Statistics of Radioactive Decay I

RT4220 – Lecture #5

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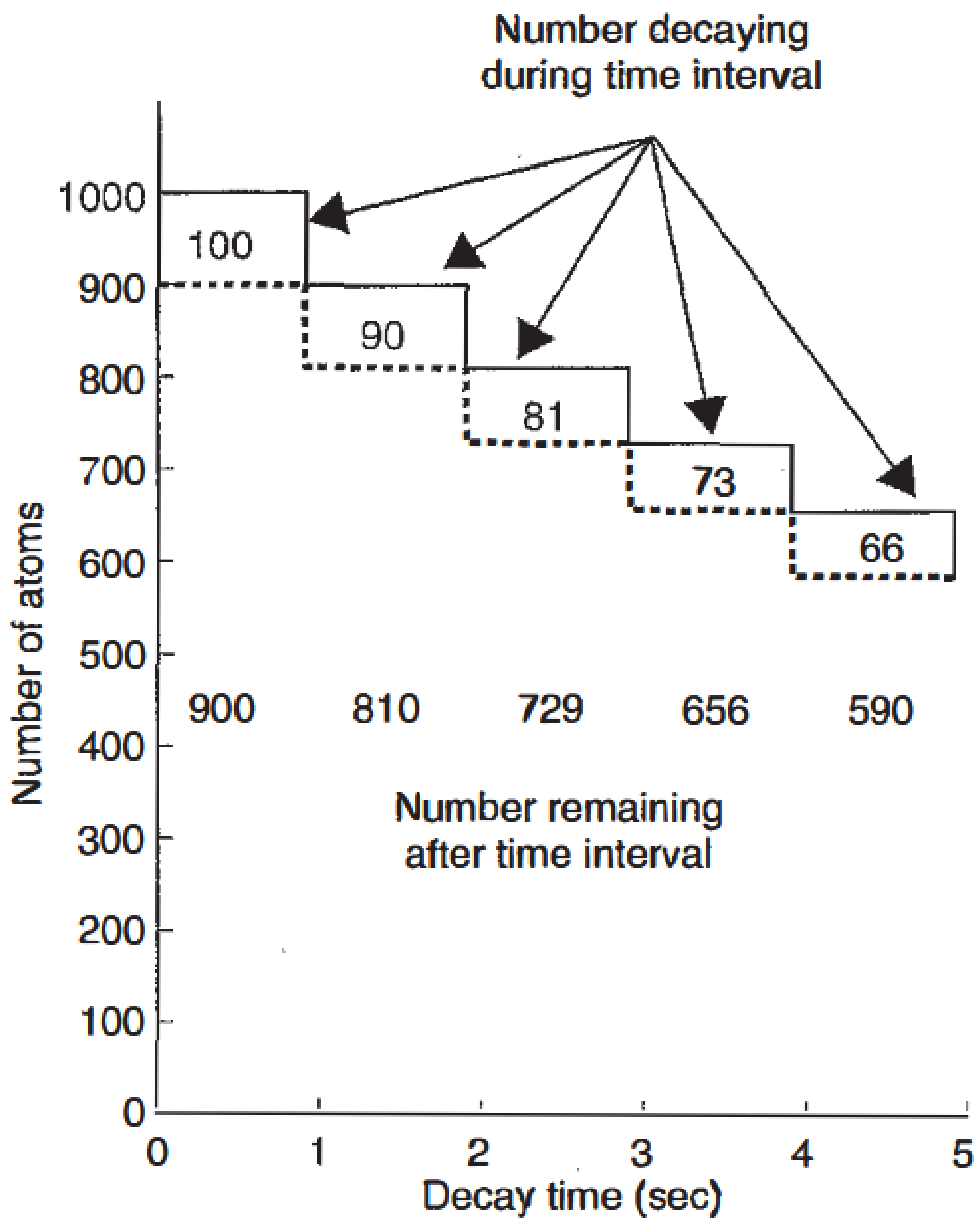
Terminology

- Radioactivity is spontaneous
  - Cannot predict PRECISE moment
  - Can model statistically
- N_x – number of remaining radioactive atoms of a given radionuclide x
- λ_x – decay constant for a given radionuclide x
  - Probability of decay of individual atom during given time period [time^-1]
  - $\lambda = 0.01 \left[\frac{1}{s}\right]$ means 1% chance of atom decay per second
- Δ __ – means change in __
  - Thus ΔN is the change in the number of remaining…
  - Δt is change in time
- $N(t)$ – means the number of remaining… at a given time t, t can be any number, positive, zero, negative, and is relative

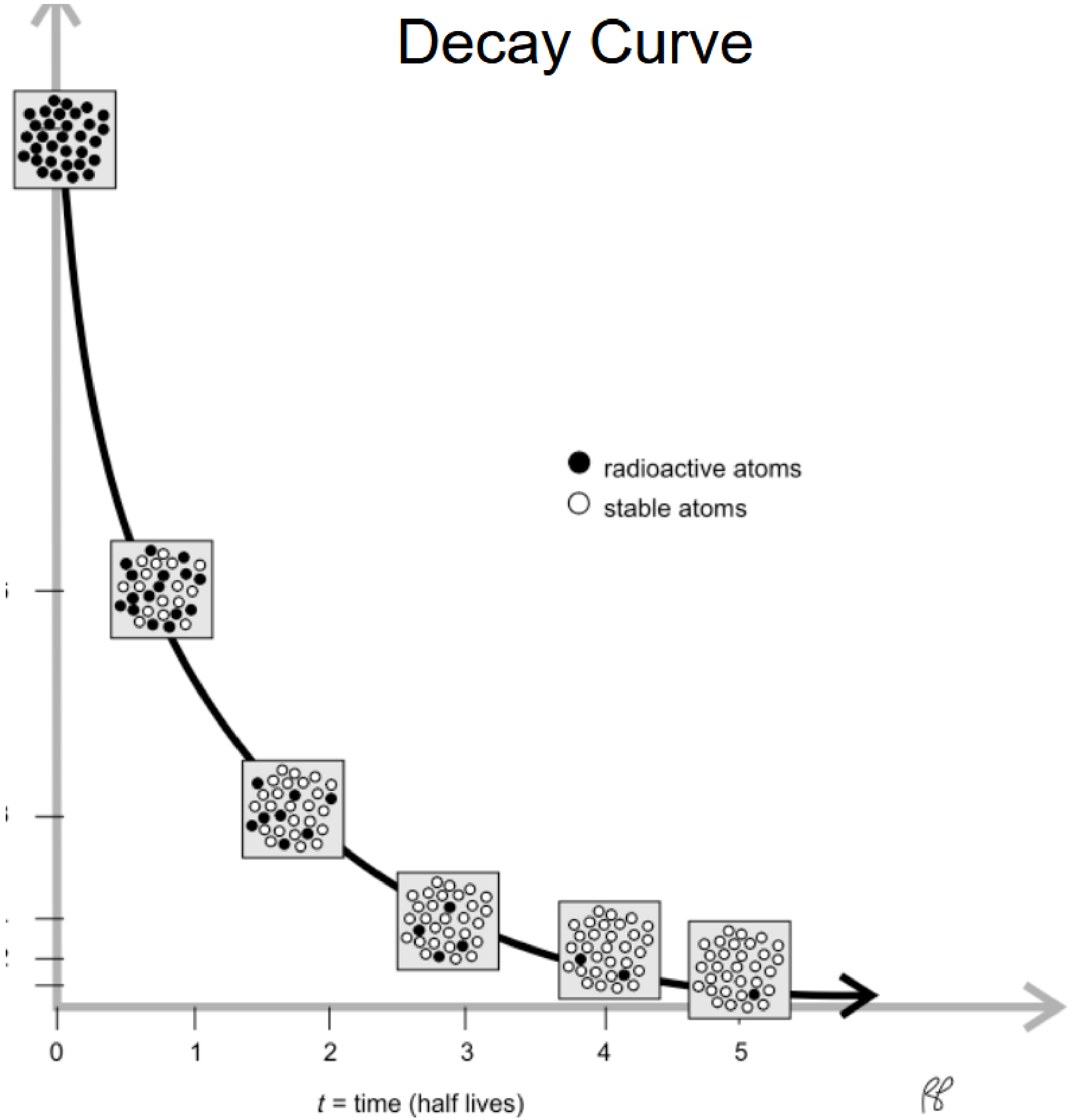
Catching the Vibe

- If for every second 10% decays, the number decaying per second will decrease
  - 100 -> 90 → 81
  - Thus, $\frac{\Delta N}{\Delta t}$ is negative
- $\frac{\Delta N}{\Delta t} = -\lambda N$ is how we represent the above and right
  - $\left|\frac{\Delta N}{\Delta t}\right| = \lambda N = A$ which is the “activity”
  - λ is the decay factor
- Through calculus we determined that $N(t) = N(0)e^{-\lambda t}$



Vibe Part 2

- As the sample decays there is an exponential pattern
- $e^{-\lambda t}$ is our exponential function
- Eventually, the whole sample decays to the daughter



Decay Constant

- Defined as the proportion of a sample that will decay during a given time period
- Characteristic of radionuclides (does NOT change for any GIVEN radionuclide)
- Also: the probability that any given atom will decay during that time period
- If there are multiple modes of decay the total decay constant is the sum of the individual constants for each mode
    - $\lambda = \lambda_1 + \lambda_2 + \lambda_3 \ldots$
- This yields the DECAY EQUATION:

$$N(t) = N(0)e^{-\lambda t}$$

Activity

- Activity is defined as the average decay rate
- Mathematically defined as $\left|\frac{\Delta N}{\Delta t}\right| = |-\lambda N| = A$
  - Activity at a given time t is represented as $A(t)$
- Units: decays per second [dps]/decays per minute [dpm] OR Curie [Ci] which is 3.7×10^{10} dps
  - dps is the definition of the Becquerel [Bq]
  - Ci originally defined as the activity of 1 g of Ra-226, was later changed very slightly
- Actual decay rate fluctuates

Decay Factor

- Decay factor is defined as the fraction of radioactive atoms REMAINING after time t
  - $\mathrm{DF} = e^{-\lambda t}$
- Reminder: $\mathrm{e} = 2.718\ldots$ This is the base of natural logarithms
- Decay factor is exponential function, as time goes on the factor gets exponentially smaller

The Activity Equation (THE MOST IMPORTANT)

$$\mathrm{A(t)} = \mathrm{A(0)}\, e^{-\lambda t}$$

- This comes from substitution of previous equations:

$$\because \mathrm{A} = \lambda \mathrm{N}$$

$$\because \mathrm{N(t)} = \mathrm{N(0)}\, e^{-\lambda t}$$

$$\lambda \times \mathrm{N(t)} = \lambda \times \mathrm{N(0)}\; e^{-\lambda t} \rightarrow$$

$$\mathrm{A(t)} = \mathrm{A(0)}\, e^{-\lambda t}$$

Half-Life 3

- Defined as the time it takes for the sample to decay to half of its INITIAL activity level
- Directly related to the DECAY CONSTANT:

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\lambda = \frac{\ln 2}{T_{1/2}}$$

- Most of the time, we use half-life instead of decay constant

Average Lifetime

- Defined as the average time that a radioactive atom sticks around until it decays
- THIS IS THE INVERSE OF **DECAY CONSTANT**… LITERALLY
  - Probability of decay of individual atom… VS. average time that a radioactive atom… until it decays
- Inverse mathematically too:

$$\tau = \frac{1}{\lambda}$$

thus

$$\tau = \frac{T_{1/2}}{\ln 2}$$

Semi-log Plots

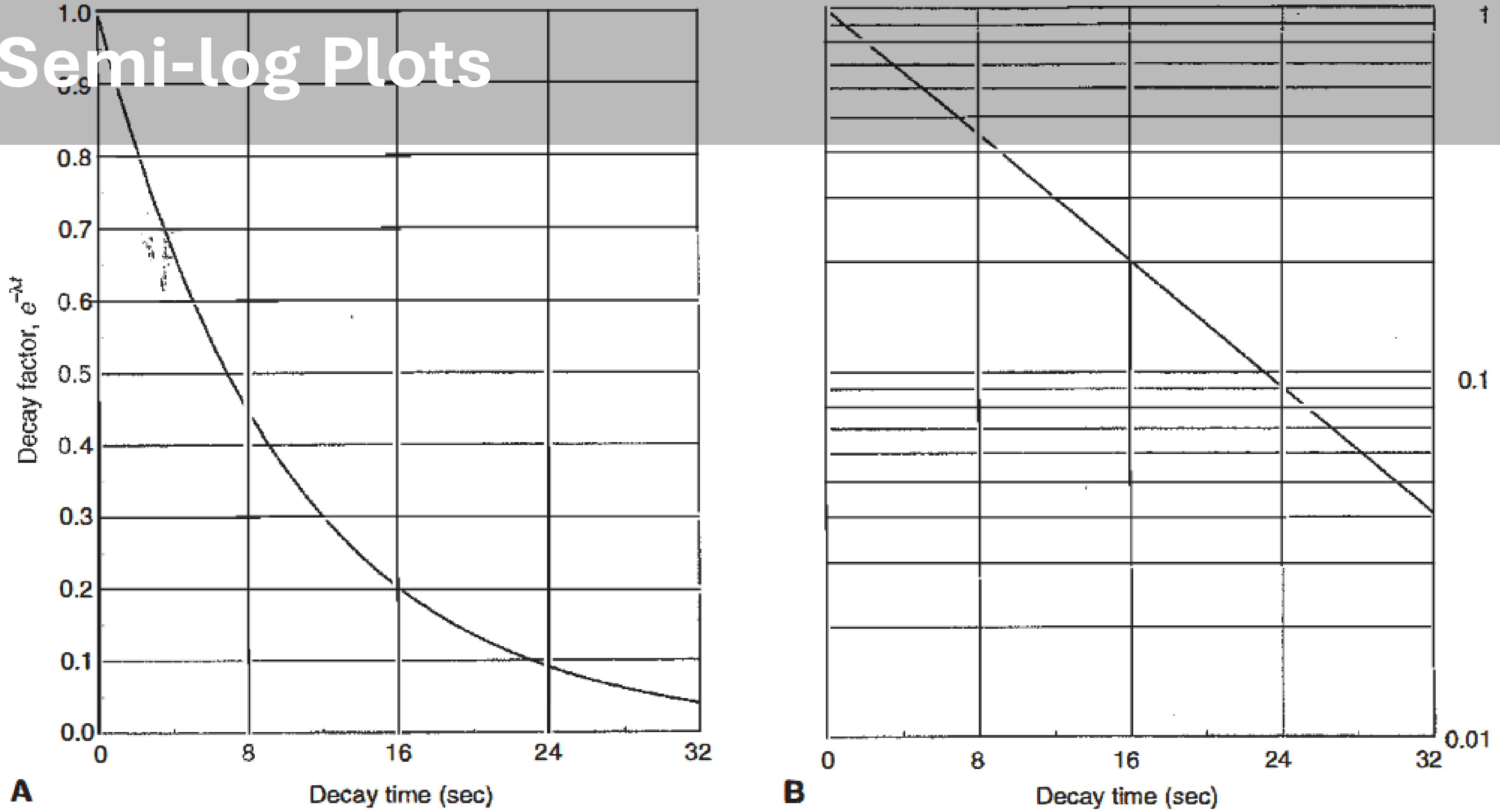


Figure 4–2. Decay factor versus time shown on linear *(A)* and semilogarithmic *(B)* plots, for radionuclide with $\lambda = 0.1\ \text{sec}^{-1}$.

Important Equations

$$\boxed{DF = e^{-\lambda t}}$$

$$\boxed{\begin{aligned} \mathrm{N(t)} &= \mathrm{N(0)}\mathrm{e}^{-\lambda \mathrm{t}} \\ \mathrm{A(t)} &= \mathrm{A(0)}e^{-\lambda t} \end{aligned}}$$

$$\boxed{\mathrm{A} = |-\lambda \mathrm{N}| = \left|\frac{\Delta \mathrm{N}}{\Delta \mathrm{t}}\right|}$$

$$\boxed{\begin{aligned} \tau &= \frac{1}{\lambda} \\ \tau &= \frac{\mathrm{T}_{1/2}}{\ln 2} \end{aligned}}$$

$$\boxed{\begin{aligned} \mathrm{T}_{1/2} &= \frac{\ln 2}{\lambda} \\ \lambda &= \frac{\ln 2}{\mathrm{T}_{1/2}} \end{aligned}}$$

Practice: Finding Half-Life

What is the half-life of an isotope with a decay constant of 2.6×10^{-6} s?

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$T_{1/2} = \frac{0.693}{2.6 \times 10^{-6}} \cong 266{,}538 \text{ s} \cong 3 \text{ days}$$

Practice: Finding Mean-Life

What is the mean-life of an isotope with a half-life of 4 days?

$$\tau = \frac{T_{1/2}}{\ln 2}$$

$$\tau = \frac{4}{\ln 2} = 4 \times 1.44 \cong 5.8 \text{ days}$$

Practice: Manipulating Decay Equation

75% of a given radioisotope sample has decayed over the course of an hour. What is the decay factor? Also, what is the half-life in minutes?

$$DF = \frac{N(60)}{N(0)} = \frac{25\%}{100\%} = 0.25 \longrightarrow$$

$$\mathrm{DF} = e^{-\lambda t} \longrightarrow 0.25 = e^{-\lambda \times 60} \rightarrow \ln 0.25 = -60\lambda \rightarrow \lambda = -\frac{\ln 0.25}{60} \rightarrow$$

$$\lambda = \frac{\ln 2}{\mathrm{T}_{1/2}} \rightarrow \frac{\ln 2}{\mathrm{T}_{1/2}} = -\frac{\ln 0.25}{60} \rightarrow \mathrm{T}_{1/2} = -\frac{60 \times \ln 2}{\ln 0.25} = 30 \text{ minutes}$$

Practice: Manipulating Activity Equation

The activity of a radioisotope sample is measured to be 370 MBq at time $\mathrm{t} = 1800$ s. What was the initial activity of the sample in mCi if the decay constant for this radioisotope is 0.04 hr^{-1}.

$$0.04\ \frac{1}{\mathrm{hr}} \times \frac{1}{3600}\frac{\mathrm{hr}}{\mathrm{s}} = 1.11 \times 10^{-5}\ \frac{1}{\mathrm{s}}$$

$$\mathrm{A}(1800) = \mathrm{A}(0)\ e^{-\lambda \times 1800} \rightarrow \mathrm{A}(0)\ [\mathrm{mCi}] \times \frac{10\ [\mathrm{mCi}]}{e^{-(1.11\times 10^{-5})\times 1800}} \cong 10.2\ \mathrm{mCi}$$