Keegan Moore
(He/him/his)

Mechanical Vibrations

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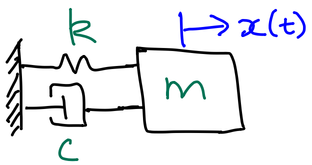
1. Vibration of Single-Degree-of-Freedom Systems

- We consider a single-degree-of-freedom (SDOF) harmonic oscillator with the equation of motion (EOM):



(1)

which can be found using Newton's method.

- Some systems that can be idealized as harmonic oscillators include:

- Given Eq. 1, we want to determine the displacement, $x(t)$, of the oscillator in response to the initial conditions (ICs):

(2)

- We start by dividing Eq. 1 by m, then place the EOM into classical form:

(3)

Where is the undamped natural frequency and ζ is the damping ratio:

(4)

- Note that the entire behavior of the oscillator is characterized entirely by only two constants: ω_n and ζ.

- We solve Eq. 3 by providing an ansatz:

Ansatz: (5)

- Plugging in the ansatz into Eq. 3, we get

(6)

Then either or (7)

We solve Eq. 7 using the quadratic formula:

(8)

(9)

- Clearly, the response (solution) depends on the value of the damping ratio, ζ:

  Case 1:

  Case 2:

  Case 3:

  Case 4:

- We focus on Case 2 here: $0 < \zeta < 1$. In this case,

(10)

Then, (11)

- Let , then (12)

(13)

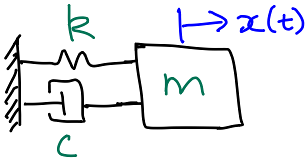
- The solution to Eq. 3 is then

(14)

- Using Euler's formula and the ICs, we get

(15)

Example 1



Given: A damped oscillator is released from equilibrium with a velocity of 0.05 m/s. The oscillator has a stiffness of 256 N/m, a mass of 1 kg, and a damping coefficient of 4 Ns/m.

Find: a) The displacement, $x(t)$, of the oscillator for the given initial conditions.
b) The maximum displacement that the oscillator achieves.

Solution

a) 1. Determine the initial conditions.

2. Substitute the ICs into Eq. 15 and simplify to find the analytical displacement.

3. Compute numerical values for ω_n, ζ, and ω_d.

4. Substitute in the numerical values for ω_n, ζ, and ω_d into the analytical expression for $x(t)$.

b) 1. The displacement is maximized whenever the velocity is equal to zero. Compute the velocity analytically. You will need the product rule.

2. The maximum displacement occurs when the velocity is zero for the first time. Let this time be t^*. Set $x(t^*) = 0$, then solve for t^* analytically, then numerically.

3. Using the numerical value for t^*, compute the maximum displacement