

Keaton Spiller
CS441
Spring 2022

Programming Assignment #1 8 Puzzle problem

Heuristic one: Number of misplaced tiles for every tile in 8 locations. Each incorrect tile would accumulate 1 more away from the goal to the state of the board currently being accessed. Worst case all 8 tiles were in the wrong spot and best case 0, none of the tiles are in the wrong spot and we've reached the goal state.

Heuristic two: Manhattan Distance by finding how many steps each tile would have to move to get to the correct location, and summing the total steps away from the goal. I achieved this by subtracting the side to side locations and the up and down locations for each point on the current state to the goal state. I used an absolute value for each point to get distance regardless of negative values, and added the total distance for the entire state.

Heuristic three: Calculating the scores of heuristic 1 and heuristic 2, I took the average of the two, and returned that as the score for each state. I initially went with whichever heuristic was the larger and more dominant score, but the scores looked too similar to Manhattan distance, and I wanted to see bigger differences in the data.

Five puzzles ranging in difficulty:

Extreme Case

initial_puzzle5= (5,6,7,4,b,8,3,2,1) puzzle_goal5= (1,2,3,8,b,4,7,6,5)

Hard Case

initial_puzzle4= (7,2,4,5,b,6,8,3,1) puzzle_goal4= (b,1,2,3,4,5,6,7,8)

Medium Case 1

initial_puzzle3= (7,8,b,4,5,6,1,2,3) puzzle_goal3= (7,4,b,6,5,8,1,2,3)

Medium Case 2

initial_puzzle2= (7,6,5,b,4,3,2,8,1) puzzle_goal2= (7,6,5,8,b,4,1,2,3)

Easy Case

initial_puzzle1= (1,2,3,4,b,5,6,7,8) puzzle_goal1= (1,2,3,b,4,5,6,7,8)

Best-first search:

Heuristic 1: Misplaced Tiles

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]

Shortest Path: 2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 2, 4, 3, 'b', 8, 1], [7, 6, 5, 2, 4, 3, 8, 'b', 1], [7, 6, 5, 2, 4, 3, 8, 1, 'b'], [7, 6, 5, 2, 4, 'b', 8, 1, 3], [7, 6, 5, 2, 'b', 4, 8, 1, 3], [7, 6, 5, 'b', 2, 4, 8, 1, 3], [7, 6, 5, 8, 2, 4, 'b', 1, 3], [7, 6, 5, 8, 2, 4, 1, 'b', 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]

Shortest Path: 10

Puzzle3: [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], [7, 5, 8, 4, 'b', 6, 1, 2, 3], [7, 5, 8, 4, 6, 'b', 1, 2, 3], [7, 5, 'b', 4, 6, 8, 1, 2, 3], [7, 'b', 5, 4, 6, 8, 1, 2, 3], ['b', 7, 5, 4, 6, 8, 1, 2, 3], [4, 7, 5, 'b', 6, 8, 1, 2, 3], [4, 7, 5, 6, 'b', 8, 1, 2, 3], [4, 'b', 5, 6, 7, 8, 1, 2, 3], [4, 5, 'b', 6, 7, 8, 1, 2, 3], [4, 5, 8, 6, 7, 'b', 1, 2, 3], [4, 5, 8, 6, 'b', 7, 1, 2, 3], [4, 'b', 8, 6, 5, 7, 1, 2, 3], ['b', 4, 8, 6, 5, 7, 1, 2, 3], [6, 4, 8, 'b', 5, 7, 1, 2, 3], [6, 4, 8, 5, 'b', 7, 1, 2, 3], [6, 4, 8, 5, 7, 'b', 1, 2, 3], [6, 4, 'b', 5, 7, 8, 1, 2, 3], [6, 'b', 4, 5, 7, 8, 1, 2, 3], [6, 7, 4, 5, 'b', 8, 1, 2, 3], [6, 7, 4, 'b', 5, 8, 1, 2, 3], ['b', 7, 4, 6, 5, 8, 1, 2, 3], [7, 'b', 4, 6, 5, 8, 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]

Shortest Path: 25

Puzzle4: [[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 5, 6, 'b', 8, 3, 1], [7, 2, 'b', 5, 6, 4, 8, 3, 1], [7, 'b', 2, 5, 6, 4, 8, 3, 1], ['b', 7, 2, 5, 6, 4, 8, 3, 1], [5, 7, 2, 'b', 6, 4, 8, 3, 1], [5, 7, 2, 6, 'b', 4, 8, 3, 1], [5, 7, 2, 6, 4, 'b', 8, 3, 1], [5, 7, 2, 6, 4, 1, 8, 3, 'b'], [5, 7, 2, 6, 4, 1, 8, 'b', 3], [5, 7, 2, 6, 4, 1, 'b', 8, 3], [5, 7, 2, 'b', 4, 1, 6, 8, 3], [5, 7, 2, 4, 'b', 1, 6, 8, 3], [5, 7, 2, 4, 1, 'b', 6, 8, 3], [5, 7, 2, 4, 1, 3, 6, 8, 'b'], [5, 7, 2, 4, 1, 3, 6, 'b', 8], [5, 7, 2, 4, 'b', 3, 6, 1, 8], [5, 7, 2, 4, 3, 'b', 6, 1, 8], [5, 7, 'b', 4, 3, 2, 6, 1, 8], [5, 'b', 7, 4, 3, 2, 6, 1, 8], ['b', 5, 7, 4, 3, 2, 6, 1, 8], [4, 5, 7, 'b', 3, 2, 6, 1, 8], [4, 5, 7, 3, 'b', 2, 6, 1, 8], [4, 'b', 7, 3, 5, 2, 6, 1, 8], [4, 7, 'b', 3, 5, 2, 6, 1, 8], [4, 7, 2, 3, 5, 'b', 6, 1, 8], [4, 7, 2, 3, 'b', 5, 6, 1, 8], [4, 'b', 2, 3, 7, 5, 6, 1, 8], ['b', 4, 2, 3, 7, 5, 6, 1, 8], [3, 4, 2, 'b', 7, 5, 6, 1, 8], [3, 4, 2, 7, 'b', 5, 6, 1, 8], [3, 4, 2, 7, 1, 5, 6, 'b', 8], [3, 4, 2, 7, 1, 5, 'b', 6, 8], [3, 4, 2, 'b', 1, 5, 7, 6, 8], ['b', 4, 2, 3, 1, 5, 7, 6, 8], [4, 'b', 2, 3, 1, 5, 7, 6, 8], [4, 1, 2, 3, 'b', 5, 7, 6, 8], [4, 1, 2, 3, 6, 5, 7, 'b', 8], [4, 1, 2, 3, 6, 5, 'b', 7, 8], [4, 1, 2, 'b', 6, 5, 3, 7, 8], ['b', 1, 2, 4, 6, 5, 3, 7, 8], [1, 'b', 2, 4, 6, 5, 3, 7, 8], [1, 6, 2, 4, 'b', 5, 3, 7, 8], [1, 6, 2, 'b', 4, 5, 3, 7, 8], [1, 6, 2, 3, 4, 5, 'b', 7, 8], [1, 6, 2, 3, 4, 5, 7, 'b', 8], [1, 6, 2, 3, 'b', 5, 7, 4, 8], [1, 'b', 2, 3, 6, 5, 7, 4, 8], ['b', 1, 2, 3, 6, 5, 7, 4, 8], [3, 1, 2, 'b', 6, 5, 7, 4, 8], [3, 1, 2, 6, 'b', 5, 7, 4, 8], [3, 1, 2, 6, 4, 5, 7, 'b', 8], [3, 1, 2, 6, 4, 5, 'b', 7, 8], [3, 1, 2, 'b', 4, 5, 6, 7, 8], ['b', 1, 2, 3, 4, 5, 6, 7, 8]]

Shortest Path: 55

Puzzle5 [solution path5]

[[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 4, 8, 'b', 3, 2, 1], [5, 6, 'b', 4, 8, 7, 3, 2, 1], [5, 'b', 6, 4, 8, 7, 3, 2, 1], ['b', 5, 6, 4, 8, 7, 3, 2, 1], [4, 5, 6, 'b', 8, 7, 3, 2, 1], [4, 5, 6, 8, 'b', 7, 3, 2, 1], [4, 5, 6, 8, 2, 7, 3, 'b', 1], [4, 5, 6, 8, 2, 7, 3, 1, 'b'], [4, 5, 6, 8, 2, 'b', 3, 1, 7], [4, 5, 'b', 8, 2, 6, 3, 1, 7], [4, 'b', 5, 8, 2, 6, 3, 1, 7], [4, 2, 5, 8, 'b', 6, 3, 1, 7], [4, 2, 5, 8, 6, 'b', 3, 1, 7], [4, 2, 5, 8, 6, 7, 3, 1, 'b'], [4, 2, 5, 8, 6, 7, 3, 'b', 1], [4, 2, 5, 8, 'b', 7, 3, 6, 1], [4, 2, 5, 8, 7, 'b', 3, 6, 1], [4, 2, 5, 8, 7, 1, 3, 6, 'b'], [4, 2, 5, 8, 7, 1, 3, 'b', 6], [4, 2, 5, 8, 'b', 1, 3, 7, 6], [4, 2, 5, 'b', 8, 1, 3, 7, 6], [4, 2, 5, 3, 8, 1, 'b', 7, 6], [4, 2, 5, 3, 8, 1, 7, 'b', 6], [4, 2, 5, 3, 8, 1, 7, 6, 'b'], [4, 2, 5, 3, 8, 'b', 7, 6, 1], [4, 2, 'b', 3, 8, 5, 7, 6, 1], [4, 'b', 2, 3, 8, 5, 7, 6, 1], ['b', 4, 2, 3, 8, 5, 7, 6, 1], [3, 4, 2, 'b', 8, 5, 7, 6, 1], [3, 4, 2, 8, 'b', 5, 7, 6, 1], [3, 4, 2, 8, 5, 'b', 7, 6, 1], [3, 4, 2, 8, 5, 1, 7, 6, 'b'], [3, 4, 2, 8, 5, 1, 7, 'b', 6], [3, 4, 2, 8, 'b', 1, 7, 5, 6], [3, 'b', 2, 8, 4, 1, 7, 5, 6], [3, 2, 'b', 8, 4, 1, 7, 5, 6], [3, 2, 1, 8, 4, 'b', 7, 5, 6], [3, 2, 1, 8, 'b', 4, 7, 5, 6], [3, 2, 1, 8, 5, 4, 7, 'b', 6], [3, 2, 1, 8, 5, 4, 7, 6, 'b'], [3, 2, 1, 8, 5, 'b', 7, 6, 4], [3, 2, 'b', 8, 5, 1, 7, 6, 4], [3, 'b', 2, 8, 5, 1, 7, 6, 4], [3, 5, 2, 8, 'b', 1, 7, 6, 4], [3, 5, 2, 8, 1, 'b', 7, 6, 4], [3, 5, 2, 8, 1, 4, 7, 6, 'b'], [3, 5, 2, 8, 1, 4, 7, 'b', 6], [3, 5, 2, 8, 'b', 4, 7, 1, 6], [3, 'b', 2, 8, 5, 4, 7, 1, 6], [3, 2, 'b', 8, 5, 4, 7, 1, 6], [3, 2, 4, 8, 5, 'b', 7, 1, 6], [3, 2, 4, 8, 'b', 5, 7, 1, 6], [3, 2, 4, 8, 1, 5, 7, 'b', 6], [3, 2, 4, 8, 1, 5, 7, 6, 'b'], [3, 2, 4, 8, 1, 'b', 7, 6, 5], [3, 2, 4, 8, 'b', 1, 7, 6, 5], [3, 'b', 4, 8, 2, 1, 7, 6, 5], ['b', 3, 4, 8, 2, 1, 7, 6, 5], [8, 3, 4, 'b', 2, 1, 7, 6, 5], [8, 3, 4, 2, 'b', 1, 7, 6, 5], [8, 3, 4, 2, 1, 'b', 7, 6, 5], [8, 3, 'b', 2, 1, 4, 7, 6, 5], [8, 'b', 3, 2, 1, 4, 7, 6, 5], [8, 1, 3, 2, 'b', 4, 7, 6, 5], [8, 1, 3, 'b', 2, 4, 7, 6, 5], ['b', 1, 3, 8, 2, 4, 7, 6, 5], [1, 'b', 3, 8, 2, 4, 7, 6, 5], [1, 2, 3, 8, 'b', 4, 7, 6, 5]]

Shortest Path: 69

Average number of steps: $(2+10+25+55+69)/5 = 32.5$

Heuristic 2: Manhattan

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]

Shortest Path: 2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 4, 'b', 3, 2, 8, 1], [7, 6, 5, 4, 8, 3, 2, 'b', 1], [7, 6, 5, 4, 8, 3, 2, 1, 'b'], [7, 6, 5, 4, 8, 'b', 2, 1, 3], [7, 6, 5, 4, 'b', 8, 2, 1, 3], [7, 6, 5, 'b', 4, 8, 2, 1, 3], [7, 6, 5, 2, 4, 8, 'b', 1, 3], [7, 6, 5, 2, 4, 8, 1, 'b', 3], [7, 6, 5, 2, 'b', 8, 1, 4, 3], [7, 6, 5, 2, 8, 'b', 1, 4, 3], [7, 6, 5, 2, 8, 3, 1, 4, 'b'], [7, 6, 5, 2, 8, 3, 1, 'b', 4], [7, 6, 5, 2, 8, 3, 'b', 1, 4], [7, 6, 5, 'b', 8, 3, 2, 1, 4], [7, 6, 5, 8, 'b', 3, 2, 1, 4], [7, 6, 5, 8, 3, 'b', 2, 1, 4], [7, 6, 5, 8, 3, 4, 2, 1, 'b'], [7, 6, 5, 8, 3, 4, 2, 'b', 1], [7, 6, 5, 8, 'b', 4, 2, 3, 1], [7, 6, 5, 8, 4, 'b', 2, 3, 1], [7, 6, 5, 8, 4, 1, 2, 3, 'b'], [7, 6, 5, 8, 4, 1, 2, 'b', 3], [7, 6, 5, 8, 4, 1, 'b', 2, 3], [7, 6, 5, 'b', 4, 1, 8, 2, 3], [7, 6, 5, 4, 'b', 1, 8, 2, 3], [7, 6, 5, 4, 1, 'b', 8, 2, 3], [7, 6, 5, 4, 1, 3, 8, 2, 'b'], [7, 6, 5, 4, 1, 3, 8, 'b', 2], [7, 6, 5, 4, 'b', 3, 8, 1, 2], [7, 6, 5, 'b', 4, 3, 8, 1, 2], [7, 6, 5, 8, 4, 3, 'b', 1, 2], [7, 6, 5, 8, 4, 3, 1, 'b', 2], [7, 6, 5, 8, 4, 3, 1, 2, 'b'], [7, 6, 5, 8, 4, 'b', 1, 2, 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]

Shortest Path: 36

Puzzle3: [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], ['b', 7, 8, 4, 5, 6, 1, 2, 3], [4, 7, 8, 'b', 5, 6, 1, 2, 3], [4, 7, 8, 5, 'b', 6, 1, 2, 3], [4, 7, 8, 5, 6, 'b', 1, 2, 3], [4, 7, 'b', 5, 6, 8, 1, 2, 3], [4, 'b', 7, 5, 6, 8, 1, 2, 3], ['b', 4, 7, 5, 6, 8, 1, 2, 3], [5, 4, 7, 'b', 6, 8, 1, 2, 3], [5, 4, 7, 6, 'b', 8, 1, 2, 3], [5, 'b', 7, 6, 4, 8, 1, 2, 3], ['b', 5, 7, 6, 4, 8, 1, 2, 3], [6, 5, 7, 'b', 4, 8, 1, 2, 3], [6, 5, 7, 4, 'b', 8, 1, 2, 3], [6, 'b', 7, 4, 5, 8, 1, 2, 3], [6, 7, 'b', 4, 5, 8, 1, 2, 3], [6, 7, 8, 4, 5, 'b', 1, 2, 3], [6, 7, 8, 4, 'b', 5, 1, 2, 3], [6, 7, 8, 'b', 4, 5, 1, 2, 3], ['b', 7, 8, 6, 4, 5, 1, 2, 3], [7, 'b', 8, 6, 4, 5, 1, 2, 3], [7, 4, 8, 6, 'b', 5, 1, 2, 3], [7, 4, 8, 6, 5, 'b', 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]

Shortest Path: 25

[[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 5, 6, 'b', 8, 3, 1], [7, 2, 'b', 5, 6, 4, 8, 3, 1], [7, 'b', 2, 5, 6, 4, 8, 3, 1], ['b', 7, 2, 5, 6, 4, 8, 3, 1], [5, 7, 2, 'b', 6, 4, 8, 3, 1], [5, 7, 2, 6, 'b', 4, 8, 3, 1], [5, 7, 2, 6, 3, 4, 8, 'b', 1], [5, 7, 2, 6, 3, 4, 'b', 8, 1], [5, 7, 2, 'b', 3, 4, 6, 8, 1], [5, 7, 2, 3, 'b', 4, 6, 8, 1], [5, 7, 2, 3, 4, 'b', 6, 8, 1], [5, 7, 2, 3, 4, 1, 6, 8, 'b'], [5, 7, 2, 3, 4, 1, 6, 'b', 8], [5, 7, 2, 3, 'b', 1, 6, 4, 8], [5, 'b', 2, 3, 7, 1, 6, 4, 8], ['b', 5, 2, 3, 7, 1, 6, 4, 8], [3, 5, 2, 'b', 7, 1, 6, 4, 8], [3, 5, 2, 7, 'b', 1, 6, 4, 8], [3, 5, 2, 7, 1, 'b', 6, 4, 8], [3, 5, 'b', 7, 1, 2, 6, 4, 8], [3, 'b', 5, 7, 1, 2, 6, 4, 8], [3, 1, 5, 7, 'b', 2, 6, 4, 8], [3, 1, 5, 'b', 7, 2, 6, 4, 8], ['b', 1, 5, 3, 7, 2, 6, 4, 8], [1, 'b', 5, 3, 7, 2, 6, 4, 8], [1, 5, 'b', 3, 7, 2, 6, 4, 8], [1, 5, 2, 3, 7, 'b', 6, 4, 8], [1, 5, 2, 3, 7, 8, 6, 4, 'b'], [1, 5, 2, 3, 7, 8, 6, 'b', 4], [1, 5, 2, 3, 'b', 8, 6, 7, 4], [1, 'b', 2, 3, 5, 8, 6, 7, 4], ['b', 1, 2, 3, 5, 8, 6, 7, 4], [3, 1, 2, 'b', 5, 8, 6, 7, 4], [3, 1, 2, 6, 5, 8, 'b', 7, 4], [3, 1, 2, 6, 5, 8, 7, 'b', 4], [3, 1, 2, 6, 5, 8, 7, 4, 'b'], [3, 1, 2, 6, 5, 'b', 7, 4, 8], [3, 1, 2, 6, 'b', 5, 7, 4, 8], [3, 1, 2, 6, 4, 5, 7, 'b', 8], [3, 1, 2, 6, 4, 5, 'b', 7, 8], [3, 1, 2, 'b', 4, 5, 6, 7, 8], ['b', 1, 2, 3, 4, 5, 6, 7, 8]]

Shortest Path: 43

Puzzle5: [[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 4, 8, 'b', 3, 2, 1], [5, 6, 'b', 4, 8, 7, 3, 2, 1], [5, 'b', 6, 4, 8, 7, 3, 2, 1], ['b', 5, 6, 4, 8, 7, 3, 2, 1], [4, 5, 6, 'b', 8, 7, 3, 2, 1], [4, 5, 6, 8, 'b', 7, 3, 2, 1], [4, 5, 6, 8, 7, 'b', 3, 2, 1], [4, 5, 'b', 8, 7, 6, 3, 2, 1], [4, 'b', 5, 8, 7, 6, 3, 2, 1], ['b', 4, 5, 8, 7, 6, 3, 2, 1], [8, 4, 5, 'b', 7, 6, 3, 2, 1], [8, 4, 5, 7, 'b', 6, 3, 2, 1], [8, 4, 5, 7, 6, 'b', 3, 2, 1], [8, 4, 'b', 7, 6, 5, 3, 2, 1], [8, 'b', 4, 7, 6, 5, 3, 2, 1], [8, 6, 4, 7, 'b', 5, 3, 2, 1], [8, 6, 4, 7, 2, 5, 3, 'b', 1], [8, 6, 4, 7, 2, 5, 3, 1, 'b'], [8, 6, 4, 7, 2, 'b', 3, 1, 5], [8, 6, 'b', 7, 2, 4, 3, 1, 5], [8, 'b', 6, 7, 2, 4, 3, 1, 5], [8, 2, 6, 7, 'b', 4, 3, 1, 5], [8, 2, 6, 7, 1, 4, 3, 'b', 5], [8, 2, 6, 7, 1, 4, 'b', 3, 5], [8, 2, 6, 'b', 1, 4, 7, 3, 5], [8, 2, 6, 1, 'b', 4, 7, 3, 5], [8, 2, 6, 1, 3, 4, 7, 'b', 5], [8, 2, 6, 1, 3, 4, 7, 5, 'b'], [8, 2, 6, 1, 3, 'b', 7, 5, 4], [8, 2, 6, 1, 'b', 3, 7, 5, 4], [8, 'b', 6, 1, 2, 3, 7, 5, 4], [8, 6, 'b', 1, 2, 3, 7, 5, 4], [8, 6, 3, 1, 2, 'b', 7, 5, 4], [8, 6, 3, 1, 2, 4, 7, 5, 'b'], [8, 6, 3, 1, 2, 4, 7, 'b', 5], [8, 6, 3, 1, 'b', 4, 7, 2, 5], [8, 'b', 3, 1, 6, 4, 7, 2, 5], ['b', 8, 3, 1, 6, 4, 7, 2, 5], [1, 8, 3, 'b', 6, 4, 7, 2, 5], [1, 8, 3, 6, 'b', 4, 7, 2, 5], [1, 8, 3, 6, 2, 4, 7, 'b', 5], [1, 8, 3, 6, 2, 4, 'b', 7, 5], [1, 8, 3, 'b', 2, 4, 6, 7, 5], ['b', 8, 3, 1, 2, 4, 6, 7, 5], [8, 'b', 3, 1, 2, 4, 6, 7, 5], [8, 2, 3, 1, 'b', 4, 6, 7, 5], [8, 2, 3, 1, 7, 4, 6, 'b', 5], [8, 2, 3, 1, 7, 4, 'b', 6, 5], [8, 2, 3, 'b', 7, 4, 1, 6, 5], [8, 2, 3, 7, 'b', 4, 1, 6, 5], [8, 2, 3, 7, 6, 4, 1, 'b', 5], [8, 2, 3, 7, 6, 4, 'b', 1, 5], [8, 2, 3, 'b', 6, 4, 7, 1, 5], ['b', 2, 3, 8, 6, 4, 7, 1, 5], [2, 'b', 3, 8, 6, 4, 7, 1, 5], [2, 6, 3, 8, 'b', 4, 7, 1, 5], [2, 6, 3, 8, 1, 4, 7, 'b', 5], [2, 6, 3, 8, 1, 4, 'b', 7, 5], [2, 6, 3, 'b', 1, 4, 8, 7, 5], [2, 6, 3, 1, 'b', 4, 8, 7, 5], [2, 'b', 3, 1, 6, 4, 8, 7, 5], ['b', 2, 3, 1, 6, 4, 8, 7, 5], [1, 2, 3, 'b', 6, 4, 8, 7, 5], [1, 2, 3, 8, 6, 4, 'b', 7, 5], [1, 2, 3, 8, 6, 4, 7, 'b', 5], [1, 2, 3, 8, 'b', 4, 7, 6, 5]]

Shortest Path: 67

Average number of steps: $(2 + 36 + 25 + 43 + 67) / 5 = 34.6$

Heuristic 3: Combined Heuristic 1 and 2

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]

Shortest Path: 2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 4, 'b', 3, 2, 8, 1], [7, 6, 5, 4, 8, 3, 2, 'b', 1], [7, 6, 5, 4, 8, 3, 'b', 2, 1], [7, 6, 5, 'b', 8, 3, 4, 2, 1], [7, 6, 5, 8, 'b', 3, 4, 2, 1], [7, 6, 5, 8, 2, 3, 4, 'b', 1], [7, 6, 5, 8, 2, 3, 4, 1, 'b'], [7, 6, 5, 8, 2, 'b', 4, 1, 3], [7, 6, 5, 8, 'b', 2, 4, 1, 3], [7, 6, 5, 'b', 8, 2, 4, 1, 3], [7, 6, 5, 4, 8, 2, 'b', 1, 3], [7, 6, 5, 4, 8, 2, 1, 'b', 3], [7, 6, 5, 4, 'b', 2, 1, 8, 3], [7, 6, 5, 4, 2, 'b', 1, 8, 3], [7, 6, 5, 4, 2, 3, 1, 8, 'b'], [7, 6, 5, 4, 2, 3, 1, 'b', 8], [7, 6, 5, 4, 'b', 3, 1, 2, 8], [7, 6, 5, 'b', 4, 3, 1, 2, 8],

[7, 6, 5, 1, 4, 3, 'b', 2, 8], [7, 6, 5, 1, 4, 3, 2, 'b', 8], [7, 6, 5, 1, 4, 3, 2, 8, 'b'], [7, 6, 5, 1, 4, 'b', 2, 8, 3], [7, 6, 5, 1, 'b', 4, 2, 8, 3], [7, 6, 5, 1, 8, 4, 2, 'b', 3], [7, 6, 5, 1, 8, 4, 'b', 2, 3], [7, 6, 5, 'b', 8, 4, 1, 2, 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]

Shortest Path: 28

Puzzle3 : [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], [7, 5, 8, 4, 'b', 6, 1, 2, 3], [7, 5, 8, 4, 6, 'b', 1, 2, 3], [7, 5, 'b', 4, 6, 8, 1, 2, 3], [7, 'b', 5, 4, 6, 8, 1, 2, 3], ['b', 7, 5, 4, 6, 8, 1, 2, 3], [4, 7, 5, 'b', 6, 8, 1, 2, 3], [4, 7, 5, 6, 'b', 8, 1, 2, 3], [4, 'b', 5, 6, 7, 8, 1, 2, 3], [4, 5, 'b', 6, 7, 8, 1, 2, 3], [4, 5, 8, 6, 7, 'b', 1, 2, 3], [4, 5, 8, 6, 'b', 7, 1, 2, 3], [4, 'b', 8, 6, 5, 7, 1, 2, 3], ['b', 4, 8, 6, 5, 7, 1, 2, 3], [6, 4, 8, 'b', 5, 7, 1, 2, 3], [6, 4, 8, 5, 'b', 7, 1, 2, 3], [6, 4, 8, 5, 7, 'b', 1, 2, 3], [6, 4, 'b', 5, 7, 8, 1, 2, 3], [6, 'b', 4, 5, 7, 8, 1, 2, 3], [6, 7, 4, 5, 'b', 8, 1, 2, 3], [6, 7, 4, 'b', 5, 8, 1, 2, 3], ['b', 7, 4, 6, 5, 8, 1, 2, 3], [7, 'b', 4, 6, 5, 8, 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]

Shortest Path: 25

Puzzle4 : [[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 5, 6, 'b', 8, 3, 1], [7, 2, 'b', 5, 6, 4, 8, 3, 1], [7, 'b', 2, 5, 6, 4, 8, 3, 1], ['b', 7, 2, 5, 6, 4, 8, 3, 1], [5, 7, 2, 'b', 6, 4, 8, 3, 1], [5, 7, 2, 6, 'b', 4, 8, 3, 1], [5, 7, 2, 6, 4, 'b', 8, 3, 1], [5, 7, 2, 6, 4, 1, 8, 3, 'b'], [5, 7, 2, 6, 4, 1, 8, 'b', 3], [5, 7, 2, 6, 4, 1, 'b', 8, 3], [5, 7, 2, 'b', 4, 1, 6, 8, 3], [5, 7, 2, 4, 'b', 1, 6, 8, 3], [5, 7, 2, 4, 1, 'b', 6, 8, 3], [5, 7, 2, 4, 1, 3, 6, 8, 'b'], [5, 7, 2, 4, 1, 3, 6, 'b', 8], [5, 7, 2, 4, 'b', 3, 6, 1, 8], [5, 7, 2, 'b', 4, 3, 6, 1, 8], ['b', 7, 2, 5, 4, 3, 6, 1, 8], [7, 'b', 2, 5, 4, 3, 6, 1, 8], [7, 4, 2, 5, 'b', 3, 6, 1, 8], [7, 4, 2, 'b', 5, 3, 6, 1, 8], ['b', 4, 2, 7, 5, 3, 6, 1, 8], [4, 'b', 2, 7, 5, 3, 6, 1, 8], [4, 5, 2, 7, 'b', 3, 6, 1, 8], [4, 5, 2, 7, 3, 'b', 6, 1, 8], [4, 5, 'b', 7, 3, 2, 6, 1, 8], [4, 'b', 5, 7, 3, 2, 6, 1, 8], ['b', 4, 5, 7, 3, 2, 6, 1, 8], [7, 4, 5, 'b', 3, 2, 6, 1, 8], [7, 4, 5, 3, 'b', 2, 6, 1, 8], [7, 'b', 5, 3, 4, 2, 6, 1, 8], ['b', 7, 5, 3, 4, 2, 6, 1, 8], [3, 7, 5, 'b', 4, 2, 6, 1, 8], [3, 7, 5, 4, 'b', 2, 6, 1, 8], [3, 'b', 5, 4, 7, 2, 6, 1, 8], [3, 5, 'b', 4, 7, 2, 6, 1, 8], [3, 5, 2, 4, 7, 'b', 6, 1, 8], [3, 5, 2, 4, 'b', 7, 6, 1, 8], [3, 5, 2, 'b', 4, 7, 6, 1, 8], ['b', 5, 2, 3, 4, 7, 6, 1, 8], [5, 'b', 2, 3, 4, 7, 6, 1, 8], [5, 4, 2, 3, 'b', 7, 6, 1, 8], [5, 4, 2, 3, 1, 7, 6, 'b', 8], [5, 4, 2, 3, 1, 7, 6, 8, 'b'], [5, 4, 2, 3, 1, 'b', 6, 8, 7], [5, 4, 2, 3, 'b', 1, 6, 8, 7], [5, 'b', 2, 3, 4, 1, 6, 8, 7], [5, 2, 'b', 3, 4, 1, 6, 8, 7], [5, 2, 1, 3, 4, 'b', 6, 8, 7], [5, 2, 1, 3, 4, 7, 6, 8, 'b'], [5, 2, 1, 3, 4, 7, 6, 'b', 8], [5, 2, 1, 3, 'b', 7, 6, 4, 8], [5, 2, 1, 3, 7, 'b', 6, 4, 8], [5, 2, 'b', 3, 7, 1, 6, 4, 8], [5, 'b', 2, 3, 7, 1, 6, 4, 8], ['b', 5, 2, 3, 7, 1, 6, 4, 8], [3, 5, 2, 'b', 7, 1, 6, 4, 8], [3, 5, 2, 7, 'b', 1, 6, 4, 8], [3, 5, 2, 7, 1, 'b', 6, 4, 8], [3, 5, 'b', 7, 1, 2, 6, 4, 8], [3, 'b', 5, 7, 1, 2, 6, 4, 8], [3, 1, 5, 7, 'b', 2, 6, 4, 8], [3, 1, 5, 'b', 7, 2, 6, 4, 8], ['b', 1, 5, 3, 7, 2, 6, 4, 8], [1, 'b', 5, 3, 7, 2, 6, 4, 8], [1, 7, 5, 3, 'b', 2, 6, 4, 8], [1, 7, 5, 3, 2, 'b', 6, 4, 8], [1, 7, 'b', 3, 2, 5, 6, 4, 8], [1, 'b', 7, 3, 2, 5, 6, 4, 8], [1, 2, 7, 3, 'b', 5, 6, 4, 8], [1, 2, 7, 3, 4, 5, 6, 'b', 8], [1, 2, 7, 3, 4, 5, 6, 8, 'b'], [1, 2, 7, 3, 4, 'b', 6, 8, 5], [1, 2, 'b', 3, 4, 7, 6, 8, 5], [1, 'b', 2, 3, 4, 7, 6, 8, 5], [1, 4, 2, 3, 'b', 7, 6, 8, 5], [1, 4, 2, 3, 7, 'b', 6, 8, 5], [1, 4, 2, 3, 7, 5, 6, 8, 'b'], [1, 4, 2, 3, 7, 5, 6, 'b', 8], [1, 4, 2, 3, 'b', 5, 6, 7, 8], [1, 'b', 2, 3, 4, 5, 6, 7, 8], ['b', 1, 2, 3, 4, 5, 6, 7, 8]]

Shortest Path: 83

Puzzle5 : [[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 4, 8, 'b', 3, 2, 1], [5, 6, 'b', 4, 8, 7, 3, 2, 1], [5, 'b', 6, 4, 8, 7, 3, 2, 1], ['b', 5, 6, 4, 8, 7, 3, 2, 1], [4, 5, 6, 'b', 8, 7, 3, 2, 1], [4, 5, 6, 8, 'b', 7, 3, 2, 1], [4, 5, 6, 8, 7, 'b', 3, 2, 1], [4, 5, 'b', 8, 7, 6, 3, 2, 1], [4, 'b', 5, 8, 7, 6, 3, 2, 1], [4, 7, 5, 8, 'b', 6, 3, 2, 1], [4, 7, 5, 8, 2, 6, 3, 'b', 1], [4, 7, 5, 8, 2, 6, 3, 1, 'b'], [4, 7, 5, 8, 2, 'b', 3, 1, 6], [4, 7, 'b', 8, 2, 5, 3, 1, 6], [4, 'b', 7, 8, 2, 5, 3, 1, 6], [4, 2, 7, 8, 'b', 5, 3, 1, 6], [4, 2, 7, 8, 1, 5, 3, 'b', 6], [4, 2, 7, 8, 1, 5, 3, 6, 'b'], [4, 2, 7, 8, 1, 'b', 3, 6, 5], [4, 2, 'b', 8, 1, 7, 3, 6, 5], [4, 'b', 2, 8, 1, 7, 3, 6, 5], ['b', 4, 2, 8, 1, 7, 3, 6, 5], [8, 4, 2, 'b', 1, 7, 3, 6, 5], [8, 4, 2, 1, 'b', 7, 3, 6, 5], [8, 'b', 2, 1, 4, 7, 3, 6, 5], [8, 2, 'b', 1, 4, 7, 3, 6, 5], [8, 2, 7, 1, 4, 'b', 3, 6, 5], [8, 2, 7, 1, 'b', 4, 3, 6, 5], [8, 2, 7, 'b', 1, 4, 3, 6, 5], [8, 2, 7, 3, 1, 4, 'b', 6, 5], [8, 2, 7, 3, 1, 4, 6, 'b', 5], [8, 2, 7, 3, 'b', 4, 6, 1, 5], [8, 2, 7, 'b', 3, 4, 6, 1, 5], ['b', 2, 7, 8,

3, 4, 6, 1, 5], [2, 'b', 7, 8, 3, 4, 6, 1, 5], [2, 3, 7, 8, 'b', 4, 6, 1, 5], [2, 3, 7, 8, 1, 4, 6, 'b', 5], [2, 3, 7, 8, 1, 4, 'b', 6, 5], [2, 3, 7, 'b', 1, 4, 8, 6, 5], [2, 3, 7, 1, 'b', 4, 8, 6, 5], [2, 3, 7, 1, 4, 'b', 8, 6, 5], [2, 3, 'b', 1, 4, 7, 8, 6, 5], [2, 'b', 3, 1, 4, 7, 8, 6, 5], ['b', 2, 3, 1, 4, 7, 8, 6, 5], [1, 2, 3, 'b', 4, 7, 8, 6, 5], [1, 2, 3, 4, 'b', 7, 8, 6, 5], [1, 2, 3, 4, 7, 'b', 8, 6, 5], [1, 2, 3, 4, 7, 5, 8, 6, 'b'], [1, 2, 3, 4, 7, 5, 8, 'b', 6], [1, 2, 3, 4, 'b', 5, 8, 7, 6], [1, 2, 3, 'b', 4, 5, 8, 7, 6], [1, 2, 3, 8, 4, 5, 'b', 7, 6], [1, 2, 3, 8, 4, 5, 7, 'b', 6], [1, 2, 3, 8, 4, 5, 7, 6, 'b'], [1, 2, 3, 8, 4, 'b', 7, 6, 5], [1, 2, 3, 8, 'b', 4, 7, 6, 5]]

Shortest Path: 57

Average number of steps: $(2 + 28 + 25 + 83 + 57) / 5 = 39.0$

A* search:

Heuristic 1: Misplaced Tiles

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]

Shortest Path: 2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 2, 4, 3, 'b', 8, 1], [7, 6, 5, 2, 4, 3, 8, 'b', 1], [7, 6, 5, 2, 4, 3, 8, 1, 'b'], [7, 6, 5, 2, 4, 'b', 8, 1, 3], [7, 6, 5, 2, 'b', 4, 8, 1, 3], [7, 6, 5, 'b', 2, 4, 8, 1, 3], [7, 6, 5, 8, 2, 4, 'b', 1, 3], [7, 6, 5, 8, 2, 4, 1, 'b', 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]

Shortest Path: 10

Puzzle3: [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], [7, 5, 8, 4, 'b', 6, 1, 2, 3], [7, 5, 8, 'b', 4, 6, 1, 2, 3], ['b', 5, 8, 7, 4, 6, 1, 2, 3], [5, 'b', 8, 7, 4, 6, 1, 2, 3], [5, 4, 8, 7, 'b', 6, 1, 2, 3], [5, 4, 8, 7, 6, 'b', 1, 2, 3], [5, 4, 'b', 7, 6, 8, 1, 2, 3], [5, 'b', 4, 7, 6, 8, 1, 2, 3], ['b', 5, 4, 7, 6, 8, 1, 2, 3], [7, 5, 4, 'b', 6, 8, 1, 2, 3], [7, 5, 4, 6, 'b', 8, 1, 2, 3], [7, 'b', 4, 6, 5, 8, 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]

Shortest Path: 15

Puzzle4: solution not found ran for 114 minutes

[[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 5, 3, 6, 8, 'b', 1], [7, 2, 4, 5, 3, 6, 8, 1, 'b'], [7, 2, 4, 5, 3, 'b', 8, 1, 6], [7, 2, 4, 5, 'b', 3, 8, 1, 6], [7, 2, 4, 5, 1, 3, 8, 'b', 6], [7, 2, 4, 5, 1, 3, 'b', 8, 6], [7, 2, 4, 'b', 1, 3, 5, 8, 6], [7, 2, 4, 1, 'b', 3, 5, 8, 6], [7, 'b', 4, 1, 2, 3, 5, 8, 6], [7, 4, 'b', 1, 2, 3, 5, 8, 6], [7, 4, 3, 1, 2, 'b', 5, 8, 6], [7, 4, 3, 1, 2, 6, 5, 8, 'b'], [7, 4, 3, 1, 2, 6, 5, 'b', 8], [7, 4, 3, 1, 2, 6, 'b', 5, 8], [7, 4, 3, 'b', 2, 6, 1, 5, 8]]

Shortest Path: Not found

Puzzle5: solution not found ran for 49 minutes

[[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 'b', 4, 8, 3, 2, 1], [5, 6, 7, 3, 4, 8, 'b', 2, 1], [5, 6, 7, 3, 4, 8, 2, 'b', 1], [5, 6, 7, 3, 'b', 8, 2, 4, 1], [5, 6, 7, 'b', 3, 8, 2, 4, 1], ['b', 6, 7, 5, 3, 8, 2, 4, 1], [6, 'b', 7, 5, 3, 8, 2, 4, 1], [6, 3, 7, 5, 'b', 8, 2, 4, 1], [6, 3, 7, 'b', 5, 8, 2, 4, 1], [6, 3, 7, 2, 5, 8, 'b', 4, 1], [6, 3, 7, 2, 5, 8, 4, 'b', 1], [6, 3, 7, 2, 'b', 8, 4, 5, 1], [6, 'b', 7, 2, 3, 8, 4, 5, 1], [6, 7, 'b', 2, 3, 8, 4, 5, 1]]

Shortest Path: Not found

Average number of steps: $2 + 10 + 15 / 3 = 9$

Heuristic 2: Manhattan

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]
Shortest Path:2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 2, 4, 3, 'b', 8, 1], [7, 6, 5, 2, 4, 3, 8, 'b', 1], [7, 6, 5, 2, 4, 3, 8, 1, 'b'], [7, 6, 5, 2, 4, 'b', 8, 1, 3], [7, 6, 5, 2, 'b', 4, 8, 1, 3], [7, 6, 5, 'b', 2, 4, 8, 1, 3], [7, 6, 5, 8, 2, 4, 'b', 1, 3], [7, 6, 5, 8, 2, 4, 1, 'b', 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]
Shortest Path:10

Puzzle3: [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], [7, 5, 8, 4, 'b', 6, 1, 2, 3], [7, 5, 8, 'b', 4, 6, 1, 2, 3], ['b', 5, 8, 7, 4, 6, 1, 2, 3], [5, 'b', 8, 7, 4, 6, 1, 2, 3], [5, 4, 8, 7, 'b', 6, 1, 2, 3], [5, 4, 8, 7, 6, 'b', 1, 2, 3], [5, 4, 'b', 7, 6, 8, 1, 2, 3], [5, 'b', 4, 7, 6, 8, 1, 2, 3], ['b', 5, 4, 7, 6, 8, 1, 2, 3], [7, 5, 4, 'b', 6, 8, 1, 2, 3], [7, 5, 4, 6, 'b', 8, 1, 2, 3], [7, 'b', 4, 6, 5, 8, 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]
Shortest Path:15

Puzzle4: [[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 'b', 5, 6, 8, 3, 1], ['b', 2, 4, 7, 5, 6, 8, 3, 1], [2, 'b', 4, 7, 5, 6, 8, 3, 1], [2, 5, 4, 7, 'b', 6, 8, 3, 1], [2, 5, 4, 7, 3, 6, 8, 'b', 1], [2, 5, 4, 7, 3, 6, 'b', 8, 1], [2, 5, 4, 'b', 3, 6, 7, 8, 1], [2, 5, 4, 3, 'b', 6, 7, 8, 1], [2, 5, 4, 3, 6, 'b', 7, 8, 1], [2, 5, 'b', 3, 6, 4, 7, 8, 1], [2, 'b', 5, 3, 6, 4, 7, 8, 1], ['b', 2, 5, 3, 6, 4, 7, 8, 1], [3, 2, 5, 'b', 6, 4, 7, 8, 1], [3, 2, 5, 6, 'b', 4, 7, 8, 1], [3, 2, 5, 6, 4, 'b', 7, 8, 1], [3, 2, 5, 6, 4, 1, 7, 8, 'b'], [3, 2, 5, 6, 4, 1, 7, 'b', 8], [3, 2, 5, 6, 4, 1, 'b', 7, 8], [3, 2, 5, 'b', 4, 1, 6, 7, 8], [3, 2, 5, 4, 'b', 1, 6, 7, 8], [3, 2, 5, 4, 1, 'b', 6, 7, 8], [3, 2, 'b', 4, 1, 5, 6, 7, 8], [3, 'b', 2, 4, 1, 5, 6, 7, 8], [3, 1, 2, 4, 'b', 5, 6, 7, 8], [3, 1, 2, 'b', 4, 5, 6, 7, 8], ['b', 1, 2, 3, 4, 5, 6, 7, 8]]
Shortest Path:27

Puzzle5: [[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 4, 8, 'b', 3, 2, 1], [5, 6, 7, 4, 8, 1, 3, 2, 'b'], [5, 6, 7, 4, 8, 1, 3, 'b', 2], [5, 6, 7, 4, 8, 1, 'b', 3, 2], [5, 6, 7, 'b', 8, 1, 4, 3, 2], ['b', 6, 7, 5, 8, 1, 4, 3, 2], [6, 'b', 7, 5, 8, 1, 4, 3, 2], [6, 7, 'b', 5, 8, 1, 4, 3, 2], [6, 7, 1, 5, 8, 'b', 4, 3, 2], [6, 7, 1, 5, 8, 2, 4, 3, 'b'], [6, 7, 1, 5, 8, 2, 4, 'b', 3], [6, 7, 1, 5, 8, 2, 'b', 4, 3], [6, 7, 1, 'b', 8, 2, 5, 4, 3], ['b', 7, 1, 6, 8, 2, 5, 4, 3], [7, 'b', 1, 6, 8, 2, 5, 4, 3], [7, 1, 'b', 6, 8, 2, 5, 4, 3], [7, 1, 2, 6, 8, 'b', 5, 4, 3], [7, 1, 2, 6, 8, 3, 5, 4, 'b'], [7, 1, 2, 6, 8, 3, 5, 'b', 4], [7, 1, 2, 6, 8, 3, 'b', 5, 4], [7, 1, 2, 'b', 8, 3, 6, 5, 4], ['b', 1, 2, 7, 8, 3, 6, 5, 4], [1, 'b', 2, 7, 8, 3, 6, 5, 4], [1, 2, 'b', 7, 8, 3, 6, 5, 4], [1, 2, 3, 7, 8, 'b', 6, 5, 4], [1, 2, 3, 7, 8, 4, 6, 5, 'b'], [1, 2, 3, 7, 8, 4, 6, 'b', 5], [1, 2, 3, 7, 8, 4, 'b', 6, 5], [1, 2, 3, 'b', 8, 4, 7, 6, 5], [1, 2, 3, 8, 'b', 4, 7, 6, 5]]
Shortest Path:31

Average number of steps: $(2+10+15+27+31)/5 = 17$

Heuristic 3: Combined Heuristic 1 and 2

Puzzle1: [[1, 2, 3, 4, 'b', 5, 6, 7, 8], [1, 2, 3, 'b', 4, 5, 6, 7, 8]]
Shortest Path: 2

Puzzle2: [[7, 6, 5, 'b', 4, 3, 2, 8, 1], [7, 6, 5, 2, 4, 3, 'b', 8, 1], [7, 6, 5, 2, 4, 3, 8, 'b', 1], [7, 6, 5, 2, 4, 3, 8, 1, 'b'], [7, 6, 5, 2, 4, 'b', 8, 1, 3], [7, 6, 5, 2, 'b', 4, 8, 1, 3], [7, 6, 5, 'b', 2, 4, 8, 1, 3], [7, 6, 5, 8, 2, 4, 'b', 1, 3], [7, 6, 5, 8, 2, 4, 1, 'b', 3], [7, 6, 5, 8, 'b', 4, 1, 2, 3]]
Shortest Path: 10

Puzzle3: [[7, 8, 'b', 4, 5, 6, 1, 2, 3], [7, 'b', 8, 4, 5, 6, 1, 2, 3], [7, 5, 8, 4, 'b', 6, 1, 2, 3], [7, 5, 8, 'b', 4, 6, 1, 2, 3], ['b', 5, 8, 7, 4, 6, 1, 2, 3], [5, 'b', 8, 7, 4, 6, 1, 2, 3], [5, 4, 8, 7, 'b', 6, 1, 2, 3], [5, 4, 8, 7, 6, 'b', 1, 2, 3], [5, 4, 'b', 7, 6, 8, 1, 2, 3], [5, 'b', 4, 7, 6, 8, 1, 2, 3], ['b', 5, 4, 7, 6, 8, 1, 2, 3], [7, 5, 4, 'b', 6, 8, 1, 2, 3], [7, 5, 4, 6, 'b', 8, 1, 2, 3], [7, 'b', 4, 6, 5, 8, 1, 2, 3], [7, 4, 'b', 6, 5, 8, 1, 2, 3]]
Shortest Path: 15

Puzzle4: [[7, 2, 4, 5, 'b', 6, 8, 3, 1], [7, 2, 4, 5, 3, 6, 8, 'b', 1], [7, 2, 4, 5, 3, 6, 'b', 8, 1], [7, 2, 4, 'b', 3, 6, 5, 8, 1], [7, 2, 4, 3, 'b', 6, 5, 8, 1], [7, 2, 4, 3, 6, 'b', 5, 8, 1], [7, 2, 'b', 3, 6, 4, 5, 8, 1], [7, 'b', 2, 3, 6, 4, 5, 8, 1], [7, 6, 2, 3, 'b', 4, 5, 8, 1], [7, 6, 2, 3, 4, 'b', 5, 8, 1], [7, 6, 2, 3, 4, 1, 5, 8, 'b'], [7, 6, 2, 3, 4, 1, 5, 'b', 8], [7, 6, 2, 3, 'b', 1, 5, 4, 8], [7, 6, 2, 3, 1, 'b', 5, 4, 8], [7, 6, 'b', 3, 1, 2, 5, 4, 8], [7, 'b', 6, 3, 1, 2, 5, 4, 8], [7, 1, 6, 3, 'b', 2, 5, 4, 8]]
Shortest Path: Not found

Puzzle5: [[5, 6, 7, 4, 'b', 8, 3, 2, 1], [5, 6, 7, 'b', 4, 8, 3, 2, 1], [5, 6, 7, 3, 4, 8, 'b', 2, 1], [5, 6, 7, 3, 4, 8, 2, 'b', 1], [5, 6, 7, 3, 'b', 8, 2, 4, 1], [5, 6, 7, 'b', 3, 8, 2, 4, 1], ['b', 6, 7, 5, 3, 8, 2, 4, 1], [6, 'b', 7, 5, 3, 8, 2, 4, 1], [6, 7, 'b', 5, 3, 8, 2, 4, 1], [6, 7, 8, 5, 3, 'b', 2, 4, 1], [6, 7, 8, 5, 'b', 3, 2, 4, 1], [6, 7, 8, 'b', 5, 3, 2, 4, 1], [6, 7, 8, 2, 5, 3, 'b', 4, 1]]
Shortest Path: Not found

Average number of steps: $(2+10+15)/3 = 9$

Conclusion:

While the A* algorithm gave more optimal solutions overall, in harder cases, the A* algorithm would take exponentially more time with the wrong heuristic than the greedy best first algorithm. It seems that in general, applications where time to get an optimal answer are needed quickly, greedy best first search would be a better option. However, if you only need to find the optimal path once, and you wanted to reuse the same path over extended periods of time I could see why someone would want to wait the extra time for the algorithm to run. If you chose the right heuristic for the job, you could significantly improve performance to find the most optimal solution.

It was interesting that the best first search had its lowest average steps from misplaced tiles. I initially thought this heuristic would do poorly compared to the others. Yet in my case this heuristic was better for the best first search algorithm. The Manhattan distance was better for the A* algorithm, and the misplaced tiles were unable to get to a solution in my harder puzzles 4 and 5 in a reasonable amount of time. However, using Manhattan distance puzzles 4 and 5 were found within 19 minutes and able to solve both the harder puzzles. The average steps of the Manhattan distance were significantly shorter with A* than Greedy best first search. In particular puzzle 5 took half the time with (31 steps instead of 67).

The combined heuristic ended up having a better overall average than the Manhattan and misplaced tiles for greedy best first search. However, had the same results as the Manhattan distance for the A* algorithm.