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- Note that the breakout rooms are not recorded.

Satisfiability Checking

04 Propositional logic III

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1 Modeling with propositional logic

Before we go on...

- Suppose we can solve the satisfiability problem... how can this help us?

- There are numerous problems in the industry that are solved via the satisfiability problem of propositional logic
  - Logistics
  - Planning
  - Electronic Design Automation industry
  - Cryptography
  - ...

Example 1: Placement of wedding guests

- Three chairs in a row: $1, 2, 3$
- We need to place Aunt, Sister and Father.
- Constraints:
  - Aunt doesn't want to sit near Father
  - Aunt doesn't want to sit in the left chair
  - Sister doesn't want to sit to the right of Father
- Q: Can we satisfy these constraints?

Example 1 (continued)

- Notation: Aunt = 1, Sister = 2, Father = 3
  Left chair = 1, Middle chair = 2, Right chair = 3

  Introduce a propositional variable for each pair (person, chair):

  $x_{p,c}$ = "person p is sited in chair c" for $1 \leq p, c \leq 3$
- Constraints:

  Aunt doesn't want to sit near Father:

$$((x_{1,1} \vee x_{1,3}) \rightarrow \neg x_{3,2}) \wedge (x_{1,2} \rightarrow (\neg x_{3,1} \wedge \neg x_{3,3}))$$

  Aunt doesn't want to sit in the left chair:

$$\neg x_{1,1}$$

  Sister doesn't want to sit to the right of Father:

$$(x_{3,1} \rightarrow \neg x_{2,2}) \wedge (x_{3,2} \rightarrow \neg x_{2,3})$$

Example 1 (continued)

Each person is placed:

$$(x_{1,1} \vee x_{1,2} \vee x_{1,3}) \wedge (x_{2,1} \vee x_{2,2} \vee x_{2,3}) \wedge (x_{3,1} \vee x_{3,2} \vee x_{3,3})$$

$$\bigwedge_{p=1}^{3} \bigvee_{c=1}^{3} x_{p,c}$$

At most one person per chair:

$$\bigwedge_{p1=1}^{3} \bigwedge_{p2=p1+1}^{3} \bigwedge_{c=1}^{3} (\neg x_{p1,c} \vee \neg x_{p2,c})$$

Example 2: Assignment of frequencies

- n radio stations
- For each station assign one of k transmission frequencies, $k < n$.
- E – set of pairs of stations, that are too close to have the same frequency.

- Q: Can we assign to each station a frequency, such that no station pairs from E have the same frequency?

Example 2 (continued)

- Notation:
  $x_{s,f}$ = "station s is assigned frequency f" for $1 \leq s \leq n$, $1 \leq f \leq k$
- Constraints:
  Every station is assigned at least one frequency:

$$\bigwedge_{s=1}^{n} \left(\bigvee_{f=1}^{k} x_{s,f} \right)$$

  Every station is assigned at most one frequency:

$$\bigwedge_{s=1}^{n} \bigwedge_{f1=1}^{k-1} \bigwedge_{f2=f1+1}^{k} (\neg x_{s,f1} \vee \neg x_{s,f2})$$

  Close stations are not assigned the same frequency:
  For each $(s1, s2) \in E$,

$$\bigwedge_{f=1}^{k} (\neg x_{s1,f} \vee \neg x_{s2,f})$$

Example 3: Seminar topic assignment

- n participants
- n topics
- Set of preferences $E \subseteq \{1, \ldots, n\} \times \{1, \ldots, n\}$
  $(p, t) \in E$ means: participant p would take topic t

- Q: Can we assign to each participant a topic which he/she is willing to take?

Example 3 (continued)

- Notation: $x_{p,t}$ = "participant p is assigned topic t"
- Constraints:

  Each participant is assigned at least one topic:

  $$\bigwedge_{p=1}^{n}\left(\bigvee_{t=1}^{n} x_{p,t}\right)$$

  Each participant is assigned at most one topic:

  $$\bigwedge_{p=1}^{n}\bigwedge_{t1=1}^{n-1}\bigwedge_{t2=t1+1}^{n}(\neg x_{p,t1} \vee \neg x_{p,t2})$$

  Each participant is willing to take his/her assigned topic:

  $$\bigwedge_{p=1}^{n}\bigwedge_{(p,t)\notin E}\neg x_{p,t}$$

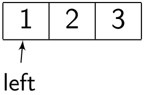
Example 3 (continued)

Each topic is assigned to at most one participant:

$$\bigwedge_{t=1}^{n} \bigwedge_{p1=1}^{n} \bigwedge_{p2=p1+1}^{n} (\neg x_{p1,t} \vee \neg x_{p2,t})$$

Annotation 5

Assume three persons A, B, C and three sequentially ordered seats (1-3 from left to right).



In the following formula, let $x_{i,j}$ denote that person i is seated in seat j:

$$
\begin{array}{ll}
 & \bigwedge_{i=1}^{3} \left(x_{A,i} \vee x_{B,i} \vee x_{C,i}\right) \\
\wedge & \left(\bigvee_{i=1}^{3} x_{A,i}\right) \wedge \left(\bigvee_{i=1}^{3} x_{B,i}\right) \wedge \left(\bigvee_{i=1}^{3} x_{C,i}\right) \\
\wedge & \bigwedge_{i=1}^{3} \left(\neg(x_{A,i} \wedge x_{B,i}) \wedge \neg(x_{A,i} \wedge x_{C,i}) \wedge \neg(x_{B,i} \wedge x_{C,i})\right) \\
\wedge & x_{B,1}
\end{array}
$$

Which of the following statements hold for **all** solutions of the above formula?

1. B sits in either seat 1 or seat 2.
2. A and C sit next to each other.
3. C does not sit on the right of A.
4. B has two neighbors.

a) 1 b) 1,2 c) 1,2,3 d) 2,3,4

Learning target

- How to encode real world problems in propositional logic?