Sheet 1

Tuesday, October 19, 2021 1:21 AM

Ex 1).

since $G=(V,E)$ is a simple graph and $deg(V) \geq \delta$ for every node $v \in V$,

there exists at least $\delta+1$ nodes

$|V| \geq \delta+1$

$\delta \geq 2$

Let G' be a subgraph of G,
$|V'|=|V|$ and $deg(V') = \delta = 2$

Then according to Lemma closed-walk,
there exits a path p of length at least $|V| \geq \delta+1$ in graph G'
and the path also exises in graph G.

Ex 2

a) a binary relation on a set V is an equivalent iff. it is reflexive symmetric and transitive

- reflexive. $\forall a \in V$, a connects a
- symmetric. Let $m, n \in V$ and m, n are connected

there exists path $p = m, V_1, \dots V_k, n$ in G

. . . . . . path $p = n, V_k, \dots V_1, m$ in G

- transitive.

Let $x, y, z \in V$ and x, y are connected

y, z are connected

there exist path from x to y

$p_1 = x, V_1, \dots, V_m, y$ in G

. . . . . . path from y to z

$p_2 = y, V_1', \dots, V_n', z$ in G

then there exists a path

$p_3 = x, V_1, \dots V_m, y, V_1', \dots V_n', z$

from x to z in G

and x, z are connected

According to above,

"being connected" defines an equivalence relation on V

b). Let $[u] = \{u \in V \mid u \sim v\}$ be the equavilence class

there induces all vertices $v \in V$ if there is a path from u to v in G,

then $[u]$ can form a connected graph

if we assume $[u]$ is not a connected component of G

then there exists a vertice $k \in V$, $k \notin [u]$

which leads to contradiction with $k \in [u]$