The tire model

Tires are perhaps the most important, but difficult to model, component of an automobile. In addition to supporting the vehicle and damping out road irregularities, the tires provide the longitudinal and lateral forces necessary to change the speed and direction of the vehicle. These forces are produced by the deformation of the tire where it contacts the road during acceleration, braking, and cornering.

Aligning Moment, M_z

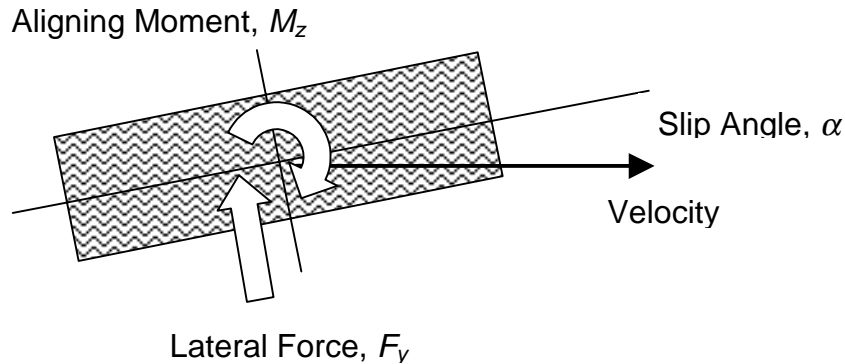Slip Angle, α

Velocity

Lateral Force, F_y

Figure 1: Tire operating at a slip angle.

In the absence of side forces, a rolling tire travels straight ahead along the wheel plane. During a cornering maneuver, however, the tire contact patch "slips" laterally while rolling such that its motion is no longer in the direction of the wheel plane (Figure 1). The angle between its direction of motion and the wheel plane is referred to as the slip angle, α. This lateral "slip" generates a lateral force, F_y, at the tire-ground interface. Because the force acts slightly behind the center of the wheel, it produces an aligning moment, M_z, which tends to realign the wheel in the direction of rolling.

Normal cornering maneuvers result in small slip angles, low lateral force, and minimal sliding of the tire. At larger slip angles lateral force increases and reaches a maximum as the tire begins to slide. Figure 2 illustrates the relationship between lateral force and slip angle for a typical tire. For small values of α—say, less than four degrees—the relationship is nearly linear. The initial slope of the curve is known as the cornering stiffness, C_α, described in units of force per degree.

There exist many tire models to describe tire behavior beyond the linear region. One model commonly used in vehicle dynamics simulations was developed by H. Pacejka of the Delft University of Technology [1]. The Pacejka tire model calculates lateral force and aligning torque based on slip angle and longitudinal force based on percent longitudinal slip. The model parameters are dependent on the normal force F_z on the tire where the normal force is given in kN.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
F_y	-22.1	1011	1078	1.82	0.208	0.000	-0.354	0.707
M_z	-2.72	-2.28	-1.86	-2.73	0.110	-0.070	0.643	-4.04
F_x	-21.3	1144	49.6	226	0.069	-0.006	0.056	0.486

Table 1: Coefficients for tire formula with load influence.

	a_9	a_{10}	a_{11}	a_{12}	a_{13}
F_y	0.028	0.000	14.8	0.022	0.000
M_z	0.015	-0.066	0.945	0.030	0.070

Table 2: Coefficients for tire formula with camber influence.

Lateral force (F_y)

For lateral force, the stiffness, shape, peak, and curvature factors are calculated as follows.

$$C = 1.30$$
$$D = a_1 F_z^{\,2} + a_2 F_z$$
$$BCD = a_3 \sin\!\left(a_4 \tan^{-1}(a_5 F_z)\right)$$
$$B = \frac{BCD}{CD}$$
$$E = a_6 F_z^{\,2} + a_7 F_z + a_8$$

The factors are slightly affected by camber angle, γ, given in degrees.

$$S_h = a_9 \gamma$$
$$S_v = \left(a_{10} F_z^{\,2} + a_{11} F_z\right)\gamma$$
$$\Delta B = -a_{12}|\gamma|B$$

Finally, the lateral force is a function of the shape factors and the slip angle, α.

$$\phi = (1 - E)(\alpha + S_h) + \frac{E}{B}\tan^{-1}(B(\alpha + S_h))$$
$$F_y = D\sin\!\left(C\tan^{-1}(B\phi)\right) + S_v$$

Note that to conform to the ISO standard definition of slip angle, the sign of the lateral force must be reversed when applied to the ISO standard vehicle model.
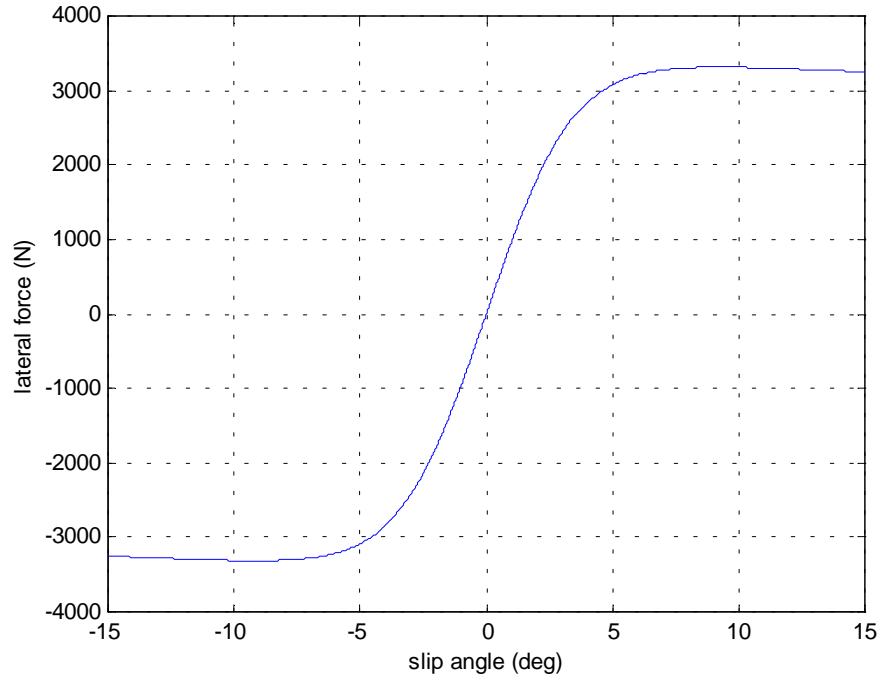
Figure 2: Lateral force versus slip angle.

Aligning moment (M_z)

For aligning moment, the stiffness, shape, peak, and curvature factors are calculated as follows.

$$C = 2.40$$

$$D = a_1 F_z^2 + a_2 F_z$$

$$BCD = \frac{a_3 F_z^2 + a_4 F_z}{e^{a_5 F_z}}$$

$$B = \frac{BCD}{CD}$$

$$E = a_6 F_z^2 + a_7 F_z + a_8$$

The factors are slightly affected by camber angle, γ, given in degrees.

$$S_h = a_9 \gamma$$

$$S_v = \left(a_{10} F_z^{\,2} + a_{11} F_z\right) \gamma$$

$$\Delta B = -a_{12}|\gamma|B$$

$$\Delta E = \frac{E}{1 - a_{13}|\gamma|} - E$$

Aligning moment is a function of the shape factors and the slip angle, α.

$$\phi = (1 - E)(\alpha + S_h) + \frac{E}{B}\tan^{-1}\left(B(\alpha + S_h)\right)$$

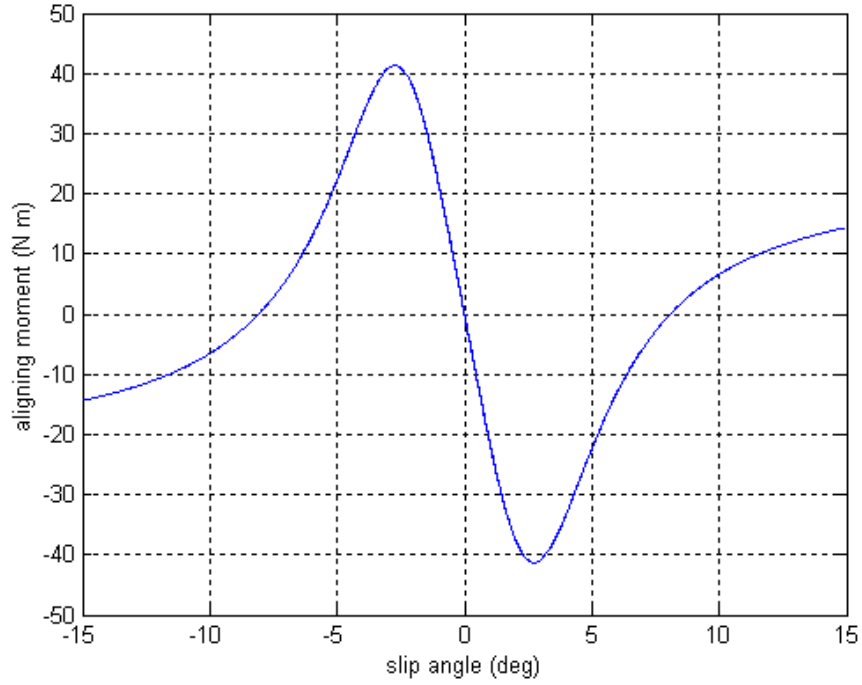$$M_z = D\sin\left(C\tan^{-1}(B\phi)\right) + S_v$$

Figure 3: Aligning moment versus slip angle.

Longitudinal force (F_x)

For longitudinal force, the stiffness, shape, peak, and curvature factors are calculated as follows.

$$C = 1.65$$

$$D = a_1 F_z^{\ 2} + a_2 F_z$$

$$BCD = \frac{a_3 F_z^{\ 2} + a_4 F_z}{e^{a_5 F_z}}$$

$$B = \frac{BCD}{CD}$$

$$E = a_6 F_z^{\ 2} + a_7 F_z + a_8$$

Longitudinal force is a function of the shape factors and the percent longitudinal slip, σ.

$$\phi = (1 - E)\sigma + \frac{E}{B}\tan^{-1}(B\sigma)$$
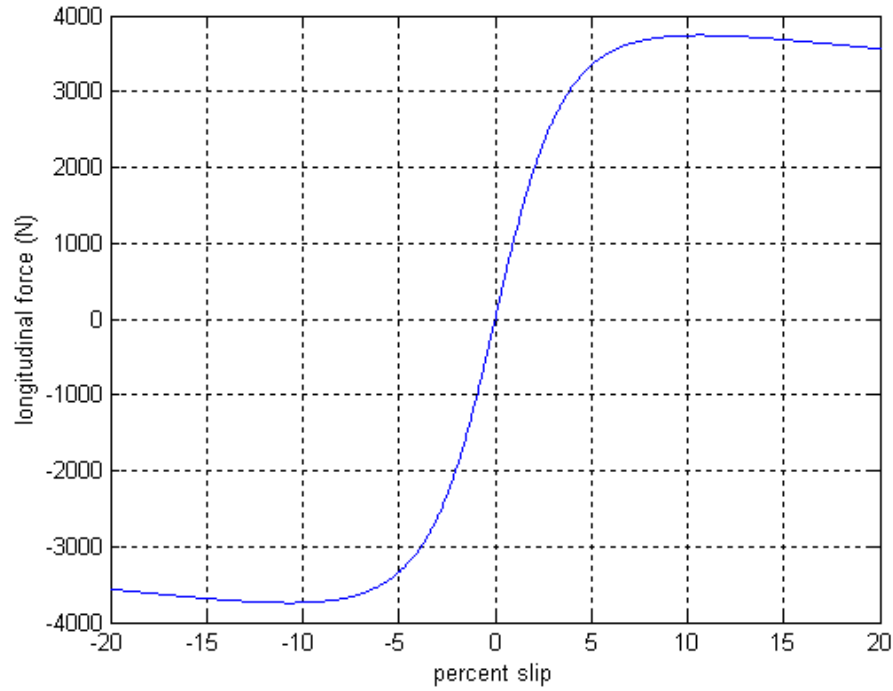
$$F_x = D\sin\left(C\tan^{-1}(B\phi)\right)$$

Figure 4: Longitudinal force versus percent slip.

Reference

[1]   H. Pacejka, E. Bakker, L. Nyborg. Tyre modelling for use in vehicle dynamics studies. *SAE* Paper No. 870421.