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1a) Show the truth table for all possible inputs given the logic below. Determine whether it is a tautology, contingency or contradiction.

i. $((p \to q) \land (\neg q)) \to p$
ii. $(\neg q \land (p \leftrightarrow q)) \to \neg(p \land q)$

(5 markah/*marks*)

i)

p	q	$p \to q$	$\neg q$	$(p\to q)\land(\neg q)$	$((p\to q)\land(\neg q))\to p$
T	T	T	F	F	T
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

contingency

ii)

p	q	$p \leftrightarrow q$	$\neg q$	$\neg q \land (p \leftrightarrow q)$	$p \land q$	$\neg(p\land q)$	Full
T	T	T	F	F	T	F	T
T	F	F	T	F	F	T	T
F	T	F	F	F	F	T	T
F	F	T	T	T	F	T	T

Tautology

b) *What is the domain/set where the statement below is true? Can the domain be Imaginary numbers?*

$\forall x \forall y:\ x = y$

(3 markah/*marks*)

no Real number
no imaginary number
singleton set ✓
$D = \{a\}$.
empty set ✓

c) Given that x,y ∈ Person in the world where K(x,y): x knows y, we can represent "Someone knows somebody", "Everyone knows everybody", "Everyone knows somebody" and "Someone knows everybody' symbolically and create 'if everyone knows somebody then somebody knows everyone'.

Determine the truth value of the propositions $\forall x \forall y K(x,y)$, $\forall x \exists y\ K(x,y)$, $\exists x \forall y\ K(x,y)$, $\exists x \exists y K(x,y)$:

(2 markah/*marks*)

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$K(x,y) =$ Knows someone

$\forall x \forall y K(x,y)$	False	
$\forall x \exists y K(x,y)$	False X	True, everyone knows someone
$\exists x \forall y K(x,y)$	False	
$\exists x \exists y K(x,y)$	True	

d) If x is an element of the Real numbers' domain determine whether the function below is valid and where does it map to?

$$f(x) = \sqrt{x}$$

(2 markah/*marks*)

$x \in \mathbb{R}$

$x \geq 0$, Range $= [0, \infty)$

$x < 0$, Range $= (0, \infty i)$ X unnecessary

The function $f(x) = \sqrt{x}$ is valid for x in the domain of real numbers where $x \geq 0$, and it maps to the range $[0, \infty)$.

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II A) a) For the relation $R = \{(x, y) | \ y = x^2 + 1, x \in \{1, 3, 5\}\}$, find the domain and range.

(1 markah/*mark*)

$R = \{(x, y) \mid y = x^2 + 1,\ x \in \{1,3,5\}\}$

Domain $= \{1, 3, 5\}$

Range $= \{2, 10, 26\}$

b) For the relation $R = \{(x, y) | \ x^2 + y^2 = 25,\ \ x, y \in \mathbb{Z}\}$, list the elements of R.

(1 markah/*mark*)

$(-4,3), (-4,-3), (4,-3)$

$R = \{(-5,0), (5,0), (0,-5), (0,5), (\cancel{-2,-3}), (\cancel{-2,3}), (\cancel{2,-3}), (\cancel{2,3})\}$

c) Consider the following relations whether they are reflexive, symmetric, and/or transitive on $X = \{a, b, c\}$.

i) $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (b, c), (c, b)\}$;

ii) $R = \{(a, a), (b, b), (c, c), (a, b), (a, c), (b, c)\}$;

iii) $R = \{(a, c), (c, b)\}$;

iv) $R = \{(a, a), (b, b), (c, c), (a, b), (b, a), (a, c), (c, a), (b, c), (c, b)\}$;

(4 markah/*marks*)

i) Reflexive, symmetry

ii) Reflexive, transitive

iii) Transitive

iv) Reflexive, symmetry, transitive

B)

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Given the matrices $A = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$, and $C = \begin{bmatrix} -6 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix}$. Check if the following can be computed. If yes, find the answer.

a) $A+C^T$

b) AB

c) det(A)

d) A^{-1}

e) CA^TA

f) Tunjukkan dengan contoh mengapa pendaraban matriks tidak komutatif.

Demonstrate with an example, why matrix multiplication is not commutative.

(5 markah/*marks*)

a) $A+C^T$

$$\begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -6 & 1 \\ -4 & 2 \\ -2 & 3 \end{bmatrix}$$

= Invalid dimension ✓

b) AB

$$= \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 0 & 0 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 2 \\ -2 & 2 \\ -2 & 2 \end{bmatrix}$$ ✓

c) det(A)

$$\begin{vmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{vmatrix} = 9[5(1)-4(2)] - 8[6(1)-4(3)] + 7[6(2)-5(3)]$$

$$= 0$$ ✓

d) A^{-1}

= Singular matrix / degenerate matrix / not invertible

single mingle ahh matrix

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e) CA^TA

$$= \begin{bmatrix} -6 & -4 & -2 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} 9 & 6 & 3 \\ 8 & 5 & 2 \\ 7 & 4 & 1 \end{bmatrix} \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1608 & -1032 & -456 \\ 708 & 456 & 204 \end{bmatrix}$$

f) Demonstrate with an example, why matrix multiplication is not commutative.

(5 markah/*marks*)

Matrix multiplication is not communitative because the order in which you multiply two matrices affects the resulting product.

Demonstrate with example

① $$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{bmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{bmatrix} = \begin{bmatrix} 84 & 90 & 96 \\ 201 & 216 & 231 \\ 318 & 342 & 366 \end{bmatrix}$$

not same

② $$\begin{bmatrix} 10 & 11 & 12 \\ 13 & 14 & 15 \\ 16 & 17 & 18 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} = \begin{bmatrix} 138 & 171 & 204 \\ 174 & 216 & 258 \\ 210 & 261 & 312 \end{bmatrix}$$

II c) Solve the following system of equations by using Cramer's method

$$\begin{cases} u+v-w &= 6 \\ 2u-3v+2w &= -4 \\ -4u+5v+w &= 9 \end{cases}$$

(2 markah/*marks*)

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$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 2 \\ -4 & 5 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ 9 \end{bmatrix}$$

$$|D| = \begin{vmatrix} 1 & 1 & -1 \\ 2 & -3 & 2 \\ -4 & 5 & 1 \end{vmatrix}$$

$$= 1[(-3)(1)-2(5)] - 1[2(1)-2(-4)] - 1[2(5)-(-3)(-4)]$$

$$= -21$$

$$|D_u| = \begin{vmatrix} 6 & 1 & -1 \\ -4 & -3 & 2 \\ 9 & 5 & 1 \end{vmatrix} = -63 \qquad |D_v| = \begin{vmatrix} 1 & 6 & -1 \\ 2 & -4 & 2 \\ -4 & 9 & 1 \end{vmatrix} = -84$$

$$|D_w| = \begin{vmatrix} 1 & 1 & 6 \\ 2 & -3 & -4 \\ -4 & 5 & 9 \end{vmatrix} = -21$$

$$u = \frac{|D_u|}{|D|} = \frac{-63}{-21} = 3 \qquad v = \frac{|D_v|}{|D|} = \frac{-84}{-21} = 4$$

$$w = \frac{|D_w|}{|D|} = \frac{-21}{-21} = 1$$

$u = 3$

$v = 4$

$w = 1$

III The Faculty of Computer Science & Information Technology, Universiti Malaya offers three types of advanced mathematics courses: *Computing Math I, Computing Math II, and Numerical Analysis. There are 120 students enrolled in at least one of these courses. The enrolment for each course is as follows:*

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- 70 pelajar telah mendaftar dalam Matematik Pengkomputeran I.

  70 students are enrolled in Computing Math I.

- 45 pelajar telah mendaftar dalam Matematik Pengkomputeran II.

  45 students are enrolled in Computing Math II.

- 60 pelajar telah mendaftar dalam Analisis Berangka.

  60 students are enrolled in Numerical Analysis.

$a = CM1$
$b = CM2$
$c = NA$

It is also known that:

- 20 pelajar telah mendaftar dalam kedua-dua Matematik Pengkomputeran I dan Matematik Pengkomputeran II.

  20 students are enrolled in both Computing Math I and Computing Math II.

- 25 pelajar telah mendaftar dalam kedua-dua Matematik Pengkomputeran I dan Analisis Berangka.

  25 students are enrolled in both Computing Math I and Numerical Analysis.

- 15 pelajar telah mendaftar dalam kedua-dua Matematik Pengkomputeran II dan Analisis Berangka.

  15 students are enrolled in both Computing Math II and Numerical Analysis.

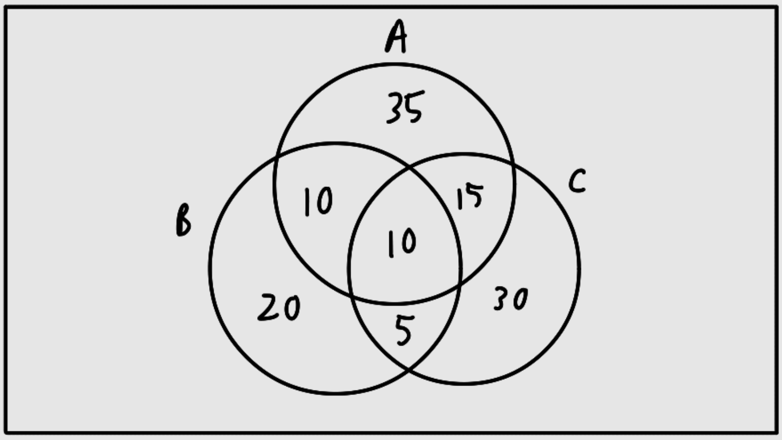
- 10 pelajar telah mendaftar dalam ketiga-tiga kursus.

  10 students are enrolled in all three courses.

a) *Given this information, the university's administration is interested in finding out the number of students who are enrolled in exactly one course to allocate appropriate resources. Calculate the number of students enrolled in exactly one course.*

(4 markah/*marks*)

Find the number of students who are enrolled in exactly one course



$a = CM1$
$b = CM2$
$c = NA$

$n(\text{only one course}) = 35 + 20 + 30$
$= 85$

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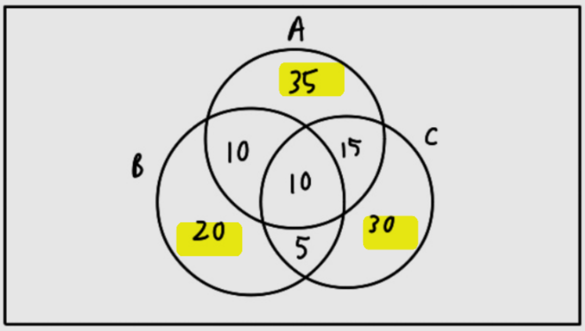
b

From the students enrolled in exactly one mathematics course at the faculty, the mathematics department decides to create a special lecture series where three guest student speakers will present on the unique challenges and insights of studying a single area of mathematics. If each speaker must be from a different one of the three courses (Computing Math I, Computing Math II, Numerical Analysis), and the order in which they speak. Find the number of different arrangements of speakers.

(4 markah/*mar*



n(exactly one) = 85

$${}^{35}C_1 \times {}^{20}C_1 \times {}^{30}C_1$$

$$= 21000$$

c

In preparation for finals, students from the same group of those who are enrolled in exactly one course decide to form study groups. If a study group can consist of any 5 students. Find the number of ways to form one study group from all the students taking one course only.

(4 markah/*marks*)

CM1

$${}^{35}C_5 = 324632$$

CM2

$${}^{20}C_5 = 15504$$

NA

$${}^{30}C_5 = 142506$$

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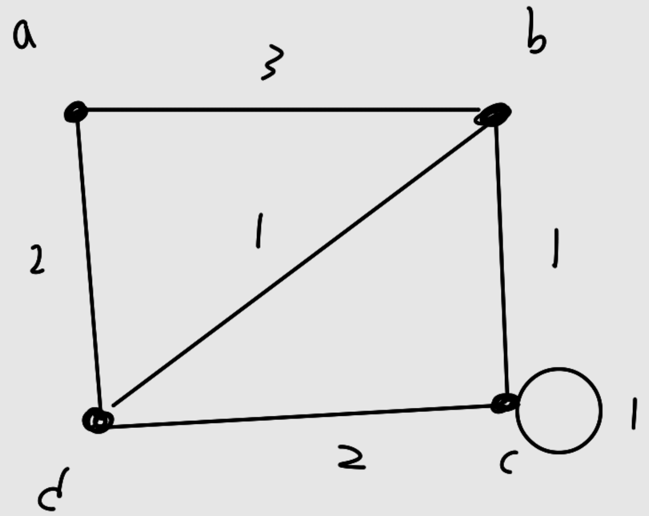
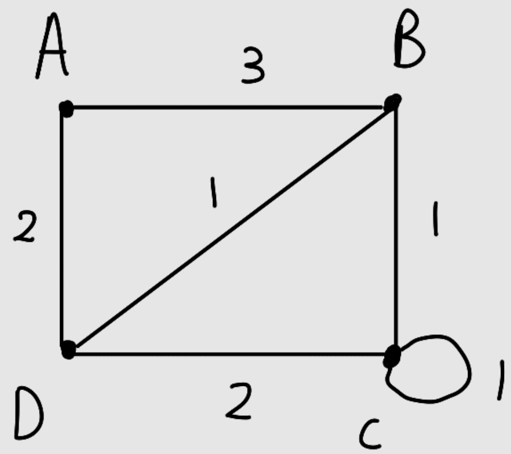
IV A)

a) Draw the graph with the following adjacency matrix with respect to the ordering of vertices a, b, c, d.

$$\begin{array}{c} \\ a \\ b \\ c \\ d \end{array}\begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

(2 markah/*marks*)





b) *Classify the above resulting graph in part a and name it.* (1 markah/*mark*)

- Undirected
- Weighted

Undirected weighted graph

c) *List the degree of each vertex of the graph in part a.* (1 markah/*mark*)

deg(a) = 2

deg(b) = 3

deg(c) = 4

deg(d) = 3

d) Can a simple graph exist with 13 vertices each of degree three? Why?

(2 markah/*mark*)

Handshaking Lemma

$$\sum \text{degrees} = 2m$$

$n = 13$

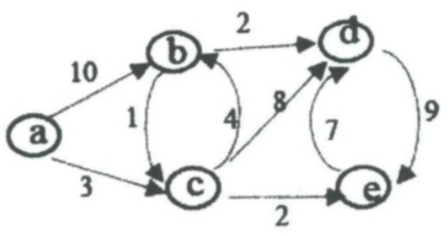
$d = 3$

$$\sum \text{degrees} = nd$$
$$= 13(3)$$
$$= 39$$
$$2m = 39$$
$$m = \frac{39}{2}$$
$$= 19.5$$

(invalid, m must be an integer)

$\therefore$ A simple graph with 13 vertices each of the degree 3 cannot exist.

B Let G = (V, E) be a weighted graph as shown in the figure below.



a) Describe the graph model.

(2 markah/*marks*)

$G = (V, E)$

$V = \{a, b, c, d, e\}$

- Directed
- Weighted
- Cyclic

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b) Using Dijkstra's algorithm, find the length of the shortest distance between the vertices a and d.

(1 markah/mark)

Vertex	Distance from A	Previous
a	0	
b	∞	
c	∞	
d	∞	
e	∞	

Vertex	Distance from A	Previous
a	0	
b	10	a
c	3	a
d	∞	
e	∞	

Vertex	Distance from A	Previous
a	0	
b	10	a
c	3	a
d	12	b
e	5	c

Vertex	Distance from A	Previous
a	0	-
b	7	c
c	3	a
d	9	b
e	5	c

$d \leftarrow b \leftarrow c \leftarrow a$

$a \rightarrow c \rightarrow b \rightarrow d$

minimum = 9

c) *Using Dijkstra's algorithm above, indicate the corresponding shortest path between a and d.*

(1 markah/*mark*)

$a \rightarrow c \rightarrow b \rightarrow d$

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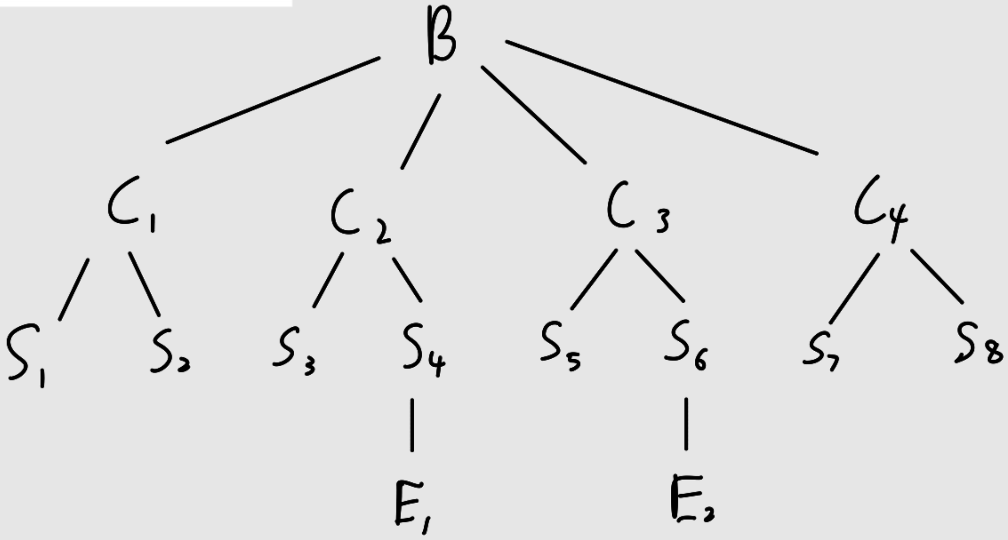
C) Let T= (V, E) be a rooted tree with root B that represents a book's contents. The book consists of four chapters (C_1, C_2, C_3, C_4). Vertices with level number 2 are the sections (S_i) within each chapter, where each chapter consists of 2 sections, respectively. S_4 and S_6 of level 2 include a single subsection (E_i) within their section that represents level 3 of the tree.

a) Draw the tree graph.



b Which vertices are leaves?

Leaf set = $\{S_1, S_2, S_3, E_1, S_5, E_2, S_7, S_8\}$

c What is the height of the tree?

(3 markah/*marks*)

3 edges (the lines)

The height of the tree is 3.

The height of the tree is defined as the number of edges on the longest path from the root to a leaf.

Donr paper.