1) A survey was conducted on a group of First-Year students to determine the factors that influenced students' decisions to choose to attend the University of Malaya (UM). The survey was based on three main factors, i.e. Peer Pressure, Parental Influence and UM's Reputation.

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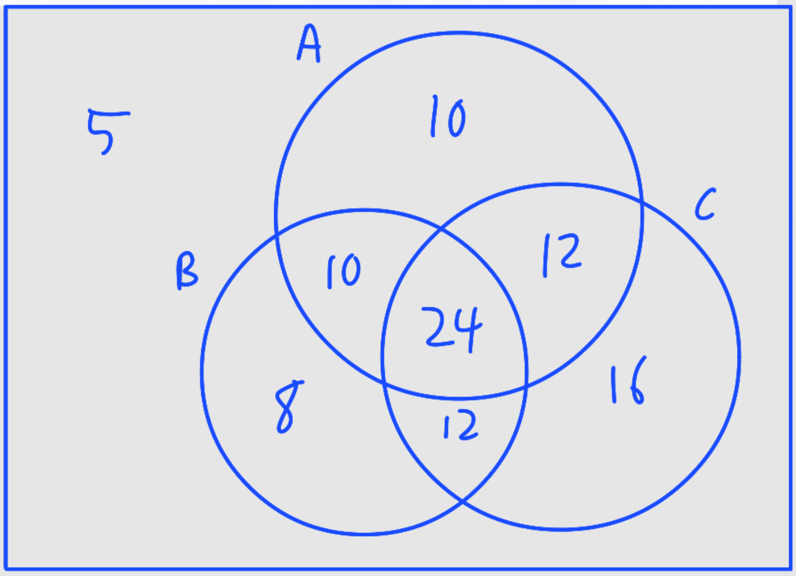
Satu analisis laporan tinjauan ini mendedahkan maklumat berikut:
An analysis of the response to the survey revealed the following information:

Faktor *Factors*	**Bilangan** *Numbers*
Tekanan Rakan Sebaya *Peer Pressure*	56
Pengaruh Ibu Bapa *Parental Influence*	54
Reputasi UM *UM's Reputation*	64
Tekanan Rakan Sebaya dan Pengaruh Ibu Bapa tetapi bukan Reputasi UM *Peer Pressure and Parental Influence but not UM's Reputation*	34
Pengaruh Ibu Bapa dan Reputasi UM *Parental Influence and UM's Reputation*	36
Tekanan Rakan Sebaya dan Reputasi UM *Peer Pressure and UM's Reputation*	36
Tekanan Rakan Sebaya, Pengaruh Ibu Bapa dan Reputasi UM *Peer Pressure, Parental Influence and UM's Reputation*	24
Tiada Faktor yang di atas *None of the above Factors*	5

i) *How many people took part in the survey?*

ξ



A = Peer pressure
B = Parental influence
C = Reputation

$5 + 10 + 10 + 24 + 12 + 8 + 12 + 16$

$= 97$

ii) *How many people were influenced by UM's Reputation, but not by their parents or peers?*

$12 + 16 = \cancel{28}$ 16

iii How many were convinced by their parents or UM's Reputation?

B C

union

$B \cup C$

$10+16=26$ XOR

$=72$

iv How many gave exactly one reason?

$10+8+16$

$=34$

v) How many gave exactly two reasons?

(10 markah/*marks*)

$10+12+12$

$=34$

2 *Consider the following relations defined on the set W*, which denotes the words in the English dictionary.

$$R = \{(x,y) \in W \times W, \text{where the words } x \text{ and } y \text{ have at least one letter in common}\}$$

Stating your reasons, determine if relation R is:

i) Refleksif
Reflexive

ii) Simetrik
Symmetric

iii) Transitif
Transitive

iv) Setara
Equivalent

(10 markah/*marks*)

i) Any word shares all of its letter with itself, every word in W will satisfy it being reflexive.

R is reflexive

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ii) If two words x, y shares atleast one letter in common, then $(x,y) \in R$ is also $(y,x) \in R$.

R is symmetric

iii) Don't hold true for all.

If for any $(x,y) \in R$ and $(y,z) \in R$, then $(x,z) \in R$.

This is false. Counter example

$x =$ apple
$y =$ tree
$z =$ wring

apple (x) and tree (y) shares the letter 'e'.

tree (y) and wring (z) shares the letter 'r'.

apple (x) and wring (z) doesn't share anything

R is not transitive

iv) R is not equivalence as it is reflexive, symmetric but not transitive.

3 Determine if the following functions are bijective. State your reasons.

i) $f: R \to R,\ f(x) = 3x - 2$

ii) $g(x) = \frac{1}{1-x}, x \in (-\infty, 1)$

iii) $h: N \to N,\ h(x) = 5 - x - x^2$ natural number 1 2 3

(10 markah/*marks*)

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i) $f(x) = 3x - 2$

Let $f(a) = f(b)$

$3a - 2 = 3b - 2$

$a = b$

Injective

$f(x)$ is bijective

$f(x) = 3x - 2$

$y = 3x - 2$

$x = \frac{y+2}{3}$

y valid for $(-\infty, \infty)$

Surjective $\mathbb{R}$

ii) $g(x) = \frac{1}{1-x}, x \in (-\infty, 1)$

Let $g(a) = g(b)$

$\frac{1}{1-a} = \frac{1}{1-b}$

$a = b$

Injective

$g(x)$ is bijective

$g(x) = \frac{1}{1-x}$

$y = \frac{1}{1-x}$

$1 - x = \frac{1}{y}$

$x = 1 - \frac{1}{y}$ ← surjective

Leith

iii) $h(x) = 5 - x - x^2$ Natural number

Let $h(a) = h(b)$

$5 - a - a^2 = 5 - b - b^2$

$a^2 + a \cancel{-5} = b^2 + b \cancel{-5}$

$a^2 + a = b^2 + b$

not injective

when $x = 2$

$h(2) = 5 - x - x^2$

$= -1$ (invalid, not natural number)

Not surjective

$h(x)$ is not bijective

4i There are 8 boys and 7 girls willing to join a committee. How many 8-member committees are possible if a committee is to contain:

a) Any number of boys and girls?

$^{15}C_8 = 6435$

b Only boys?

$^{8}C_8 = 1$

c At least one member of each gender?

$6435 - 1 = 6434$

only one combination for full boys (girls only 7)

Leith

ii) a

Proof that:

C(n + r, n - r) = C(n + r, 2r)

$$\frac{(n+r)!}{(n-r)!\,(n+r-(n-r))!} = \frac{(n+r)!}{(2r)!\,(n+r-2r)!}$$

$$\frac{(n+r)!}{(n-r)!\,(2r)!} = \frac{(n+r)!}{(n-r)!\,(2r)!}$$

$$\therefore \binom{n+r}{n-r} = \binom{n+r}{2r}$$

b

if

P(n,r) = n,

there is only 1 possible value for r.

(10 markah/*marks*)

$$\frac{n!}{(n-r)!} = n \qquad \text{find } r$$

$$n! = n(n-r)!$$

$$\frac{n!}{n} = (n-r)!$$

$$(n-1)! = (n-r)!$$

$$r = 1$$

$$\frac{n!}{n} = (n-1)!$$

$$\frac{5!}{5} = 4 \cdot 3 \cdot 2 \cdot 1 \quad \text{eg.}$$

Leith

5i

The following is a system of linear equations with a parameter s.

$$\begin{cases} 3sx - 2y = 4 \\ -6x + sy = 1 \end{cases}$$

a) Determine the value of x and y in term of s using Cramer method.

$$\begin{bmatrix} 3s & -2 \\ -6 & s \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 3s & -2 \\ -6 & s \end{vmatrix} \qquad |D_x| = \begin{vmatrix} 4 & -2 \\ 1 & s \end{vmatrix} \qquad |D_y| = \begin{vmatrix} 3s & 4 \\ -6 & 1 \end{vmatrix}$$

$$= 3s^2 - (-2)(-6) \qquad = 4s + 2 \qquad = 3s + 24$$

$$= 3s^2 - 12$$

$$x = \frac{|D_x|}{|D|} \qquad y = \frac{|D_y|}{|D|}$$

$$= \frac{4s + 2}{3s^2 - 12} \qquad = \frac{3s + 24}{3s^2 - 12}$$

b) If the system of equations has unique solution, i.e. there is a unique value of y for an x given, determine the range of s.

$$3s^2 - 12 \neq 0$$

$$3s^2 \neq 12$$

$$s^2 \neq \frac{12}{3}$$

$$\neq 4$$

$$s \neq -2, \quad s \neq 2$$

Range of $s = (-\infty, \infty) \setminus \{-2, 2\}$

$= \mathbb{R} \setminus \{-2, 2\}$

Leith

ii /Given that $A = \begin{pmatrix} 5 & 3 & -7 \\ 0 & -3 & 2 \\ 2 & -2 & 2 \end{pmatrix}$ dan/and $B = \begin{pmatrix} 9 & -3 & 8 \\ 2 & -1 & -1 \\ 5 & 0 & 5 \end{pmatrix}$.

a) $2A - B$

b) Transpos/*Transpose of A*

c) B^2

d) Det(A) + Det(B)

(10 markah/*marks*)

a) $2A - B$

$= 2\begin{bmatrix} 5 & 3 & -7 \\ 0 & -3 & 2 \\ 2 & -2 & 2 \end{bmatrix} - \begin{bmatrix} 9 & -3 & 8 \\ 2 & -1 & -1 \\ 5 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 1 & 9 & -22 \\ -2 & -5 & 5 \\ -1 & -4 & -1 \end{bmatrix}$

b) A^T

$= \begin{bmatrix} 5 & 3 & -7 \\ 0 & -3 & 2 \\ 2 & -2 & 2 \end{bmatrix}^T$

$= \begin{bmatrix} 5 & 0 & 2 \\ 3 & -3 & -2 \\ -7 & 2 & 2 \end{bmatrix}$

c) B^2

$= \begin{bmatrix} 9 & -3 & 8 \\ 2 & -1 & -1 \\ 5 & 0 & 5 \end{bmatrix} \begin{bmatrix} 9 & -3 & 8 \\ 2 & -1 & -1 \\ 5 & 0 & 5 \end{bmatrix}$

$= \begin{bmatrix} 9(9)-3(2)+8(5) & 9(-3)-3(-1)+8(0) & 9(8)-3(-1)+8(5) \\ 2(9)-1(2)-1(5) & 2(-3)-1(-1)-1(0) & 2(8)-1(-1)-1(5) \\ 5(9)+0(2)+5(5) & 5(-3)+0(-1)+5(0) & 5(8)+0(-1)+5(5) \end{bmatrix}$

$= \begin{bmatrix} 115 & -24 & 115 \\ 11 & -5 & 12 \\ 70 & -15 & 65 \end{bmatrix}$

d) Det(A) + Det(B)

Leith

$$= \begin{vmatrix} 5 & 3 & -7 \\ 0 & -3 & 2 \\ 2 & -2 & 2 \end{vmatrix} + \begin{vmatrix} 9 & -3 & 8 \\ 2 & -1 & -1 \\ 5 & 0 & 5 \end{vmatrix}$$

$$= 5[-3(2)-2(-2)] - 3[0(2)-2(2)] - 7[0(-2)-(-3)(2)] + 9[-1(5)$$
$$-(-1)(0)] + 3[2(5)-(-1)(5)] + 8[2(0)-(-1)(5)]$$

$$= -40 + 40$$

$$= 0$$

Peraturan Mantik
The Laws of Logic

$p \vee \neg\neg p = T$	Negation Law
$p \wedge \neg\neg p = F$	Negation Law
$\neg\neg p = p$	Double Negation
$p \wedge TRUE = p$	Identity
$p \vee FALSE = p$	Identity
$p \vee TRUE = TRUE$	Domination Law
$p \wedge FALSE = FALSE$	Domination Law
$p \vee p = p$	Idempotent Law
$p \wedge p = p$	Idempotent Law
$p = p$	Idempotent Law
$p \vee q = q \vee p$	Commutative Law
$p \wedge q = q \wedge p$	Commutative Law
$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	Associative Law
$(p \vee q) \vee r = p \vee (q \vee r)$	Associative Law
$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	Distributive Law
$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$	Distributive Law
$p \rightarrow q = \neg p \vee q$	Law of Implication
$p \wedge (p \vee q) = p$	Absorption Law
$p \vee (p \wedge q) = p$	Absorption Law
$\neg(p \vee q) = \neg p \wedge \neg q$	De Morgan's

6

Peraturan Mantik:

Determine if the following propositions are equivalent using the Laws of Logic.

i) $\neg\{(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)\} = F$

ii) $\neg(p \wedge \neg(\neg p \wedge q)) \equiv \neg p$

iii) $(p \vee \neg q) \vee ((\neg p \wedge \neg r) \wedge q) \equiv (p \vee \neg r) \vee \neg q$

Zeith

i) $\neg\{(\neg p \wedge \neg q) \rightarrow (\neg p \vee \neg q)\} = F$

LHS

$\neg\{\neg(\neg p \wedge \neg q) \vee (\neg p \vee \neg q)\}$

$= \neg\neg(\neg p \wedge \neg q) \wedge \neg(\neg p \vee \neg q)$

$= (\neg p \wedge \neg q) \wedge (\neg\neg p \vee \neg\neg q)$

$= (\neg p \wedge \neg q) \wedge (p \vee q)$

$= p \wedge (\neg p \wedge \neg q) \vee q \wedge (\neg p \wedge \neg q)$

$= (p \wedge \neg p) \wedge \neg q) \vee (\neg p \wedge q \wedge \neg q)$

$\quad 0 \qquad\qquad\qquad 0$

$= F \vee F$

$\equiv F$ ∴ Matches RHS

ii $\neg(p \wedge \neg(\neg p \wedge q)) \equiv \neg p$

LHS $\neg(p \wedge \neg(\neg p \wedge q))$

$= (\neg p \vee \neg\neg(\neg p \wedge q))$

$= \neg p \vee (\neg p \wedge q)$ (absorption law)

$\equiv \neg p$ ∴ matches RHS

iii $(p \vee \neg q) \vee ((\neg p \wedge \neg r) \wedge q) \equiv (p \vee \neg r) \vee \neg q$

Zeiti

LHS

$(p \vee \neg q) \vee ((\neg p \wedge \neg r) \wedge q)$

$= (p \vee \neg q) \vee (\neg p \wedge \neg r \wedge q)$

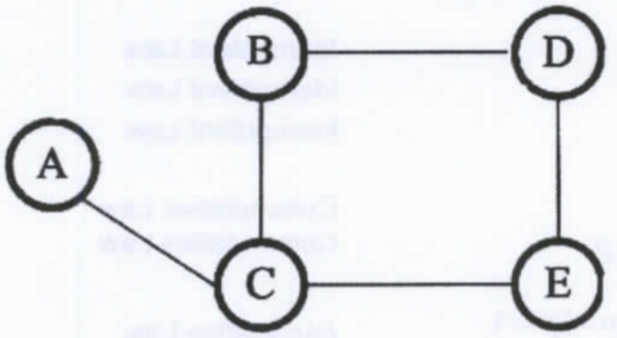
$= ((p \vee \neg q) \vee \neg p) \wedge ((p \vee \neg q) \vee \neg r) \wedge ((p \vee \neg q) \vee q)$

$= (p \vee \neg q \vee \neg p) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee q)$

$= 1 \wedge (p \vee \neg q \vee \neg r) \wedge 1$

$\equiv (p \vee \neg r) \vee \neg q$ $\quad \therefore$ Matches RHS

7 *Let G = (V, E) be the undirected graph as shown in Figure 1.*



Rajah 1: Graf G

Figure 1 Graph G

(10 markah/*marks*)

i) *Find the degree of each vertex of Graph G.*

deg(A) = 1

deg(D) = 2

deg(B) = 2

deg(E) = 2

deg(C) = 3

Zeith

ii Build the incidence matrix for G.

	A,C	B,C	C,E	D,E	B,D
A	1	0	0	0	0
B	0	1	0	0	1
C	1	1	1	0	0
D	0	0	0	1	1
E	0	0	1	1	0

iii Give the adjacency lists of each vertex.

A	C
B	C D
C	A B E
D	B E
E	C D

Done paper