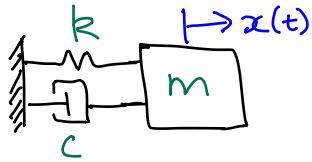


1. Vibration of Single-Degree-of-Freedom Systems

- We consider a single-degree-of-freedom (SDOF) harmonic oscillator with the **equation of motion (EOM)**:



(1)

which can be found using Newton's method.

- Some systems that can be idealized as harmonic oscillators include:

- Given Eq. 1, we want to determine the **displacement**, $x(t)$, of the oscillator in response to the **initial conditions (ICs)**:

(2)

- We start by dividing Eq. 1 by m , then place the EOM into **classical form**:

(3)

Where **is the undamped natural frequency** and **ξ is the damping ratio**:

(4)

- Note that the entire behavior of the oscillator is characterized entirely by only two constants: ω_n and ξ .

- We solve Eq. 3 by providing an **ansatz**:

Ansatz: (5)

- Plugging in the ansatz into Eq. 3, we get

(6)

Then either

or

(7)

We solve Eq. 7 using the quadratic formula:

(8)

(9)

- Clearly, the response (solution) depends on the value of the damping ratio, ξ :

Case 1:

Case 2:

Case 3:

Case 4:

- We focus on Case 2 here: $0 < \xi < 1$. In this case,

(10)

Then,

(11)

- Let

, then

(12)

(13)

- The solution to Eq. 3 is then

(14)

- Using Euler's formula and the ICs, we get



(15)

