

FINANCIAL MODELING PROJECT

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PURPOSE OF THE PROJECT:

PART I:

- To estimate and perform a comprehensive analysis of a portfolio of 3 bonds – government bond A, corporate bond B and municipal bond C – with respect to risk associated with each bond. All three bonds have different time to maturity, coupons and yield to maturity (YTM).
- To identify various factors associated with risk measures.
- To understand and quantify these risk factors that the portfolio might be exposed based on tools and strategies learned in class.

PART II:

- To perform due-diligence and detailed stock valuation of equity stocks in the S&P 500 index. These stocks are identified as – Alphabet Inc. Class A (GOOGL) and Eli Lilly & Co. (LLY) using the free cash flow (FCF) valuation model.
- To construct an optimal portfolio from the following stocks of the S&P 500 index:
 - Amazon (AMZN)
 - Nvidia Corp (NVDA)
 - Tesla Inc. (TSLA)
 - UnitedHealth Group Inc. (UNH)
 - Eli Lilly & Co. (LLY)
- To construct an efficient frontier from the above listed stocks and locate the global mean-variance portfolio (GMVP) and the maximum Sharpe ratio portfolio (tangent portfolio) given the current risk free rate i.e. 4.22%, as on December 3, 2023.
- To understand, analyze and present the effect of no short selling i.e. sum of weights equals 1 and within bounds $[0, 1]$ and, the effect of short selling i.e. sum of weights equals 1 and within bounds $[-1, 1]$ based on the above given five stocks in our optimum portfolio.

PART I:

Portfolio Overview:

As shown below, our portfolio is a combination of Government, Corporate, and Municipal bonds with varying maturities, coupons and yield to maturities. Bond A and B are being sold at a discount, while Bond C is sold at premium. Additionally, we can infer that our Corporate bond is investment-grade (BBB rated or above) because its current YTM is lower than the government bond's YTM. If this was larger, we would assume that investors attribute significant risk premium to that corporation. The same thing cannot be said for the Municipal bond, however, because we must consider potential tax incentives that investors receive for investing in Munis. Our total portfolio has a market value of \$1,359,642 with a face value of \$1,500,000 and modified duration of 9.03.

Security	Face Value	Maturity	Annual Coupon Rate	Current YTM	Bond Price	Duration	Modified Duration	Convexity	Portfolio Weight	Weighted Duration
Government Bond A	\$600,000	20 Yr	2.50%	5%	\$411,729	14.80	14.44	266.96	30.28%	4.37
Corporate Bond B	\$400,000	5 Yr	3.80%	4%	\$396,407	4.60	4.51	25.83	29.16%	1.31
Municipal Bond C	\$500,000	10 Yr	4.20%	3%	\$551,506	8.37	8.24	84.03	40.56%	3.34
Total	\$1,500,000				\$1,359,642				100.00%	9.03

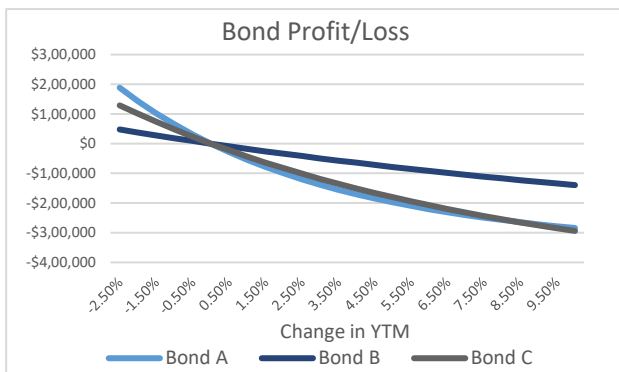
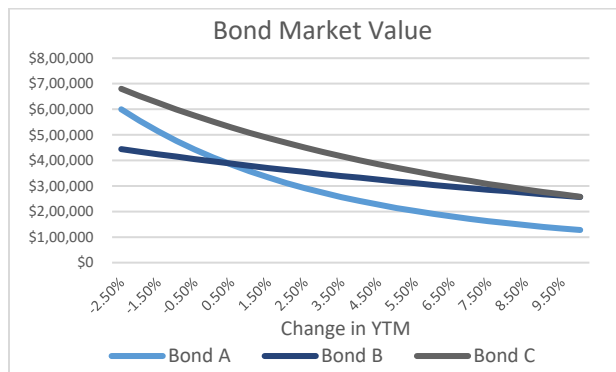
Portfolio Duration 9.03

Liquidity Risk:

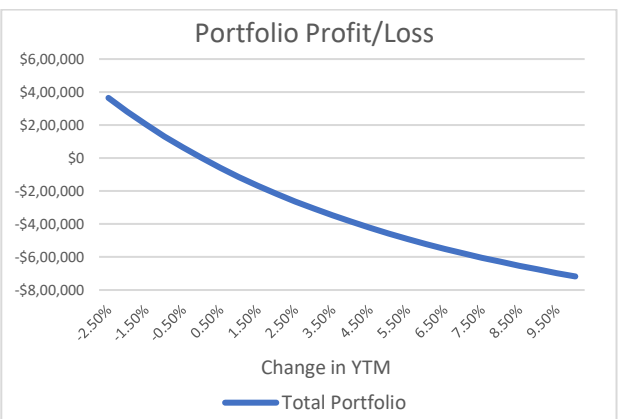
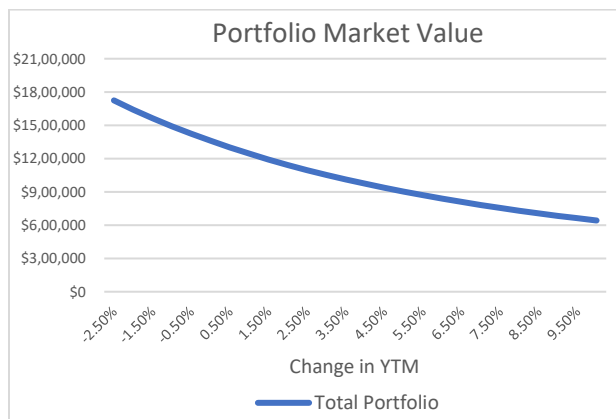
Our portfolio should not suffer from significant liquidity risk. Treasury bonds are the most actively traded bonds in the market so getting in and out of that position shouldn't cause any issue. The Corporate bond should have higher liquidity risk comparatively, but since we assume it to be investment-grade there should be sufficient liquidity in the market to facilitate changes in our position. Spreads will be wider than Government bonds, but this should have minimal effect on our overall portfolio. On the other hand, the Municipal bond market has far less liquidity than the other two, mainly because of the sheer amount of Muni issuers. Although there's not enough information to determine how liquid this specific bond is, based on Muni market qualities we expect this position to be our most likely candidate for liquidity risk. As portfolio managers, we should focus on making small adjustments to our portfolio using Government bonds, medium adjustments using a combination of Government and Corporate bonds, and only sell or add Municipal positions if required to.

Interest Rate Risk:

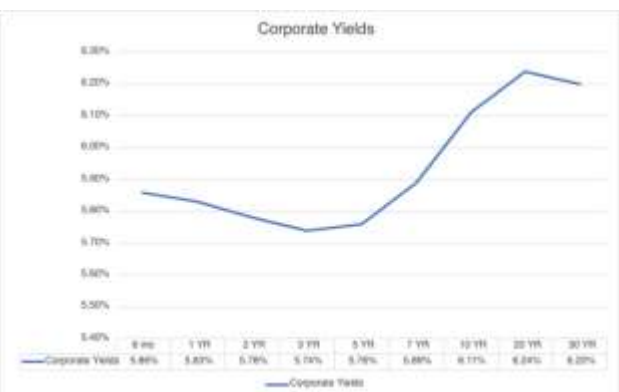
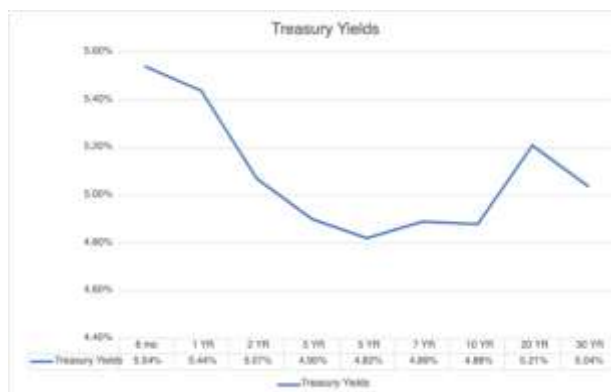
To analyze the bonds' sensitivity to interest rates, we ran a sensitivity analysis to show the change in the dollar value, should rates rise or fall. The graph on the left shows the overall change in market value given changes in rates, while the chart on the right represents the profit or loss by changes in rates. Due to bond duration, we can expect longer dated and lower coupon securities to change more in price as rates rise or fall relative to others. This is true with our portfolio – Bond A is the longest maturity, lowest coupon, and changes the most, followed by Bond C. Bond B has the lowest maturity and duration which is why large changes in YTM have a relatively low effect on the position's value.

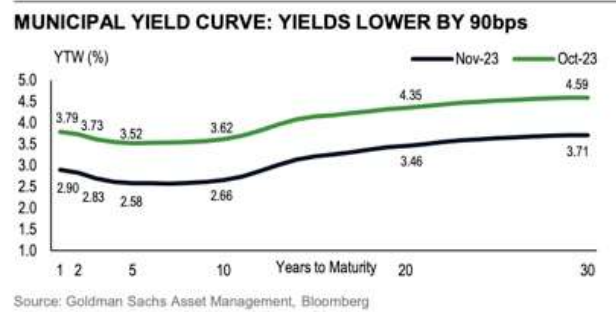


Due to diversification, our portfolio's weighted average duration is 9.03. This measure shows how sensitive the portfolio is to changes in rates. Using the Fidelity U.S. Bond Index as a proxy for the Bloomberg Aggregate Index, we can see that our portfolio is longer duration than the market which has a duration of 6.03 years. The chart below shows the estimated change in our portfolio's value given changes in rates. For example, if rates increase by 1%, our portfolio value will decrease by \$115,176.



Looking at our bonds versus the current term structures shown below, our government and municipal bonds YTM's are below the current market rates. This would imply our bonds would be sold at a loss in the market as rates have risen and investors require more yield. On the contrary, our municipal bond has accreted in value as current market rates are below our YTM.





*Treasury and Corporate Bond Yield Curves sourced from the U.S. Department of the Treasury, retrieved from FRED, Federal Reserve Bank of St. Louis

Immunization:

Since we are portfolio managers at BVS, we assume that our investors intend to be long risk. Additionally, we do not have enough information to know of any longer-term liabilities at BVS Bank. Due to these reasons, there is no immunization strategy required for this portfolio.

PART II:

Out of the 10 stocks, we have decided to value Alphabet Inc. Class A (GOOGL) and Eli Lilly & Co. (LLY) for stock valuation. Here are a few reasons for considering Alphabet and Eli Lilly for stock valuation.

Alphabet Inc. Class A (GOOGL):

1. **Strong growth history and future potential:** Google has demonstrated an exceptionally high growth rate in its core business of digital advertising. The revenue from advertising constitutes to approximately 80% of Google's revenue and the CAGR stands at 19.83%.
2. **High profitability:** Google operates with very high profit margins thanks to the scalability of its advertising platform. Looking at the Income statement of Google from FY 2013-2022, the company's gross margin has always been above 55%. This provides security for investors.
3. **Providing Essential services:** Digital services such as Gmail and Google Maps are necessary on a day-to-day basis. This in turn generates consistent demand that is unlikely to decline.
4. **Technical Leadership:** Google leads development in AI, machine learning, and other key technologies expected to shape the future and drive profits.

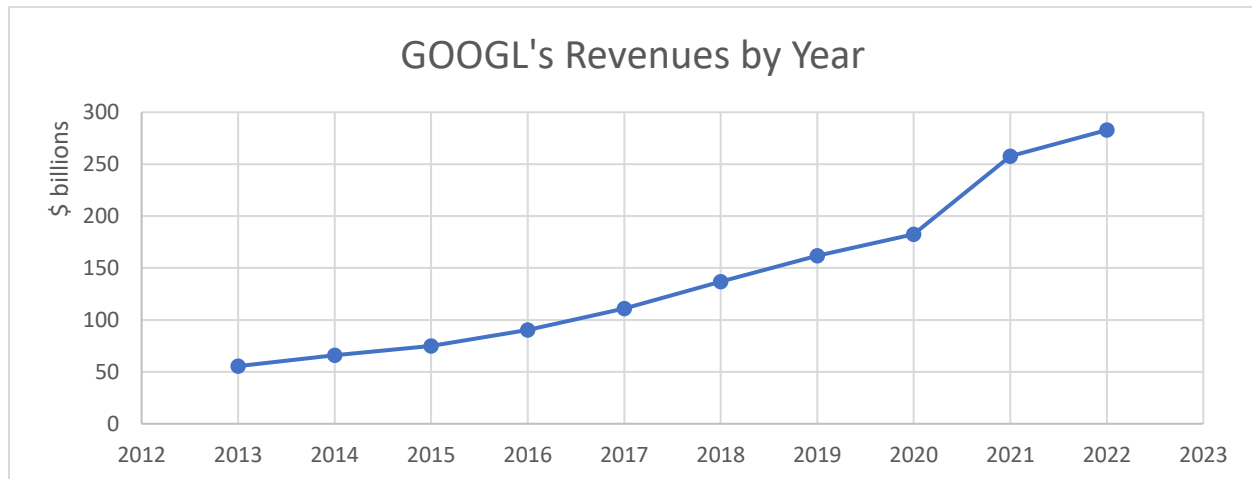
Eli Lilly & Co. (LLY):

1. **Innovative drug pipeline:** Eli Lilly has invested heavily in the R&D to build and to perform clinical trials of new drugs to address diabetes, cancer, and obesity. Recently the company got FDA approval for Zepbound (weight loss drug). This pipeline will support future revenue growth globally.
2. **Strong financial position:** The company generates strong cash flows from patent-protected drugs like Trulicity, Humalog and Cialis. This funds the company's research and acquisitions.
3. **Strong Dividend History:** Eli Lilly has a payout history of around 52% to its shareholders providing a stable income element and consistent dividends to shareholders.
4. **Diversification across domains and geographies:** Eli Lilly has products spanning major disease areas and a global customer base, minimizing risk exposure to any particular drug or market.

Google and Eli Lilly work together to provide defensive health sector stability and technological progress, both of which are essential for maintaining an ideal portfolio balance.

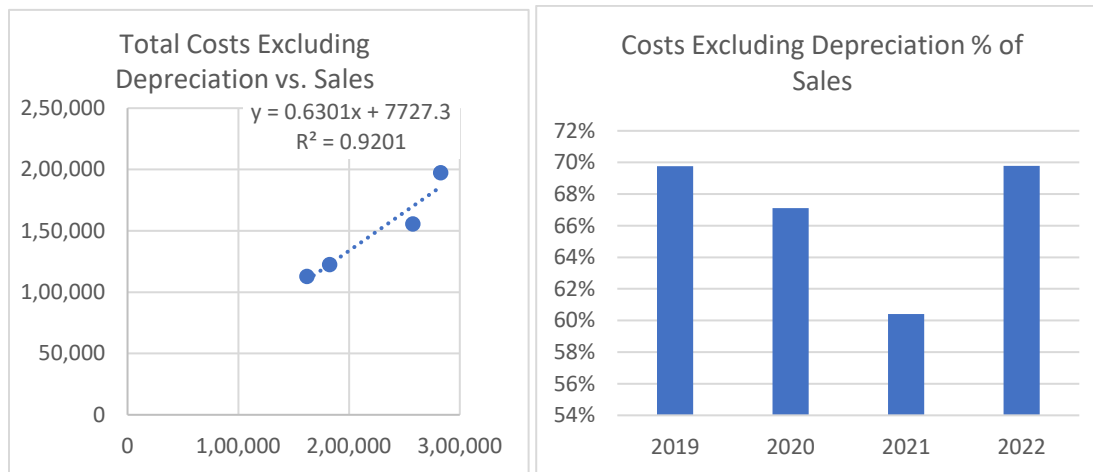
Key Analysis for Alphabet Inc. Class A (GOOGL):

1. YoY Sales Growth Analysis:



Over the last nine years, the company's sales have kept growing steadily. They went from making \$55,500 million in 2013 to a big \$282,800 million in 2022. On average, they grew by almost 20% each year, which is a strong and consistent increase in their sales. The increase in revenue can be attributed to growth in search engines and increased traffic on YouTube.

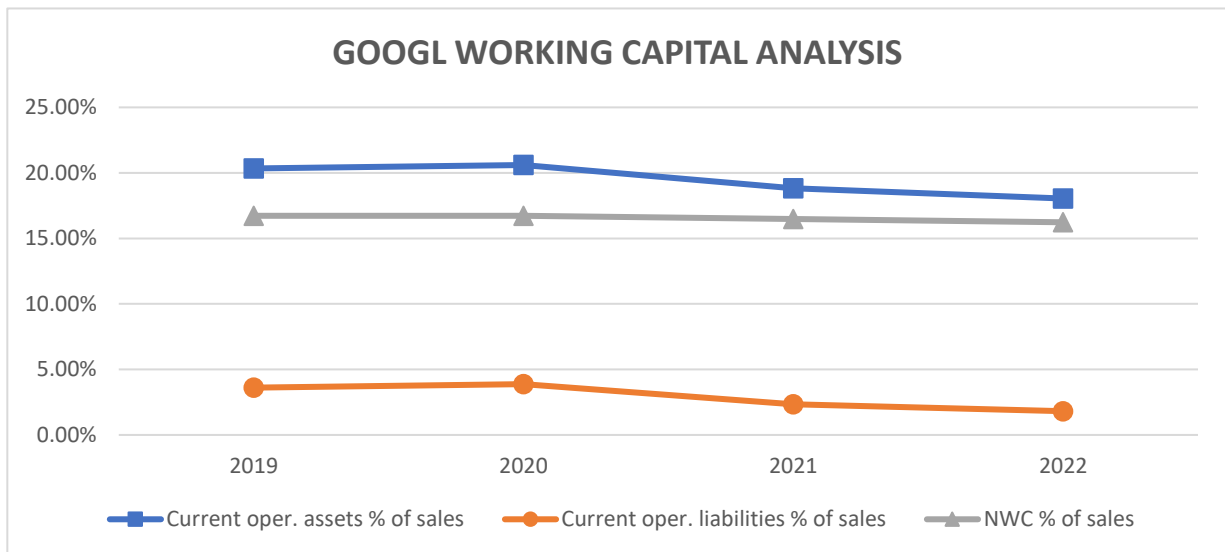
2. Operating Cost Analysis:



The analysis of operating costs from 2019 to 2022 indicates notable fluctuations and trends:

- The cost of sales showed a consistent increase, rising from \$71,900 million in 2019 to \$126,200 million in 2022. This represents a significant 75% increase over the period. In 2022 costs attributed to ~70% of sales.
- The intercept of 7,727 suggests that when sales are zero, the estimated costs (excluding depreciation) would be \$7,727.
- The slope of 0.63 indicates that for every unit increase in sales, costs excluding depreciation are estimated to increase by 0.63 units. This implies a positive relationship between sales and costs, suggesting that as sales increase, costs also tend to increase, although not proportionally.
- Overall, the Cost of Sales notably increased, while Other Costs and Expenses fluctuated, leading to changes in the percentage of costs concerning total sales.

3. Net Working Capital Analysis:

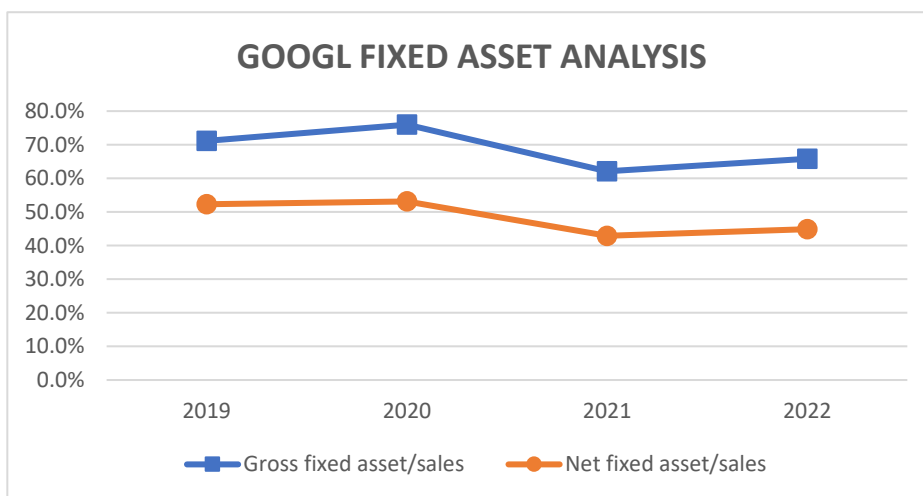


The percentage of current operating assets concerning sales remained relatively steady from 2019 to 2022, ranging between 18.04% and 20.60%. This consistency suggests a stable proportion of assets utilized for day-to-day operations relative to sales revenue.

Conversely, the percentage of current operating liabilities concerning sales experienced a decline from 3.61% in 2019 to 1.81% in 2022. This reduction signifies a decreasing reliance on short-term liabilities to fund operational activities, possibly indicating improved financial management or increased liquidity.

The net working capital as a percentage of sales remained relatively constant, fluctuating marginally between 16.23% and 16.73% over the four years. This stability suggests a maintained balance between current assets and liabilities to support day-to-day operations.

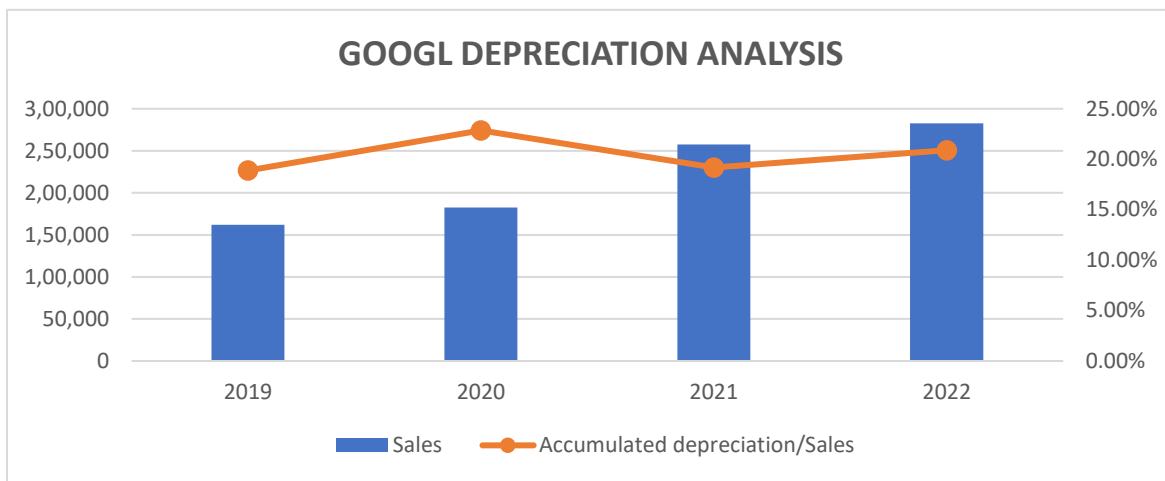
4. Fixed Asset Analysis:



The fluctuations in both gross and net fixed assets as a percentage of sales indicate changing asset valuations or capital investments relative to revenue over the four-year period.

The decline in the proportion of net fixed assets relative to sales, compared to gross fixed assets, implies the impact of depreciation or amortization on the asset values.

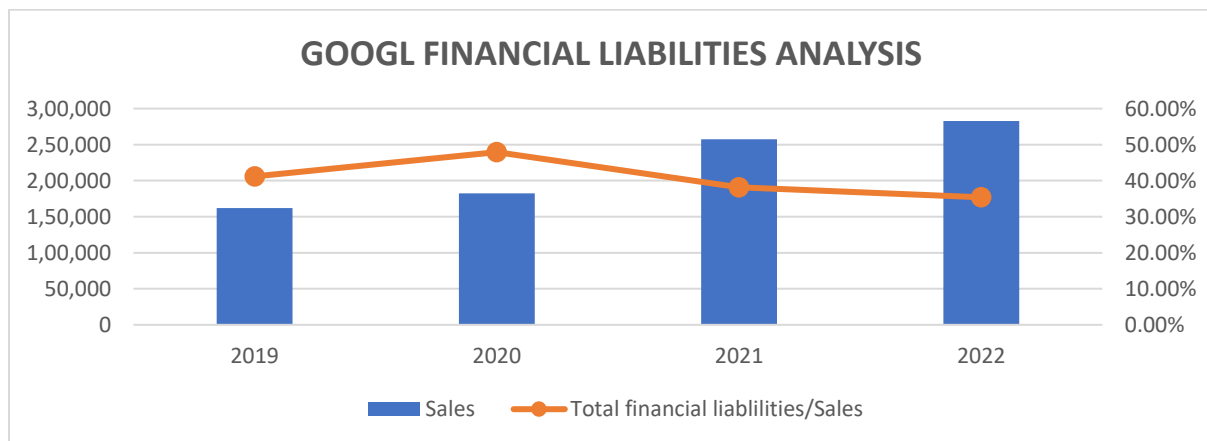
5. Depreciation Analysis:



A higher percentage of accumulated depreciation relative to sales indicates a significant impact of depreciation expenses on overall revenue. This impact might affect the company's profitability by reducing reported income.

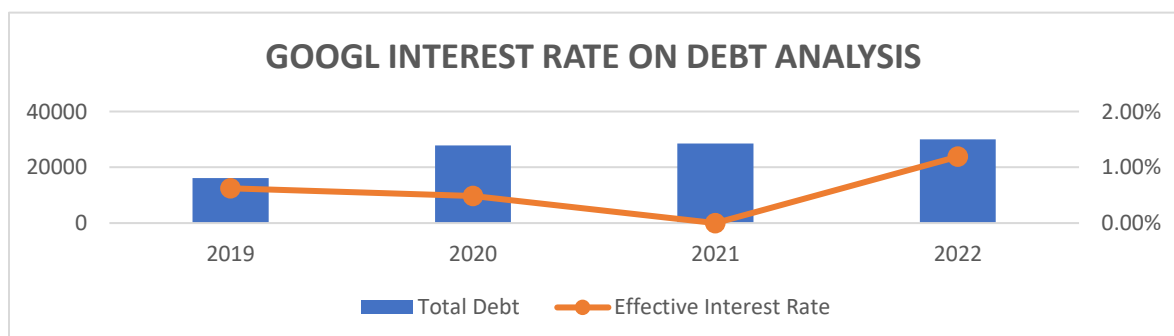
There were variations in the accumulated depreciation percentage as a percentage of sales from 2019 to 2022, with a range of 18.88% to 22.85%. Over time, there was a stable proportion of depreciation charges relative to sales income.

6. Total Liability Analysis:



Between 2019 and 2022, the percentage of total financial liabilities related to sales revenue varied, ranging from 35.39% to 47.91%. This illustrates the proportion of sales revenue that the company must cover its debts. Accrued and other current liabilities comprised ~55% of the total financial liabilities.

7. Interest rate on debt Analysis:



A higher interest expense and effective interest rate could indicate increased borrowing costs, impacting on the company's financial expenses. The interest expense ranged from 0.48% in 2020 to 1.21% in 2021 and then stabilized to 1.19% in 2022.

8. Income Tax rate analysis:

The effective tax rate of Income ranged from 13.3% to 15.9% from 2019 to 2022. This reflects the tax obligations based on the company's Income. The weighted average of tax comes to 15.7%

9. Risk-free yield curve:

The return on an investment that is thought to carry no risks is known as the risk-free yield. It acts as a standard for evaluating the possible return on an investment devoid of any risk considerations, including market volatility or default risk. Generally, it refers to the yields on government bonds like US government bonds. The risk-free rate is taken from the following data.

<https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/default.aspx>

10. Calculating the WACC:

The company had 12580 million shares outstanding and the price per share on December 31, 2022 was \$88.23 bringing the total value of Equity to \$1100000 million. The company has zero net debt because the company has more cash and marketable securities as compared to debt. Thus, WACC = cost of Equity. We have calculated the Re (cost of equity) using the CAPM method, where the formula is:

$$\underline{R_f + (R_m - R_f) * \beta}$$

R_f = risk free rate (10-year treasury bonds)

(R_m - R_f) = E(r_m) = Market risk premium

β = stock's sensitivity to market's risk

The risk-free rate of 10-year US government bond is 3.88% as on 31st December, 2022. We have used Gordon's growth model to calculate the expected market return i.e., 10% and the β factor of 1.03 is extracted from Yahoo Finance.

Taking into consideration all these factors, we have arrived at a WACC of **10.18%**

11. Building a proforma Model and valuing the firm:

Google does not currently pay out dividends on its stock. The company has never paid dividends since its founding and recent management commentary does not indicate plans to initiate dividends. Thus, the dividend discount model has not been considered and instead the free cash flow model has been considered. The free cash flow model better captures the intrinsic value of Google by quantifying how much excess cash the company can generate each year to fund growth opportunities. Our analysis takes a conservative 5-year cash flow projection and applies an appropriate discount rate to value Google today based on harnessing the growth and cash generation potential in its business model.

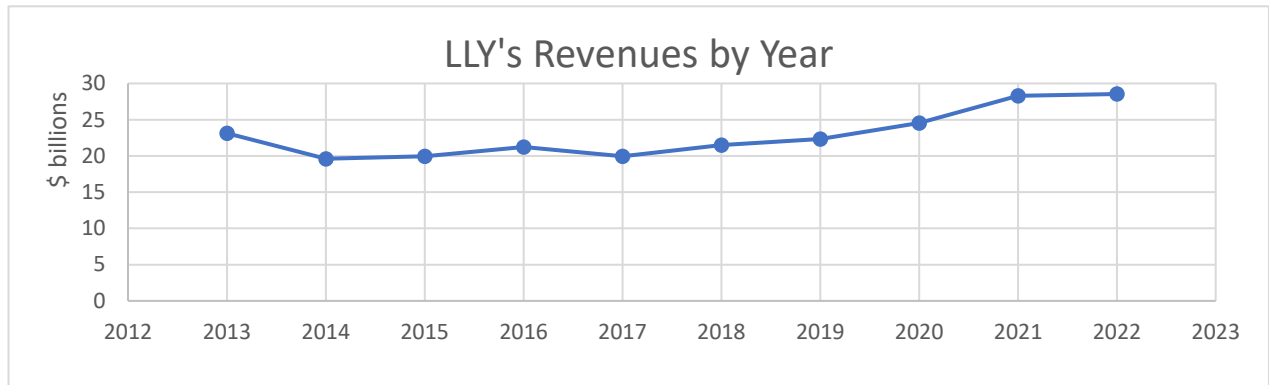
To calculate the Enterprise value, we have taken a conservative approach for forecasting long term growth rate to be 6%. The long-term growth rate is necessary for analyzing the present value of the terminal value. Taking this into consideration, our Enterprise value stood at \$1783460 million. On adding back, the excess assets and subtracting the net debt we arrived at the Equity value of the firm i.e., \$1839514 million. The company had 12580 million shares outstanding. Thus, to derive the fair value of the share we divided the equity value by the total number of shares outstanding arriving at a fair value of \$146.23.

Upon evaluating the current market price of the stock at \$130 alongside its fair value assessment of \$146.23, it is discerned that the stock is currently undervalued relative to its perceived intrinsic worth.

Considering this assessment, the proposed investment strategy entails holding the stock until it either reaches its fair value target of \$146.23 or reaches a one-year horizon value, whichever occurs earlier.

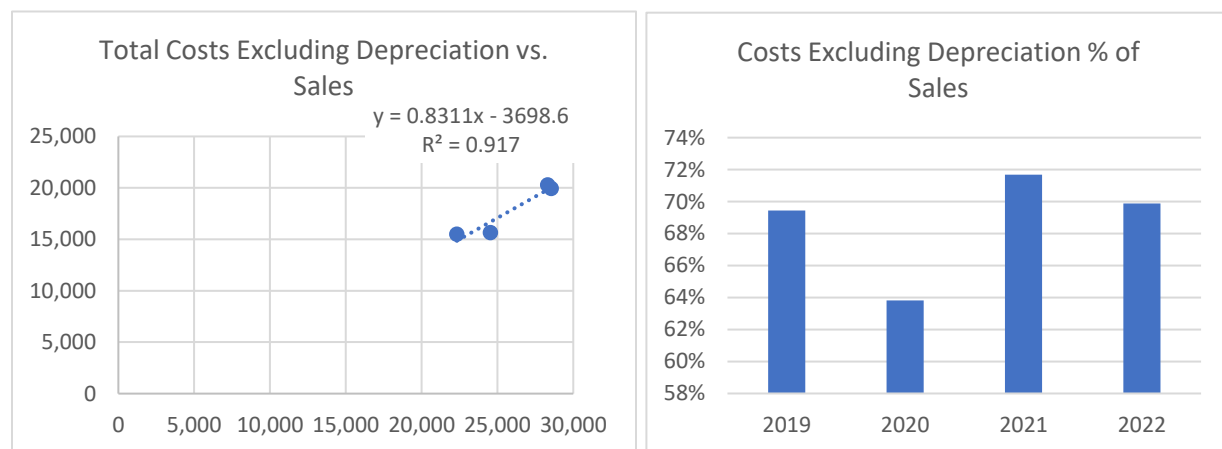
Key Analysis for Eli Lilly & Co. (LLY):

1. YoY sales growth Analysis:



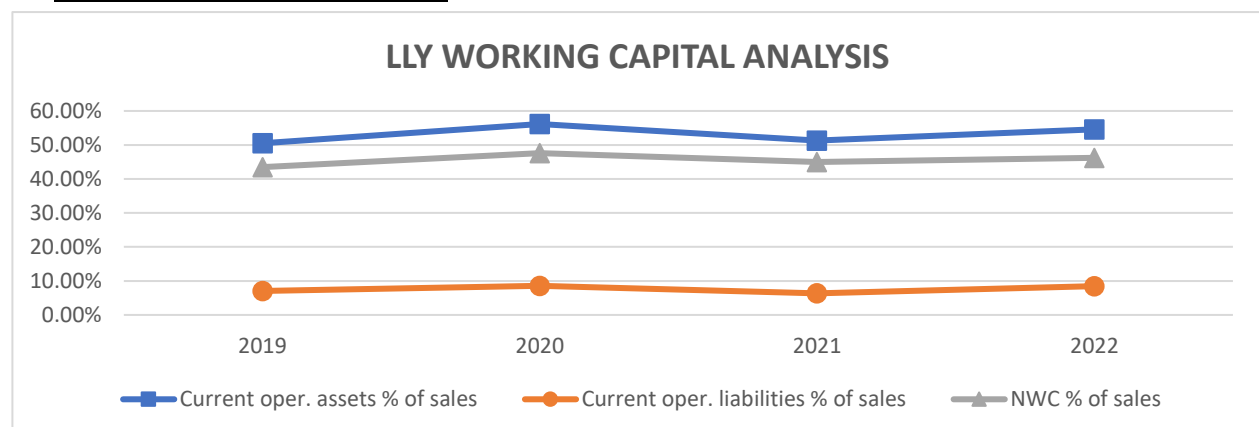
The revenue trend reflects a significant dip in 2014 followed by a period of erratic growth patterns until 2017. From 2017 onwards, there's a steadier and more consistent upward trajectory in revenue growth. The CAGR of 2.37% provides a broad average of the annual growth rate, showcasing a moderate overall growth performance over the past decade.

2. Operating Cost Analysis:



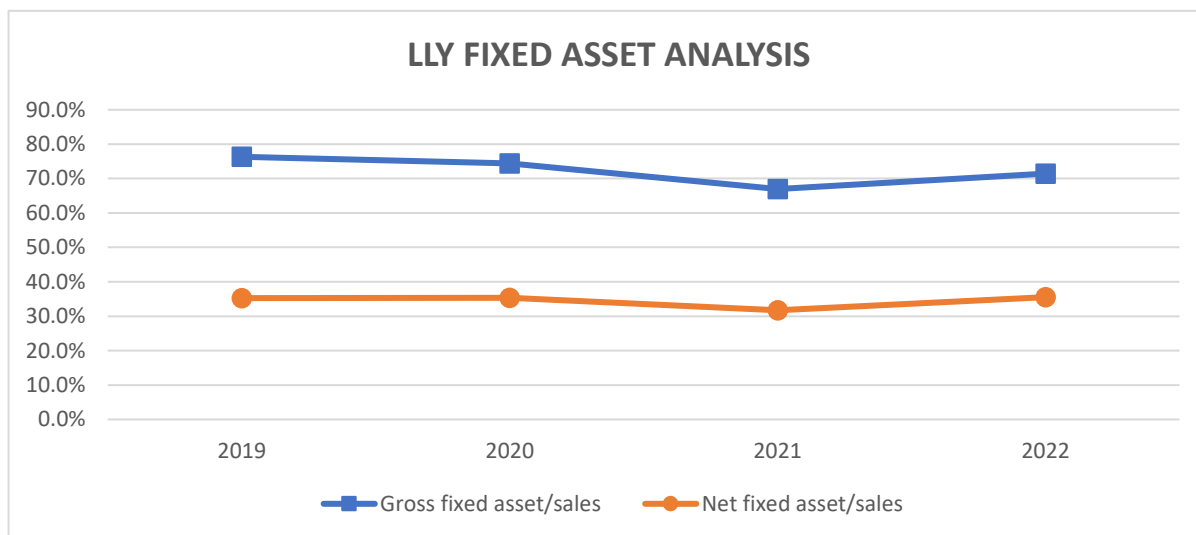
The ratio of costs excluding depreciation to sales varied between 63.8% and 71.7% over the period, peaking in 2021. Despite fluctuations, the costs as a percentage of sales remained within a moderate range, indicating relative stability in managing operational expenses relative to sales revenue.

3. Net Working Capital Analysis:



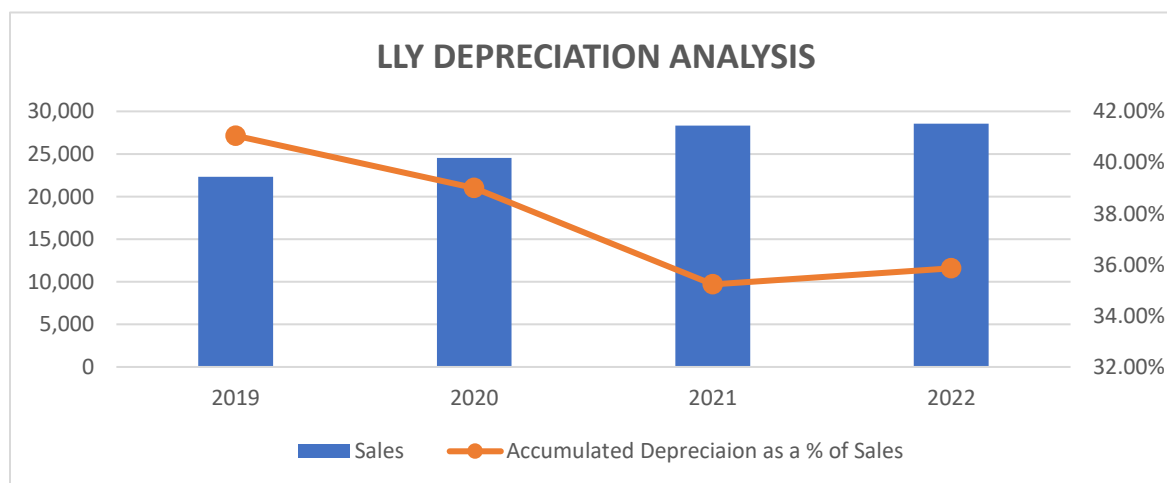
The NWC to sales ratio ranged from 43.48% to 46.20%, reflecting the proportion of NWC concerning sales revenue. Showed a relatively stable percentage, signifying the consistent efficiency in managing working capital concerning sales. The stable NWC percentage implies efficient management of working capital concerning sales revenue over the specified period.

4. Fixed Asset Analysis:



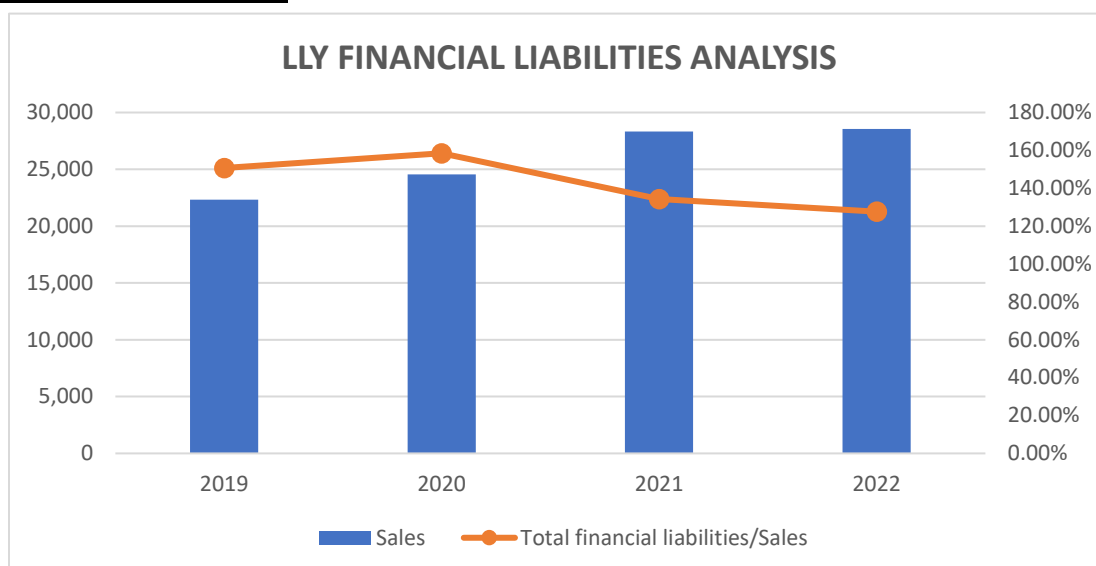
The Gross Fixed Asset/Sales ratio fluctuated over the years, ranging from 76.3% in 2019 to 67.0% in 2021. The Net Fixed Asset/Sales ratio fluctuated as well, ranging from 31.7% in 2021 to 35.5% in 2022. The analysis of Gross Fixed Asset/Sales and Net Fixed Asset/Sales ratios highlights fluctuations in both total and net fixed asset proportions concerning sales revenue over the specified period. These fluctuations might indicate changes in investment strategies, asset utilization, or management approaches concerning fixed assets.

5. Depreciation Analysis:



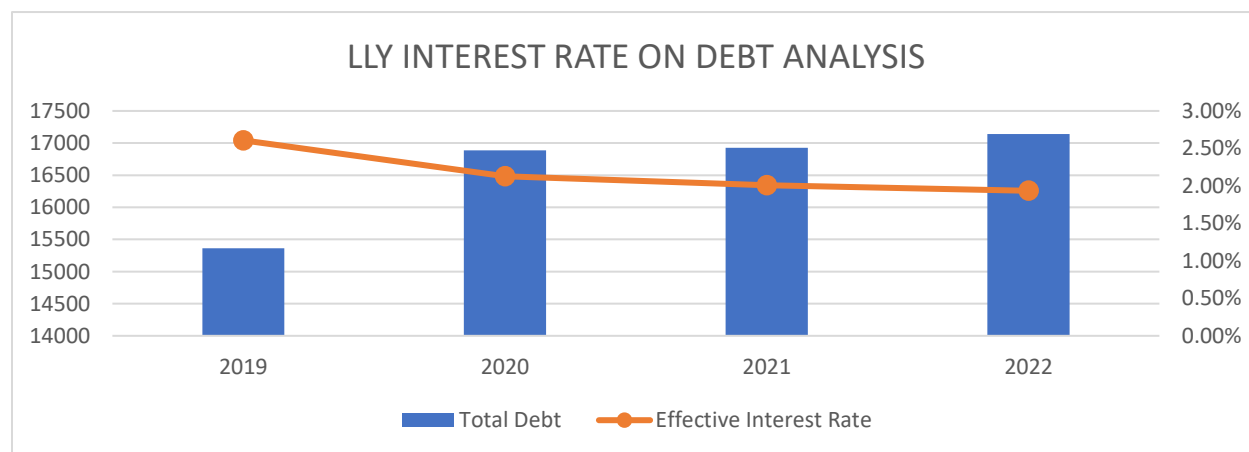
There is a consistent downward trend in the percentage of Accumulated Depreciation concerning sales from 2019 to 2021, indicating a reduction in the proportion of depreciation expenses relative to sales revenue. The depreciation % of sales revenue was 41% in the year 2019 and then it consistently shown downward trend till 2021 at 35%. Decreasing percentages suggest potential improvements in asset utilization or more efficient management of depreciable assets relative to sales revenue.

6. Total Liability Analysis:



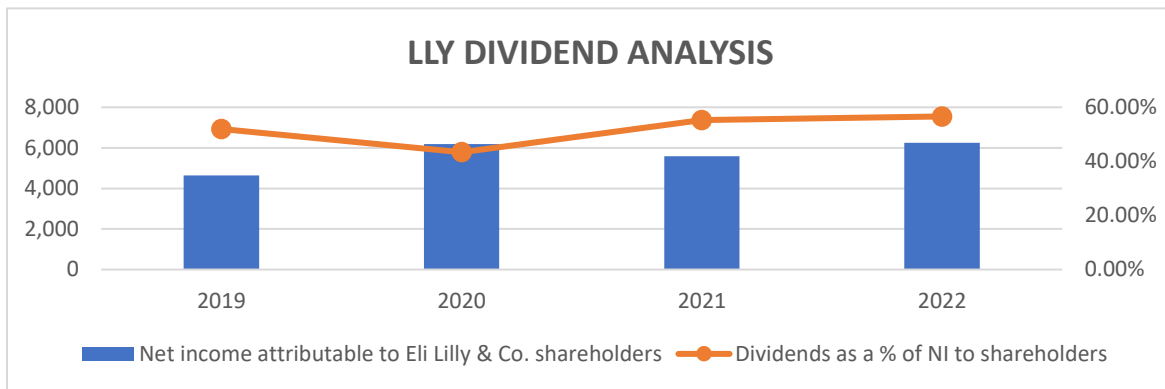
The data shows that the company's total financial liabilities have increased from \$33,619 million in 2019 to \$36,434 million in 2022. However, sales have also increased from \$22,320 million in 2019 to \$28,541 million in 2022. This means that the company's financial liabilities have grown at a slower rate than its sales. The total liabilities to sales stood at 160% in 2020 and then showed a reverse trend till 2022 standing at 120% of total sales. This suggests that the company is growing its business and taking on more debt to finance its growth. However, the company is also reducing some of its long-term liabilities. Overall, the financial liabilities and sales table suggests that the company is growing its business and taking on more debt to finance its growth.

7. Interest rate on debt analysis:



The average effective interest rate decreased from 2.61% in 2019 to 1.93% in 2022, as the table illustrates. This implies that in recent years, the business has been able to borrow money at cheaper interest rates. This could be due to various reasons such as the company's increasing creditworthiness and the overall decline in interest rates. The company's interest expense has decreased due to the decrease in the average effective interest rate. The company spent \$401 million in interest costs in 2019. In 2022, this dropped to \$332 million. This equates to an annual savings of \$69 million.

8. Dividend Analysis:



Dividends paid to stockholders have increased from 51.96% 2019 to 56.62% in 2022. The company's financial performance has been strong in recent years, with net income and dividends paid to stockholders both growing at an average annual rate of over 13%. This is likely due to several factors, including the company's strong portfolio of products, its global reach, and its commitment to innovation.

9. Income Tax rate analysis:

The effective tax rate of Income ranged from 11.9% to 8.3% from 2019 to 2022. This reflects the tax obligations based on the company's Income. The weighted average of tax comes to 11%.

10. Computing Market risk premium $E(r_M)$ using Market Multiple:

The stock was trading at \$362.27 on average in December 2022. The equity cash flow payout ratio was 81.88% (<https://stockanalysis.com/stocks/lly/statistics/>) and the anticipated growth of market was 14.54% (<https://stockanalysis.com/stocks/lly/statistics/>) bringing the $E(r_M)$ to **14.80%**.

11. Calculating the WACC:

The company had 949000 million shares outstanding and the price per share on December 31, 2022 was \$362.27 bringing the total value of Equity to \$3439000 million. The company has net debt of \$33000 million.

Thus, **WACC = cost of Equity (Re)*E/V + cost of debt (Rd)*D/V**. We have calculated the Re (cost of equity) using the CAPM method, where the formula is: **$R_f + (R_m - R_f) * \beta$** and we have factorized the tax benefit of interest in the cost of debt calculation.

- R_f = risk free rate (10-year treasury bonds)
- $(R_m - R_f) = E(r_M)$ = Market risk premium
- β = stock's sensitivity to market's risk

The risk-free rate of 10-year US government bond is 3.88% as on 31st December, 2022. We have used Gordon's growth model to calculate the expected market return i.e., 14.80% and the β factor of 0.33 is extracted from Yahoo Finance.

Taking into consideration all these factors, we have arrived at a WACC of **7.48%**

12. Building a proforma model and valuing the firm:

While Eli Lilly has an established track record of paying rising dividends to shareholders for several years, relying on a dividend discount model (DDM) to value Eli Lilly poses some complications versus a free cash flow model:

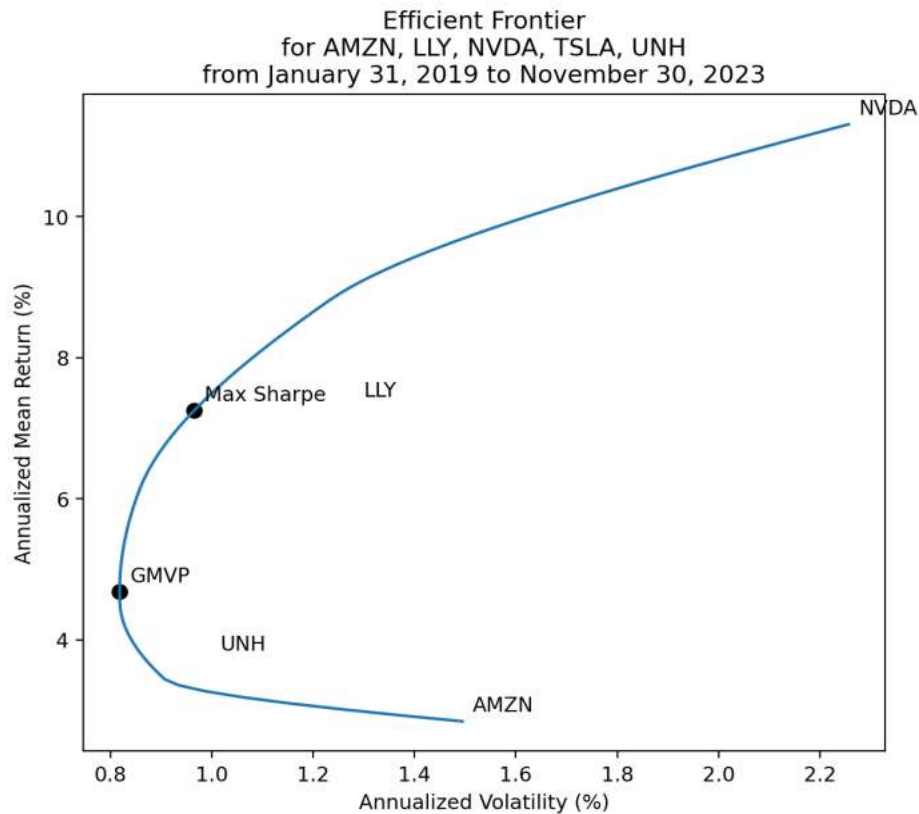
- **Variability in payouts:** Year-to-year variability in Eli Lilly's dividend payout ratio means dividend growth forecasts carry more uncertainty versus forecasting total free cash flow directly. Over the past 5 years, the payout ratio has ranged from 43% to 57% of earnings.
- **Cyclical industry:** The pharmaceutical industry growth is impacted by drug development cycles, generic erosion, and regulatory changes - factors not easily captured in long-term dividend projections. This uncertainty can affect the dividend payout in the long run.
- **Investments mute dividends:** Despite maturity, Eli Lilly still needs to aggressively invest in its drug pipeline through acquisitions and R&D spending to drive future revenue. These investments compete with maintaining stable, high dividend growth.

On the contrary, Free cash flow provides excess cash to deploy into investments and dividends in a flexible manner without overcommitting to stable dividend benchmarks each quarter. This financial flexibility is crucial amidst ongoing industry changes.

To calculate the Enterprise value, we have taken a conservative approach for forecasting long term growth rate to be 3.5%. The long-term growth rate is necessary for analyzing the present value of the terminal value. Taking this into consideration, our Enterprise value stood at \$563162329 million. On adding back, the excess assets and subtracting the net debt we arrived at the Equity value of the firm i.e., \$ 563138369 million. The company had 949310 million shares outstanding. Thus, to derive the fair value of the share we divided the equity value by the total number of shares outstanding arriving at a fair value of \$593.21.

Upon evaluating the current market price of the stock at \$589.25 alongside its fair value assessment of \$593.21, it is discerned that the stock is currently almost at par with its perceived intrinsic worth.

Efficient Frontier – No Short Sell



This code performs portfolio optimization and visualizes the efficient frontier using stock data fetched from Yahoo Finance. Here's a step-by-step breakdown:

1. **Importing the data:** Imported essential libraries such as matplotlib, NumPy, pandas, and SciPy. Optimized for plotting, numerical computing, data handling, and optimization.
2. **Fetching the Stock Data:** Utilizing the yahoo finance library, we downloaded the historical stock data for Amazon, Nvidia, Tesla, UnitedHealth, and Eli Lilly from December 2018 to December 2023. Took the risk-free rate of 4.22% (assumed). Resampled the data to monthly intervals and converted it into a Data Frame for analysis. We took adjusted close of monthly returns because the adjusted close accounts for dividend as well.
3. **Calculating the log returns:** Monthly log returns are calculated from the adjusted closing price of the stocks. The script computes the logarithm of the ratio between the monthly consecutive prices.
4. **Portfolio Optimization functions:** The function port_vol calculates the portfolio volatility. This matrix contains the pairwise covariances between different assets' returns. We multiply it by 252 assuming that there are 252 trading days in a year. This entire computation aims to derive the portfolio's volatility based on the provided weights and the covariance matrix of returns, assuming these weights determine the asset allocations in the portfolio. The function port_mean computes the expected annual return of the portfolio based on the provided weights and historical average returns of assets. It multiplies the weighted average return by 252 to represent the annualized return. The neg_sharpe_ratio represents the risk-adjusted

performance of the portfolio. A higher (less negative) Sharpe ratio indicates better risk adjusted returns.

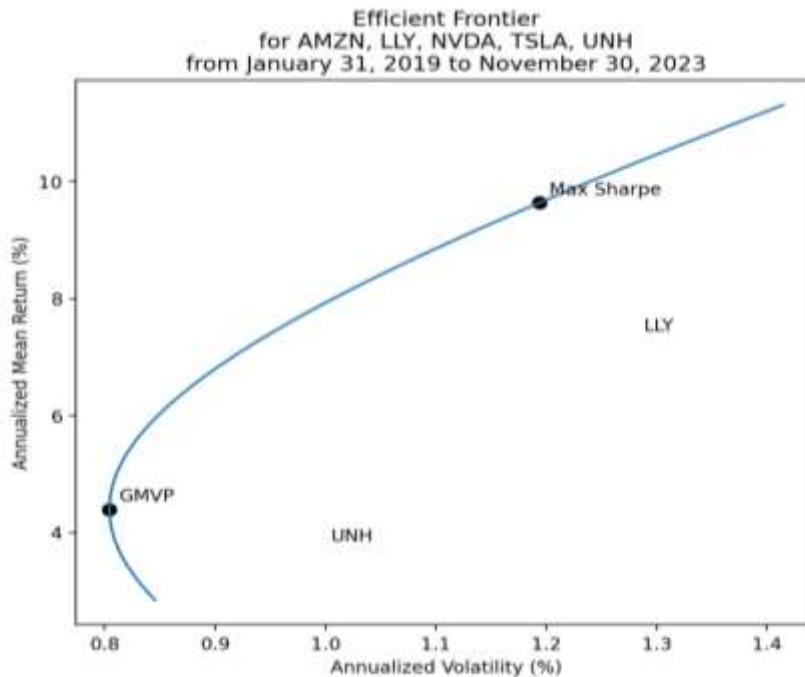
5. **Global Minimum Variance Portfolio (GMVP):** The Global Minimum Variance Portfolio is the portfolio allocation that minimizes the risk of the portfolio given the constraints of the optimization problem. The annualized return of this portfolio is 4.6811. This represents the expected gain or loss of the portfolio over a year based on historical data. The volatility, or standard deviation of returns, is 0.8185. This metric measures the dispersion of returns. In this case, it's the lowest volatility achievable among the considered portfolios. The Sharpe ratio is 5.6673. This is a measure of risk-adjusted return. A higher Sharpe ratio signifies better performance concerning the risk taken.

There are allocations(weights) of different assets in the portfolio. For this Global Minimum Variance Portfolio: Amazon (AMZN) comprises 17.20% of the portfolio. Eli Lilly (LLY) comprises 27.83% of the portfolio. NVIDIA (NVDA) comprises 1.56% of the portfolio. Tesla (TSLA) has a weight of 0%.

UnitedHealth (UNH) comprises 53.41% of the Portfolio. This allocation suggests that based on historical data and optimization parameters, the algorithm recommends a portfolio with a substantial allocation towards UnitedHealth and a reasonable allocation to Eli Lilly, with a smaller allocation to Amazon and NVIDIA, while Tesla is completely excluded from this optimized allocation.

6. **Maximizing the Sharpe ratio portfolio:** It aims to find the portfolio allocation which represents the trade-off between risk and return. The resulting portfolio is considered optimal as it maximizes the risk-adjusted return. The annualized return of 7.2502 represents the expected gain of the portfolio over a year based on historical data. The volatility of 0.9655 implies lower risk. The Sharpe ratio of 7.4652 indicates higher returns relative to the risk taken. The analysis recommends a portfolio heavily skewed towards Eli Lilly (47.17%) and UnitedHealth (28.59%), with NVIDIA (24.25%) having a significant but lesser weight, and Amazon and Tesla being completely excluded from this optimized allocation.
7. **Constructing an efficient frontier:** By using “sco. minimize” function we found the optimal portfolio weights that would minimize the portfolio risk for a specific target return. We minimized the risk of portfolio volatility using (port_vol). Since short selling is prohibited, we have kept the bounds (Weights) from 0 to 1 and two constraints were imposed stating that the portfolio weight must be equal to 1 and the portfolio's expected return should match the target return(t). In summary, this code performed a series of optimizations, aiming at finding the portfolio composition that minimizes risk while achieving a specific target return.
8. **Analysis of the Efficient frontier Graph:** The frontier is located above the market risk premium, indicating that the four stocks offer the potential to generate excess returns over the risk-free rate. The GMVP indicates that at 0.81% volatility(lowest) the returns are 4.68% which is above the risk-free rate of 4.22% with proper diversification. The Maximum Sharpe ratio portfolio aims to achieve the maximum possible return for the minimum level of risk. The maximum returns of 7.25% can be achieved at 0.96% volatility by considering LLY (47.17%), UNH (28.59%) and NVDA (24.25%) in the portfolio.

Efficient Frontier – Short Sell



Rest everything remaining the same, the bounds (weights allocation) change from -1 to +1. Using these bounds,

1. **GMVP:** The performance of global minimum variance portfolio indicates a return of 4.3925% with the lowest volatility of 0.8042% and the Sharpe ratio of 5.4095. These returns could be achieved by allocating the following weights (long position) – UNH (53.19%), LLY (27.07%), AMZN (21.99%), NVDA (3.86%) and taking a short position in Tesla (-6.11%).
2. **Maximum Sharpe Ratio:** The resulting Sharpe ratio portfolio is considered optimal as it maximizes the risk-adjusted return. The annualized return of 9.6393 represents the expected gain of the portfolio over a year based on historical data. The volatility of 1.1940 implies lower risk. The Sharpe ratio of 8.0380 indicates higher returns relative to the risk taken. The analysis recommends a portfolio heavily skewed towards LLY (56.12%), NVDA (42.26%), UNH (39.68%), Tesla (5.02%) and taking a short position in AMZN (-43.08%)

Thus, we can conclude that if short selling is prohibited, our optimal portfolio will generate a return of 7.25% at 0.96% volatility; whereas if short selling is allowed our optimal portfolio will generate a return of 9.64% with 1.19%.