Homework 6

Due date: March 3, 2017, by 8p.m. at the homework box.

Reading assignment. Chapter 10 of the text book.

Exercise 1. Write a function in your favorite programming language to simulate a discrete system whose input signal $x[n]$, and output signal $y[n]$ satisfy a linear constant-coefficient difference equation of the form

$$y[n] + \sum_{k=1}^{N} a_k\, y[n-k] = \sum_{k=0}^{M-1} b_k\, x[n-k].$$

You can assume the system is LTI and causal (initial rest assumption). You can also assume that $x[n]=0$ for $n<0$. Hence $y[n]=0$ for $n<0$. We will assume that the input signal $x[n]$ is provided for $0 \leqslant n < K$. Your function must be able to compute the output signal $y[n]$ for $0 \leqslant n < K$. Your function must take as input an array of length N for the a_k's, an array of length M for the b_k's, and an array of length K representing the input signal $x[n]$. Your function must return an array of length K holding the output signal $y[n]$.

Exercise 2. In this exercise you will use the code you wrote in Exercise 1. The input signal $x[n]$ and output $y[n]$ of a causal LTI system satisfy the equation

$$y[n] - \frac{1}{21}(4\,y[n-1] + 2\,y[n-2] + y[n-3]) = \frac{1}{3}(2\,x[n] - 4\,x[n-1] + 2\,x[n-2]).$$

Using your code from Exercise 1 make representative plots for $x[n]$ and $y[n]$ when

i. $x[n] = \delta[n]$.

ii. $x[n]$ is a periodic square wave (right-going only) of period 6, period 10 and period 20 (3 separate signals).

iii. $x[n]$ is a periodic triangular wave (right-going only) of period 6, period 10 and period 20.

iv. $x[n]$ is a periodic parabolic wave (right-going only) of period 6, period 10 and period 20.

You can use your own *suitable* definitions of square, triangular and parabolic discrete periodic waves (see Homework 2 for help). Note that these waves begin at time 0 and extend to the right only. So there is no problem using your code.

What do you think this filter does? Look carefully at the plots of the period 20 input-output pairs. (You may need to use three (or more) periods to see what is going on.)

Exercise 3. In this exercise you will use the code you wrote in Exercise 1 to find the frequency response for the LTI system presented in Exercise 2. First make sure that your code can work with complex input signals $x[n]$. Now let $L=50$ and let l denote an integer between 0 (inclusive) and L (exclusive). For each l from 0 to $L-1$, consider an input signal of the form $x[n] = e^{j\,2\pi l n/L} u[n]$, and use your code to find $y[n]$. Plot the function $y[n]/x[n]$. Look at the graph of $y[n]/x[n]$ and notice that the ratio becomes almost constant as n becomes large. This constant is usually denoted as $H(e^{j\,2\pi l/L})$. It tells you how much the pure harmonic at frequency $2\,\pi\,l/L$ is amplified and shifted by the LTI system. For each l, find the value of this constant. Plot the **absolute** value of this constant as a function of l. Would you characterize the LTI system of Exercise 2 as a low-pass filter or a high-pass filter?

Exercise 4. Find the z-transforms of the following functions along with their region of convergence.

i. $a^n\, u[n]$

ii. $a^n\, u[-n-1]$

iii. $n\, a^n\, u[n]$

iv. z_0^n (careful)

Exercise 5. If $\mathcal{X}(z)$ is the z-transform of $x[n]$ with ROC $a < |z| < b$, what are the z-transforms and ROC's of the following functions

i. $x[n - n_0]$

ii. $x[-n]$

iii. $z_0^n\, x[n]$

iv. $n\, x[n]$

Justify all your answers.

Exercise 6. Find the discrete signal $x[n]$, whose z-tranform is given by $\mathcal{X}(z) = (z^4 + 1)^{-1}$, with ROC $|z| < 1$.