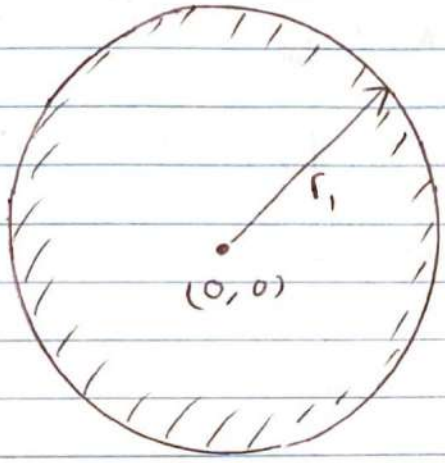
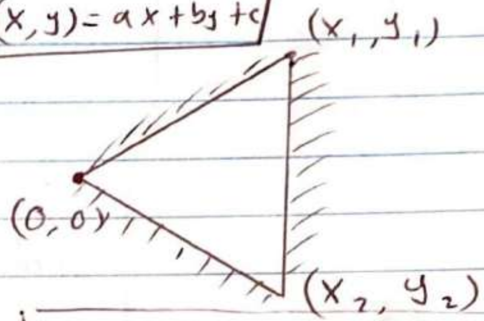
Exercise 1

Lets assume some coordinates. [using Linear Primitive f]

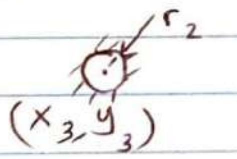


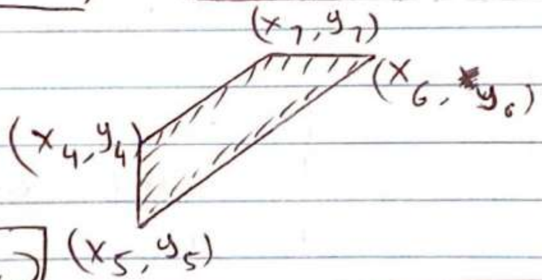
$$\boxed{f(x,y) = ax + by + c}$$



$$\boxed{H_a \Rightarrow \left\{ x^2 + y^2 - r_1^2 \leq 0 \right\}}$$

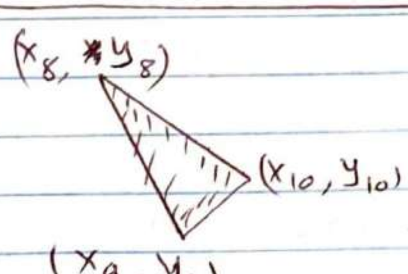
$$\boxed{H_b = \left\{ \begin{array}{l} -f\left[(x_2 - 0), (y_2 - 0)\right] \leq 0 \\ -f\left[(x_1 - 0), (y_1 - 0)\right] \leq 0 \\ -f\left[(x_2 - x_1), (y_2 - y_1)\right] \leq 0 \end{array} \right\}}$$





$$\boxed{H_c \Rightarrow \left\{ -\left[(x - x_3)^2 + (y - y_3)^2 - r_2^2\right] \leq 0 \right\}}$$

$$\boxed{H_d \Rightarrow \left\{ \begin{array}{l} f\left[(x_5 - x_4), (y_5 - y_4)\right] \leq 0 \\ f\left[(x_6 - x_5), (y_6 - y_5)\right] \leq 0 \\ f\left[(x_7 - x_6), (y_7 - y_6)\right] \leq 0 \\ f\left[(x_4 - x_7), (y_4 - y_7)\right] \leq 0 \end{array} \right\}}$$



$$\boxed{H_e \Rightarrow \left\{ \begin{array}{l} f\left[(x_9 - x_8), (y_9 - y_8)\right] \leq 0 \\ f\left[(x_{10} - x_9), (y_{10} - y_9)\right] \leq 0 \\ f\left[(x_8 - x_{10}), (y_8 - y_{10})\right] \leq 0 \end{array} \right\}}$$

Algebraic Set

$$\boxed{X = (H_a \cap H_b \cap H_c) \cup H_d \cup H_e}$$

Exersise 2

a) $R(\alpha, \beta, \gamma) = R_z(\gamma)\, R_y(\beta)\, R_z(\alpha)$

$$= \begin{bmatrix} \cos(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \cos(\beta) & 0 & \sin(\beta) \\ 0 & 1 & 0 \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \times$$

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\gamma)\cos(\beta) & -\sin(\gamma) & \cos(\gamma)\sin(\beta) \\ \sin(\gamma)\cos(\beta) & \cos(\gamma) & \sin(\gamma)\sin(\beta) \\ -\sin(\beta) & 0 & \cos(\beta) \end{bmatrix} \times$$

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Ans.

$$= \begin{bmatrix} \cos(\gamma)\cos(\beta)\cos(\alpha) - \sin(\gamma)\sin(\alpha) & -\cos(\gamma)\cos(\beta)\sin(\alpha) - \sin(\gamma)\cos(\alpha) & \cos(\gamma)\sin(\beta) \\ \sin(\gamma)\cos(\beta)\cos(\alpha) + \cos(\gamma)\sin(\alpha) & \cos(\gamma)\cos(\alpha) - \sin(\gamma)\cos(\beta)\sin(\alpha) & \sin(\gamma)\sin(\beta) \\ -\sin(\beta)\cos(\alpha) & \sin(\beta)\sin(\alpha) & \cos(\beta) \end{bmatrix}$$

b) Show $R(\alpha, \beta, \gamma) = R(\alpha - \pi, -\beta, \gamma - \pi)$

From Previous question we got $R(\alpha, \beta, \gamma)$

$\therefore R(\alpha - \pi, -\beta, \gamma - \pi)$

* Changes in the signs of sin and cos are listed below

$\sin(\alpha) \to \sin(\alpha - \pi) = -\sin\alpha$
$\cos(\alpha) \to \cos(\alpha - \pi) = -\cos\alpha$
$\sin(\beta) \to \sin(-\beta) = -\sin\beta$
$\cos(\beta) \to \cos(-\beta) = \cos\beta$
$\sin(\gamma) \to \sin(\gamma - \pi) = -\sin\gamma$
$\cos(\gamma) \to \cos(\gamma - \pi) = -\cos\gamma$

$\therefore R(\alpha - \pi, -\beta, \gamma - \pi)$

$$= \begin{bmatrix} (-\cos(\gamma))(\cos\beta)(-\cos\alpha) - (-\sin\gamma)(-\sin\alpha) & (-\cos\gamma)(\cos\beta)(-\sin\alpha) - (-\sin\gamma)(-\cos\alpha) & (-\cos\gamma)(-\sin\beta) \\ (-\sin\gamma)(\cos\beta)(-\cos\alpha) + (-\cos\gamma)(-\sin\alpha) & (-\cos\gamma)(-\cos\alpha) - (-\sin\gamma)(\cos\beta)(-\sin\alpha) & (-\sin\gamma)(-\sin\beta) \\ -(-\sin\beta)(-\cos\alpha) & (-\sin\beta)(-\sin\alpha) & \cos\beta \end{bmatrix}$$

$= R(\alpha, \beta, \gamma)$

c) R' is a given rotation Matrix.

$\therefore \quad R'(3,3) = \cos(\beta)$ — ①

$R'(3,2) = \sin(\beta)\sin(\alpha)$ — ②

$R'(2,3) = \sin(\gamma)\sin(\beta)$ — ③

$\therefore \quad \boxed{\beta = \cos^{-1}(R'(3,3))}$ From ①

Put β in ②

$\therefore \quad R'(3,2) = \sin(\cos^{-1}(R'(3,3)))\sin(\alpha)$

$\therefore \quad \boxed{\alpha = \sin^{-1}\left[\frac{R'(3,2)}{\sin(\cos^{-1}(R'(3,3)))}\right]}$

Put β in ③

$\therefore \quad R'(2,3) = \sin(\gamma)\sin(\cos^{-1}(R'(3,3)))$

$\therefore \quad \boxed{\gamma = \sin^{-1}\left[\frac{R'(2,3)}{\sin(\cos^{-1}(R'(3,3)))}\right]}$

Exercise 3

a) From the code for question 7.~~(a)~~ [forward]

We got positions of the joints as.

$(0,0)$, $(5.657, 5.657)$, $(0, 11.314)$, $(-2.071, 19.041)$

$$a = \frac{(5.657, 5.657) + (0,0)}{2}$$

$$\boxed{a = (2.828, \ 2.828)}$$

$$b = (0, 11.314) + (1 \times \cos(\theta_1 + \theta_2), \ 1 \times \sin(\theta_1 + \theta_2))$$

$$= (0, 11.314) + (-0.707, \ 0.707)$$

$$\boxed{b = (-0.707, \ 12.021)}$$

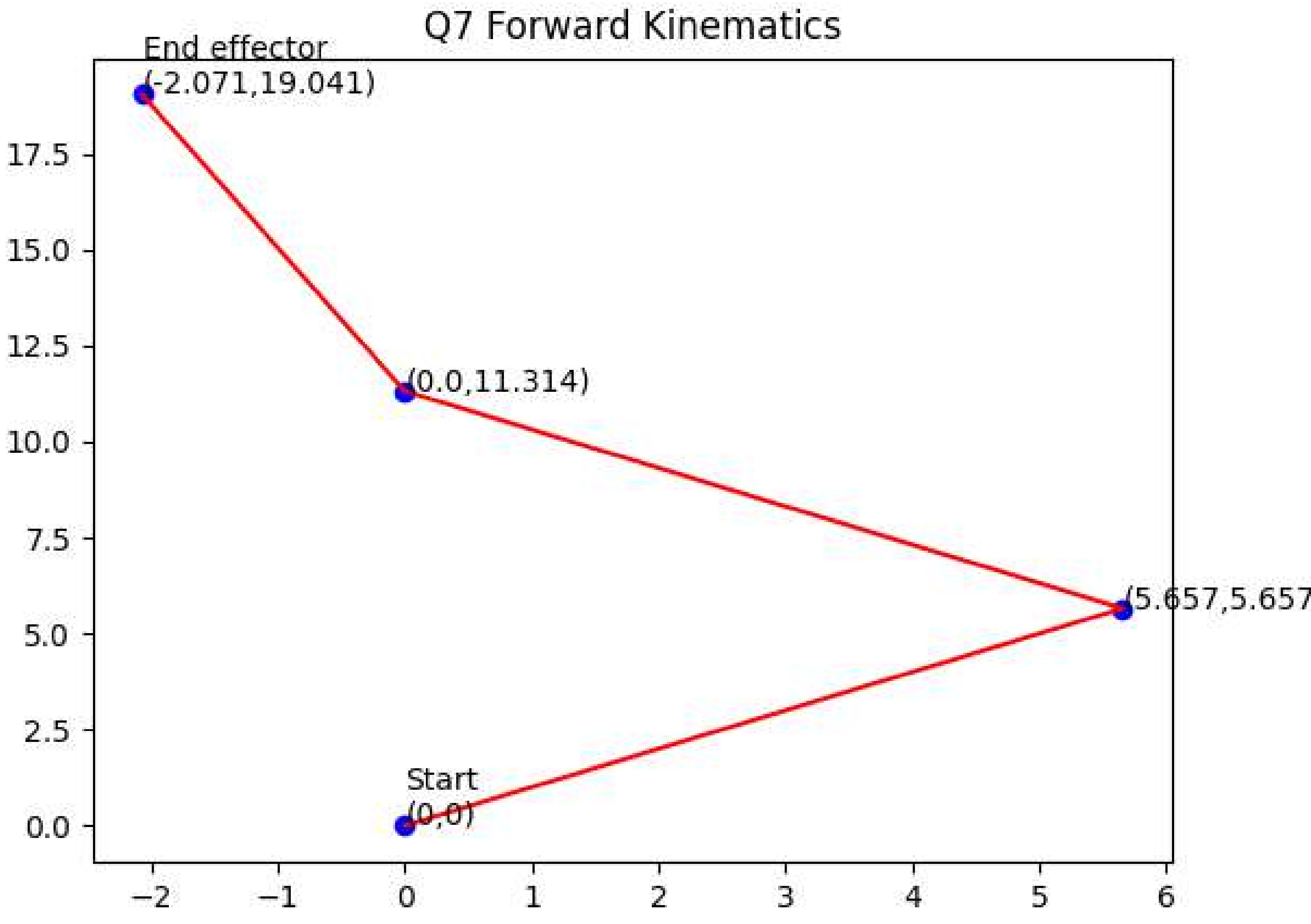
$$c = \left(-2.071, 19.041\right) + \left(\sqrt{2} \times \cos\left(\theta_1 + \theta_2 + \theta_3 + \frac{\pi}{4}\right), \ \sqrt{2} \times \sin\left(\theta_1 + \theta_2 + \theta_3 + \frac{\pi}{4}\right)\right)$$

$$= (-2.071, 19.041) + \left(-\frac{\sqrt{6}}{2}, \ \frac{\sqrt{2}}{2}\right)$$

$$\boxed{c = (-3.296, \ 19.748)}$$

b) From the code for question 7 [inverse] configuration are $\boxed{(163.0^\circ, -81.456^\circ, -125.544^\circ)}$

Exercise 3 a

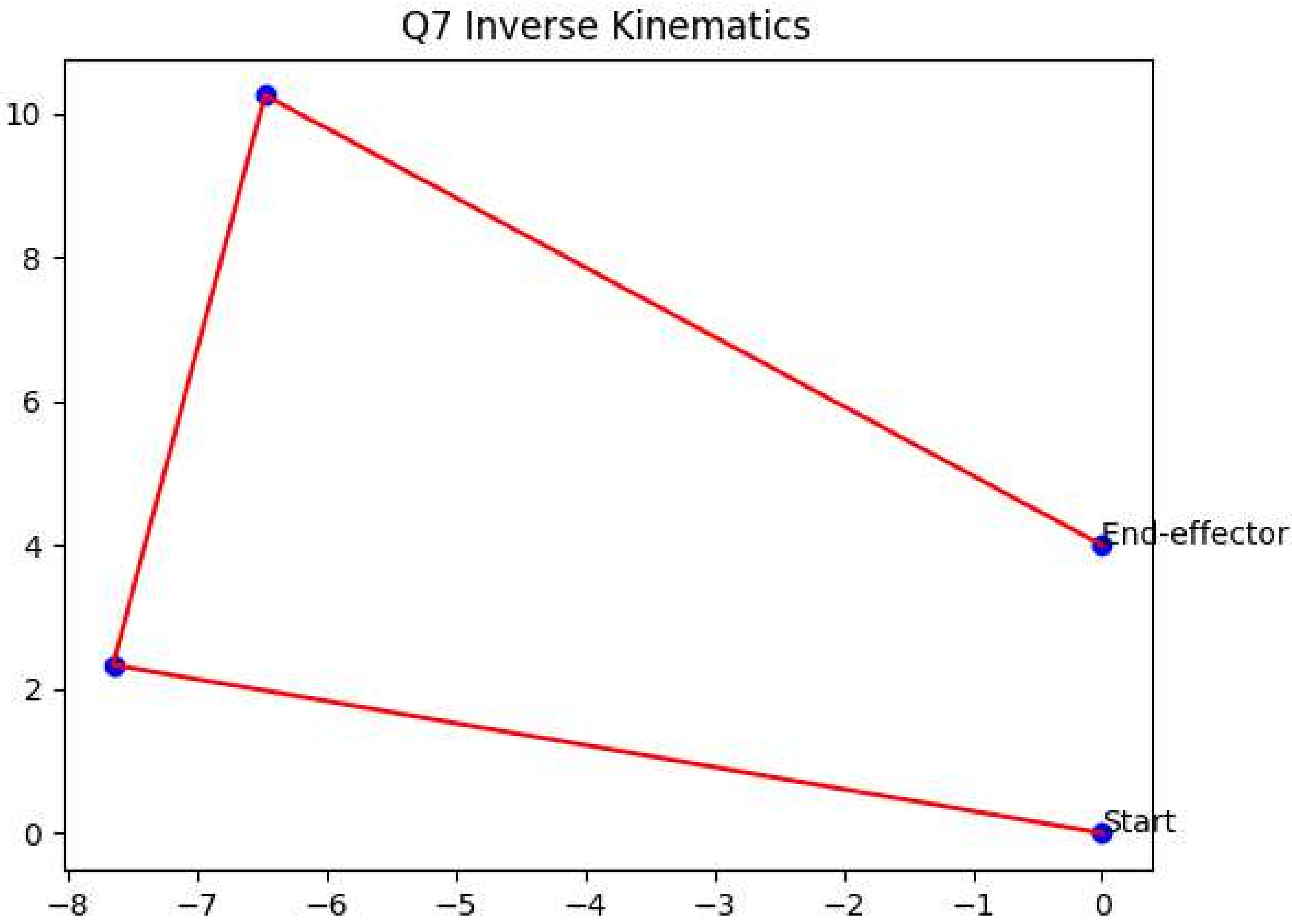


Q7 Forward Kinematics
End effector
(-2.071,19.041)
(0.0,11.314)
(5.657,5.657
Start
(0,0)
17.5
15.0
12.5
10.0
7.5
5.0
2.5
0.0
−2
−1
0
1
2
3
4
5
6

Forward Kinematics

```
Enter the length of the first link: 8

Enter its angle (degrees) wrt the ground: 45

Enter the length of the first link: 8

Enter its angle (degrees) wrt the previous link: 90

Enter the length of the first link: 8

Enter its angle (degrees) wrt the previous link: -30

Point 0 is: (0,0)

Point 1 is: (5.657,5.657)

Point 2 is: (0.0,11.314)

Final position is: (-2.071,19.041)
```

Exercise 3 b

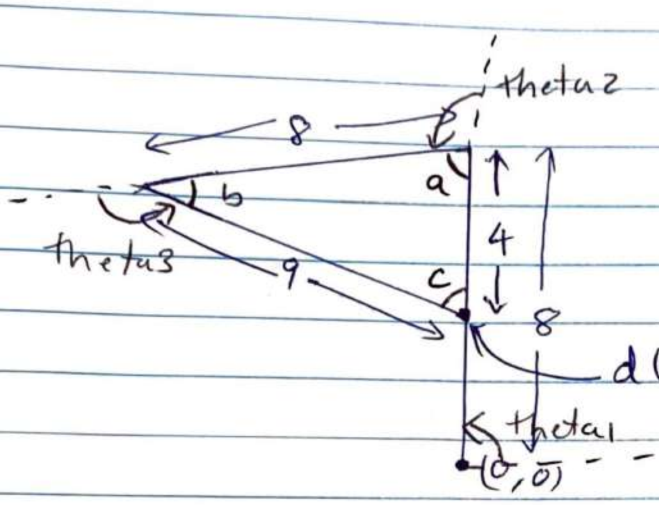


Q7 Inverse Kinematics
End-effector
Start
10
8
6
4
2
0
−8
−7
−6
−5
−4
−3
−2
−1
0

Inverse Kinematics

```
Enter the length of the first link: 8

Enter the length of the second link: 8

Enter the length of the third link: 9

Enter end-effector x value: 0

Enter end-effector y value: 4
found
found
found
found
One of the configuration is:  163.0 -81.46 -125.54  degrees
```

An other configuration can be



From the cosine rule we get

$a = 90.895^\circ$, $b = 26.384^\circ$, $c = 62.72^\circ$

$\therefore$ In this configuration

$$\boxed{\begin{aligned} theta1 &= 90^\circ \\ theta2 &= 89.105^\circ \\ theta3 &= 153.616^\circ \end{aligned}}$$

Exercise 4

a) $R^1 \times R^1 \rightarrow R^2$

The two trains have <u>independent</u> motion along seperate tracks

b) $R^2 \times SO(2) \rightarrow SE(2)$

R^2 is for translation in 2D and $SO(2)$ is for rotation

c) $SE(2) \times SE(2)$

Two seperate set for two independent robots.

d) $R^2 \times SO(2) \rightarrow SE(2)$ or $SE(2) \times T^2$ [If there are revolute joints on robot]

The two planar robots are joined rigidly so there is no relative motion between them It is a single rigid body.

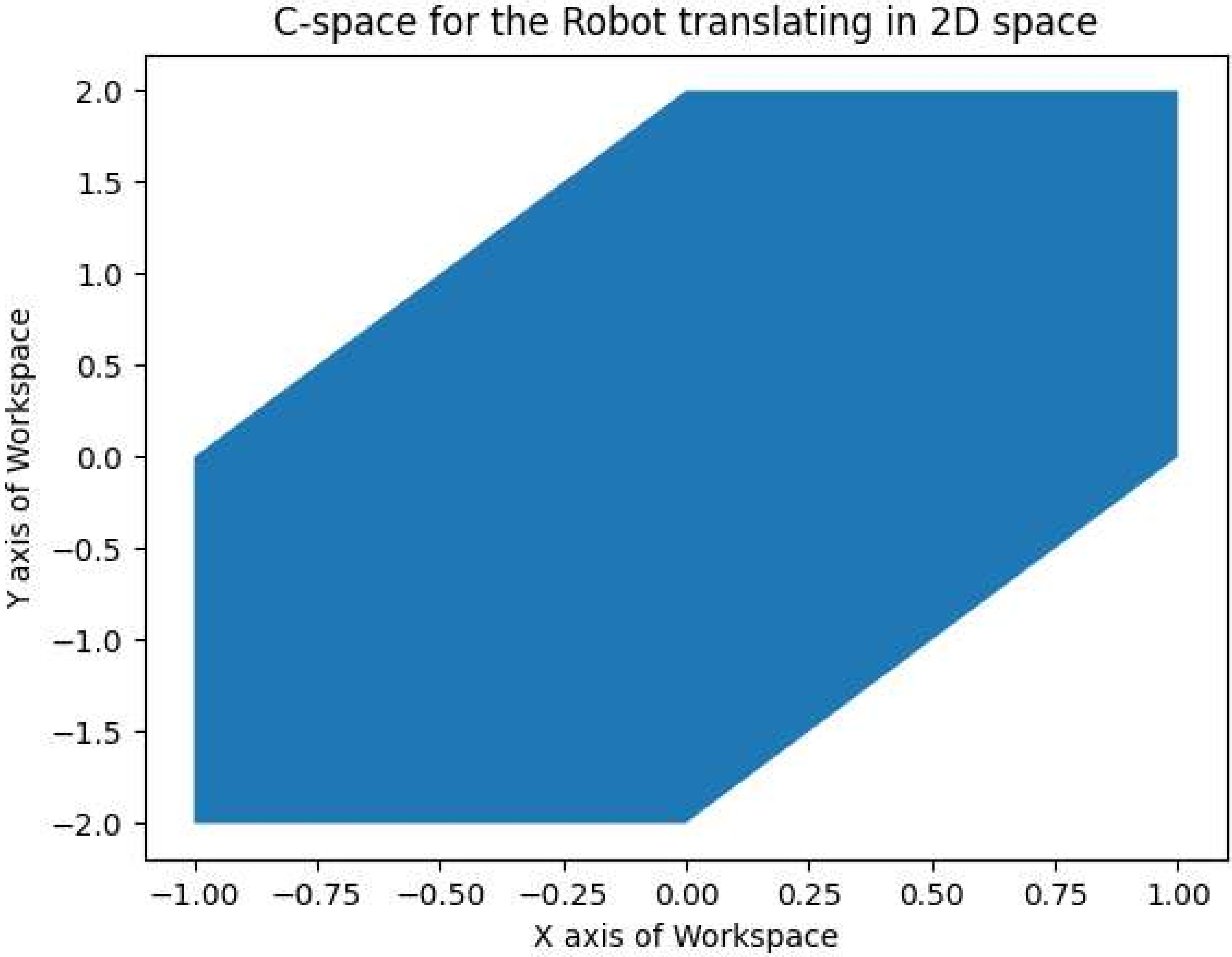
e) $R^3 \times SO(2)$

One of the rotation is constrained hence it is equivalent to 2D rotation.

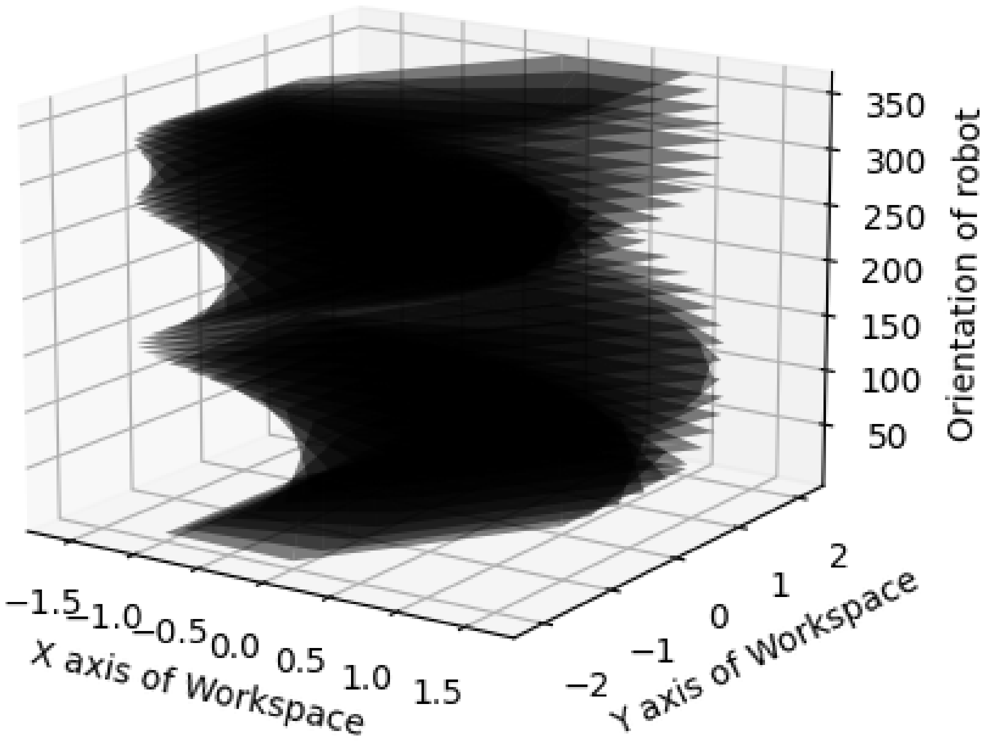
f) $\underbrace{SE(3)}_{\text{Space craft}} \times \underbrace{S^1 \times S^1 \times S^1}_{\text{links}}$

g) $S^1 \times S^1 \times S^1 \times S^1 \times S^1 \times S^1 \times S^1$ - 7 revolute joints.

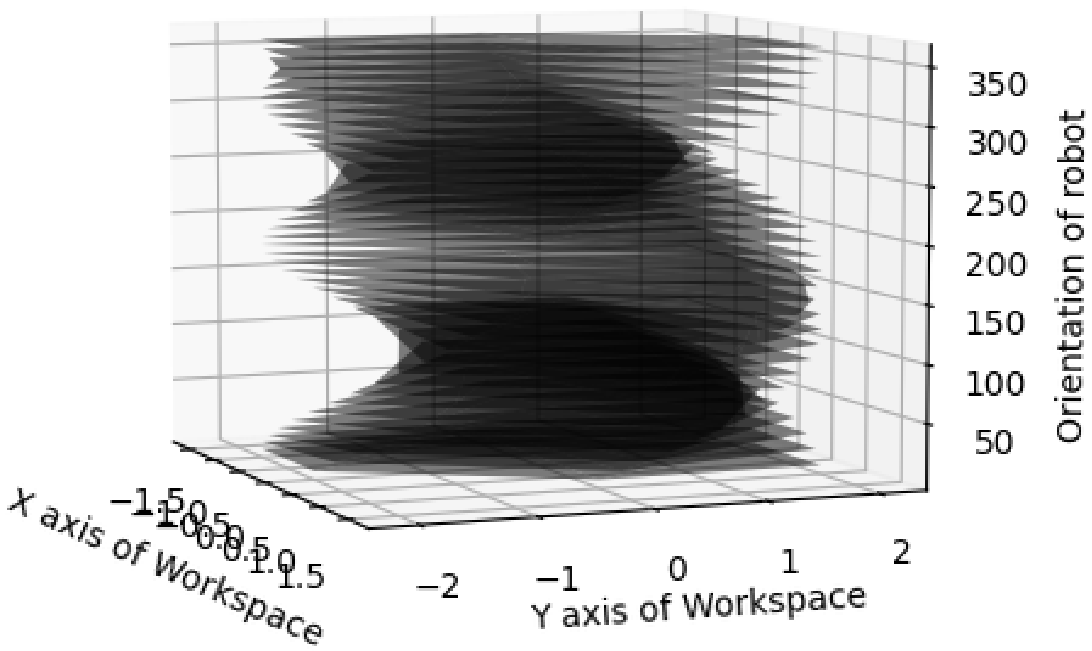
Exercise 5


C-space for the Robot translating in 2D space
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Y axis of Workspace
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
X axis of Workspace

C-space for the Robot translating and rotating in 2D space



C-space for the Robot translating and rotating in 2D space



Exercise 6

A set(S) is convex if
$\forall$ points in the set (i, j) satisfy.

$$t\, S(i_1, j_1) + (1-t)\, S(i_2, j_2) \in S, \quad t = [0, 1]$$

$\therefore$ for Robot R is convex

$$t\, R(i_1, j_1) + (1-t)\, R(i_2, j_2) \in R \quad [\text{for } 2D]$$

$$t\, R(i_1, j_1, k_1 \ldots) + (1-t)\, R(i_2, j_2, k_2 \ldots) \in R^n \quad [\text{for } \mathbb{R}^n]$$

$\therefore$ for Obstacle O is convex

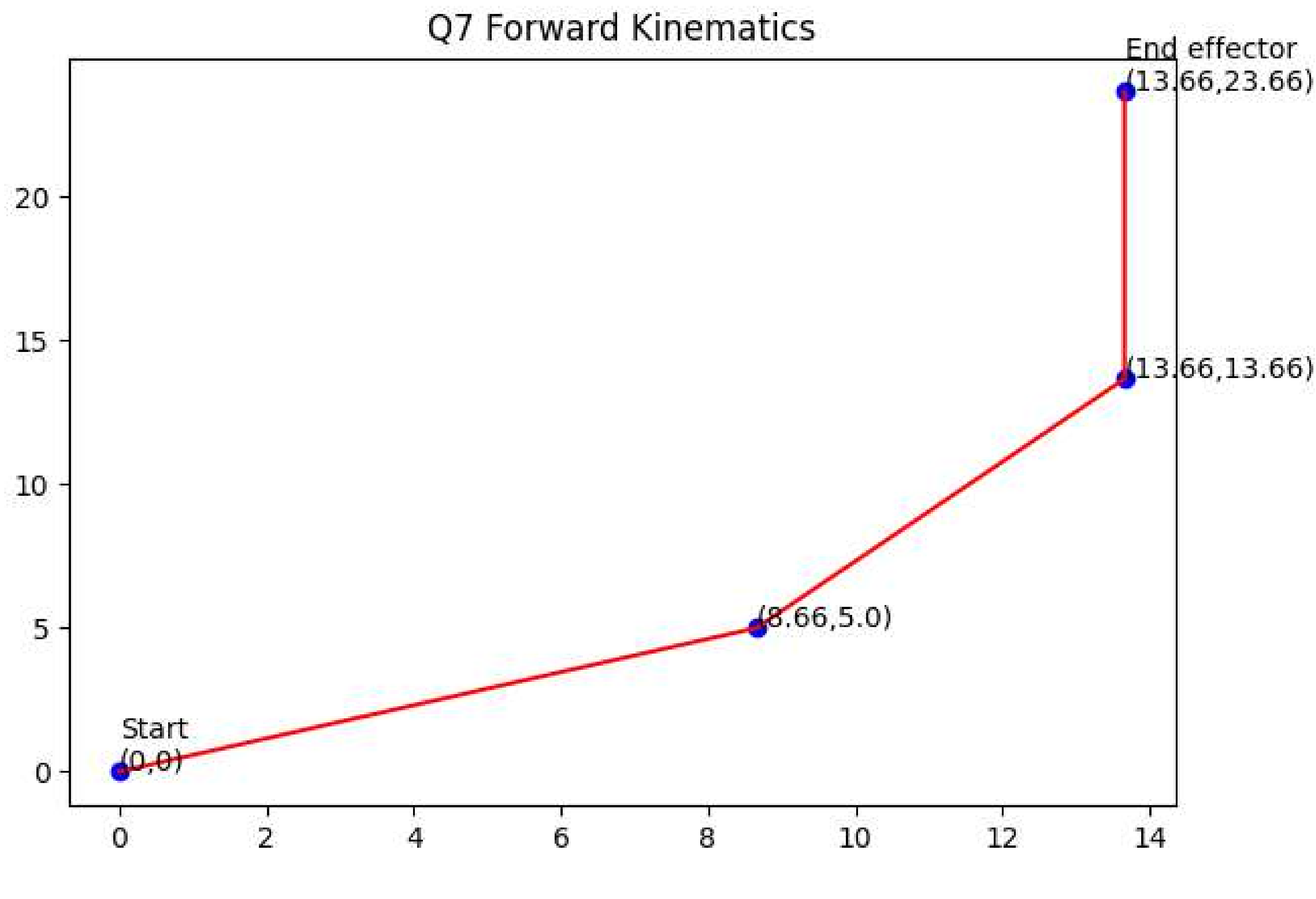
$$t\, O(i_1, j_1, k_1 \ldots) + (1-t)\, O(i_2, j_2, k_2 \ldots) \in O \quad [\text{for } \mathbb{R}^n]$$

Minkowski Sum $= R(i, j, k \ldots) + O(i, j, k \ldots)$
$\forall$ points in the set $(i, j, k \ldots)$

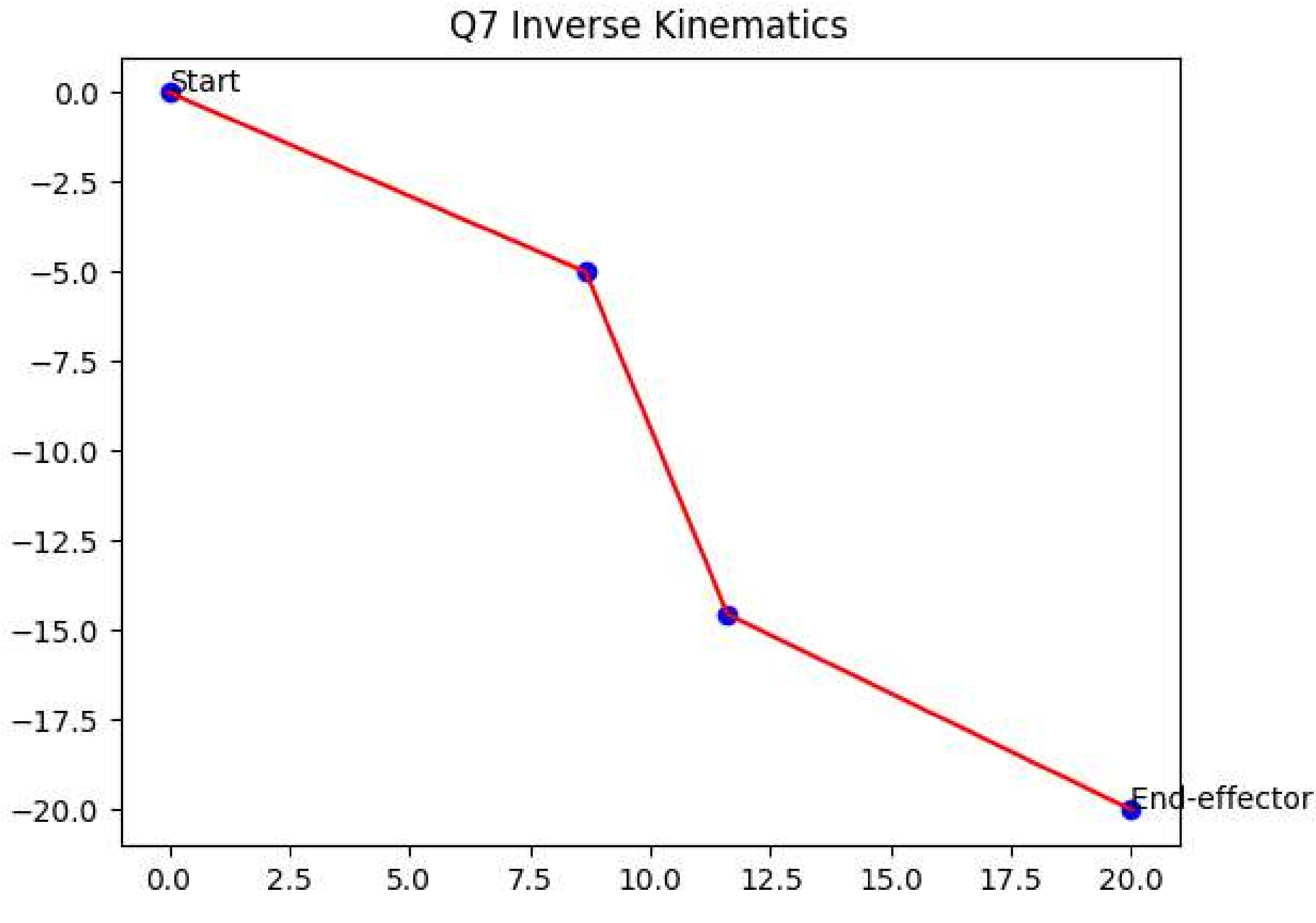
$$R \oplus O = [t\, R(i, j, k \ldots) + (1-t)\, R(i, j, k \ldots)]$$
$$\oplus [t\, O(i, j, k \ldots) + (1-t)\, O(i, j, k \ldots)]$$
$$= t\, R \oplus O(i, j, k \ldots) + (1-t)\, R \oplus O(i, j, k \ldots)$$

Hence the set $R \oplus O$ is convex

Exercise 7



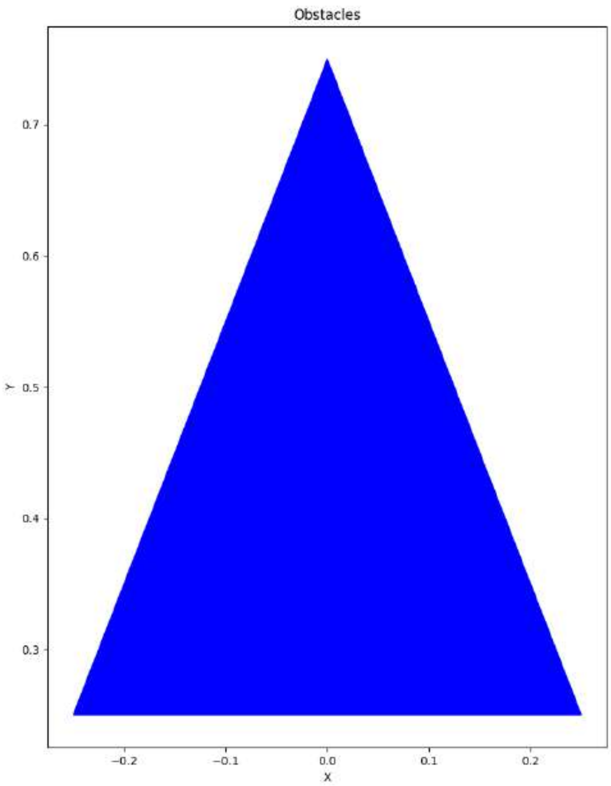
Q7 Forward Kinematics
End effector
(13.66,23.66)
(13.66,13.66)
(8.66,5.0)
Start
(0,0)
0
5
10
15
20
0
2
4
6
8
10
12
14

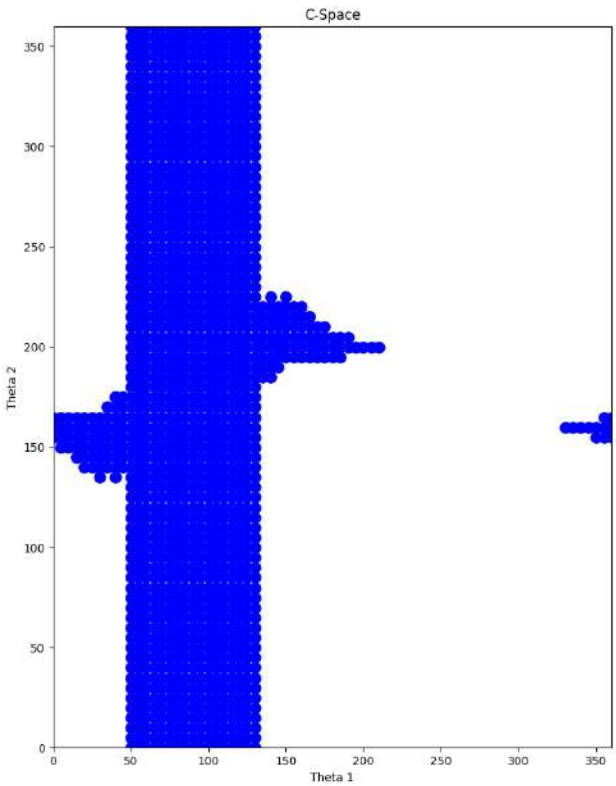


Q7 Inverse Kinematics
Start
End-effector
0.0
−2.5
−5.0
−7.5
−10.0
−12.5
−15.0
−17.5
−20.0
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0

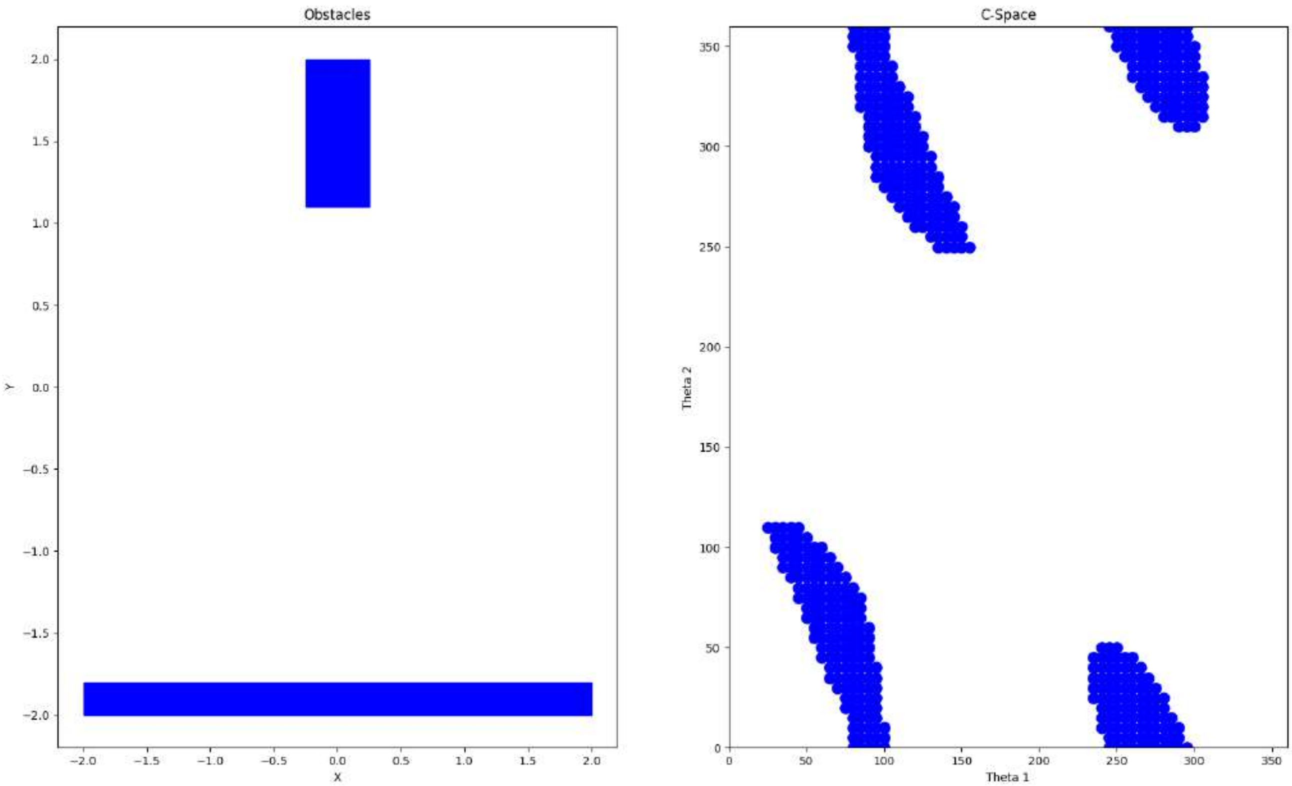
Exercise 8

Obstacle group 1





Obstacle group 2



Obstacle group 3