Statistics of Radioactive Decay II

RT4220 – Lecture #5

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WSU

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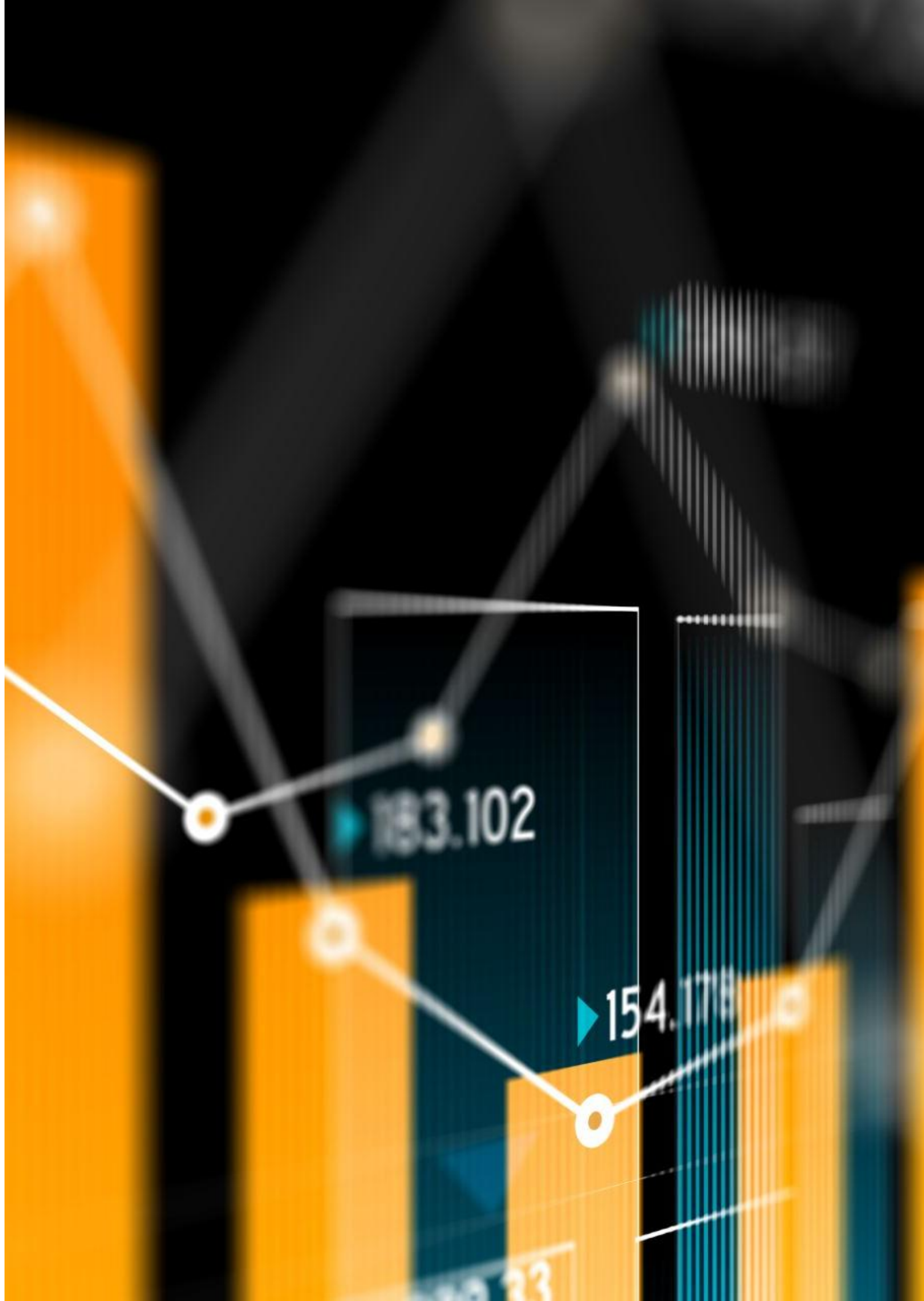


Specific Activity Terminology

Carrier is the STABLE portion of radioactive sample

I-131 radioactive

I-132 stable → carrier

If there is NO stable portion it is **carrier-free**

Specific activity is the ratio of ACTIVITY OVER MASS of the element present

Carrier-free specific activity is the highest possible specific activity (100% radioactive)

High specific activity is typically desirable

If its injected: less to inject!

If its in a machine: smaller machine!

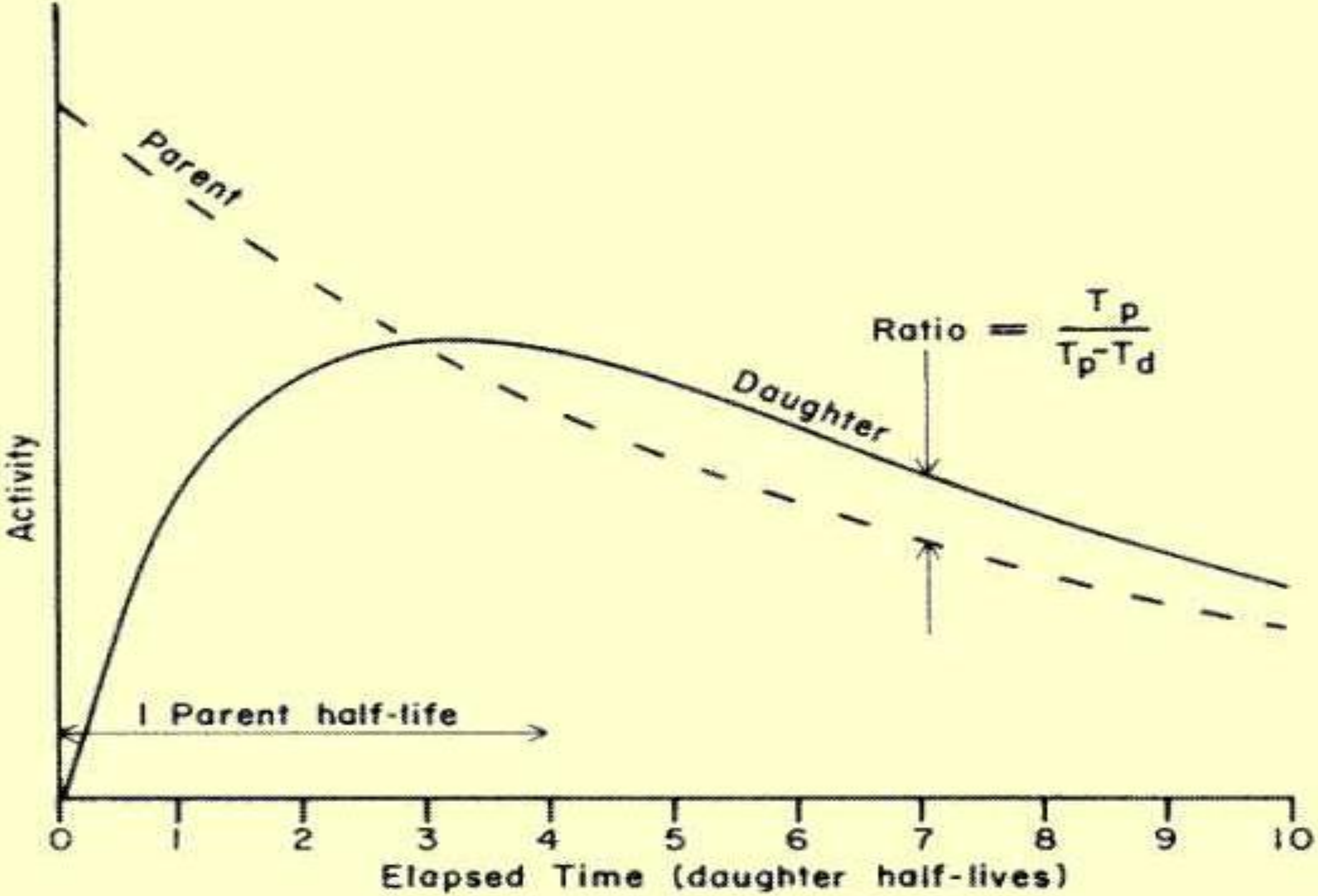
Practice: Specific Activity

Given two samples of Myoview (brand name Tc-99m) with the following values determine the empty values:

Sample	Activity (mCi)	Mass (g)	Spec. Act. (mCi/g)
1	0.1	100	0.001
2	200	20	10

If 10 mCi is required for a gastric clearance study, how many grams are required to be injected? If the samples all have a density of 1 g/mL, how many mLs must be injected?

Sample 1 would require a 1,000 mL or 1 L INJECTION WOW. Sample 2 would require a 1 mL injection *wow*

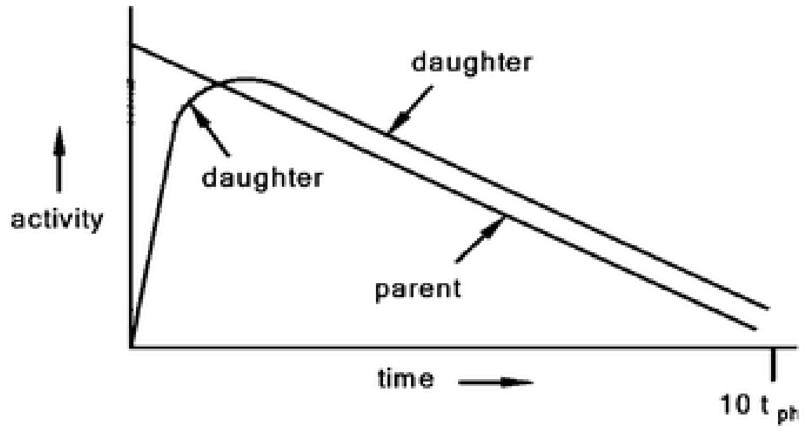

Parent
Daughter
Ratio = $\frac{T_p}{T_p - T_d}$
Activity
1 Parent half-life
0
1
2
3
4
5
6
7
8
9
10
Elapsed Time (daughter half-lives)

Equilibrium

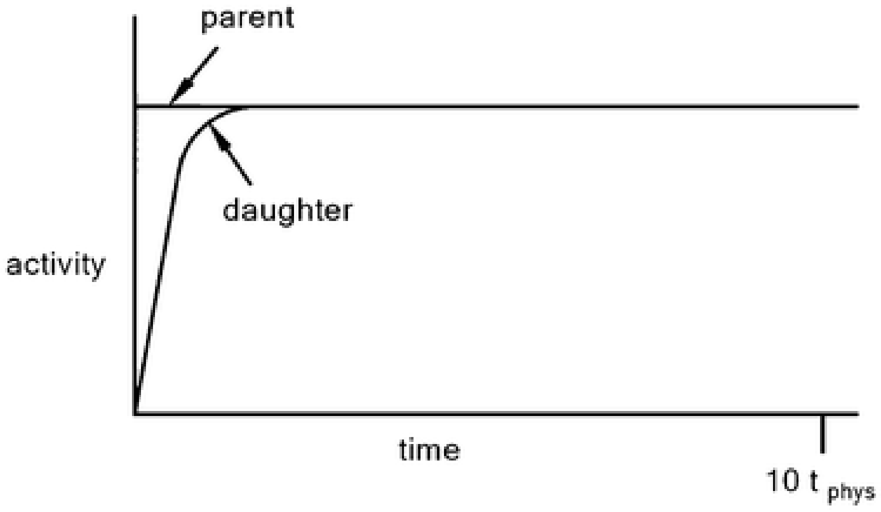
Shan’t Equivocate this Equilibrium

Equilibrium Basics

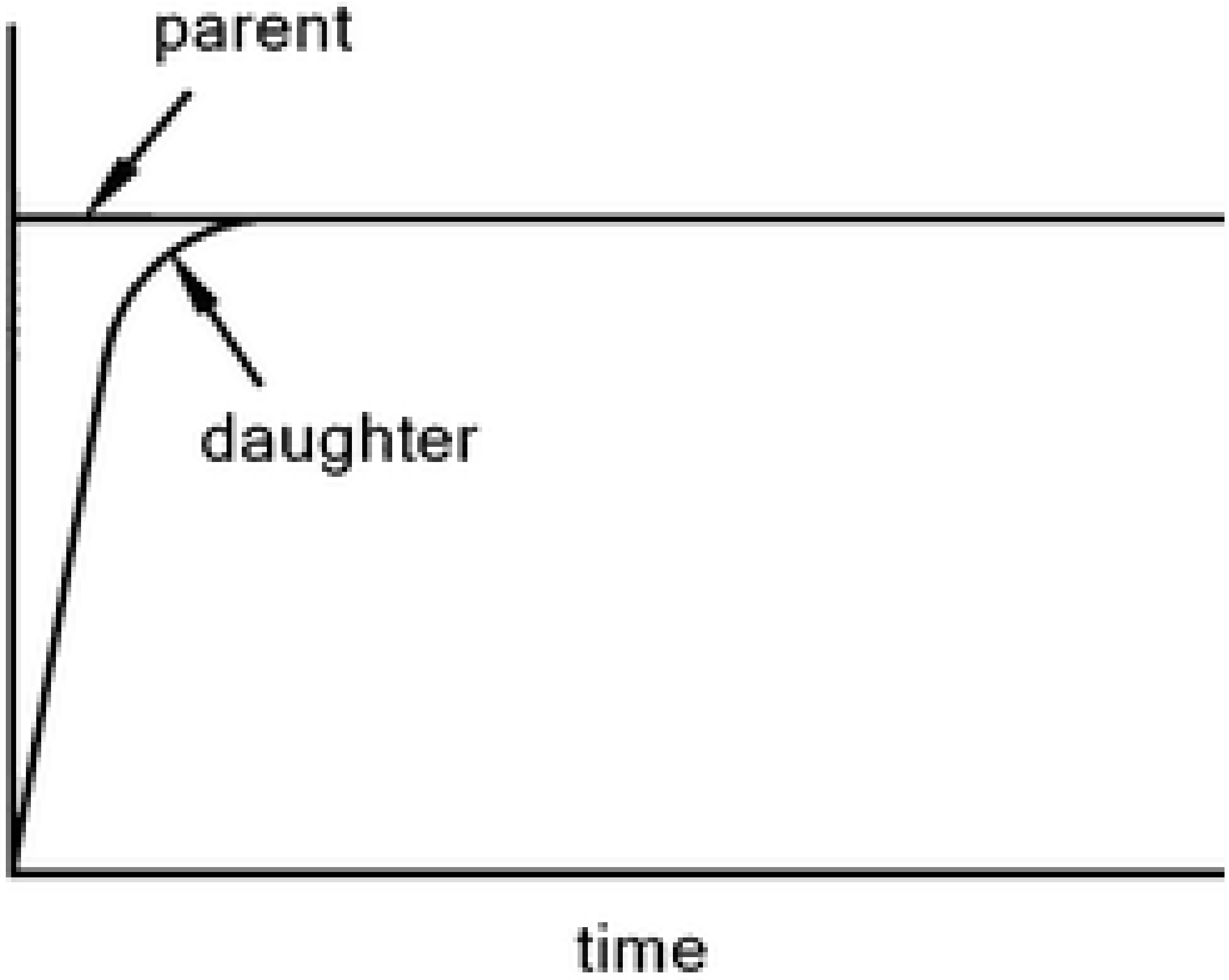
- What happens if the daughter is radioactive?
- They decay at different rates → a race!
- This leads us to equilibria, three types:
  - **Secular**: Half-life of parent >>> daughter's
  - **Transient**: Half-life of parent > daughter's
  - **No Equilibrium**: Half-life of parent <= daughter's



(a) transient equilibrium



(b) secular equilibrium



(b) secular equilibrium

Secular Equilibrium

- If the parent half-life is *much* longer than the daughter half-life, then the equilibrium established is called **secular equilibrium**

- The activity of the daughter closely matches the parent because there is a bottleneck
    - Not enough production
    - Much consumption
    - Seize the means

Secular Equilibrium Example

$$^{226}_{88}\mathrm{Ra} \xrightarrow{T_{1/2}=1602\ \mathrm{y}} {}^{222}_{86}\mathrm{Rn} \xrightarrow{T_{1/2}=3.8\ \mathrm{d}} {}^{218}_{84}\mathrm{Po}$$

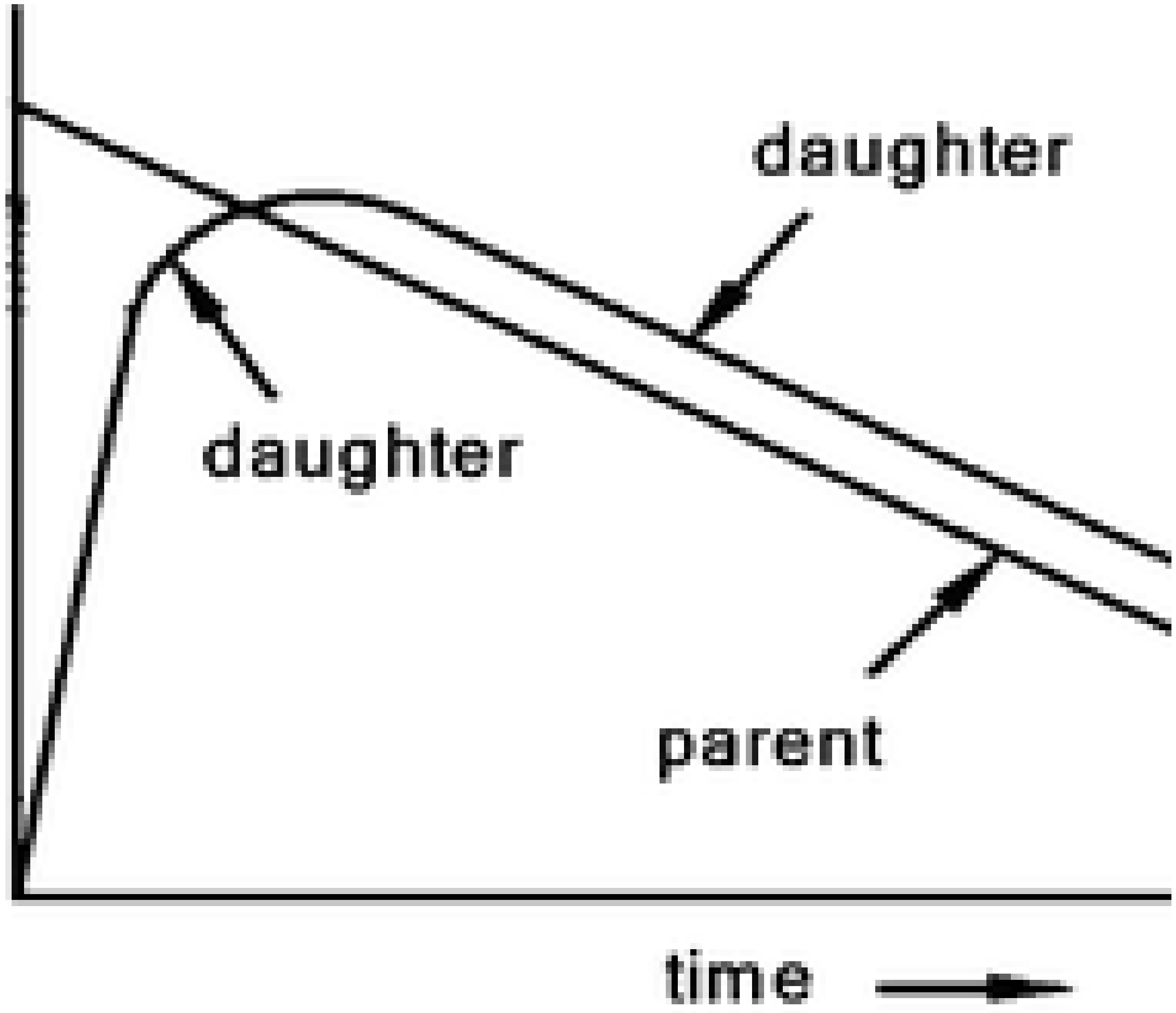
- Radium’s half life >>> daughter → quick buildup period → maximum → same decay rate

Transient Equilibrium

- If the half-life of the parent is longer than the daughter half-life then the equilibrium established is called **transient equilibrium**

- After some maximum where equilibrium occurs (rate of production = rate of consumption) the RATIO of parent:daughter activity becomes CONSTANT

- The decay rate of the daughter is dependent upon the decay rate of the parent



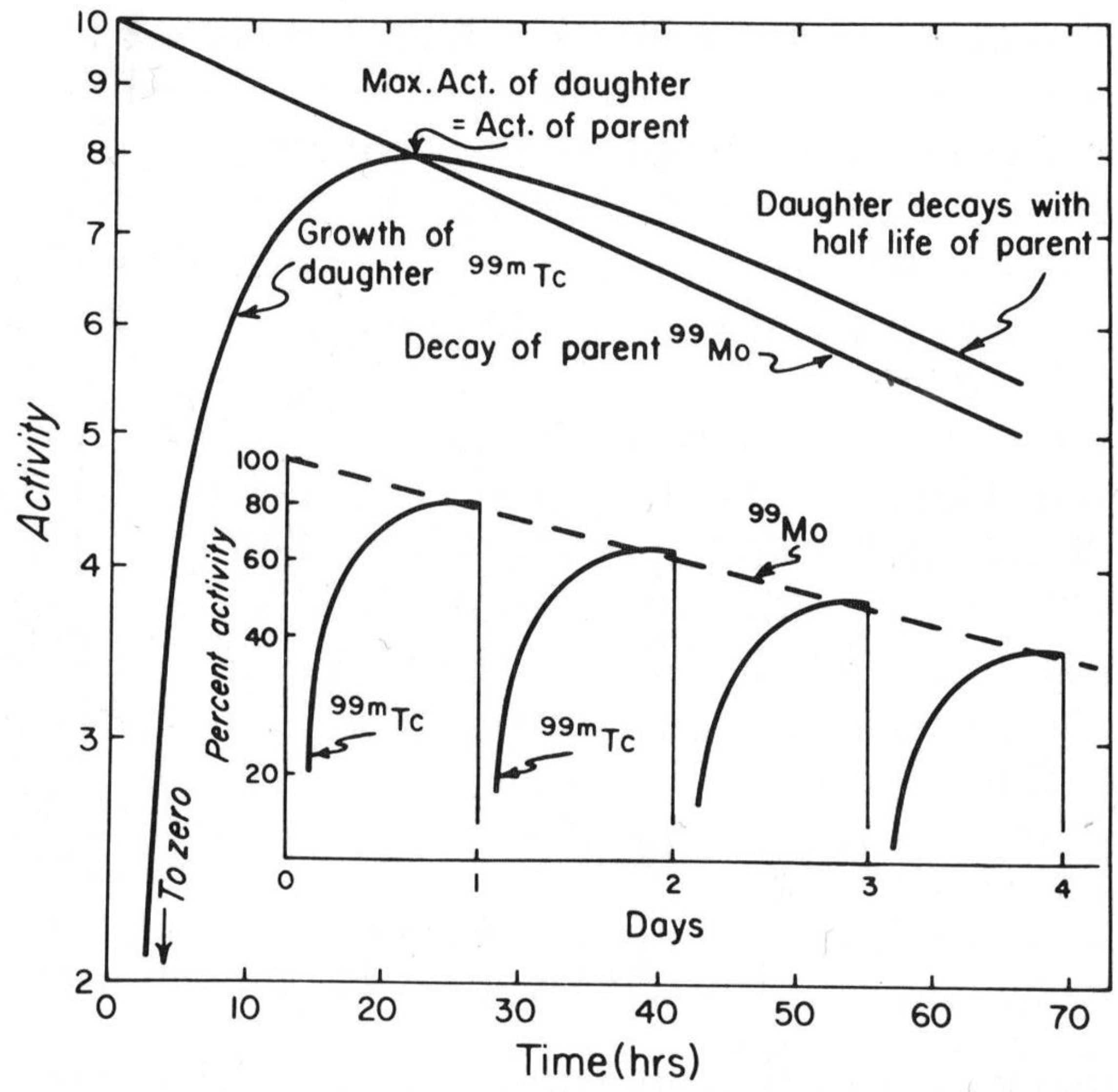
Transient Equilibrium Example

$$ {}^{99}_{42}\mathrm{Mo} \xrightarrow{T_{1/2}=66.7\ \mathrm{h}} {}^{99\mathrm{m}}_{43}\mathrm{Tc} \xrightarrow{T_{1/2}=6.03\ \mathrm{h}} {}^{99}_{43}\mathrm{Tc} $$

- Technetium is ubiquitous in nuclear medicine departments for imaging
  - 140 keV gammas perfect for imaging
  - Short half-life makes patients safe shortly after procedure
- A Molybdenum generator is referred to as a “Moly-Cow”
  - Cows are milked
  - Do not milk bulls

Transient Equilibrium Example Plot

- At the right is a plot of the activity inside the Moly-Cow
- Pattern of transient equilibriu is apparent, obfuscated by th milking process
- Hospitals typically have their own Moly Cow
- Time for maximum activity is large compared to the half-lif



Parent-Daughter Dual Decay

- When both the parent and daughter are radioactive, modeling the daughter activity becomes more difficult

- The ACTIVITY EQUATION remains the same for the PARENT:
  - $\mathrm{A(t)} = \mathrm{A(0)}e^{-\lambda t}$

- The ACTIVITY EQUATION for the DAUGHTER **depends on the parent** activity

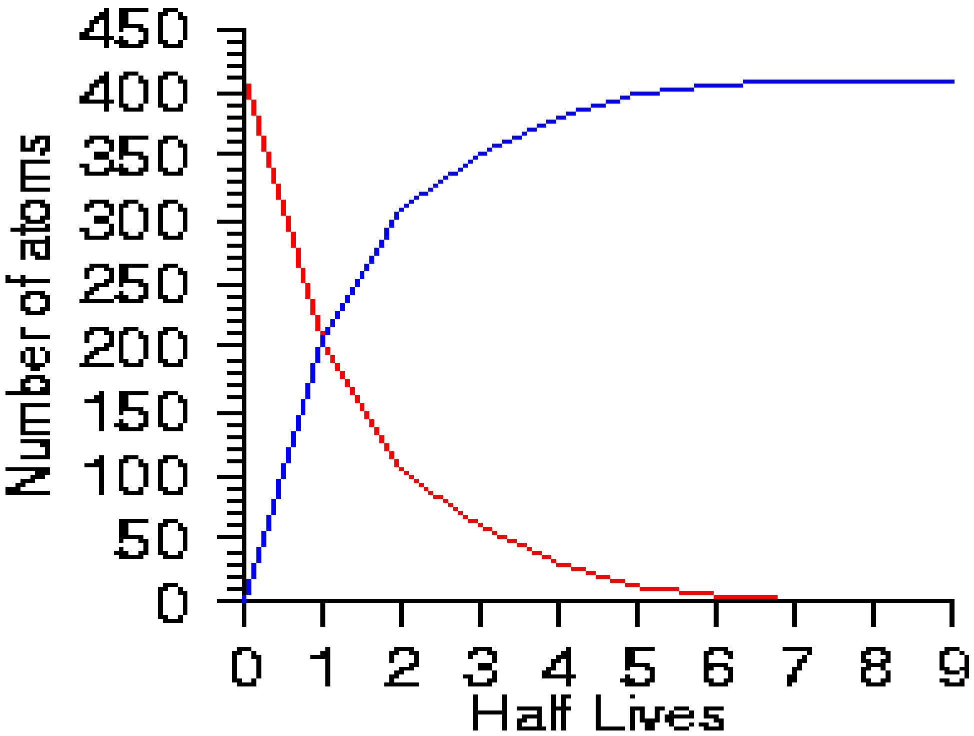
No Equilibrium

- At no point is the daughter production/decay rates equivalent

- Daughter activity will rise as it decays slower than the parent
  - Parent half-life < Daughter half-life
  - Daughter production rate < Daughter decay rate

Parent

Daughter



Daughter Activity Equation

$$1 \xrightarrow{\lambda_1} 2 \xrightarrow{\lambda_2} 3$$

$$\frac{\Delta \mathrm{N}_1}{\Delta \mathrm{t}} = -\lambda_1 \mathrm{N}_1$$

$$\frac{\Delta \mathrm{N}_2}{\Delta \mathrm{t}} = \lambda_1 \mathrm{N}_1 - \lambda_2 \mathrm{N}_2$$

$$\mathrm{A}_1(\mathrm{t}) = \left(\frac{\lambda_2}{\lambda_2 - \lambda_1}\right) \times \mathrm{A}_1(0) \times \left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right)$$

Time to Maximum Daughter Activity

- As the parent creates daughter and the daughter decays, there is a race.
- For every set of decay constants (characteristic) there will be a time where the daughter reaches a maximum NUMBER of atoms
- We can predict this using the following equation:

$$t_m = \frac{\ln {}^{\lambda_2}/_{\lambda_1}}{\lambda_2 - \lambda_1}$$