Time domain analysis

Standard test signals-

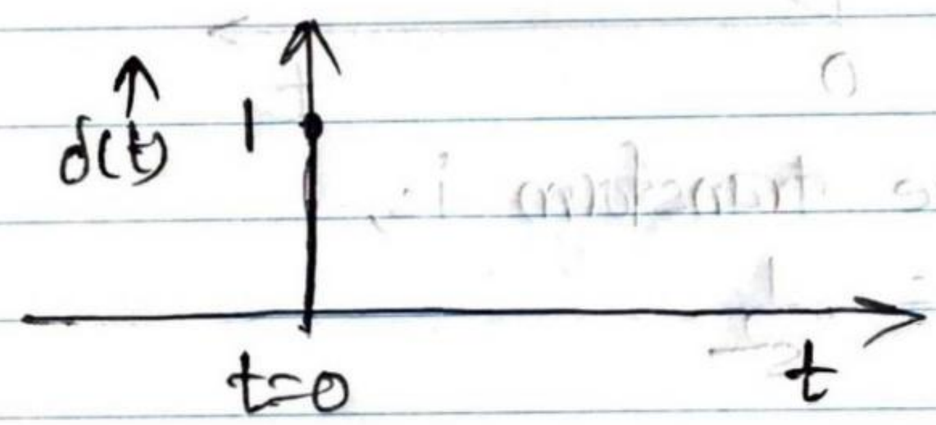
The following signals are used as test signals for time domain analysis of linear time invariant systems.

1. Unit impulse signal-

The unit impulse signal is represented as,

$$\delta(t) = 1, \quad t = 0$$
$$= 0, \quad t \neq 0$$

It can be sketched as,



Its Laplace transform is,

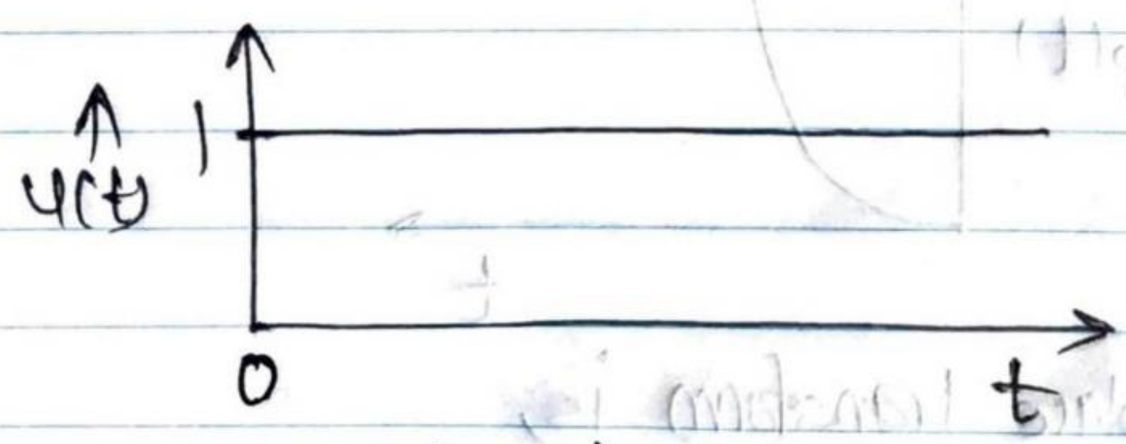
$$\delta(s) = 1$$

2. Unit step signal-

The unit step signal is represented as,

$$u(t) = 1, \quad t \geq 0$$
$$= 0, \quad t < 0$$

It can be sketched as,



Its Laplace transform is,

$$U(s) = \frac{1}{s}$$

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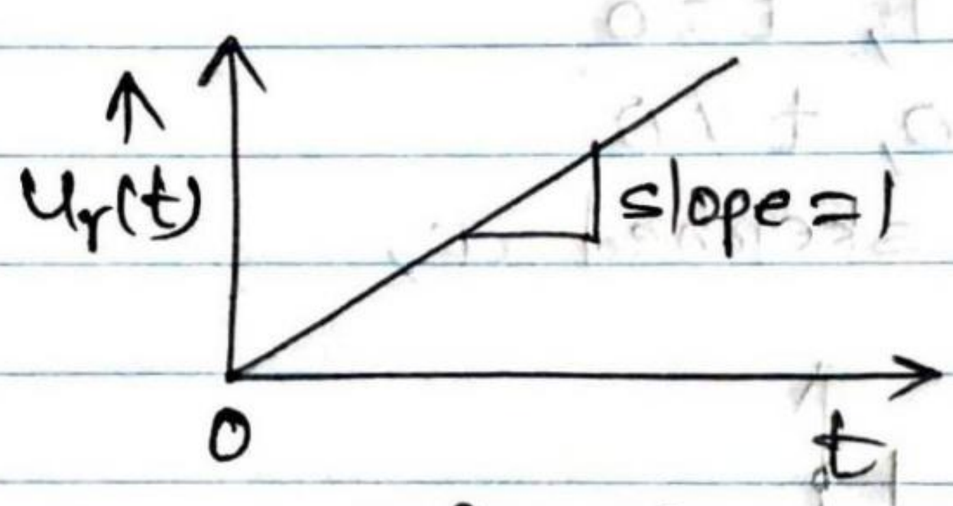
3. <u>Unit ramp signal</u>-

Unit ramp signal is represented as,

$$u_r(t) = t, \quad t \geq 0$$
$$= 0, \quad t < 0$$

It can be sketched as,



Its Laplace transform is,

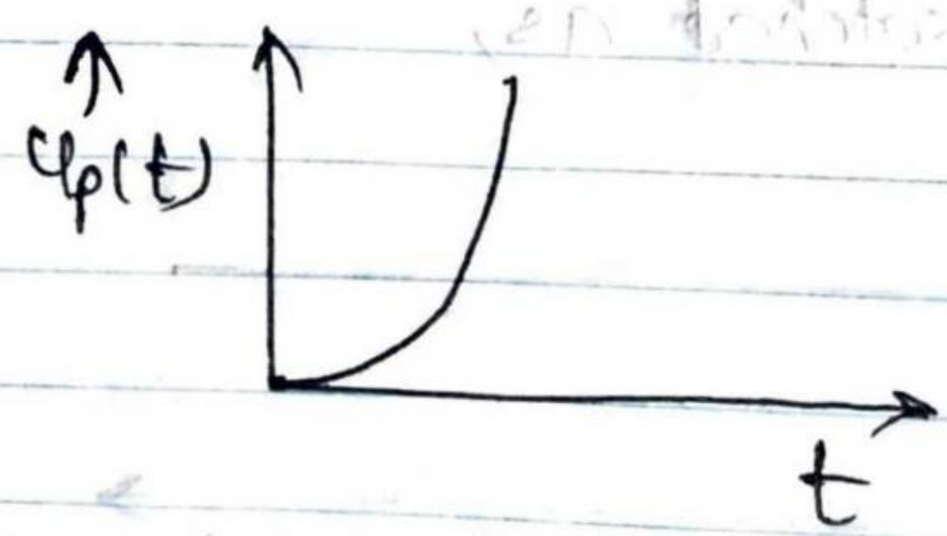
$$U_r(s) = \frac{1}{s^2}$$

4. <u>Unit parabolic signal</u>-

Unit parabolic signal is represented as,

$$u_p(t) = \frac{t^2}{2}, \quad t \geq 0$$
$$= 0, \quad t < 0$$

It can be sketched as,



Its Laplace transform is,

$$U_p(s) = \frac{1}{s^3}$$

The standard test signals are used to simulate time domain analysis of system under study.

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Order of a system– Order of a system is the highest power of 's' in the denominator of transfer function of the system.

It is equal to number of poles of the system.

~~Type of~~ Type of a system– Type of a system is number of poles of the system at origin.

For example– For a system with transfer function

$$G(s) = \frac{K(s+z_1)(s+z_2)\cdots(s+z_m)}{s^j(s+p_1)(s+p_2)\cdots(s+p_{n-j})}, \quad m \le n$$

Type = No. of poles at origin = j

order = No. of poles = n

Examples on type and order–

1. Determine type and order of system with transfer function $G(s) = \dfrac{s^3+4s^2+7s+3}{s^4+4s^3+2s^2+5s+4}$

Sol^n order = No. of poles = 4

type = No. of poles at origin = 0

2. Determine type and order of system with transfer function $G(s) = \dfrac{s^2+5s+7}{s^3+4s^2+5s}$

Sol^n $G(s) = \dfrac{s^2+5s+7}{s(s^2+4s+5)}$

$\therefore$ order = No. of poles = 3

Type = No. of poles at origin = 1

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3. Determine order and type of system with transfer function $G(s) = \dfrac{s(s+2)(s+5)}{s^2(s+4)(s+7)+10}$

Sol^n ~~order = Highest power of s~~

$$G(s) = \frac{s(s+2)(s+5)}{s^2(s^2+11s+28)+10}$$

$$= \frac{s(s+2)(s+5)}{s^4+11s^3+28s^2+10}$$

$\therefore$ Order = Highest power of s in denominator = 4

type = No. of poles at origin = 0

4. Determine order and type of system with transfer function $G(s) = \dfrac{(s+1)(s^2+2s+2)}{s(s+2)(s+5)}$

Sol^n Order = No. of poles = 3

Type = poles at origin = 1

5. Determine order and type of system with transfer function $G(s) = \dfrac{(s+1)(s^2+2s+2)}{s(s+2)(s+5)+4}$

Sol^n Order = Highest power of s in denominator = 3

Type = poles at origin = 0

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Impulse response of first order system-

The first order system can be represented by transfer function $G(s) = \frac{1}{1+s\tau}$ (unity gain system)

$$\therefore \frac{Y(s)}{R(s)} = \frac{1}{1+s\tau}$$

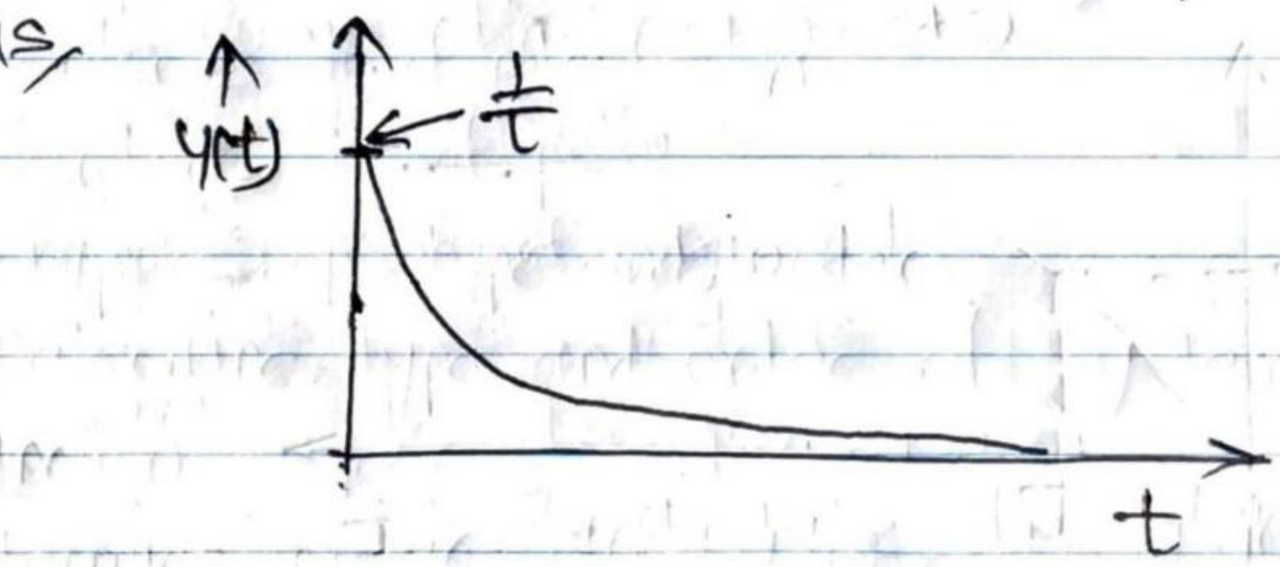
For impulse response, $R(s) = 1$

$$\therefore Y(s) = \frac{1}{1+s\tau}$$

$$= \frac{1/\tau}{s+1/\tau}$$

$$\therefore y(t) = \frac{1}{\tau} e^{-t/\tau}$$ ---impulse response

The impulse response of first order system can be sketched as,



Step response of first order system-

We have the transfer function of first order system as,

$$G(s) = \frac{1}{1+s\tau}$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{1}{1+s\tau}$$

For step response, $R(s) = \frac{1}{s}$

$$\therefore Y(s) = G(s)\,R(s) = \frac{1}{s} \cdot \frac{1}{1+s\tau}$$

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$$\therefore Y(s) = \frac{1/\tau}{s(s+1/\tau)}$$

$$= \frac{A}{s} + \frac{B}{s+1/\tau}$$

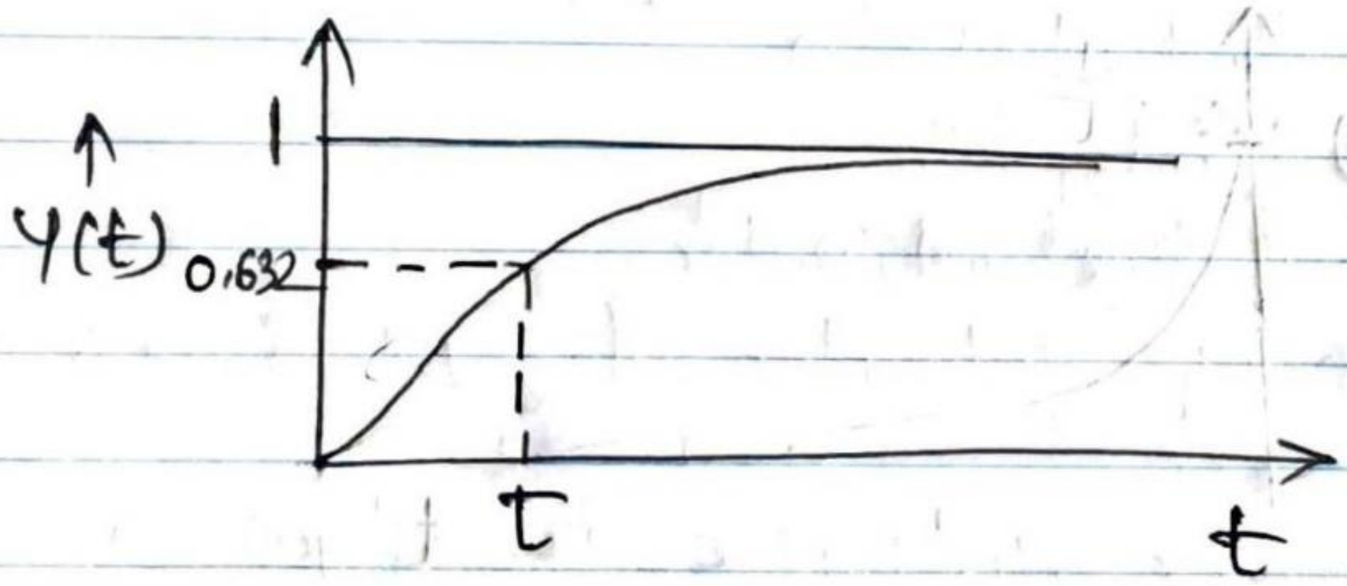
$$A = \left.\frac{1/\tau}{s+1/\tau}\right|_{s=0} = 1$$

$$B = \left.\frac{1/\tau}{s}\right|_{s=-1/\tau} = -1$$

$$\therefore Y(s) = \frac{1}{s} - \frac{1}{s+1/\tau}$$

$$\therefore y(t) = 1 - e^{-t/\tau}$$ → step response

It can be sketched as



At $t = \tau$, $y(t) = 0.632$ where τ is time constant.

<u>Time constant</u> – It is the time taken by the system to reach 63.2% of final steady state output.

Similarly,

At $t = 2\tau$, $y(t) = 0.86$

At $t = 3\tau$, $y(t) = 0.95$

At $t = 4\tau$, $y(t) = 0.98$

At $t = 5\tau$, $y(t) = 0.99$

At $t = \infty$, $y(t) = 1$

The $y(t)$ reaches steady state at $t = \infty$. However, practically steady state considered at $t = 4\tau$ where o/p enters in band of 2%.

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Second order system-

A typical open loop transfer function of second order system is,

$$G_{OL}(s) = \frac{\omega_n^2}{s(s+2\xi\omega_n)}$$

Thus its closed loop transfer function with unity feedback is,

$$G_{CL}(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

where, ω_n is the natural frequency
ξ is the damping factor

Depending upon value of damping factor ξ, the systems are classified as,

$\xi = 0 \longrightarrow$ undamped system
$0<\xi<1 \longrightarrow$ under damped system
$\xi = 1 \longrightarrow$ critically damped system
$\xi > 1 \longrightarrow$ overdamped System

Depending upon ξ the pole locations are,

$\xi = 0 \longrightarrow$ Purely imaginary
$0<\xi<1 \longrightarrow$ Complex conjugate in left half of s-plane
$\xi = 1 \longrightarrow$ Real and equal
$\xi > 1 \longrightarrow$ Real and unequal

Depending upon ξ the system has different nature of output as,

$\xi = 0 \longrightarrow$ Oscillations with constant amplitude
$0<\xi<1 \longrightarrow$ Oscillations with decreasing amplitude
$\xi = 1 \longrightarrow$ Non oscillatory
$\xi > 1 \longrightarrow$ No oscillatory

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Impulse response of second order system –

The standard transfer function of second order system is,

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \longrightarrow \text{unity gain system}$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

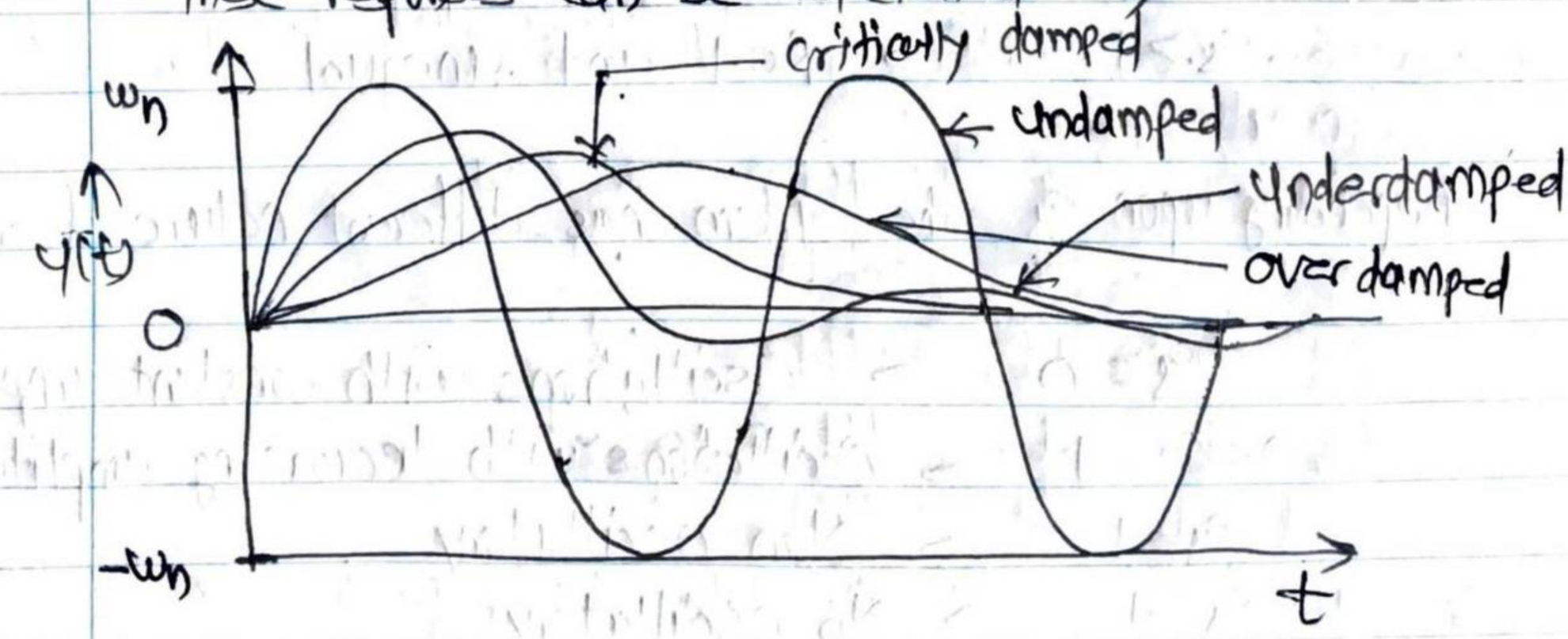
For impulse response, $R(s)=1$

$$\therefore Y(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

For different ranges of ξ, impulse response is,

Damping factor	Impulse response
$\xi = 0$	$y(t) = \omega_n \sin \omega_n t$
$0<\xi<1$	$y(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin(\omega_n \sqrt{1-\xi^2}\, t)$
$\xi = 1$	$y(t) = \omega_n^2 t e^{-\omega_n t}$
$\xi > 1$	$y(t) = \frac{\omega_n}{2\sqrt{\xi^2-1}} \left[e^{-(\xi\omega_n - \omega_n\sqrt{\xi^2-1})t} - e^{-(\xi\omega_n + \omega_n\sqrt{\xi^2-1})t} \right]$

These responses can be sketched as,



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Unit step response of second order system -

The transfer function of unity gain second order system is,

$$G(s) = \frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

For step response, $R(s) = \frac{1}{s}$

$$\therefore Y(s) = \frac{\omega_n^2}{s(s^2 + 2\xi\omega_n s + \omega_n^2)}$$

For different values of ξ, the step response is,

$\xi = 0$ $\quad y(t) = 1 - \cos\omega_n t$

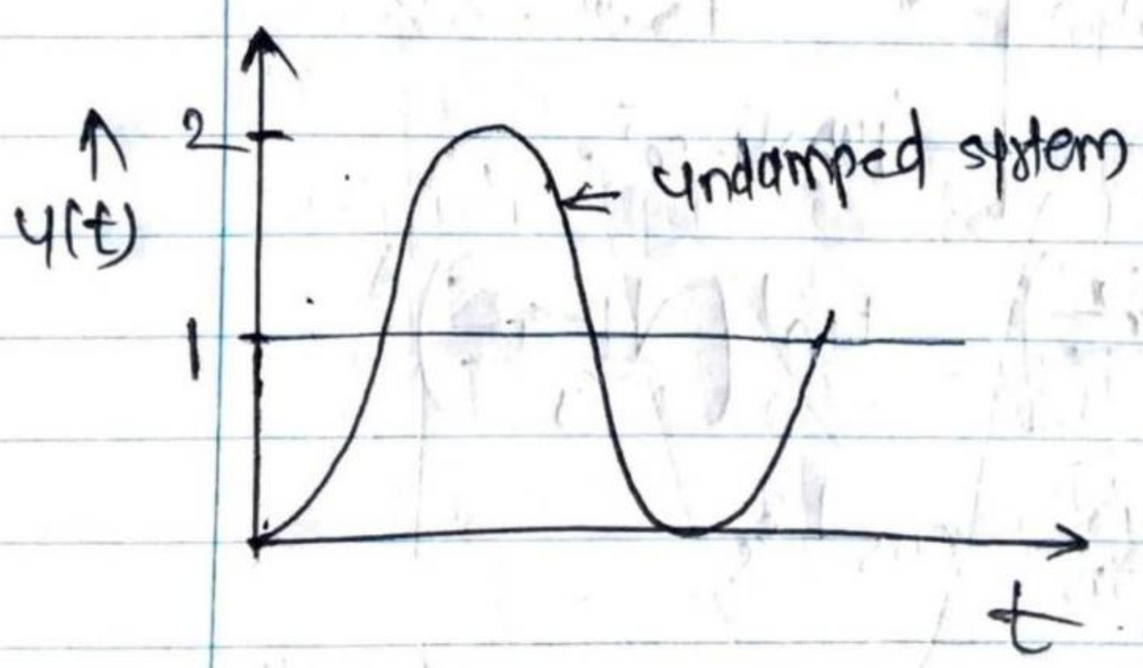
$0 < \xi < 1$ $\quad y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$

$\xi = 1$ $\quad y(t) = 1 - e^{-\omega_n t}[1 + \omega_n t]$

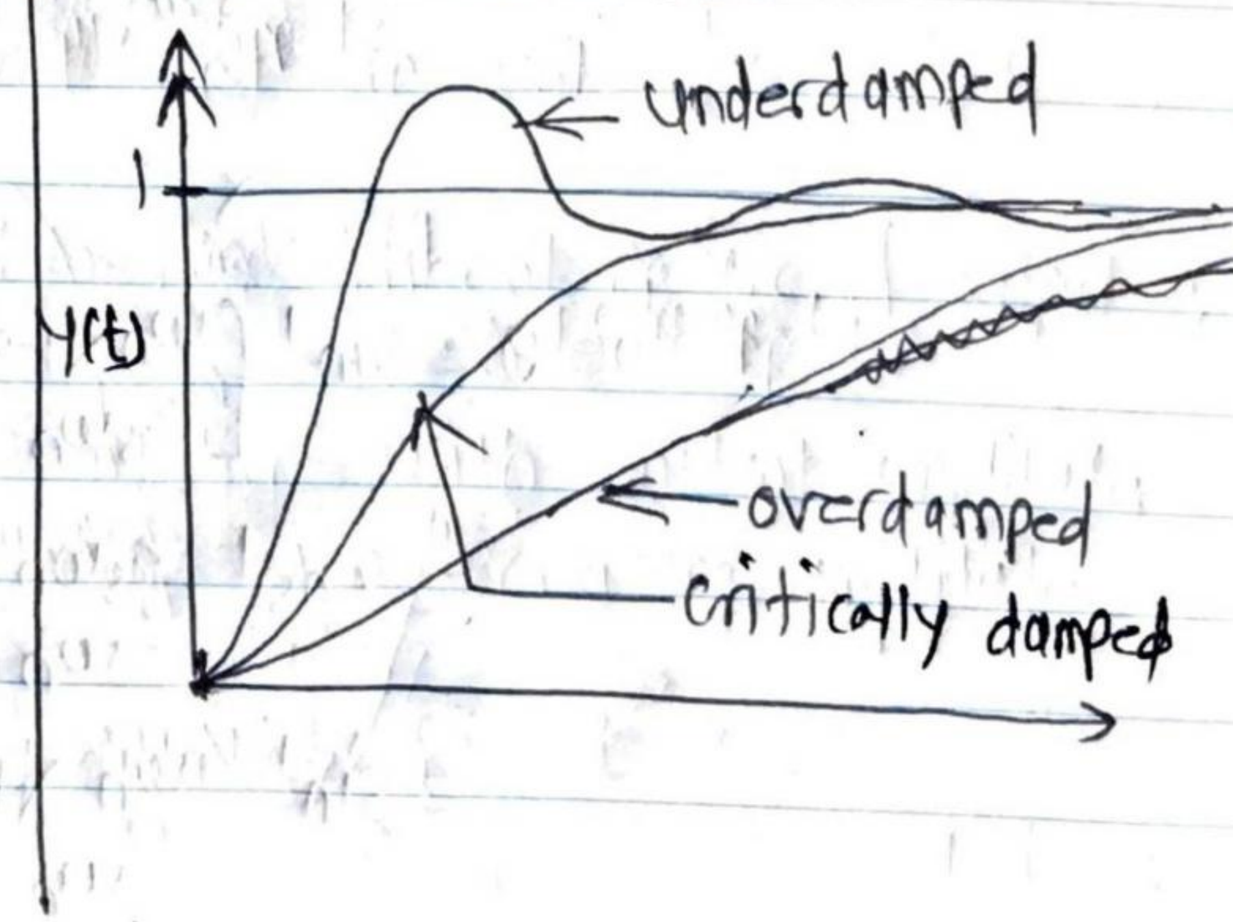
$\xi > 1$ $\quad y(t) = 1 + \frac{1}{2\sqrt{\xi^2-1}\,(\xi + \sqrt{\xi^2-1})} e^{-(\xi+\sqrt{\xi^2-1})\omega_n t} - \frac{1}{2\sqrt{\xi^2-1}\,(\xi - \sqrt{\xi^2-1})} e^{-(\xi-\sqrt{\xi^2-1})\omega_n t}$

These responses can be sketched as,

For $\xi = 0$



Other three cases



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The step response of underdamped system and its time domain specifications have huge importance in control system analysis and design. The step response of underdamped system is,

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

where ω_d is damped frequency and is given by,

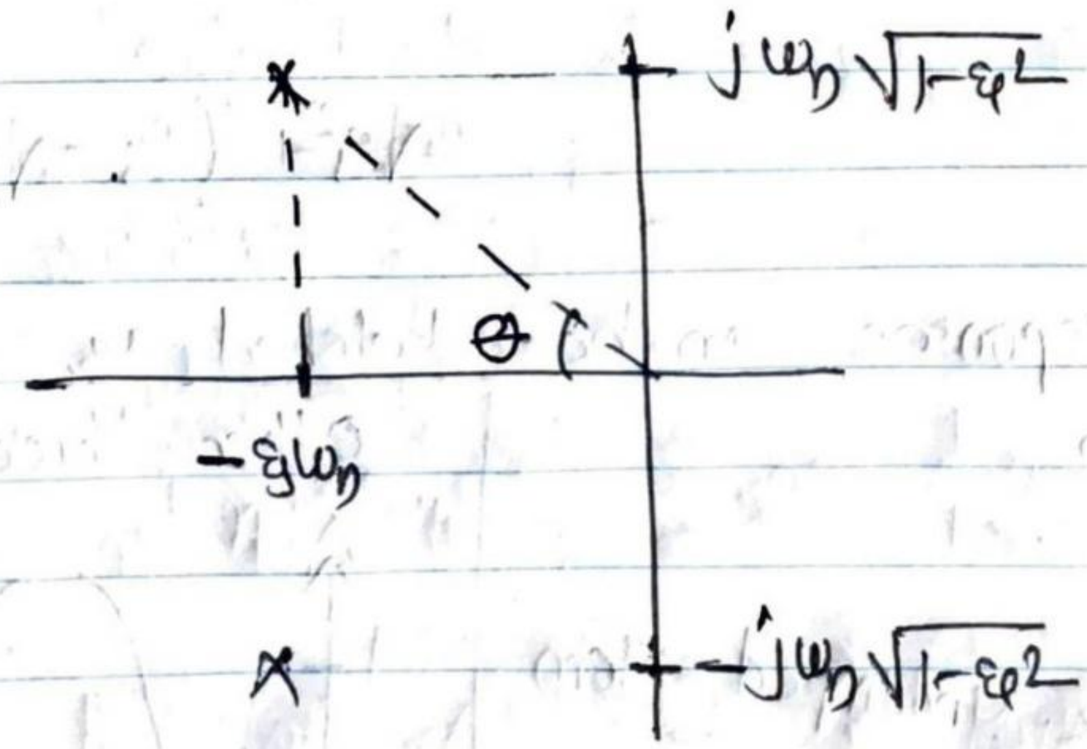
$$\omega_d = \omega_n \sqrt{1-\xi^2}$$

and θ is given as $\theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = \cos^{-1}\xi$

$$= \sin^{-1}\sqrt{1-\xi^2}$$

The poles of underdamped system are located at

$$s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2} = -\xi\omega_n \pm j\omega_d$$



$$\theta = \tan^{-1}\left[\frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}\right] = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

$$= \cos^{-1}\frac{\xi\omega_n}{\omega_n} = \cos^{-1}\xi$$

$$= \sin^{-1}\frac{\omega_n\sqrt{1-\xi^2}}{\omega_n} = \sin^{-1}\sqrt{1-\xi^2}$$

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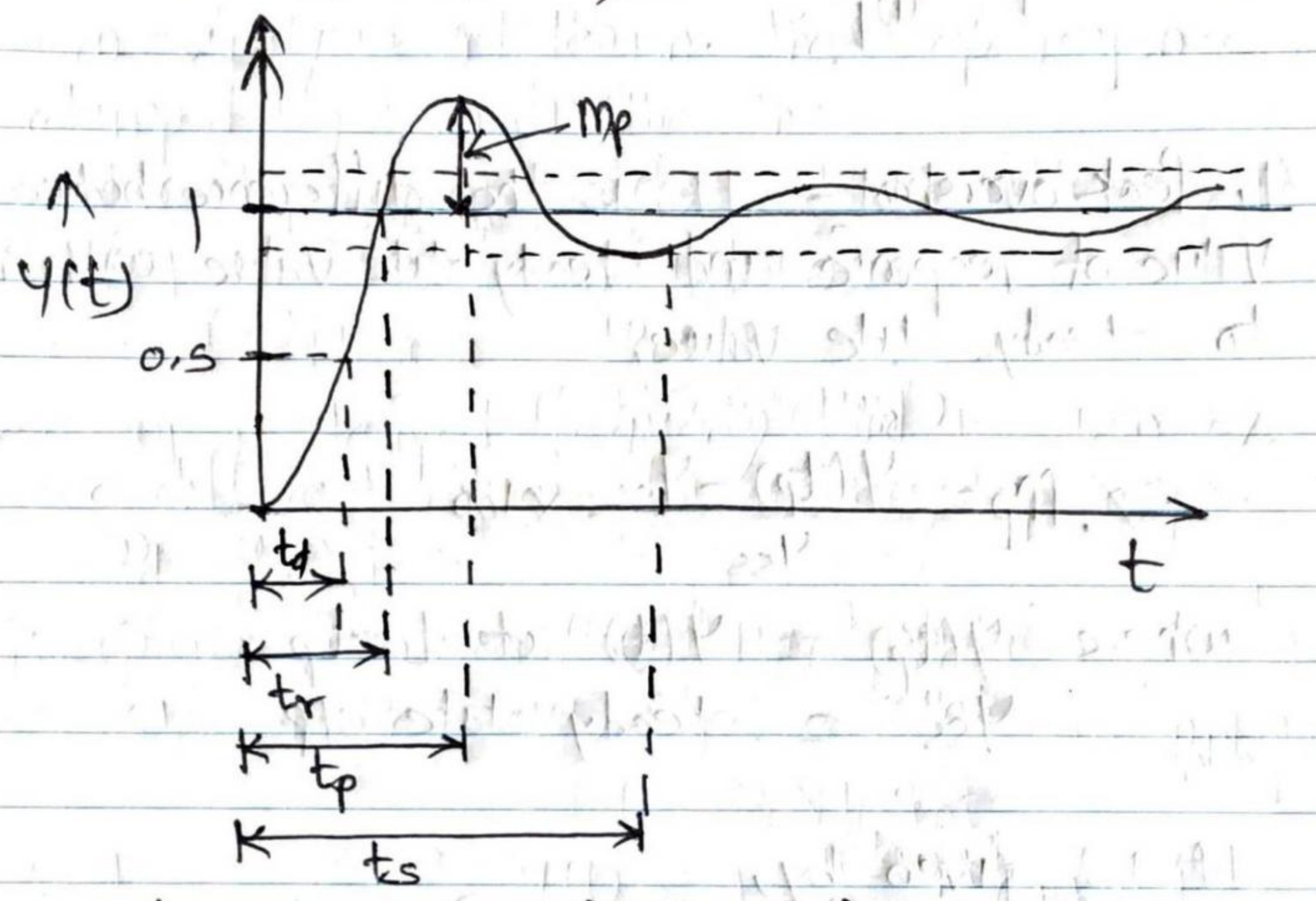
The step response of second order under damped system can be sketched as,



The time domain specifications of second order underdamped system are

1. <u>Delay time (t_d)</u> – It is the time taken by the system to reach 50% of the final steady state output. It is expressed as,

$$t_d = \frac{1+0.7\xi}{\omega_n}$$

2. <u>Rise time (t_r)</u> – For underdamped system, it is the time taken by the system to reach from 0% to 100% of steady state output in first attempt. It is expressed as,

$$t_r = \frac{\pi-\theta}{\omega_d}$$

3. <u>Peak time (t_p)</u> – It is the time taken by the system to reach highest value of response. It is expressed

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as,

$$t_p = \frac{\pi}{\omega_d}$$

4. <u>Peak overshoot</u> – It is the difference between highest value of response and steady state value with reference to steady state value.

$$\%\, M_p = \frac{y(t_p) - y_{ss}}{y_{ss}} \times 100$$

where $y(t_p) = y(t)$ at $t = t_p$
y_{ss} = steady state o/p

It is given by,

$$\%\, M_p = \left[e^{-\pi \xi / \sqrt{1-\xi^2}} \right] \times 100$$

5. <u>Settling time</u> – It is the time taken by the system response to enter in the predefined tolerance band around steady state and stay within the band there after.

It is expressed as,

For 5% settling band

$$t_s = \frac{3}{\xi \omega_n}$$

For 2% settling band

$$t_s = \frac{4}{\xi \omega_n}$$

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Derivation for rise time-

For second order underdamped system,

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

At $t = t_r$, $y(t_r) = 1$

$$\therefore y(t_r) = 1 = 1 - \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta)$$

$$\therefore \frac{e^{-\xi\omega_n t_r}}{\sqrt{1-\xi^2}} \sin(\omega_d t_r + \theta) = 0$$

$$\therefore \sin(\omega_d t_r + \theta) = 0$$

$$\therefore \omega_d t_r + \theta = n\pi$$

For rise time, $n = 1$

$$\therefore \omega_d t_r + \theta = \pi$$

$$\therefore \omega_d t_r = \pi - \theta$$

$$\therefore t_r = \frac{\pi - \theta}{\omega_d}$$

Derivation for peak time-

For second order underdamped system,

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

At $t = t_p$, $y(t_p)$ is maximum. Hence t_p can be obtained by equating $\frac{dy(t)}{dt} = 0$

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$$\frac{dy(t)}{dt} = \frac{\xi\omega_n e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta) - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \cdot \omega_d \cos(\omega_d t + \theta)$$

At $t = t_p$, $\frac{dy(t)}{dt} = 0$

$$\therefore \frac{\xi\omega_n e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta) - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \omega_d \cos(\omega_d t_p + \theta) = 0$$

$$\therefore \xi\omega_n \sin(\omega_d t_p + \theta) - \omega_d \cos(\omega_d t_p + \theta) = 0$$

$$\therefore \xi\omega_n \sin(\omega_d t_p + \theta) = \omega_d \cos(\omega_d t_p + \theta)$$

$$\therefore \tan(\omega_d t_p + \theta) = \frac{\omega_n \sqrt{1-\xi^2}}{\xi\omega_n}$$

$$\therefore \tan(\omega_d t_p + \theta) = \frac{\sqrt{1-\xi^2}}{\xi}$$

$$\therefore \omega_d t_p + \theta = \tan^{-1} \frac{\sqrt{1-\xi^2}}{\xi} = \theta$$

$$\therefore \omega_d t_p = n\pi$$

For t_p, $n = 1$

$$\therefore \omega_d t_p = \pi$$

$$\therefore t_p = \frac{\pi}{\omega_d}$$

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Derivation for peak overshoot-

For second order underdamped system we have,

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

At $t = t_p$

$$y(t_p) = 1 - \frac{e^{-\xi\omega_n t_p}}{\sqrt{1-\xi^2}} \sin(\omega_d t_p + \theta)$$

$$= 1 - \frac{e^{-\xi\omega_n \pi/\omega_d}}{\sqrt{1-\xi^2}} \sin\left(\omega_d \frac{\pi}{\omega_d} + \theta\right)$$

$$= 1 - \left[\frac{e^{-\pi\xi\omega_n/\omega_n(\sqrt{1-\xi^2})}}{\sqrt{1-\xi^2}} \sin(\pi + \theta)\right]$$

we have $\sin(\pi + \theta) = -\sin\theta$

$$\therefore y(t_p) = 1 - \left[-\frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} \sin\theta\right]$$

we have, $\sin\theta = \sqrt{1-\xi^2}$

$$\therefore y(t_p) = 1 + \frac{e^{-\pi\xi/\sqrt{1-\xi^2}}}{\sqrt{1-\xi^2}} (\sqrt{1-\xi^2})$$

$$= 1 + e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$M_p = \frac{y(t_p) - y_{ss}}{y_{ss}}$$

$$= \frac{1 + e^{-\pi\xi/\sqrt{1-\xi^2}} - 1}{1} = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$\therefore \% M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} \times 100$$

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Derivation for t_s –

For 5% settling

$t_s = 3\tau$ ... First order system approximation

and for 2% settling

$t_s = 4\tau$ ... First order system approximation

For second order system the time constant is approximated as, $\tau = \frac{1}{\xi\omega_n}$

∴ For 5% ~~settl~~ settling,

$$t_s = 3\tau = \frac{3}{\xi\omega_n}$$

For 2% settling

$$t_s = 4\tau = \frac{4}{\xi\omega_n}$$

Another approach –

~~Thus~~ For second order under damped system we have,

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \theta)$$

At settling time t_s, it is approximated that the oscillations in the system output are almost diminished

∴ $\omega_d = 0$

∴ At $t = t_s$

$$y(t_s) = 1 - \frac{e^{-\xi\omega_n t_s}}{\sqrt{1-\xi^2}} \sin\theta$$

we have, $\sin\theta = \sqrt{1-\xi^2}$

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$\therefore y(t_s) = 1 - e^{-\xi\omega_n t_s}$

For 5% settling,

$y(t_s) = 0.95$

$\therefore 0.95 = 1 - e^{-\xi\omega_n t_s}$

$\therefore e^{-\xi\omega_n t_s} = 0.05$

$\therefore -\xi\omega_n t_s = \ln(0.05) = -2.9957$

$\therefore t_s = \frac{2.9957}{\xi\omega_n} \approx \frac{3}{\xi\omega_n}$

similarly for 2% settling

$y(t_s) = 0.98$

$\therefore 0.98 = 1 - e^{-\xi\omega_n t_s}$

$\therefore e^{-\xi\omega_n t_s} = 0.02$

$\therefore -\xi\omega_n t_s = \ln(0.02) = -3.9120$

$\therefore t_s = \frac{3.9120}{\xi\omega_n} \approx \frac{4}{\xi\omega_n}$

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Examples on time domain specifications of second order system-

1. Consider a second order system with closed loop transfer function $G(s) = \frac{25}{s^2+8s+25}$. Determine all time domain specifications.

Soln we have,

$$G(s) = \frac{25}{s^2+8s+25}$$

comparing with standard transfer function,

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$ we get

$$\omega_n^2 = 25$$

$\therefore \omega_n = 5$ rad/sec

$2\xi\omega_n = 8$

$\therefore 10\xi = 8$

$\therefore \xi = 0.8$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$= 5\sqrt{1-(0.8)^2} = 3 \text{ rad/sec}$$

Delay time $t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.8)}{5} = 0.312$ sec

Rise time $t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}\xi}{\omega_d} = \frac{\pi-0.6435}{3} = 0.8327$ sec

Peak time $t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3} = 1.0472$ sec

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{4} = 1 \text{ sec}$$

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$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.8\pi/\sqrt{1-(0.8)^2}}$$

$$= 0.0152 = 1.52\%$$

2. Consider unity feedback system with open loop transfer function $G(s) = \frac{16}{s(s+4)}$. Determine $\omega_n, \xi, \omega_d, t_d, t_r, t_p, M_p$ and t_s.

<u>soln</u> The unity feedback system has O.L.T.F.

$$G(s) = \frac{16}{s(s+4)}$$

$$\therefore G_{CL}(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\frac{16}{s(s+4)}}{1+\frac{16}{s(s+4)}} = \frac{16}{s^2+4s+16}$$

$\therefore \omega_n^2 = 16$

$\therefore \omega_n = 4$ rad/sec

$2\xi\omega_n = 4$

$\therefore 8\xi = 4$

$\therefore \xi = 0.5$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \cancel{4\sqrt{1-(0.5)^2}}\ 4\sqrt{1-(0.5)^2} = 3.4641 \text{ rad/sec}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.5)}{4} = 0.3375 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}0.5}{3.4641} = 0.6046 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.4641} = 0.9069 \text{ sec}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{2} = 2 \text{ sec}$$

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$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.5\pi/\sqrt{1-(0.5)^2}}$$

$$= 0.1630 = 16.30\%$$

3. Consider a unity feedback system with open loop transfer function $G(s) = \frac{12}{(s+1)(s+4)}$. Determine all time domain specifications.

<u>soln</u> We have O.L.T.F.

$$G(s) = \frac{12}{(s+1)(s+4)}$$

$\therefore$ C.L.T.F. is,

$$G_{CL}(s) = \frac{\frac{12}{(s+1)(s+4)}}{1+\frac{12}{(s+1)(s+4)}} = \frac{12}{s^2+5s+16}$$

$\therefore$ The system is not in standard form.

$\therefore$ Compare denominator of $G_{CL}(s)$ with $s^2+2\xi\omega_n s+\omega_n^2$

$\therefore$ $\omega_n^2 = 16$

$\therefore$ $\omega_n = 4$ rad/sec

$2\xi\omega_n = 5$

$\therefore 8\xi = 5$

$\therefore \xi = 0.625$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 4\sqrt{1-(0.625)^2} = 3.1225 \text{ rad/sec}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.625)}{4} = 0.3594 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}0.625}{3.1225} = 0.7193 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.1225} = 1.0061 \text{ sec}$$

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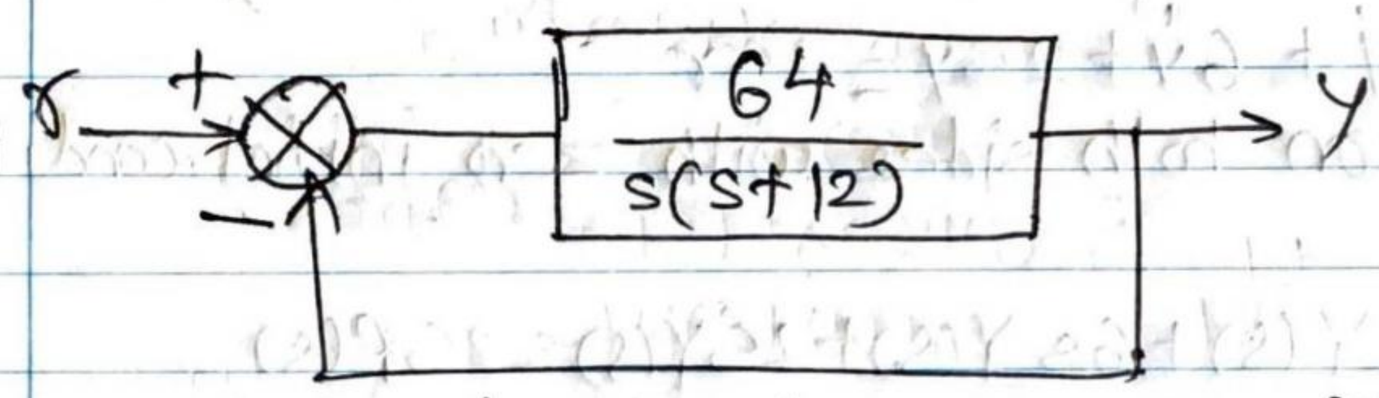
$$t_s = \frac{4}{\xi \omega_n} = \frac{4}{2.5} = 1.6 \text{ sec}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.625\pi/\sqrt{1-(0.625)^2}}$$

$$= 0.0808 = 8.08\%$$

4. For the system shown in figure below determine closed loop transfer function and all time domain specifications



<u>sol^n</u> For the given system, C.L.T.F. is.

$$G_{CL}(s) = \frac{\frac{64}{s(s+12)}}{1+\frac{64}{s(s+12)}} = \frac{64}{s^2+12s+64}$$

$\therefore \omega_n^2 = 64$

$\therefore \omega_n = 8$ rad/sec

$2\xi\omega_n = 12$

$\therefore \xi = \frac{12}{16} = 0.75$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 8\sqrt{1-(0.75)^2} = 5.2915 \text{ rad/sec}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.75)}{8} = 0.1906 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}0.75}{5.2915} = 0.4571 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{5.2915} = 0.5937 \text{ sec}$$

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$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{6} = 0.6667 \text{ sec}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.75\pi/\sqrt{1-(0.75)^2}}$$

$$= 0.0284 = 2.84\%$$

5. A closed loop system is represented by differential equation $\ddot{y} + 6\dot{y} + 25y = 25r$. Determine all time domain specifications of this system.

<u>soln</u> The closed loop differential equation is,

$$\ddot{y} + 6\dot{y} + 25y = 25r$$

Taking L.T. on both sides with zero initial condition we get,

$$s^2 Y(s) + 6s\,Y(s) + 25Y(s) = 25R(s)$$

$$\therefore [s^2 + 6s + 25]\,Y(s) = 25R(s)$$

$$\therefore \frac{Y(s)}{R(s)} = \frac{25}{s^2+6s+25} = G(s)$$

$$\therefore \omega_n^2 = 25$$

$$\therefore \omega_n = 5 \text{ rad/sec}$$

$$2\xi\omega_n = 6$$

$$\therefore \xi = 6/10 = 0.6$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 5\sqrt{1-(0.6)^2} = 4 \text{ rad/sec}$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+(0.7)(0.6)}{5} = 0.284 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi - \cos^{-1}\xi}{4} = 0.5536 \text{ sec}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{4} = 0.7854 \text{ sec}$$

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$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{3} = 1.3333 \text{ sec}$$

$$m_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.6\pi/\sqrt{1-(0.6)^2}}$$

$$= 0.0948 = 9.48\%$$

6. Consider a system with closed loop transfer function $G(s) = \frac{k_2}{s^2 + k_1 s + k_2}$. Determine k_1 and k_2 such that peak overshoot is 15% and peak time is 2 sec. Also determine t_d, t_r and t_s.

<u>soln</u> We have

$$G(s) = \frac{k_2}{s^2 + k_1 s + k_2}$$

Also, $m_p = 15\%$.

$$\therefore e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.15$$

$$\therefore \frac{-\pi\xi}{\sqrt{1-\xi^2}} = -1.8971$$

$$\therefore \frac{\xi}{\sqrt{1-\xi^2}} = 0.6039$$

$$\therefore \frac{\xi^2}{1-\xi^2} = 0.3647$$

$$\therefore \xi^2 = 0.3647 - 0.3647\xi^2$$

$$\therefore \xi^2 = \frac{0.3647}{1.3647} = 0.2672$$

$$\therefore \xi = 0.5170$$

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$$t_p = \frac{\pi}{\omega_d}$$

$$\therefore \omega_d = \frac{\pi}{t_p} = \frac{\pi}{2} = 1.5708 \text{ rad/sec}$$

$$\therefore \omega_n \sqrt{1-\xi^2} = 1.5708$$

$$\therefore \omega_n = \frac{1.5708}{\sqrt{1-(0.5170)^2}} = 1.8351 \text{ rad/sec}$$

$$\therefore K_2 = \omega_n^2 = 3.3675$$

$$K_1 = 2\xi\omega_n = 2(0.5170)(1.8351) = 1.8975$$

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.5170)}{1.8351} = 0.7421 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}0.5170}{1.5708} = 1.3459 \text{ sec}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{1.0478} = 3.8174 \text{ sec}$$

7. Unity feedback system has open loop transfer function $G(s) = \frac{K}{s(1+sT)}$. Determine K and T such that peak overshoot is 10% and 2% settling time is 2 sec.

<u>soln</u> The O.L.T.F. is,

$$G(s) = \frac{K}{s(1+sT)}$$

$$\therefore G_{CL}(s) = \frac{\frac{K}{s(1+sT)}}{1+\frac{K}{s(1+sT)}} = \frac{K}{Ts^2+s+K} = \frac{K/T}{s^2+\frac{s}{T}+K/T}$$

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We have $M_p = 10\%$

$\therefore e^{-\pi\xi/\sqrt{1-\xi^2}} = 0.1$

$\therefore -\frac{\pi\xi}{\sqrt{1-\xi^2}} = -2.3026$

$\frac{\xi}{\sqrt{1-\xi^2}} = 0.7329$

$\frac{\xi^2}{1-\xi^2} = 0.5372$

$\therefore \xi^2 = 0.3495$

$\therefore \xi = 0.5912$

2% settling time, $t_s = 2$ sec

$\therefore \frac{4}{\xi\omega_n} = 2$

$\therefore \omega_n = \frac{4}{2(0.5912)} = 3.3829$ rad/sec

$\therefore 2\xi\omega_n = \frac{1}{T} = 2(0.5912)(3.3829) = 4$

$\therefore T = \frac{1}{4} = 0.25$

Also,

$\omega_n^2 = \frac{K}{T}$

$\therefore K = \omega_n^2 T = (3.3829)^2(0.25)$

$= 2.8610$

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8. Unity feedback system has open loop transfer function $G(s) = \frac{k}{s(s+5)}$. Determine k such that damping factor, $\xi = 0.8$. Also determine t_d, t_r, t_p, t_s, M_p.

Soln The O.L.T.F. is,

$$G(s) = \frac{k}{s(s+5)}$$

$$\therefore G_{CL}(s) = \frac{\frac{k}{s(s+5)}}{1+\frac{k}{s(s+5)}} = \frac{k}{s^2+5s+k}$$

$\therefore 2\xi\omega_n = 5$

$\therefore 1.6\omega_n = 5$

$\therefore \omega_n = 3.125$ rad/sec

$k = \omega_n^2 = 9.7656$ rad/sec

$$t_d = \frac{1+0.7\xi}{\omega_n} = \frac{1+0.7(0.8)}{3.125} = \cancel{0.1597\text{ sec}}\ 0.4992 \text{ sec}$$

$$t_r = \frac{\pi-\theta}{\omega_d} = \frac{\pi-\cos^{-1}0.8}{3.125\sqrt{1-(0.8)^2}} = 1.3323$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{1.875} = 1.6755 \text{ sec}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{2.5} = 1.6 \text{ sec}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$$

$$= e^{-0.8\pi/\sqrt{1-(0.8)^2}}$$

$$= 0.0152 = 1.52\%$$

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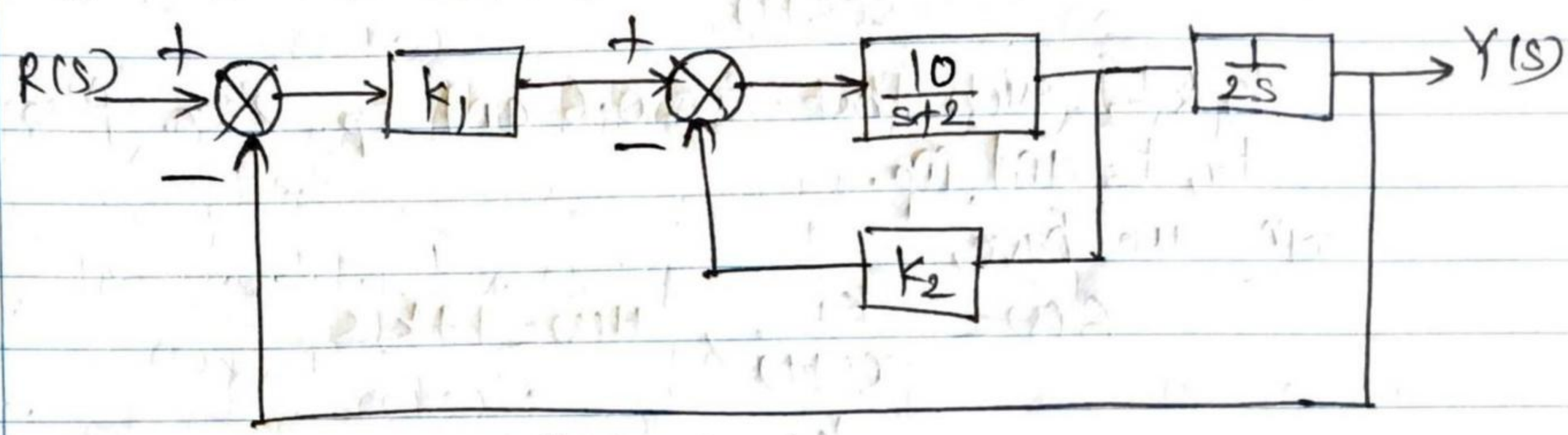
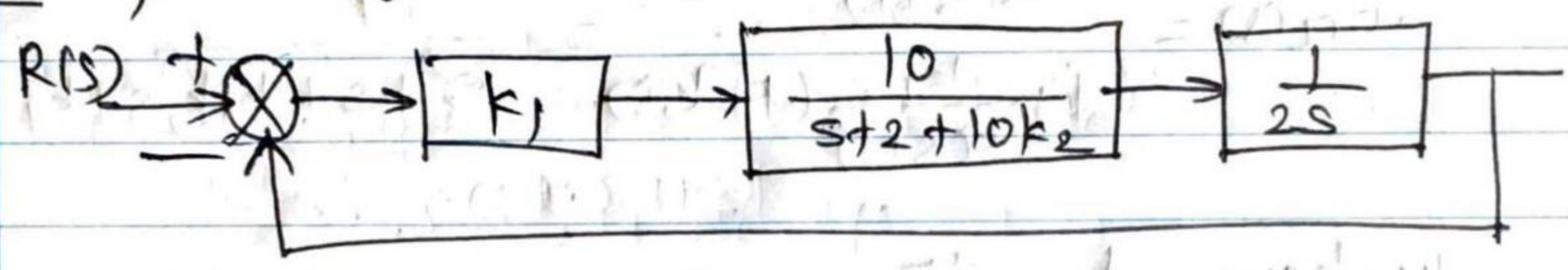
9. For the system shown below determine k_1 and k_2 such that $\xi = 0.7$ and $t_s = 0.1$ sec (2% settling)



Sol^n By reducing block diagram we get



$$\therefore G_{CL}(S) = \frac{\frac{5k_1}{s(s+2+10k_2)}}{1+\frac{5k_1}{s(s+2+10k_2)}} = \frac{5k_1}{s^2+2s+10k_2s+5k_1}$$

we have, $t_s = 0.1$ & $\xi = 0.7$

$$\therefore \frac{4}{\xi\omega_n} = 0.1$$

$$\therefore \omega_n = \frac{4}{(0.7)(0.1)} = 57.1429$$

$$\therefore \omega_n^2 = 5k_1 = 3265.3061$$

$$\therefore k_1 = 653.06$$

$$2\xi\omega_n = 2+10k_2$$

$$\therefore \xi\omega_n = 1+5k_2$$

$$\therefore 40 = 1+5k_2$$

$$\therefore k_2 = \frac{39}{5} = 7.8$$

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10. For the ~~unity~~ negative feedback system with open loop transfer function $G(s) = \frac{k_1}{s(s+1)}$ and feedback $H(s) = 1 + k_2 s$, determine k_1 & k_2 such that $\xi = 0.6$ and $t_p = 2$ sec. Also determine t_r, t_s and M_p.

<u>soln</u> we have,

$$G(s) = \frac{k_1}{s(s+1)}, \quad H(s) = 1 + k_2 s$$

$$\therefore G_c(s) = \frac{\frac{k_1}{s(s+1)}}{1 + \frac{k_1}{s(s+1)}(1+k_2 s)} = \frac{k_1}{s^2 + s + k_1 k_2 s + k_1}$$

we have, $t_p = \frac{\pi}{\omega_d} = 2$

$$\therefore \omega_d = \frac{\pi}{2} = 1.5708 \text{ rad/sec}$$

$$\therefore \omega_n \sqrt{1-\xi^2} = 1.5708$$

$$\therefore \omega_n = \frac{1.5708}{\sqrt{1-(0.6)^2}} = 1.9635$$

$$k_1 = \omega_n^2 = 3.8553$$

$$2\xi\omega_n = 1 + k_1 k_2$$

$$\therefore k_2 = \frac{2\xi\omega_n - 1}{k_1} = \frac{2(0.6)(1.9635) - 1}{3.8553} = 0.3518$$

$$t_r = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \cos^{-1} 0.6}{1.5708} = 1.4097 \text{ sec}$$

$$t_s = \frac{4}{\xi\omega_n} = \frac{4}{0.6(1.9635)} = 3.3953 \text{ sec}$$

$$M_p = e^{-\pi\xi/\sqrt{1-\xi^2}} = e^{-0.6\pi/\sqrt{1-(0.6)^2}} = 0.0948$$

$$= 9.48\%$$

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Steady state error and static error coefficients-

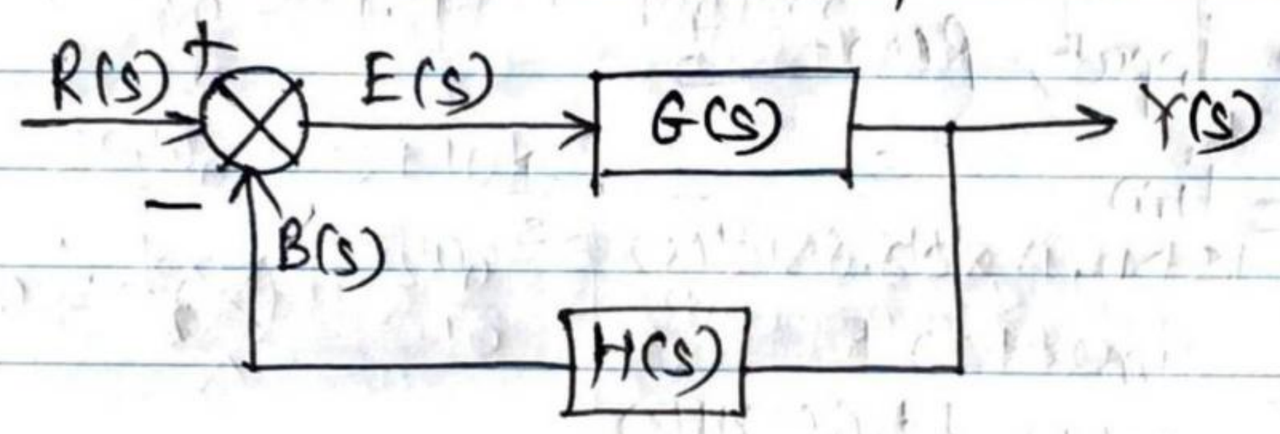
Steady state error is the difference between system output and set point under steady state conditions. It is expressed as,

$$e_{ss} = \lim_{t \to \infty} r(t) - y(t)$$

It is the measure of steady state performance of the system. Ideally its value should be zero.

Expression for steady state error in feedback control system-

Consider a feedback system shown below.



For the above system we have,

$$E(s) = R(s) - B(s)$$

& $B(s) = E(s)\,G(s)\,H(s)$

$$\therefore E(s) = R(s) - E(s)\,G(s)\,H(s)$$

$$\therefore E(s)[1 + G(s)H(s)] = R(s)$$

$$\therefore \frac{E(s)}{R(s)} = \frac{1}{1+G(s)H(s)}$$

$$\therefore E(s) = \frac{R(s)}{1+G(s)H(s)}$$

We have,

$$e_{ss} = \lim_{t \to \infty} e(t)$$

By final value theorem of Laplace transform we get,

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{s\,R(s)}{1+G(s)H(s)}$$

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Static error coefficients-

Position error constant (k_p), velocity error constant (k_v) and acceleration error constant (k_a) are the three static error constants related to steady state error e_{ss}. Larger the values of these coefficients, smaller is the steady state error.

Position error constant (k_p)-

We have,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}$$

For step input, $R(s) = \frac{1}{s}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s}{1+G(s)H(s)} \cdot \frac{1}{s}$$

$$= \lim_{s \to 0} \frac{1}{1+G(s)H(s)}$$

$$= \frac{1}{1+\lim_{s \to 0} G(s)H(s)} = \frac{1}{1+k_p}$$

where,

$$k_p = \lim_{s \to 0} G(s)H(s)$$

Velocity error constant (k_v)-

We have,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1+G(s)H(s)}$$

For ramp input, $R(s) = 1/s^2$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s}{1+G(s)H(s)} \cdot \frac{1}{s^2}$$

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$$\therefore e_{ss} = \lim_{s \to 0} \frac{1}{s + sG(s)H(s)}$$

$$= \frac{1}{\lim_{s \to 0} sG(s)H(s)} = \frac{1}{k_v}$$

where,

$$k_v = \lim_{s \to 0} sG(s)H(s)$$

<u>Acceleration error constant (k_a)-</u>

We have,

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)H(s)}$$

For parabolic input, $R(s) = 1/s^3$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s}{1 + G(s)H(s)} \cdot \frac{1}{s^3}$$

$$= \lim_{s \to 0} \frac{1}{s^2 + s^2 G(s)H(s)}$$

$$= \frac{1}{\lim_{s \to 0} s^2 G(s)H(s)} = \frac{1}{k_a}$$

where,

$$k_a = \lim_{s \to 0} s^2 G(s)H(s)$$

Thus for different inputs the steady state error interms of static error constants is

Input	e_{ss}	Input	e_{ss}
step	$\frac{1}{1+k_p}$	Parabolic	$\frac{1}{k_a}$
Ramp	$\frac{1}{k_v}$		

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Values of static error constants for systems with type 0, type 1, type 2 and higher -

Position error constant -

$$k_p = \lim_{s \to 0} G(s) H(s)$$

For type 0

k_p = constant

For type 1 or higher

$k_p = \infty$

Velocity error constant -

$$k_v = \lim_{s \to 0} sG(s) H(s)$$

For type 0

$k_v = 0$

For type 1

k_v = constant

For type 2 or higher

$k_v = \infty$

Acceleration error constant -

$$k_a = \lim_{s \to 0} s^2 G(s) H(s)$$

For type 0, 1

$k_a = 0$

For type 2

k_a = constant

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For type 3 and higher

$k_a = \infty$

From this we get steady state error for different inputs and type of system as follows

<u>e_{ss} -</u>

Input	Type 0	Type 1	Type 2
Unit step	$\frac{1}{1+k_p}$	0	0
Unit Ramp	∞	$\frac{1}{k_v}$	0
Unit Parabolic	∞	∞	$\frac{1}{k_a}$

<u>Examples on steady state error and static error constants -</u>

1. Consider unity feedback system with open loop transfer function $G(s) = \frac{10(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)}$. Determine static error constants and steady state error for ramp input.

<u>Sol^n</u> We have, $G(s) = \frac{10(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)}$, $H(s) = 1$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{10(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)} = \infty$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{10s(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)}$$

$$= \frac{10(2)(5)}{10} = 10$$

$$K_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{10s^2(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)} = 0$$

$$e_{ss} = \lim_{s\to 0} \frac{sR(s)}{1+G(s)H(s)}$$

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For ramp input, $R(s) = \frac{1}{s^2}$

$$\therefore e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s^2}{1 + \frac{10(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)}}$$

$$= \lim_{s \to 0} \frac{1}{s + \frac{10s(s+2)(s^2+2s+5)}{s(s+1)(s^2+4s+10)}}$$

$$= \lim_{s \to 0} \frac{(s+1)(s^2+4s+10)}{10(s+2)(s^2+2s+5)}$$

$$= \frac{10}{100} = 0.1$$

<u>Another approach for e_{ss} -</u>

For ramp input ~~with~~ e_{ss} is,

$$e_{ss} = \frac{1}{K_v} = \frac{1}{10} = 0.1$$

2. For unity feed back system with open loop transfer function $G(s) = \frac{1}{s(s+2)}$, determine static error constants and steady state error for step, ramp and parabolic input.

<u>Soln</u> We have, $G(s) = \frac{1}{s(s+2)}$, $H(s) = 1$

$$K_p = \lim_{s \to 0} G(s)H(s) = \lim_{s \to 0} \frac{1}{s(s+2)} = \infty$$

$$K_v = \lim_{s \to 0} s\,G(s)H(s) = \lim_{s \to 0} \frac{s}{s(s+2)} = \lim_{s \to 0} \frac{1}{s+2} = 0.5$$

$$K_a = \lim_{s \to 0} s^2 G(s)H(s) = \lim_{s \to 0} \frac{s^2}{s(s+2)} = \lim_{s \to 0} \frac{s}{s+2} = 0$$

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steady state error for step input is,

$$e_{ss} = \frac{1}{1+k_p} = \frac{1}{1+\infty} = 0$$

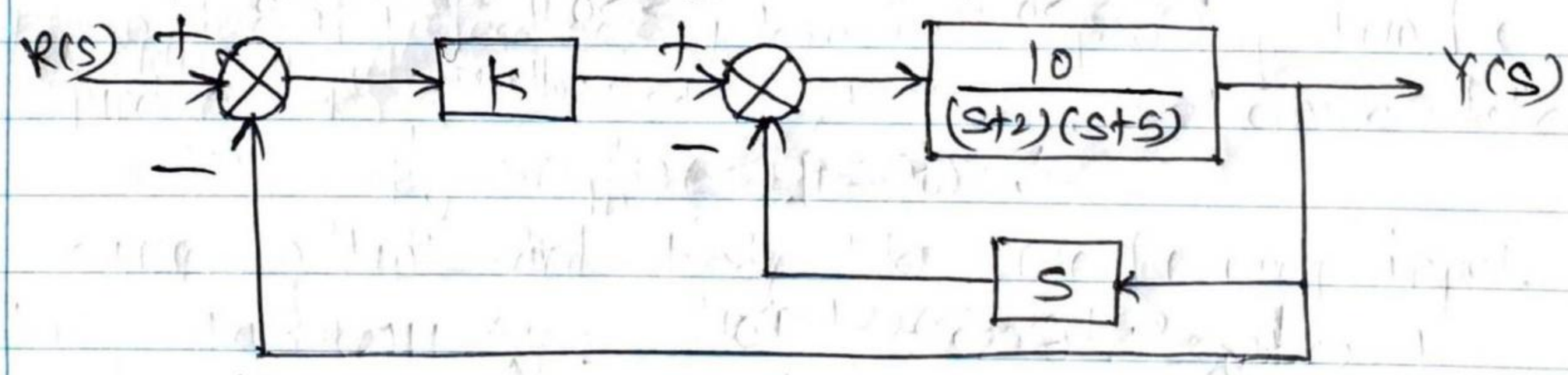
For ramp input,

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0.5} = 2$$

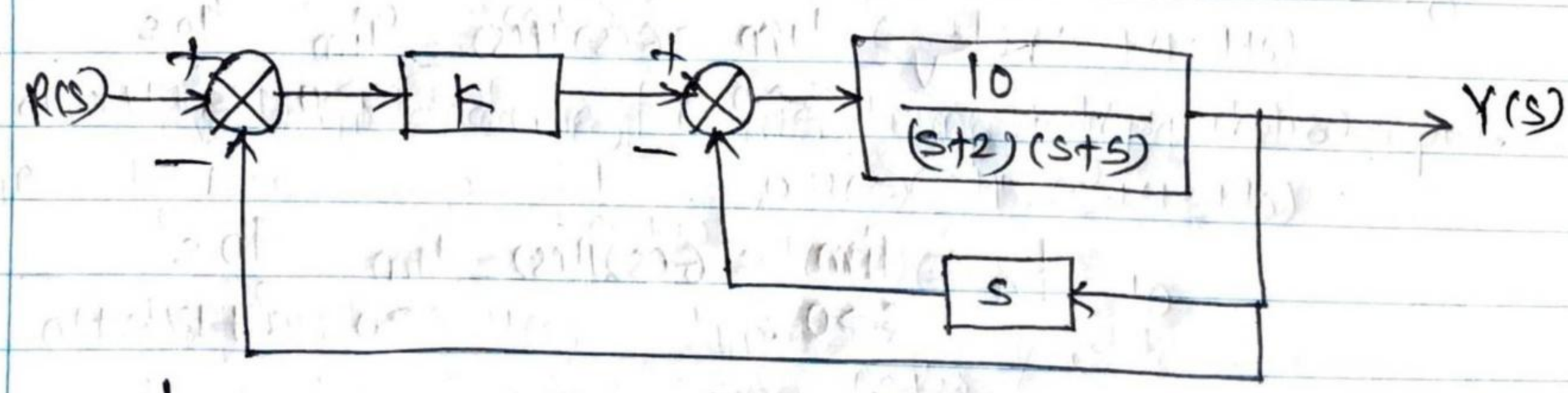
For parabolic input,

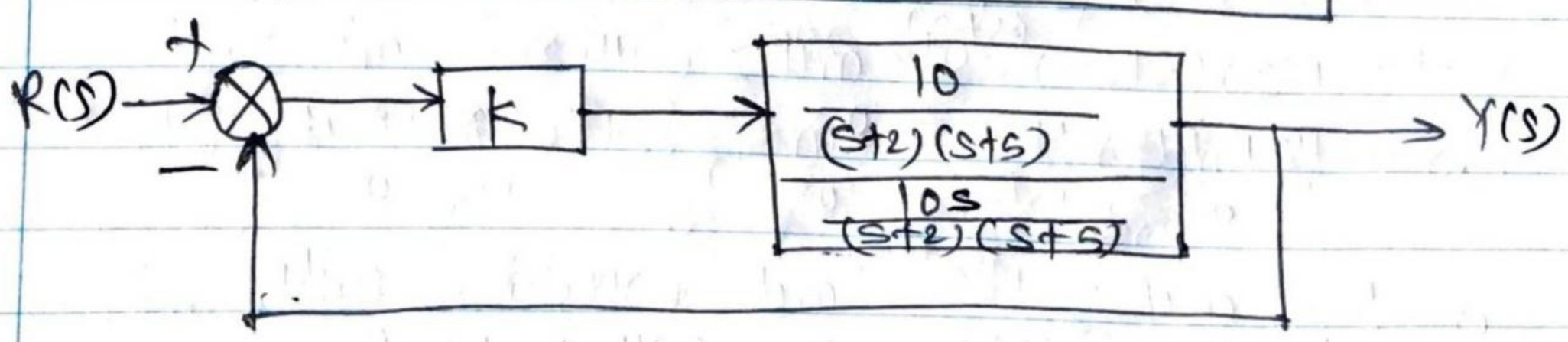
$$e_{ss} = \frac{1}{K_a} = \frac{1}{0} = \infty$$

3. For the system shown below, determine value of k such that steady state error for step input is 0.1 when system is subjected to step input. For this value of k determine static error constants k_p, k_v and k_a.



<u>soln</u> we have the system,





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$$\therefore G(s) = \frac{10K}{s^2+17s+10}, \quad H(s)=1$$

For step input ess is,

$$e_{ss} = \frac{1}{1+k_p}$$

$$\therefore 0.1 = \frac{1}{1+k_p}$$

$$\therefore 0.1 + 0.1k_p = 1$$

$$\therefore 0.1k_p = 0.9$$

$$\therefore k_p = 9$$

$$k_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{10K}{s^2+17s+10} = 9$$

$$\therefore K = 9$$

$$\therefore G(s) = \frac{90}{s^2+17s+10}, \quad H(s)=1$$

$$\therefore k_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{90s}{s^2+17s+10} = 0$$

$$k_a = \lim_{s\to 0} s^2G(s)H(s) = \lim_{s\to 0} \frac{90s^2}{s^2+17s+10} = 0$$

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4. The unity feedback system has open loop transfer function $G(s) = \frac{k}{s^2(s+2)(s+4)}$. Determine the value of k such that steady state error for input $r(t) = 1+3t+4t^2$ is 6.

<u>soln</u> we have,

$$G(s) = \frac{k}{s^2(s+2)(s+4)}, \quad H(s) = 1$$

$$\therefore k_p = \lim_{s \to 0} G(s)H(s) = \infty$$

$$k_v = \lim_{s \to 0} s\,G(s)H(s) = \infty$$

$$k_a = \lim_{s \to 0} s^2 G(s)H(s) = \frac{s^2 k}{s^2(s+2)(s+4)} = \frac{k}{8}$$

The input is $r(t) = 1+3t+4t^2 = 1+3t+8\frac{t^2}{2}$

$$\therefore e_{ss} = \frac{1}{1+k_p} + \frac{3}{k_v} + \frac{8}{k_a}$$

$$= \frac{1}{1+\infty} + \frac{3}{8} + \frac{8}{k/8} = 0 + 0 + \frac{64}{k}$$

$$= \frac{64}{k}$$

$$\therefore \frac{64}{k} = 6$$

$$\therefore k = \frac{64}{6} = 10.6667$$

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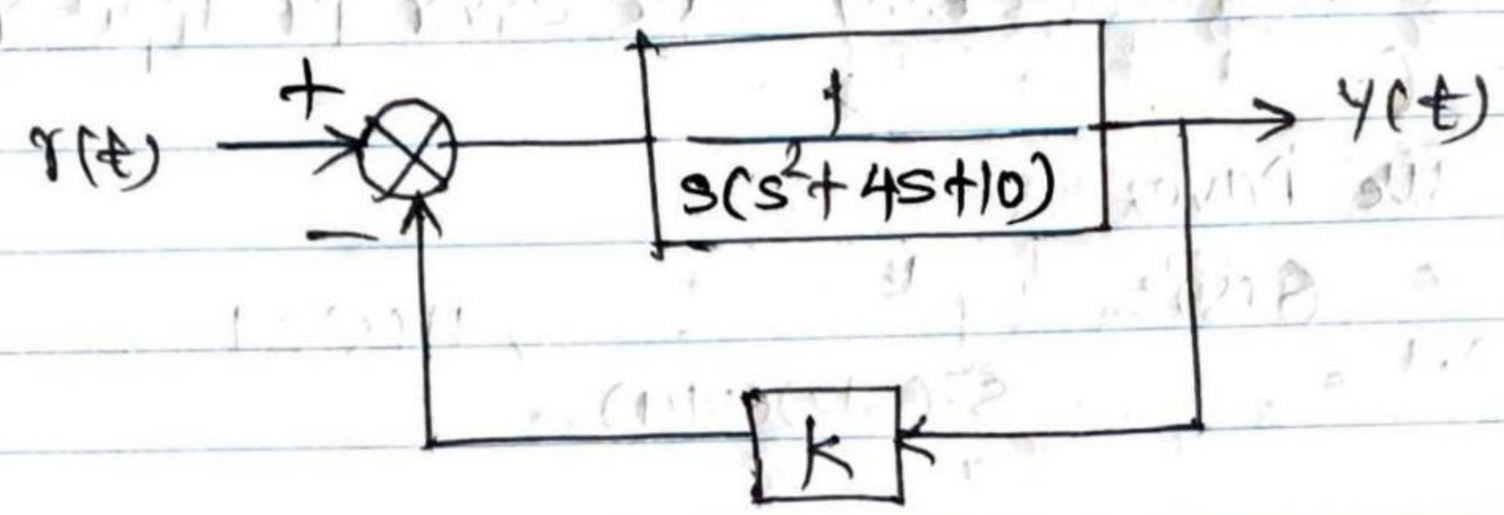
5. For the system shown below determine k when the steady state error is 0.2 if system is subjected to input $r(t) = 1+3t$



Sol^n We have, $G(s) = \dfrac{1}{s(s^2+4s+10)}$, $H(s) = k$

$$\therefore K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{k}{s(s^2+4s+10)} = \infty$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{ks}{s(s^2+4s+10)} = \frac{k}{10}$$

For input $r(t) = 1+3t$

$$e_{ss} = \frac{1}{1+K_p} + \frac{3}{K_v} = \frac{1}{1+\infty} + \frac{3}{k/10} = \frac{30}{k}$$

$$\therefore \frac{30}{k} = 0.2$$

$$\therefore k = \frac{30}{0.2} = 150.$$

6. For the system shown below

(i) determine K_p, K_v, K_a when $k=1$ and e_{ss} if system is subjected to step input.

(ii) determine k such that, closed loop damping factor is 0.65 and find K_p, K_v, K_a and e_{ss} for step input in this case.

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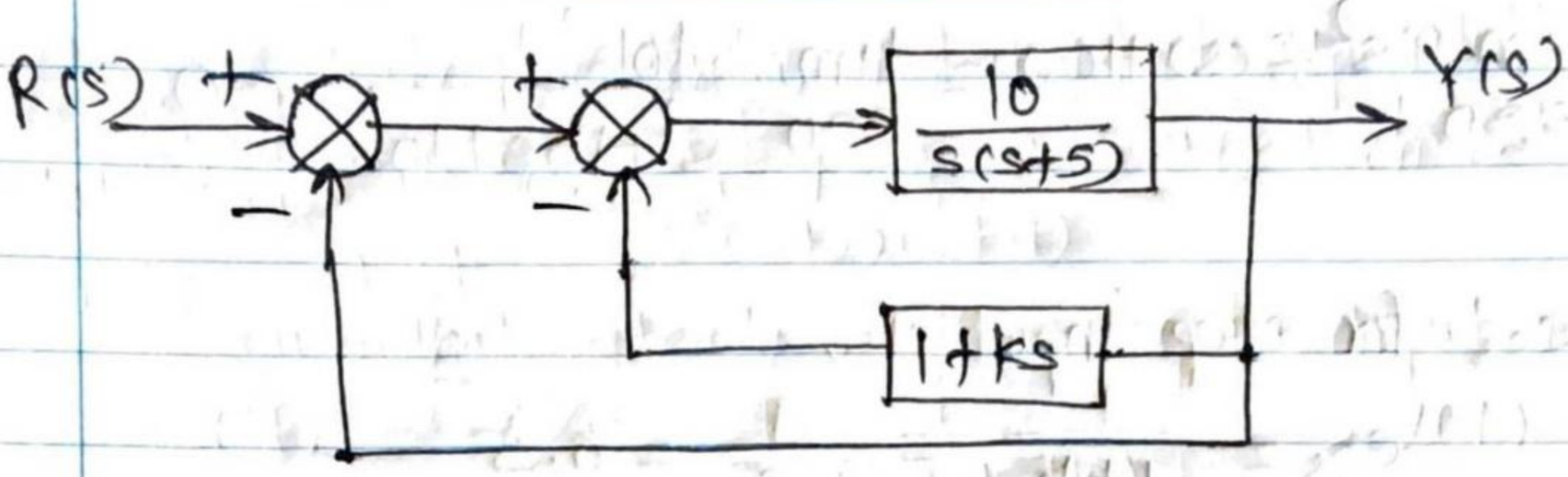
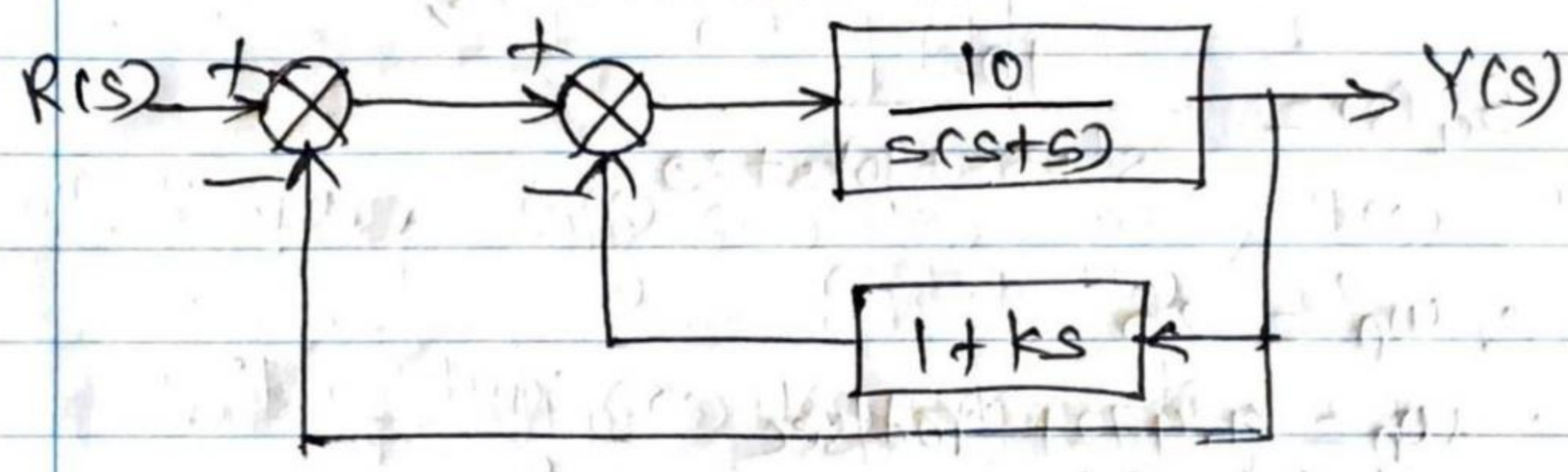
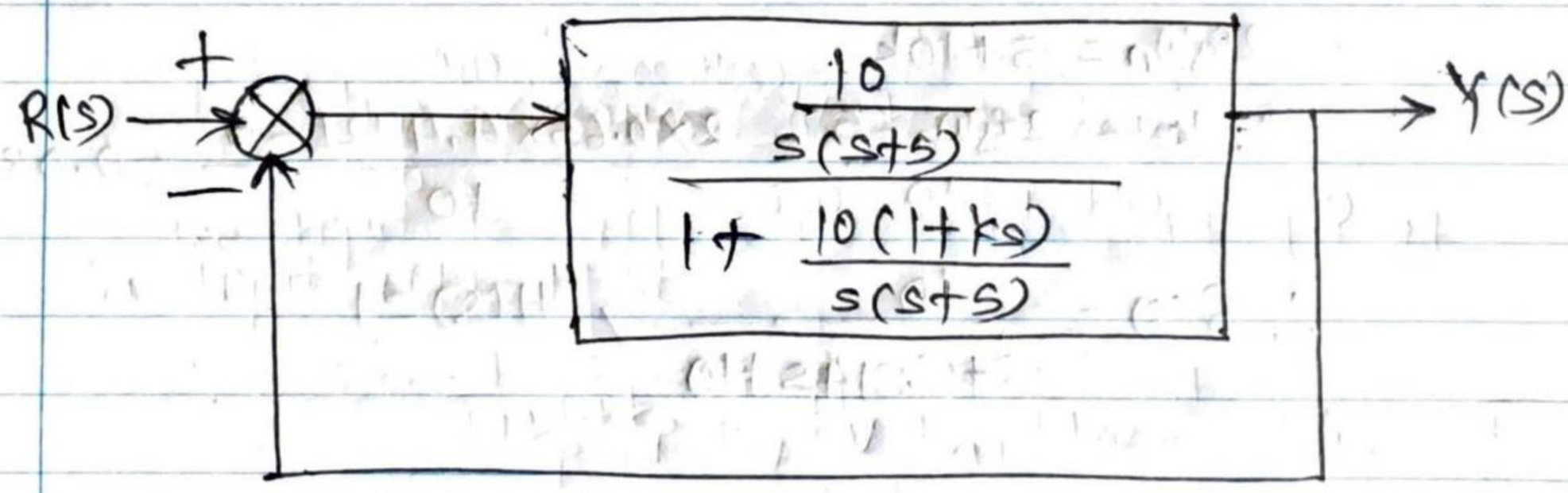


<u>soln</u> We have the system as,





$$\therefore G(s) = \frac{10}{s^2+5s+10ks+10}, \quad H(s) = 1$$

~~$K_p = \lim_{s\to 0} G(s)H(s) = \lim$~~

a] $k = 1$

$$\therefore G(s) = \frac{10}{s^2+15s+10}, \quad H(s) = 1$$

$$K_p = \lim_{s\to 0} G(s)H(s) = \lim_{s\to 0} \frac{10}{s^2+15s+10} = 1$$

$$K_v = \lim_{s\to 0} sG(s)H(s) = \lim_{s\to 0} \frac{10s}{s^2+15s+10} = 0$$

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$$K_a = \lim_{s \to 0} s^2 G(s) H(s) = \lim_{s \to 0} \frac{10s^2}{s^2+15s+10} = 0$$

e_{ss} for step input is,

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{2} = 0.5$$

b] For the given system C.L.T.F. is,

$$G_{CL}(s) = \frac{10}{s^2+5s+10ks+20}$$

$\therefore \omega_n^2 = 20$

$\omega_n = 4.4721$ rad/sec

$2\xi\omega_n = 5+10k$

$$\therefore k = \frac{2\xi\omega_n - 5}{10} = \frac{2\times 0.65\times 4.4721 - 5}{10} = 0.0814$$

$$\therefore G(s) = \frac{10}{s^2+5.814s+10}, \quad H(s) = 1$$

$$K_p = \lim_{s \to 0} G(s) H(s) = \lim_{s \to 0} \frac{10}{s^2+5.814s+10} = 1$$

$$K_v = \lim_{s \to 0} sG(s)H(s) = 0$$

$$K_a = \lim_{s \to 0} s^2 G(s) H(s) = 0$$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{2} = 0.5$$

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Importance of dominant closed loop poles-

Concept of dominant pole- A pole of the system which is closest to imaginary axis is dominant pole. It has the highest impact on the system output and hence called as dominant pole. A pole away from imaginary axis has less impact on system response. A pole far away from imaginary axis as almost negligible impact of system response. This fact is very useful in control system design.

Importance of closed loop poles in control system design-

In the design of control system, the objective is to

1. stabilize closed loop system.
2. obtain desired transient (dynamic) performance
3. obtain desired steady state performance

To ensure closed loop stability all closed loop poles are selected such that they are in left half of s-plane. To obtain desired transient performance the poles are placed at desired location.

For higher order systems, the closed loop performance specifications are expressed in terms of time domain specifications like peak time, rise time, peak overshoot, settling time and steady state specifications like k_p, k_v, k_a, e_{ss}.

In design process, for the given time domain specifications the values of ξ and ω_n are calculated and the controller is designed to achieve these values. It results in a complex conjugate pole pair at $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$. Then to have less impact of other poles, they are selected at least 5 times

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away from the imaginary axis.

The design based on lower order system offers simplicity with minor effect on performance of system due to impact of less dominant poles on system output.

For example, if desired time domain specifications of a 5th order system are resulting in say $s = -2 \pm j0.8$ then the remaining three poles can be selected at $s = -10$ or left to $s = -10$ and the controller can be designed for $s = -2 \pm j0.8$.

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