Find a general solution of the following differential equation.

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0. {4}$$

$$z^5 = i$$
, $z \in \mathbb{C}$.

- a) Solve the equation, giving the roots in the form $re^{i\theta}$, r > 0, $-\pi < \theta \le \pi$. (5)
- b) Plot the roots of the equation as points in an Argand diagram. (1)

a) Sketch the graph of $y = \operatorname{arsech} x$, defined for $0 < x \le 1$. (3)

b) Show clearly that

$$\frac{dy}{dx} = -\frac{1}{x\sqrt{1-x^2}} \,. \tag{4}$$

c) Hence evaluate

$$\int_{\frac{1}{2}}^{1} \operatorname{arsech} x \ dx.$$

Give the answer in the form $\lambda \left[2\pi - 3\ln(2 + \sqrt{3}) \right]$, where λ is a rational number to be found. (8)

By showing formally all the limiting processes evaluate the following integral

$$\int_0^{\frac{1}{4}\pi} \frac{1}{x} - \frac{\sin 2x}{1 - \cos 2x} \ dx \ .$$

Give the answer in the form
$$\ln \left[\frac{\pi \sqrt{2}}{n} \right]$$
, where *n* is a positive integer to be found. (8)

Consider the following infinite convergent series.

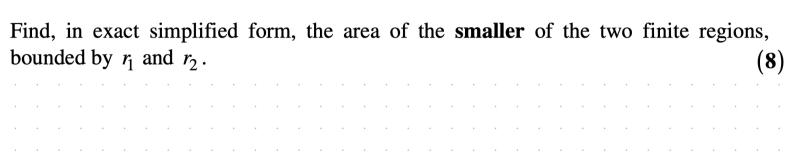
$$\frac{3}{1\times2} - \frac{5}{2\times3} + \frac{7}{3\times4} - \frac{9}{4\times5} + \frac{11}{5\times6} - \dots$$

- a) Use the method of differences, to find the sum of this series. (8)
- b) Verify the answer of part (a) by using a method based on the Maclaurin expansion of $\ln(1+x)$.

The following polar equations are given.

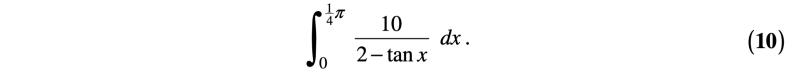
$$r_1 = \cos \theta$$
, $0 \le \theta \le \pi$.

$$r_2 = \frac{1}{\cos\theta - \sin\theta}, \quad -\frac{1}{4}\pi \le \theta \le \frac{5}{4}\pi.$$





Use appropriate integration techniques to find an exact simplified value for



$$f(x) = 2\arcsin\sqrt{x} - \arcsin(2x-1), \quad 0 \le x \le 1.$$

By considering f'(x) sketch the graph of f(x). (8)