Appendix A
Notations

For $X_i \in \mathbb{R}^{n_i \times m}$, let $\text{col}(X_1, \ldots, X_N) = [X_1^\top, \ldots, X_N^\top]^\top$. $A \otimes B$ denotes the Kronecker product of matrices A and B. $A > 0$ means that A is a positive definite matrix, and $A < 0$ means that A is a negative definite matrix.

We recall that a function $\eta : [0, a) \to [0, \infty), a \in \mathbb{R}^+$ is a class $\mathcal{K}$ function if it is continuous, strictly increasing and $\eta(0) = 0$. If a class $\mathcal{K}$ function η also satisfies $\eta(s) \to \infty$ as $s \to \infty$, then it is a class $\mathcal{K}_\infty$ function. A continuous function $\gamma : [0, a) \times [0, \infty) \to [0, \infty)$ is a class $\mathcal{KL}$ function if for each fixed t, the function $\gamma(\cdot, t)$ is a class $\mathcal{K}$ function, and for each fixed s, $\gamma(s, \cdot)$ is decreasing and satisfies $\gamma(s, t) \to 0$ as $t \to \infty$.

For a vector $x \in \mathbb{R}^n$, $\|x\|$ denote the Euclidean norm, and $\|x\|_\infty$ denote the infinity norm:

$$\|x\| = \left(\sum_{i=1}^{n} x_i^2 \right)^{1/2} = (x^\top x)^{1/2}$$

$$\|x\|_\infty = \max_{i=1,\ldots,n} |x_i|$$

Given a Lebesgue measurable signal $x(t) : I \to \mathbb{R}^n$, $\|x\|_{\mathcal{L}_p}^I$ denotes its $\mathcal{L}_p$ norm defined as [1], [2]:

$$\|x\|_{\mathcal{L}_p}^I = \left(\int_I \|x(t)\|_p^p dt \right)^{1/p} < \infty, p \in [1, \infty)$$

$$\|x\|_{\mathcal{L}_\infty}^I = \sup_{t \in I} \|x(t)\|_\infty$$

Lemma 1: (Young's Inequality [3]) If a and b are nonnegative real numbers and θ and q are positive real numbers such that $\frac{1}{\theta} + \frac{1}{q} = 1$, then $ab \leq \frac{a^\theta}{\theta} + \frac{b^q}{q}$.

Appendix B
Hautus Test for Stabilizability

In this appendix, we use the Hautus test to verify the stabilizability of the system represented by the state-space pair (A, B). The system dynamics are given by:

$$\dot{\xi}_i(t) = A\xi_i(t) + Bu_i(t) + D\omega_i(t)$$

where the matrix A is:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \otimes I_2$$

The characteristic equation $\det(A - \lambda I) = 0$ yields the eigenvalues:

$$\lambda_1 = 0, \quad \lambda_2 = 0$$

To test stabilizability for $\lambda = 0$, we construct the matrix $[A - \lambda I \mid B]$, where B is defined as:

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \otimes I_2$$

This gives the augmented matrix:

$$[A - \lambda I \mid B] = [A \mid B] = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

It can be checked that $\text{rank}([A - \lambda I \mid B]) = 4 = n$, where n denotes the number of states in the system. Thus, the system is stabilizable.

References

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