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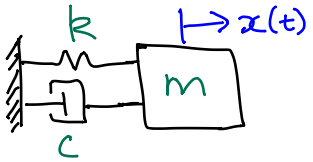
Mechanical Vibrations

Digital Copy:

1. Vibration of Single-Degree-of-Freedom Systems

- We consider a single-degree-of-freedom (SDOF) harmonic oscillator with the equation of motion (EOM):



(1)

Which can be found using Newton's method.

- Some systems that can be idealized as harmonic oscillators include:

- Given Eq. 1, we want to determine the displacement, $x(t)$, of the oscillator in response to the initial conditions (ICs):

(2)

- We start by dividing Eq. 1 by m, then place the EOM into classical form:

(3)

Where is the undamped natural frequency and ζ is the damping ratio:

(4)

- Note that the entire behavior of the oscillator is characterized entirely by only two constants: ω_n and ζ.

- We solve Eq. 3 by providing an ansatz:

Ansatz: (5)

- Plugging in the ansatz into Eq. 3, we get

(6)

Then either or (7)

We solve Eq. 7 using the quadratic formula:

(8)

(9)

- Clearly, the response (solution) depends on the value of the damping ratio, ζ:

  Case 1:
  Case 2:
  Case 3:
  Case 4:

- We focus on Case 2 here: $0 < \zeta < 1$. In this case,

(10)

Then, (11)

- Let , then (12)

(13)

- The solution to Eq. 3 is then

(14)

- Using Euler's formula and the ICs, we get

(15)