Project Problem Statement: Describing the motion of a 2-Stage Rocket

In this project I will describe the motion of the 2nd stage of a 2-stage rocket that is fired vertically from rest at s = 0 by calculating Velocity and Displacement using the acceleration info as seen below:

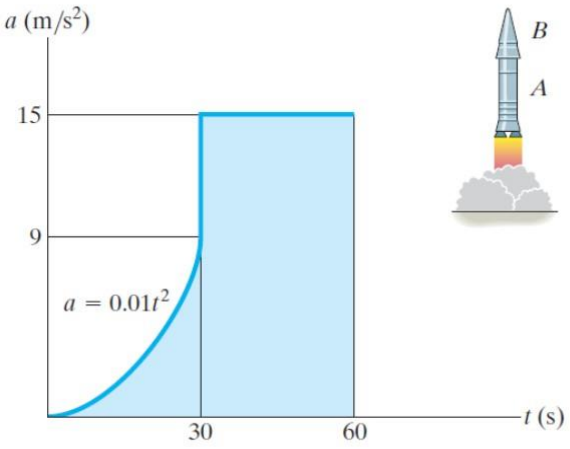

Figure 1.1: The initial acceleration time graph started with

The first stage A ignites at t=0 and burns out at t=30s from which stage B ignites. my main objective is to describe the motion of stage B. The strategy is to first find out what the full acceleration equation is, and everything will fall in place. Under thorough investigation, I surmised that the acceleration equation is in the form of a piecewise function from 0-30s which represents stage A being $a = 0.01\ t^2$ then from 30-60s, it is a constant acceleration of 15m/s^2. After further investigation using graphing programs like Desmos, I arrived at the acceleration equation seen below.

$$acceleration := t \rightarrow \begin{cases} 0.01\ t^2 & 0 \le t \le 30 \\ 15 & 30 < 60 \end{cases}$$

Next, all I had to do was use Maple to first generate the acceleration function, test whether it produces a plot similar to Figure 1.1 then use Maples' in-built math function tools to integrate it to produce the Velocity and Displacement Functions easily as seen below.
Table 1 and 2 clearly show why I chose Maple as it is straightforward, intuitive use of math functions and minimal coding is needed to achieve the objective which is to describe the motion of Stage B by deriving the equations, producing the columns of data corresponding to the Time, Acceleration, Velocity and Displacement as well as plotting the data in graphs against time as seen from Figure 1.2 –1.4.

$$velocity := t \rightarrow \begin{cases} 15.\ t & t \le 0. \\ 0.003333333333\ t^3 & t \le 30. \\ 15.\ t - 360. & 30. < t \end{cases}$$

$$displacement := t \rightarrow \begin{cases} 7.500000000\ t^2 & t \le 0. \\ 0.0008333333332\ t^4 & t \le 30. \\ 7.500000000\ t^2 - 360.\ t + 4725.000000 & 30. < t \end{cases}$$

Let acceleration be f(t) and define it;

$$f := \begin{cases} 0.01\,t^2 & 0 \le t \le 30 \\ 15 & 30 < 60 \end{cases}$$

$$f := \begin{cases} 0.01\,t^2 & 0 \le t \le 30 \\ 15 & otherwise \end{cases} \quad (1)$$

Next integrate f(t) in terms of t using maples in built function tools;

$$f := \begin{cases} 0.01\,t^2 & 0 \le t \le 30 \\ 15 & 30 < 60 \end{cases} \xrightarrow{\text{integrate w.r.t. t}} \begin{cases} 15.\,t & t \le 0. \\ 0.003333333333\,t^3 & t \le 30. \\ 15.\,t - 360. & 30. < t \end{cases}$$

Let the new integrated function be defined as g(t);

$$g := \begin{cases} 15.\,t & t \le 0. \\ 0.003333333333\,t^3 & t \le 30. \\ 15.\,t - 360. & 30. < t \end{cases}$$

$$g := \begin{cases} 15.\,t & t \le 0. \\ 0.003333333333\,t^3 & t \le 30. \\ 15.\,t - 360. & 30. < t \end{cases} \quad (2)$$

From this integrate g(t) in terms of t in the same process as f(t);

$$g := \begin{cases} 15.\,t & t \le 0. \\ 0.003333333333\,t^3 & t \le 30. \\ 15.\,t - 360. & 30. < t \end{cases} \xrightarrow{\text{integrate w.r.t. t}}$$

$$\begin{cases} 7.500000000\,t^2 & t \le 0. \\ 0.0008333333332\,t^4 & t \le 30. \\ 7.500000000\,t^2 - 360.\,t + 4725.000000 & 30. < t \end{cases}$$

Let the new integrated function be defined as h(t);

$$h := \begin{cases} 7.500000000\,t^2 & t \le 0. \\ 0.0008333333332\,t^4 & t \le 30. \\ 7.500000000\,t^2 - 360.\,t + 4725.000000 & 30. < t \end{cases}$$

$$h := \begin{cases} 7.500000000\,t^2 & t \le 0. \\ 0.0008333333332\,t^4 & t \le 30. \\ 7.500000000\,t^2 - 360.\,t + 4725.000000 & 30. < t \end{cases} \quad (3)$$

Finally, have f(t),g(t) and h(t) equivalent to Acceleration, Velocity and Displacement rescpectively

Table 1: Process of deriving Velocity and Displacement Equations from Acceleration

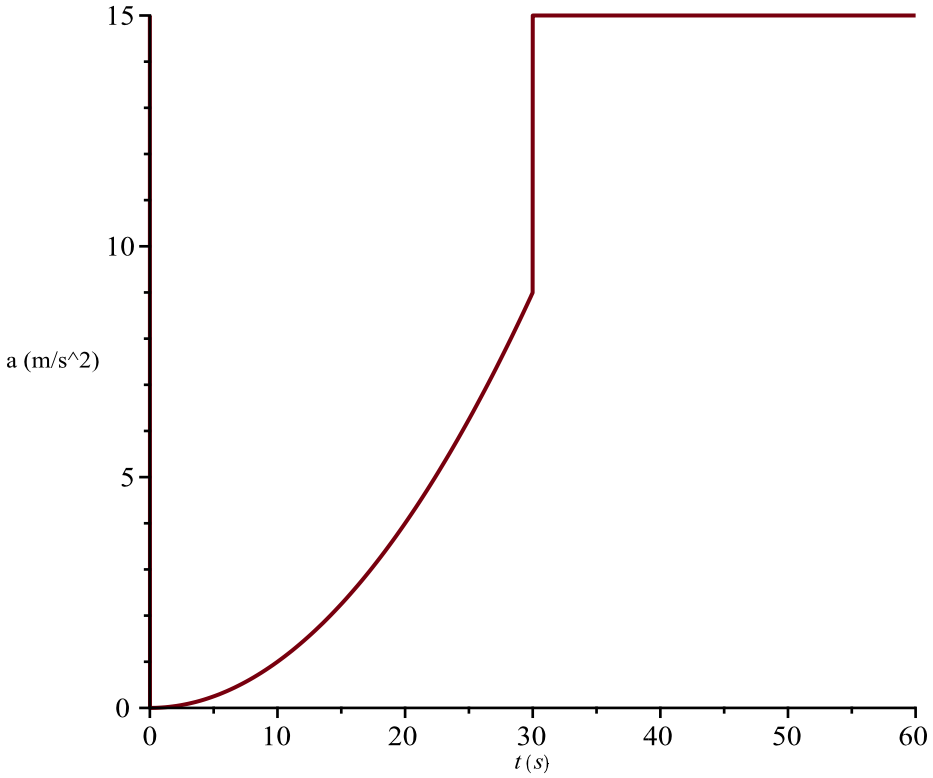

Figure 1.2: Acceleration Time graph

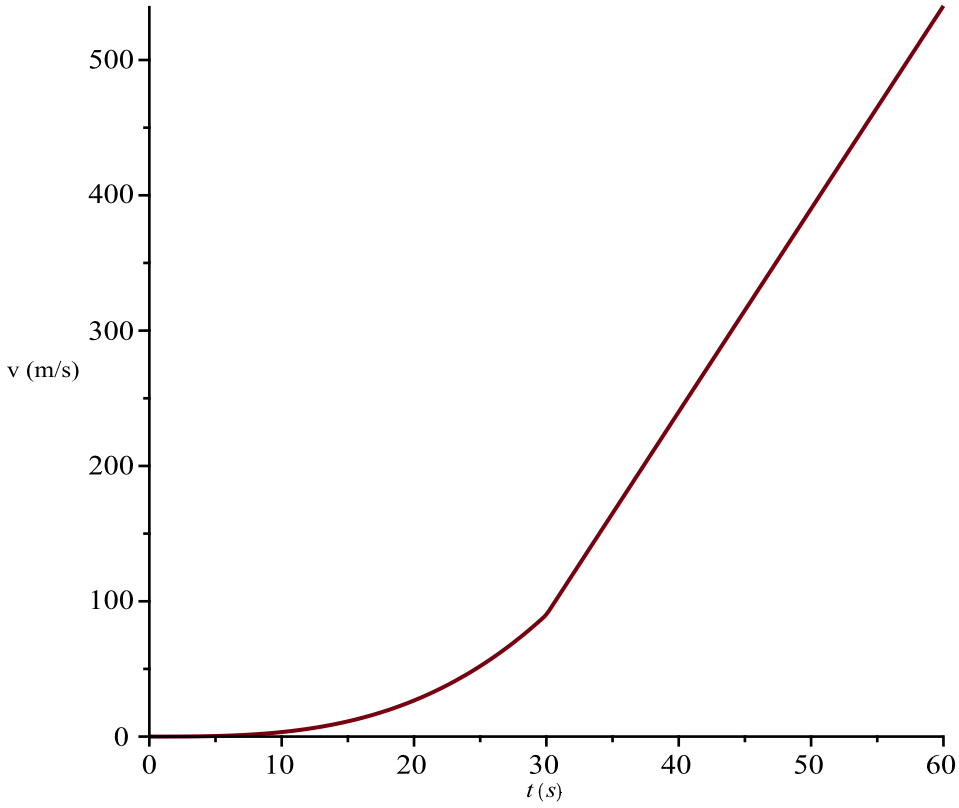

Figure 1.3: Velocity Time graph

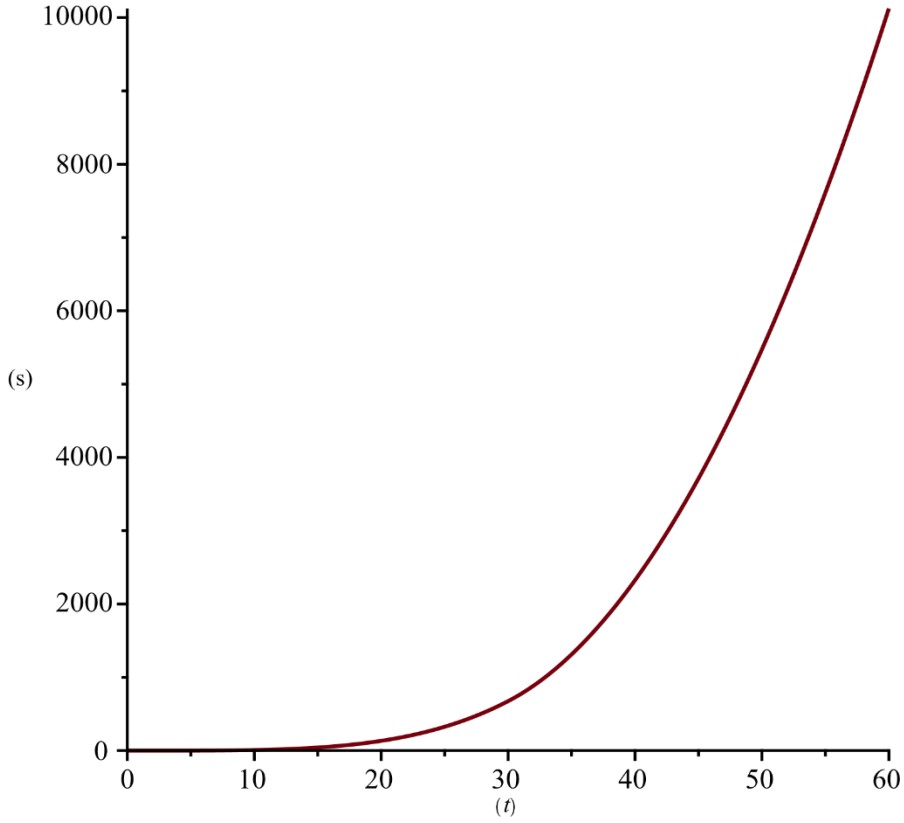

Figure 1.4: Displacement Time graph

To generate the data values of acceleration, velocity and displacement against time I used maples coding as seen in Table 1 which goes through the steps i made from declaring each function to printing them.

$$acceleration := t \rightarrow \begin{cases} 0.01\,t^2 & 0 \le t \le 30 \\ 15 & 30 < 60 \end{cases}$$

$$acceleration := t \mapsto \begin{cases} 0.01 \cdot t^2 & 0 \le t \le 30 \\ 15 & 30 < 60 \end{cases} \quad \textbf{(4)}$$

$$velocity := t \rightarrow \begin{cases} 15.\,t & t \le 0. \\ 0.003333333333\,t^3 & t \le 30. \\ 15.\,t - 360. & 30. < t \end{cases}$$

(5)

$$velocity := t \mapsto \begin{cases} 15. \cdot t & t \le 0. \\ 0.003333333333 \cdot t^3 & t \le 30. \\ 15. \cdot t - 360. & 30. < t \end{cases} \quad \textbf{(5)}$$

$$displacement := t \rightarrow \begin{cases} 7.500000000\,t^2 & t \le 0. \\ 0.0008333333332\,t^4 & t \le 30. \\ 7.500000000\,t^2 - 360.\,t + 4725.000000 & 30. < t \end{cases}$$

$$displacement := t \mapsto \begin{cases} 7.500000000 \cdot t^2 & t \le 0. \\ 0.0008333333332 \cdot t^4 & t \le 30. \\ 7.500000000 \cdot t^2 - 360. \cdot t + 4725.000000 & 30. < t \end{cases} \quad \textbf{(6)}$$

```
print(Time, Displacement, Velocity, Acceleration);
for t from 0 by 0.5 to 60 do
  print(t, displacement(t), velocity(t), acceleration(t));
end do;
```

Table 2: The coding used to generate the acceleration, Velocity and Displacement values against time.

From the data that was printed, I organised it such that it is easily legible and discernible what the values of each are as seen in the data values below.

Table 3: Generated Values for Acceleration, Velocity and Displacement

Time (s)	Displacement(m)	Velocity (m/s)	Acceleration(m/s^2)
0	0	0	0
0.5	0.00005208333332	0.000416667	0.0025
1	0.000833333	0.003333333	0.01
1.5	0.00421875	0.01125	0.0225
2	0.013333333	0.026666667	0.04
2.5	0.032552083	0.052083333	0.0625
3	0.0675	0.09	0.09
3.5	0.125052083	0.142916667	0.1225
4	0.213333333	0.213333333	0.16
5	0.520833333	0.416666667	0.25
5.5	0.762552083	0.554583333	0.3025
6	1.08	0.72	0.36
6.5	1.487552083	0.915416667	0.4225
7	2.000833333	1.143333333	0.49
7.5	2.63671875	1.40625	0.5625
8	3.413333333	1.706666666	0.64
8.5	4.350052083	2.047083333	0.7225
9	5.467499999	2.43	0.81
9.5	6.787552082	2.857916666	0.9025
10	8.333333332	3.333333333	1
10.5	10.12921875	3.85875	1.1025
11	12.20083333	4.436666666	1.21
11.5	14.57505208	5.069583333	1.3225
12	17.28	5.759999999	1.44
12.5	20.34505208	6.510416666	1.5625
13	23.80083333	7.323333333	1.69
13.5	27.67921875	8.201249999	1.8225
14	32.01333333	9.146666666	1.96
14.5	36.83755208	10.16208333	2.1025
15	42.18749999	11.25	2.25
15.5	48.10005208	12.41291667	2.4025
16	54.61333332	13.65333333	2.56
16.5	61.76671874	14.97375	2.7225
17	69.60083332	16.37666667	2.89
17.5	78.15755207	17.86458333	3.0625
18	87.47999999	19.44	3.24
18.5	97.61255207	21.10541666	3.4225
19	108.6008333	22.86333333	3.61
19.5	120.4917187	24.71625	3.8025
20	133.3333333	26.66666666	4
20.5	147.1750521	28.71708333	4.2025
21	162.0675	30.87	4.41

21.5	178.0625521	33.12791666	4.6225
22	195.2133333	35.49333333	4.84
22.5	213.5742187	37.96875	5.0625
23	233.2008333	40.55666666	5.29
23.5	254.150052	43.25958333	5.5225
24	276.48	46.08	5.76
24.5	300.250052	49.02041666	6.0025
25	325.5208333	52.08333333	6.25
25.5	352.3542187	55.27124999	6.5025
26	380.8133333	58.58666666	6.76
26.5	410.962552	62.03208333	7.0225
27	442.8674999	65.60999999	7.29
27.5	476.595052	69.32291666	7.5625
28	512.2133333	73.17333333	7.84
28.5	549.7917187	77.16374999	8.1225
29	589.4008332	81.29666666	8.41
29.5	631.112552	85.57458332	8.7025
30	674.9999999	89.99999999	9
30.5	721.875	97.5	15
31	772.5	105	15
31.5	826.875	112.5	15
32	885	120	15
32.5	946.875	127.5	15
33	1012.5	135	15
33.5	1081.875	142.5	15
34	1155	150	15
34.5	1231.875	157.5	15
35	1312.5	165	15
35.5	1396.875	172.5	15
36	1485	180	15
36.5	1576.875	187.5	15
37	1672.5	195	15
37.5	1771.875	202.5	15
38	1875	210	15
38.5	1981.875	217.5	15
39	2092.5	225	15
39.5	2206.875	232.5	15
40	2325	240	15
40.5	2446.875	247.5	15
41	2572.5	255	15
41.5	2701.875	262.5	15
42	2835	270	15
42.5	2971.875	277.5	15
43	3112.5	285	15

43.5	3256.875	292.5	15
44	3405	300	15
44.5	3556.875	307.5	15
45	3712.5	315	15
45.5	3871.875	322.5	15
46	4035	330	15
46.5	4201.875	337.5	15
47	4372.5	345	15
47.5	4546.875	352.5	15
48	4725	360	15
48.5	4906.875	367.5	15
49	5092.5	375	15
49.5	5281.875	382.5	15
50	5475	390	15
50.5	5671.875	397.5	15
51	5872.5	405	15
51.5	6076.875	412.5	15
52	6285	420	15
52.5	6496.875	427.5	15
53	6712.5	435	15
53.5	6931.875	442.5	15
54	7155	450	15
54.5	7381.875	457.5	15
55	7612.5	465	15
55.5	7846.875	472.5	15
56	8085	480	15
56.5	8326.875	487.5	15
57	8572.5	495	15
57.5	8821.875	502.5	15
58	9075	510	15
58.5	9331.875	517.5	15
59	9592.5	525	15
59.5	9856.875	532.5	15
60	10125	540	15