Theorem 1: The ISSS property of the platoon system can be achieved by the ILLC strategy if $\Upsilon = R^{-1}D^\top$ and $K = -B^\top \bar{E}^\top P$ with $D \triangleq BF$, $\bar{D} \triangleq \bar{E}D$, $\bar{B} \triangleq \bar{E}B$, and $0 < c_2 < \gamma_0$, $c_1 > 0$. The positive definite matrices P and R are, respectively, solutions to the following two matrix inequalities:

$$\begin{aligned} PA + A^\top P - c_1\sigma_i P\bar{B}\bar{B}^\top P + [\lambda_{\max}(P\bar{B}\bar{B}^\top P) \\ + \lambda_{\max}(P\bar{D}\bar{D}^\top P)]I_N < 0 \end{aligned}$$

$$R\mathcal{S} + \mathcal{S}^\top R - 2D^\top D + \lambda_{\max}^2(\mathcal{H})I_N < 0$$

By choosing $c_2 \geq \gamma_0$, the impact of the time-varying external disturbance $u_0(t)$ can be fully rejected, with P a solution to the following two matrix inequality:

$$PA + A^\top P - c_1\sigma_i P\bar{B}\bar{B}^\top P + \lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N < 0$$

Proof. Substituting $K = -B^\top \bar{E}^\top P$ into (14):

$$\begin{aligned} \dot{V}_1 =& {\delta^\xi}^\top \left[I_N \otimes (A^\top \bar{E}^\top P + P\bar{E}A) - c_1\Sigma \otimes P\bar{E}BB^\top \bar{E}^\top P\right]\delta^\xi \\ & - 2{\delta^\xi}^\top(\mathcal{A}_0\mathbf{1}_N \otimes P\bar{E}B)u_0 + 2c_2{\delta^\xi}^\top(\mathcal{H} \otimes P\bar{E}B)G(\delta^\xi) \\ & + 2{\delta^\xi}^\top(\mathcal{H} \otimes P\bar{E}D)\delta^\omega + 2{\delta^\omega}^\top[I_N \otimes (R\mathcal{S} - R\Upsilon D)]\delta^\omega \end{aligned} \tag{15}$$

where $\Sigma \triangleq \mathcal{H}^\top + \mathcal{H}$ is positive definite. Denote the eigenvalues of Σ by σ_i. Since Σ is symmetric and positive definite, there exists a unitary matrix M such that $M^\top \Sigma M = \mathrm{diag}\{\sigma_1, \sigma_2, \cdots, \sigma_N\}$. Define a state transformation $\epsilon(t) = (M^\top \otimes I_n)\delta^\xi(t)$, where $\epsilon = \mathrm{col}(\epsilon_1, \cdots, \epsilon_N)$. We then analyze each term of (15) separately:

$$\begin{aligned} -2{\delta^\xi}^\top(\mathcal{A}_0\mathbf{1}_N \otimes P\bar{E}B)u_0 =& -2\sum_{i=1}^N a_{i0}{\delta_i^\xi}^\top P\bar{E}Bu_0 \\ \leq& \quad 2\sum_{i=1}^N a_{i0}\left\|B^\top \bar{E}^\top P\delta_i^\xi\right\| \|u_0\|_\infty \\ \leq& \quad 2\gamma_0\sum_{i=1}^N a_{i0}\left\|B^\top \bar{E}^\top P\delta_i^\xi\right\| \end{aligned}$$

From Young's inequality, $\bar{D} \triangleq \bar{E}D$, $\bar{B} \triangleq \bar{E}B$, we have:

$$\begin{aligned} 2{\delta^\xi}^\top(\mathcal{H}\otimes P\bar{D})\delta^\omega \leq& {\delta^\xi}^\top(I_N\otimes P\bar{D}\bar{D}^\top P)\delta^\xi + \lambda_{\max}^2(\mathcal{H})I_N{\delta^\omega}^\top\delta^\omega \\ \leq& \lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N{\delta^\xi}^\top\delta^\xi + \lambda_{\max}^2(\mathcal{H})I_N{\delta^\omega}^\top\delta^\omega \end{aligned}$$

According to (**??**), for $K\delta_i^\xi(t) \neq 0$, the following holds:

$$\begin{aligned} {\delta_i^\xi}^\top P\bar{E}B\hat{g}\left(\delta_i^\xi\right) =& -\frac{{\delta_i^\xi}^\top P\bar{E}BB^\top \bar{E}^\top P\delta_i^\xi}{\|B^\top \bar{E}^\top P\delta_i^\xi\|} \\ =& -\left\|B^\top \bar{E}^\top P\delta_i^\xi\right\| \end{aligned} \tag{16}$$

By the well-known Cauchy-Schwarz inequality, we can obtain:

$$\begin{aligned}
-\delta_i^{\xi\top} P\bar{E}B\hat{\mathrm{g}}\left(\delta_j^\xi\right) &\leq \left|\delta_i^{\xi\top} P\bar{E}B\hat{\mathrm{g}}\left(\delta_j^\xi\right)\right| \\
&\leq \left\|B^\top \bar{E}^\top P\delta_i^\xi\right\| \left\|\hat{\mathrm{g}}\left(\delta_j^\xi\right)\right\| \\
&\leq \left\|B^\top \bar{E}^\top P\delta_i^\xi\right\|
\end{aligned} \tag{17}$$

According to (16) and (17), we have $\delta_i^{\xi\top} P\bar{E}B\hat{\mathrm{g}}\left(\delta_i^\xi\right) - \delta_i^{\xi\top} P\bar{E}B\hat{\mathrm{g}}\left(\delta_j^\xi\right) \leq 0$. Therefore, we can conclude:

$$\begin{aligned}
&2c_2\delta^{\xi\top}(\mathcal{H}\otimes P\bar{E}B)G(\delta^\xi) \\
&= 2c_2\sum_{i=1}^N \delta_i^{\xi\top} P\bar{E}B\sum_{j=0}^N a_{ij}\left(\hat{\mathrm{g}}\left(\delta_i^\xi\right) - \hat{\mathrm{g}}\left(\delta_j^\xi\right)\right) \\
&= 2c_2\sum_{i=1}^N \delta_i^{\xi\top} P\bar{E}B\left[a_{i0}\left(\hat{\mathrm{g}}\left(\delta_i^\xi\right) - \hat{\mathrm{g}}\left(\delta_0^\xi\right)\right)\right. \\
&\quad \left. + a_{i1}\left(\hat{\mathrm{g}}\left(\delta_i^\xi\right) - \hat{\mathrm{g}}\left(\delta_1^\xi\right)\right) + \cdots + a_{iN}\left(\hat{\mathrm{g}}\left(\delta_i^\xi\right) - \hat{\mathrm{g}}\left(\delta_N^\xi\right)\right)\right] \\
&\leq -2c_2\sum_{i=1}^N a_{i0}\left\|B^\top \bar{E}^\top P\delta_i^\xi\right\|
\end{aligned}$$

When $c_2 \geq \gamma_0$, choose c_1 and a positive definite matrix P such that

$$\begin{aligned}
&P\bar{E}A + A^\top \bar{E}^\top P + \lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N \\
&\qquad - c_1\sigma_i P\bar{B}\bar{B}^\top P = -Q_1 < 0,
\end{aligned}$$

let $\Upsilon \triangleq R^{-1}D^\top$, and

$$R\mathcal{S} + \mathcal{S}^\top R - 2D^\top D + \lambda_{\max}^2(\mathcal{H})I_N = -\hat{Q}_1 < 0.$$

Note that $\|\epsilon\| = \|\delta^\xi\|$, thus:

$$\begin{aligned}
\dot{V}_1 \leq\ & \delta^{\xi\top}\left[I_N\otimes(A^\top \bar{E}^\top P + P\bar{E}A) + I_N\otimes\lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N\right. \\
& \left. - c_1\Sigma\otimes P\bar{B}\bar{B}^\top P\right]\delta^\xi + 2(\gamma_0 - c_2)\sum_{i=1}^N a_{i0}\left\|B^\top \bar{E}^\top P\delta_i^\xi\right\| \\
& + \delta^{\omega\top}\left[I_N\otimes(R\mathcal{S} + \mathcal{S}^\top R - R\Upsilon D - D^\top \Upsilon^\top R\right. \\
& \left. + \lambda_{\max}^2(\mathcal{H})I_N)\right]\delta^\omega \\
\leq\ & \sum_{i=1}^N \epsilon_i^\top\left[P\bar{E}A + A^\top \bar{E}^\top P + \lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N\right. \\
& \left. - c_1\sigma_i P\bar{B}\bar{B}^\top P\right]\epsilon_i + \delta^{\omega\top}\left[I_N\otimes(R\mathcal{S} + \mathcal{S}^\top R - 2D^\top D\right. \\
& \left. + \lambda_{\max}^2(\mathcal{H})I_N)\right]\delta^\omega \\
\leq & -\sum_{i=1}^N \epsilon_i^\top Q_1\epsilon_i + \delta^{\omega\top}\left[I_N\otimes(R\mathcal{S} + \mathcal{S}^\top R - 2D^\top D\right. \\
& \left. + \lambda_{\max}^2(\mathcal{H})I_N)\right]\delta^\omega
\end{aligned}$$

where $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimum and maximum eigenvalues of corresponding matrix. Thus $\delta^{\xi}(t) \to 0$ and $\delta^{\omega}(t) \to 0$, i.e., the tracking discrepancy converge to zero, and disturbance observer error converge to zero. In this case, regardless of the bounded acceleration form of the leading CAV, the following HDVs can track the trajectory of the leading CAV under any initial conditions of $\delta^{\xi}(0)$ and $\delta^{\omega}(0)$. Furthermore, the disturbance observer $\hat{\omega}_i(t)$ completely rejects external disturbances.

Another case considers the control strategy under the condition $0 < c_2 < \gamma_0$. In this case, c_2 attenuates, rather than completely rejects, the impact of the time-varying input (external disturbance) u_0. Assume $c_2 = \kappa_1\gamma_0$ and $\|u_0(t)\| = \kappa_2(t)\gamma_0$, where $\kappa_1 \in (0, 1)$ and $\kappa_2(t) \in [0, 1]$. Then,

$$\begin{aligned}\|u_0(t)\| - c_2 =& \|u_0(t)\| - \frac{\kappa_1}{\kappa_2(t)}\|u_0(t)\| \\ =& \kappa(t)\|u_0(t)\|\end{aligned}$$

where $\kappa(t) = 1 - \frac{\kappa_1}{\kappa_2(t)} < 1$. Choose c_1 such that

$$\begin{aligned}&P\bar{E}A + A^{\top}\bar{E}^{\top}P - c_1\sigma_i P\bar{B}\bar{B}^{\top}P \\ &\quad + [\lambda_{\max}(P\bar{B}\bar{B}^{\top}P) + \lambda_{\max}(P\bar{D}\bar{D}^{\top}P)]I_N = -Q_2 < 0,\end{aligned}$$

let $\Upsilon \triangleq R^{-1}D^{\top}$, and

$$R\mathcal{S} + \mathcal{S}^{\top}R - 2D^{\top}D + \lambda_{\max}^2(\mathcal{H})I_N = -\hat{Q}_2 < 0.$$

By using Young's Inequality:

$$
\begin{aligned}
\dot{V}_1 \leq & \sum_{i=1}^{N} \epsilon_i^\top \left(P\bar{E}A + A^\top \bar{E}^\top P - c_1\sigma_i P\bar{B}\bar{B}^\top P\right)\epsilon_i \\
& + 2(\|u_0\| - c_2)\sum_{i=1}^{N} a_{i0}\|B^\top \bar{E}^\top P\delta_i^\xi\| \\
& + 2{\delta^\xi}^\top(\mathcal{H}\otimes P\bar{E}D)\delta^\omega + 2{\delta^\omega}^\top(I_N\otimes(R\mathcal{S}-R\Upsilon D))\delta^\omega \\
= & \sum_{i=1}^{N} \epsilon_i^\top \left(P\bar{E}A + A^\top \bar{E}^\top P - c_1\sigma_i P\bar{B}\bar{B}^\top P\right)\epsilon_i \\
& + 2\kappa\|u_0\|\sum_{i=1}^{N} a_{i0}\|B^\top \bar{E}^\top P\delta_i^\xi\| + 2{\delta^\xi}^\top(\mathcal{H}\otimes P\bar{D})\delta^\omega \\
& + 2{\delta^\omega}^\top(I_N\otimes(R\mathcal{S}-R\Upsilon D))\delta^\omega \\
\leq & \sum_{i=1}^{N} \epsilon_i^\top \left(P\bar{E}A + A^\top \bar{E}^\top P - c_1\sigma_i P\bar{B}\bar{B}^\top P\right)\epsilon_i + \kappa^2\|u_0\|^2 \\
& + {\delta^\xi}^\top(I_N\otimes P\bar{D}\bar{D}^\top P)\delta^\xi + \sum_{i=1}^{N}\|B^\top \bar{E}^\top P\delta_i^\xi\|^2 \\
& + {\delta^\omega}^\top(I_N\otimes(R\mathcal{S} + \mathcal{S}^\top R - 2D^\top D + \lambda_{\max}^2(\mathcal{H})I_N))\delta^\omega \\
\leq & \sum_{i=1}^{N} \epsilon_i^\top \big[P\bar{E}A + A^\top \bar{E}^\top P + \lambda_{\max}(P\bar{B}\bar{B}^\top P)I_N \\
& + \lambda_{\max}(P\bar{D}\bar{D}^\top P)I_N - c_1\sigma_i P\bar{B}\bar{B}^\top P\big]\epsilon_i + \kappa^2\|u_0\|^2 \\
& + {\delta^\omega}^\top(I_N\otimes\hat{Q}_2)\delta^\omega \\
\leq & -\sum_{i=1}^{N}\epsilon_i^\top Q_2\epsilon_i + \kappa^2\|u_0\|^2 - \lambda_{\min}(\hat{Q}_2)\|\delta^\omega\|^2 \\
\leq & -\lambda_{\min}(Q_2)\|\delta^\xi\|^2 + \kappa^2\|u_0\|^2 - \lambda_{\min}(\hat{Q}_2)\|\delta^\omega\|^2 \\
< & \kappa^2\gamma_0^2
\end{aligned}
\tag{18}
$$

According to the result obtained from (18):

$$
\begin{aligned}
&\alpha_1(\|\delta^\xi\|) = \lambda_1\|\delta^\xi\|^2, && \alpha_2(\|\delta^\xi\|) = \lambda_2\|\delta^\xi\|^2 \\
&\alpha_3(\|\delta^\xi\|) = \lambda_3\|\delta^\xi\|^2 + \lambda_{\min}(\hat{Q}_2)\|\delta^\omega\|^2, && \alpha_4(\|u\|) = \kappa^2\|u_0\|^2
\end{aligned}
$$

where $\lambda_1 = \lambda_{\min}(P)$, $\lambda_2 = \lambda_{\max}(P)$, and $\lambda_3 = \lambda_{\min}(Q_2)$.

$\square$