Control systems - The systems intended for controlling the behaviour of plant variables are control systems.

Classification of control systems -

Depending upon different parameters such as input - output relationship, control strategy the control systems are classified as

A) Depending on input - output relationship
1. Linear v/s non linear
2. Time invariant v/s time variant
3. Causal v/s non causal
4. Stable v/s unstable
5. Single input single output v/s multiple input multiple output

B) Depending on control strategy
1. Open loop v/s closed loop
2. Feedback v/s feed forward
3. Tracking v/s regulation
4. Manual v/s automatic control

Linear v/s non linear control system - A control system which obeys superposition theorem and law of homogenity is called as linear control system, otherwise it is non linear control system.

Time invariant v/s time variant control system - A control system for which input output relationship does not change with time is called as time invariant control system, otherwise it is time variant control system.

Causal v/s non causal control system - A control system is

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causal if its output depends only on present and/or past input and not on the future input, otherwise the control system is non causal.

<u>Stable V/s unstable control system</u>- A control system is stable if it produces bounded output for every bounded input, otherwise it is unstable control system.

<u>Note</u>- Every control system must be causal and stable.

<u>SISO V/s MIMO control system</u>- A control system may have single input and single output or multiple inputs and outputs. Accordingly the control system may be SISO or MIMO.

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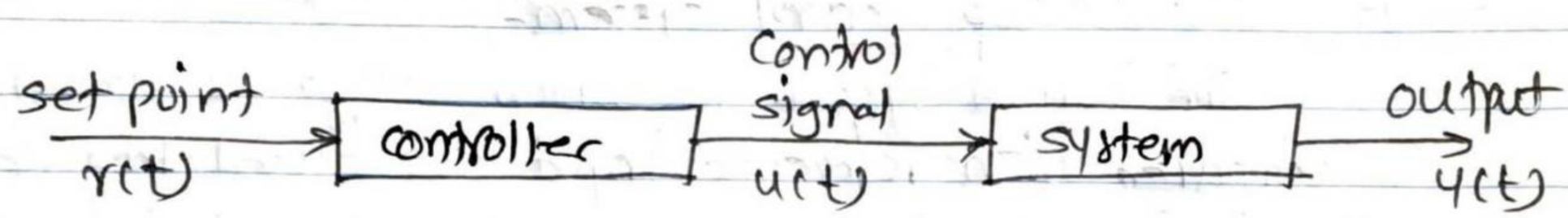
Open loop ~~system~~ control system

The system in which output is neither measured nor fed back for comparision with set point (reference input) is open loop control system. The figure below shows block schematic of open loop control system.



The control signal in open loop system does not depend on the system output. Control signal depends on set point and for every set point the control signal profile is predefined from the caliberation of system.

Open loop systems are highly sensitive to disturbances and uncertainities. Open loop systems can perform satisfactorily only if the system behaviour is perfectly known and the system is not affected by the disturbances.

Features of open loop control systems -

1. It is highly sensitive to disturbances and uncertainities.
2. The control signal is independent of system output
3. Output is not fed back for comparision with set point
4. Accuracy is less.
5. Bandwidth is less.
6. Stability is not the major problem.
7. Complexity is less.
8. Cost is less.

Examples- All timer based systems are open loop systems because in these systems the system output is not

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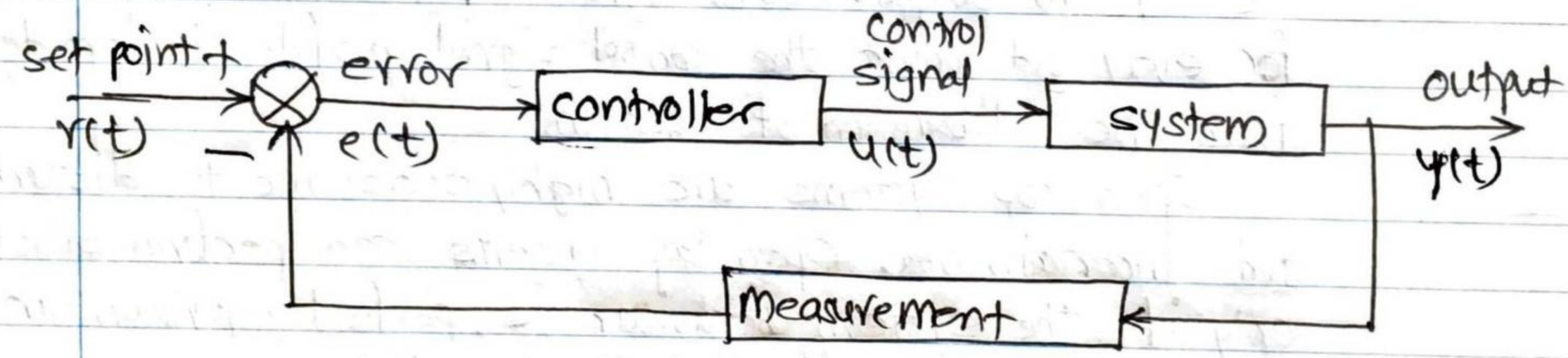
measured. Traffic signal, timer based washing machine are the examples of open loop control system as in these systems, the outputs that is traffic and cleanliness of cloths are not measured.

Closed loop control system-

The control system in which output is measured and compared with reference input is closed loop control system. The figure below shows block schematic of closed loop control system.



In this system, the output is continuously fed back, measured and compared with set point to generate error signal. The error signal is applied to controller. The controller then generates control signal which is applied to system to keep output of system as close as possible to set point despite of external disturbances.

Features of closed loop system-

1. Control ~~system~~ signal is dependent of system output
2. It is less sensitive to external disturbances.
3. Output is continuously fed back, measured and compared with set point.
4. Accuracy is more.

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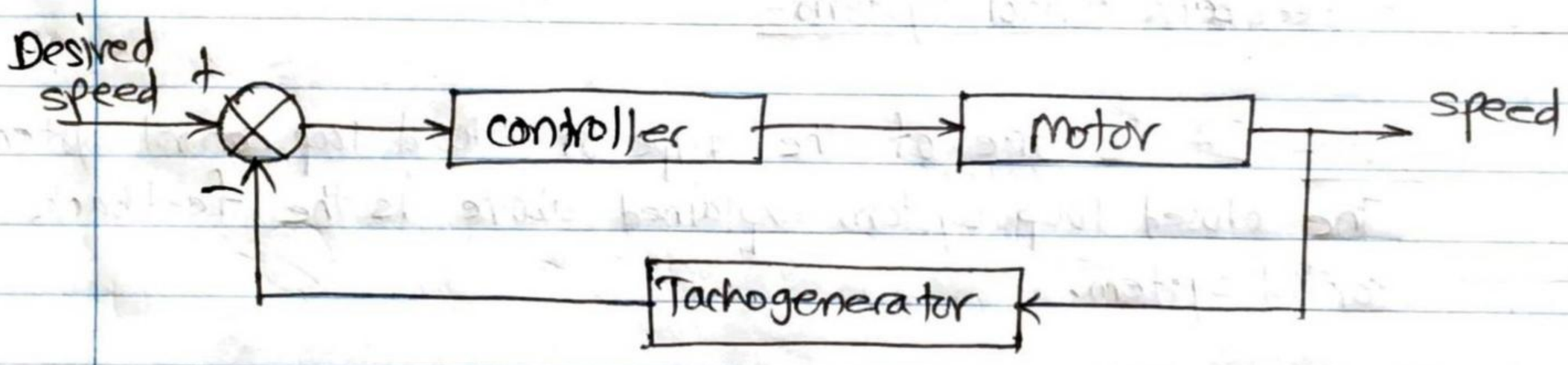
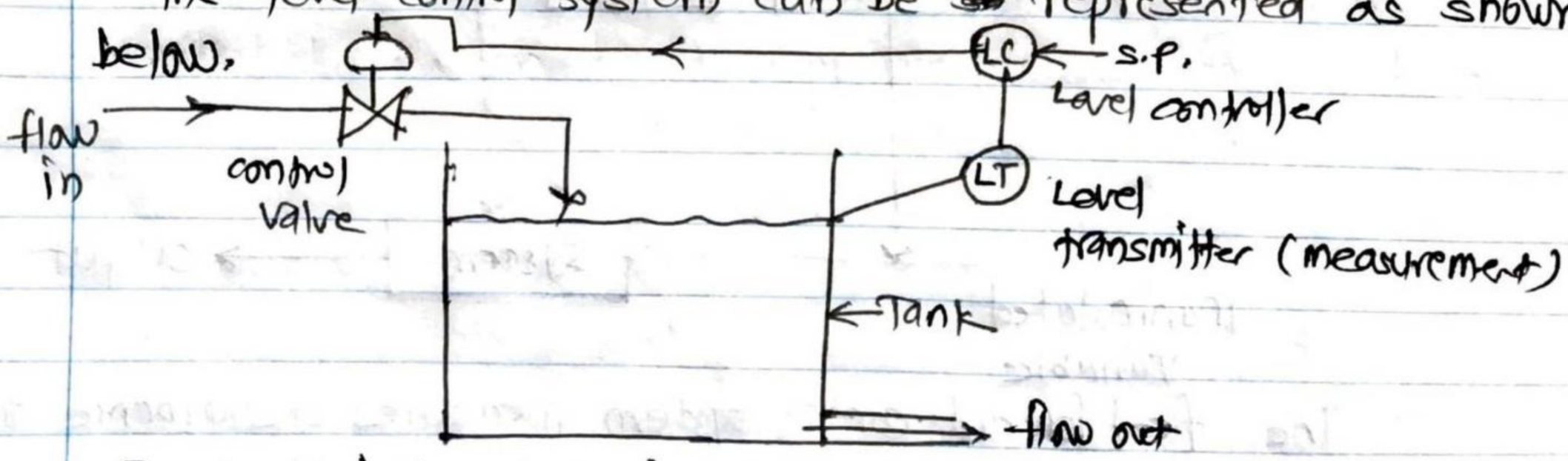
5. Bandwidth is ~~less~~ more.
6. Complexity is more.
7. Stability is the most important issue.
8. Cost is more.

Examples- Speed control of D.C. motor, control of liquid level in a tank are the examples of closed loop control system. The figure below shows block schematic of speed control system.



In this system, the speed of motor is continuously measured by a tachogenerator and is compared with the desired speed. The error signal is applied to controller which generates the control signal. This control signal is applied to motor to control the speed, by adjusting armature voltage.

The level control system can be represented as shown below.



The level inside tank is continuously measured and corresponding signal is transmitted by level transmitter to level controller. The set point is applied to level controller. Level controller

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Level controller compares setpoint with measured level and generates control signal depending upon error signal. The control signal is applied to control valve which adjusts opening of the valve and in turn the ~~inlet~~ inlet flow rate. The controller reduces error to zero and brings level inside tank at set point.

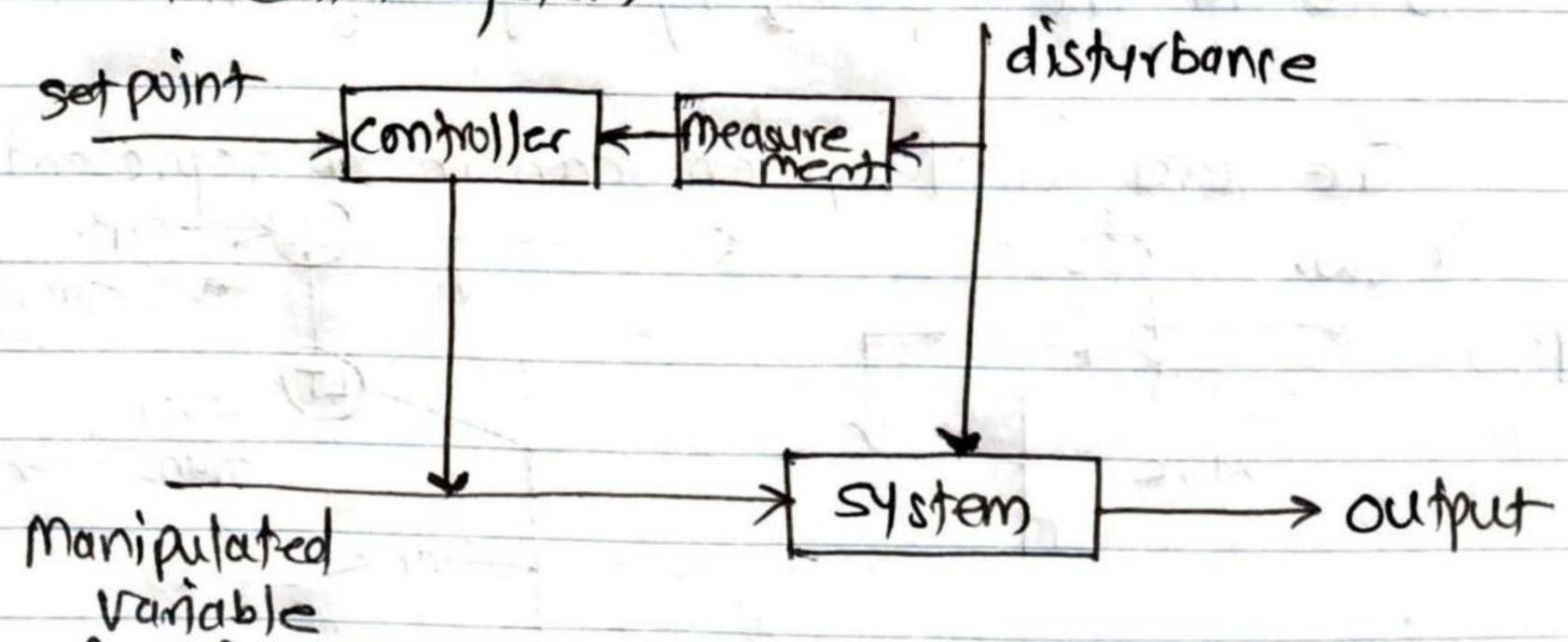
~~Feed~~

Feedback control system-

Note- It is one of the type of closed loop control system. The closed loop system explained above is the feedback control system.

Feed forward control system-

Feed forward control system measures load disturbance and takes corrective action to eliminate the effect of disturbance on system output. The figure below shows feed forward control system.



The feed forward control system measures disturbance and not the system output. It measures the disturbance and sends the signal to controller. The controller then formulates the effect of disturbance on output and

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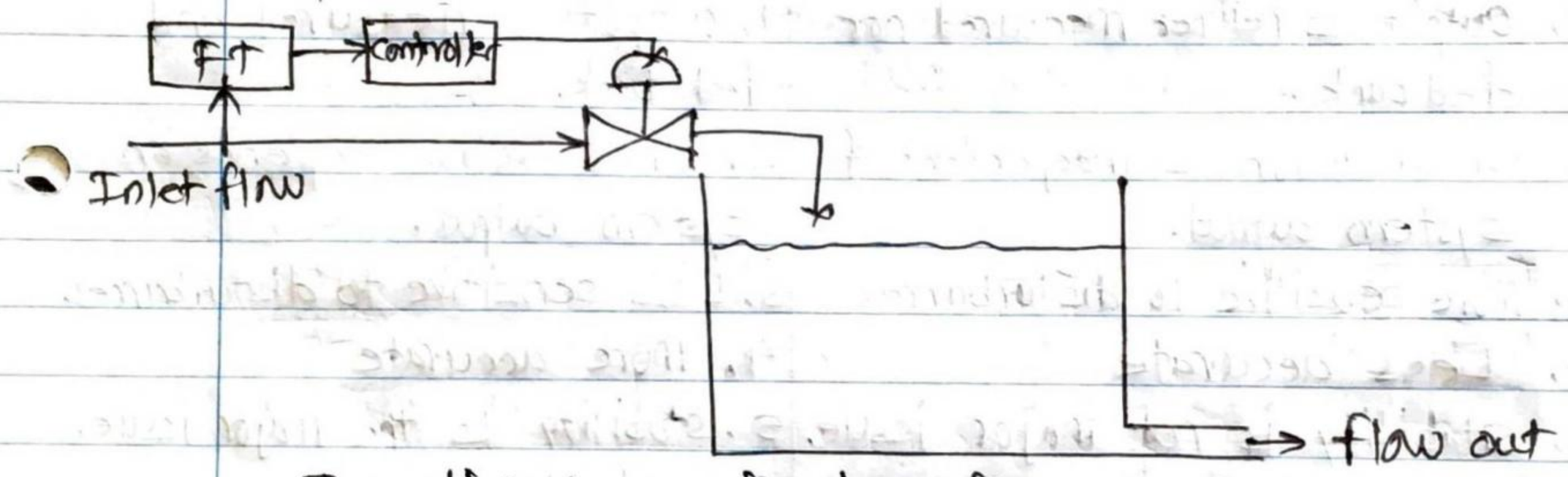
generates the control signal such that the effect of disturbance is nullified / reduced before the disturbance seriously affects the output. Thus it is an anticipatory control action.

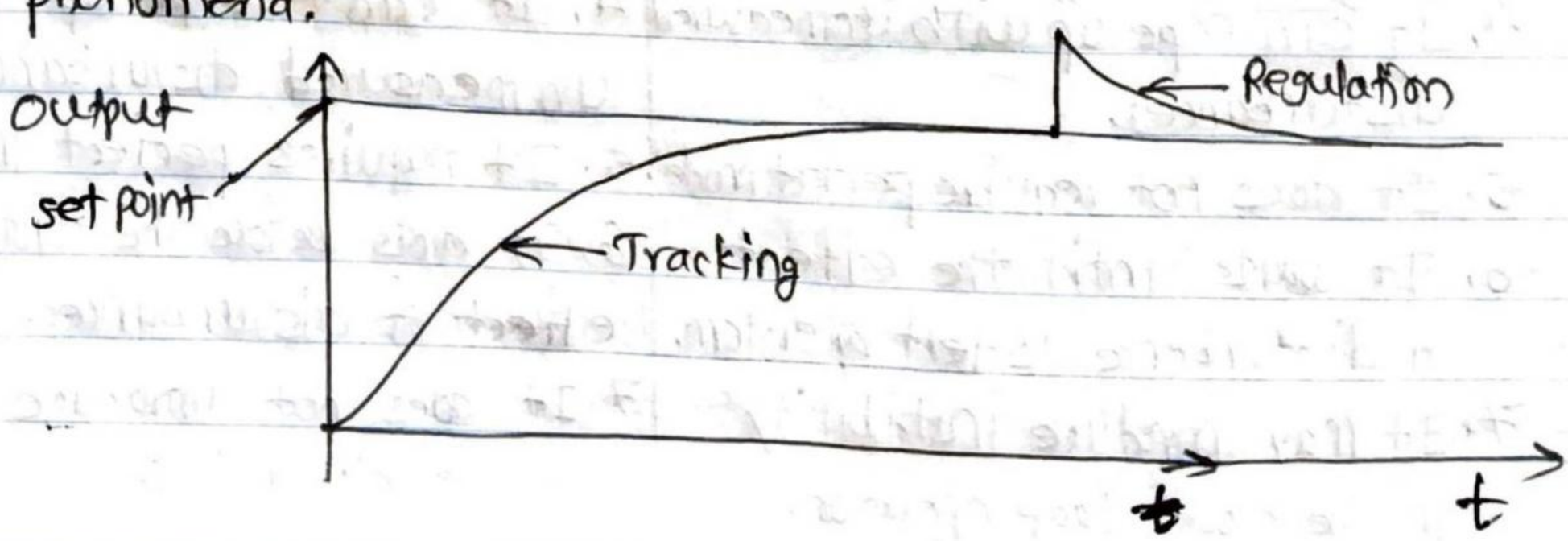
<u>Example</u>- The figure below shows feed forward control for liquid level system.



The disturbance in inlet flow rate is measured and given to feed forward controller. The feed forward controller formulates the effect of change in flow rate on level inside tank and adjusts the valve position to cancel the effect of change in flow rate.

<u>Tracking v/s regulatory control system</u>-

A system may require change in set point of system output. Also it may experience the change in output due to external disturbances. In both cases, it is required to bring the ~~set point~~ system output to setpoint. The figure below shows these phenomena.



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Concept of tracking the setpoint by the system output is known as tracking control and nullifying the effect of disturbance is regulation control.

Comparision between open loop and closed loop control systems -

Open loop system	Closed loop system
1. Output is neither measured nor fed back.	1. Output is measured and fed back.
2. Control action is independent of system output.	2. Control action depends on system output.
3. More sensitive to disturbances.	3. Less sensitive to disturbances.
4. Less accurate	4. More accurate
5. Stability is not major issue.	5. Stability is the major issue.
6. Bandwidth is less	6. Bandwidth is more
7. Less complex	7. More complex
8. Less cost	8. ~~Less~~ More cost
9. Exact System knowledge is essential	9. Exact System knowledge is not essential.

Comparison between feedback and feed forward control systems

Feedback control systems	Feed forward control systems
1. It does not measure disturbance	1. It measures disturbance.
2. It is not sensitive to model errors	2. It is sensitive to model errors.
3. It is unsatisfactory for slow systems.	3. It is good for slow systems.
4. It can cope up with unmeasured disturbances.	4. It can't cope up with unmeasured disturbances.
5. It does not require perfect model	5. It requires perfect model.
6. It waits until the effect of disturbance is felt by system.	6. It acts before the system feels effect of disturbances.
7. It may introduce instability in the closed loop operation.	7. It does not introduce instability in the closed loop.

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Transfer function -

Transfer function is defined as the ratio of Laplace transform of output to Laplace transform of input with zero initial conditions.

$$G(s) = \left.\frac{Y(s)}{R(s)}\right|_{\text{zero initial conditions}}$$

$Y(s) = L\{y(t)\}$ where $y(t)$ is system output

$R(s) = L\{r(t)\}$ where $r(t)$ is system input.

In general, for n^{th} order system the transfer function can be written as,

$$G(s) = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{s^n + a_1 s^{n-1} + \dots + a_{n-1} s + a_n}, \quad m \le n$$

Poles of system - The roots of denominator of transfer function are called as poles of system.

Zeros of system - The roots of numerator of transfer function are called as zeros of system.

Example - $G(s) = \dfrac{(s+1)(s+4)}{s(s^2+2s+2)}$

Poles can be obtained by equating denominator of T.F. to zero.

$\therefore s(s^2+2s+2) = 0$

$\therefore s = 0$ and $s = -1 \pm j$

zeros can be obtained by equating numerator of T.F. to zero.

$\therefore (s+1)(s+4) = 0$

$\therefore s = -1$ and $s = -4$

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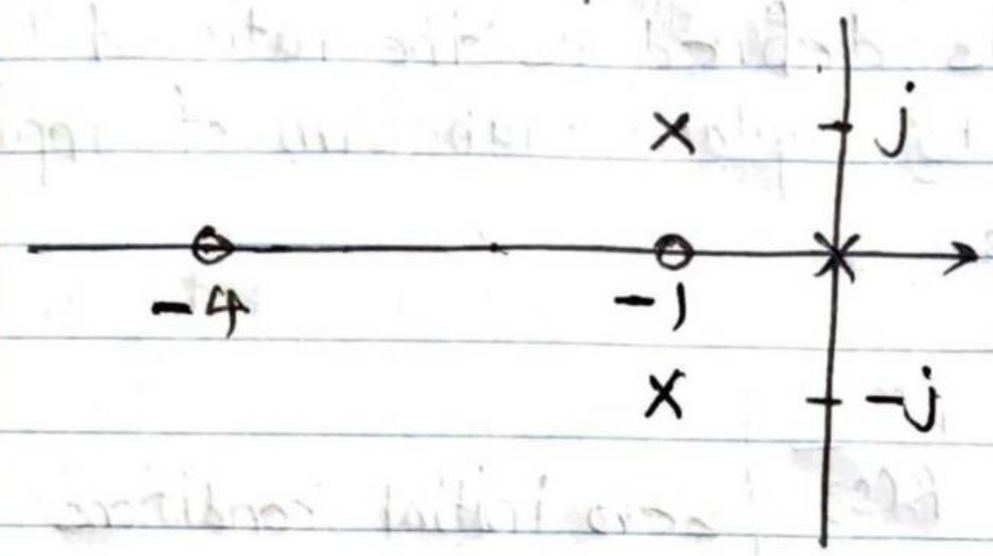
The pole zero plot can be sketched as.



Poles denoted by X
zeros denoted by O

$$G(s) = \frac{s^2+4s+4}{s^3+8s^2+19s+12}$$

For poles, $s^3+8s^2+19s+12=0$

$\therefore (s+1)(s+3)(s+4)=0$

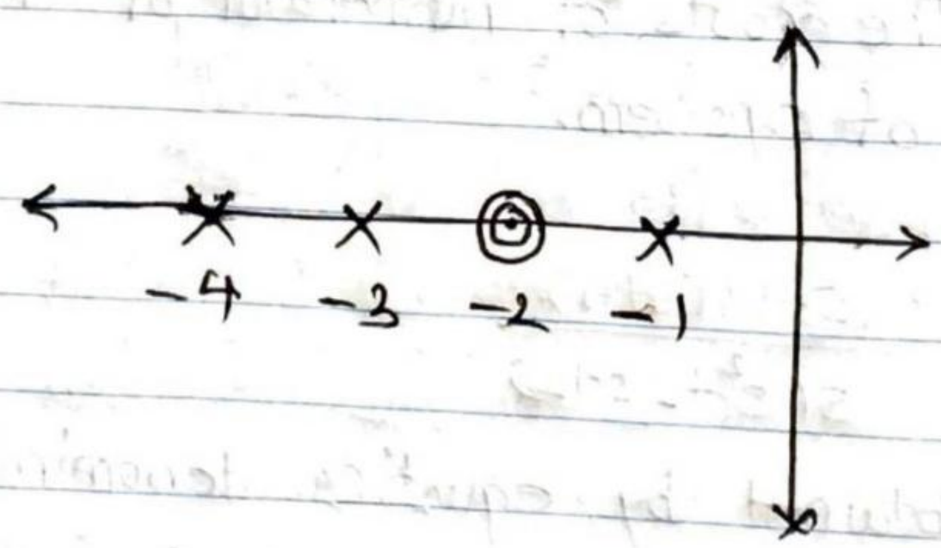
$\therefore s=-1,-3,-4$ are the poles

For zeros, $s^2+4s+4=0$

$\therefore (s+2)^2=0$

$\therefore s=-2,-2$ are the zeros

The ~~pole~~ pole zero plot is as shown below.



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Modeling of LTI system-

Modeling of LTI systems can be done applying fundamental physical and/or chemical laws to get system model in the form of linear differential equation. The system model can be obtained in the form of transfer function by applying Laplace transform to differential equations.

The generalized n^{th} order differential equation is

$$\frac{d^n y(t)}{dt^n} + a_1 \frac{d^{n-1} y(t)}{dt^{n-1}} + a_2 \frac{d^{n-2} y(t)}{dt^{n-2}} + \dots + a_{n-1} \frac{dy(t)}{dt} + a_n y(t) =$$

$$b_0 \frac{d^m u(t)}{dt^m} + b_1 \frac{d^{m-1} u(t)}{dt^{m-1}} + \dots + b_{m-1} \frac{du(t)}{dt} + b_m u(t) \quad , \; m \leq n$$

Taking L.T. on both sides with zero initial conditions we get,

$$s^n Y(s) + a_1 s^{n-1} Y(s) + a_2 s^{n-2} Y(s) + \dots + a_{n-1} s Y(s) + a_n Y(s) =$$
$$b_0 s^m U(s) + b_1 s^{m-1} U(s) + \dots + b_{m-1} s U(s) + b_m U(s)$$

$$\therefore \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \dots + b_{m-1} s + b_m}{s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n} \quad , \; m \leq n$$

In this way, the models of electrical systems, mechanical systems can be obtained by applying KCL, KVL and laws of motion respectively.

Modeling of electrical systems-

The electrical systems have three basic components as resistor (R), inductor (L) and capacitor (C). The voltages across them and currents through them can be written as.

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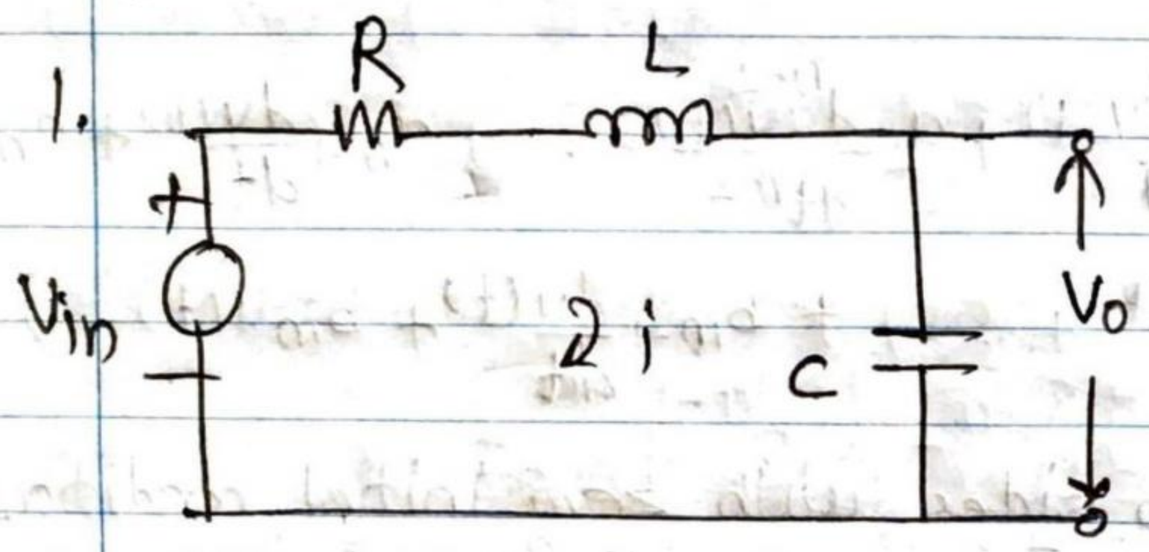
	Voltage across	Current through
R	$V = IR$	$I = V/R$
L	$V = L\frac{dI}{dt}$	$I = \frac{1}{L}\int V\,dt$
C	$V = \frac{1}{C}\int I\,dt$	$I = C\frac{dV}{dt}$

Examples on modeling of Electrical systems-

1. 

Determine the transfer function $\frac{V_o(s)}{V_{in}(s)}$.

Soln Applying KVL we get

$$V_{in} = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$$

Taking L.T. on both sides we get

$$V_{in}(s) = RI(s) + Ls\,I(s) + \frac{1}{Cs}I(s)$$

$$= \left[Ls + R + \frac{1}{Cs}\right]I(s)$$

$$= \left[\frac{LCs^2 + RCs + 1}{Cs}\right]I(s)$$

The output voltage is

$$V_o = \frac{1}{C}\int i\,dt$$

$$\therefore V_o(s) = \frac{1}{Cs}I(s)$$

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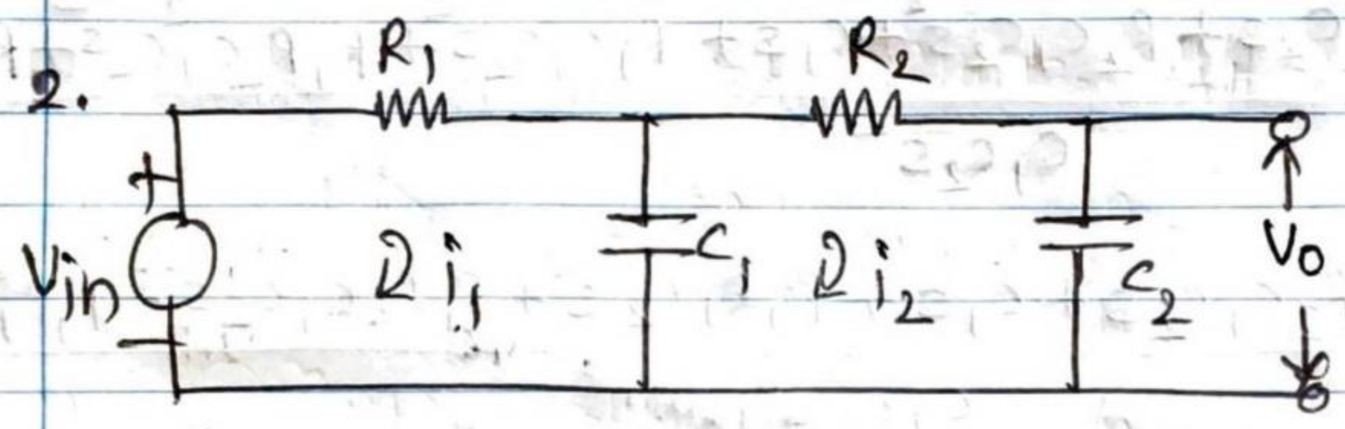
$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{Cs}}{\frac{LCs^2+RCs+1}{Cs}}$$

$$= \frac{1}{LCs^2+RCs+1} = \frac{\frac{1}{LC}}{s^2+\frac{R}{L}s+\frac{1}{LC}}$$

2.



Determine the transfer function $\frac{V_o(s)}{V_{in}(s)}$

<u>soln</u> Applying KVL to loop I we get

$$V_{in} = i_1 R_1 + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$\therefore V_{in}(s) = R_1 I_1(s) + \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right]$$

$$= \left[R_1 + \frac{1}{C_1 s}\right] I_1(s) - \frac{1}{C_1 s} I_2(s)$$

$$= \left[\frac{1+R_1C_1s}{C_1 s}\right] I_1(s) - \frac{1}{C_1 s} I_2(s)$$

Applying KVL to loop II we get

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = R_2 i_2 + \frac{1}{C_2}\int i_2\,dt$$

$$\therefore \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right] = R_2 I_2(s) + \frac{1}{C_2 s} I_2(s)$$

$$\therefore \frac{1}{C_1 s} I_1(s) = I_2(s)\left[\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s}\right]$$

$$\therefore \frac{1}{C_1 s} I_1(s) = \left[\frac{C_2 s + R_2 C_1 C_2 s^2 + C_1 s}{C_1 C_2 s^2}\right] I_2(s)$$

$$\therefore I_1(s) = \left[\frac{C_2 s + R_2 C_1 C_2 s^2 + C_1 s}{C_2 s}\right] I_2(s)$$

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$$\therefore V_{in}(s) = \left[\frac{1+R_1C_1s}{C_1s}\right]\left[\frac{C_2s+R_2C_1C_2s^2+C_1s}{C_2s}\right]I_2(s) - \frac{1}{C_1s}I_2(s)$$

$$= \frac{(1+R_1C_1s)(C_2s+R_2C_1C_2s^2+C_1s)-C_2s}{C_1C_2s^2}\,I_2(s)$$

$$= \left[\frac{C_2s+R_2C_1C_2s^2+C_1s+R_1C_1C_2s^2+R_1R_2C_1^2C_2s^3+R_1C_1^2s^2-C_2s}{C_1C_2s^2}\right]I_2$$

$$= \frac{R_2C_1C_2s^2+C_1s+R_1C_1C_2s^2+R_1R_2C_1^2C_2s^3+R_1C_1^2s^2}{C_1C_2s^2}\,I_2(s)$$

$$= \left[\frac{R_2C_2s+1+R_1C_2s+R_1R_2C_1C_2s^2+R_1C_1s}{C_2s}\right]I_2(s)$$

$$V_o = \frac{1}{C_2}\int i_2\,dt$$

$$\therefore V_o(s) = \frac{1}{C_2s}I_2(s)$$

$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{C_2s}I_2(s)}{\frac{R_1R_2C_1C_2s^2+R_1C_1s+R_1C_2s+R_2C_2s+1}{C_2s}\,I_2(s)}$$

$$= \frac{1}{R_1R_2C_1C_2s^2+R_1C_1s+R_1C_2s+R_2C_2s+1}$$

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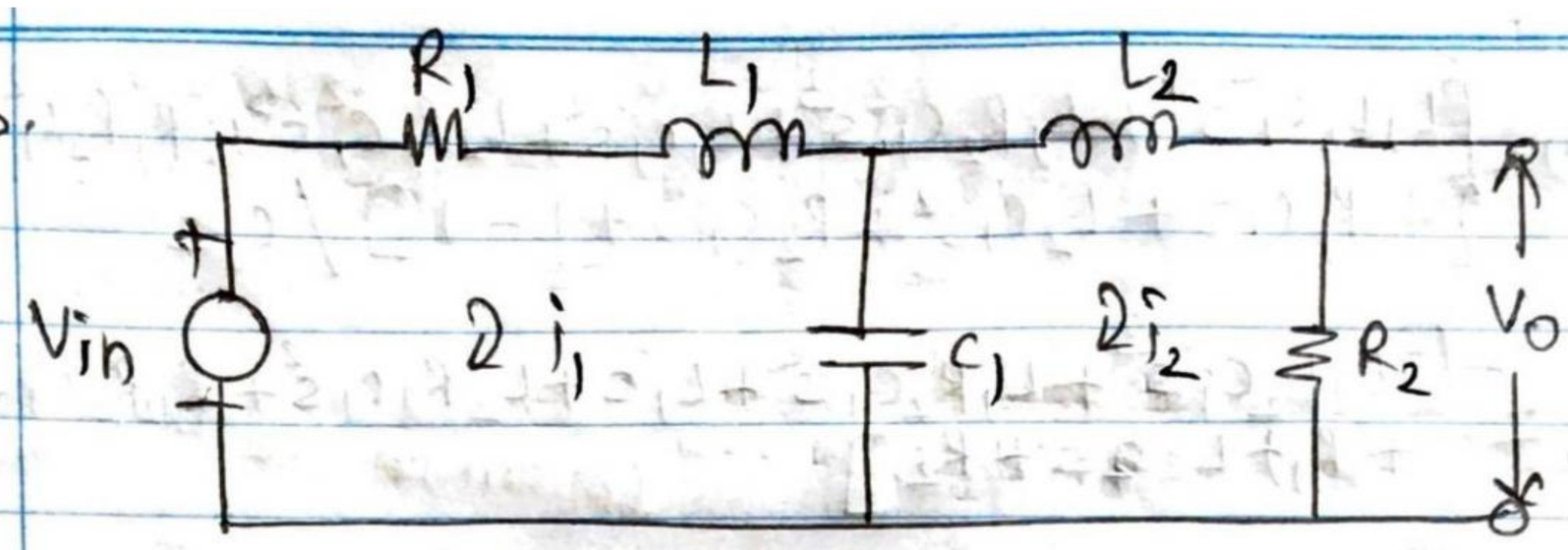
3. 

Determine the T.F. $\frac{V_o(s)}{V_{in}(s)}$

<u>soln</u> Applying KVL to loop I we get

$$V_{in} = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (i_1 - i_2)\,dt$$

$$\therefore V_{in}(s) = R_1 I_1(s) + L_1 s I_1(s) + \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right]$$

$$= \left[R_1 + L_1 s + \frac{1}{C_1 s}\right] I_1(s) - \frac{1}{C_1 s} I_2(s)$$

$$= I_1(s)\left[\frac{L_1 C_1 s^2 + R_1 C_1 s + 1}{C_1 s}\right] - \frac{1}{C_1 s} I_2(s)$$

Applying KVL to loop II we get

$$\frac{1}{C_1}\int (i_1 - i_2)\,dt = L_2 \frac{di_2}{dt} + R_2 i_2$$

$$\therefore \frac{1}{C_1 s}\left[I_1(s) - I_2(s)\right] = L_2 s\, I_2(s) + R_2 I_2(s)$$

$$\therefore \frac{1}{C_1 s} I_1(s) = I_2(s)\left[L_2 s + R_2 + \frac{1}{C_1 s}\right]$$

$$\therefore \frac{1}{C_1 s} I_1(s) = I_2(s)\left[\frac{L_2 C_1 s^2 + R_2 C_1 s + 1}{C_1 s}\right]$$

$$\therefore I_1(s) = \left(L_2 C_1 s^2 + R_2 C_1 s + 1\right) I_2(s)$$

$$\therefore V_{in}(s) = \left[\frac{(L_1 C_1 s^2 + R_1 C_1 s + 1)(L_2 C_1 s^2 + R_2 C_1 s + 1) - 1}{C_1 s}\right] I_2(s)$$

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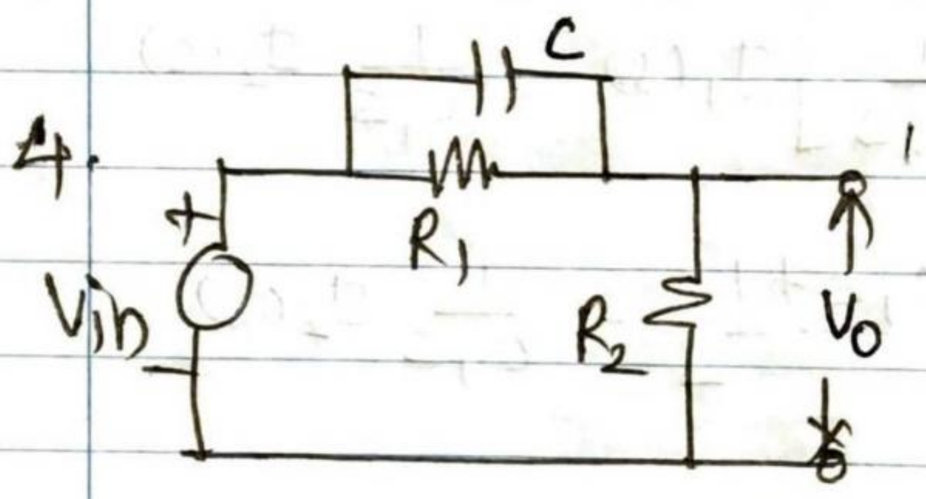
$$\therefore V_{in}(s) = \left\{ \begin{matrix} L_1L_2C_1^2s^4 + L_1R_2C_1^2s^3 + L_1C_1s^2 + L_2R_1C_1^2s^3 + R_1R_2C_1^2s^2 \\ [+ R_1C_1s + L_2C_1s^2 + R_2C_1s + 1 - 1] / C_1s \end{matrix} \right\} I_2(s)$$

$$= [L_1L_2C_1s^3 + L_1R_2C_1s^2 + L_1s + L_2R_1C_1s^2 + R_1R_2C_1s + R_1 + L_2s + R_2]$$

$$V_0 = i_2R_2$$

$$\therefore V_0(s) = I_2(s)\,R_2$$

$$\therefore \frac{V_0(s)}{V_{in}(s)} = \left[\frac{R_2}{L_1L_2C_1s^3 + L_1R_2C_1s^2 + L_2R_1C_1s^2 + R_1R_2C_1s + L_1s + L_2s + R_1 + R_2} \right]$$

4. 

Determine the T.F. $\frac{V_0(s)}{V_{in}(s)}$

<u>soln</u> Applying KCL we get,

$$I_{R_1} + I_{C_1} = I_{R_2}.$$

$$\therefore \frac{V_{in} - V_0}{R_1} + C\frac{d}{dt}(V_{in} - V_0) = \frac{V_0}{R_2}$$

$$\therefore \frac{V_{in}(s)}{R_1} - \frac{V_0(s)}{R_1} + Cs\,V_{in}(s) - Cs\,V_0(s) = \frac{V_0(s)}{R_2}$$

$$\therefore V_{in}(s)\left[\frac{1}{R_1} + Cs\right] = V_0(s)\left[\frac{1}{R_1} + \frac{1}{R_2} + Cs\right]$$

$$\therefore V_{in}(s)\left[\frac{1 + R_1Cs}{R_1}\right] = V_0(s)\left[\frac{R_2 + R_1 + R_1R_2Cs}{R_1R_2}\right]$$

$$\therefore V_{in}(s)\left[1 + R_1Cs\right] = V_0(s)\left[\frac{R_1 + R_2 + R_1R_2Cs}{R_2}\right]$$

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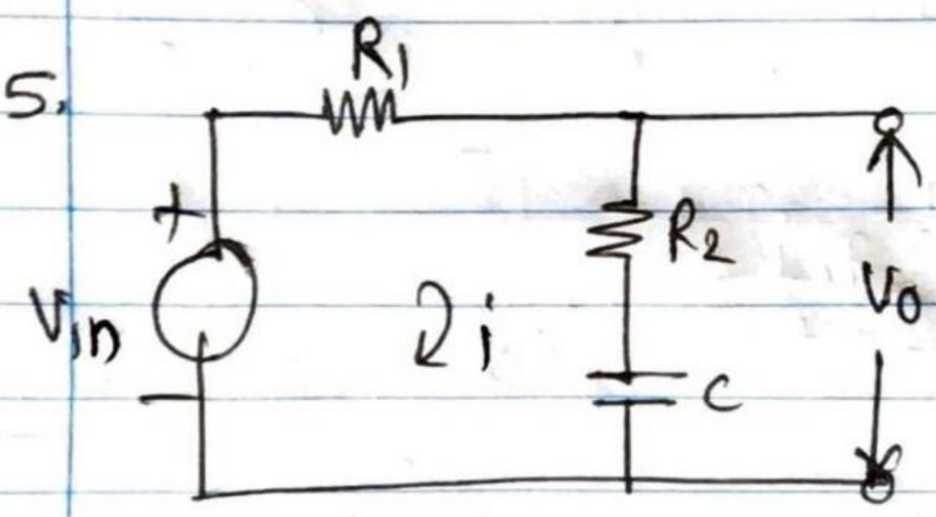
$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{R_1 + R_2 + R_1 R_2 Cs}{R_2(1 + R_1 Cs)}$$

5. 

Determine the T.F. $\frac{V_o(s)}{V_{in}(s)}$

Applying KVL we get,

$$V_{in} = iR_1 + iR_2 + \frac{1}{C}\int i\,dt$$

$$\therefore V_{in}(s) = \left[R_1 + R_2 + \frac{1}{Cs}\right] I(s)$$

$$= \left[\frac{1 + (R_1 + R_2)Cs}{Cs}\right] I(s)$$

$$V_o = iR_2 + \frac{1}{C}\int i\,dt$$

$$V_o(s) = R_2 I(s) + \frac{1}{Cs} I(s)$$

$$V_o(s) = \left[R_2 + \frac{1}{Cs}\right] I(s)$$

$$\therefore V_o(s) = \left[\frac{R_2(Cs+1)}{Cs}\right] I(s)$$

$$\therefore \frac{V_o(s)}{V_{in}(s)} = \frac{\frac{R_2(Cs+1)}{Cs}}{\frac{(R_1+R_2)Cs+1}{Cs}} = \frac{1 + R_2 Cs}{1 + (R_1 + R_2)Cs}$$

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Modeling of Mechanical Systems-

Mechanical systems can be classified as per type of motion as translational systems and rotational systems.

Translational mechanical systems displace linearly upon application of a force. These systems have 3 basic elements Mass, damper and spring. The relation between force and their motion is as under.

Mass-

For mass the force is directly proportional to acceleration.

$$F \propto a$$

$$\therefore F = ma = m\frac{d^2x}{dt^2} = m\ddot{x}$$

Damper-

For damper (friction) the force is directly proportional to velocity.

$$F \propto V$$

$$\therefore F = BV = B\frac{dx}{dt} = B\dot{x}$$

spring-

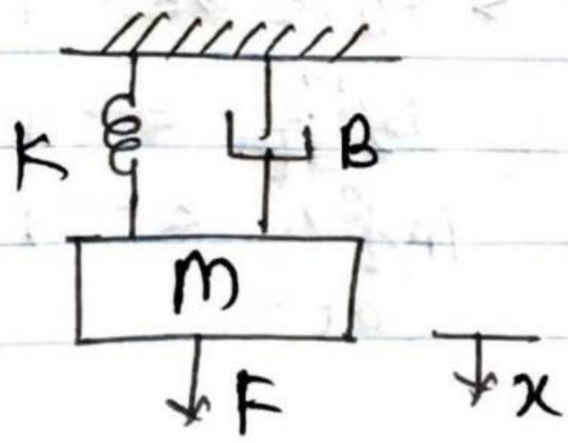
For spring the force is directly proportional to displacement

$$F \propto x$$

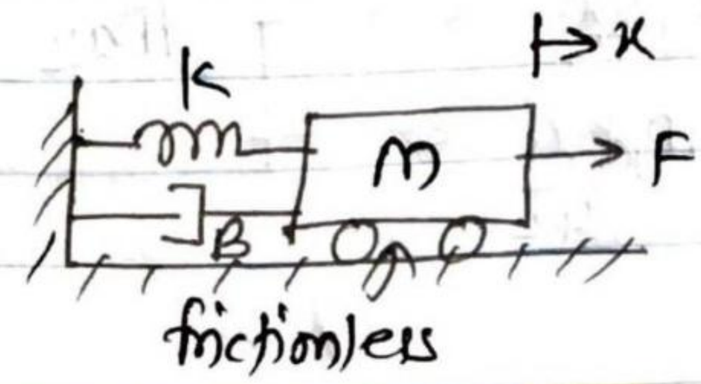
$$F = kx$$

Examples on modeling of translational mechanical systems-

1. Determine the T.F. $\frac{X(s)}{F(s)}$



or



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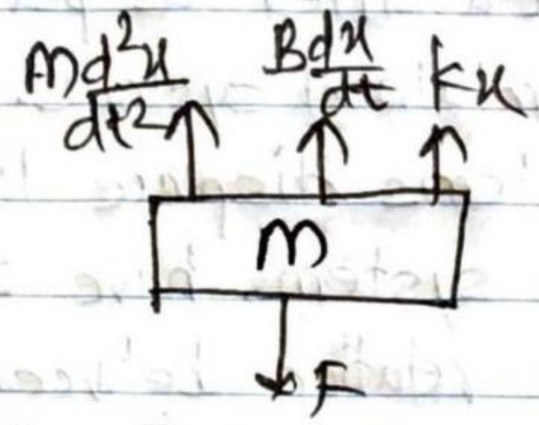
sol^n ~~Applying~~ The free body diagram is as shown below



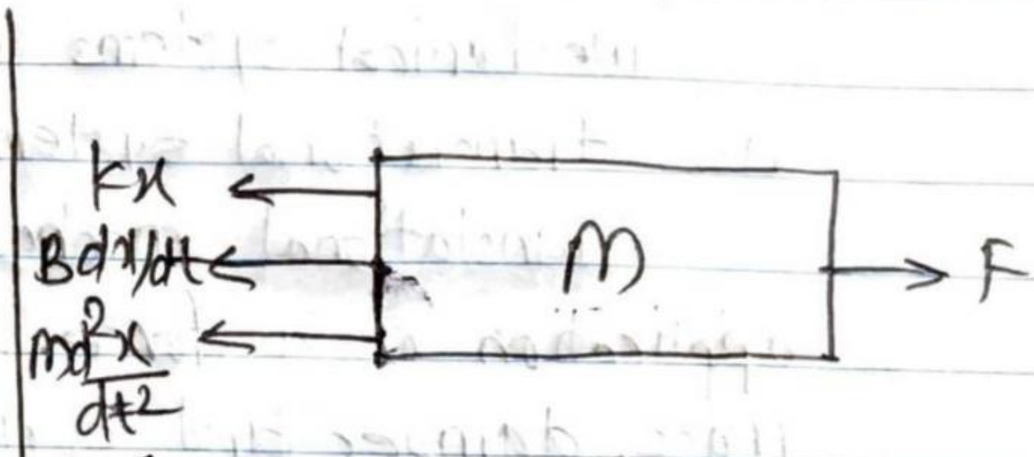
Applying force balance we get

$$F = m\frac{d^2x}{dt^2} + B\frac{dx}{dt} + kx$$

$$\therefore F(s) = ms^2X(s) + BsX(s) + kX(s)$$

$$\therefore F(s) = [ms^2 + Bs + k]X(s)$$

$$\therefore \frac{X(s)}{F(s)} = \frac{1}{ms^2 + Bs + k}$$



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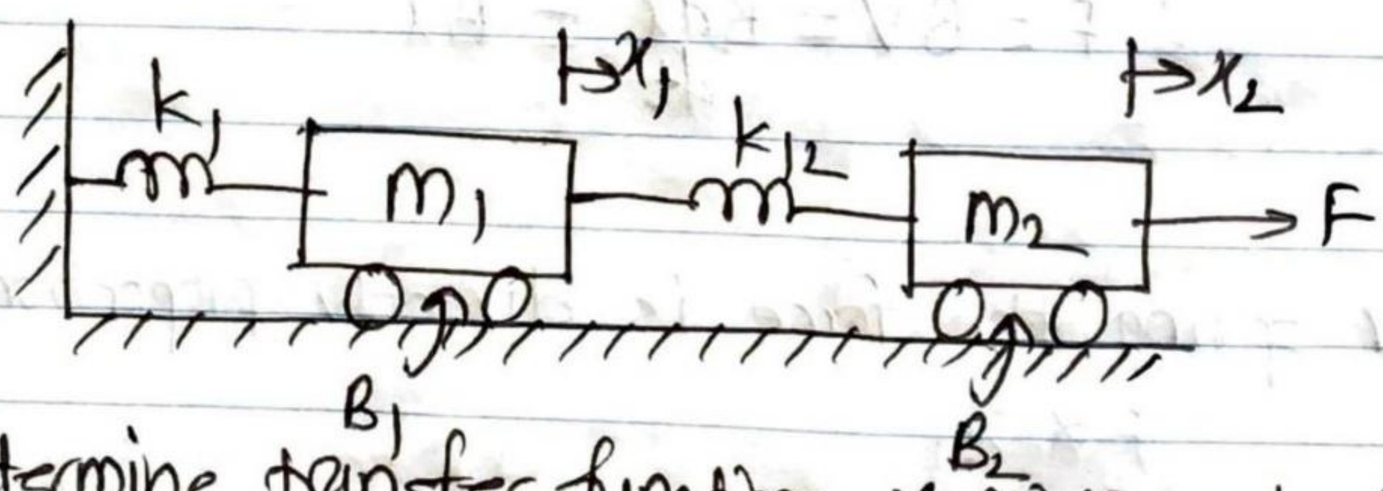
$$F(s) = [ms^2 + Bs + k]X(s)$$

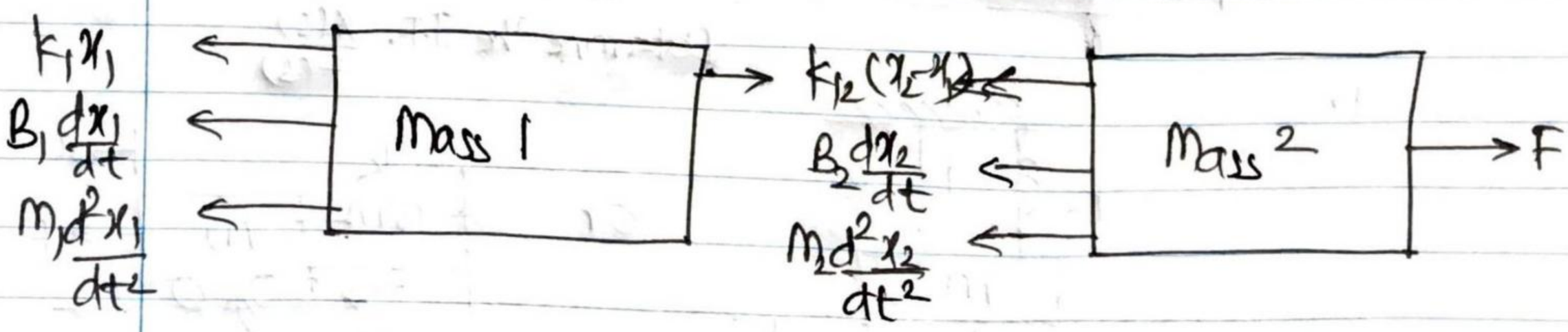
$$\therefore \frac{X(s)}{F(s)} = \frac{1}{ms^2 + Bs + k}$$

2.



Determine transfer functions $X_1(s)/F(s)$ & $X_2(s)/F(s)$

sol^n The free body diagram is as shown below



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Applying force balance to body 2 we get,

$$F = m_2 \frac{d^2 x_2}{dt^2} + B_2 \frac{dx_2}{dt} + k_{12}(x_2 - x_1)$$

$$\therefore F(s) = m_2 s^2 X_2(s) + B_2 s X_2(s) + k_{12}[X_2(s) - X_1(s)]$$

$$= [m_2 s^2 + B_2 s + k_{12}] X_2(s) - k_{12} X_1(s)$$

Applying force balance to body 1 we get

$$k_{12}(x_2 - x_1) = m_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + k_1 x_1$$

$$\therefore k_{12}[X_2(s) - X_1(s)] = m_1 s^2 X_1(s) + B_1 s X_1(s) + k_1 X_1(s)$$

$$\therefore k_{12} X_2(s) = [m_1 s^2 + B_1 s + k_1 + k_{12}] X_1(s)$$

$$\therefore X_2(s) = \frac{m_1 s^2 + B_1 s + k_1 + k_{12}}{k_{12}} X_1(s)$$

$$\& \quad X_1(s) = \frac{k_{12}}{m_1 s^2 + B_1 s + k_1 + k_{12}} X_2(s)$$

$$\therefore F(s) = \left[\frac{(m_2 s^2 + B_2 s + k_{12})(m_1 s^2 + B_1 s + k_1 + k_{12})}{k_{12}} - k_{12}\right] X_1(s)$$

$$= \left[\frac{(m_2 s^2 + B_2 s + k_{12})(m_1 s^2 + B_1 s + k_1 + k_{12}) - k_{12}^2}{k_{12}}\right] X_1(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{k_{12}}{(m_2 s^2 + B_2 s + k_{12})(m_1 s^2 + B_1 s + k_1 + k_{12}) - k_{12}^2}$$

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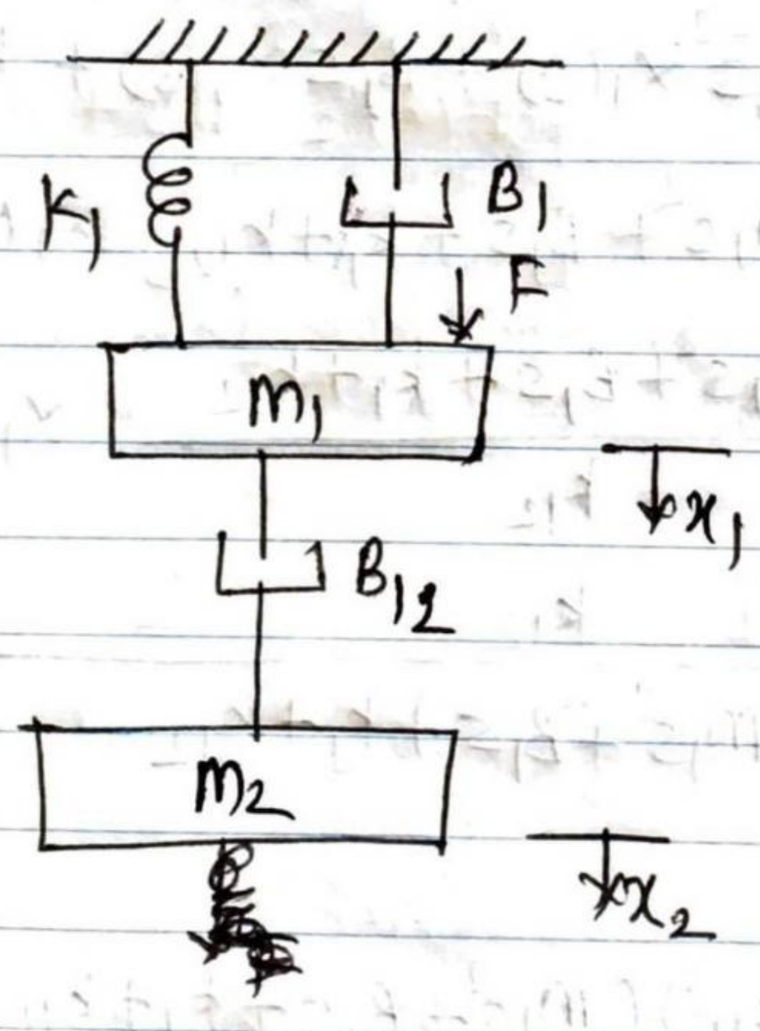
Also,

$$F(s) = (m_2 s^2 + B_2 s + k_{12}) X_2(s) - k_{12}\left[\frac{k_{12}}{m_1 s^2 + B_1 s + k_1 + k_{12}}\right] X_2(s)$$

$$= \left[\frac{(m_2 s^2 + B_2 s + k_{12})(m_1 s^2 + B_1 s + k_1 + k_{12}) - k_{12}^2}{m_1 s^2 + B_1 s + k_1 + k_{12}}\right] X_2(s)$$

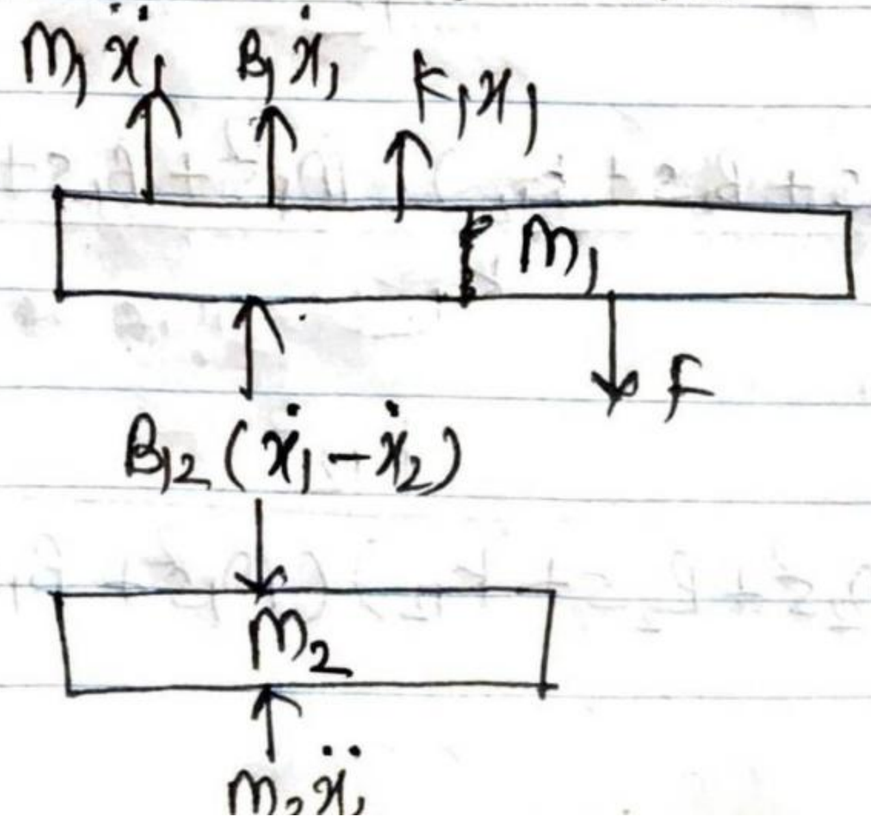
$$\therefore \frac{X_2(s)}{F(s)} = \frac{m_1 s^2 + B_1 s + k_1 + k_{12}}{(m_2 s^2 + B_2 s + k_{12})(m_1 s^2 + B_1 s + k_1 + k_{12}) - k_{12}^2}$$

3.



Determine transfer functions $\frac{X_1(s)}{F(s)}$ & $\frac{X_2(s)}{F(s)}$

Soln The F.B.D. is as shown below



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Applying force balance to body 1 we get,

$$F = m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1 + B_{12}(\dot{x}_1 - \dot{x}_2)$$

$$\therefore F(s) = m_1s^2X_1(s) + B_1sX_1(s) + k_1X_1(s) + B_{12}s[X_1(s) - X_2(s)]$$

$$\therefore F(s) = [m_1s^2 + B_1s + B_{12}s + k_1]X_1(s) - B_{12}sX_2(s)$$

Applying force balance to body 2 we get,

$$B_{12}(\dot{x}_1 - \dot{x}_2) = m_2\ddot{x}_2$$

$$\therefore B_{12}s[X_1(s) - X_2(s)] = m_2s^2X_2(s)$$

$$\therefore B_{12}sX_1(s) = [m_2s^2 + B_{12}s]X_2(s)$$

$$\therefore X_1(s) = \left[\frac{m_2s^2 + B_{12}s}{B_{12}s}\right]X_2(s)$$

$$\& \quad X_2(s) = \left[\frac{B_{12}s}{m_2s^2 + B_{12}s}\right]X_1(s)$$

$$\therefore F(s) = [m_1s^2 + B_1s + B_{12}s + k_1]X_1(s) - B_{12}s\left[\frac{B_{12}s}{m_2s^2 + B_{12}s}\right]X_1(s)$$

$$= \left[\frac{(m_1s^2 + B_1s + B_{12}s + k_1)(m_2s^2 + B_{12}s) - B_{12}^2s^2}{m_2s^2 + B_{12}s}\right]X_1(s)$$

$$\therefore \frac{X_1(s)}{F(s)} = \frac{m_2s^2 + B_{12}s}{(m_1s^2 + B_1s + B_{12}s + k_1)(m_2s^2 + B_{12}s) - B_{12}^2s^2}$$

Also,

$$F(s) = \left[\frac{(m_1s^2 + B_1s + B_{12}s + k_1)(m_2s^2 + B_{12}s) - B_{12}s}{B_{12}s}\right]X_2(s)$$

$$\therefore \frac{X_2(s)}{F(s)} = \frac{B_{12}s}{(m_1s^2 + B_1s + B_{12}s + k_1)(m_2s^2 + B_{12}s) - B_{12}^2s^2}$$

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Modeling of rotational mechanical systems-

The rotational mechanical systems rotate upon application of torque. The three components involved are inertia, friction and shaft stiffness. The relation between torque and their motion is given as,

Inertia (J) - The torque is directly proportional to angular acceleration (α)

$$T \propto \alpha$$

$$\therefore T = J\alpha = J\frac{d^2\theta}{dt^2} = J\ddot{\theta}$$

Friction (f/B) - The torque is directly proportional to angular velocity (ω)

$$T \propto \omega$$

$$T = B\omega = B\frac{d\theta}{dt} = B\dot{\theta}$$ ~~Also denoted as~~

It is also denoted as,

$$T = f\omega = f\frac{d\theta}{dt} = f\dot{\theta}$$

Shaft stiffness (K) - The torque is directly proportional to angular displacement θ

$$T \propto \theta$$

$$T = k\theta$$

All these elements oppose the motion due to active torque T.

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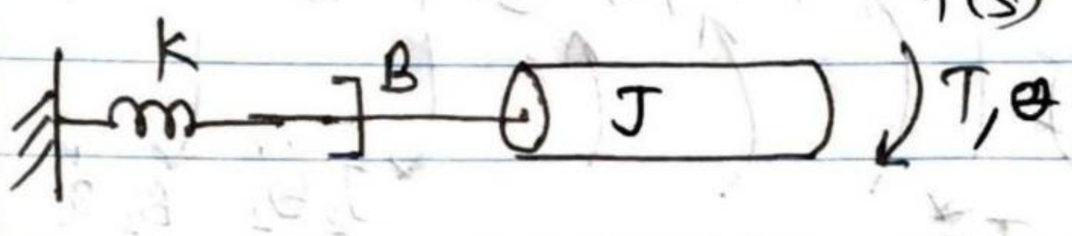
Examples on modeling of rotational mechanical systems -

1. Determine the T.F. $\frac{\theta(s)}{T(s)}$



Sol^n Applying torque balance equations we get,

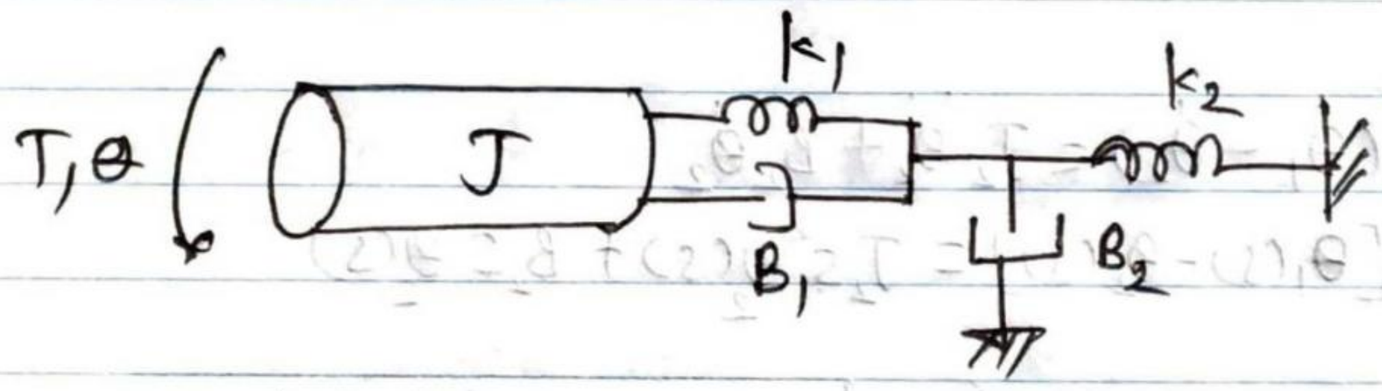
$$T = J\ddot{\theta} + B\dot{\theta} + K\theta$$

$$\therefore T(s) = Js^2\theta(s) + Bs\theta(s) + K\theta(s)$$

$$= [Js^2 + Bs + K]\theta(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + Bs + K}$$

2. Determine the T.F. $\frac{\theta(s)}{T(s)}$



Sol^n Applying torque balance equation we get,

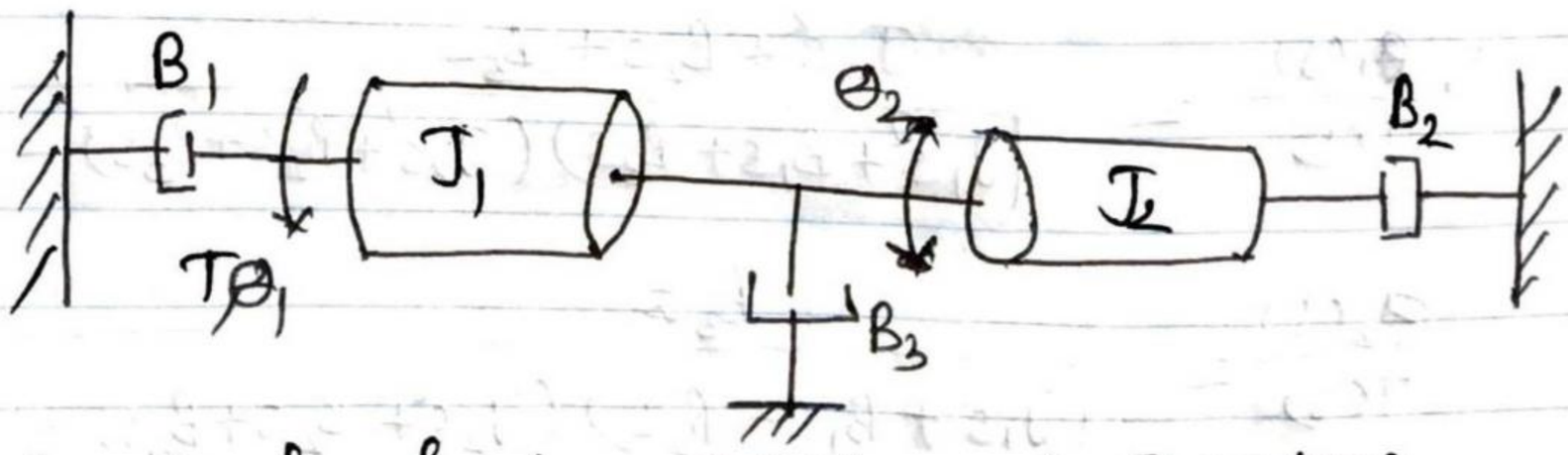
$$T = J\ddot{\theta} + B_1\dot{\theta} + B_2\dot{\theta} + K_1\theta + K_2\theta$$

$$\therefore T(s) = Js^2\theta(s) + B_1 s\theta(s) + B_2 s\theta(s) + K_1\theta(s) + K_2\theta(s)$$

$$= [Js^2 + B_1 s + B_2 s + K_1 + K_2]\theta(s)$$

$$\therefore \frac{\theta(s)}{T(s)} = \frac{1}{Js^2 + B_1 s + B_2 s + K_1 + K_2}$$

3.



Determine transfer functions $\theta_1(s)/T(s)$ & $\theta_2(s)/T(s)$

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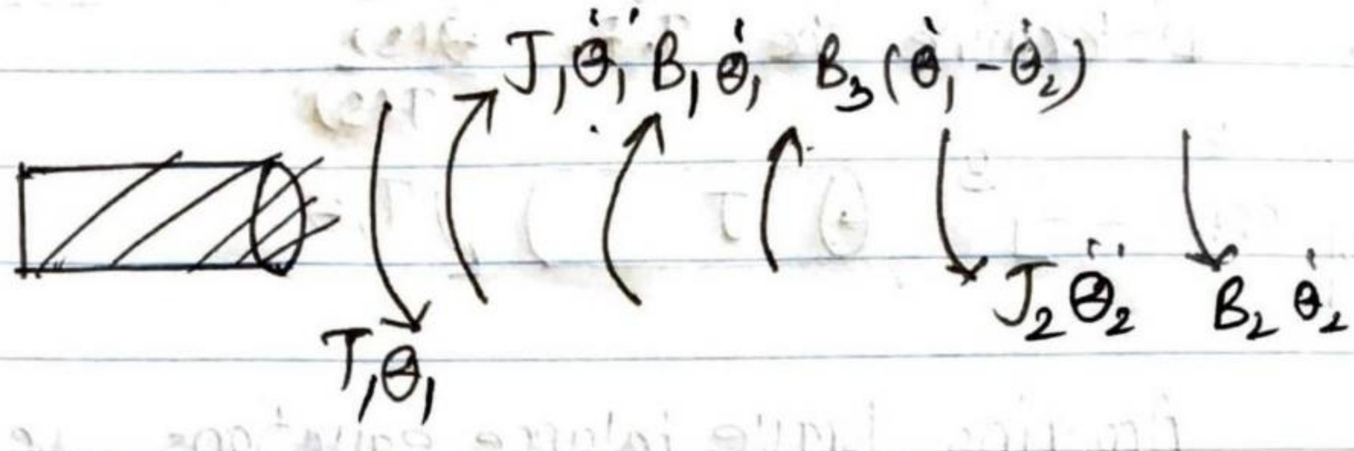
Sol^n The F.B.D. is as shown below



Applying torque balance we get,

$$T = J_1\ddot{\theta}_1 + B_1\dot{\theta}_1 + B_3(\dot{\theta}_1 - \dot{\theta}_2)$$

$$\therefore T(s) = J_1 s^2\theta_1(s) + B_1 s\,\theta_1(s) + B_3 s\,\theta_1(s) - B_3 s\,\theta_2(s)$$

$$= (J_1 s^2 + B_1 s + B_3 s)\,\theta_1(s) - B_3 s\,\theta_2(s)$$

Also,

$$B_3(\dot{\theta}_1 - \dot{\theta}_2) = J_2\ddot{\theta}_2 + B_2\dot{\theta}_2$$

$$\therefore B_3 s[\theta_1(s) - \theta_2(s)] = J_2 s^2\theta_2(s) + B_2 s\,\theta_2(s)$$

$$\therefore B_3 s\,\theta_1(s) = [J_2 s^2 + B_2 s + B_3 s]\,\theta_2(s)$$

$$\therefore \theta_1(s) = \left[\frac{J_2 s^2 + B_2 s + B_3 s}{B_3 s}\right]\theta_2(s) \quad \Bigg| \quad \theta_2(s) = \frac{B_3 s\,\theta_1(s)}{J_2 s^2 + B_2 s + B_3 s}$$

$$\therefore T(s) = [J_1 s^2 + B_1 s + B_3 s]\,\theta_1(s) - B_3(s)\left[\frac{B_3 s}{J_2 s^2 + B_2 s + B_3 s}\right]\theta_1(s)$$

$$= \left[\frac{(J_1 s^2 + B_1 s + B_3 s)(J_2 s^2 + B_2 s + B_3 s) - B_3^2 s^2}{J_2 s^2 + B_2 s + B_3 s}\right]\theta_1(s)$$

$$\therefore \frac{\theta_1(s)}{T(s)} = \frac{J_2 s^2 + B_2 s + B_3 s}{(J_1 s^2 + B_1 s + B_3 s)(J_2 s^2 + B_2 s + B_3 s) - (B_3 s)^2}$$

$$\frac{\theta_2(s)}{T(s)} = \frac{B_3 s}{(J_1 s^2 + B_1 s + B_2 s)(J_2 s^2 + B_2 s + B_3 s) - (B_3 s)^2}$$

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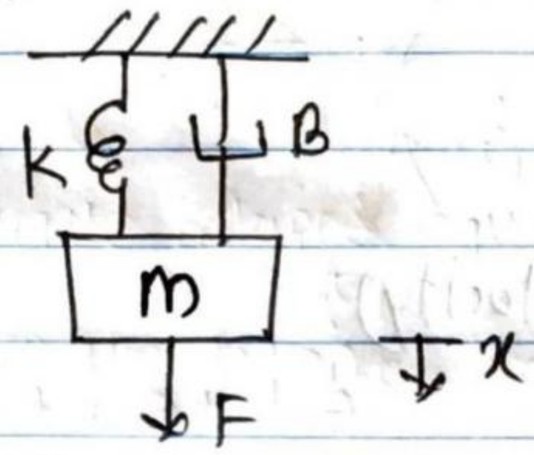
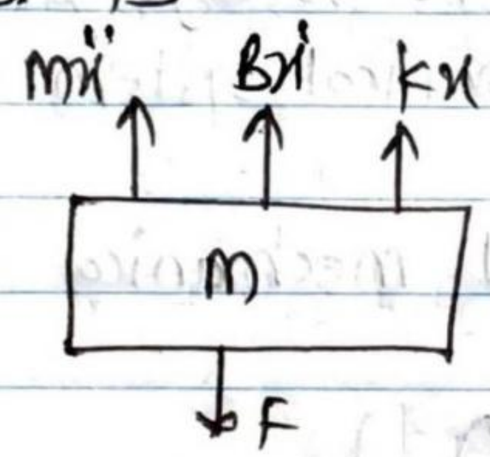
Force to voltage analogy –

Consider a second order translational mechanical system as shown below.

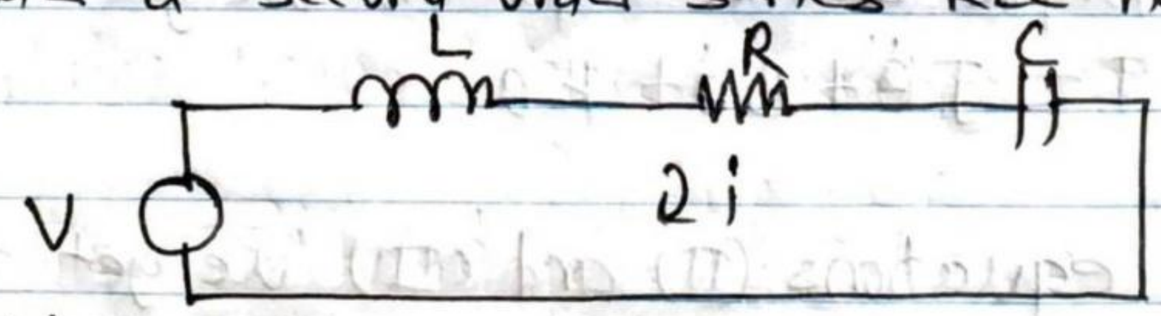


Its F.B.D. is



$$\therefore \; F = m\ddot{x} + B\dot{x} + kx \qquad (I)$$

consider a second order series RLC network as shown below



Applying KVL we get

$$V = L\frac{di}{dt} + Ri + \frac{1}{C}\int i\,dt$$

we have,

$$i = \frac{dq}{dt}$$

$$\therefore \; V = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q \qquad (II)$$

Comparing equations (I) and (II) we get F–V analogy as,

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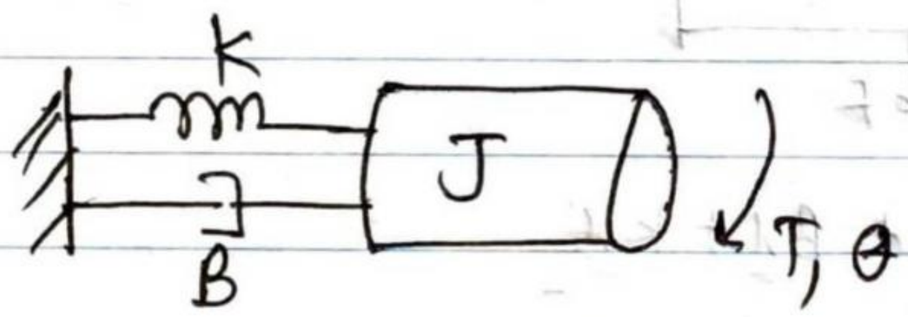
Mechanical system	Electrical system
F	V
M	L
B	R
K	1/C
x	q
u (velocity)	i (current)

similary ~~force~~ torque to voltage analogy can be developed between rotational mechanical system and electrical system.

consider a rotational mechanical system shown below



Applying torque balance equation we get

$$T = J\ddot{\theta} + B\dot{\theta} + K\theta \qquad (III)$$

comparing equations (II) and (III) we get torque to voltage analogy as

Mechanical system	Electrical system
T	V
J	L
B	R
K	1/C
θ	q
ω	i

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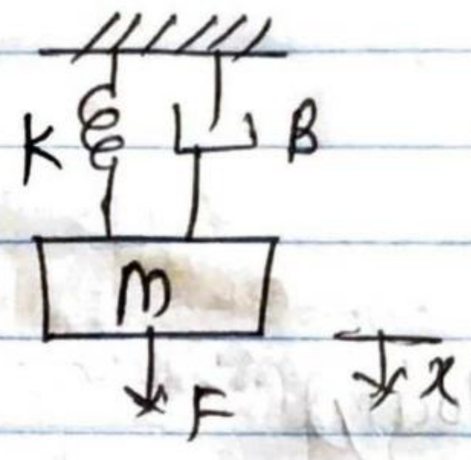
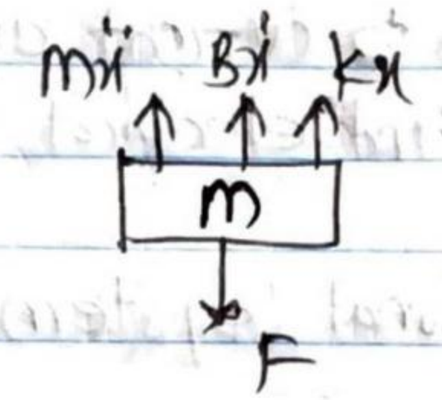
Force to current analogy-

consider a second order mechanical system as

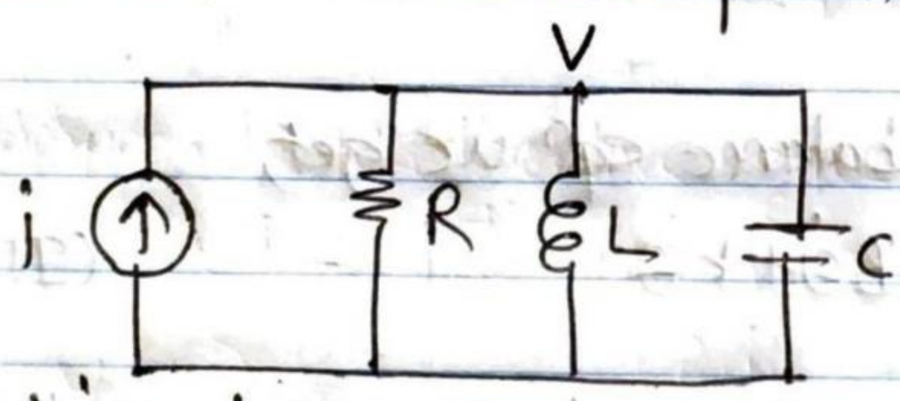


The F.B.D. is as shown below



$$\therefore \; F = m\ddot{x} + B\dot{x} + Kx \qquad \text{— (I)}$$

consider a second order parallel RLC network as



Applying KCL we get

$$i = C\frac{dv}{dt} + \frac{v}{R} + \frac{1}{L}\int v\,dt$$

we have, $v = \frac{d\phi}{dt}$

$$\therefore \; i = C\frac{d^2\phi}{dt^2} + \frac{1}{R}\frac{d\phi}{dt} + \frac{1}{L}\phi \qquad \text{— (II)}$$

comparing equations (I) & (II) we get force to current analogy between translational mechanical and electrical system as

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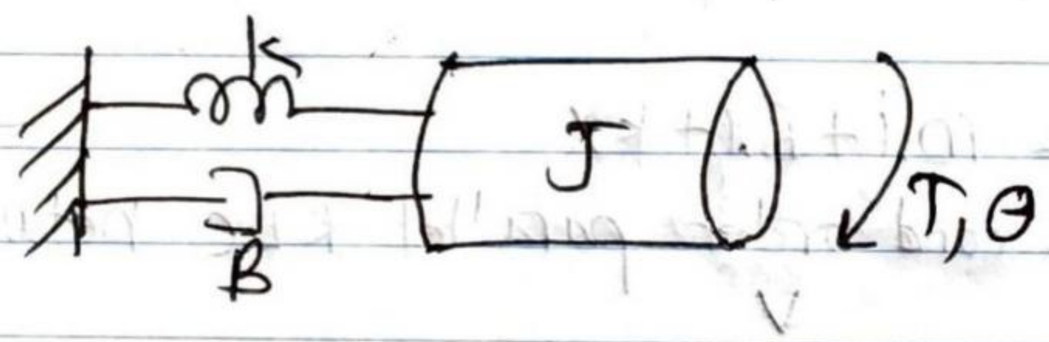
Mechanical system	Electrical system
F	i
m	C
B	1/R
k	1/L
x	ϕ
v (velocity)	V

similarly the torque to current analogy between rotational mechanical system and electrical system can be obtained.

consider a rotational system shown below.



Applying torque balance eqn we get,

$$T = J\ddot{\theta} + B\dot{\theta} + k\theta \qquad \text{(III)}$$

comparing equations (II) and (III) we get the torque to current analogy as,

Mechanical system	Electrical system
T	i
m	C
B	1/R
k	1/L
θ	ϕ
ω	V

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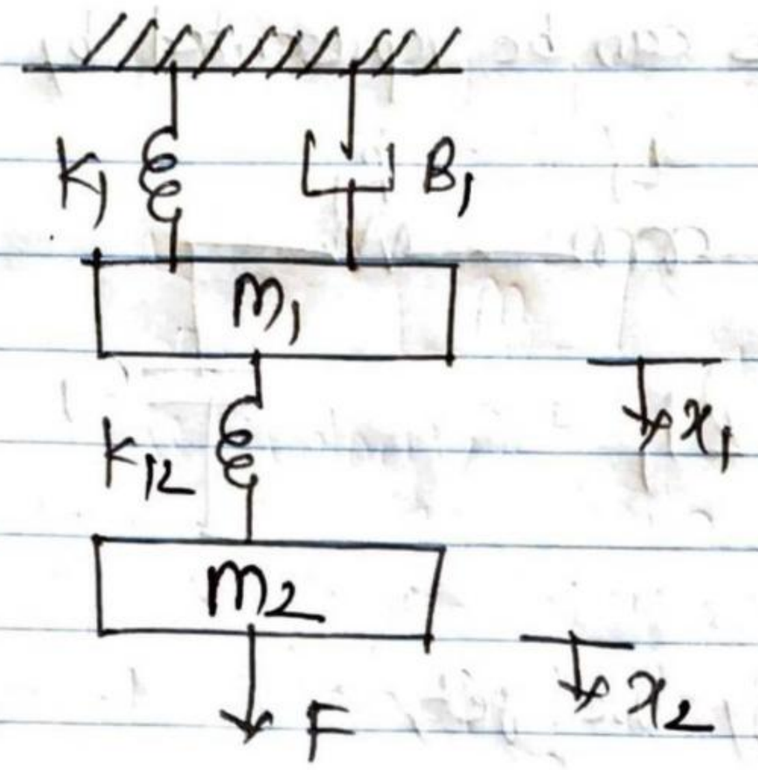
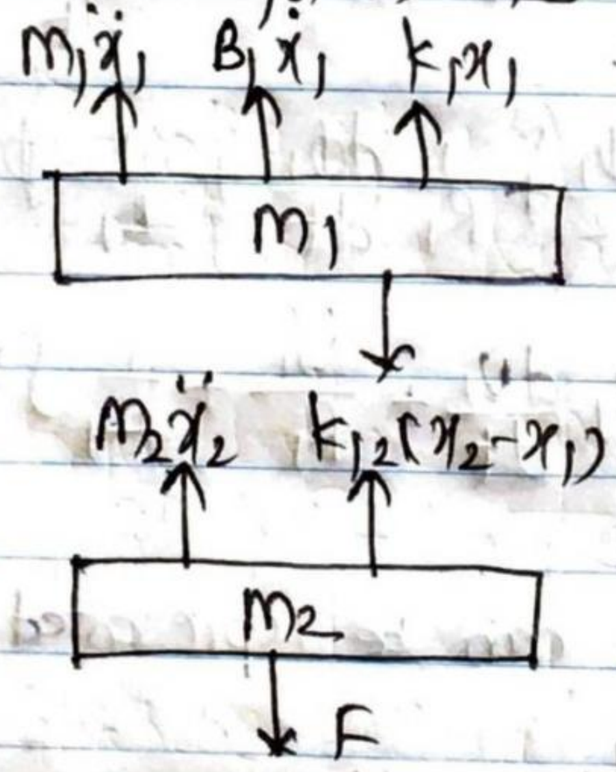
Examples on F-V and F-I analogy-

1. 

Obtain F-V and F-I analogous differential equations & networks.

Sol^n F.B.D. of the system is



Applying force balance we get,

$$F = m_2\ddot{x}_2 + k_{12}(x_2-x_1)$$

$$k_{12}(x_2-x_1) = m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x_1$$

Applying F-V analogy we get

$$V = L_2\ddot{q}_2 + \frac{1}{C_{12}}(q_2-q_1) = L_2\frac{di_2}{dt} + \frac{1}{C_{12}}\int(i_2-i_1)\,dt$$

$$\frac{1}{C_{12}}(q_2-q_1) = L_1\ddot{q}_1 + R_1\dot{q}_1 + \frac{1}{C_1}q_1$$

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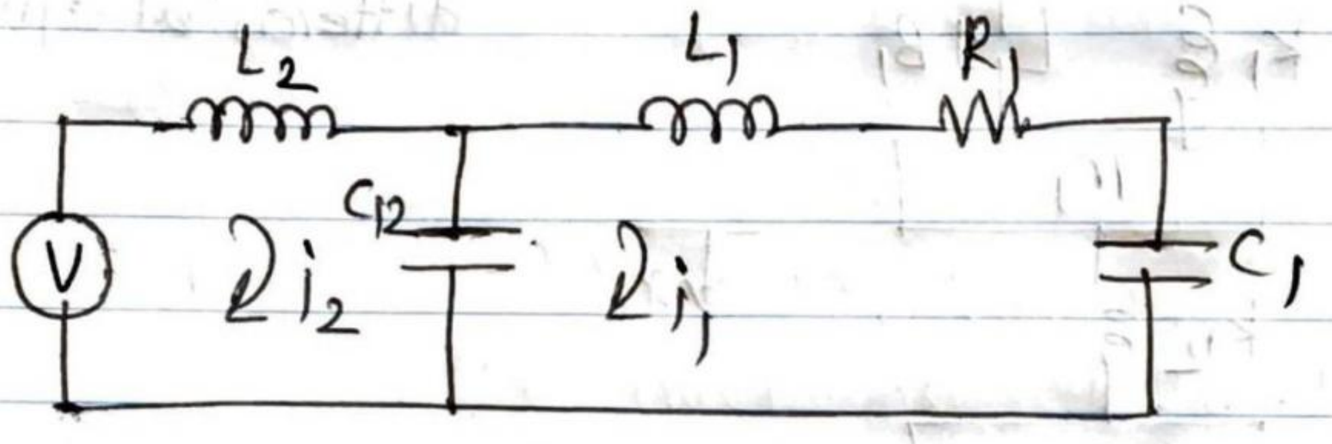
$$\therefore \frac{1}{C_{12}} \int (i_2 - i_1)\,dt = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1\,dt$$

The above equations can be represented by the circuit



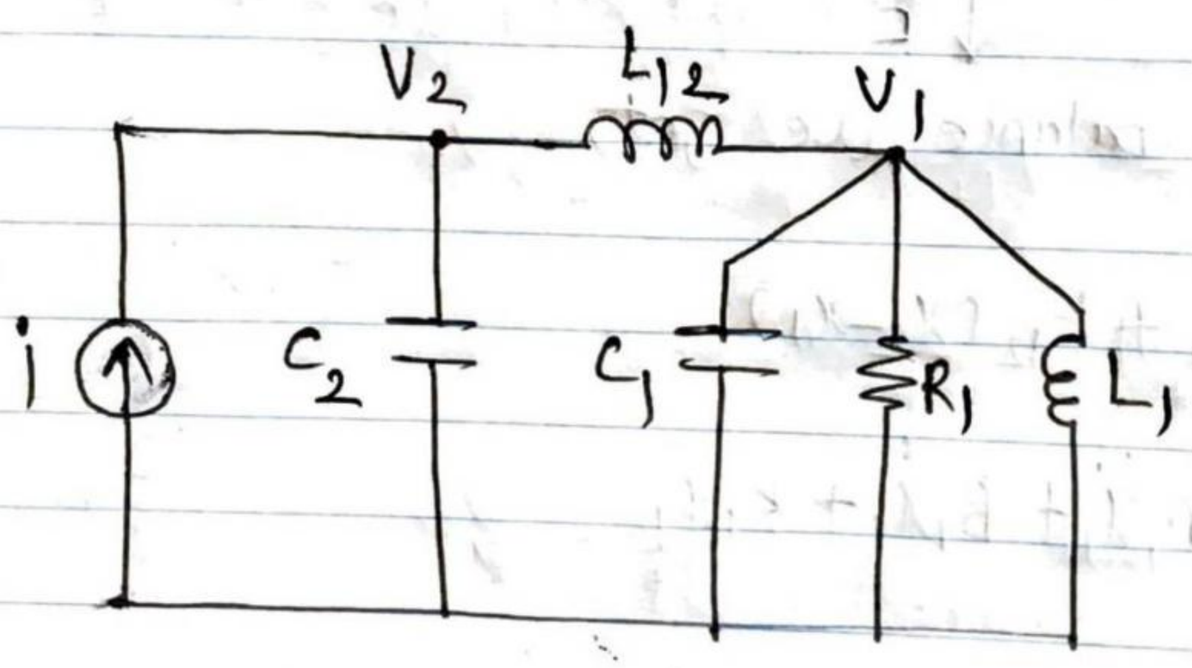
Applying F-I analogy we get,

$$i = C_2 \frac{d^2\phi_2}{dt^2} + \frac{1}{L_{12}}(\phi_2 - \phi_1) = C_2 \frac{dv_2}{dt} + \frac{1}{L_{12}} \int (v_2 - v_1)\,dt$$

$$\frac{1}{L_{12}}(\phi_2 - \phi_1) = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{R_1}\frac{d\phi_1}{dt} + \frac{1}{L_1}\phi_1$$

$$\therefore \frac{1}{L_{12}} \int (v_2 - v_1)\,dt = C_1 \frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1} \int v_1\,dt$$

The above equations can be represented by the circuit



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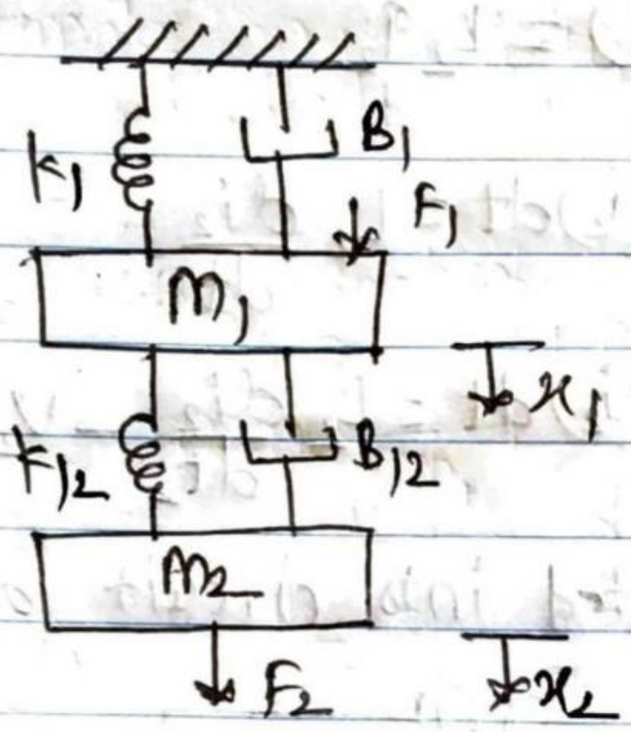
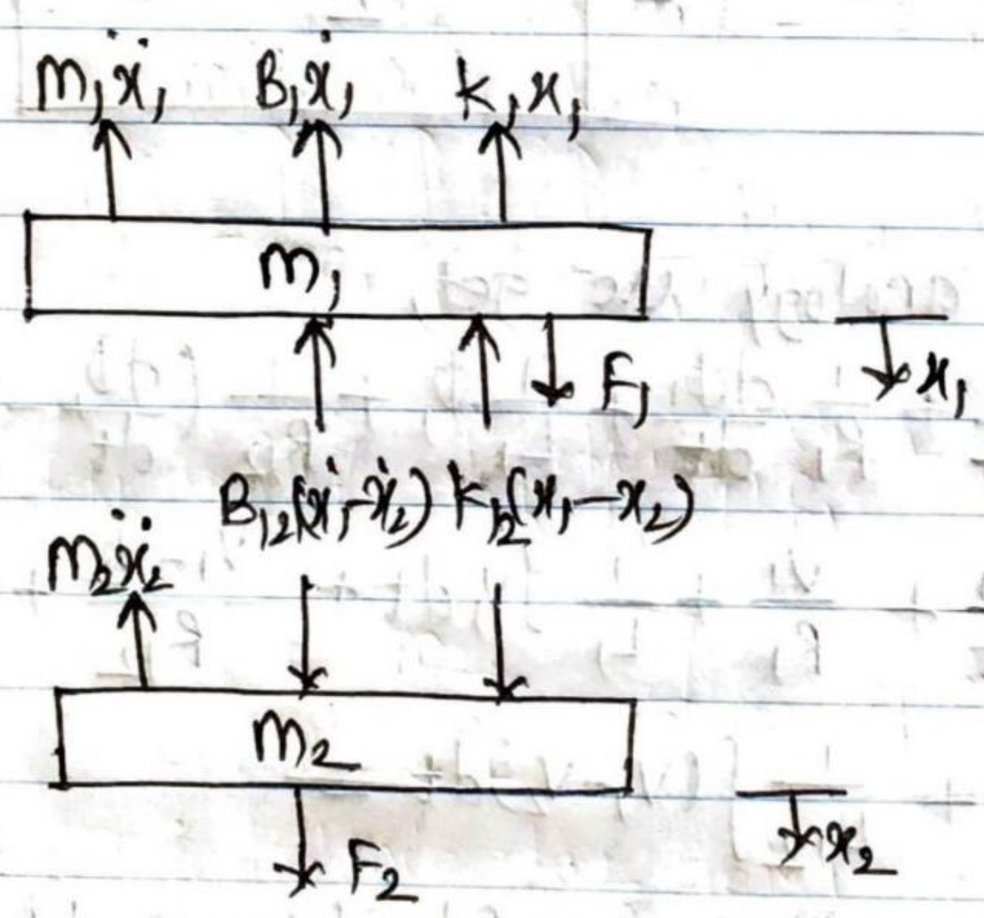
2. 

obtain F-V & F-I analogous differential equations and networks.

<u>soln</u> F.B.D. of system is



Applying force balance we get,

$$F_1 = m_1\ddot{x}_1 + B_1\dot{x}_1 + k_1x + B_{12}(\dot{x}_1-\dot{x}_2) + k_{12}(x_1-x_2)$$

&

$$F_2 + B_{12}(\dot{x}_1-\dot{x}_2) + k_{12}(x_1-x_2) = m_2\ddot{x}_2$$

Applying F-V analogy we get,

$$V_1 = L_1\ddot{q}_1 + R_1\dot{q}_1 + \frac{1}{C_1}q_1 + R_{12}(\dot{q}_1-\dot{q}_2) + \frac{1}{C_{12}}(q_1-q_2)$$

$$= L_1\frac{di_1}{dt} + R_1i_1 + \frac{1}{C_1}\int i_1\,dt + R_{12}(i_1-i_2) + \frac{1}{C_{12}}\int(i_1-i_2)\,dt$$

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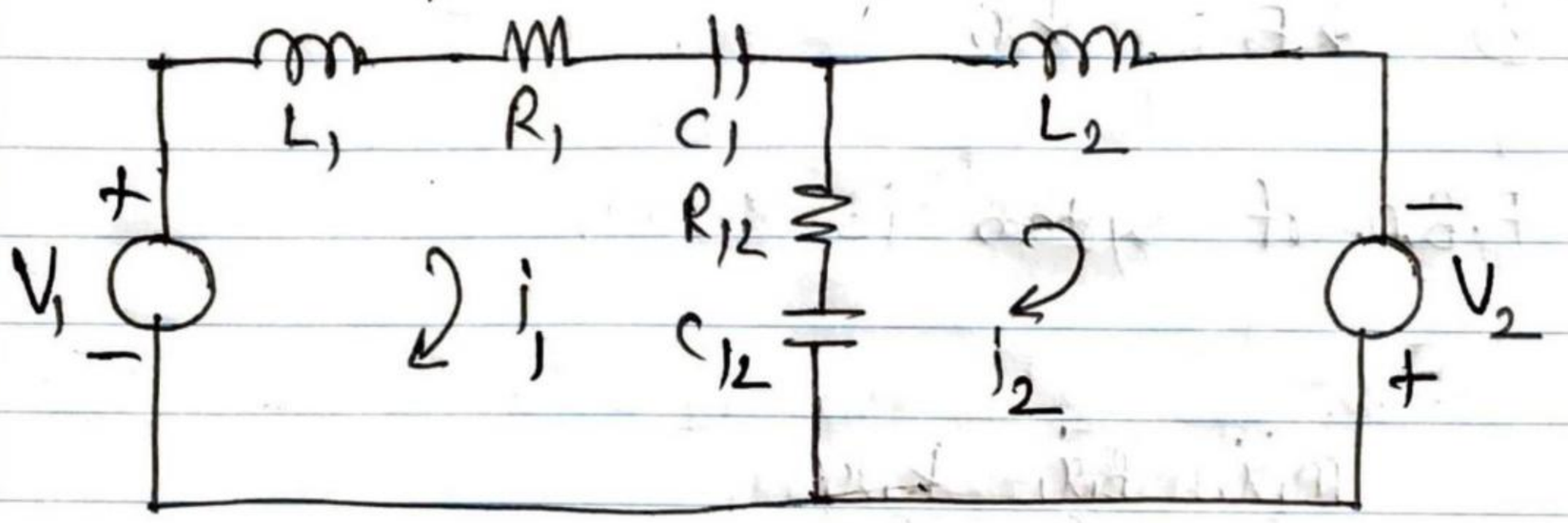
$$V_2 + R_{12}(\dot{q}_1 - \dot{q}_2) + \frac{1}{C_{12}}(q_1 - q_2) = L_2 \ddot{q}_2$$

$$\therefore V_2 + R_2(i_1 - i_2) + \frac{1}{C_{12}}\int (i_1 - i_2)\,dt = L_2 \frac{di_2}{dt}$$

$$\therefore R_2(i_1 - i_2) + \frac{1}{C_{12}}\int (i_1 - i_2)\,dt = L_2 \frac{di_2}{dt} - V_2$$

The above eq^{n}s can be converted into circuit as,



Applying F-I analogy we get,

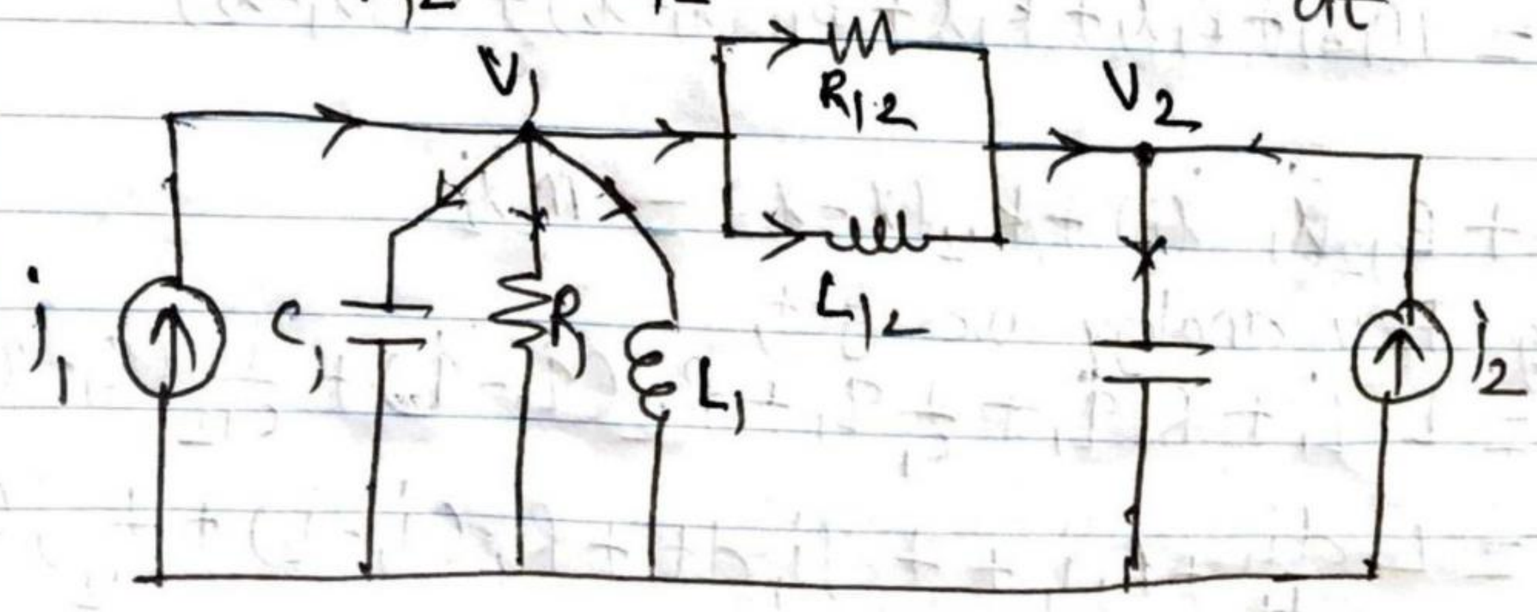
$$i_1 = C_1 \frac{d^2\phi_1}{dt^2} + \frac{1}{R_1}\frac{d\phi_1}{dt} + \frac{1}{L_1}\phi_1 + \frac{1}{R_{12}}\left(\frac{d\phi_1}{dt} - \frac{d\phi_2}{dt}\right) + \frac{1}{L_{12}}(\phi_1 - \phi_2)$$

$$= C_1 \frac{dv_1}{dt} + \frac{v_1}{R_1} + \frac{1}{L_1}\int v_1\,dt + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt$$

$$\cancel{\therefore i_2 + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt =}$$

$$i_2 + \frac{1}{R_{12}}\left(\frac{d\phi_1}{dt} - \frac{d\phi_2}{dt}\right) + \frac{1}{L_{12}}(\phi_1 - \phi_2) = C_2 \frac{d^2\phi_2}{dt^2}$$

$$\therefore i_2 + \frac{v_1 - v_2}{R_{12}} + \frac{1}{L_{12}}\int (v_1 - v_2)\,dt = C_2 \frac{dv_2}{dt}$$



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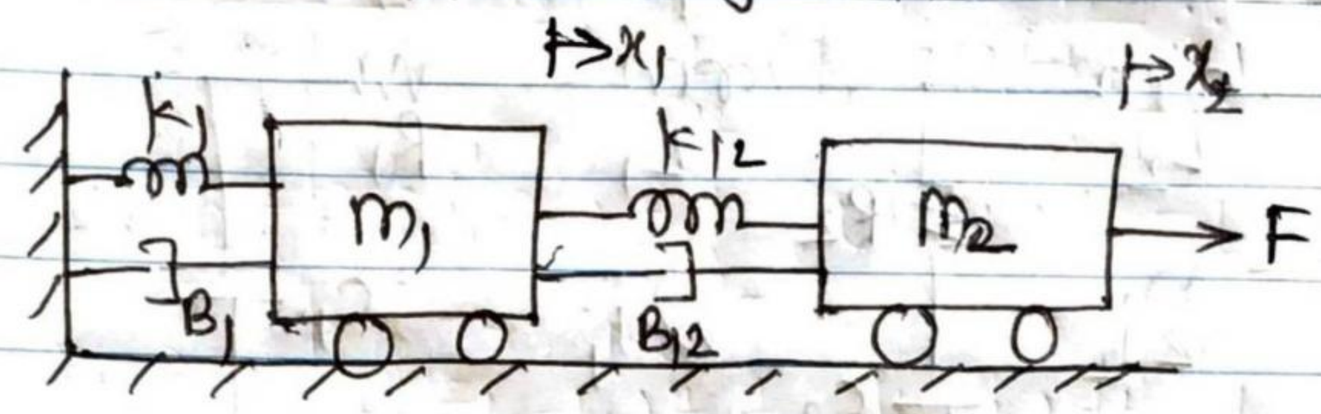
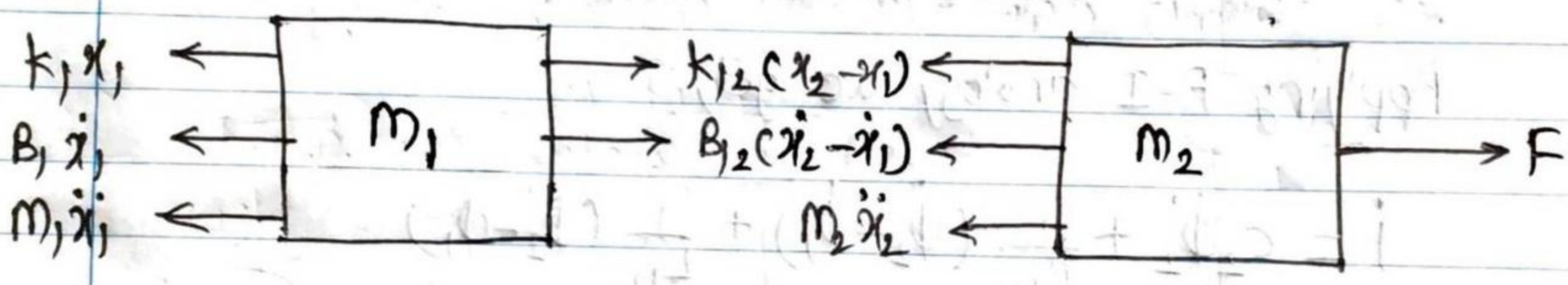
3. obtain F-V and F-I analogous ckts.



Soln

F.B.D of the system is



Applying force balance we get,

$$F = m_2 \ddot{x}_2 + B_{12}(\dot{x}_2 - \dot{x}_1) + k_{12}(x_2 - x_1)$$

&

$$B_{12}(\dot{x}_2 - \dot{x}_1) + k_{12}(x_2 - x_1) = m_1 \ddot{x}_1 + B_1 \dot{x}_1 + k_1 x_1$$

Applying F-V analogy we get,

$$V = L_2 \ddot{q}_2 + R_{12}(\dot{q}_2 - \dot{q}_1) + \frac{1}{C_{12}}(q_2 - q_1)$$

$$= L_2 \frac{di_2}{dt} + R_{12}(i_2 - i_1) + \frac{1}{C_{12}} \int (i_2 - i_1)\, dt$$

&

$$\cancel{R_{12}(i_2 - i_1) + \frac{1}{C_{12}} \int (i_2 - i_1) = L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{1}{C_1} q_1}$$

$$\therefore R_{12}(\dot{q}_2 - \dot{q}_1) + \frac{1}{C_{12}}(q_2 - q_1) = L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{1}{C_1} q_1$$

$$\therefore R_{12}(i_2 - i_1) + \frac{1}{C_{12}} \int (i_2 - i_1)\, dt = L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1\, dt$$

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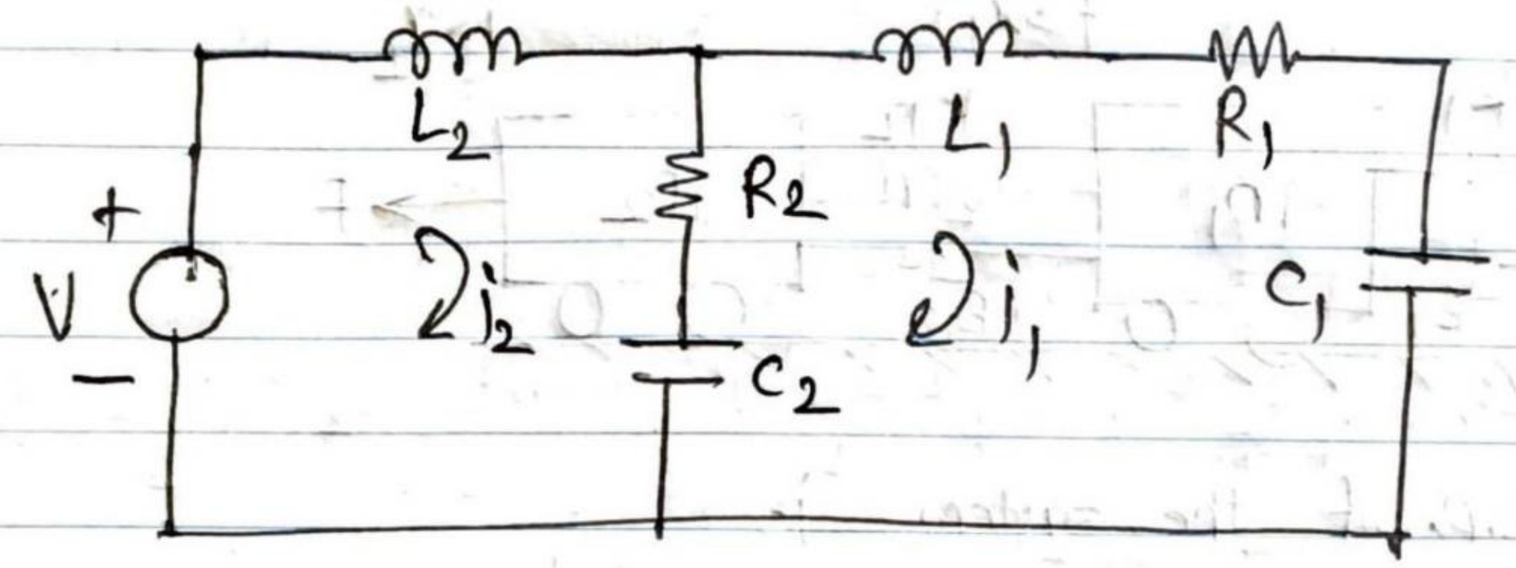
The above equations can be represented by a circuit.



Applying F-I analogy we get,

$$i = C_2\ddot{\phi}_2 + \frac{1}{R_{12}}(\dot{\phi}_2 - \dot{\phi}_1) + \frac{1}{L_{12}}(\phi_2 - \phi_1)$$

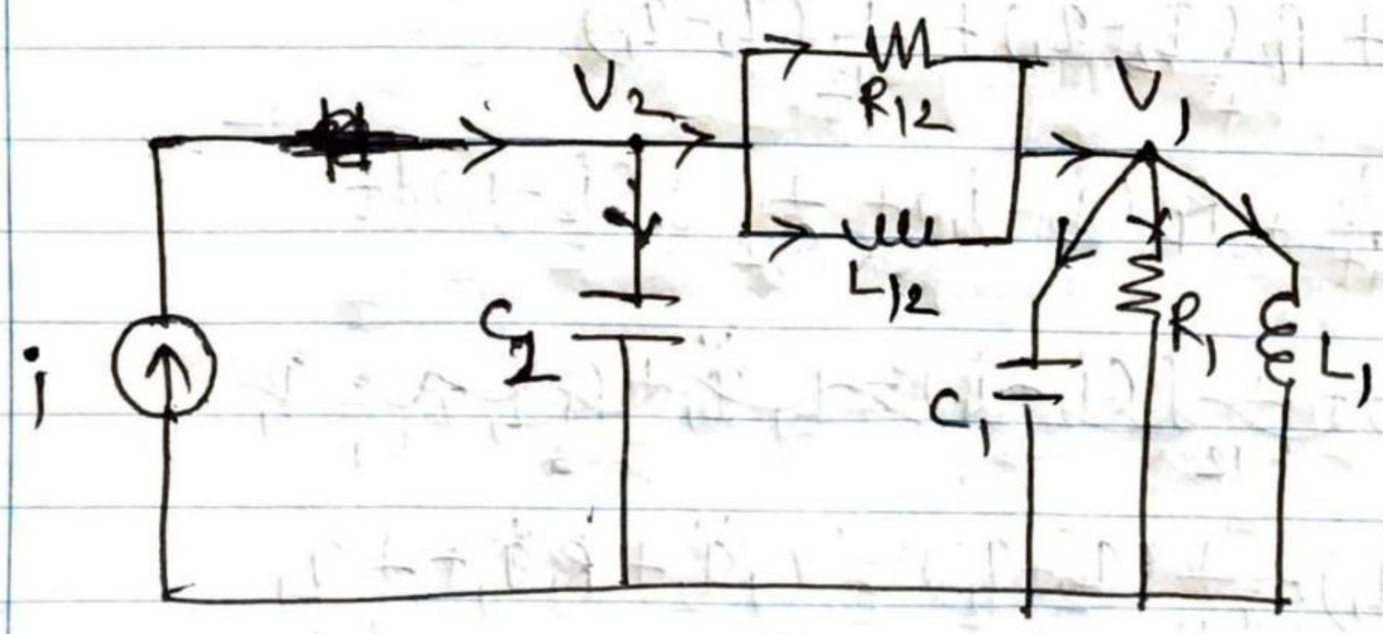
$$\therefore i = C_2\frac{dV_2}{dt} + \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_{12}}\int(V_2 - V_1)\,dt$$

&

$$\frac{1}{R_{12}}(\dot{\phi}_2 - \dot{\phi}_1) + \frac{1}{L_{12}}(\phi_2 - \phi_1) = C_1\ddot{\phi}_1 + \frac{1}{R_1}\dot{\phi}_1 + \frac{1}{L_1}\phi_1$$

$$\therefore \frac{V_2 - V_1}{R_{12}} + \frac{1}{L_{12}}\int(V_2 - V_1)\,dt = C_1\frac{dV_1}{dt} + \frac{V_1}{R_1} + \frac{1}{L_1}\int V_1\,dt$$

The above equations can be converted into the ckt.



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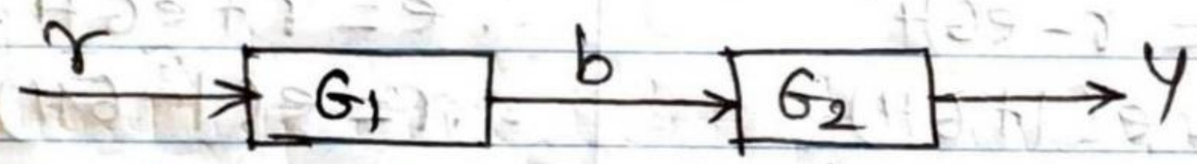
Block diagram algebra-

A system is physical structure developed to perform certain task. A system can be constructed by collecting ~~seve~~ one or more components. Every component can be modeled and represented by a block mentioning its transfer function. These blocks are interconnected to form a system. The system with multiple blocks can be converted into single block to get overall transfer function of system by using block diagram algebra.

Rules of block diagram algebra for block diagram reduction-

1. Blocks in series get multiplied.

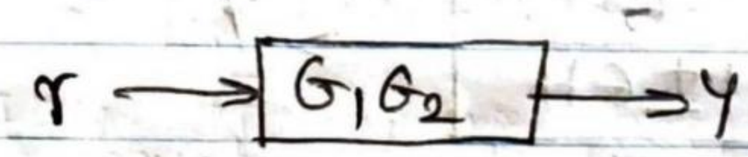


For above system we have

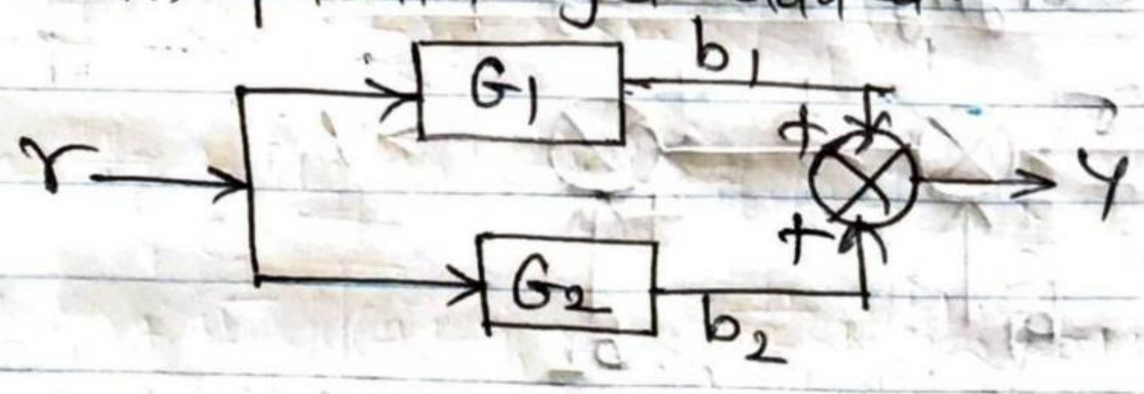
$$b = rG_1$$

$$y = bG_2 = rG_1G_2$$

∴ Equivalent diagram is



2. Blocks in parallel get added.



For above system we have

$$b_1 = rG_1$$

$$b_2 = rG_2$$

$$y = b_1 + b_2 = rG_1 + rG_2 = r(G_1 + G_2)$$

∴ Equivalent diagram is

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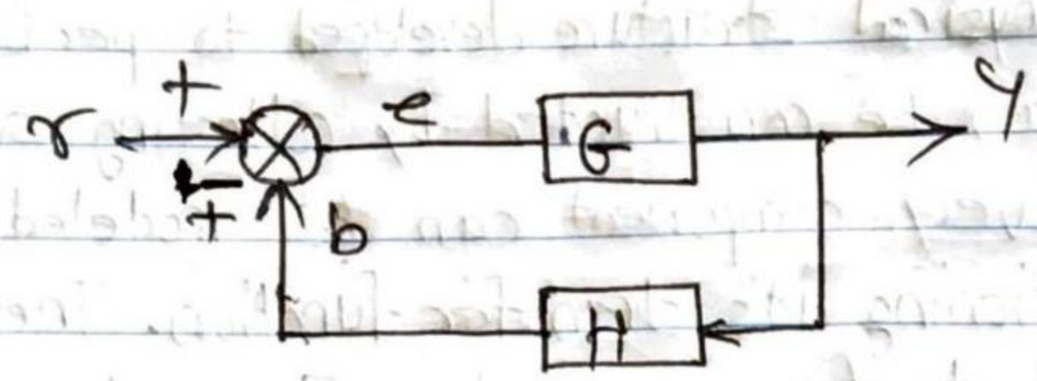
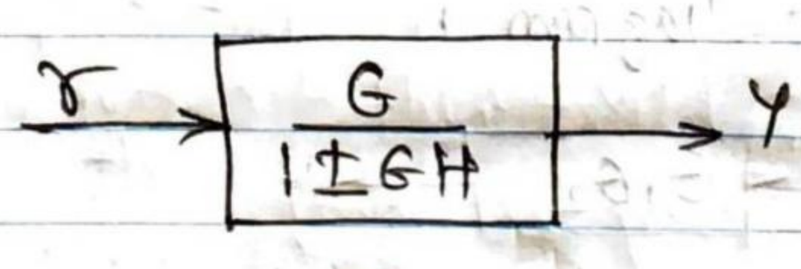
3. Resolving feedback loop -



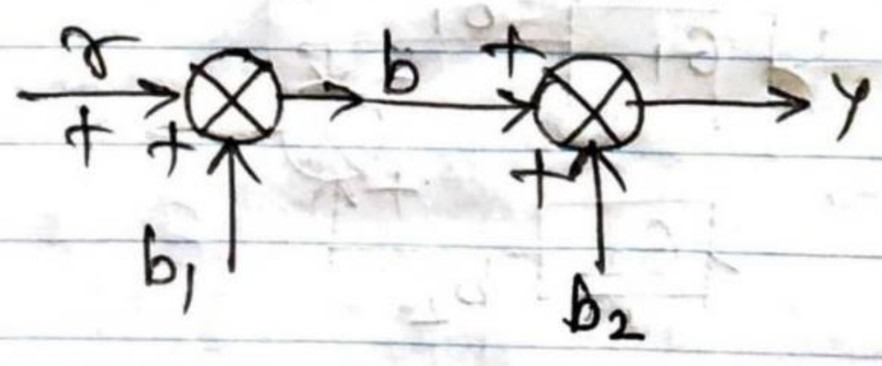
For above system we have, ~~for negative feedback~~

For negative feedback	For positive feedback
$e = r - b$	$e = r + b$
$y = eG$	$y = eG$
$b = eGH$	$b = eGH$
$\therefore e = r - eGH$	$\therefore e = r + eGH$
$\therefore r = e(1+GH)$	$\therefore r = e(1-GH)$
$\therefore \frac{y}{r} = \frac{G}{1+GH}$	$\therefore \frac{y}{r} = \frac{G}{1-GH}$

$\therefore$ Equivalent diagram is,



4. Interchanging summing points -

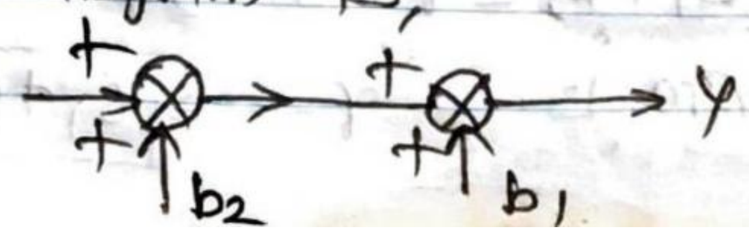


For the above system we have,

$$b = r + b_1$$

$$y = b + b_2 = r + b_1 + b_2$$

$\therefore$ Equivalent diagram is,



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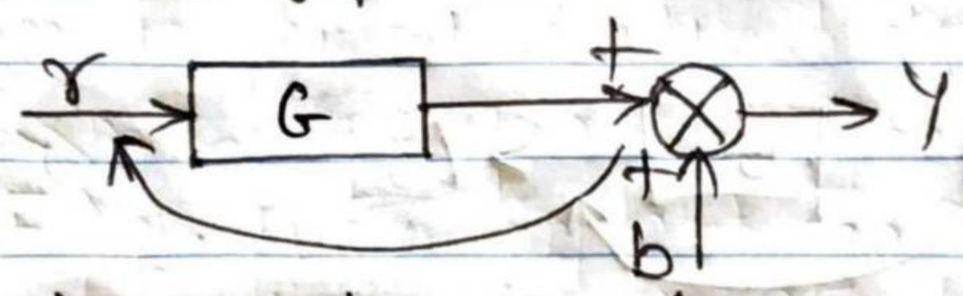
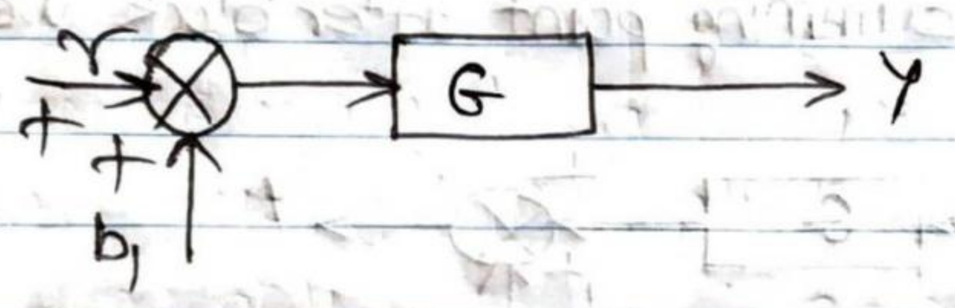
5. <u>shifting summing point before block-</u>



For the above system we have,

$$Y = rG + b$$

By shifting summing point before block we get,



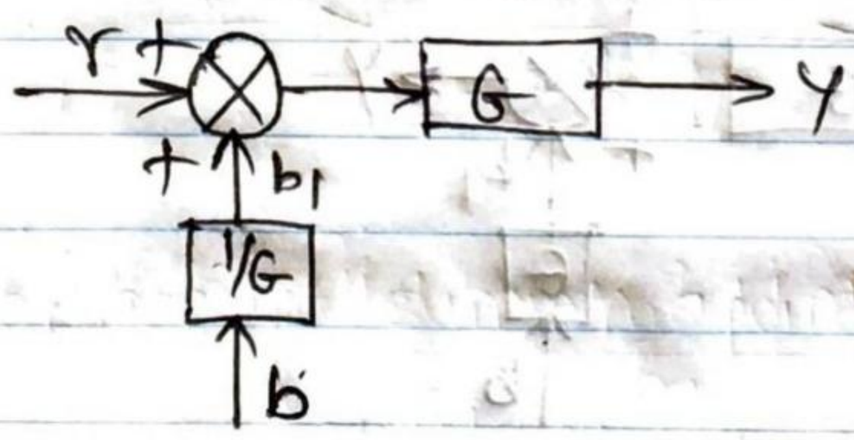
$$Y = (r + b_1)G = rG + b_1G$$

For the two systems to be equivalent we get,

$$b = b_1G$$

$$\therefore b_1 = \frac{1}{G} b$$

$\therefore$ The equivalent diagram is



For the above system we have

$$Y = \left(r + \frac{b}{G}\right)G = rG + b$$ which is same as initial system.

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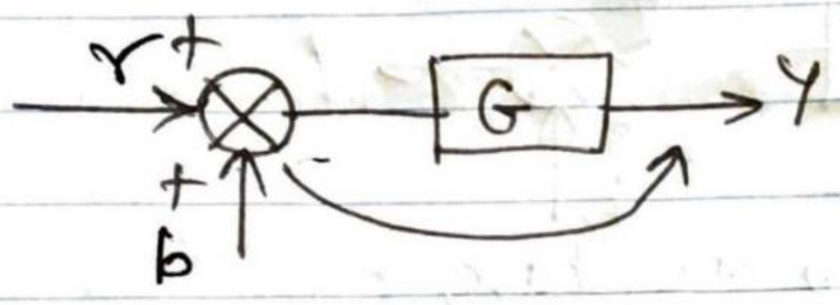
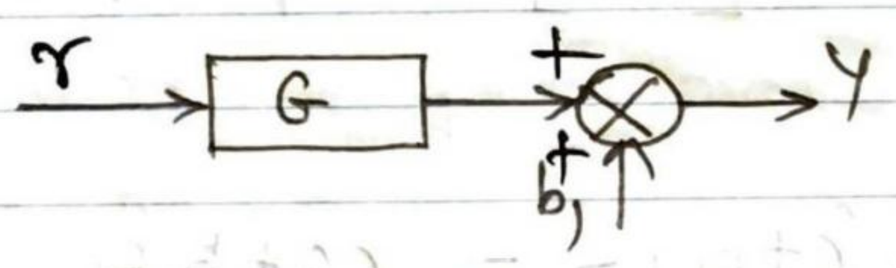
6. <u>Shifting summing point after block</u>-



For the above system we have,

$$y = (r+b)G = rG + bG$$

By shifting summing point after block we get,

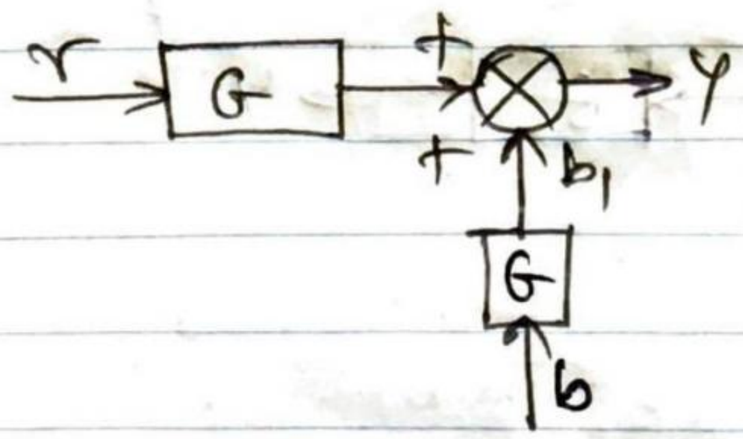


$$y = rG + b_1$$

For both systems to be equivalent

$$b_1 = bG$$

∴ The equivalent diagram is,



For the above system we ~~have~~ get,

$y = rG + bG$ which is same as the initial system.

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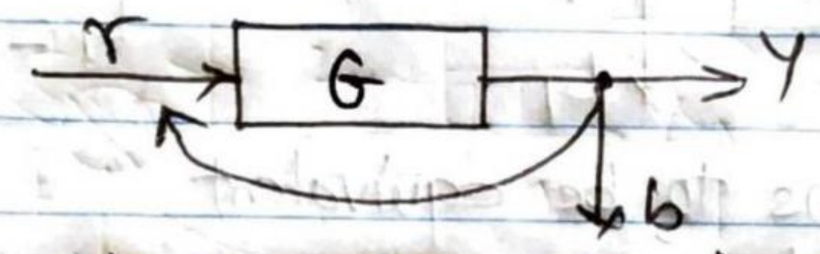
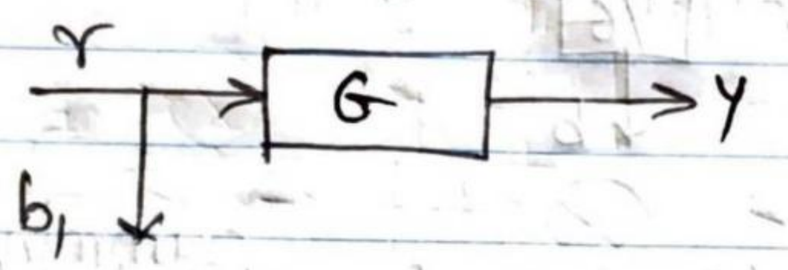
7. <u>Shifting take off point before block</u>-



For the above system we have

$$Y = rG$$
$$b = Y = rG$$

By shifting take off point before block we get

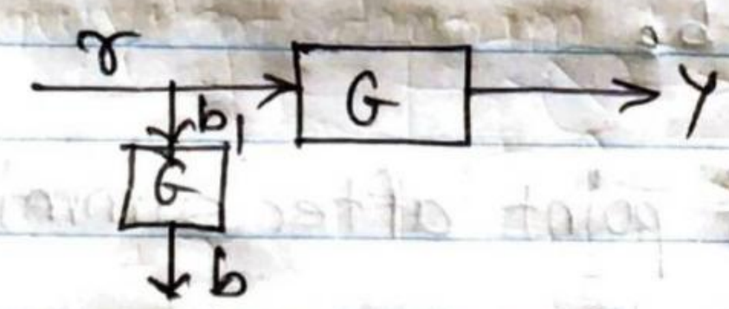


$$Y = rG$$
$$b_1 = r$$

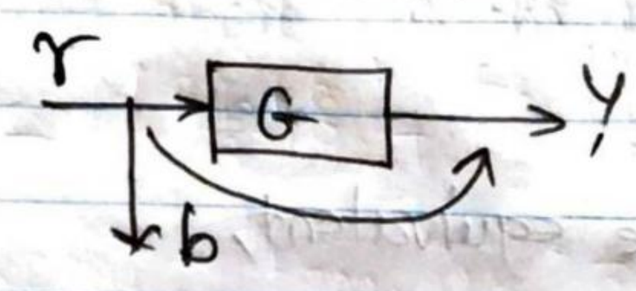
For both systems to be equivalent

$$b = b_1 G$$

∴ The equivalent system is



8. <u>Shifting take off point after block</u>-



For above system we have

$$Y = rG$$
$$b = r$$

By shifting take off after block we get,

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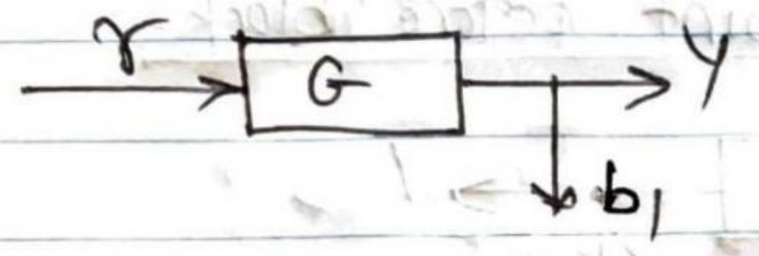


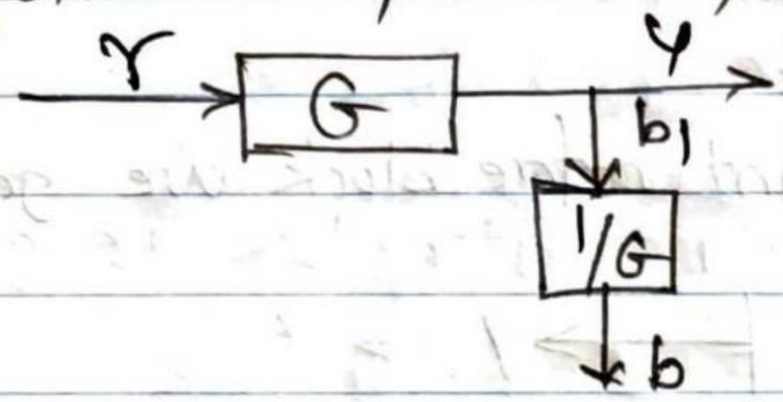
$\therefore$ $y = rG$

$b_1 = y = rG$

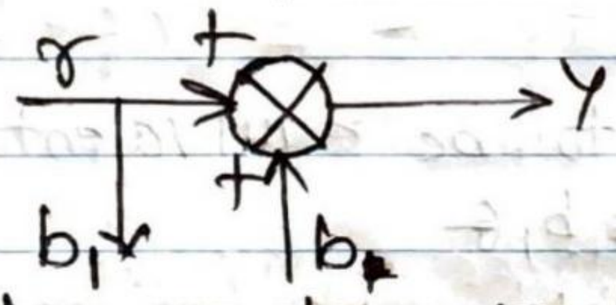
For both systems to be equivalent

$$b = b_1/G$$

$\therefore$ Equivalent system is,



9. <u>Shifting take off point after summing point</u>

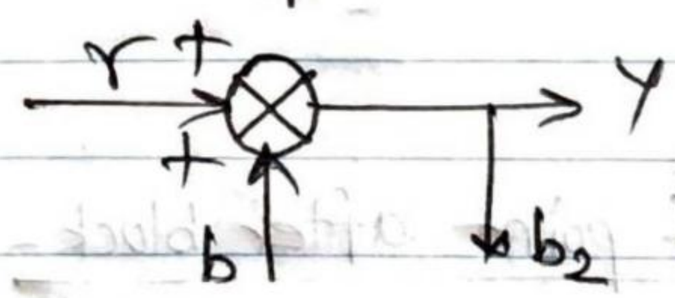


For the above system we have

$$y = r + b$$

$$b_1 = r$$

By shifting take off point after summing point we get



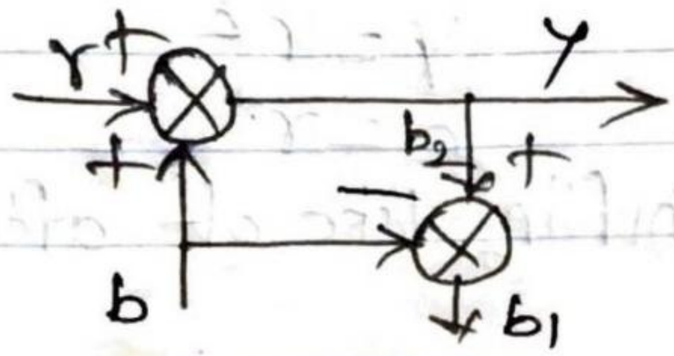
$$y = r + b$$

$$b_2 = y = r + b$$

For both systems to be equivalent,

$$b_1 = b_2 - b$$

$\therefore$ Equivalent diagram is,



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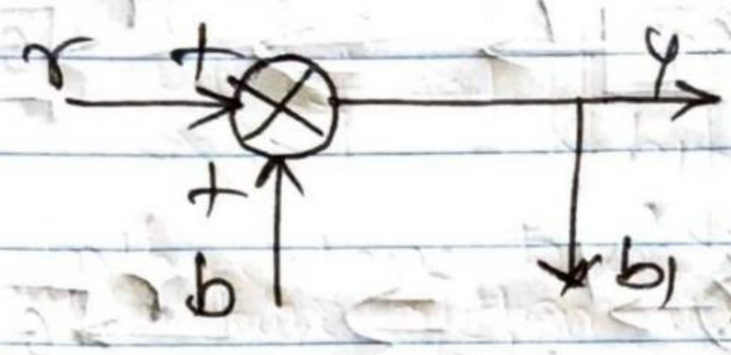
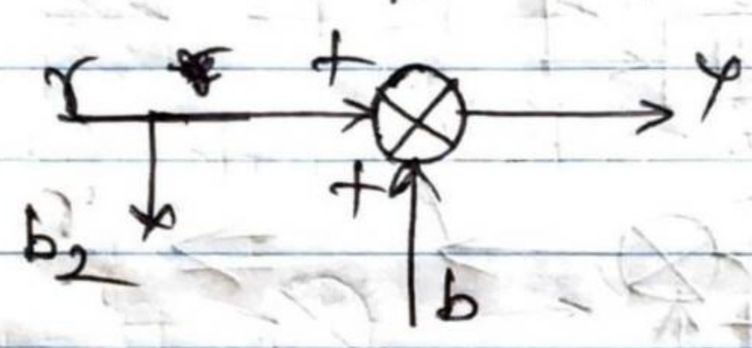
10. <u>Shifting take off point before summing point</u>–



For the above system we have,

$$y = r + b$$
$$b_1 = y = r + b$$

By shifting take off point before summing point we get,

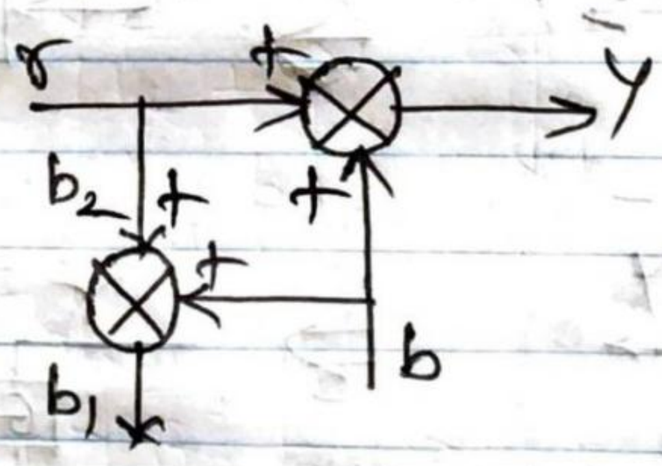


$$\therefore\ y = r + b$$
$$b_2 = r$$

For both systems to be equivalent

$$b_1 = b_2 + b$$

$\therefore$ Equivalent diagram is,



<u>Summary</u>–

Elements of block diagram

1. Block representing transfer function

2. Summing point for adding signals

3. Take off point

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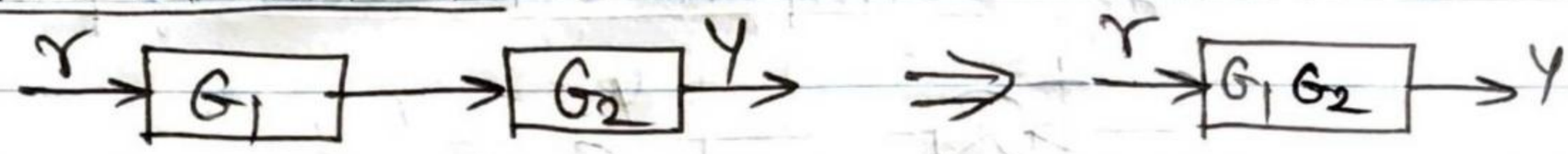
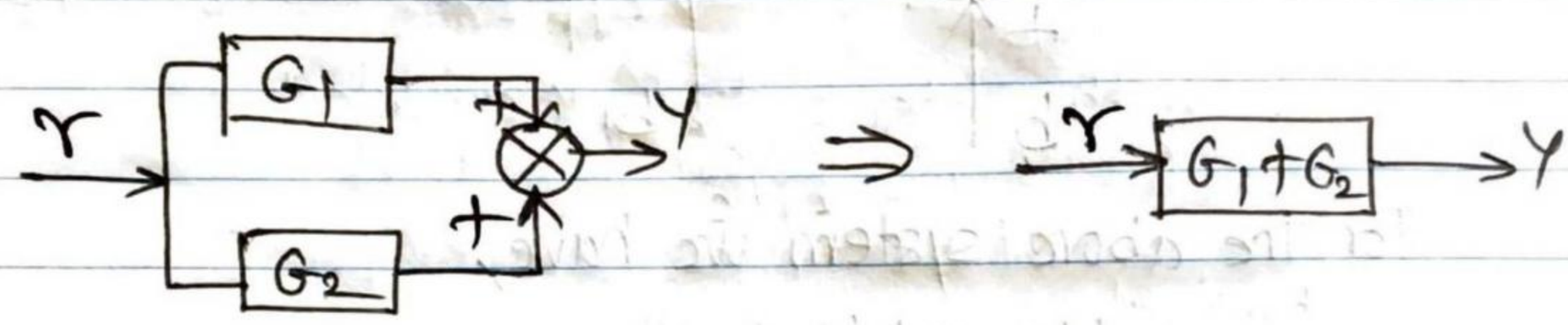
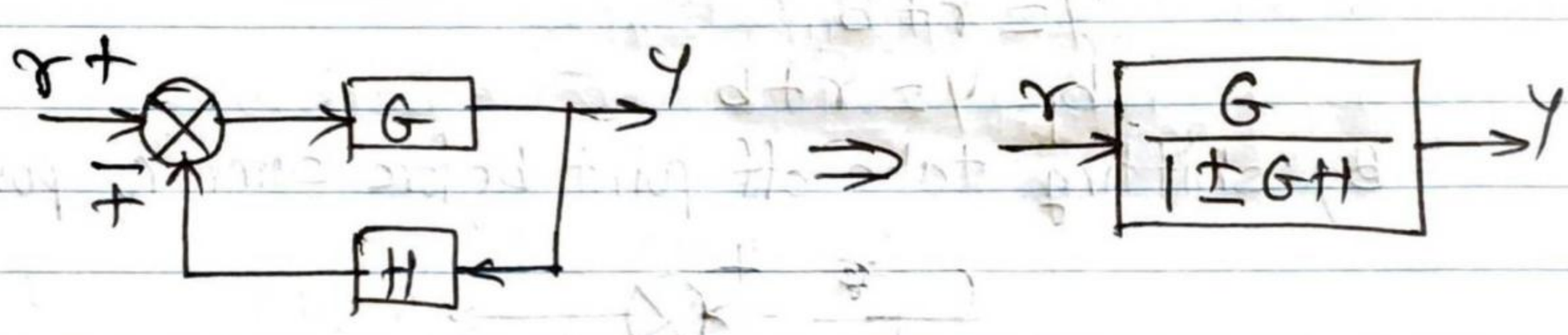
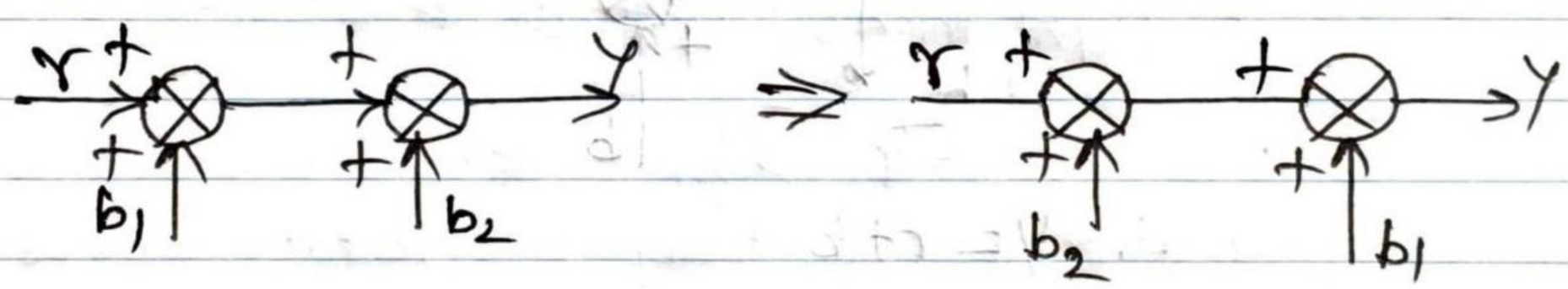
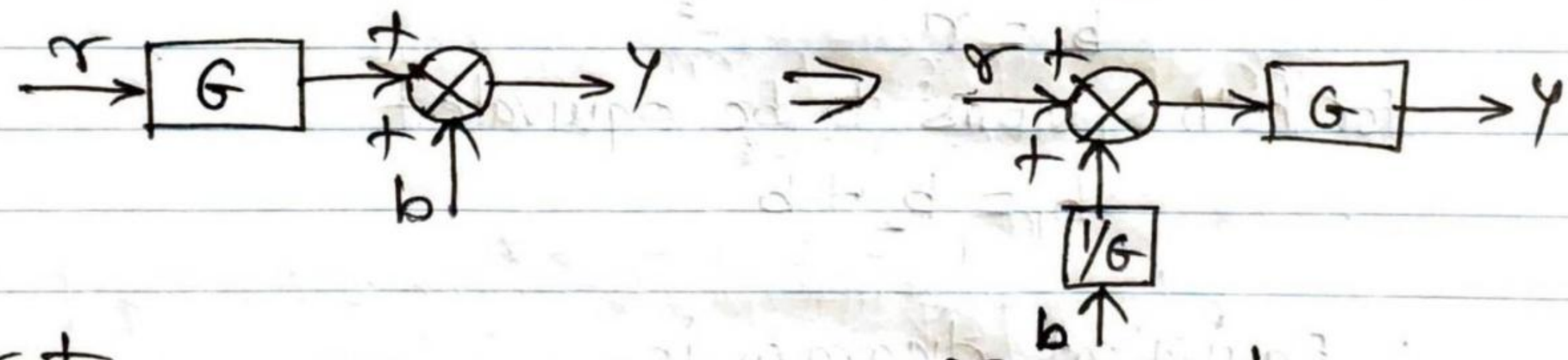
Rules summerized-

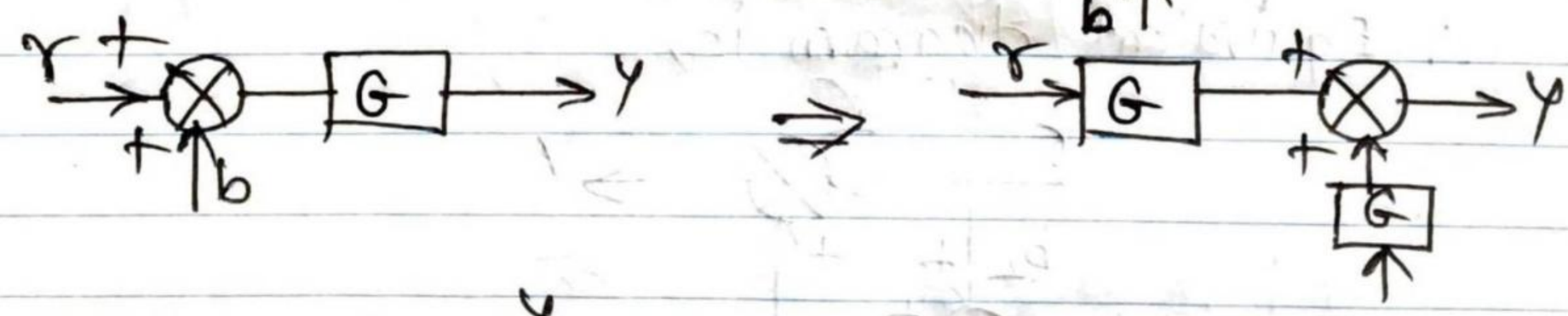
1. 

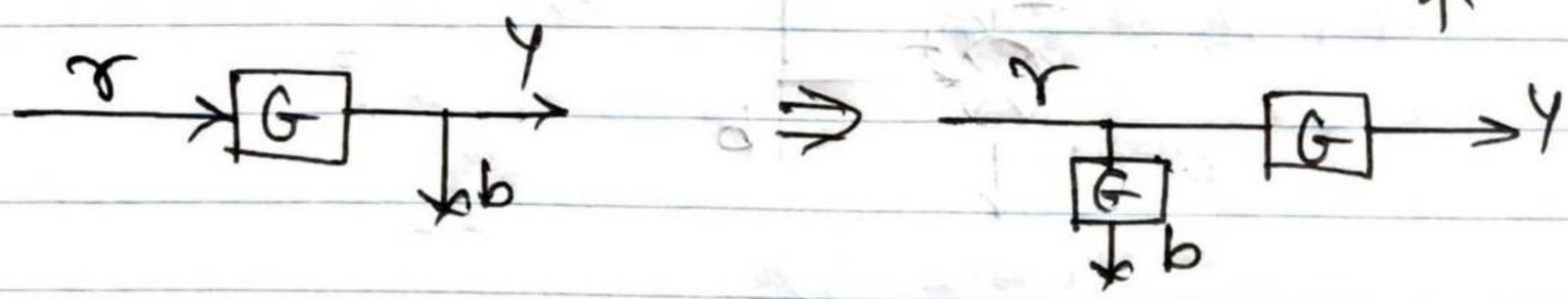
2. 

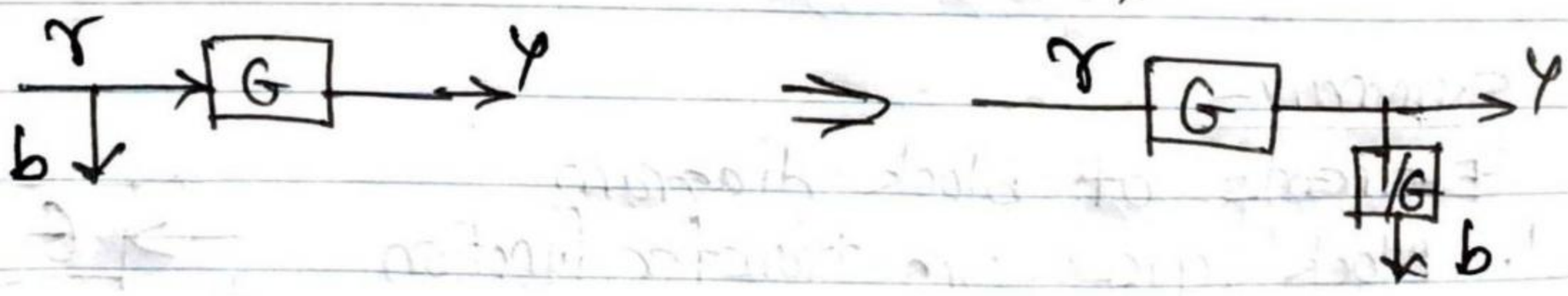
3. 

4. 

5. 

6. 

7. 

8. 

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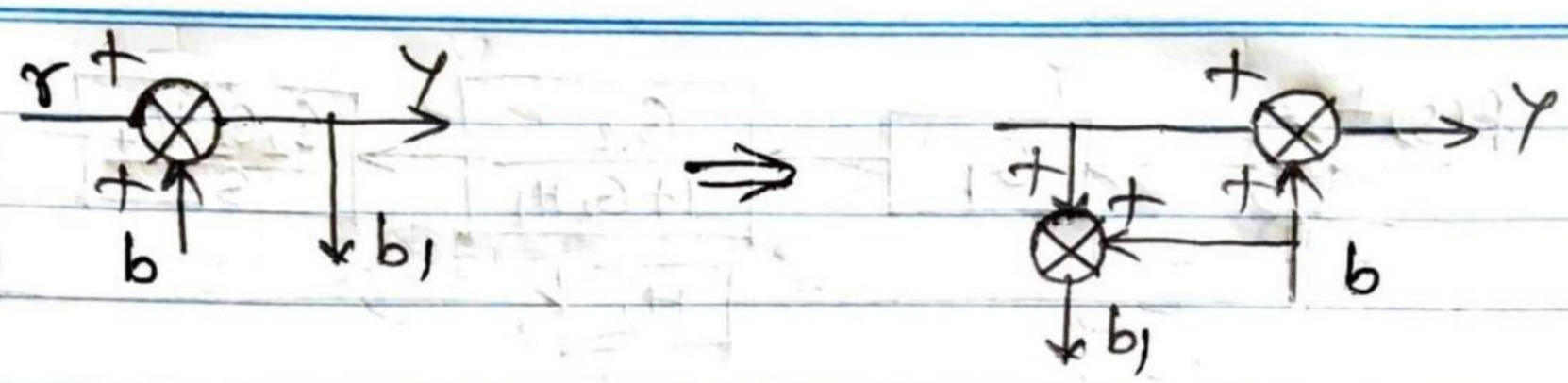
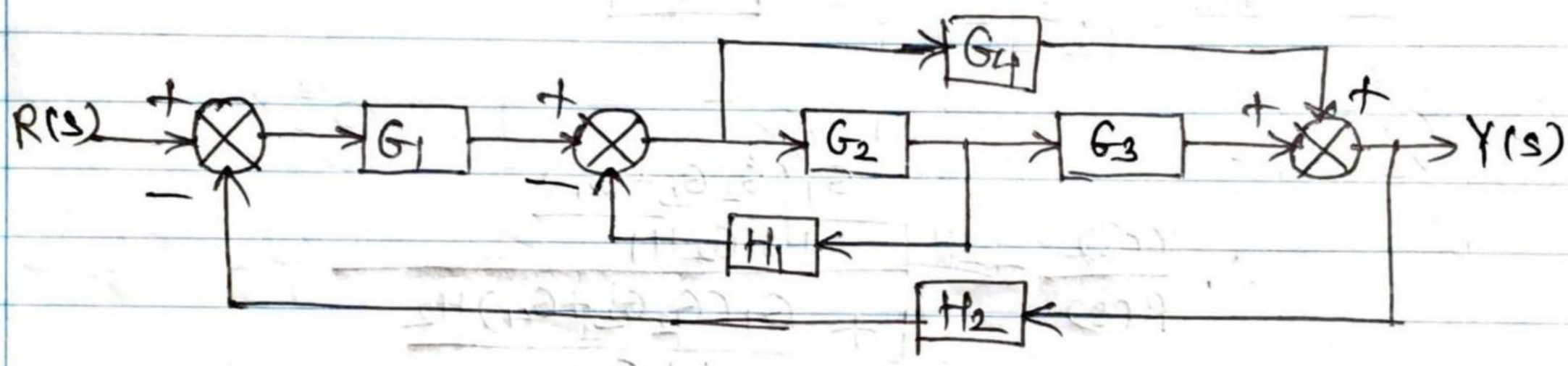
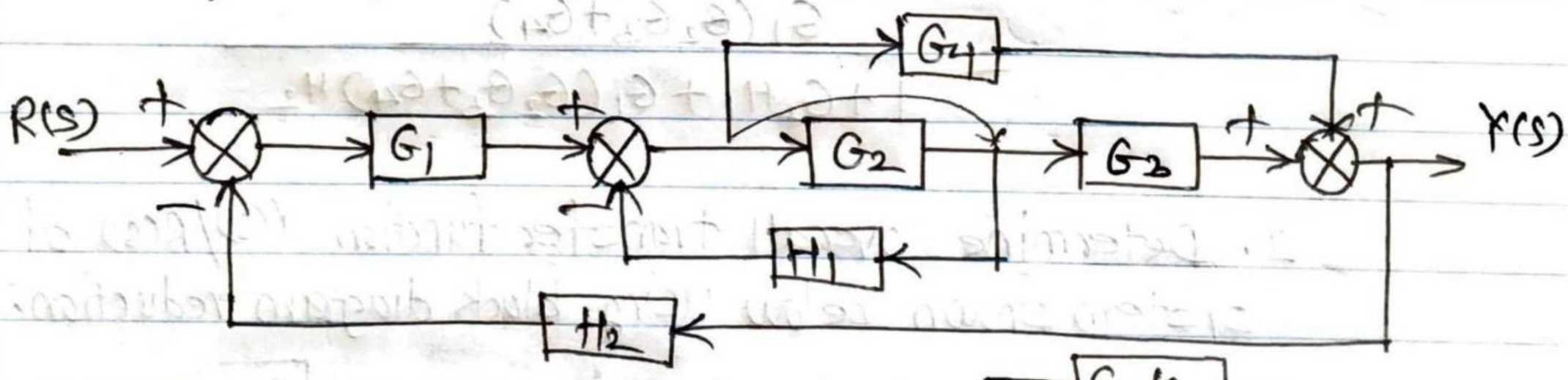
10. 

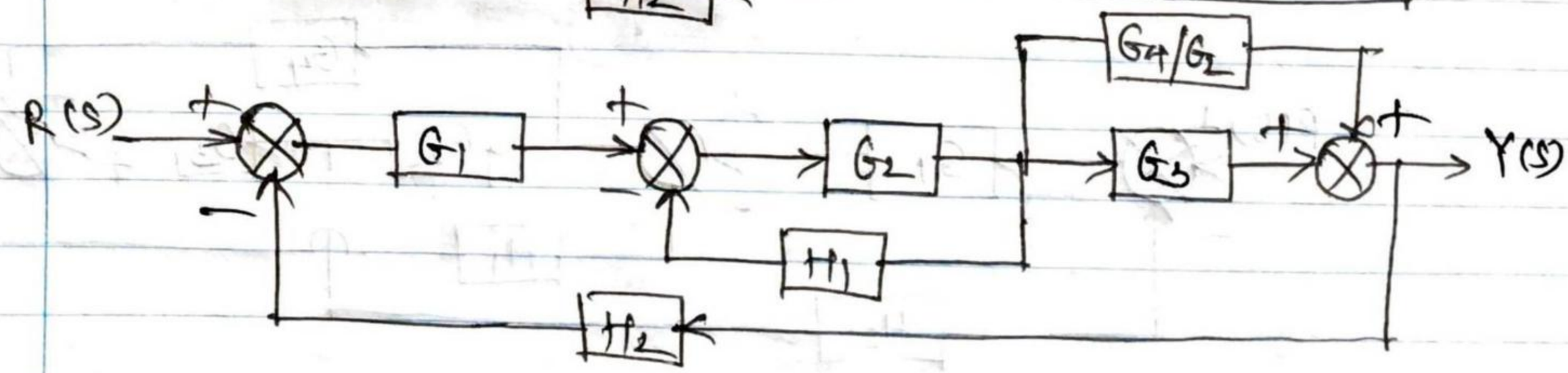
Examples of block diagram reduction-

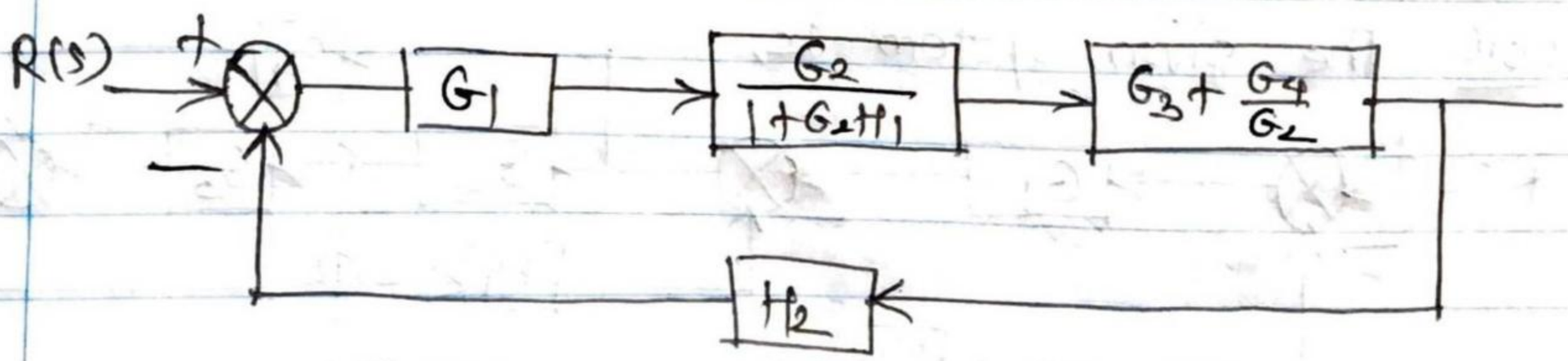
1. Determine overall transfer function $Y(s)/R(s)$ using block diagram reduction rules.



Soln The given system is,







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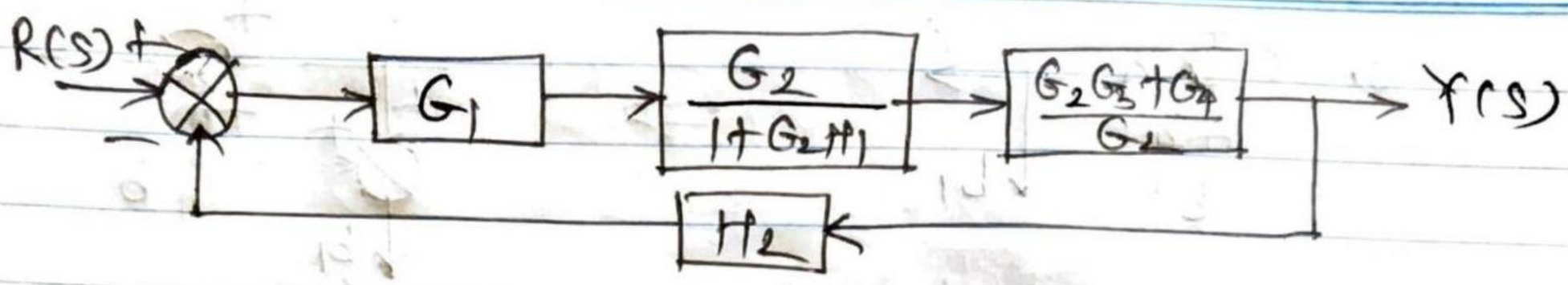
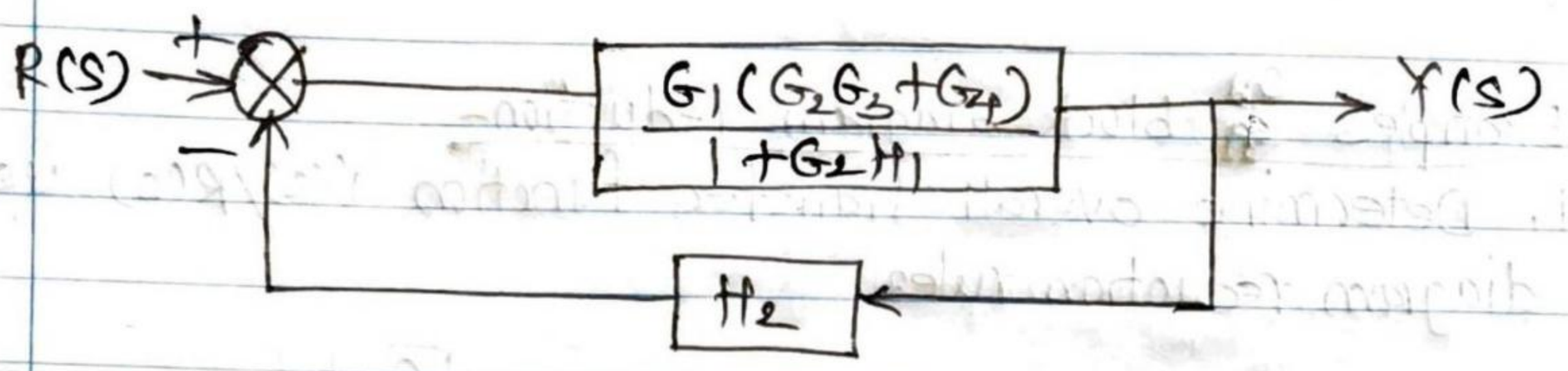
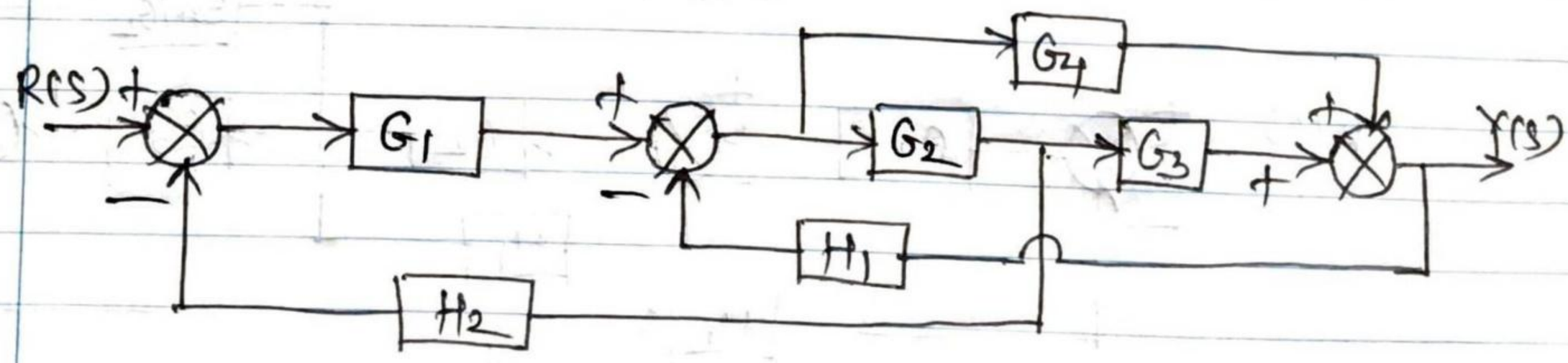




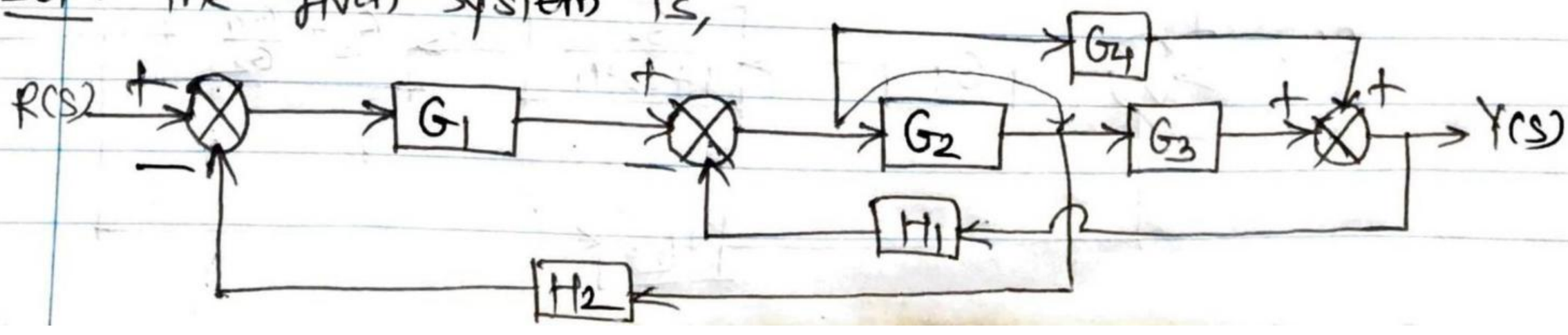
$$\therefore \frac{Y(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+G_2H_1}}{1+\dfrac{G_1(G_2G_3+G_4)H_2}{1+G_2H_1}}$$

$$= \frac{G_1(G_2G_3+G_4)}{1+G_2H_1+G_1(G_2G_3+G_4)H_2}$$

2. Determine overall transfer function $Y(s)/R(s)$ of the system shown below using block diagram reduction.



<u>soln</u> The given system is,



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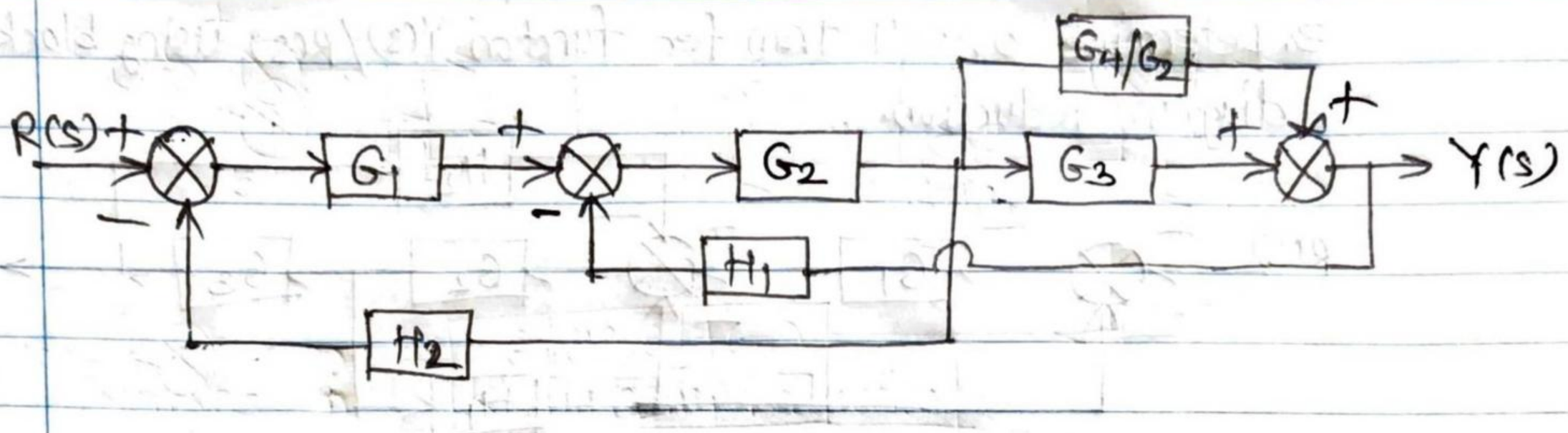

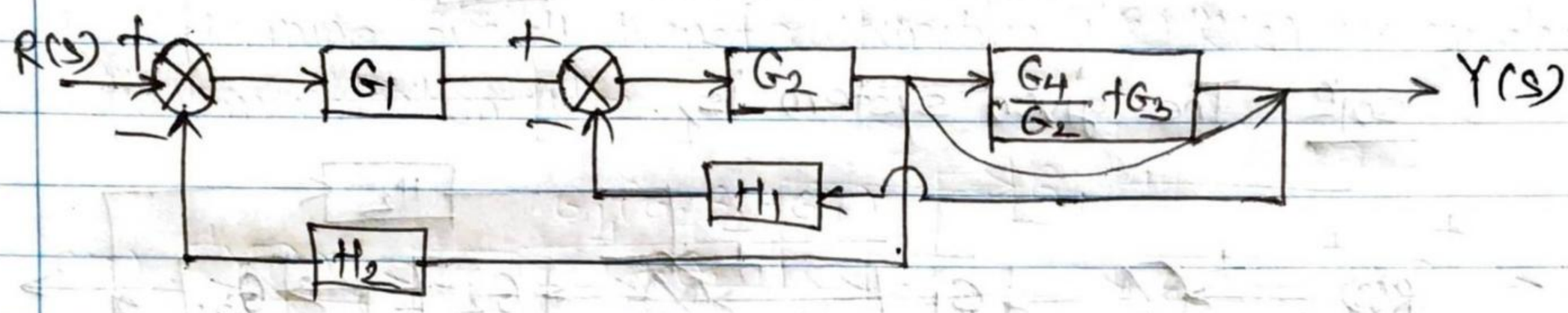

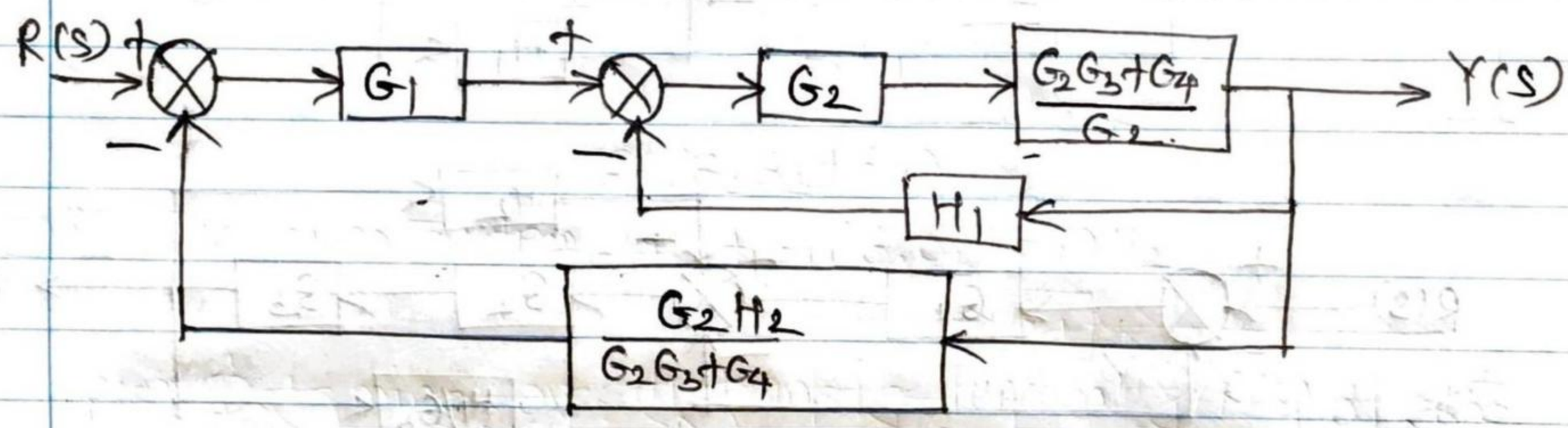

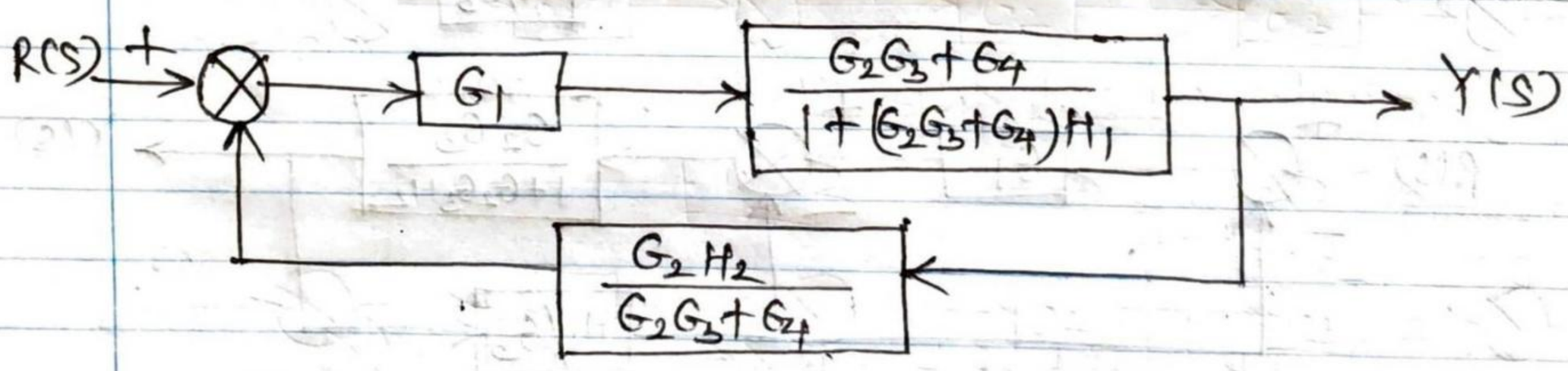

$$\therefore \frac{Y(s)}{R(s)} = \frac{\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_1}}{1+\dfrac{G_1(G_2G_3+G_4)}{1+(G_2G_3+G_4)H_1}\cdot\dfrac{G_2H_2}{G_2G_3+G_4}}$$

$$= \frac{G_1(G_2G_3+G_4)}{1+G_2G_3H_1+G_4H_1+G_1G_2H_2}$$

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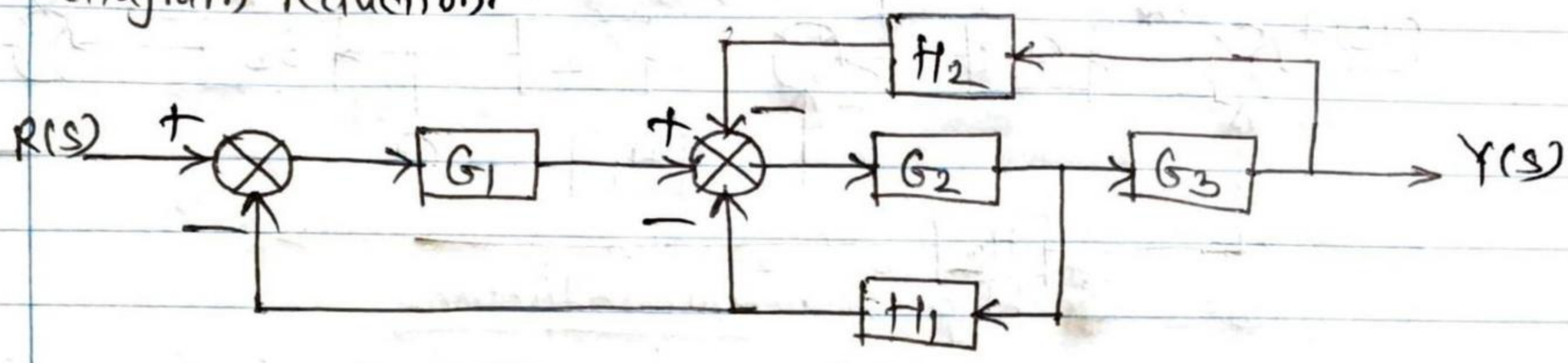
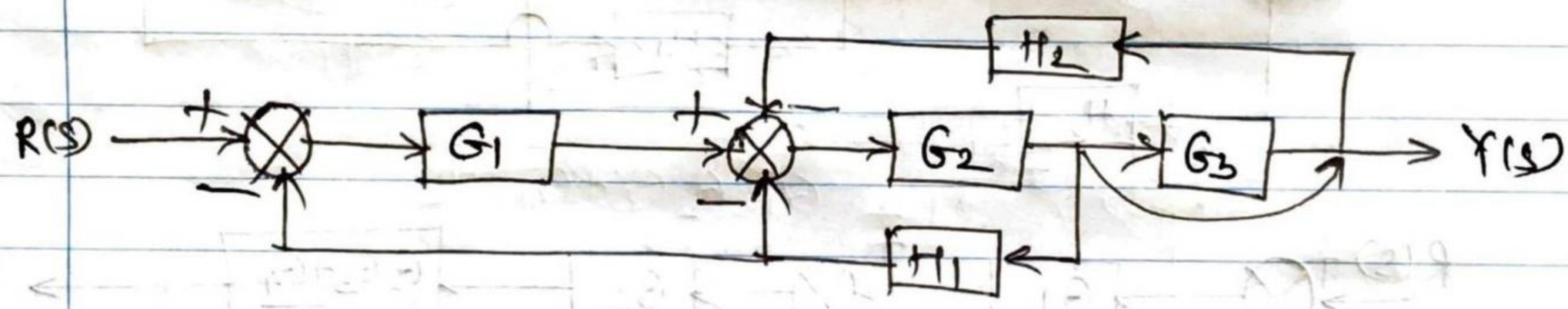
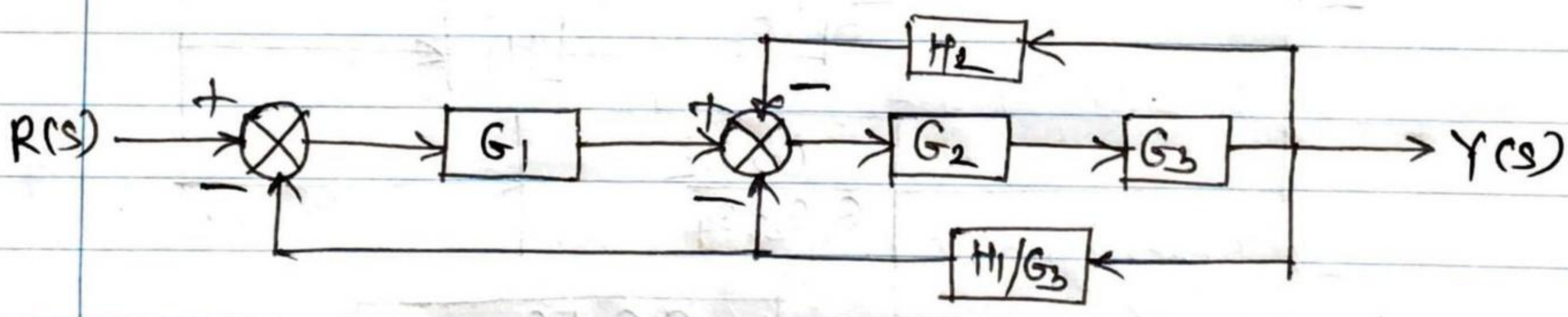
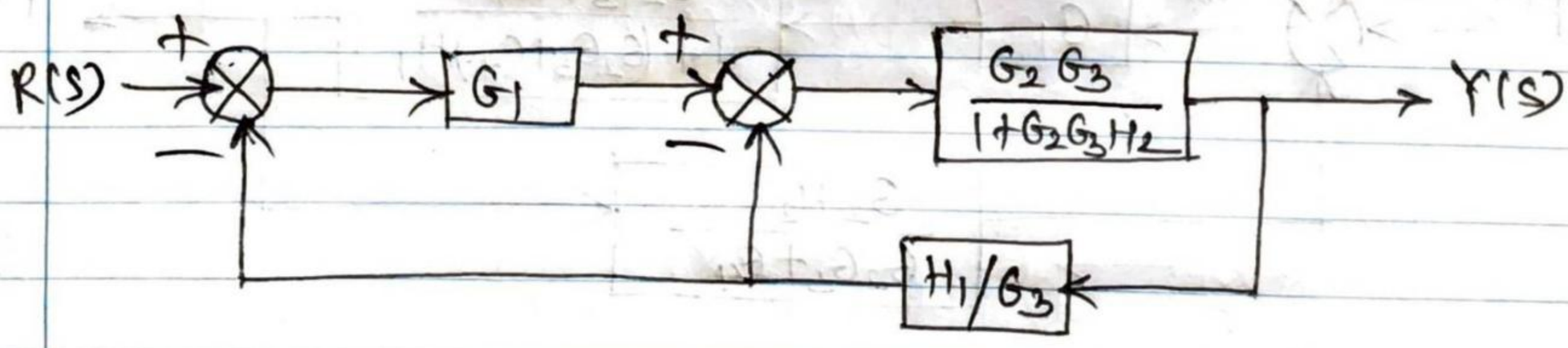
3. Determine overall transfer function $Y(s)/R(s)$ using block diagram reduction.

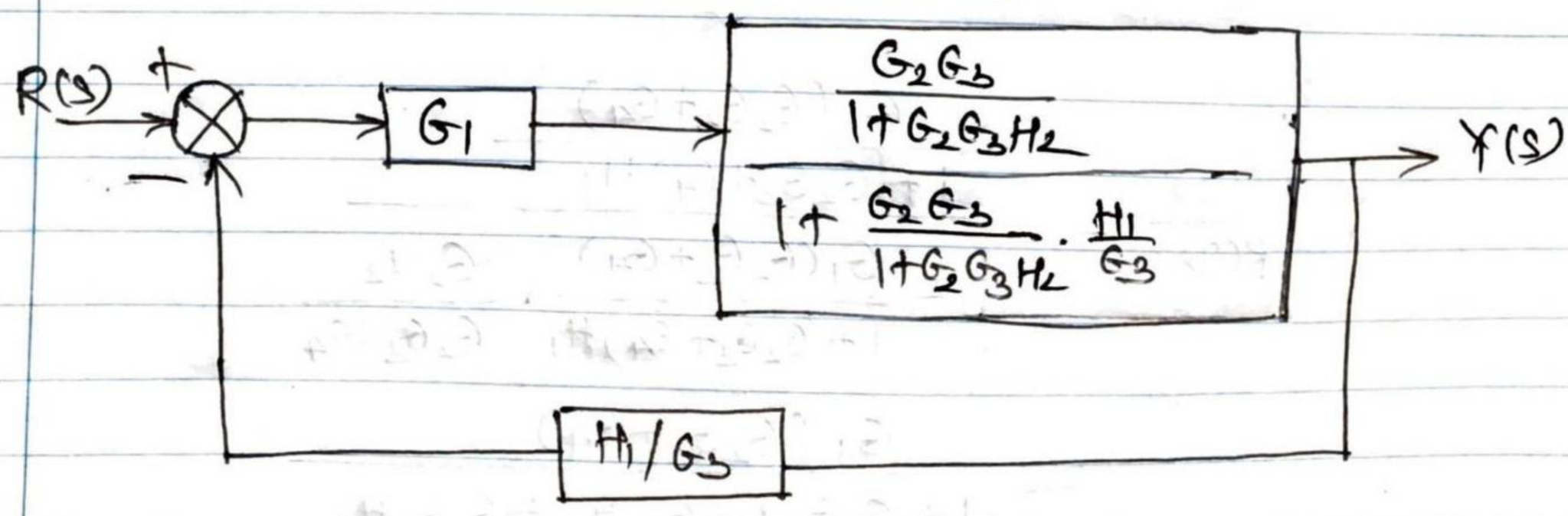


<u>soln</u> The given system is,









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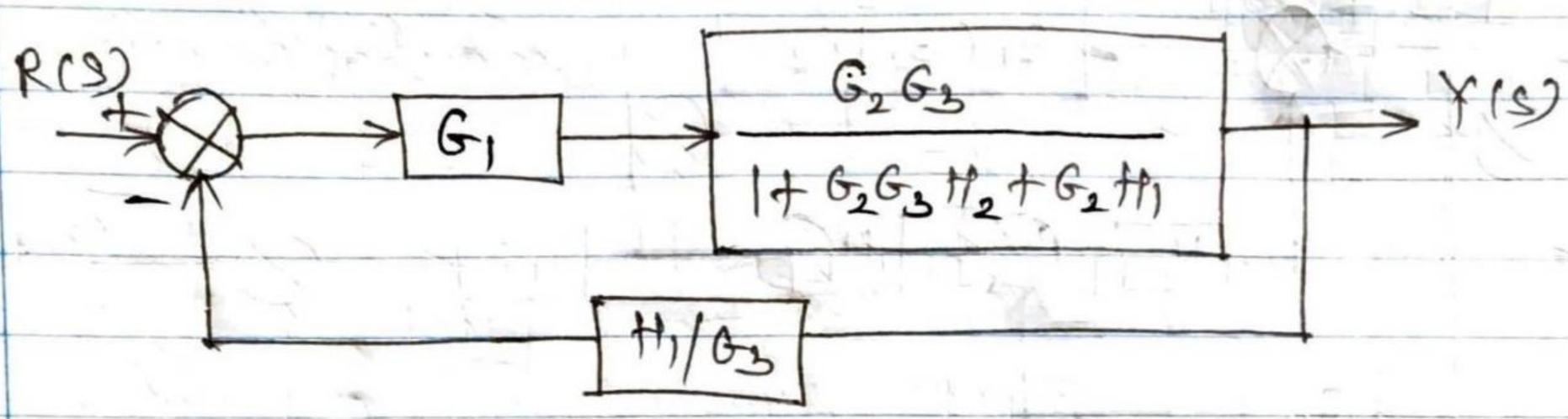
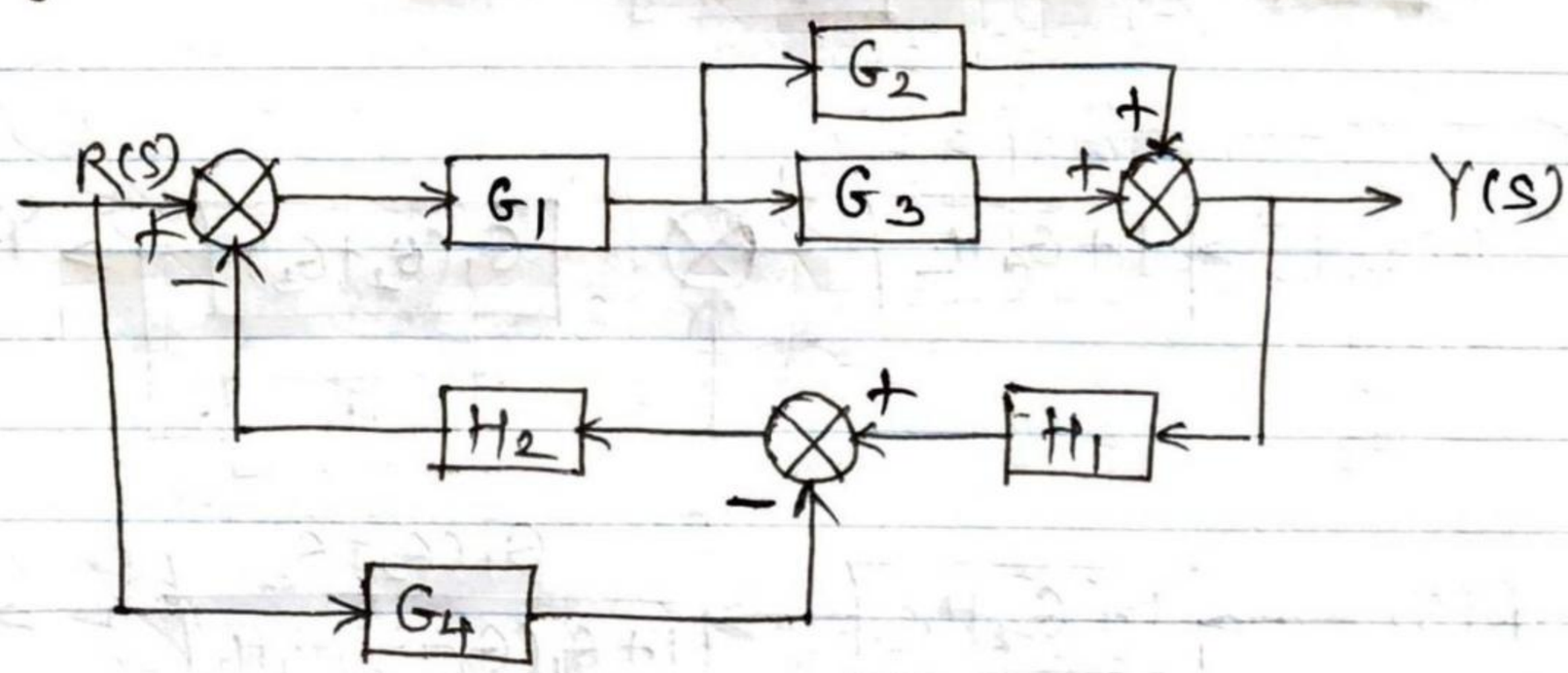


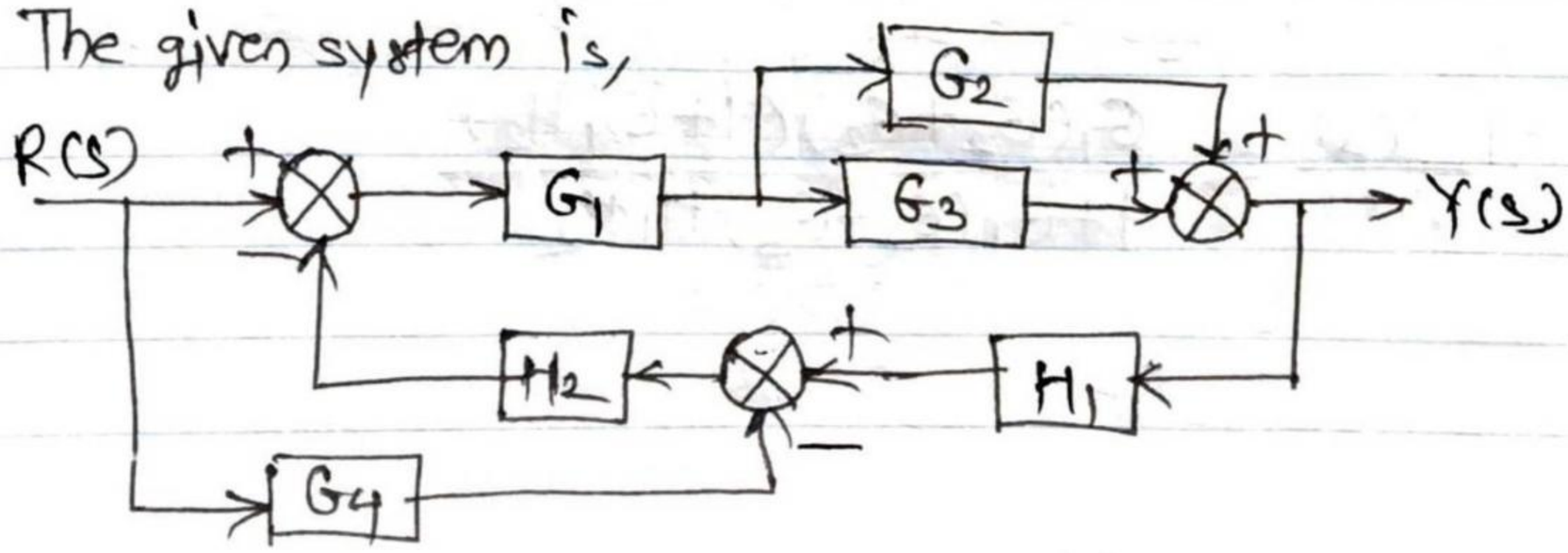
$$\therefore \frac{Y(s)}{R(s)} = \frac{\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_2H_1}}{1+\dfrac{G_1G_2G_3}{1+G_2G_3H_2+G_2H_1}\cdot\dfrac{H_1}{G_3}}$$

$$= \frac{G_1G_2G_3}{1+G_2G_3H_2+G_2H_1+G_1G_2H_1}$$

4. Determine overall transfer function $Y(s)/R(s)$ using block diagram reduction.



<u>soln</u> The given system is,



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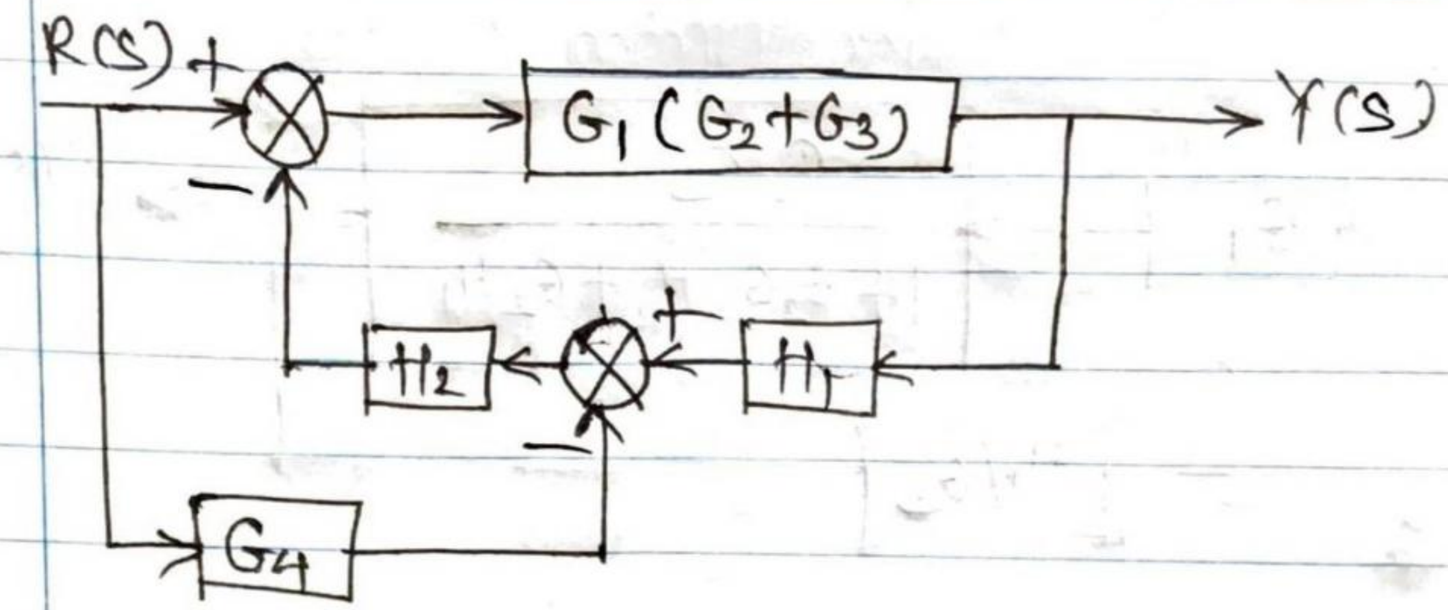

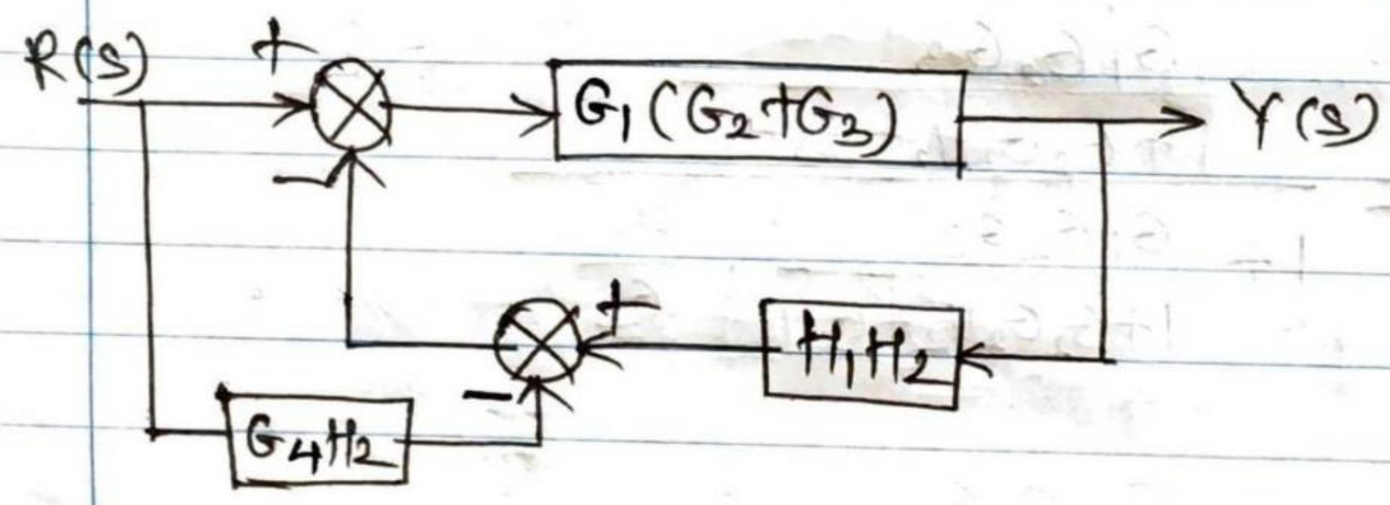

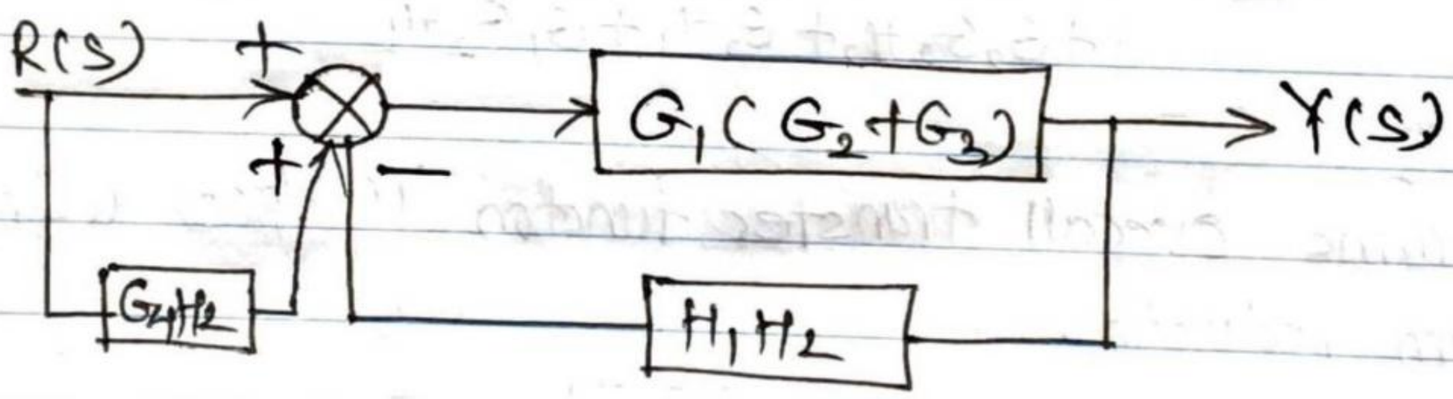

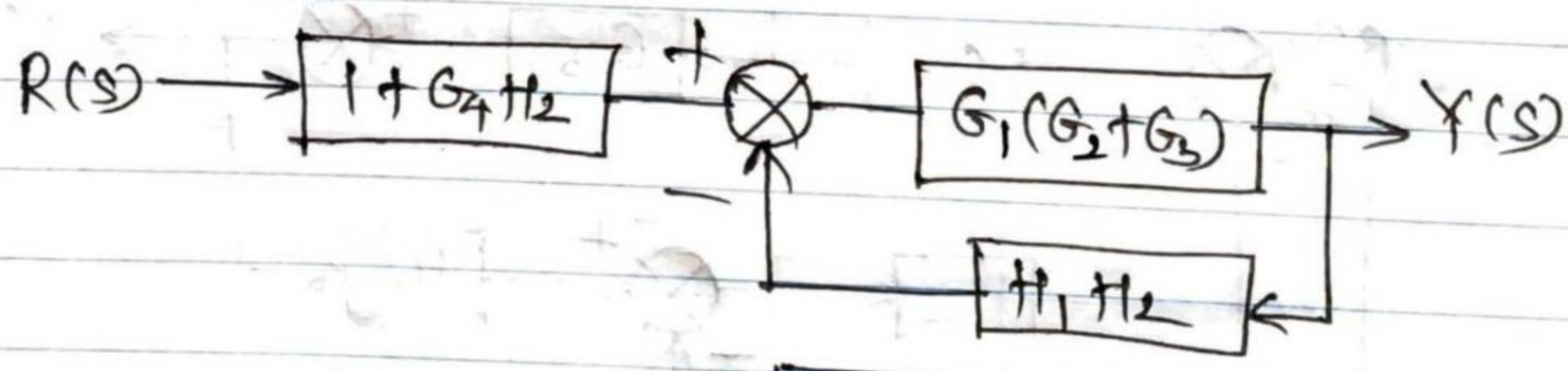

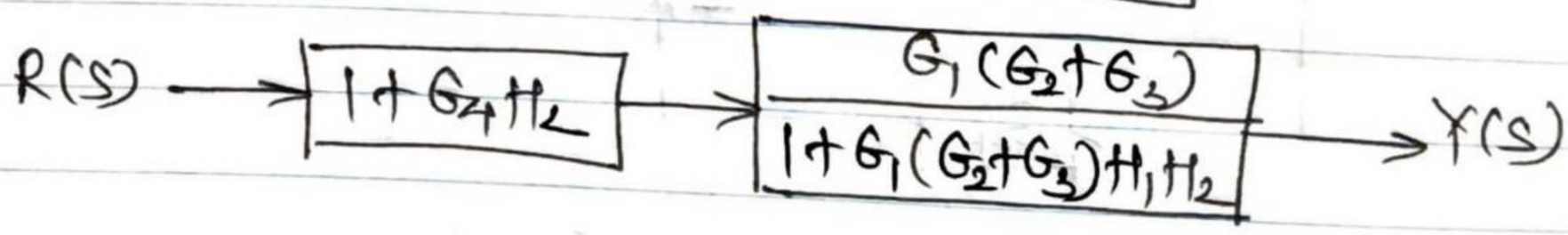

$$\therefore \frac{Y(S)}{R(S)} = \frac{G_1(G_2+G_3)(1+G_4H_2)}{1+G_1(G_2+G_3)H_1H_2}$$

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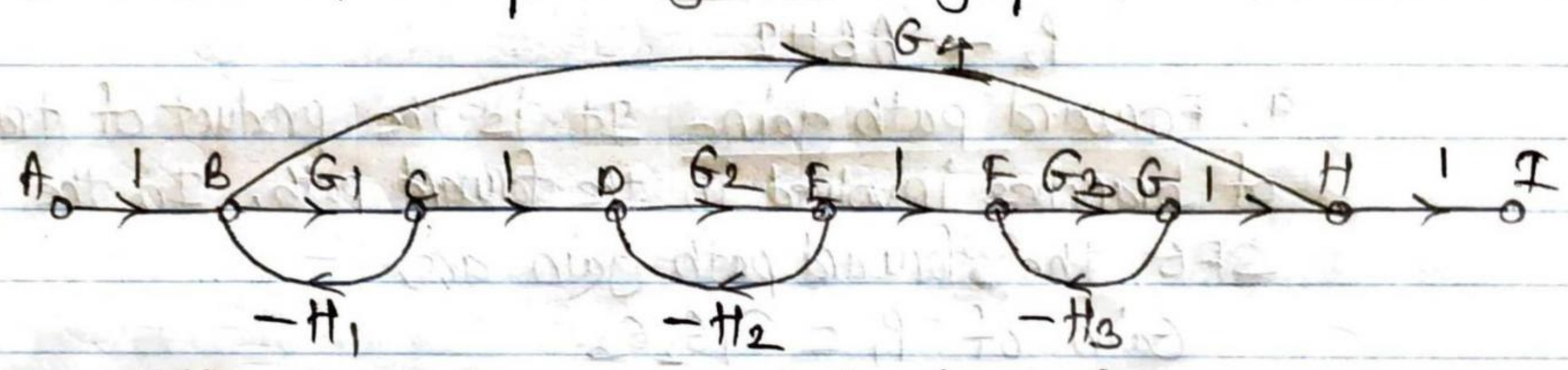
Signal flow graph-

Signal flow graph is a graphical representation of a set of simultaneous linear algebriac equations. It can be used to represent the systems.

Terminology of signal flow graph-

Consider a sample signal flow graph as shown below.



The different terms related to signal flow graph are

1. Node- A point representing a signal is called as node. The nodes are of three types as input node, output node & mixed node. In the above S.F.G., A, B, C, D, E, F, G, H, I are the nodes. Every node contains sum of all incoming signals to it.
2. Branch- A branch is a line with direction which joins two nodes. In the given S.F.G., AB, BC, CD, DE, EF, FG, GH, HI, BH, CB, ED, GF are the branches.
3. Branch transmittance- Branch transmittance is the gain offered by the system. In the given S.F.G., transmittance of branch BC is G_1 and of branch DE is G_2.
4. Input node- The node with only outgoing branches is known as input node. In the given S.F.G., A is the input node.
5. Output node- The node with only incoming branches is known as output node. In the given S.F.G., I is the output node.
6. Mixed node- The node with incoming as well as outgoing

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branches is known as mixed node. In the given S.F.G, except A and I all nodes are mixed nodes.

7. <u>Path</u>- A path is the traversal of connected branches in the direction of signal (indicated by arrows) such that no node is traversed more than once.

8. <u>Forward path</u>- A forward path is a path from input node to output node. In the given S.F.G. there are two forward paths.

$P_1 \rightarrow ABCDEFGHI$

$P_2 \rightarrow ABHI$

9. <u>Forward path gain</u>- It is the product of transmittances of branches involved in the forward path. In the given SFG, the forward path gain are,

Gain of $P_1 = G_1G_2G_3$

Gain of $P_2 = G_4$

10. <u>Loop</u>- Loop is the closed path. In the given S.F.G. there are three loops as,

$L_1 = BCB$

$L_2 = DED$

$L_3 = FGF$

11. <u>Loop gain</u>- Loop gain is the product of transmittances of branches involved in the loop. In the given S.F.G., the loop gains are,

Gain of $L_1 = -G_1H_1$

Gain of $L_2 = -G_2H_2$

Gain of $L_3 = -G_3H_3$

12. <u>Non touching loops</u>:- The loops that do not have common node between them are called as non touching loops. In the given S.F.G, L_1, L_2 and L_3 are non touching loops.

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Mason's gain formula for determination of transfer function-

Mason's gain formula is given by,

$$T.F. = \frac{\Sigma P_k \Delta_k}{\Delta}$$

where, P_k is the gain of kth forward path.

Δ is the determinant of the graph which is given by,

Δ = 1 − sum of all individual loop gain + sum of product of all two non touching loops − sum of product of three non touching loops + . . .

Δ_k is determinant of subgraph and is the value of Δ for the part not touching to k^{th} forward path.

For the given S.F.G, the T.F. can be obtained as,

No. of forward path and their gain-

$P_1 = G_1 G_2 G_3$

$P_2 = G_4$

No. of loops and their loop gain-

$L_1 = -G_1 H_1$

$L_2 = -G_2 H_2$

$L_3 = -G_3 H_3$

No. of two non touching loops and their gain product-

$L_1 L_2 = G_1 G_2 H_1 H_2$

$L_2 L_3 = G_2 G_3 H_2 H_3$

$L_1 L_3 = G_1 G_3 H_1 H_3$

No. of three non touching loops and their gain product-

$L_1 L_2 L_3 = -G_1 G_2 G_3 H_1 H_2 H_3$

$$\therefore \Delta = 1 - (-G_1 H_1 - G_2 H_2 - G_3 H_3) + G_1 G_2 H_1 H_2 + G_2 G_3 H_2 H_3 + G_1 G_3 H_1 H_3 - (-G_1 G_2 G_3 H_1 H_2 H_3)$$

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$\therefore \Delta = 1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_2H_1H_2 + G_1G_3H_1H_3 + G_2G_3H_2H_3 + G_1G_2G_3H_1H_2H_3$

$\Delta_1 = 1 - (0) = 1$

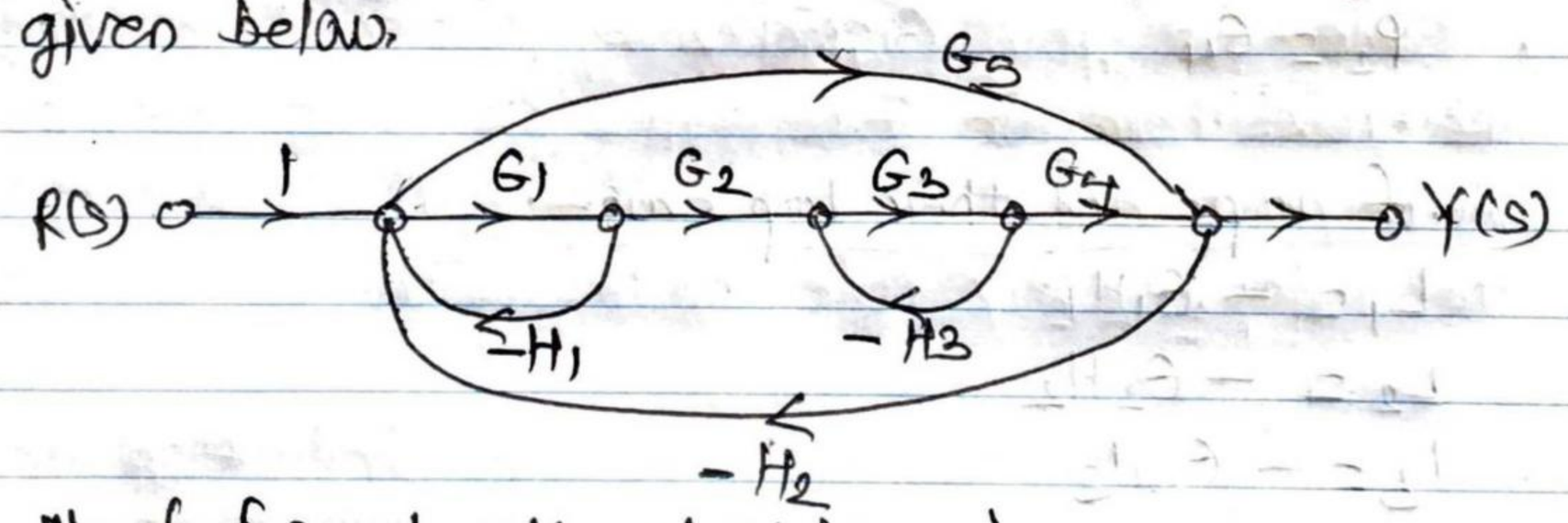
$\Delta_2 = 1 - (-G_2H_2 - G_3H_3) + G_2G_3H_2H_3$

$= 1 + G_2H_2 + G_3H_3 + G_2G_3H_2H_3$

$$\therefore G(s) = \frac{G_1G_2G_3 + G_4(1 + G_2H_2 + G_3H_3 + G_2G_3H_2H_3)}{[1 + G_1H_1 + G_2H_2 + G_3H_3 + G_1G_2H_1H_2 + G_2G_3H_2H_3 + G_1G_3H_1H_3 + G_1G_2G_3H_1H_2H_3]}$$

Examples on signal flow graph-

1. Determine the transfer function $Y(s)/R(s)$ using Mason's gain formula for the system shown in signal flow graph given below.



Soln No. of forward path and their gain-

$P_1 = G_1G_2G_3G_4$

$P_2 = G_5$

No. of loops and their gain-

$L_1 = -G_1H_1$

$L_2 = -G_3H_3$

$L_3 = -G_1G_2G_3G_4H_2$

$L_4 = -G_5H_2$

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No. of two non touching loops and product of their gain-

$L_1 L_2 = G_1 G_3 H_1 H_3$

$L_2 L_4 = G_3 G_5 H_2 H_3$

$\therefore \Delta = 1 + G_1H_1 + G_3H_3 + G_1G_2G_3G_4H_2 + G_5H_2 + G_1G_3H_1H_3 + G_3G_5H_2H_3$

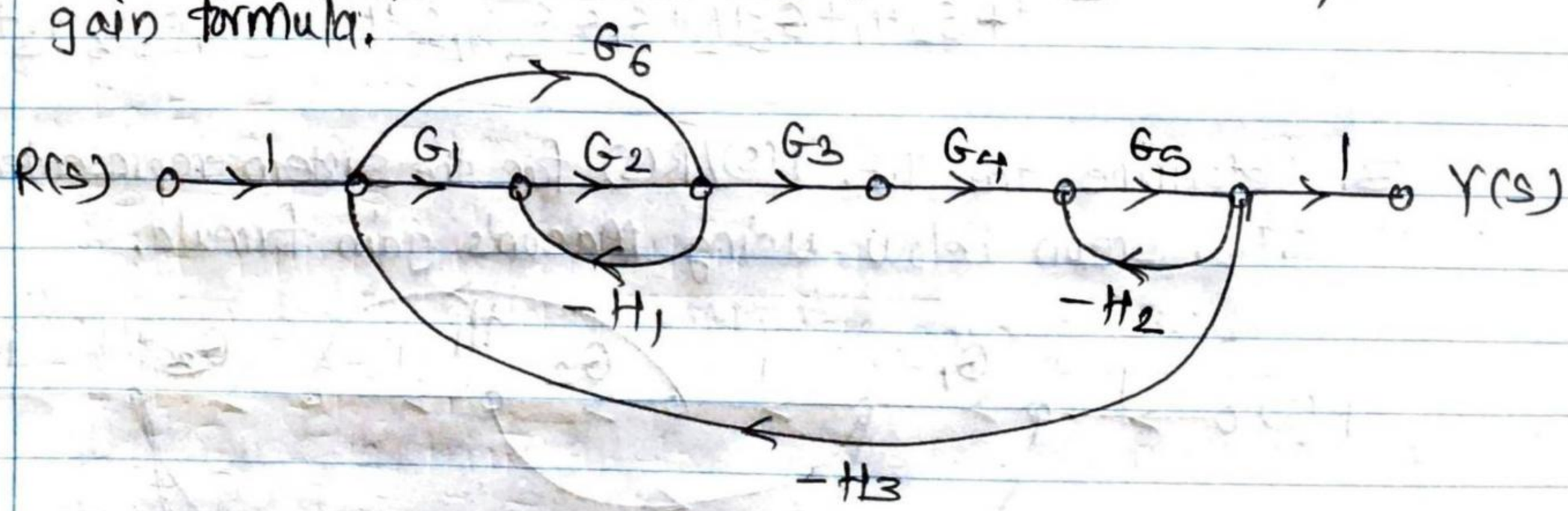
$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 + G_3H_3$

$$G(s) = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4 + G_5(1 + G_3H_3)}{1 + G_1H_1 + G_3H_3 + G_1G_2G_3G_4H_2 + G_5H_2 + G_1G_3H_1H_3 + G_3G_5H_2H_3}$$

2. Determine the transfer function $Y(s)/R(s)$ for the system represented by signal flow graph shown below by Mason's gain formula.



Soln No. of forward path and their gain-

$P_1 = G_1G_2G_3G_4G_5$

$P_2 = G_3G_4G_5G_6$

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No. of loops and their gain -

$L_1 = -G_2H_1$

$L_2 = -G_5H_2$

$L_3 = -G_1G_2G_3G_4G_5H_3$

$L_4 = -G_3G_4G_5G_6H_3$

Two non touching loops and product of their gain -

$L_1L_2 = G_2G_5H_1H_2$

$\therefore \Delta = 1 + G_2H_1 + G_5H_2 + G_1G_2G_3G_4G_5H_3 + G_3G_4G_5G_6H_3 + G_2G_5H_1H_2$

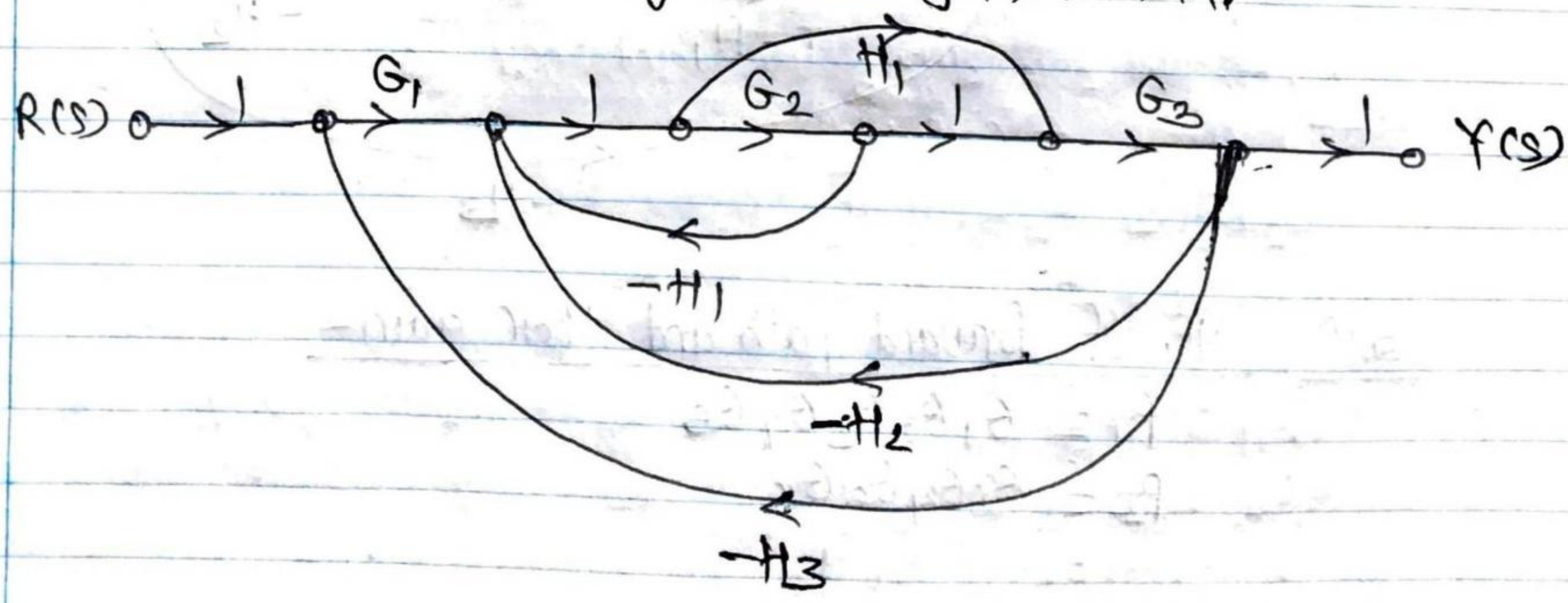
$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$$\therefore G(s) = \frac{\sum P_k\Delta_k}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3G_4G_5 + G_3G_4G_5G_6}{1 + G_2H_1 + G_5H_2 + G_1G_2G_3G_4G_5H_3 + G_3G_4G_5G_6H_3 + G_2G_5H_1H_2}$$

3. Determine the T.F. $Y(s)/R(s)$ for the system represented by S.F.G. shown below using Mason's gain formula.



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Sol^n No. of forward path and their gain -

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 H_1 G_3$

No. of loops and their gain -

$L_1 = -G_2 H_1$

$L_2 = -G_2 G_3 H_2$

$L_3 = -G_1 G_2 G_3 H_3$

$L_4 = -G_1 H_1 G_3 H_3$

$L_5 = -H_1 G_3 H_2$

~~There~~ All loops are touching each other.

$\therefore \Delta = 1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 H_3 + G_1 H_1 G_3 H_3 + H_1 G_3 H_2$

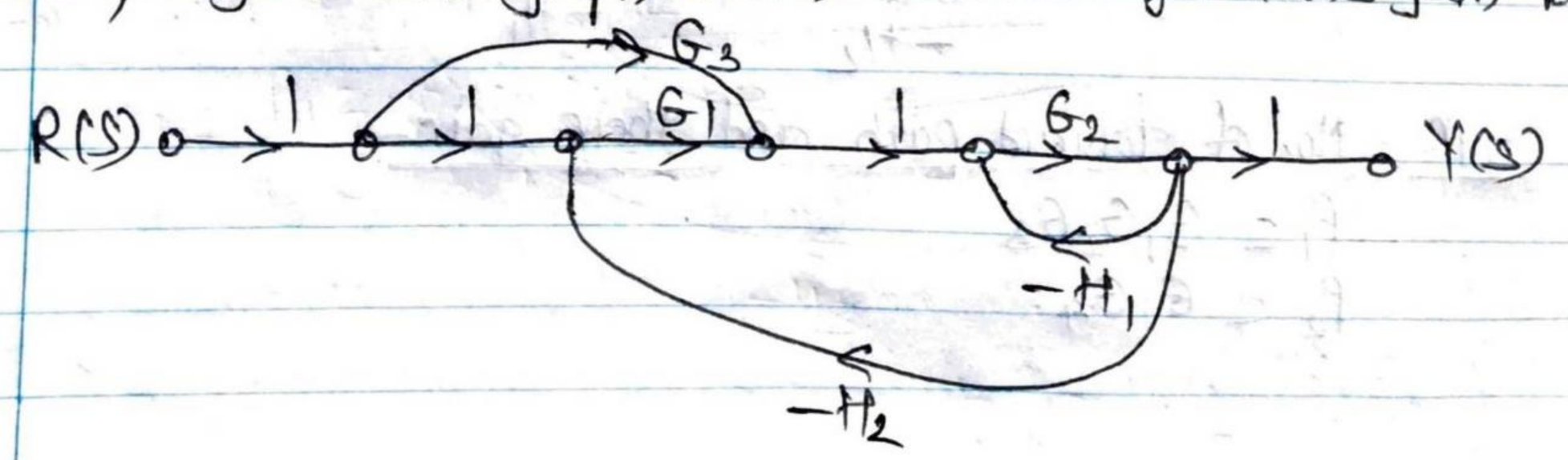
$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$$\therefore G(s) = \frac{\sum P_k \Delta_k}{\Delta} = P_1 \Delta_1 + P_2 \Delta_2$$

$$= \frac{G_1 G_2 G_3 + G_1 H_1 G_3}{1 + G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 H_3 + G_1 H_1 G_3 H_3 + H_1 G_3 H_2}$$

4. Determine transfer function $Y(s)/R(s)$ for the system represented by signal flow graph shown below using Mason's gain formula.



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Sol^n No. of forward path and their gain–

$P_1 = G_1 G_2$

$P_2 = G_2 G_3$

No. of loops and their gain–

$L_1 = -G_1 G_2 H_2$

$L_2 = -G_2 H_1$

$\therefore \Delta = 1 + G_1 G_2 H_2 + G_2 H_1$

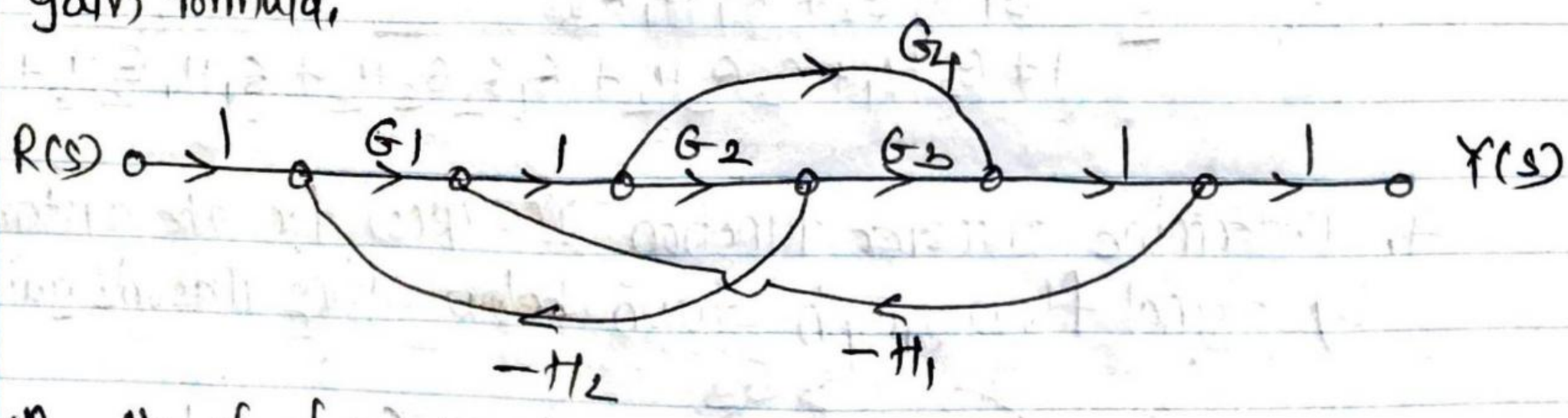
$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$$\therefore G(s) = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 + G_2 G_3}{1 + G_1 G_2 H_2 + G_2 H_1}$$

5. Determine the transfer function $Y(s)/R(s)$ of the system represented by the signal flow graph shown below by Mason's gain formula.



Sol^n No. of forward path and their gain–

$P_1 = G_1 G_2 G_3$

$P_2 = G_1 G_4$

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No. of loops and their gain-

$L_1 = -G_1 G_2 H_2$

$L_2 = -G_2 G_3 H_1$

$L_3 = -G_4 H_1$

$\therefore \Delta = 1 + G_1 G_2 H_2 + G_2 G_3 H_1 + G_4 H_1$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - 0 = 1$

$$\therefore G(s) = \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 G_3 + G_1 G_4}{1 + G_1 G_2 H_2 + G_2 G_3 H_1 + G_4 H_1}$$

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