Leita

1a) Given $M = \begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$. Find:

i) $2M$
ii) M^2
iii) $3M^2 - 2M$

(3 markah/*marks*)

i) $2M$

$$= 2\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 4 & 2 \\ 10 & 6 & 8 \\ 2 & 2 & 4 \end{bmatrix}$$

ii) M^2

$$= \begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4(4)+2(5)+1(1) & 4(2)+2(3)+1(1) & 4(1)+2(4)+1(2) \\ 5(4)+3(5)+4(1) & 5(2)+3(3)+4(1) & 5(1)+3(4)+4(2) \\ 1(4)+1(5)+2(1) & 1(2)+1(3)+2(1) & 1(1)+1(4)+2(2) \end{bmatrix}$$

$$= \begin{bmatrix} 27 & 15 & 14 \\ 39 & 23 & 25 \\ 11 & 7 & 9 \end{bmatrix}$$

iii) $3M^2 - 2M$

$$= 3\begin{bmatrix} 27 & 15 & 14 \\ 39 & 23 & 25 \\ 11 & 7 & 9 \end{bmatrix} - 2\begin{bmatrix} 4 & 2 & 1 \\ 5 & 3 & 4 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 73 & 41 & 40 \\ 107 & 63 & 67 \\ 31 & 19 & 23 \end{bmatrix}$$

b) Refer to the following matrix:

$$A = \begin{pmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{pmatrix}$$

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i Without using a calculator, find the determinant of A and explain how you can identify A is an invertible matrix.

(3 markah/*marks*)

$$|A| = \begin{vmatrix} 1 & -3 & 7 \\ 1 & 1 & 1 \\ 1 & -2 & 3 \end{vmatrix}$$

$$= 1[1(3) - 1(-2)] + 3[1(3) - 1(1)] + 7[1(-2) - 1(1)]$$

$$= -10$$

A is an invertible matrix because its determinant is not 0.

ii) Find the minors, cofactors and adjoint of the matrix. Hence, find the inverse of the matrix.

(4 markah/*marks*)

Matrix minors

$$\begin{bmatrix} 1(3)-1(-2) & 1(3)-1(1) & 1(-2)-1(1) \\ -3(3)-7(-2) & 1(3)-7(1) & 1(-2)-(-3)(1) \\ -3(1)-7(1) & 1(1)-7(1) & 1(1)-(-3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 2 & -3 \\ 5 & -4 & 1 \\ -10 & -6 & 4 \end{bmatrix}$$

Matrix cofactor

$$\begin{bmatrix} 1_+ & -3_- & 7_+ \\ 1_- & 1_+ & 1_- \\ 1_+ & -2_- & 3_+ \end{bmatrix} = \begin{bmatrix} 1 & 3 & 7 \\ -1 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$$

Matrix adjoint (minors + cofactor + tranpose

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$$\begin{bmatrix} 5_{+} & 2_{-} & -3_{+} \\ 5_{-} & -4_{+} & 1_{-} \\ -10_{+} & -6_{-} & 4_{+} \end{bmatrix}^{T} = \begin{bmatrix} 5 & -2 & -3 \\ -5 & -4 & -1 \\ -10 & 6 & 4 \end{bmatrix}^{T}$$

$$\mathrm{Adj}(A) = \begin{bmatrix} 5 & -5 & -10 \\ -2 & -4 & 6 \\ -3 & -1 & 4 \end{bmatrix}$$

Inverse of matrix

$$A^{-1} = \frac{1}{|A|} \mathrm{Adj}(A)$$

$$= \frac{1}{-10} \begin{bmatrix} 5 & -5 & -10 \\ -2 & -4 & 6 \\ -3 & -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 0.5 & 1 \\ 0.2 & 0.4 & -0.6 \\ 0.3 & 0.1 & -0.4 \end{bmatrix}$$

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2a) *Refer to the following hypothesis:*

P R Q

"Mary Poppins will throw her umbrella if she knows that Mickey have his holiday with Woody but Toothless wants to fly with Hiccup. Toothless is looking forward to fly with Hiccup and Mary Poppins is not going to throw her umbrella."

R $\wedge$ $\bar{P}$

The propositions in the hypothesis have been symbolized as follows:

P : Mary Poppins will throw her umbrella
Q : Mickey does have his holiday with Woody
R : Toothless wants to fly with Hiccup

i) *Rewrite the hypothesis in symbolic form.*

(3 markah/*marks*)

$(Q \wedge R) \to P$ (if Q and R, then P)

$R \wedge \neg P$

ii) *Using rule of inferences, determine whether the conclusion "Mickey does not have his holiday with Woody" is valid or not.* $\neg Q$

(3 markah/*marks*)

if then

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

$R \wedge \neg P$

R is true

P is false

Modus Tollens

$(Q \wedge R) \to P$

False False

$Q \wedge R$ false

True

False

∴ Mickey does not have his holiday with Woody.

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b Given that both x and y are real numbers, we define:
$P(x,y)$: x is divisible by y

$Q(x,y) : \frac{x}{y} = 1$

Determine the truth value of each of the following:

i) $\forall x \forall y\ Q(x,y)$
ii) $\forall x \exists y\ P(x,y)$
iii) $\forall x \forall y\ (\ Q(x,y) \rightarrow P(x,y)\ )$
iv) $\forall x \forall y\ (\ P(x,y) \rightarrow Q(x,y)\ )$

(4 markah/*marks*)

i) False, $\forall x \forall y\ Q(x,y)$ does not hold when $x < y$.

ii) True

iii) True

iv) False, for $\frac{x}{y}$ not guaranteed to be 1.

3a A lecturer has seven (7) programming books with different titles on a bookshelf. Four (4) of these books are on 'Java' and the rest are on 'Pascal'. In how many ways can the lecturer arrange these books on the shelf if:

4 Java
3 Pascal

i) there is no restriction, the lecturer can arrange the books in any way he wants;

$7! = 5040$

ii) all the 'Pascal' books must be next to each other;

_ J _ J _ J _ J _

${}^5P_5 \times {}^3P_3 = 720$ ways

Pascal + 4 Java

Pascal shuffled

iii) the books should be arranged according to their title in an ascending order, from left to right.

(6 markah/*marks*)

???? huh

1 way

Leitm

b

Seven philosophers, Kant, Hegel, Marx, Weber, Simmel, Heidegger and Nietzsche are dining at a round table. In how many ways can these philosophers be seated?

(2 markah/*marks*)

$7-1=6$

${}^{6}P_{6} = 720$

c

In how many ways can 9 distinct books be distributed to two philosophers, Kant and Hegel, if Kant gets 6 books and Hegel gets 3 books?

(2 markah/*marks*)

Kant $= {}^{9}C_{6}$

$= 84$

Hegel $= {}^{3}C_{3}$ Remaining

$= 1$ (you don't have to count this, Hegel is forced to get it, redundant)

Number of ways

$= 84$

4a

Given A = {1, 2, 3, 4, 5}, S = {(x,y): x∈A, y∈A}. Find the ordered pairs which satisfy the conditions given below. For each of the following, state and explain whether it is reflexive, symmetric, transitive and equivalence relation:

i) $x + y = 5$

(3 markah/*marks*)

$R = \{(1,4),(2,3),(3,2),(4,1)\}$

Symmetric

ii) $x + y < 5$

(3 markah/*marks*)

$R = \{(1,1),(1,2),(2,1),(2,2),(1,3),(3,1)\}$

~~Reflexive~~

Symmetric

b If $f : R \to R$ is defined as $f(x) = 2x^2 - 9$, identify whether it is:

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i injective (one-to-one)

Assume $f(x_1) = f(x_2)$

$f(x) = 2x^2 - 9$

$f(x_1) = f(x_2)$

$2x_1^2 - 9 = 2x_2^2 - 9$

$x_1^2 = x_2^2$

Case 1:

$x_1 = x_2$

This is injective

Case 2:

$x_1 = -x_2$

This makes it not injective

$\therefore$ Not injective

ii surjective

(4 markah/*marks*)

$f(x) = 2x^2 - 9$

$2x^2 - 9 = y$

$x^2 = \frac{y+9}{2}$ $\quad x^2 \geq 0$

so, $y + 9 \geq 0$

$y \geq -9$

The function $f(x) = 2x^2 - 9$ can only take values $y \geq 9$. Thus, the function is not surjective because it cannot map to any $y < -9$ in $\mathbb{R}$.

$\therefore$ Not surjective

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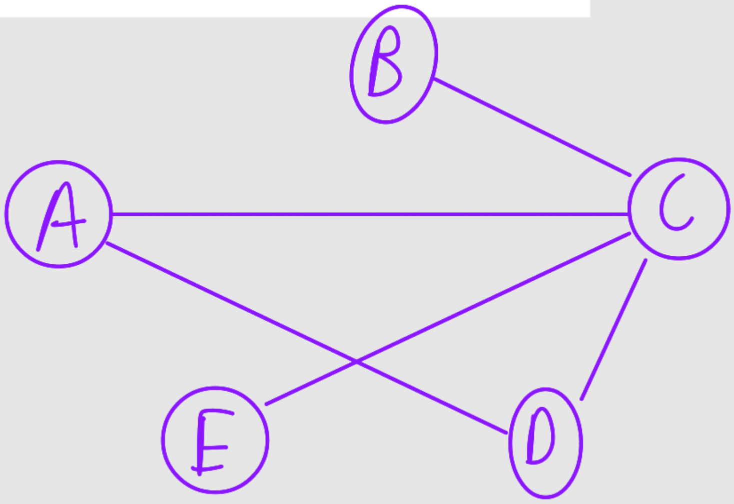
5 Suppose that a group consist of 5 people: A, B, C, D, and E, the following pairs of people are acquainted with each other:

- A and C
- A and D
- B and C
- C and D
- C and E

i Draw a graph G to represent this situation.



ii) List the vertex set, and the edge set, using set notation. In other words, show sets V and E for the vertices and edges, respectively, in G = {V, E}.

$V = \{A, B, C, D, E\}$

$E = \{\{A,C\}, \{B,C\}, \{A,D\}, \{C,E\}, \{C,D\}\}$

iii) Draw an adjacency matrix for G.

(10 markah/*marks*)

	A	B	C	D	E
A	0	0	1	1	0
B	0	0	1	0	0
C	1	1	0	1	1
D	1	0	1	0	0
E	0	0	1	0	0