Optimization B

WS 2021/22

Prof. Dr. Christina Büsing
Tabea Krabs, Timo Gersing
combiopt@combi.rwth-aachen.de

Combinatorial Optimization
Teaching and Reasearch Area

Exercise Sheet 1

Submission until: 20.10.2021, 8:00 o'clock

Exercise 1 **3 Points**

Let $G = (V, E)$ be a simple graph and $\delta \in N$ with $\delta \geq 2$ and $\deg(v) \geq \delta$ for every node $v \in V$. Proof that there exists a circle C in G of length at least $\delta + 1$.

Exercise 2 **2 Points**

Let $G = (V, E)$ be a graph. Two vertices $v, w \in V$ *connected* iff there exists a path from v to w in G. Let $V' \subseteq V$. The *induced subgraph* of V' is defined as $G' := (V', \{vw \in E \mid v, w \in V'\})$.

a) Prove that "being connected" defines an equivalence relation on V.

b) Prove that the subgraphs which are induced by the equivalence classes are the connected components of G.

Exercise 3 **4 Points**

Let $G = (V, E)$ be a graph with $|V| \geq 2$. Then, the following statements are equivalent:

1) G is a tree.

2) G is connected and acyclic.

3) G is acyclic and $|E| = n - 1$.

4) G is connected and $|E| = n - 1$.

5) For every pair of nodes $u, v \in V$ with $u \neq v$ there exists exactly one u, v-path.

6) G is acyclic and if an arc $\{u, v\}$ with $u, v \in V$ and $\{u, v\} \notin E$ is added to G, there exists exactly one circle.

7) G is connected and for all $e \in E$, the subgraph $G - e = (V, E \backslash \{e\})$ is not connected.

The equivalencies 1) $\Leftrightarrow$ 2) $\Leftrightarrow$ 3) $\Leftrightarrow$ 4) were shown in the lecture (Definition + Theorem 15). Complete the proof by showing:

a) 4) $\Rightarrow$ 5)

b) 5) $\Rightarrow$ 6)

c) 6) $\Rightarrow$ 7)

d) 7) $\Rightarrow$ 4)

Exercise 4 **1 Points**

Prove the following theorem from the lecture: Trees with at least two vertices have at least two leaves.