Bode plot- Bode plot is a graphical representation of frequency response. It comprises of two plots on the same paper.

1. Magnitude plot (Gain in dB V/s frequency)
2. Phase plot (Phase in degree V/s frequency)

In magnitude plot, the magnitude of system in dB is plotted against frequency on logarithmic scale. In phase plot, phase of system is plotted against frequency on logarithmic scale.

Both plots are drawn on same semi log paper.

Bode plots have following applications-

1. From Bode plot we get graphical representation of frequency response of system.
2. Stability analysis.
3. Controller tuning and compensator design.
4. Transfer function of system can be obtained from recorded frequency domain data.

For a system with transfer function,

$$G(s)H(s) = \frac{k(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}, \quad m \le n$$

the frequency response function is

$$G(j\omega)H(j\omega) = \frac{k(j\omega+z_1)(j\omega+z_2)\cdots(j\omega+z_m)}{(j\omega+p_1)(j\omega+p_2)\cdots(j\omega+p_n)}$$

Then magnitude response function is

$$|G(j\omega)H(j\omega)| = \frac{k|j\omega+z_1||j\omega+z_2|\cdots|j\omega+z_m|}{|j\omega+p_1||j\omega+p_2|\cdots|j\omega+p_n|}$$

$\therefore$ Magnitude response in dB is

$$20\log|G(j\omega)H(j\omega)| = 20\log k + 20\log|j\omega+z_1| + 20\log|j\omega+z_2| + \cdots + 20\log|j\omega+z_m| - 20\log|j\omega+p_1| - 20\log|j\omega+p_2| - 20\log|j\omega+p_n|$$

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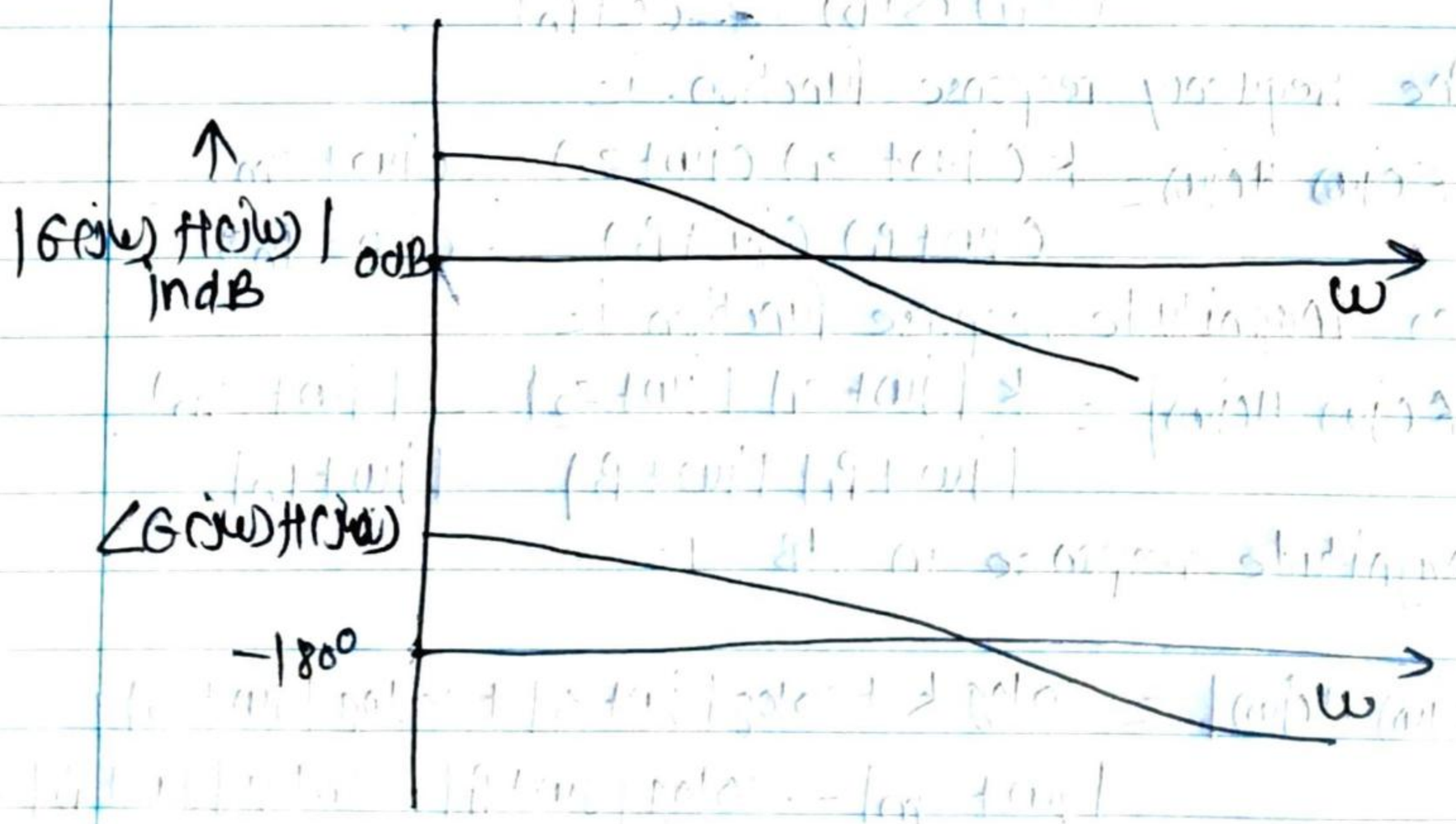
The phase response function is,

$$\angle G(j\omega)H(j\omega) = \angle j\omega+z_1 + \angle j\omega+z_2 + \dots + \angle j\omega+z_m - \angle j\omega+p_1 - \angle j\omega+p_2 - \dots - \angle j\omega+p_n$$

Then the functions $20\log|G(j\omega)H(j\omega)|$ and $\angle G(j\omega)H(j\omega)$ are tabulated for different frequencies as shown below

| $\omega$ | $20\log\|G(j\omega)H(j\omega)\|$ | $\angle G(j\omega)H(j\omega)$ |
|---|---|---|
| 0.1 | | |
| 0.2 | | |
| ⋮ | | |
| 10 | | |
| 20 | | |
| 50 | | |
| ⋮ | | |
| $\infty$ | | |

The values in table are then plotted on semilog paper to get Bode plot as shown below




Typical Bode plot

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## Asymptotic Bode plot-

In asymptotic Bode plots, the effect of poles and zeros before their corner frequencies is neglected and after corner frequency it is considered as −20dB/dec for every pole and 20dB/dec for every zero.

## Different factors of transfer function and their effect on asymptotic Bode plot-

The transfer function has following different factors

1. Gain k
2. Pole at origin
3. Zero at origin
4. Multiple poles at origin
5. Multiple zeros at origin
6. Pole at $s=-a$
7. zero at $s=-a$
8. Multiple poles at $s=-a$
9. Multiple zeros at $s=-a$
10. Complex conjugate pole pair
11. Complex conjugate zero pair

The effect of these factors on asymptotic Bode plot is as follows.

1. Gain k- Gain k has effect $20\log k$ for all $\omega$ on gain plot and no effect on phase plot.

2. Pole at origin-

Effect on gain plot-

For pole at origin,

$$G(s) = \frac{1}{s}$$

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$\therefore G(j\omega) = \frac{1}{j\omega}$

$\therefore |G(j\omega)| = \frac{1}{\omega} = -20\log\omega$ dB

Thus the gain decreases by 20 dB for every decade of frequency. Hence it is a line with slope −20dB/dec from origin.

<u>Effect on phase plot</u>-

$\angle G(j\omega) = -90^\circ$ for all $\omega$

3. <u>Zero at origin</u>-

<u>Effect on gain plot</u>-

for zero at origin,

$G(s) = s$

$\therefore G(j\omega) = j\omega$

$|G(j\omega)| = \omega = 20\log\omega$ dB

Thus gain increases by 20 dB for every decade of freq. Hence it is a line with slope 20 dB/dec from origin.

<u>Effect on phase plot</u>-

$\angle G(j\omega) = 90^\circ$ for all $\omega$.

4. <u>Multiple poles at origin</u>-

For 'n' poles at origin,

$G(s) = \frac{1}{s^n}$

$\therefore G(j\omega) = \frac{1}{(j\omega)^n}$

<u>Effect on gain plot</u>-

$|G(j\omega)| = \frac{1}{\omega^n} = -20n\log\omega$ dB

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Thus it is a line with $-20n$ dB/dec slope from origin.

<u>Effect on phase plot</u>–

$\angle G(j\omega) = (-90)^\circ n$ for all $\omega$

5. <u>Multiple zeros at origin</u>–

For 'n' zeros at origin,

$$G(s) = s^n$$

$$\therefore G(j\omega) = (j\omega)^n$$

<u>Effect on gain plot</u>–

$$|G(j\omega)| = \omega^n = 20 \cdot n \log \omega \text{ dB}$$

Thus it is a straight line with slope $20n$ dB/dec from origin.

<u>Effect on phase plot</u>–

$\angle G(j\omega) = (90)^\circ n$ for all $\omega$

6. <u>Pole at $s = -a$</u>

For pole at $s = -a$,

$$G(s) = \frac{a}{s+a}$$

$$\therefore G(j\omega) = \frac{a}{j\omega + a}$$

<u>Effect on gain plot</u>–

$$|G(j\omega)| = \frac{a}{\sqrt{\omega^2 + a^2}} = \cancel{-20 \log \sqrt{\omega^2+a^2}}$$

$$= \frac{1}{\sqrt{1+(\omega/a)^2}} = -20 \log \sqrt{1+(\omega/a)^2} \text{ dB}$$

For $\omega \ll a$, $|G(j\omega)| = -20 \log \sqrt{1+(\omega/a)^2} \approx 0$

For $\omega \gg a$, $|G(j\omega)| = -20 \log \sqrt{1+(\omega/a)^2} \approx -20 \log \omega/a$

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Thus gain decreases by 20dB/dec from $\omega = a$. Thus the effect is approximated as,

For $\omega < a$ effect is 0 dB/dec

For $\omega \geq a$ effect is $-20$dB/dec.

<u>Effect on phase plot</u>–

$$\angle G(j\omega) = -\tan^{-1} \omega/a$$

7. <u>Zero at $s = -a$</u>

For zero at $s = -a$

$$G(s) = \frac{1}{a}[s+a]$$

$$\therefore G(j\omega) = \frac{1}{a}[j\omega + a] = 1 + j\,\omega/a$$

<u>Effect on gain plot</u>–

$$|G(j\omega)| = \sqrt{1+\left(\frac{\omega}{a}\right)^2} = 20\log\sqrt{1+\left(\frac{\omega}{a}\right)^2}\ dB$$

For $\omega << a$,

$$|G(j\omega)| \simeq 0\,dB$$

For $\omega >> a$

$$|G(j\omega)| = 20\log\frac{\omega}{a}\ dB$$

Thus gain increases by 20 dB/dec from $\omega = a$ & the effect is approximated as,

For $\omega < a$ effect is 0 dB/dec

For $\omega \geq a$ effect is 20 dB/dec

~~Similarly for multiple poles at $s = -a$~~

<u>Effect on phase plot</u>–

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{a}$$

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8. Multiple poles at $s=-a$

For multiple poles at $s=-a$ the effect on gain and phase plots is as follows

Effect on gain plot-

For $\omega < a$, effect is 0 dB/dec

For $\omega \geq a$, effect is $-20n$ dB/dec (for 'n' poles)

Effect on phase plot-

$\angle G(j\omega) = -n \tan^{-1} \omega/a$

9. Multiple zeros at $s=-a$-

For multiple zeros at $s=-a$ the effect on gain and phase plots is as follows

Effect on gain plot-

For $\omega < a$, effect is 0 dB/dec

For $\omega \geq a$, effect is $20n$ dB/dec (for 'n' zeros)

Effect on phase plot-

$\angle G(j\omega) = n \tan^{-1} \omega/a$

10. Complex conjugate pole pair-

For complex conjugate pole pair at $s=-\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$ the transfer function is,

$$G(s) = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$\therefore G(j\omega) = \frac{\omega_n^2}{(j\omega)^2+2\xi\omega_n(j\omega)+\omega_n^2}$$

$$= \frac{1}{1+2\xi\left(\frac{j\omega}{\omega_n}\right)+\left(\frac{j\omega}{\omega_n}\right)^2}$$

Effect on gain plot-

$$|G(j\omega)| = \frac{1}{\left[\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2\right]^{1/2}}$$

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$$\therefore |G(j\omega)| = -20 \log \sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}$$

For $\omega << \omega_n$

$|G(j\omega)| \approx 0$ dB

For $\omega >> \omega_n$

$$|G(j\omega)| \approx -20 \log \left(\frac{\omega}{\omega_n}\right)^2 = -40 \log \frac{\omega}{\omega_n} \text{ dB}$$

$\therefore$ It is a line with slope $-40$ dB/dec from $\omega = \omega_n$. with resonant peak at $\omega_r$.

<u>Effect on phase plot</u>

$$\angle G(j\omega) = -\tan^{-1}\left[\frac{2\xi\, \omega/\omega_n}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

11. <u>Complex conjugate zero pair at $s = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$</u>

Similar to complex conjugate pole pair the effect on gain plot is,

For $\omega < \omega_n$ effect is 0 dB/dec

For $\omega > \omega_n$ effect is 40 dB/dec with resonant peak at $\omega_r$

The effect on phase plot is,

$$\angle G(j\omega) = \tan^{-1}\left[\frac{2\xi\, \omega/\omega_n}{1-\left(\frac{\omega}{\omega_n}\right)^2}\right]$$

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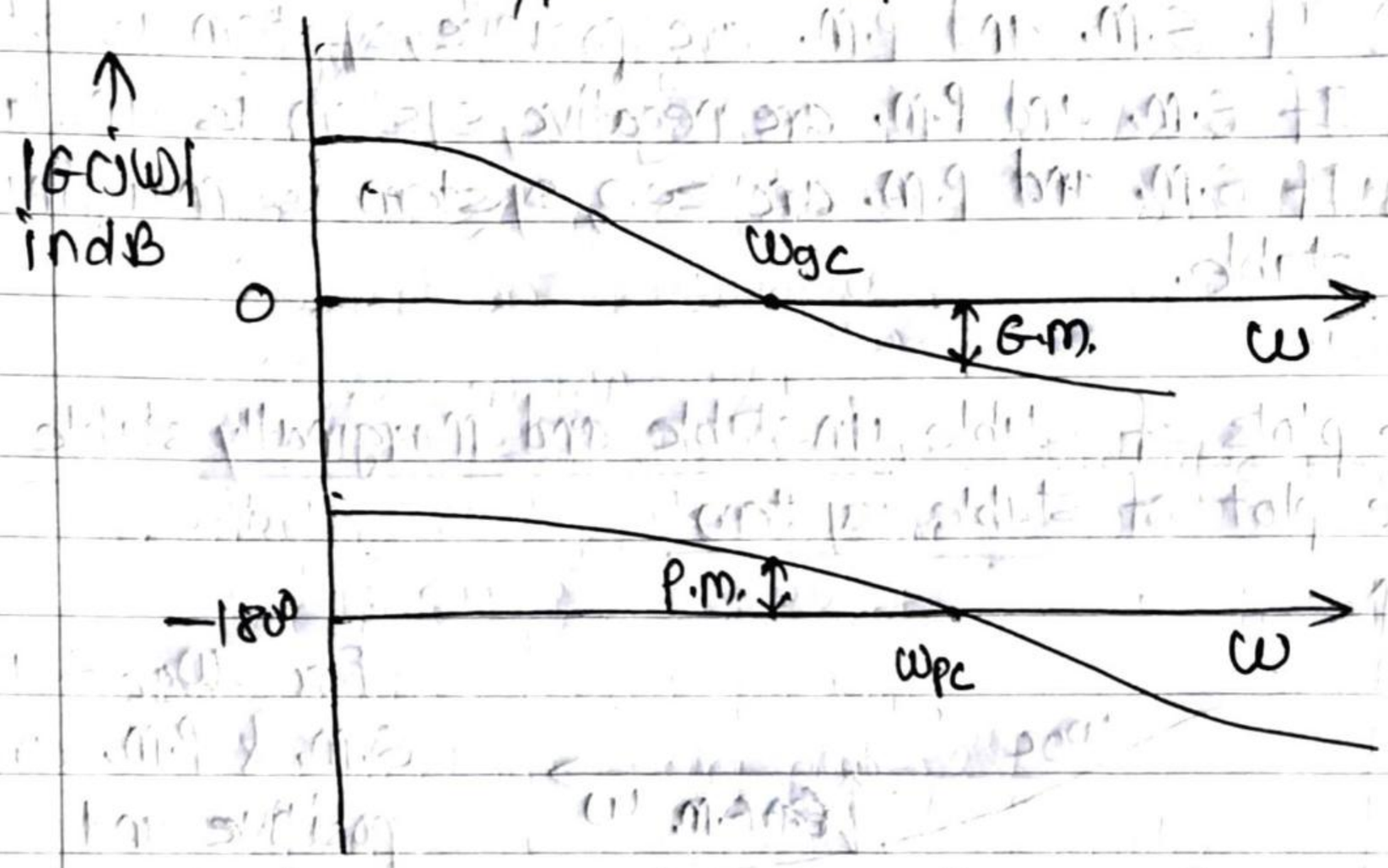
## Stability analysis using Bode plot -

### Determination of $\omega_{gc}$, $\omega_{pc}$, G.M. and P.M. from Bode plot

Consider a typical Bode plot as shown below.




Gain crossover frequency is the frequency at which gain plot cuts 0 dB line. Phase crossover frequency is the frequency at which phase plot cuts $-180^\circ$ line

The gain margin is determined as,

$$G.M. = 0 - |G(j\omega)|_{\omega=\omega_{pc}}$$

The phase margin is determined as,

$$P.M. = 180^\circ + \angle G(j\omega)|_{\omega=\omega_{pc}}$$

The system is stable if gain margin and phase margin are positive. The system is unstable if gain margin and phase margin are negative. The system is marginally stable if gain margin and phase margin are zero.

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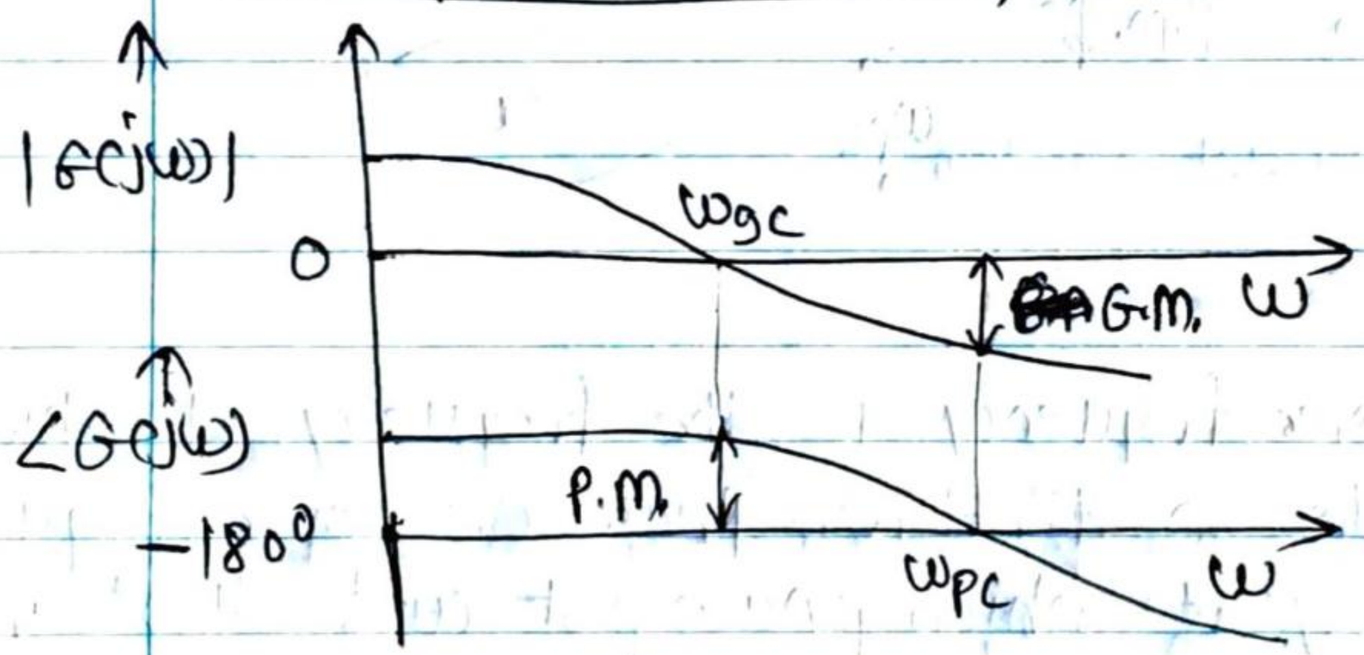
Thus the steps in stability analysis are

1. Draw Bode plot for given system.
2. Determine $\omega_{gc}$, $\omega_{pc}$, G.M. and P.M. from Bode plot.
3. a] If G.M. and P.M. are positive, system is stable.
   b] If G.M. and P.M. are negative, system is unstable.
   c] If G.M. and P.M. are zero, system is marginally stable.

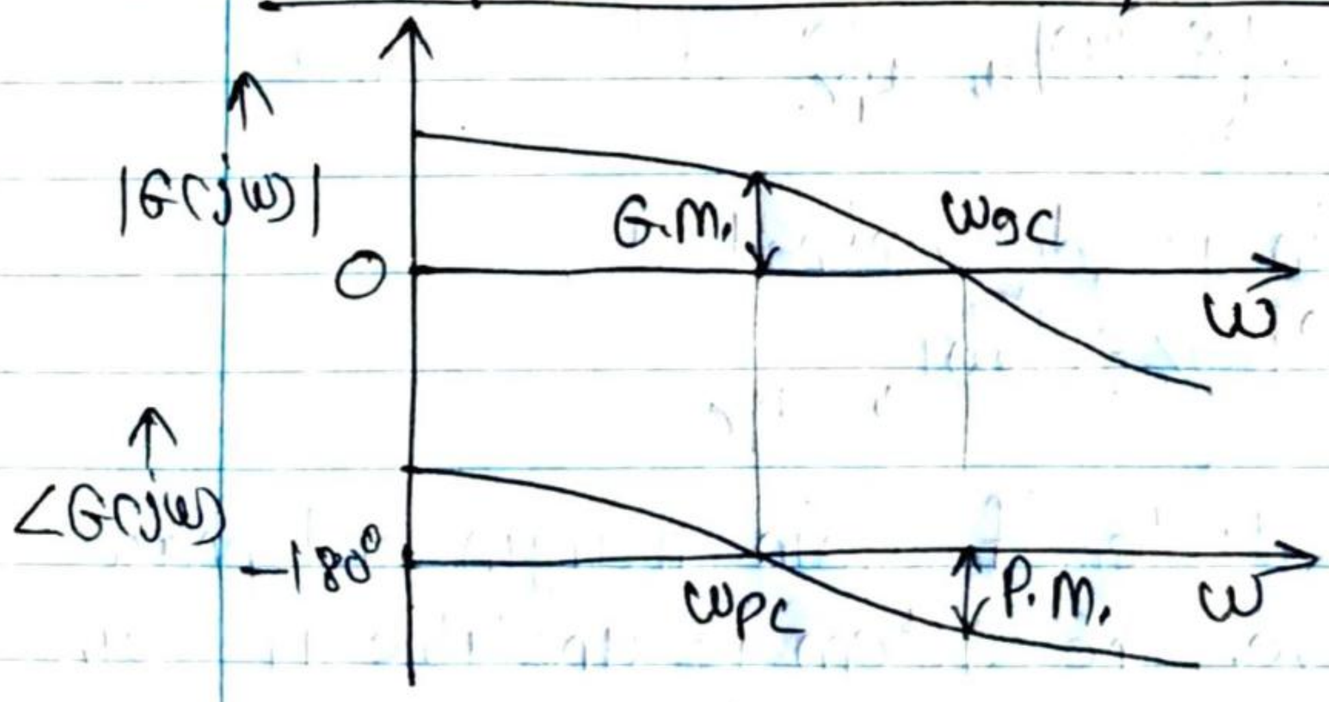
## Bode plots of stable, unstable and Marginally stable systems

### Bode plot of stable system




For $\omega_{gc} < \omega_{pc}$ G.M. & P.M. are positive and system is stable

### Bode plot of unstable system




For $\omega_{gc} > \omega_{pc}$ G.M. & P.M. are negative and system is unstable

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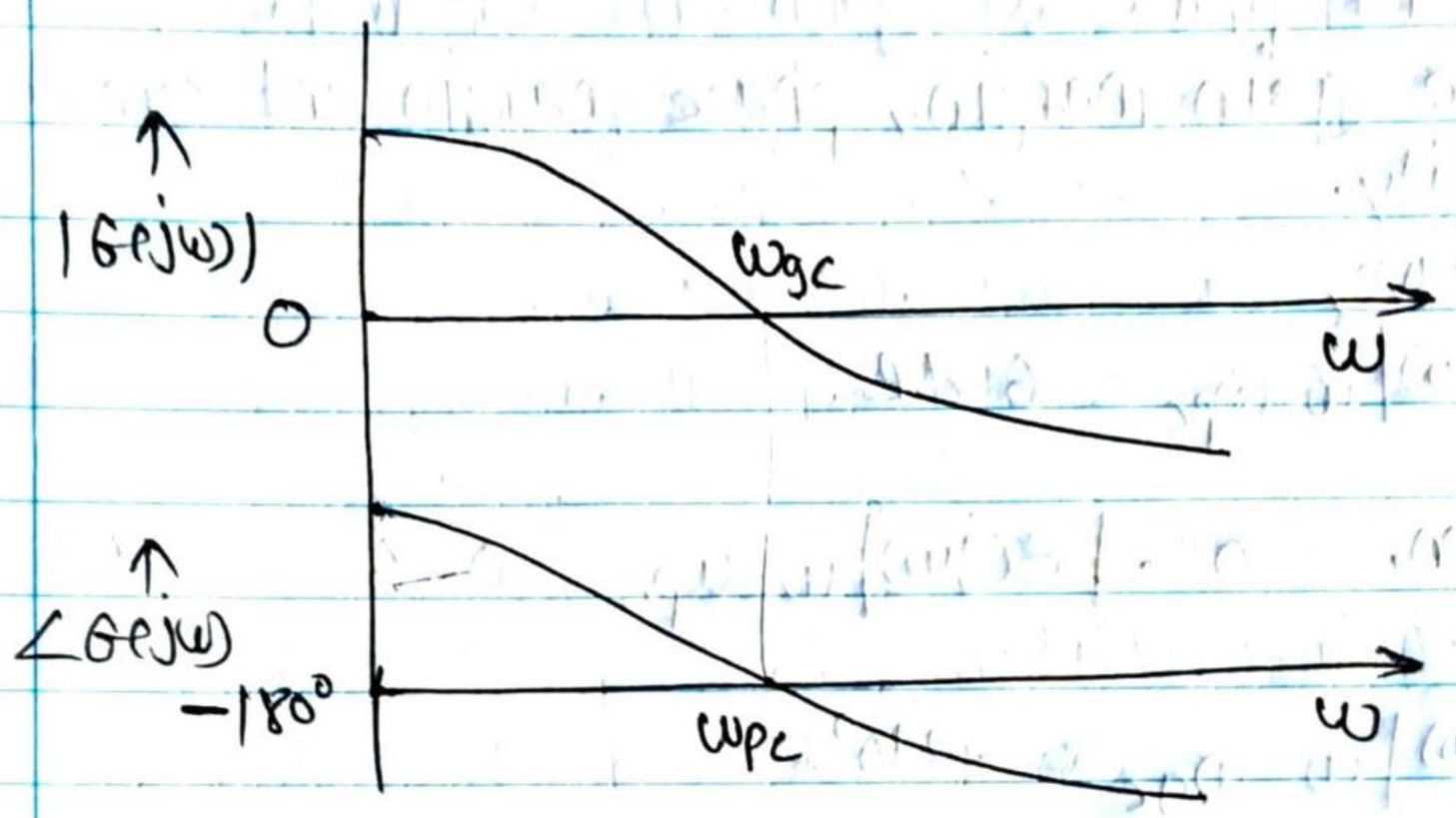
## Bode plot of marginally stable system –




For $\omega_{gc} = \omega_{pc}$

G.M. and P.M. are zero and system is marginally stable.

## Examples on stability analysis using Bode plot –

1. If the system has phase at gain crossover frequency $-140^\circ$ and gain at phase cross over frequency $-8$ dB. Determine gain margin, phase margin and comment on stability.

Sol$^n$ We have,

$$|G(j\omega)|_{\omega=\omega_{pc}} = -8\,dB$$

$$\therefore G.M. = 0 - |G(j\omega)|_{\omega=\omega_{pc}} = 0-(-8)$$
$$= 8\,dB$$

$$\angle G(j\omega)|_{\omega=\omega_{gc}} = -140^\circ$$

$$\therefore P.M. = 180^\circ + \angle G(j\omega)|_{\omega=\omega_{gc}}$$
$$= 180^\circ + (-140^\circ) = 40^\circ$$

$\therefore$ The system is stable because G.M. and P.M. are positive.

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2. The system has gain at phase cross over frequency 6 dB and phase at gain crossover frequency $-210^0$. Determine gain margin, phase margin and comment on stability.

<u>soln</u> We have,

$$|G(j\omega)|_{\omega=\omega_{pc}} = 6 \text{ dB}$$

$$\therefore G.M. = 0 - |G(j\omega)|_{\omega=\omega_{pc}} = 0-6 = -6 \text{ dB}$$

$$\angle G(j\omega)|_{\omega=\omega_{gc}} = -210^0$$

$$\therefore P.M. = 180^0 + \angle G(j\omega)|_{\omega=\omega_{gc}} = 180^0 + (-210^0) = -30^0$$

$\therefore$ The system is unstable as G.M. & P.M. are negative.

3. Comment on the stability of following systems with

a] $\omega_{gc} = 4$ rad/sec $\omega_{pc} = 8$ rad/sec

b] $\omega_{gc} = 6$ rad/sec $\omega_{pc} = 5$ rad/sec

c] $\omega_{gc} = 10$ rad/sec $\omega_{pc} = 10$ rad/sec

<u>soln</u> a] The system has $\omega_{gc} = 4$ rad/sec, $\omega_{pc} = 8$ rad/sec
The system is stable because $\omega_{gc} < \omega_{pc}$.

b] The system has $\omega_{gc} = 6$ rad/sec, $\omega_{pc} = 5$ rad/sec
The system is unstable because $\omega_{gc} > \omega_{pc}$.

c] The system has $\omega_{gc} = 10$ rad/sec, $\omega_{pc} = 10$ rad/sec
The system is marginally stable because $\omega_{gc} = \omega_{pc}$.

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## Examples on Bode plot–

1. Draw the Bode plot of system with open loop transfer function $G(s) = \frac{10}{s(s+2)(s+5)}$. Determine $\omega_{gc}, \omega_{pc}, G.M., P.M.$ and comment on stability.

**Sol^n** The given system has open loop transfer function

$$G(s) = \frac{10}{s(s+2)(s+5)}$$

$$\therefore G(j\omega) = \frac{10}{j\omega(j\omega+2)(j\omega+5)}$$

$$= \frac{10}{10j\omega(1+j\omega/2)(1+j\omega/5)}$$

$$= \frac{1}{j\omega(1+j\omega/2)(1+j\omega/5)}$$

### Analysis for gain plot–

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $K=1$ | $20\log K = 0\,dB$ | |
| 2. Pole at origin | $-20\,dB/dec$ from origin | $-20\,dB/dec$ from origin upto $\omega=2$ |
| 3. Pole at $\omega=2$ | $-20\,dB/dec$ from $\omega=2$ | $-40\,dB/dec$ from $\omega=2$ upto $\omega=5$ |
| 4. Pole at $\omega=5$ | $-20\,dB/dec$ from $\omega=5$ | $-60\,dB/dec$ from $\omega=5$ onwards |

### Analysis for phase plot–

$$\phi = -90^\circ - \tan^{-1}\omega/2 - \tan^{-1}\omega/5$$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-94^\circ$ | 5 | $-203.20^\circ$ |
| 0.2 | $-98^\circ$ | 10 | $-232.13^\circ$ |
| 0.5 | $-109.75^\circ$ | 20 | $-250.25^\circ$ |
| 1 | $-127.87^\circ$ | 50 | $-262^\circ$ |
| 2 | $-156.80^\circ$ | $\infty$ | $-270^\circ$ |

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From Bode plot we get,

$\omega_{gc} = 1 \text{ rad/sec}$ $\quad$ G.M. $= 14\,dB$

$\omega_{pc} = 3.2 \text{ rad/sec}$ $\quad$ P.M. $= 54^\circ$

As G.M. and P.M. are positive the system is stable.

2. Draw Bode plot of system with open loop transfer function $G(s) = \dfrac{100}{(s+1)(s+2)(s+5)}$ and comment on its stability.

<u>soln</u> The given system has open loop transfer function,

$$G(s) = \frac{100}{(s+1)(s+2)(s+5)}$$

$$\therefore G(j\omega) = \frac{100}{(j\omega+1)(j\omega+2)(j\omega+5)}$$

$$= \frac{100}{10(1+j\omega)(1+j\omega/2)(1+j\omega/5)}$$

$$= \frac{10}{(1+j\omega)(1+j\omega/2)(1+j\omega/5)}$$

<u>Analysis for gain plot</u>

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $k = 10$ | $20\log k = 20\,dB$ | |
| 2. Pole at $\omega = 1$ | $-20\,dB/dec$ from $\omega = 1$ | $-20\,dB/dec$ from $\omega = 1$ upto $\omega = 2$ |
| 3. Pole at $\omega = 2$ | $-20\,dB/dec$ from $\omega = 2$ | $-40\,dB/dec$ from $\omega = 2$ upto $\omega = 5$ |
| 4. Pole at $\omega = 5$ | $-20\,dB/dec$ from $\omega = 5$ | $-60\,dB/dec$ from $\omega = 5$ onwards |

<u>Analysis for phase plot</u>

$$\phi = -\tan^{-1}\omega - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{5}$$

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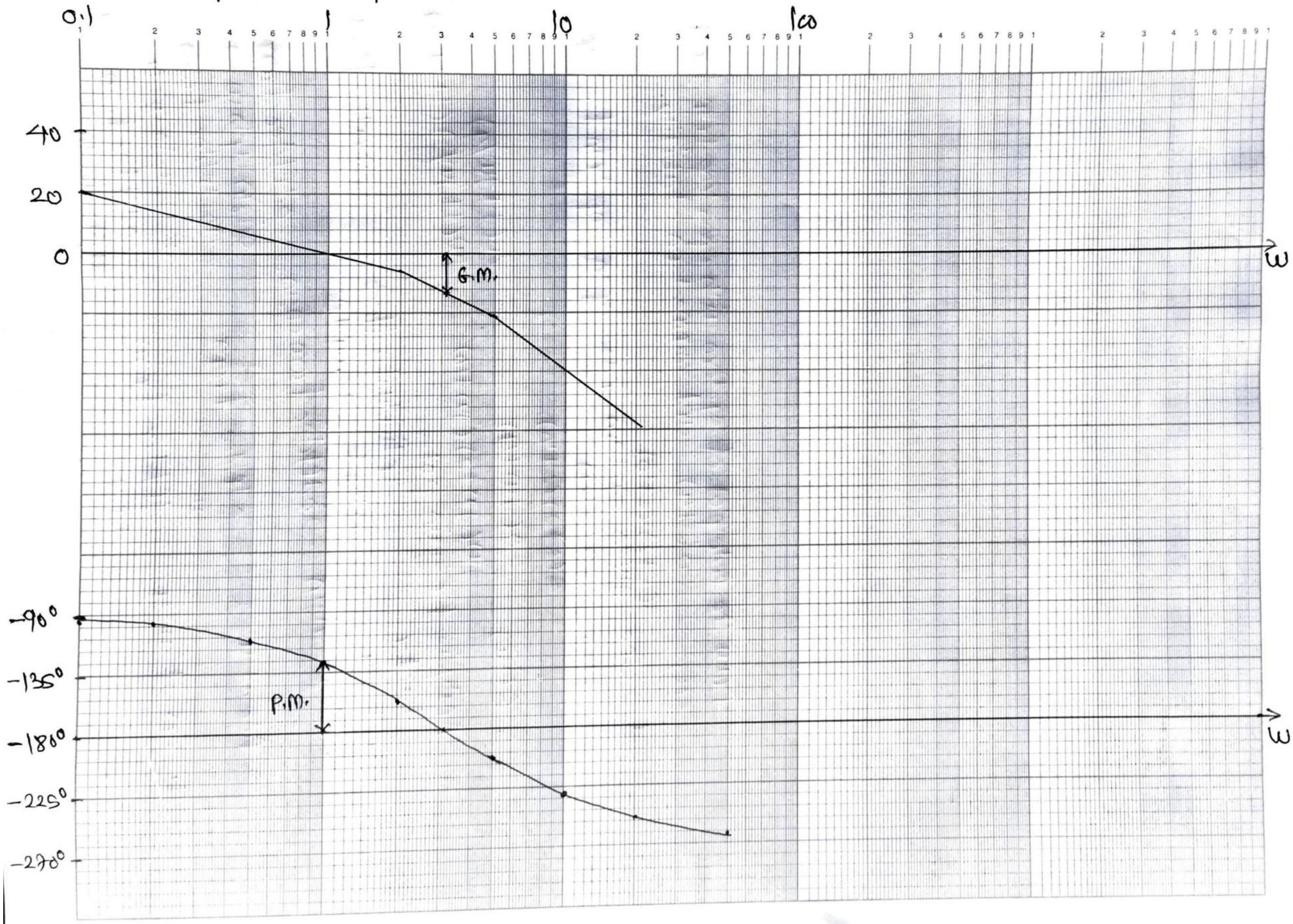
NOTES Bode plot of example 1

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| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-9.72^\circ$ | 5 | $-191.89^\circ$ |
| 0.2 | $-19.31^\circ$ | 10 | $-226.41^\circ$ |
| 0.5 | $-46.31^\circ$ | 20 | $-247.39^\circ$ |
| 1 | $-82.87^\circ$ | 50 | $-260.85^\circ$ |
| 2 | $-130.24^\circ$ | $\infty$ | $-270^\circ$ |

From Bode plot we get,

$\omega_{gc} = 4.2$ rad/sec $\quad$ G.M. $= 0$ dB

$\omega_{pc} = 4.2$ rad/sec $\quad$ P.M. $= 0^\circ$

As G.M. $= 0$ and P.M. $= 0^\circ$, the system is marginally stable.

3. Draw Bode plot of the system with open loop transfer function $G(s) = \dfrac{5}{s(1+2s)(1+4s)}$ and comment on stability.

<u>Soln</u> The system has open loop transfer function

$$G(s) = \frac{5}{s(1+2s)(1+4s)}$$

$$\therefore G(j\omega) = \frac{5}{j\omega(1+j2\omega)(1+j4\omega)}$$

$$= \frac{5}{j\omega(1+j\omega/0.5)(1+j\omega/0.25)}$$

<u>Analysis for gain plot</u>

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $K = 5$ | $20\log K = 13.98$ dB | |
| 2. Pole at origin | $-20$ dB/dec from origin | $-20$ dB/dec from origin upto $\omega = 0.25$ |
| 3. Pole at $\omega = 0.25$ | $-20$ dB/dec from $\omega = 0.25$ | $-40$ dB/dec from $\omega = 0.25$ upto $\omega = 0.5$ |
| 4. Pole at $\omega = 0.5$ | $-20$ dB/dec from $\omega = 0.5$ | $-60$ dB/dec from $\omega = 0.5$ onwards |

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Analysis for phase plot-

$\phi = -90^\circ - \tan^{-1} 2\omega - \tan^{-1} 4\omega$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-123.11^\circ$ | 2 | $-248.84^\circ$ |
| 0.2 | $-150.46^\circ$ | 5 | $-261.43^\circ$ |
| 0.5 | $-198.43^\circ$ | 10 | $-265.71^\circ$ |
| 1 | $-229.40^\circ$ | $\infty$ | $-270^\circ$ |

From Bode plot we get,

$\omega_{gc} = 0.85$ rad/sec  G.M. $= -18$ dB

$\omega_{pc} = 0.38$ rad/sec  P.M. $= -40^\circ$

As G.M. and P.M. are negative the system is unstable.

4. Draw the Bode plot of system with open loop transfer function $G(s) = \dfrac{20(s+4)}{s(s+2)(s+10)}$ and comment on stability.

Sol$^n$ The given system has open loop transfer function,

$$G(s) = \frac{20(s+4)}{s(s+2)(s+10)}$$

$$\therefore G(j\omega) = \frac{20(j\omega+4)}{j\omega(j\omega+2)(j\omega+10)} = \frac{4(1+j\omega/4)}{j\omega(1+j\omega/2)(1+j\omega/10)}$$

Analysis for gain plot-

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $K=4$ | $20\log K = 12.04$ dB | |
| 2. Pole at origin | $-20$ dB/dec from origin | $-20$ dB/dec from origin upto $\omega=2$ |
| 3. Pole at $\omega=2$ | $-20$ dB/dec from $\omega=2$ | $-40$ dB/dec from $\omega=2$ upto $\omega=4$ |
| 4. zero at $\omega=4$ | 20 dB/dec from $\omega=4$ | $-20$ dB/dec from $\omega=4$ upto $\omega=10$ |
| 5. Pole at $\omega=10$ | $-20$ dB/dec from $\omega=10$ | $-40$ dB/dec from $\omega=10$ onwards |

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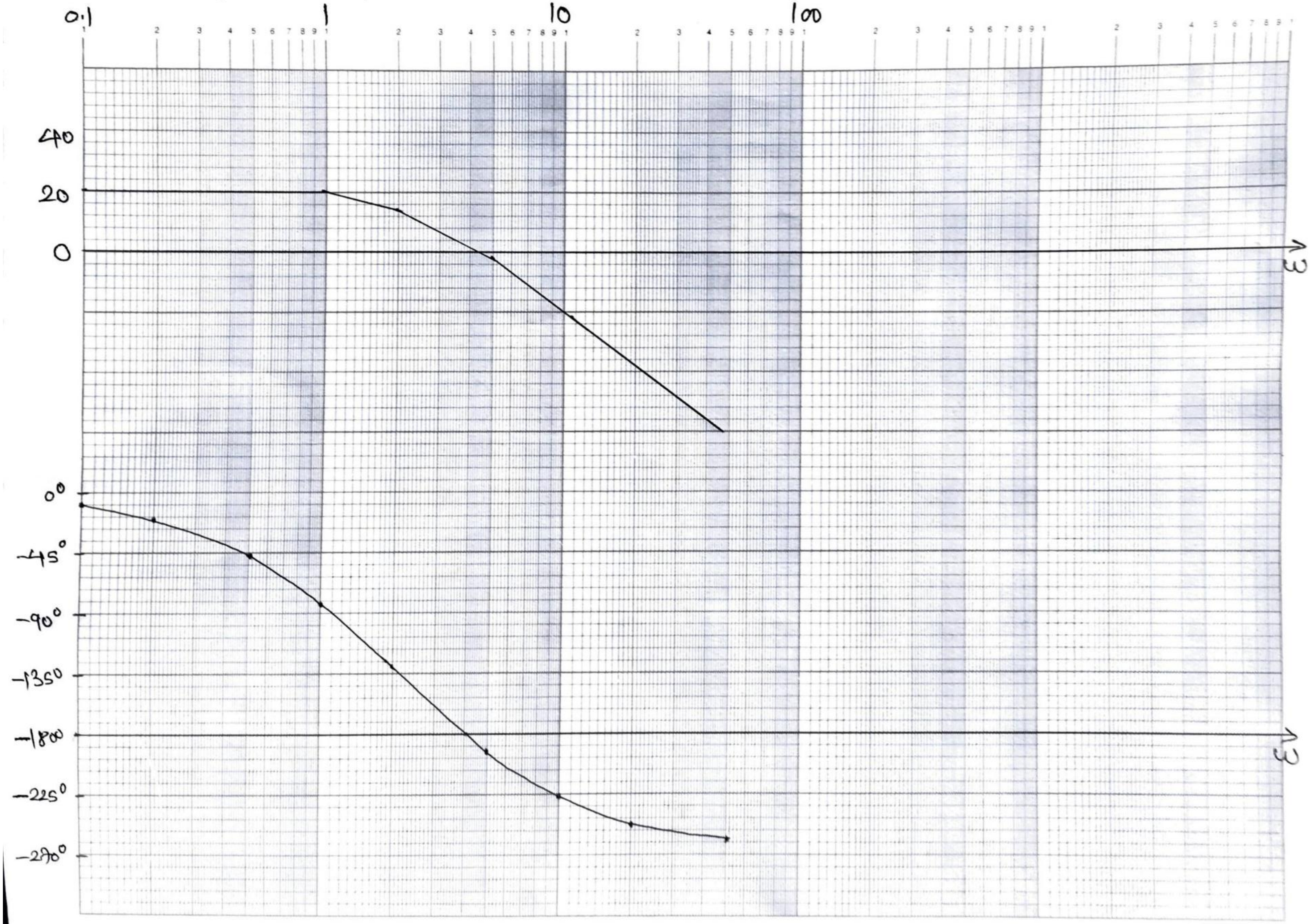
NOTES Bode plot of example 2

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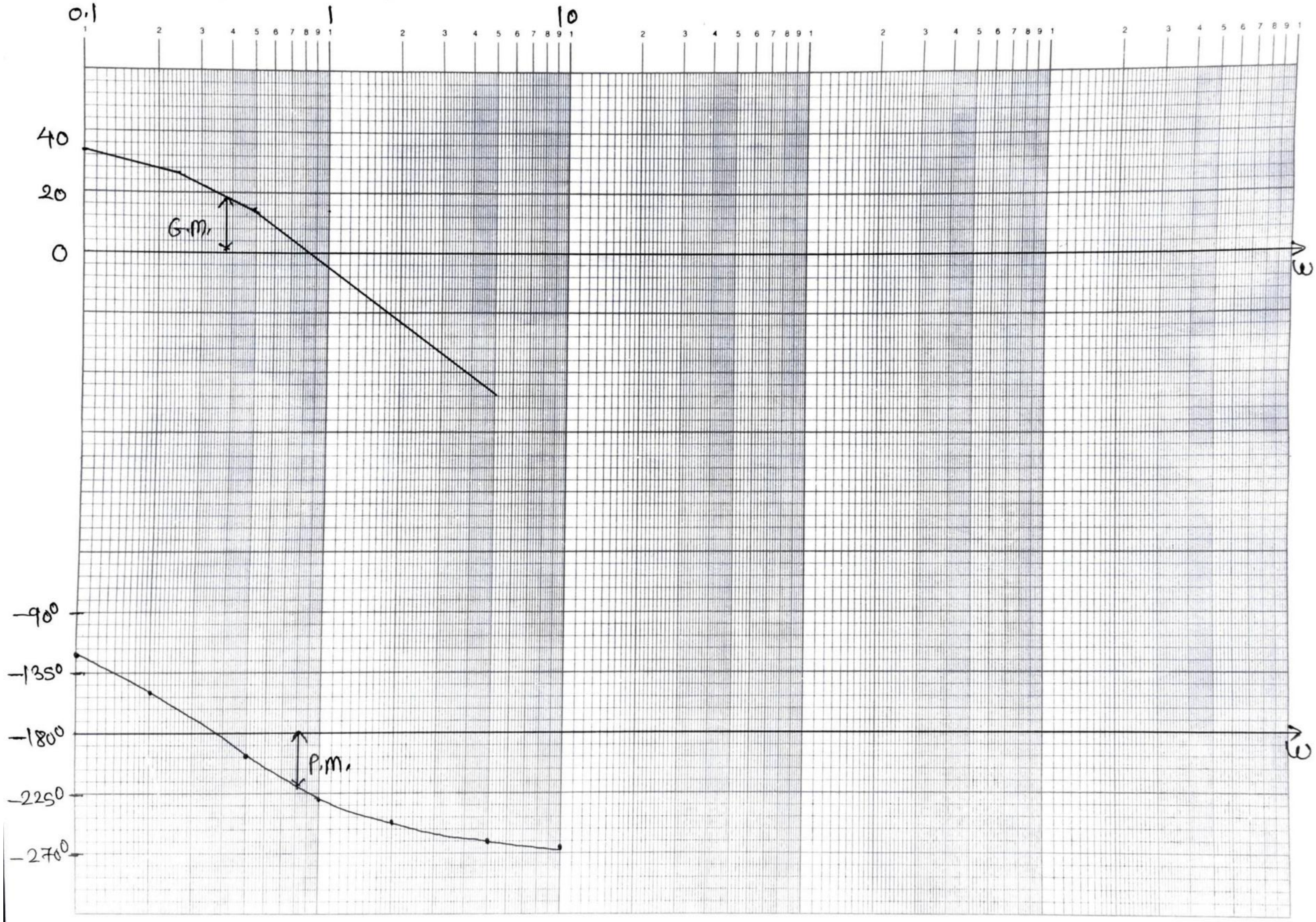
NOTES Bode plot of example 3

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Analysis for phase plot-

$$\phi = -90 - \tan^{-1}\frac{\omega}{2} + \tan^{-1}\frac{\omega}{4} - \tan^{-1}\frac{\omega}{10}$$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-92^\circ$ | 5 | $-133.42^\circ$ |
| 0.2 | $-94^\circ$ | 10 | $-145.49^\circ$ |
| 0.5 | $-99.77^\circ$ | 20 | $-159.03^\circ$ |
| 1 | $-108.24^\circ$ | 50 | $-170.97^\circ$ |
| 2 | $-119.74^\circ$ | $\infty$ | $-180^\circ$ |

From Bode plot we get,

$\omega_{gc} = 2.9$ rad/sec $\quad$ G.M. $= \infty$

$\omega_{pc} = \infty$ rad/sec $\quad$ P.M. $= 54^\circ$

As G.M. and P.M. are positive the system is stable.

5. Draw Bode plot of system with open loop transfer function $G(s) = \dfrac{50(s+2)}{s(s+5)(s+10)}$ and comment on stability.

soln The open loop transfer function of system is,

$$G(s) = \frac{50(s+2)}{s(s+5)(s+10)}$$

$$\therefore G(j\omega) = \frac{50(j\omega+2)}{j\omega(j\omega+5)(j\omega+10)}$$

$$= \frac{100(1+j\omega/2)}{50j\omega(1+j\omega/5)(1+j\omega/10)}$$

$$= \frac{2(1+j\omega/2)}{j\omega(1+j\omega/5)(1+j\omega/10)}$$

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## Analysis for gain plot-

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $K=2$ | $20\log K = 6.02$ dB | |
| 2. Pole at origin | $-20$dB/dec from origin | $-20$dB/dec from origin upto $\omega=2$ |
| 3. zero at $\omega=2$ | 20dB/dec from $\omega=2$ | 0dB/dec from $\omega=2$ upto $\omega=5$ |
| 4. Pole at $\omega=5$ | $-20$dB/dec from $\omega=5$ | $-20$dB/dec from $\omega=5$ upto $\omega=10$ |
| 5. Pole at $\omega=10$ | $-20$dB/dec from $\omega=10$ | $-40$ dB/dec from $\omega=10$ onwards |

## Analysis for phase plot-

$$\phi = -90^\circ + \tan^{-1}\omega/2 - \tan^{-1}\omega/5 - \tan^{-1}\omega/10$$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-88.86^\circ$ | 5 | $-93.37^\circ$ |
| 0.2 | $-87.73^\circ$ | 10 | $-119.74^\circ$ |
| 0.5 | $-84.54^\circ$ | 20 | $-145.11^\circ$ |
| 1 | $-80.46^\circ$ | 50 | $-165.27^\circ$ |
| 2 | $-78.11^\circ$ | $\infty$ | $-180^\circ$ |

From Bode plot we get,

$\omega_{gc} = 2$ rad/sec $\qquad$ G.M. $= \infty$ dB

$\omega_{pc} = \infty$ rad/sec $\qquad$ P.M. $= 102^\circ$

As G.M. and P.M. are positive the system is stable.

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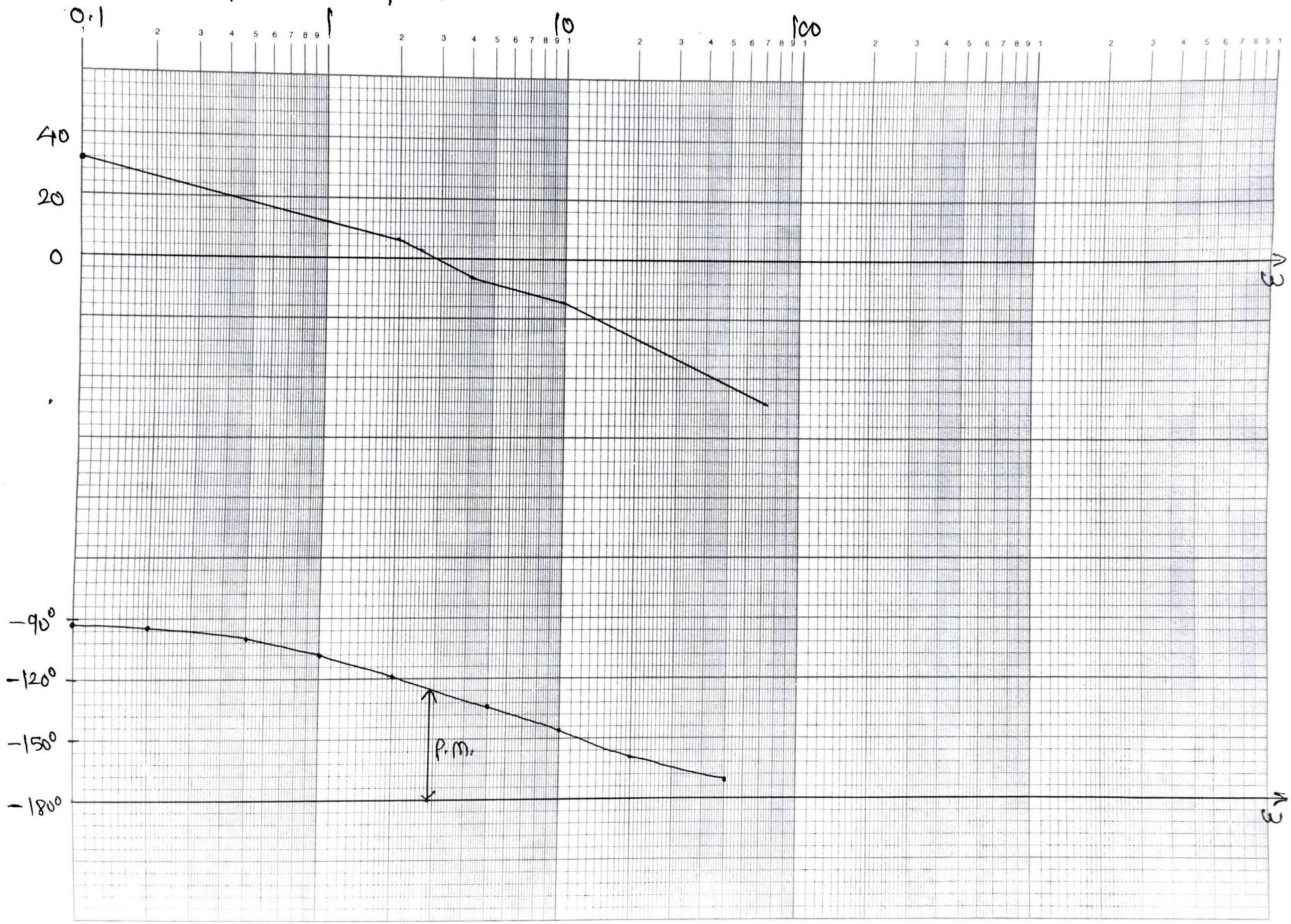
NOTES Bode plot of example 4

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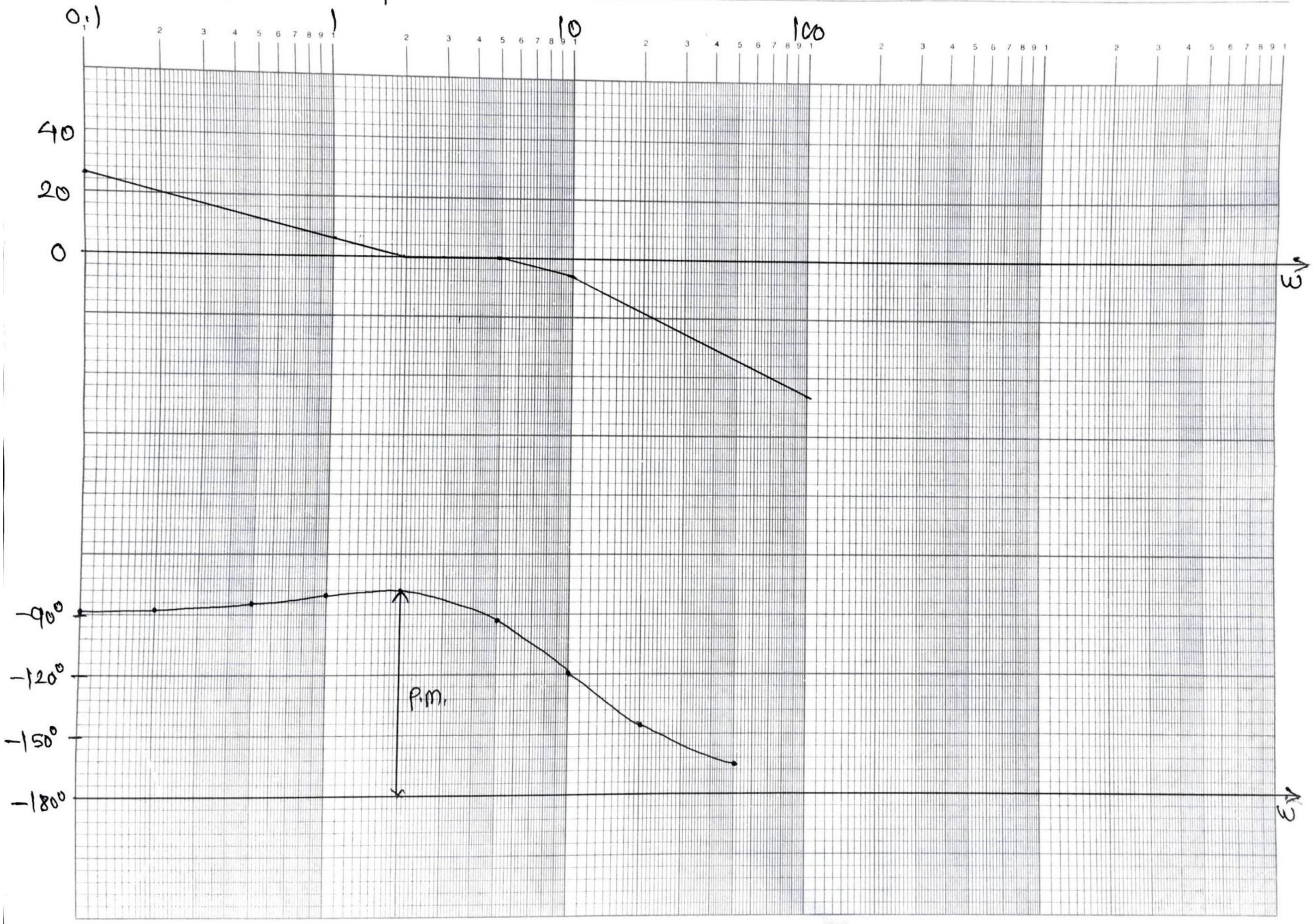
NOTES Bode plot of examples

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6. For a system with open loop transfer function $G(s) = \frac{40(s+1)}{s^2(s+4)(s+10)}$

sketch Bode plot and comment on stability.

Soln The system has open loop transfer function,

$$G(s) = \frac{40(s+1)}{s^2(s+4)(s+10)}$$

$$\therefore G(j\omega) = \frac{40(j\omega+1)}{(j\omega)^2(j\omega+4)(j\omega+10)} = \frac{(1+j\omega)}{(j\omega)^2(1+j\omega/4)(1+j\omega/10)}$$

Analysis for gain plot-

| Factor | Individual effect | Overall effect |
|---|---|---|
| 1. $K=1$ | $20\log K = 0dB$ | |
| 2 Double pole at origin | $-40dB/dec$ from origin | $-40dB/dec$ from origin upto $\omega=1$ |
| 3. zero at $\omega=1$ | $20dB/dec$ from $\omega=1$ | $-20dB/dec$ from $\omega=1$ upto $\omega=4$ |
| 4. Pole at $\omega=4$ | $-20dB/dec$ from $\omega=4$ | $-40dB/dec$ from $\omega=4$ upto $\omega=10$ |
| 5 Pole at $\omega=10$ | $-20dB/dec$ from $\omega=10$ | $-60dB/dec$ from $\omega=10$ onwards |

Analysis for phase plot-

$$\phi = -180^\circ + \tan^{-1}\omega - \tan^{-1}\omega/4 - \tan^{-1}\omega/10$$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-176.29^\circ$ | 5 | ~~$-208.90^\circ$~~ $-179.22^\circ$ |
| 0.2 | $-172.70^\circ$ | 10 | $-208.90^\circ$ |
| 0.5 | $-163.42^\circ$ | 20 | $-234.99^\circ$ |
| 1 | $-154.74^\circ$ | 50 | $-255.26^\circ$ |
| 2 | ~~$-179.28^\circ$~~ $-154.44^\circ$ | $\infty$ | $-270^\circ$ |

From Bode plot we get,

$\omega_{gc} = 1 rad/sec$ $\omega_{pc} = 5 rad/sec$ $G.M. = 16dB$ $P.M. = 24^\circ$

As ~~gain~~ G.M. & P.M. are positive system is stable.

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7. Draw Bode plot of system with open loop transfer function $G(s) = \frac{k}{s(s+1)(s+5)}$ and determine k if

(i) system is marginally stable, (ii) G.M. = 10 dB
(iii) $\omega_{gc} = 4$ rad/sec, (iv) P.M. = 40°

<u>Sol^n</u> The given system has open loop transfer function

$$G(s) = \frac{k}{s(s+1)(s+5)}$$

$$G(j\omega) = \frac{k}{j\omega(j\omega+1)(j\omega+5)} = \frac{k/5}{j\omega(1+j\omega)(1+j\omega/5)}$$

Let $k_1 = k/5$

For drawing Bode plot take $k_1 = 1$

<u>Analysis for gain plot-</u>

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $k_1 = 1$ | $20\log k_1 = 0$ dB | |
| 2. Pole at origin | −20 dB/dec from origin | −20 dB/dec from origin upto $\omega = 1$ |
| 3. Pole at $\omega = 1$ | −20 dB/dec from $\omega = 1$ | −40 dB/dec from $\omega = 1$ upto $\omega = 5$ |
| 4. Pole at $\omega = 5$ | −20 dB/dec from $\omega = 5$ | −60 dB/dec from $\omega = 5$ onwards |

<u>Analysis for phase plot-</u>

$$\phi = -90° - \tan^{-1}\omega - \tan^{-1}\omega/5$$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | −96.86° | 5 | −213.69° |
| 0.2 | −103.60° | 10 | −237.72° |
| 0.5 | −122.28° | 20 | −253.10° |
| 1 | −146.31° | 50 | −263.14° |
| 2 | −175.24° | ∞ | −270° |

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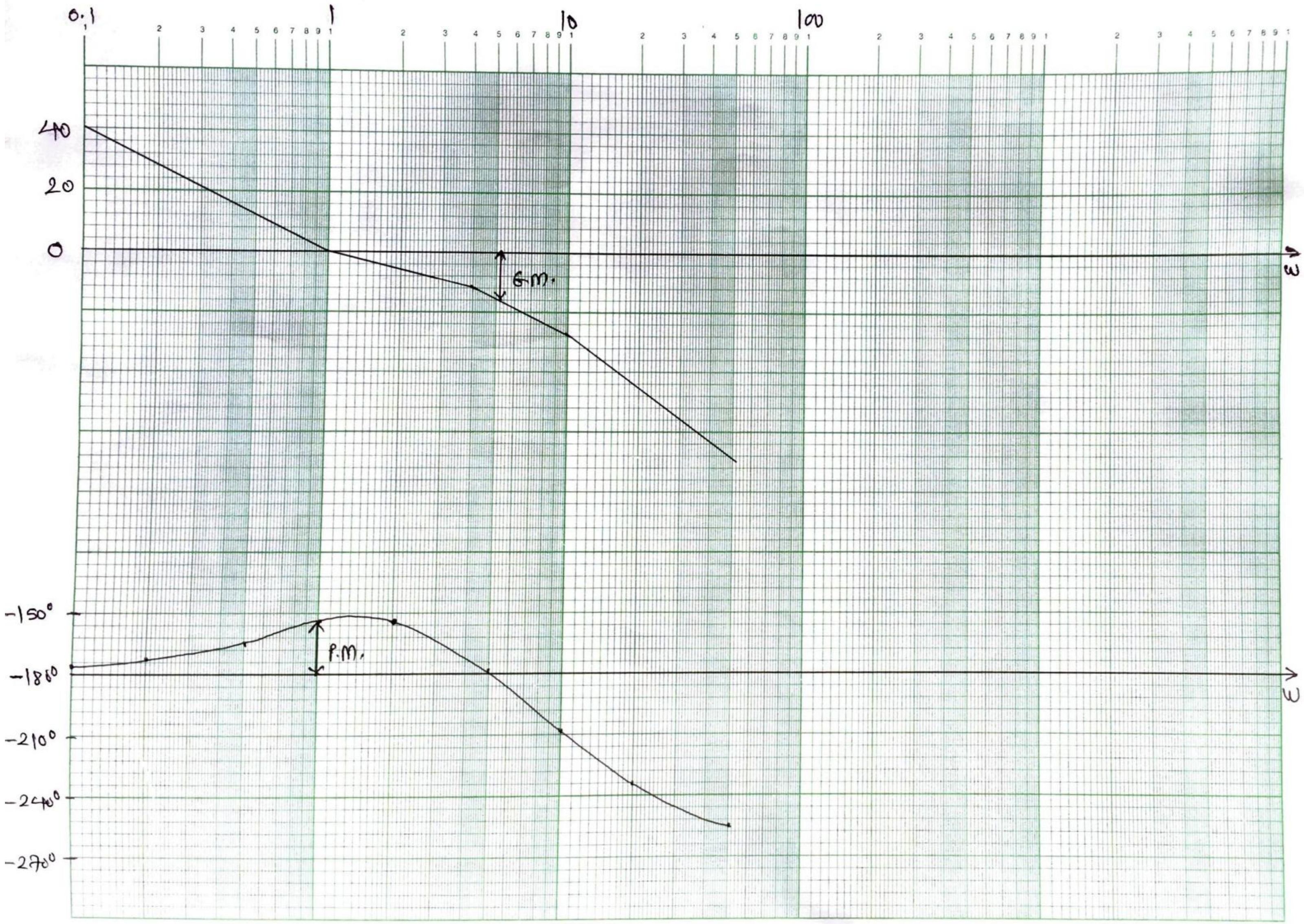
NOTES Bode plot of example 6

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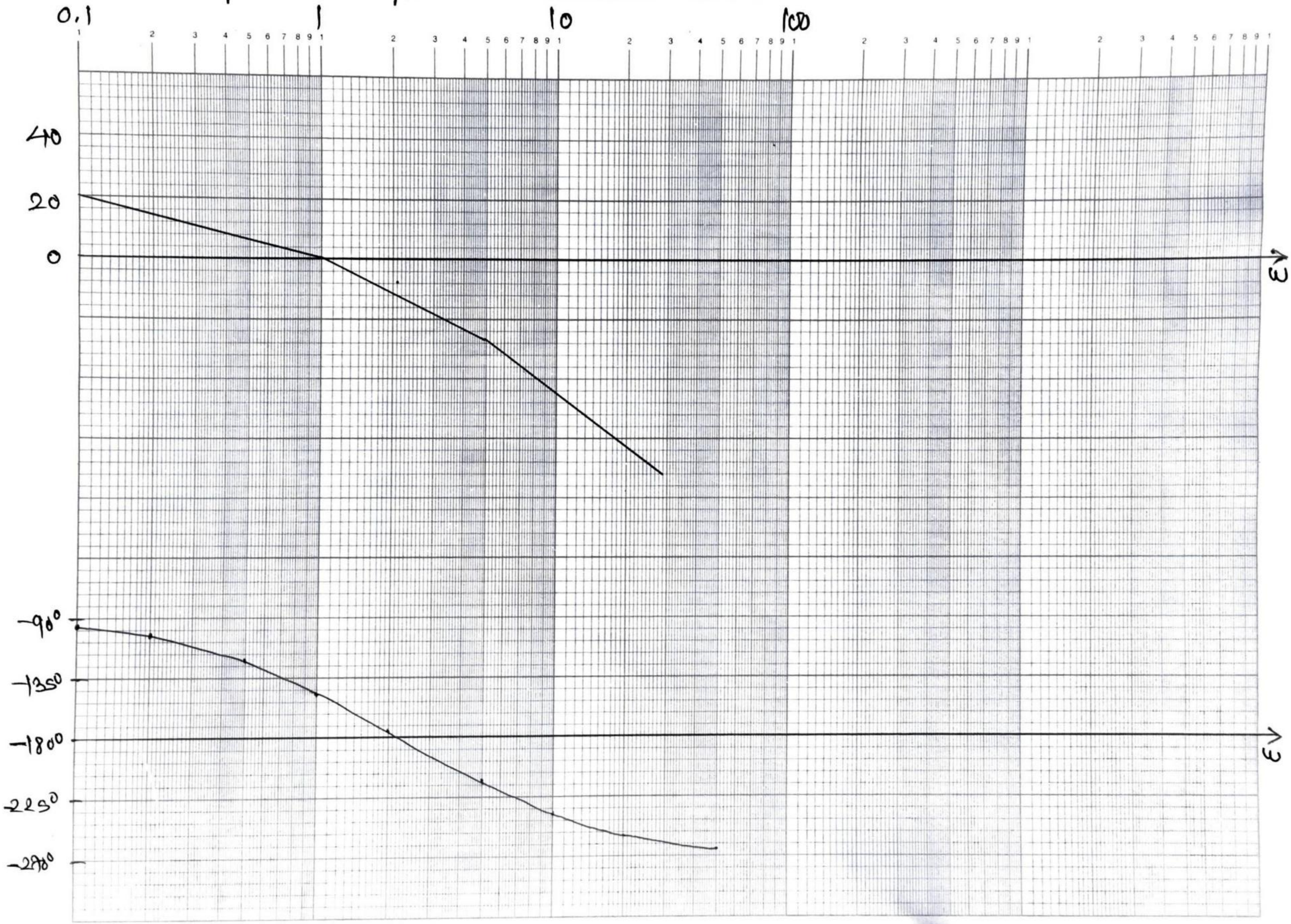
NOTES Bode plot of example 7

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From Bode plot for $k_1 = 1$

$\omega_{gc} = 1$ rad/sec, $\omega_{pc} = 2.2$ rad/sec, G.M. = 13dB, P.M. = 30°

(i) For system to be marginally stable

$$\omega_{gc} = \omega_{pc} = 2.2 \text{ rad/sec}$$

At $\omega = \omega_{gc}$, $|G(j\omega)| = -13$dB

$\therefore 20\log k_1 = 13$ dB

$\therefore k_1 = 4.4668$

$\therefore k = 5k_1 = 22.33$

(ii) For G.M. = 10dB, we can shift gain plot by 13−10=3dB

$\therefore 20\log k_1 = 3$

$\therefore k_1 = 1.4125$

$\therefore k = 5k_1 = 7.0627$

(iii) For $\omega_{gc} = 4$ rad/sec

At $\omega = 4$ rad/sec, $|G(j\omega)| = 24$dB

$\therefore 20\log k_1 = 24$

$\therefore k_1 = 15.8489$

$\therefore k = 5k_1 = 79.2445$

(iv) For P.M. = 40°

$\angle G(j\omega) = -140°$ at $\omega = 0.85$ rad/sec

$\therefore \omega_{gc\,new} = 0.85$ rad/sec. At $\omega = 0.85$, $|G(j\omega)| = -2$dB

$\therefore 20\log k_1 = -2$

$\therefore k_1 = 0.7943$

$\therefore k = 5k_1 = 3.9716$

8. Draw Bode plot of system with open loop transfer function $G(s) = \dfrac{k}{s(1+2s)(1+5s)}$ and determine k for system to be marginally stable.

Soln The open loop system has transfer function

$$G(s) = \frac{k}{s(1+2s)(1+5s)}$$

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$\therefore G(j\omega) = \dfrac{k}{j\omega(1+2j\omega)(1+5j\omega)} = \dfrac{k}{j\omega(1+j\omega/0.5)(1+j\omega/0.2)}$

For drawing Bode plot, let $k=1$

Analysis for gain plot-

| Factor | Individual effect | overall effect |
|---|---|---|
| 1. $k=1$ | $20\log k = 0$ dB | |
| 2. Pole at origin | $-20$ dB/dec from origin | $-20$ dB/dec from origin upto $\omega=0.2$ |
| 3. Pole at $\omega=0.2$ | $-20$ dB/dec from $\omega=0.2$ | $-40$ dB/dec from $\omega=0.2$ upto $\omega=0.5$ |
| 4. Pole at $\omega=0.5$ | $-20$ dB/dec from $\omega=0.5$ | $-60$ dB/dec from $\omega=0.5$ onwards |

Analysis for phase plot-

$\phi = -90^\circ - \tan^{-1}2\omega - \tan^{-1}5\omega$

| $\omega$ | $\phi$ | $\omega$ | $\phi$ |
|---|---|---|---|
| 0.1 | $-127.88^\circ$ | 2 | $-250.25^\circ$ |
| 0.2 | $-156.80^\circ$ | 5 | $-262^\circ$ |
| 0.5 | $-203.20^\circ$ | 10 | $-266^\circ$ |
| 1 | $-232.13^\circ$ | $\infty$ | $-270^\circ$ |

From Bode plot for $k=1$

$\omega_{gc} = 0.44$ rad/sec $\omega_{pc} = 0.32$ rad/sec G.M. $= -6$dB P.M. $= -16^\circ$

For Marginally stable system

$\omega_{gc} = \omega_{pc} = 0.32$ rad/sec

At $\omega = 0.32$ $|G(j\omega)| = 6$

$\therefore 20\log k = -6$

$\therefore k = 0.5012$

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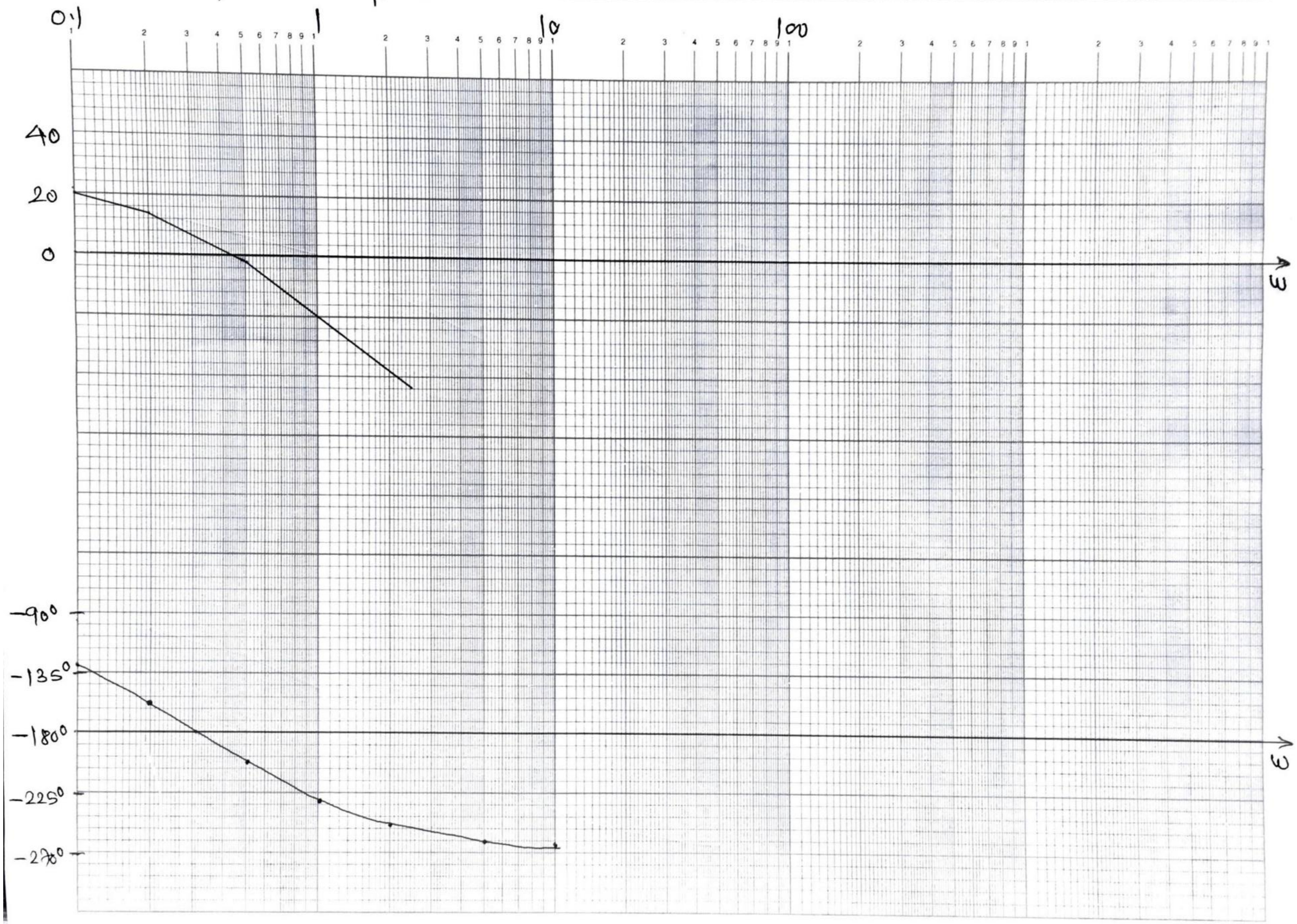
NOTES Bode plot of example 8

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