Chapter 4

The Continuous-Time Fourier Transform

Review

$$x(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = ?$$

$$\delta(t - t_0) \xrightarrow{LTI} h\left(t - t_0\right)$$

Chapter 2: $$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau$$

$$y(t) = x(t) * h(t)$$

$$= \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

Time-Domain:Convolution

Review

$$x(t) \longrightarrow \boxed{\text{LTI}} \longrightarrow y(t) = ?$$

Chapter 3:

$$e^{st} \xrightarrow{LTI} H(S)e^{st} \qquad z^n \xrightarrow{LTI} H(z)z^n$$

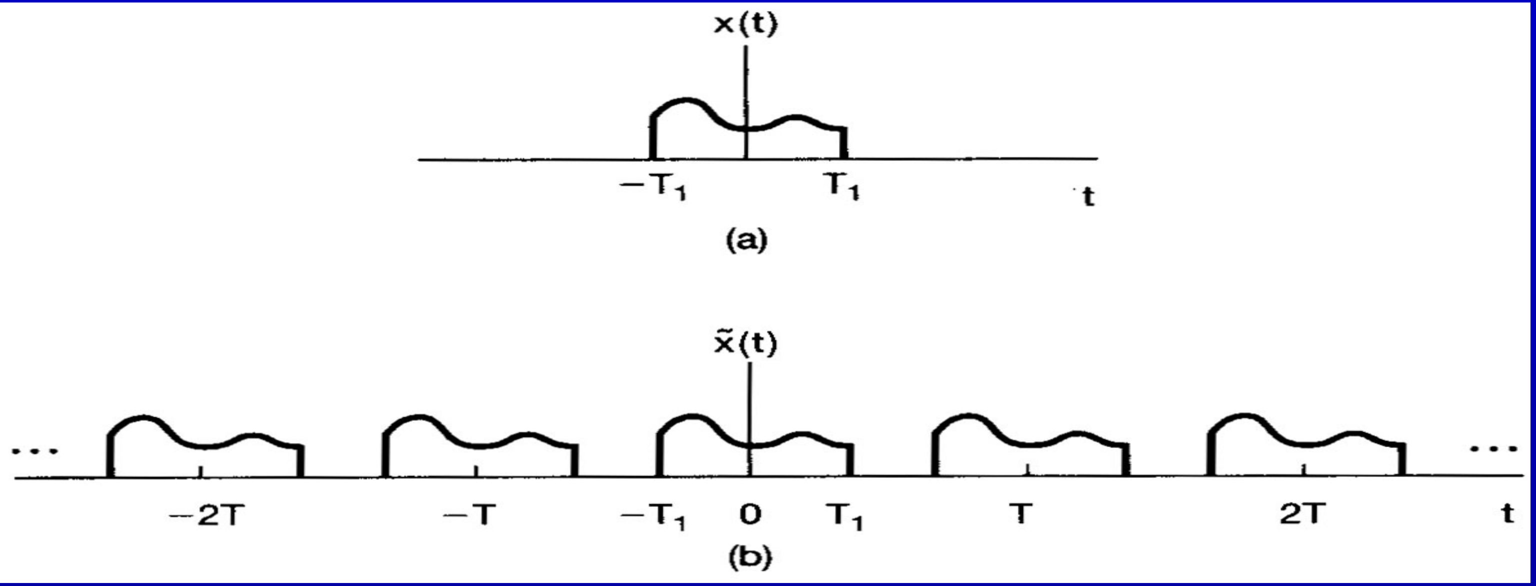
$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} \qquad x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n}$$

$$y(t) = \sum_{k=-\infty}^{+\infty} a_k H(jkw_0) e^{jk\omega_0 t} \qquad y[n] = \sum_{k=\langle N \rangle} a_k H(e^{jkw}) e^{jk\omega_0 n}$$

Frequency-Domain:frequency response

Signals & Systems

4.1 Representation of aperiodic signal:the continuous-time Fourier transform



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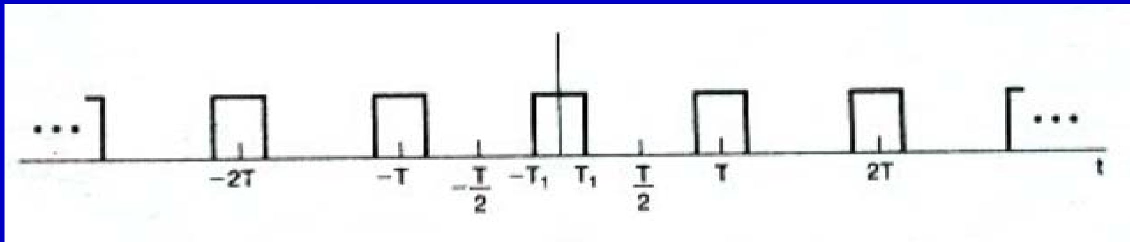
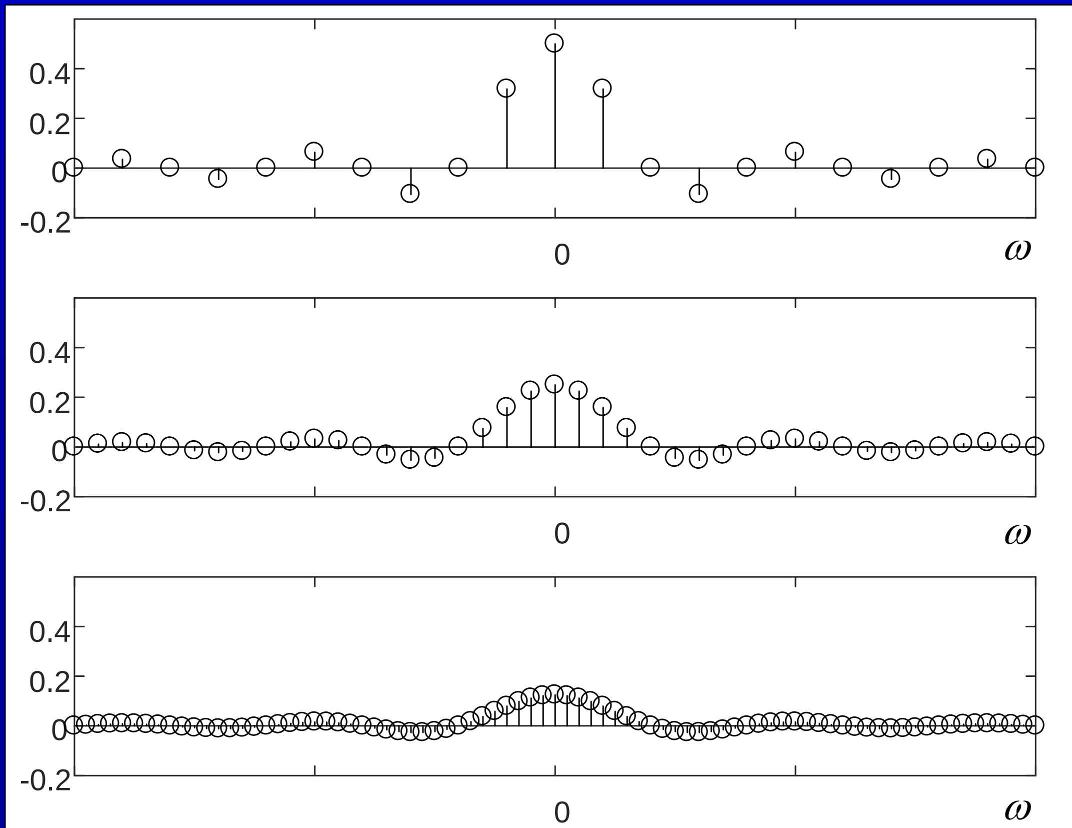


$$a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T}$$



(a)$T=4T_1$

(b)$T=8T_1$

(c)$T=16T_1$

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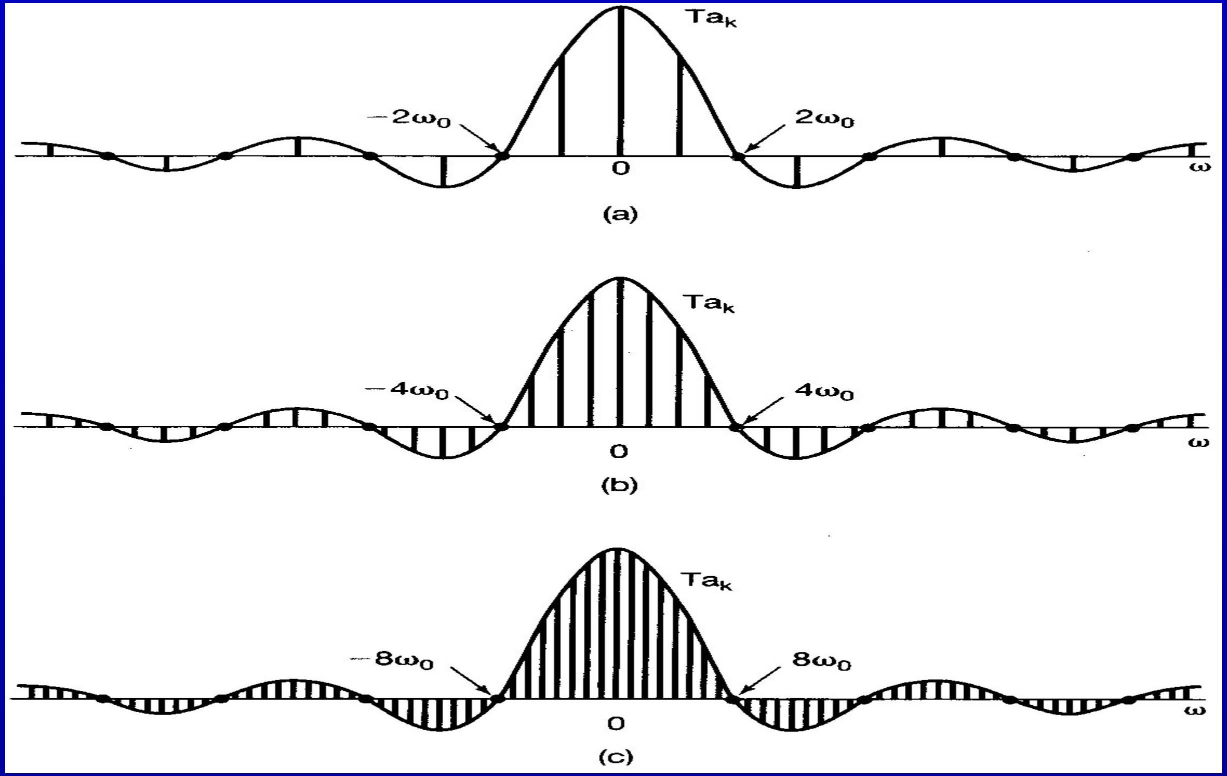
Envelop of a_k

$$a_k = \frac{2\sin(k\omega_0 T_1)}{k\omega_0 T} \qquad Ta_k = \left.\frac{2\sin\omega T_1}{\omega}\right|_{\omega=k\omega_0}$$



(a) $T=4T_1$

(b) $T=8T_1$

(c) $T=16T_1$

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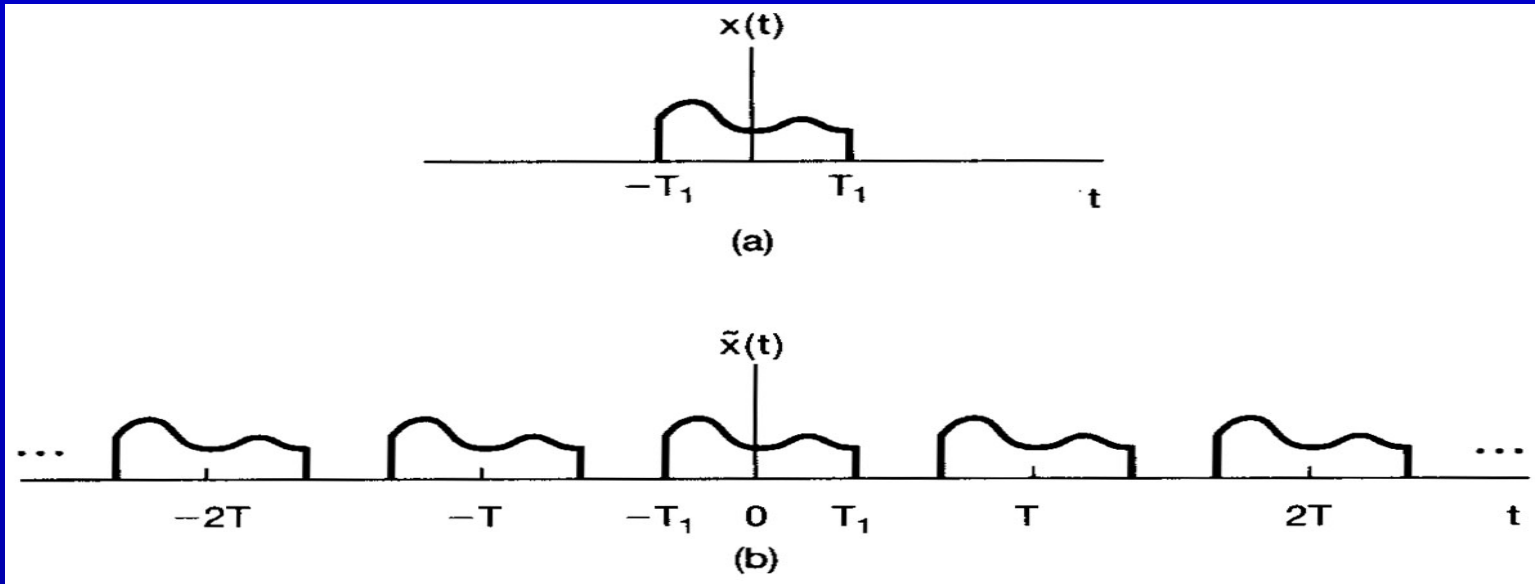
Development



$$T \longrightarrow \infty \Rightarrow \hat{x}(t) \longrightarrow x(t)$$

$$Ta_K = ?$$

Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$x(t) \xleftrightarrow{FT} X(jw)$$

Summary

- Fourier series ⟶ periodic signal
- Fourier transform ⟶ aperiodic signal
- $$a_k = \frac{X(jw)}{T}\bigg|_{\omega = k\omega_0}$$
- a_k ——spectral coefficients

  $X(jw)$ ——spectrum

4.1.2 Convergence of Fourier transform

Signals & Systems

- View of energy
- finite energy(square integrable)

$$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

- The dirichlet conditions

(1)x(t) absolutely integrable;

(2) finite number of maxima and minima with any finite interval;

(3) finite number of discontinuities with any finite interval;each of discontinuities must be finite

4.1.3 Example of continuous-time FT

Example 4.1

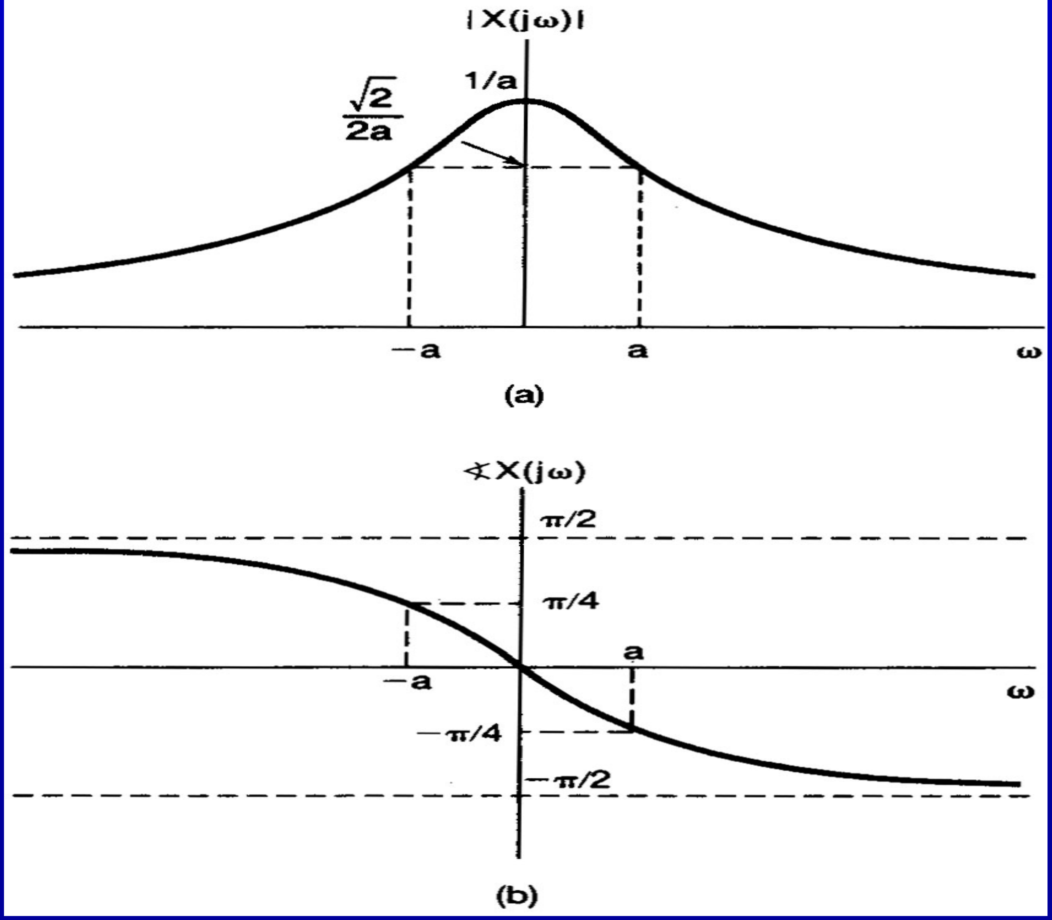
$$x(t) = e^{-at}u(t) \qquad a > 0$$

a is real,please determine the FT of x(t)

$$X(j\omega) = \frac{1}{a + j\omega}, \quad a > 0.$$

$$|X(j\omega)| = \frac{1}{\sqrt{a^2 + \omega^2}}, \quad \angle X(j\omega) = -\tan^{-1}\left(\frac{\omega}{a}\right)$$

$$x(t) = e^{-at}u(t) \xleftrightarrow{FT} X(j\omega) = \frac{1}{a + j\omega} \qquad a > 0$$



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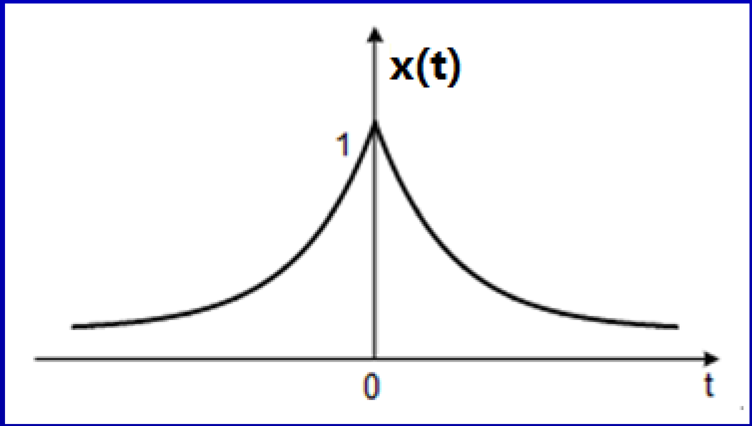
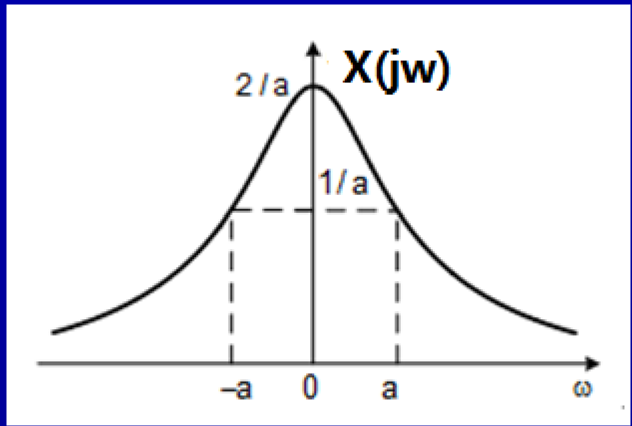
Example 4.2

$$x(t) = e^{-a|t|} \qquad a > 0$$

a is real,please determine the FT of x(t)



$$X(j\omega) = \frac{2a}{a^2 + \omega^2}$$



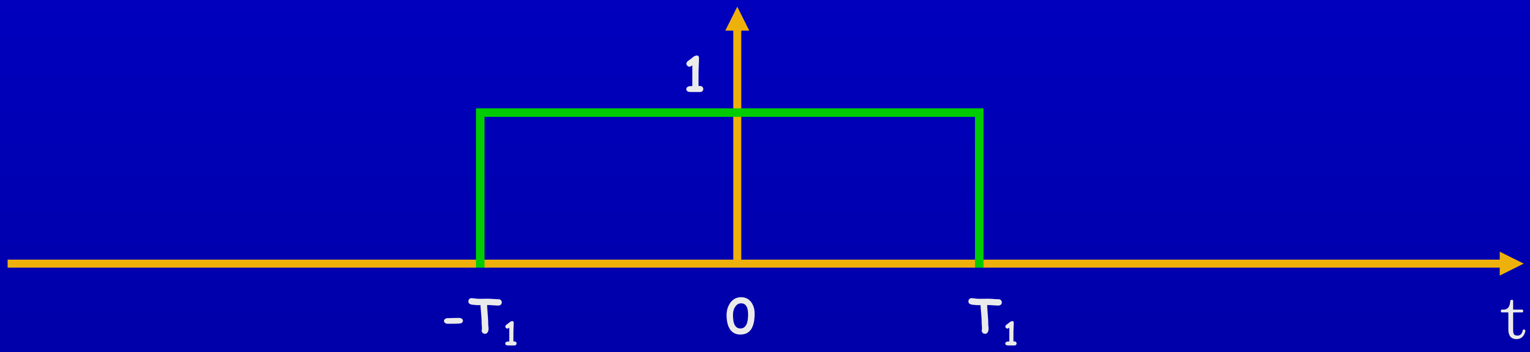
Example 4.3

$x\left(t\right)=\delta(t)$,please determine the $X(jw)$

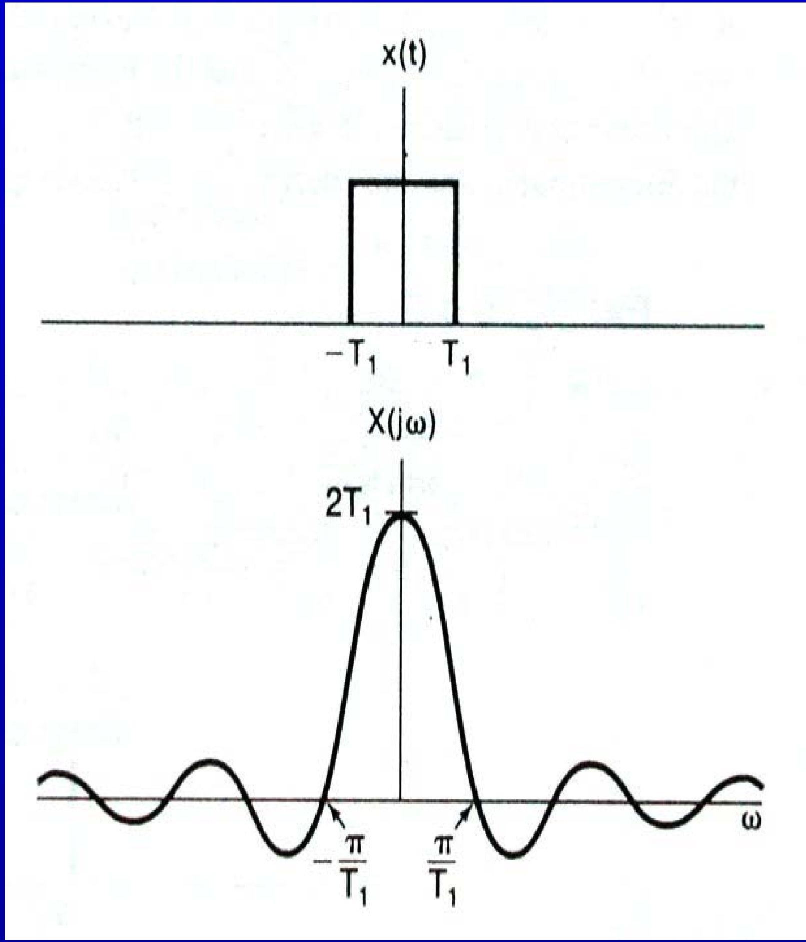
$$X(j\omega)=\int_{-\infty}^{+\infty}\delta(t)e^{-j\omega t}dt=1$$

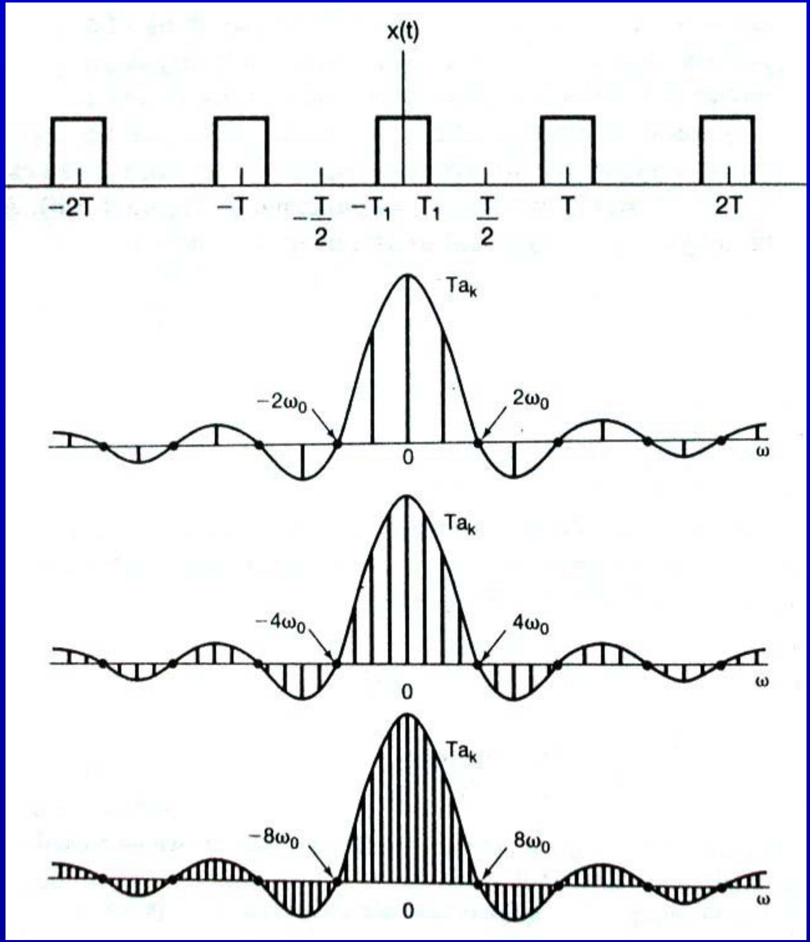
Example 4.4

$$x(t)=\begin{cases}1, & |t|<T_1\\ 0, & |t|>T_1\end{cases}\qquad \text{determine} \quad X(jw)$$



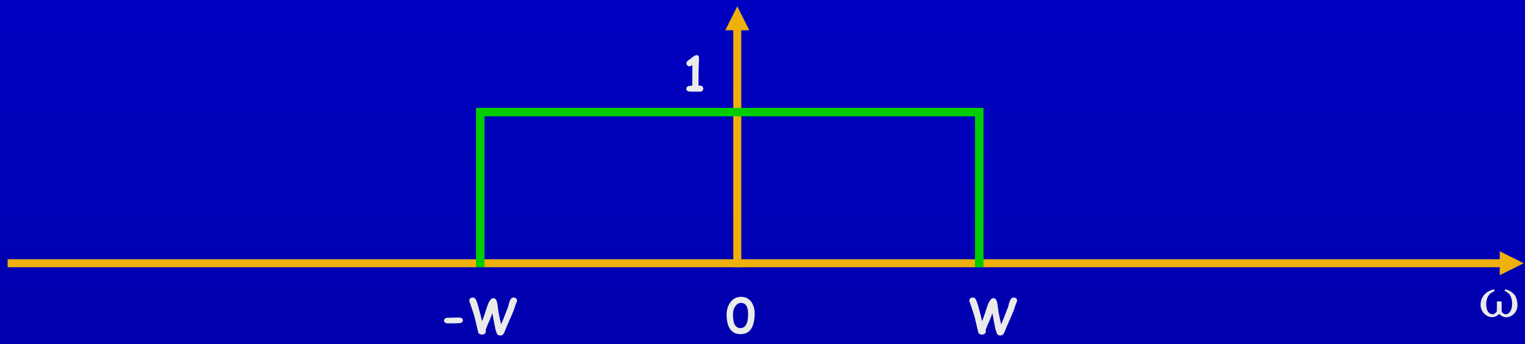
$$X(j\omega)=2\frac{\sin\omega T_1}{\omega}$$


x(t)
−T1
T1
X(jω)
2T1
ω
−π/T1
π/T1

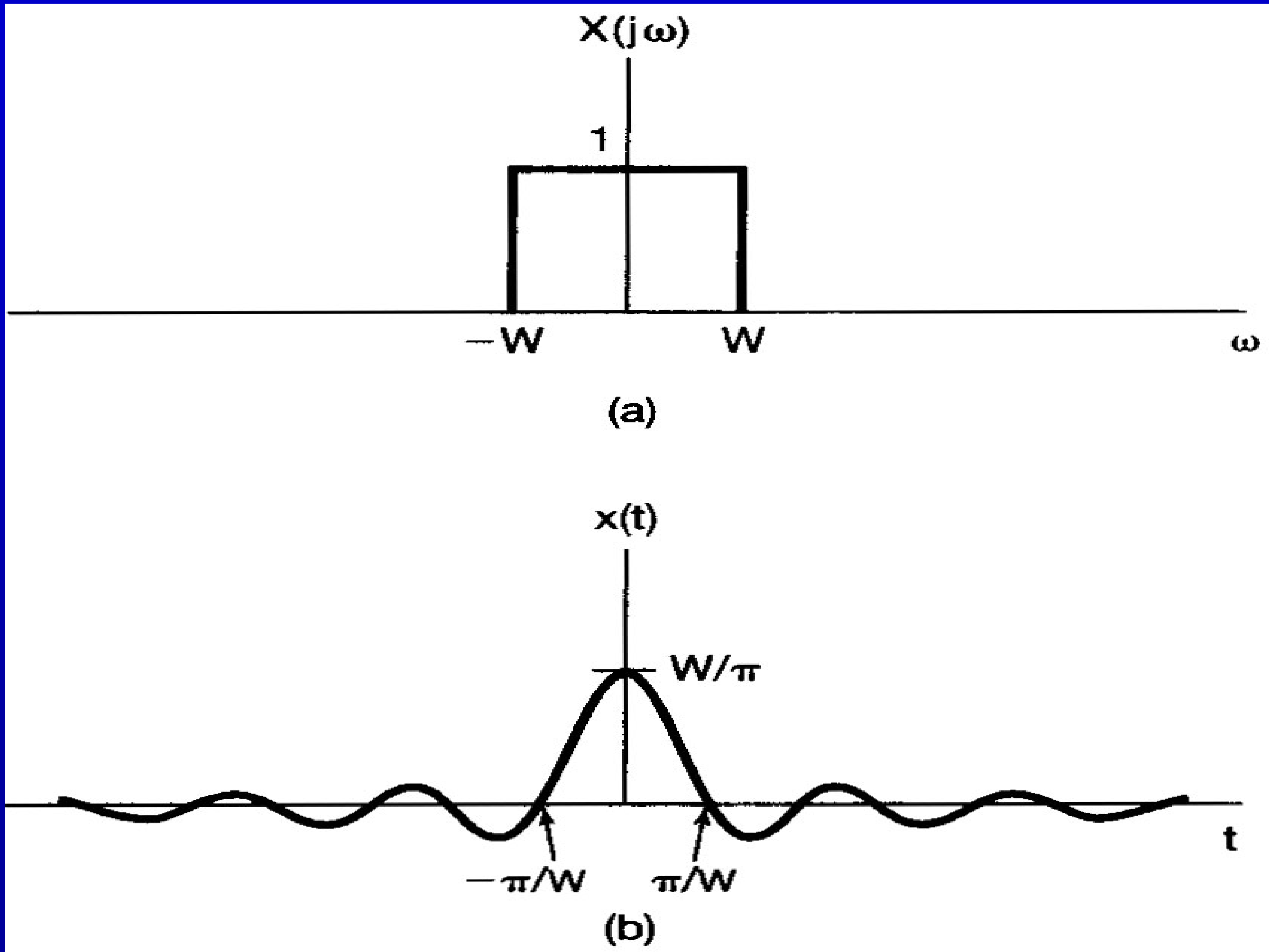

x(t)
−2T
−T
−T/2
−T1
T1
T/2
T
2T
Tak
−2ω0
2ω0
0
ω
Tak
−4ω0
4ω0
0
ω
Tak
−8ω0
8ω0
0
ω

Example 4.5

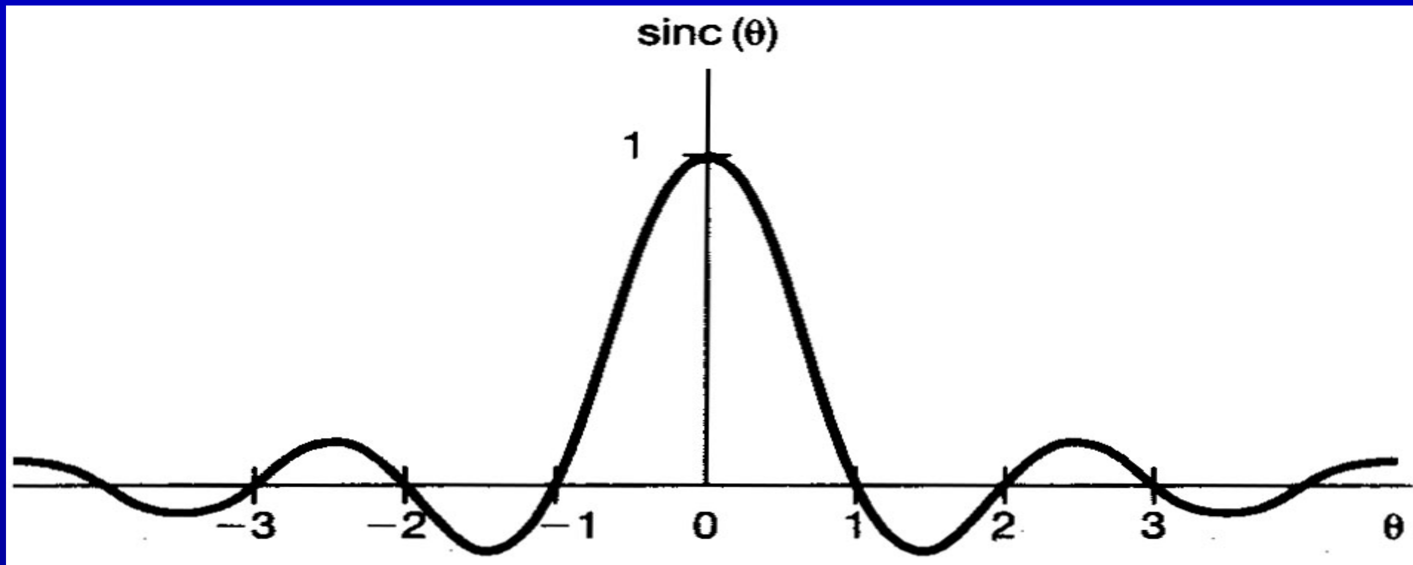
$$X(j\omega)=\begin{cases}1, & |\omega|<W\\0, & |\omega|>W\end{cases}\quad \text{determine } x(t)$$

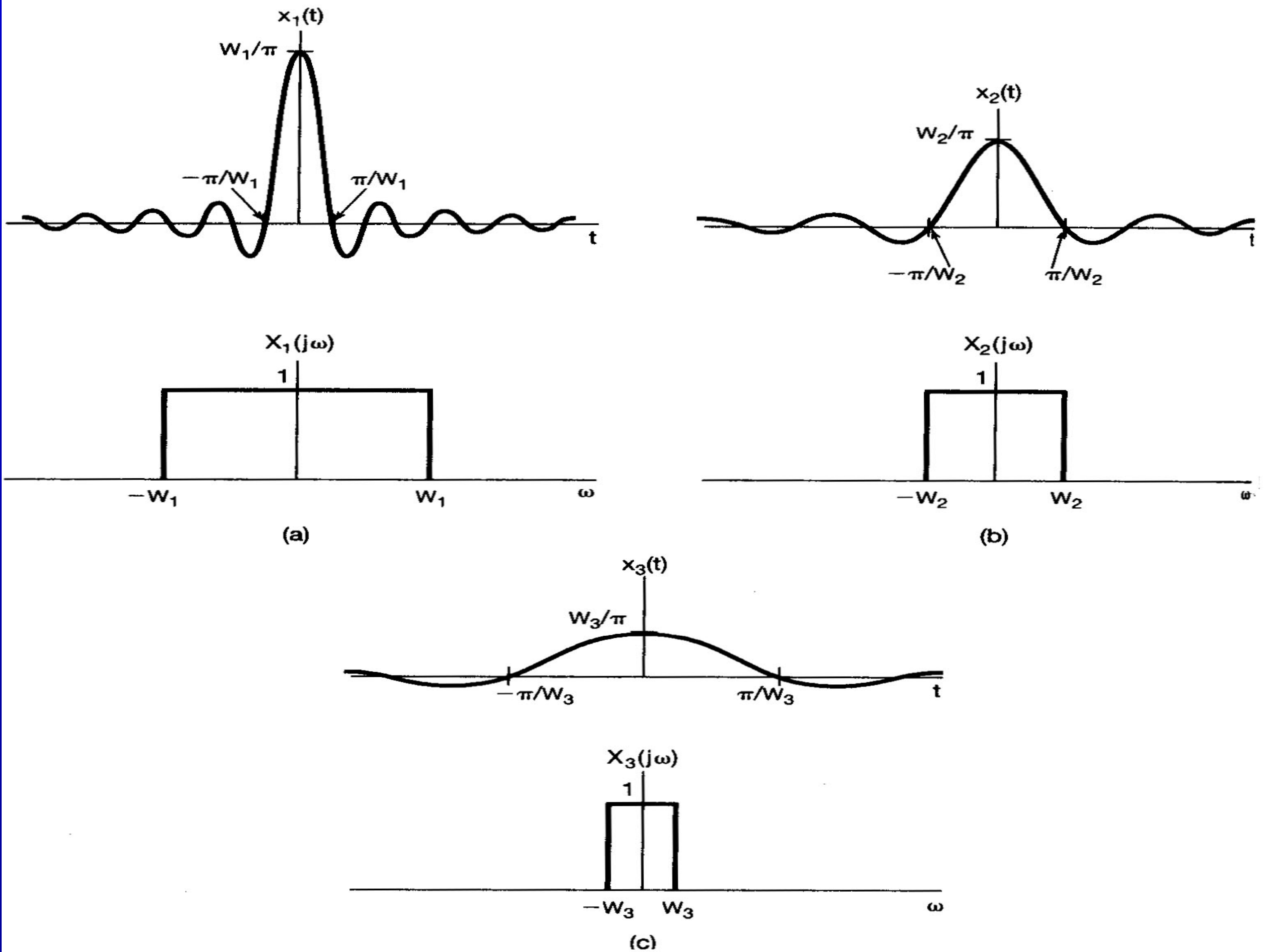


$$x(t)=\frac{\sin Wt}{\pi t}$$


X(jω)
1
−W
W
ω
(a)
x(t)
W/π
−π/W
π/W
t
(b)

$$\mathrm{sinc}(\theta) = \frac{\sin \pi\theta}{\pi\theta}$$





$x_1(t)$
W_1/π
$-\pi/W_1$
π/W_1
t
$X_1(j\omega)$
1
$-W_1$
W_1
ω
(a)
$x_2(t)$
W_2/π
$-\pi/W_2$
π/W_2
t
$X_2(j\omega)$
1
$-W_2$
W_2
ω
(b)
$x_3(t)$
W_3/π
$-\pi/W_3$
π/W_3
t
$X_3(j\omega)$
1
$-W_3$
W_3
ω
(c)

Signals & Systems

4.2 The Fourier transform for periodic signals

$$x(t)=\sum_{k=-\infty}^{+\infty}a_k e^{jk\omega_0 t}$$

$$F\{x(t)\}=?$$

The key: $F\{e^{jkw_0 t}\}=?$

$$2\pi\delta(\omega-\omega_0)\xleftrightarrow{F^{-1}T}e^{j\omega_0 t}$$

$$X(j\omega)=\sum_k 2\pi a_k\delta(\omega-k\omega_0)\leftrightarrow x(t)=\sum_k a_k e^{jk\omega_0 t}$$

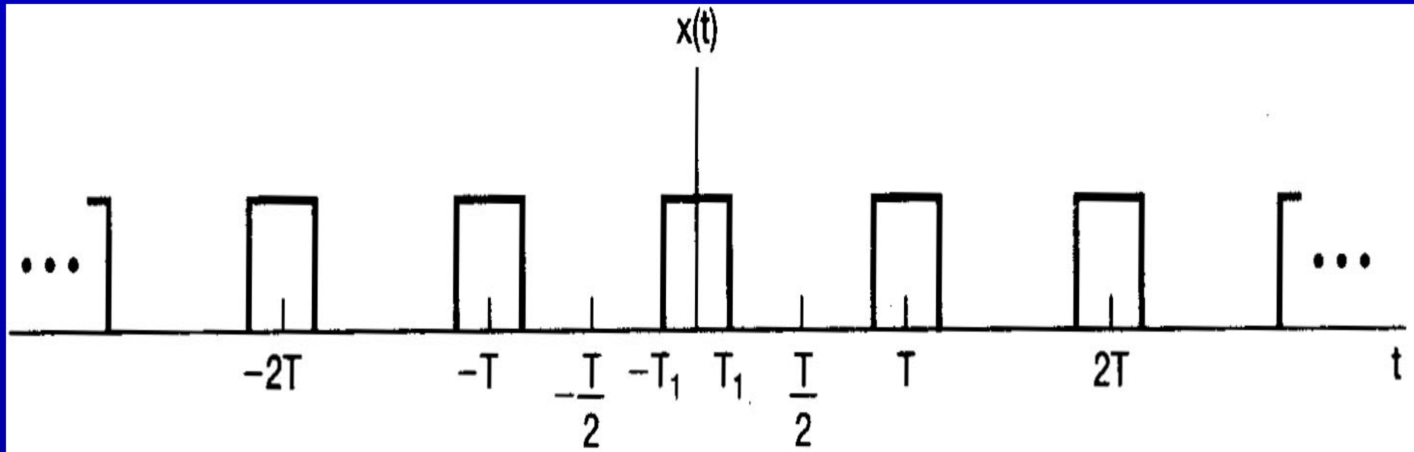
$$1\leftrightarrow 2\pi\delta(\omega)$$

Signals & Systems

Example 4.6

$$x(t)=\begin{cases}1, & |t|<T_1\\ 0, & T_1<|t|<T/2\end{cases} \qquad period\ is\ T$$

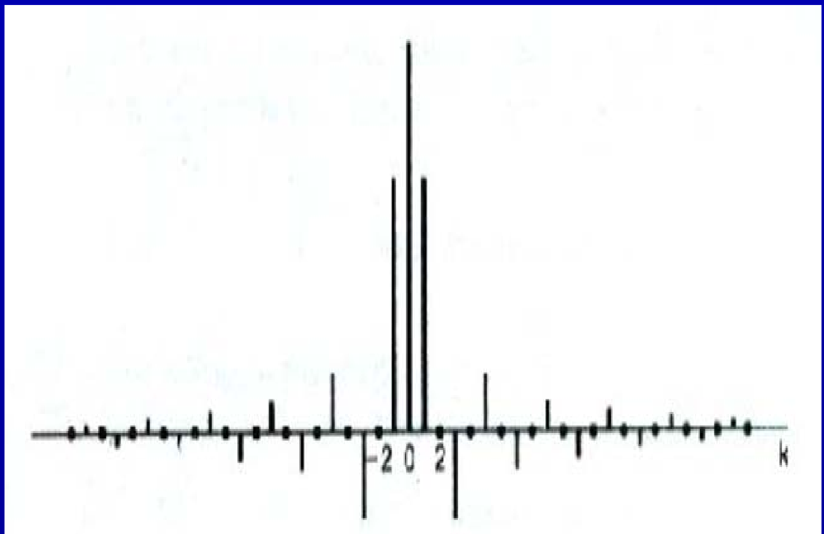
determine $X(jw)$

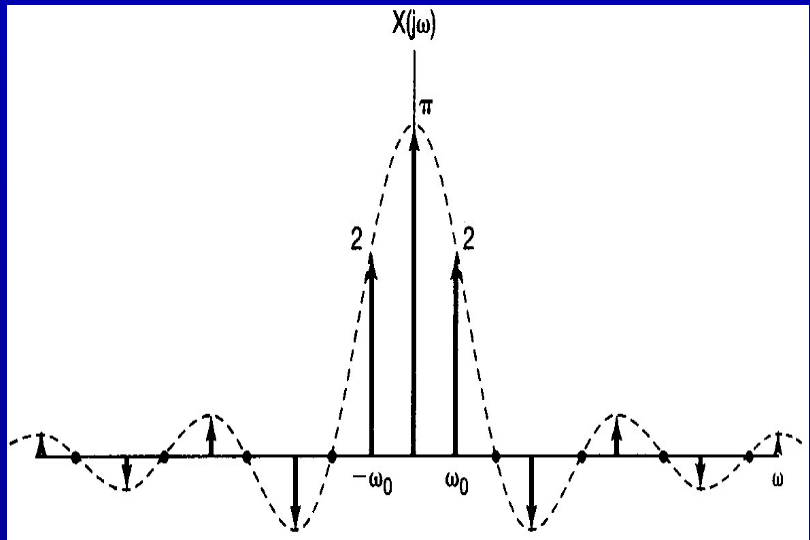


$$k=0 \quad a_0=\frac{2T_1}{T} \qquad k\neq 0 \quad a_k=\frac{2\sin\left(k\omega_0T_1\right)}{k\omega_0T}$$

$$k = 0 \quad a_0 = \frac{2T_1}{T} \qquad k \neq 0 \quad a_k = \frac{2\sin\left(k\omega_0 T_1\right)}{k\omega_0 T}$$

$$x(t) = \sum_{k=-\infty}^{+\infty} \frac{2\sin\left(k\omega_0 T_1\right)}{k\omega_0 T} e^{jk\omega_0 t} \quad X(j\omega) = \sum_{k=-\infty}^{+\infty} \frac{4\pi\sin(k\omega_0 T_1)}{k\omega_0 T}\delta(\omega - k\omega_0)$$





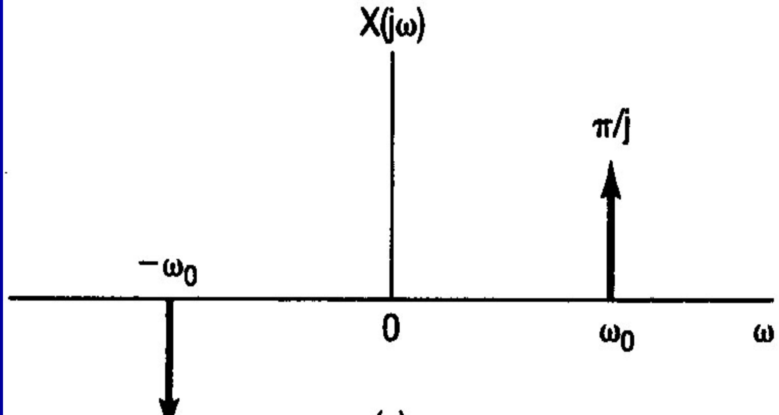
Example 4.7

$$x_1(t) = \sin w_0 t \qquad x_2(t) = \cos w_0 t$$

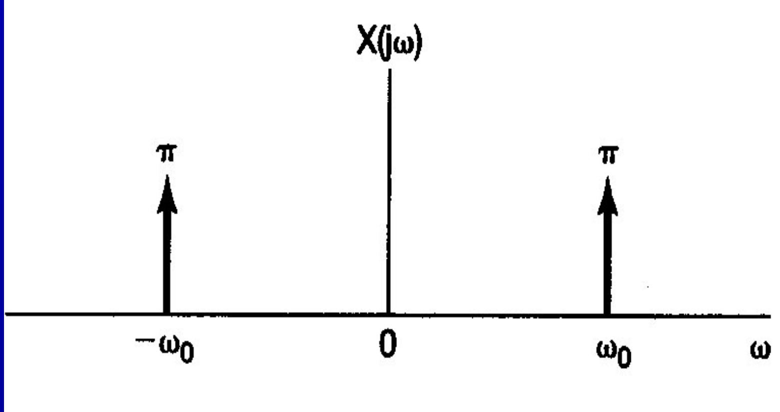
determine $X_1(jw)$ *and* $X_2(jw)$

$$X_1(jw) = \frac{\pi}{j}\delta(w - w_0) - \frac{\pi}{j}\delta(w + w_0)$$

$$X_2(jw) = \pi\delta(w - w_0) + \pi\delta(w + w_0)$$



(a)

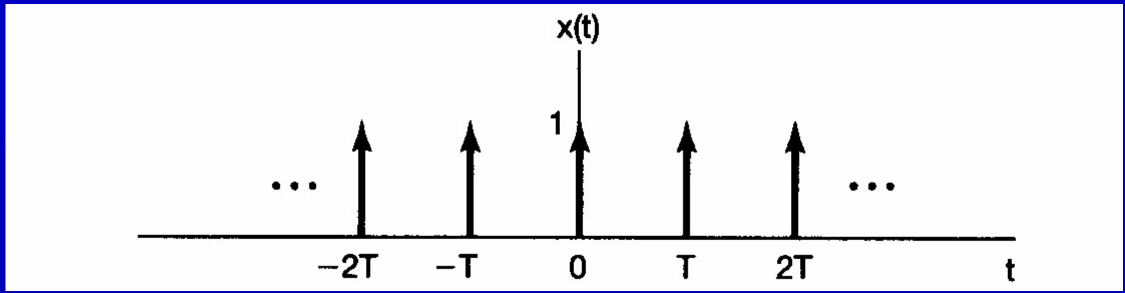


(b)

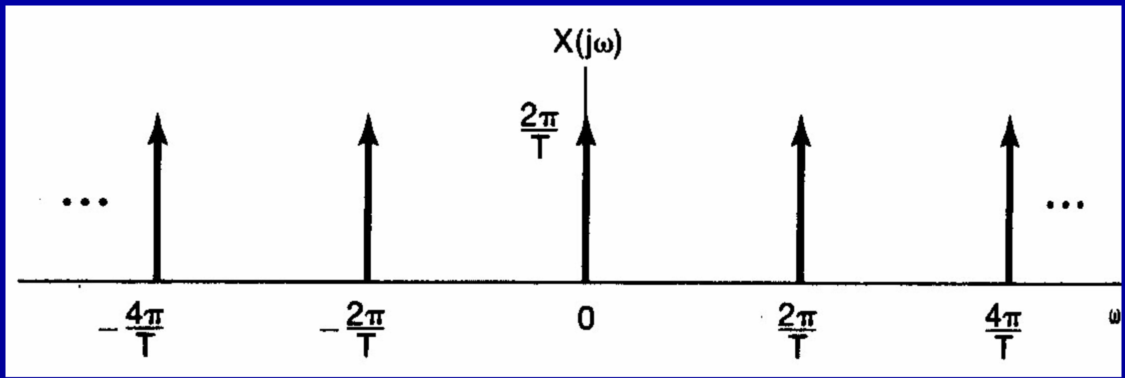
Signals & Systems

Example 4.8

$$x(t)=\sum_{k=-\infty}^{+\infty}\delta(t-kT),\text{determine } X(jw)$$



$$X(j\omega)=\frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta\left(\omega-\frac{2\pi k}{T}\right)$$



Signals & Systems

4.3 Properties of the continuous-time Fourier transform

4.3.1 Linearity

$$ax(t)+by(t)\xleftrightarrow{F}aX(j\omega)+bY(j\omega)$$

4.3.2 Time Shifting

$$x(t-t_0)\xleftrightarrow{F}e^{-j\omega t_0}X(j\omega)$$

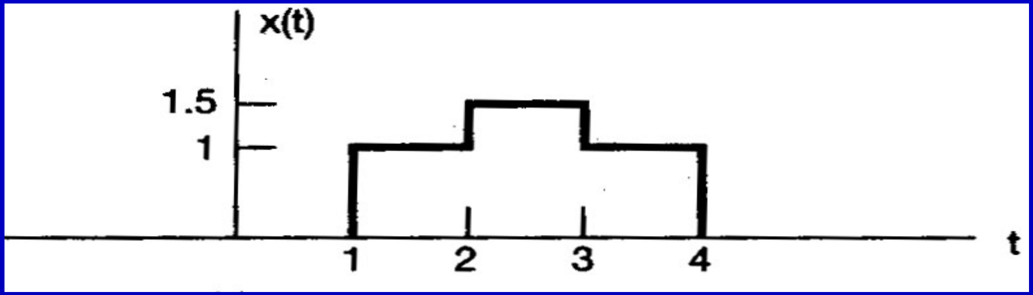
$$\left|X(j\omega)e^{-j\omega t_0}\right|=\left|X(j\omega)\right|$$

$$\measuredangle e^{-j\omega t_0}X(j\omega)=\measuredangle X(j\omega)-\omega t_0$$

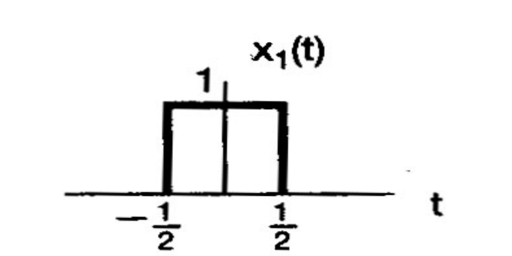
Signals & Systems

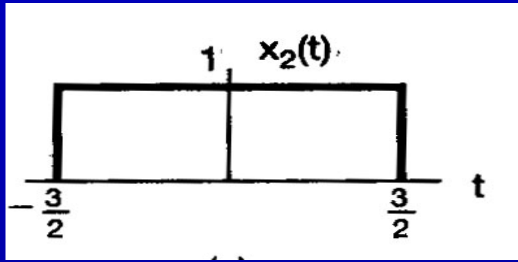
Example 4.9

Determine the FT of x(t)



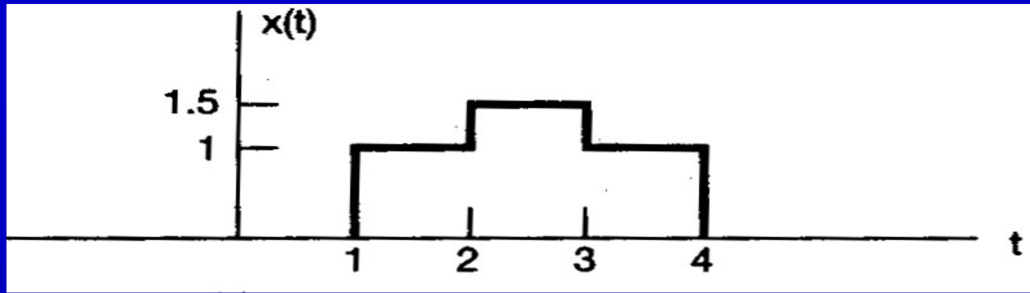
- **Method 1:**



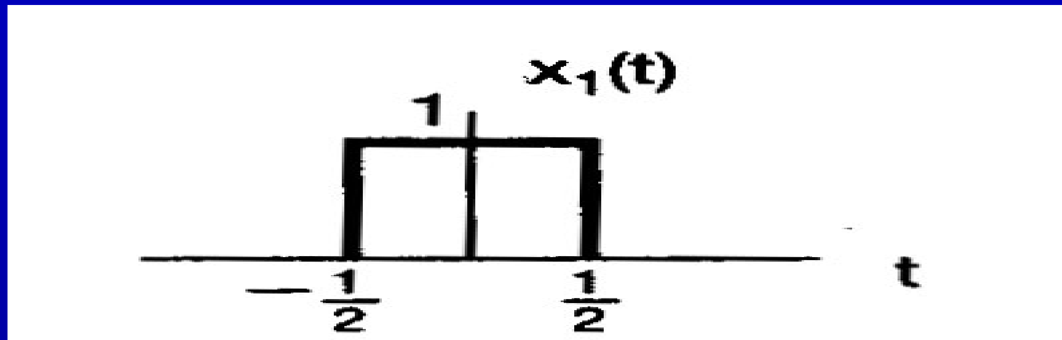


$$x(t)=\frac{1}{2}x_1(t-2.5)+x_2(t-2.5)$$

$$X(j\omega)=e^{-j5\omega/2}\left\{\frac{\sin(\omega/2)+2\sin(3\omega/2)}{\omega}\right\}$$



Method 2:



$$x(t) = x_1(t - \frac{3}{2}) + \frac{3}{2} x_1(t - \frac{5}{2}) + x_1(t - \frac{7}{2})$$

Signals & Systems

4.3.3 Conjugation and conjugate symmetry

$$x^*(t) \stackrel{F}{\longleftrightarrow} X^*(-j\omega)$$

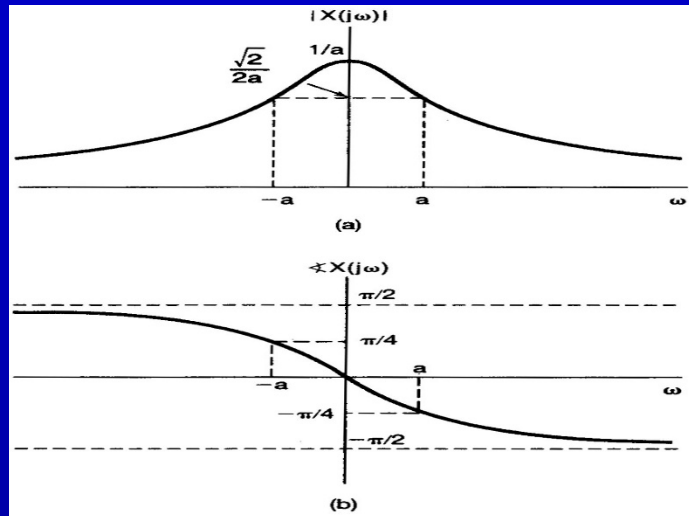
$If \;\; x(t) \; is \; real \quad x(t) = x^*(t)$

$$X(-j\omega) = X^*(j\omega)$$

Conjugation and conjugate symmetry

$$If\ \ x(t)\ is\ real\quad x(t) = x^*(t)$$

$$X(-j\omega) = X^*(j\omega)$$



Example

$$e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a + j\omega}$$

$$X(j\omega) = |X(j\omega)| \cdot e^{j\measuredangle X(j\omega)}$$

$$|X(j\omega)| = |X(-j\omega)|$$

$$\measuredangle X(j\omega) = -\measuredangle X(-j\omega)$$

x(t) is real and even

$$X(-j\omega) = X^*(j\omega)$$

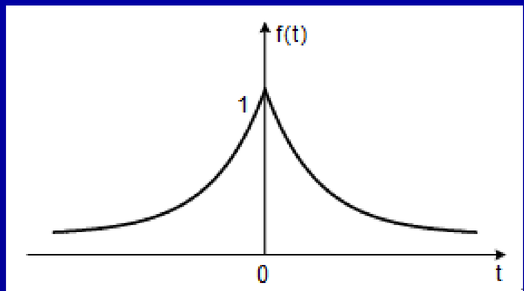
$$X(j\omega) = \mathrm{Re}\{X(j\omega)\} + j\,\mathrm{Im}\{X(j\omega)\}$$

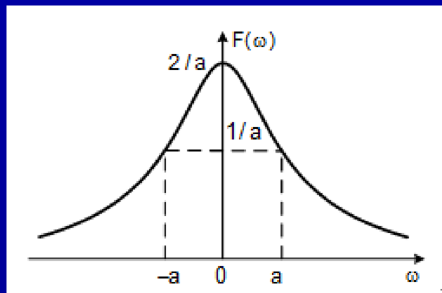
$$\boldsymbol{X(jw) = X(-jw) = X^*(jw)}$$

$$X(j\omega) = X(-j\omega) = \mathrm{Re}\{X(j\omega)\}$$

Example

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+\omega^2}$$





x(t) is real and odd

$$X(-j\omega) = X^*(j\omega)$$

$$X(j\omega) = \mathrm{Re}\{X(j\omega)\} + j\,\mathrm{Im}\{X(j\omega)\}$$

$$\boldsymbol{X(jw) = -X(-jw) = -X^*(jw)}$$

$$X(j\omega) = j\,\mathrm{Im}\{X(j\omega)\}$$

$$\mathrm{Im}\{X(j\omega)\} = -\mathrm{Im}\{X(-j\omega)\}$$

Signals & Systems

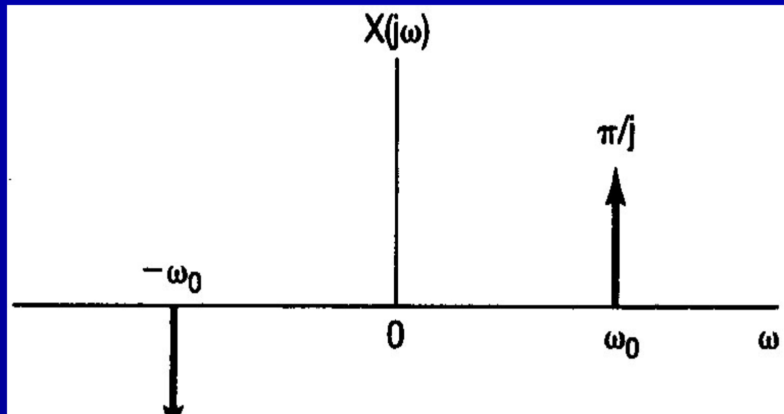
x(t) is real and odd

$$X(jw) = -X(-jw) = -X^*(jw)$$

$$X(j\omega) = j\,\mathrm{Im}\{X(j\omega)\}$$

$$\mathrm{Im}\{X(j\omega)\} = -\mathrm{Im}\{X(-j\omega)\}$$

$$\sin(\omega_0 t) \xleftrightarrow{FT} \frac{\pi}{j}(\delta(\omega-\omega_0)-\delta(\omega+\omega_0))$$



(a)

$$If\ \ x(t)\ is\ real\quad x(-t)\xleftrightarrow{FT}X(-j\omega)=X^{*}(j\omega)$$

$$x_e(t)=\frac{x(t)+x(-t)}{2}\qquad x_o(t)=\frac{x(t)-x(-t)}{2}$$

$$x_e(t)\xleftrightarrow{FT}\mathrm{Re}[X(j\omega)]$$

$$x_o(t)\xleftrightarrow{FT}j\,\mathrm{Im}[X(j\omega)]$$

Signals & Systems

Example 4.10

$$x(t)=e^{-a|t|} \quad a>0, \text{determine } X(jw)$$

$$x_e(t) \xleftrightarrow{FT} \mathrm{Re}[X(j\omega)]$$

$$x(t)=e^{-a|t|}=e^{-at}u(t)+e^{at}u(-t)$$

$$=2Ev\{e^{-at}u(t)\}$$

$$F\{x(t)\}=2\mathrm{Re}\left\{F\left\{e^{-at}u(t)\right\}\right\}$$

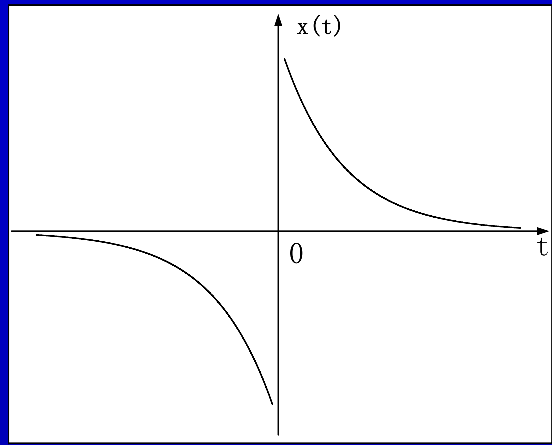
$$=2\mathrm{Re}\left\{\frac{1}{a+jw}\right\}$$

$$=\frac{2a}{a^2+w^2}$$

Signals & Systems

Example

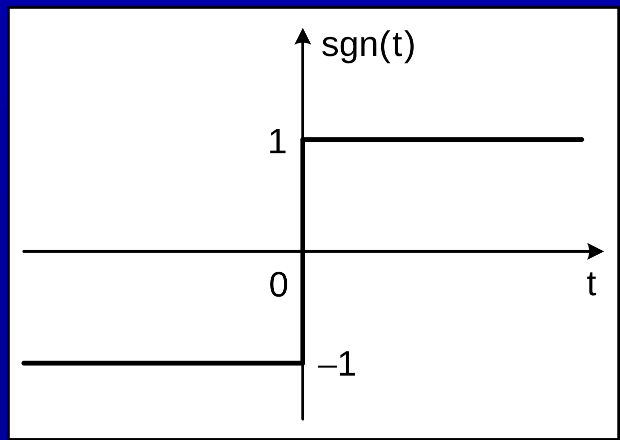
$$x(t) = e^{-at}u(t) - e^{at}u(-t)$$



$$x_o(t) \xleftrightarrow{FT} j\,\mathrm{Im}[X(j\omega)]$$

$$x(t) = e^{-at}u(t) - e^{at}u(-t) \xleftrightarrow{FT} \frac{-2jw}{a^2 + w^2}$$

$$a \to 0$$

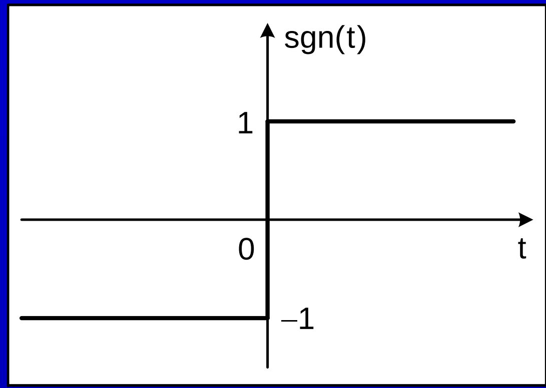


Example

Signals & Systems

Sign function

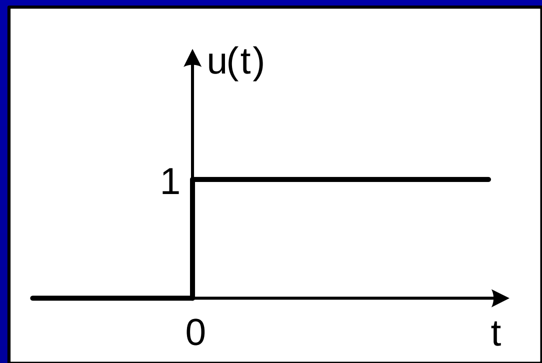
$$\operatorname{sgn}(t)=\begin{cases}1, & t>0\\ -1, & t<0\end{cases}$$



$$\operatorname{sgn}(t)\xleftrightarrow{FT}\lim_{a\to 0}\frac{-2j\omega}{a^2+\omega^2}=\frac{2}{j\omega}$$

$$u(t)=\frac{1}{2}(\operatorname{sgn}(t)+1)$$

$$u(t)\xleftrightarrow{FT}\frac{1}{j\omega}+\pi\delta(\omega)$$



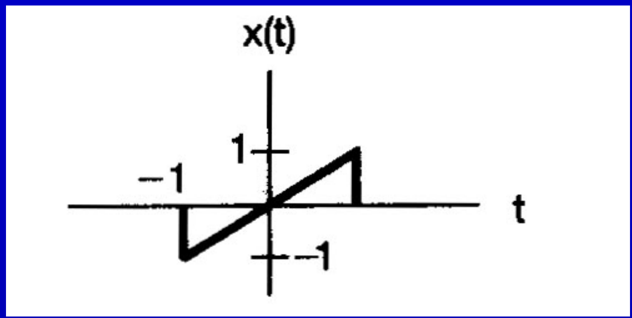
Signals & Systems

4.3.4 Differential and integral

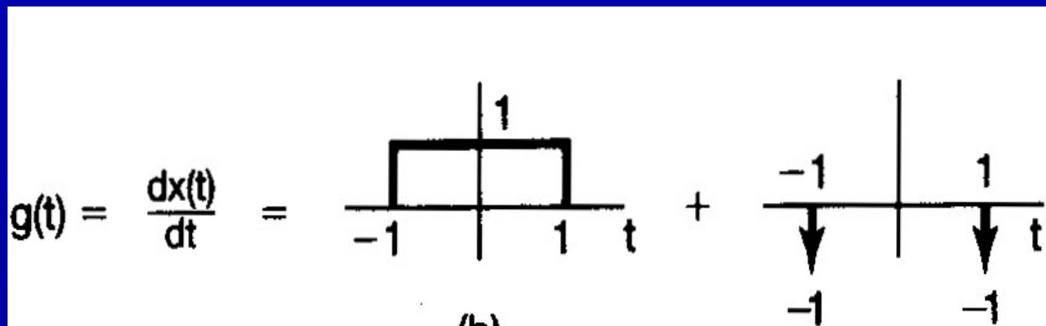
$$\frac{dx(t)}{dt} \xleftrightarrow{F} j\omega X(j\omega)$$

$$\int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{F} \frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$$

Example 4.12



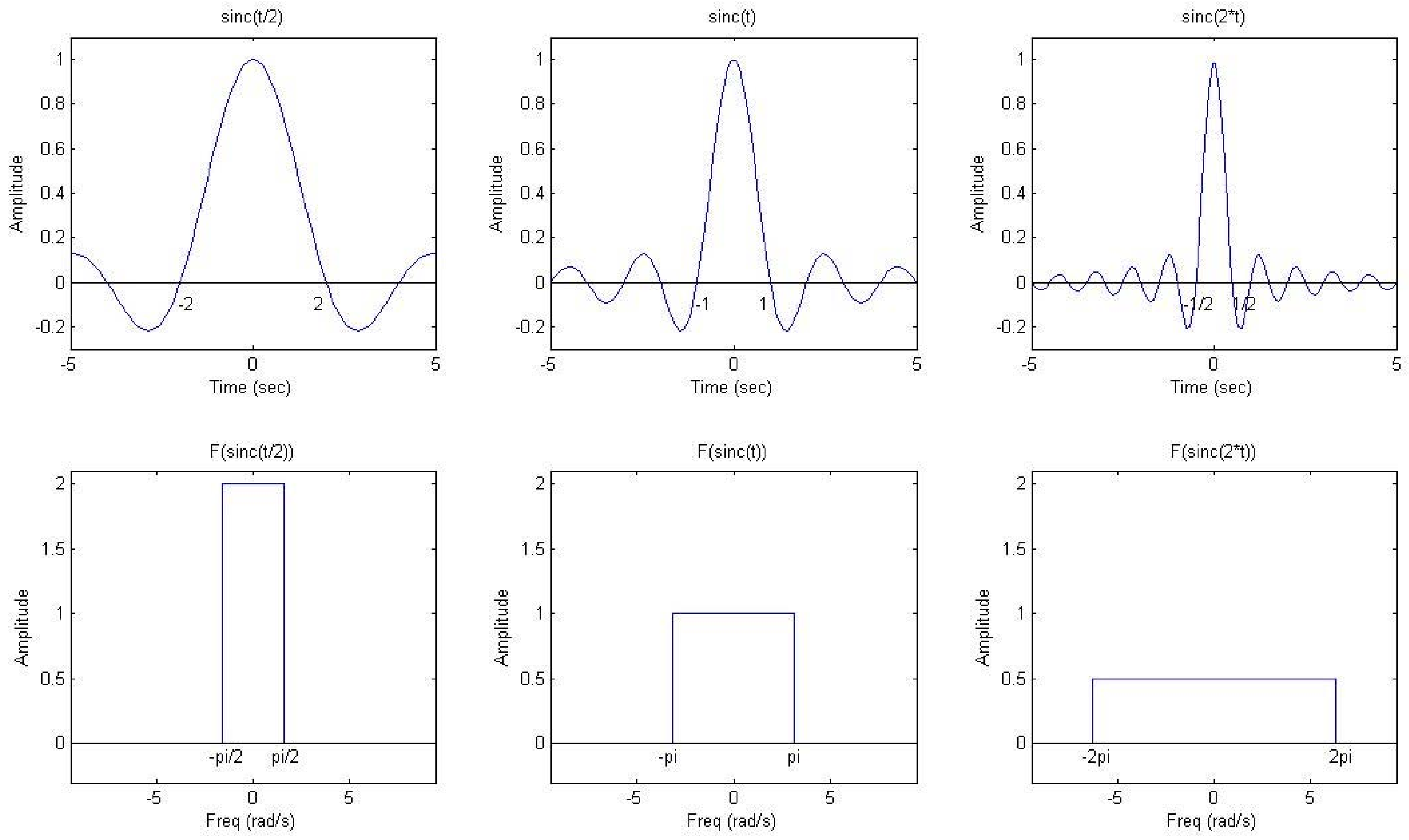
$\Rightarrow$



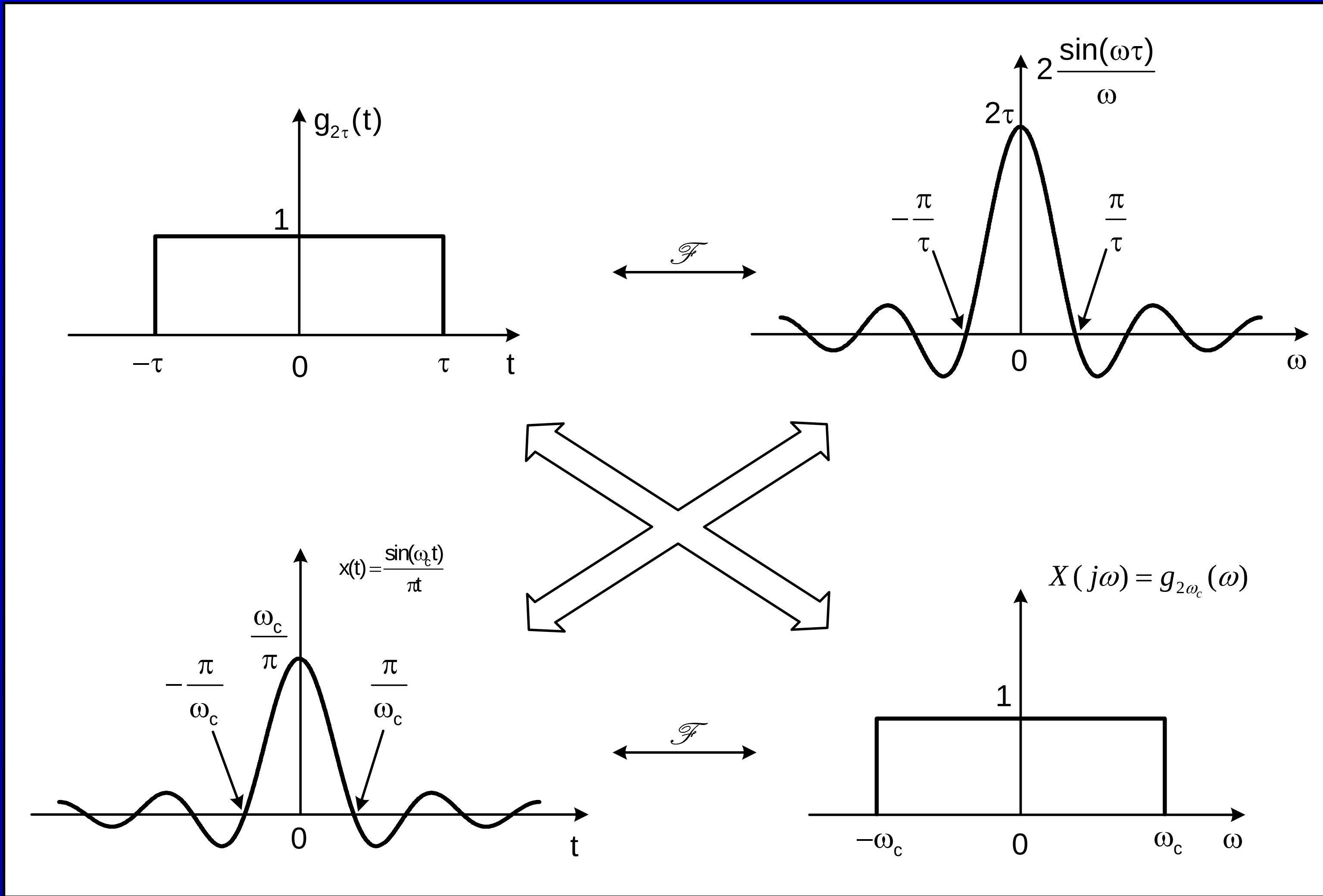
Signals & Systems

4.3.4 Time and frequency scaling

$$x(at) \overset{F}{\longleftrightarrow} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$$



4.3.5 Duality



$g_{2\tau}(t)$
1
$-\tau$
0
τ
t
$\mathscr{F}$
$2\frac{\sin(\omega\tau)}{\omega}$
2τ
$-\frac{\pi}{\tau}$
$\frac{\pi}{\tau}$
0
ω
$x(t)=\frac{\sin(\omega_c t)}{\pi t}$
$\frac{\omega_c}{\pi}$
$-\frac{\pi}{\omega_c}$
$\frac{\pi}{\omega_c}$
0
t
$\mathscr{F}$
$X(j\omega)=g_{2\omega_c}(\omega)$
1
$-\omega_c$
0
ω_c
ω

$$X(jt) \xleftrightarrow{F} 2\pi x(-w)$$

Example 4.13

$$g(t) = \frac{2}{1+t^2}, \text{determine } G(jw)$$

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+w^2}$$

$$\frac{2}{1+t^2} \xleftrightarrow{FT} 2\pi e^{-|w|}$$

Signals & Systems

Some useful properties

$$\begin{cases} -jtx(t) \xleftrightarrow{FT} \dfrac{dX(jw)}{dw} \\ \dfrac{dx(t)}{dt} \xleftrightarrow{FT} jwX(jw) \end{cases}$$

$$\begin{cases} e^{jw_0 t}x(t) \xleftrightarrow{FT} X\left(j\left(w-w_0\right)\right) \\ x\left(t-t_0\right) \xleftrightarrow{FT} e^{-jwt_0}X(jw) \end{cases}$$

$$\begin{cases} -\dfrac{1}{jt}x(t)+\pi x(0)\delta(t) \xleftrightarrow{FT} \int_{-\infty}^{w} X(\eta)d\eta \\ \int_{-\infty}^{t} x(\tau)d\tau \xleftrightarrow{FT} \dfrac{1}{jw}X(jw)+\pi X(0)\delta(w) \end{cases}$$

4.3.7 Parseval's relation

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(jw)|^2 dw$$

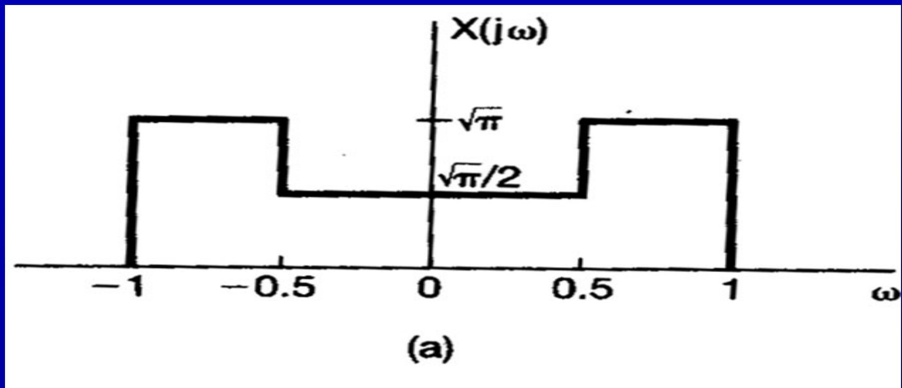
$|X(jw)|^2$ —— *energy density spectrum*

$X(jw)$ —— *frequency spectrum*

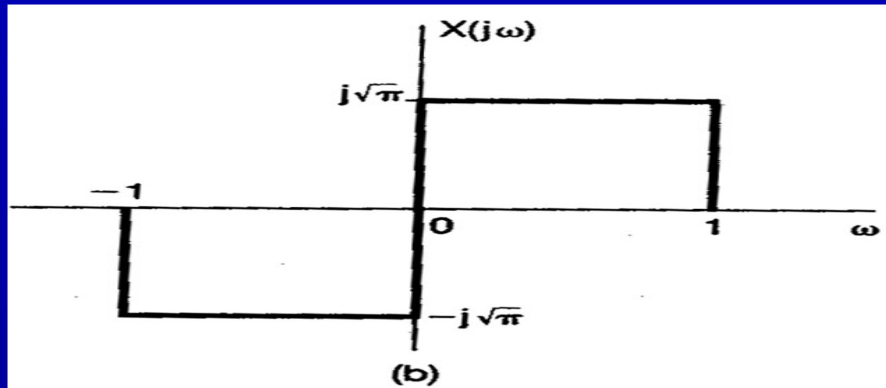
Example 4.14

Determine $E = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt$

$$D = \frac{d}{dt} x(t)\Big|_{t=0}$$



(a)



(b)

Signals & Systems

4.4 The convolution property

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = ?$$

$$y(t) = x(t) * h(t)$$

$$X(jw) \longrightarrow \boxed{H(jw)} \longrightarrow Y(jw) = ?$$

$$Y(jw) = X(jw)H(jw)$$

$$x(t) * y(t) \xleftrightarrow{FT} X(jw)Y(jw)$$

The response of cascade system

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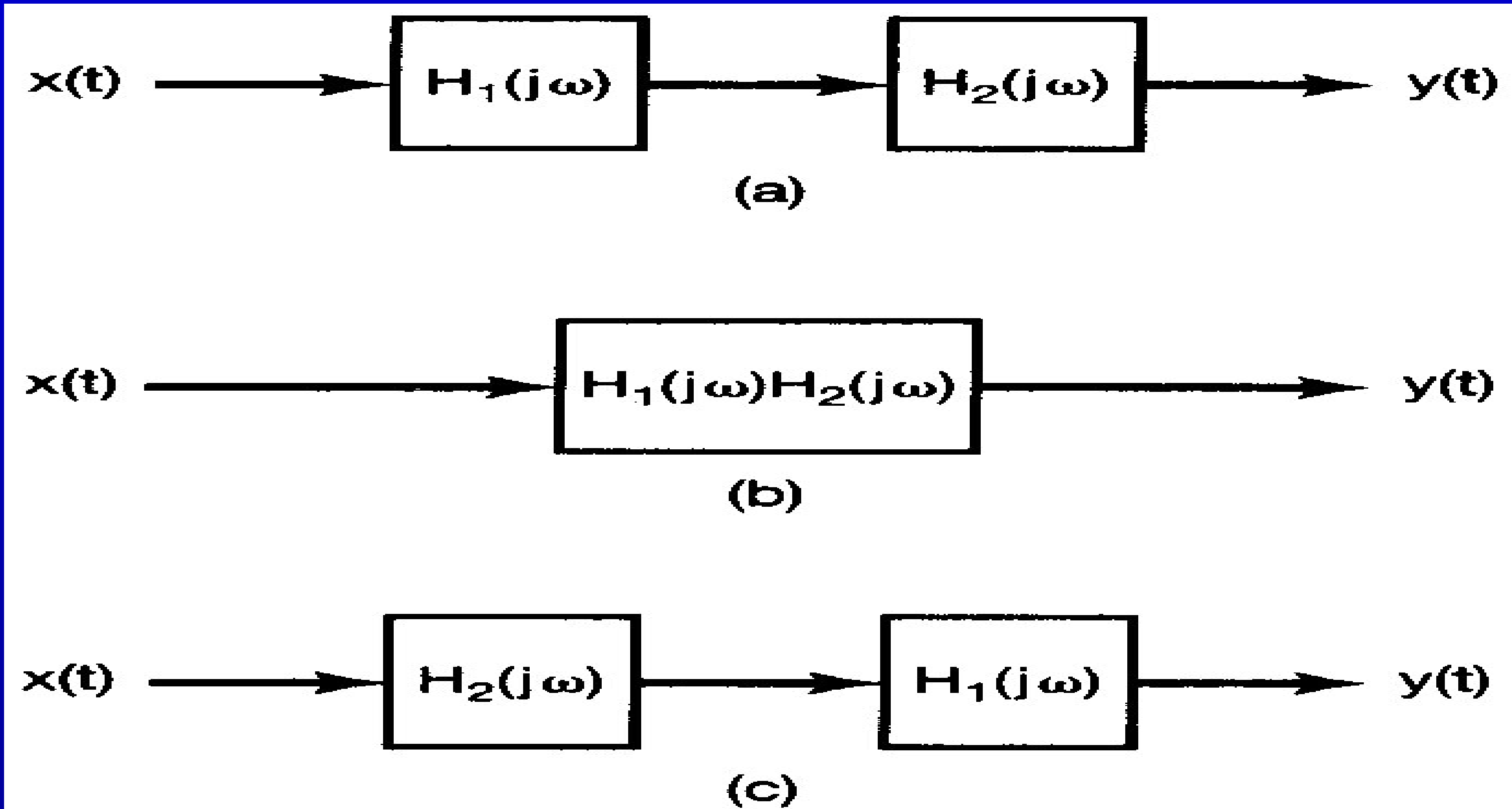


$$H(jw) = H_1(jw)H_2(jw)$$

The representation of LTI system

Signals & Systems

$$h(t) \xleftrightarrow{FT} H(jw)$$

Example 4.15

$An\ LTI\ system, h(t) = \delta(t - t_0)$, determine $y(t)$

Example 4.16

$$y(t) = \frac{dx(t)}{dt}, \text{determine } H(jw)$$

Example 4.19

$$x(t)=e^{-bt}u(t), b>0 \text{ and } h(t)=e^{-at}u(t), a>0$$

determine $y(t)$

$$x(t)=e^{-bt}u(t) \xleftrightarrow{FT} \frac{1}{b+jw}=X(jw)$$

$$h(t)=e^{-at}u(t) \xleftrightarrow{FT} \frac{1}{a+jw}=H(jw)$$

$$Y(jw)=X(jw)H(jw)=\frac{1}{a+jw}\frac{1}{b+jw}$$

$$\begin{cases} y(t)=\dfrac{1}{b-a}\left\{e^{-at}u(t)-e^{-bt}u(t)\right\} & a\neq b \\ y(t)=te^{-at}u(t) & a=b \end{cases}$$

Partial fraction expansion

$$H(w)=\frac{b_{n-1}w^{n-1}+b_{n-2}w^{n-2}+\cdots+b_1w+b_0}{w^n+a_{n-1}w^{n-1}+\cdots+a_1w+a_0}$$

- The roots of denominator polynomial are different

$$H(w)=\frac{A_1}{w-a}+\frac{A_2}{w-b}+\frac{A_3}{w-c}$$

$$(w-a)H(w)=A_1+\frac{A_2(w-a)}{w-b}+\frac{A_3(w-a)}{w-c}$$

$$A_1=(w-a)H(w)\Big|_{w=a}$$

- The roots of denominator polynomial are same

$$H(w)=\frac{A_{11}}{w-a}+\frac{A_{12}}{(w-a)^2}+\frac{A_{21}}{w-b}$$

$$(w-a)^2H(w)=A_{11}(w-a)+A_{12}+\frac{A_{21}(w-a)^2}{w-b}$$

$$\frac{d}{dw}\left[(w-a)^2H(w)\right]=A_{11}+A_{21}\left[\frac{2(w-a)(w-b)-(w-a)^2}{(w-b)^2}\right]$$

Example

$$x(t)=e^{-t}u(t),\ and\ h(t)=\frac{1}{2}e^{-t}u(t)+\frac{1}{2}e^{-3t}u(t)$$

determine $y(t)$

Example 4.20

$$x(t)=\frac{\sin w_i t}{\pi t}\ \textit{and}\ h(t)=\frac{\sin w_c t}{\pi t}\ \text{determine}\ y(t)$$

$$x(t)=\frac{\sin w_i t}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & |w|\le w_i\\ 0 & |w|>w_i\end{cases}=X(jw)$$

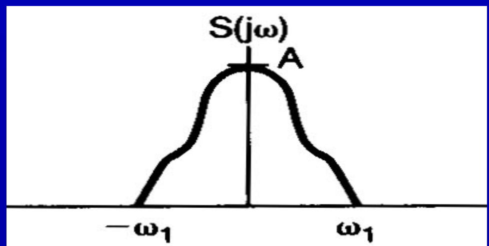
$$h(t)=\frac{\sin w_c t}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & |w|\le w_c\\ 0 & |w|>w_c\end{cases}=H(jw)$$

$$y(t)=\begin{cases}x(t) & w_i\le w_c\\ h(t) & w_i\ge w_c\end{cases}$$

4.5 The multiplication property

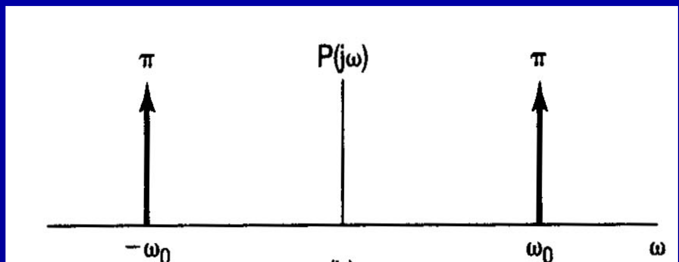
$$s(t)\cdot p(t) \xleftrightarrow{FT} \frac{1}{2\pi}[S(j\omega) * P(j\omega)]$$

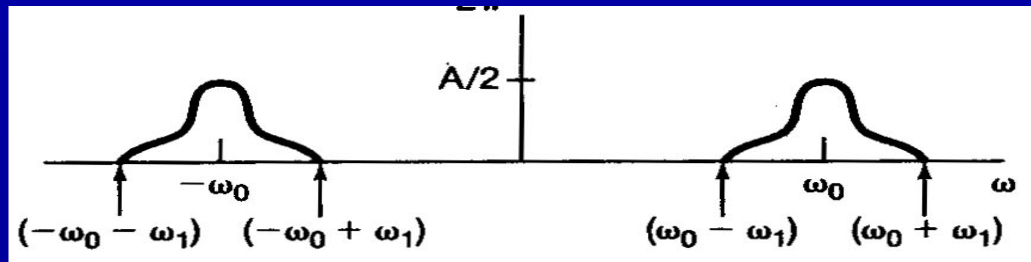
Example 4.21



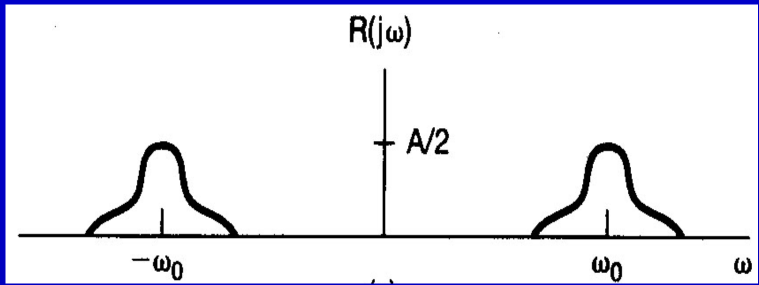
$p(t) = \cos w_0 t$

determine *the FT of* $p(t)s(t)$





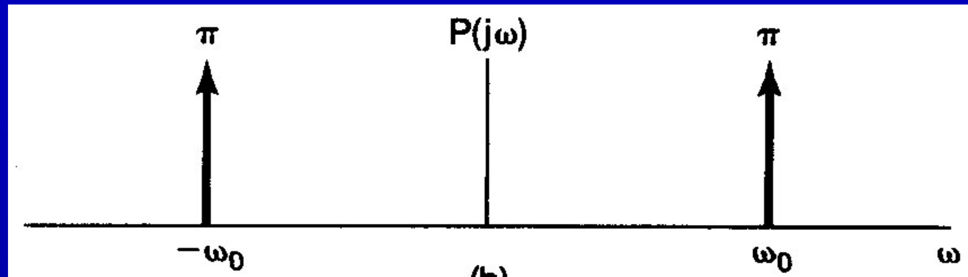
Example 4.22



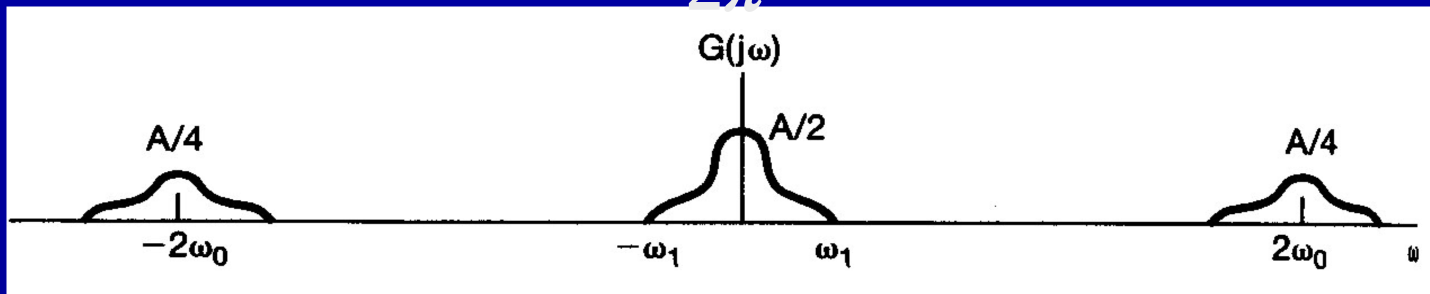
$$p(t) = \cos w_0 t$$

determine *the FT of* $p(t)r(t)$

$$p(t) \xleftrightarrow{FT}$$



$$r(t)\,p(t) \xleftrightarrow{FT} \frac{1}{2\pi} R(jw) * P(jw)$$



Example 4.23

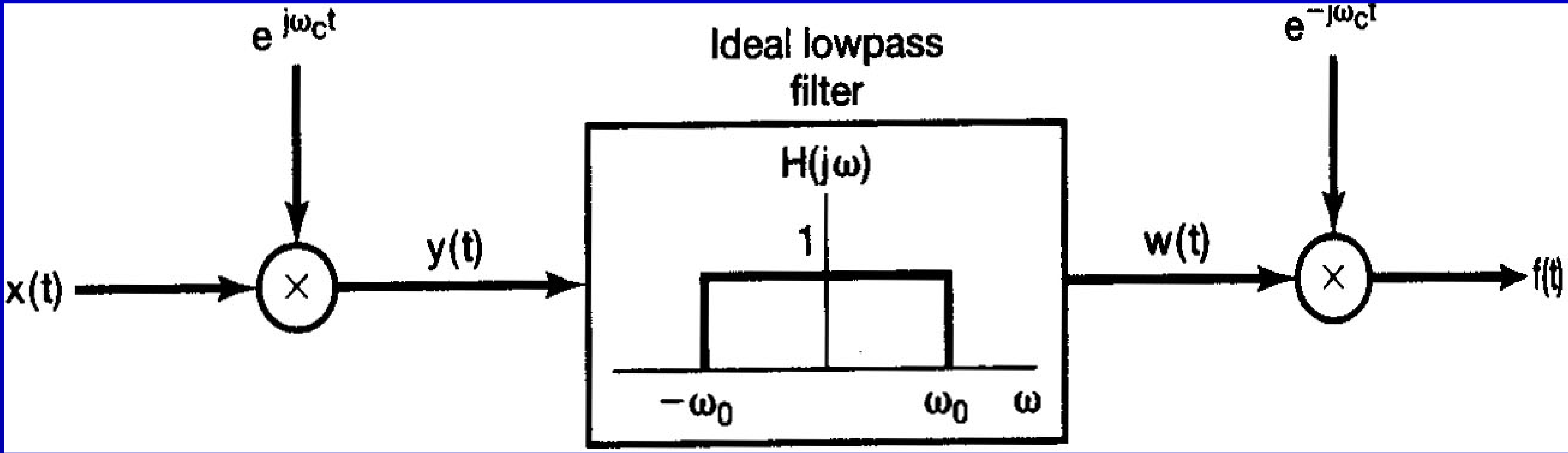
$$x(t)=\frac{\sin(t)\sin\left(\frac{t}{2}\right)}{\pi t^2},\text{determine } X(jw)$$

$$x_1(t)=\frac{\sin(t)}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & |w|\le 1\\ 0 & |w|>1\end{cases}=X_1(jw)$$

$$x_2(t)=\frac{\sin(\frac{1}{2}t)}{\pi t}\xleftrightarrow{FT}\begin{cases}1 & |w|\le \frac{1}{2}\\ 0 & |w|>\frac{1}{2}\end{cases}=X_2(jw)$$

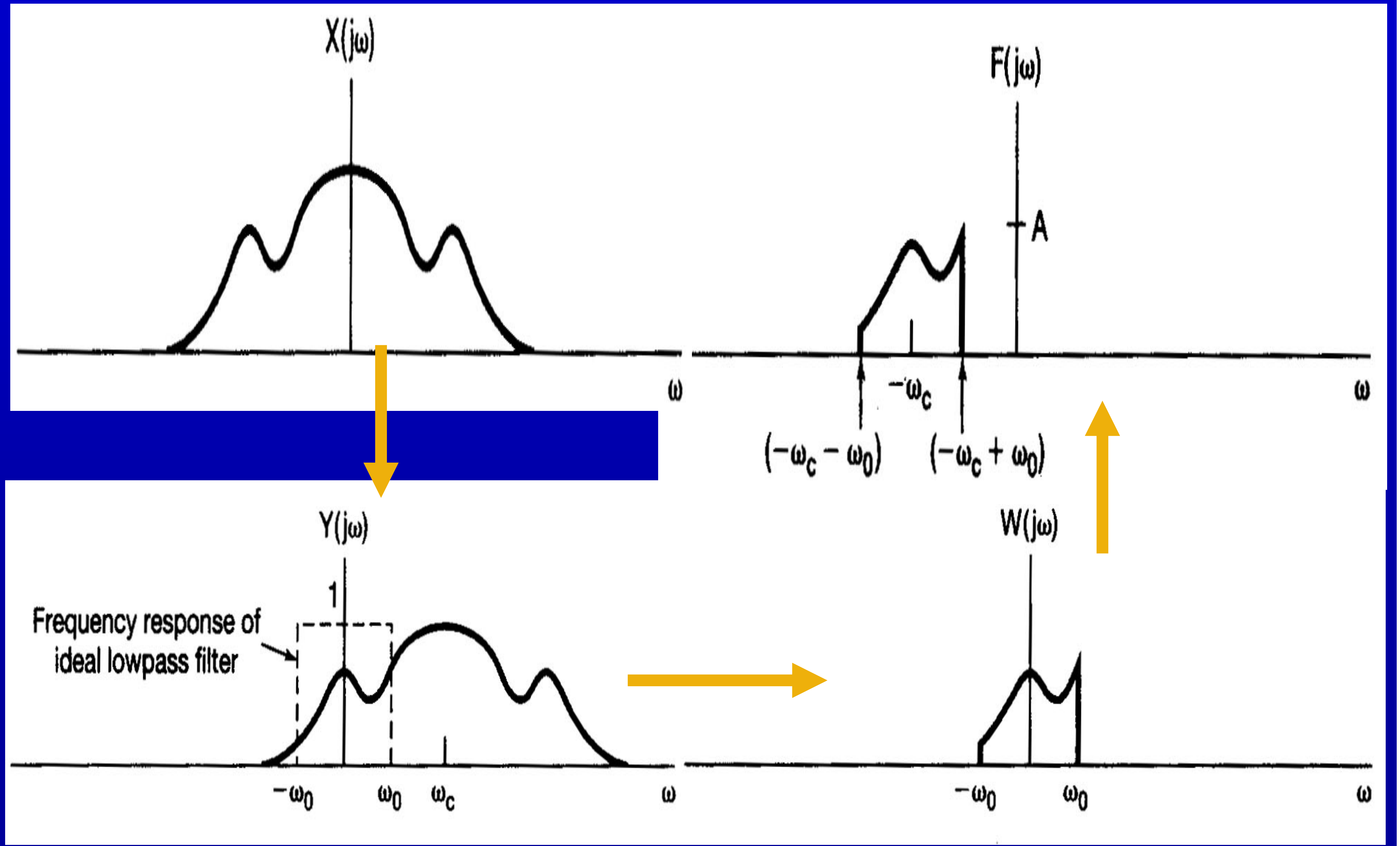
$$x(t)=\pi\cdot x_1(t)x_2(t)\xleftrightarrow{FT}\frac{1}{2\pi}\cdot\pi X_1(jw)*X_2(jw)$$

4.5.1 Frequency-selective filtering with variable center frequency



$$x(t)e^{jw_ct} = y(t) \xleftrightarrow{FT} Y(jw) = X\left(j\left(w - w_c\right)\right)$$

$$w(t)e^{-jw_ct} = f(t) \xleftrightarrow{FT} F(jw) = W\left(j\left(w + w_c\right)\right)$$


X(jω)
ω
F(jω)
A
−ω_c
(−ω_c − ω_0)
(−ω_c + ω_0)
ω
Y(jω)
1
Frequency response of
ideal lowpass filter
−ω_0
ω_0
ω_c
ω
W(jω)
−ω_0
ω_0
ω

4.7 System characterized by linear constant-coefficient differential equations

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+2x(t)$$

$$\sum_{k=0}^{2}a_k\frac{d^ky(t)}{dt^k}=\sum_{k=0}^{1}b_k\frac{d^kx(t)}{dt^k}$$

$$a_2=1,\ \ a_1=4,\ \ a_0=3$$
$$b_1=1,\ \ b_0=2$$

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

$$F\left\{\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k}\right\} = F\left\{\sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}\right\}$$

$$\sum_{k=0}^{N} a_k (jw)^k Y(jw) = \sum_{k=0}^{M} b_k (jw)^k X(jw)$$

$$H(jw) = \frac{Y(jw)}{X(jw)} = \frac{\sum_{k=0}^{M} b_k (jw)^k}{\sum_{k=0}^{N} a_k (jw)^k}$$

Example 4.24

$$\frac{dy(t)}{dt} + ay(t) = x(t) \quad a > 0, \text{determine } h(t)$$

Example 4.25

$$\frac{d^2y(t)}{dt^2} + 4\frac{dy(t)}{dt} + 3y(t) = \frac{dx(t)}{dt} + 2x(t)$$

determine $h(t)$

Signals & Systems

Example 4.26

$x(t)=e^{-t}u(t)$ and

$$\frac{d^2y(t)}{dt^2}+4\frac{dy(t)}{dt}+3y(t)=\frac{dx(t)}{dt}+2x(t)$$

determine $y(t)$

Example

let X(jw) denote the FT of the signal x(t), x(t) depicted as below

a) find $\angle X(jw)$

b) find $\int_{-\infty}^{+\infty} X(jw)dw = ?$

c) find $X(jw)\big|_{w=0} = ?$