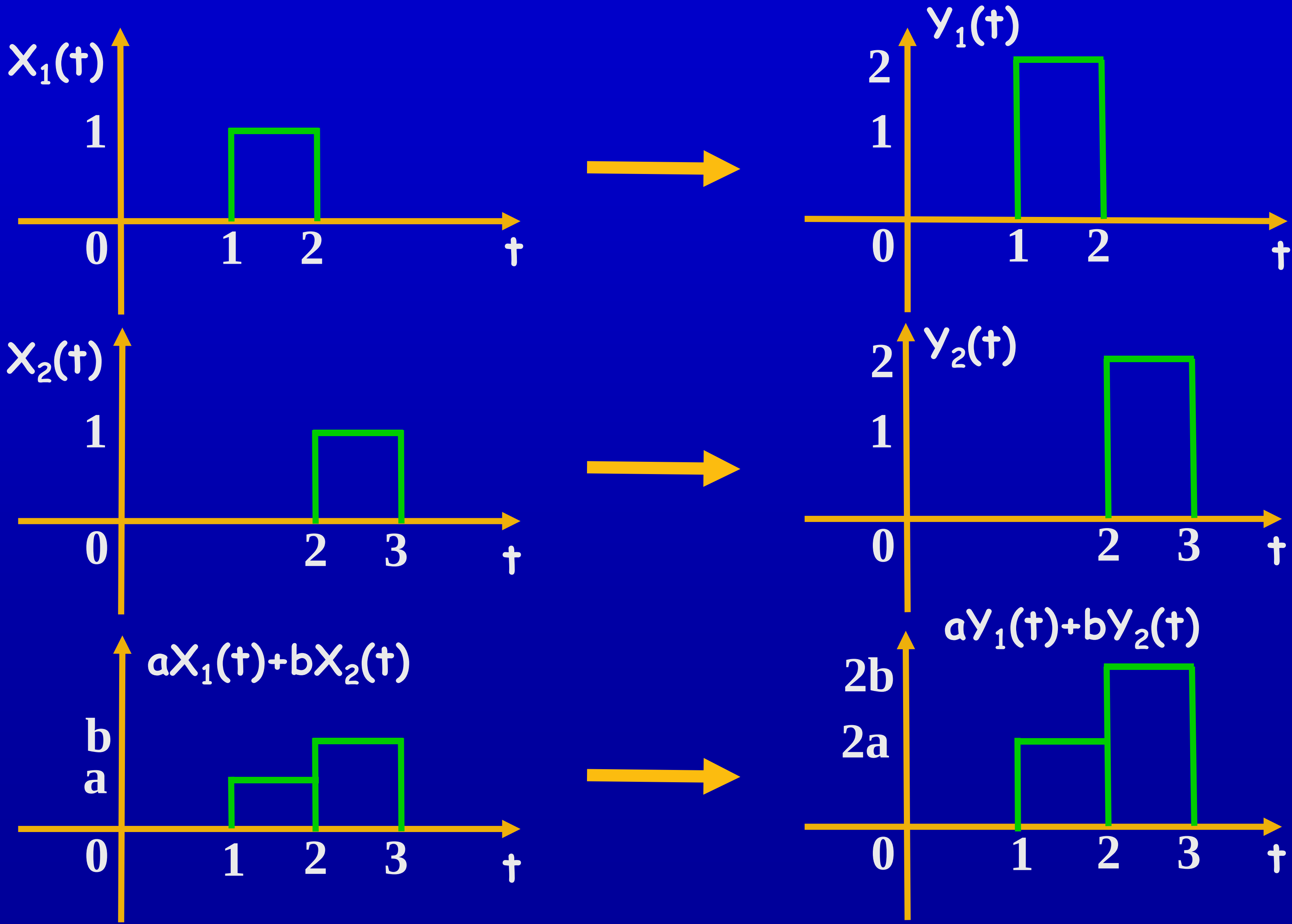
$$y(t) = \sin[x(t)] \quad y[n] = nx[n] \quad y(t) = x(2t)$$

1.6.6 Linearity

- Linear system

  Linearity ----include additivity and Scaling or homogeneity

Linearity


x1(t)
1
0
1
2
t
y1(t)
2
1
0
1
2
t
x2(t)
1
0
2
3
t
y2(t)
2
1
0
2
3
t
ax1(t)+bx2(t)
b
a
0
1
2
3
t
ay1(t)+by2(t)
2b
2a
0
1
2
3
t

Linearity

$$ax_1(t)+bx_2(t)\rightarrow ay_1(t)+by_2(t)$$

$$ax_1[n]+bx_2[n]\rightarrow ay_1[n]+by_2[n]$$

$$x[n]=\sum_k a_k x_k[n]\rightarrow y[n]=\sum_k a_k y_k[n]$$

Example

$$y(t)=t\cdot x(t) \qquad y(t)=x^2(t)$$

$$y[n]=\mathrm{Re}\{x[n]\} \qquad y[n]=2x[n]+3$$

incrementally linear system