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Question from last lecture

May NNF conversion lead to an exponential increase in formula size for formulas with nested if and only if (\Leftrightarrow) ?

Satisfiability Checking 06 SAT solving examples

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WS 21/22

06 SAT solving examples

SAT solving: Example1

- We use enumeration and propagation in DPLL-style for the search, and CDCL-style conflict resolution for backtracking.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be $x_1 < x_2 < x_3 < x_4$.

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1: c_1, c_3 \neg x_1: x_2: c_1, c_2 \neg x_2: c_3 x_3: c_4 \neg x_3: x_4:
```

 $\neg x_4$: c_2, c_4

```
Activities Trail: (increment=1):  x_1 \quad 0 \\ x_2 \quad 0 \\ x_3 \quad 0 \\ x_4 \quad 0
```

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1 : c_1, c_3
\neg x_1 : x_2 : c_1, c_2
\neg x_2 : c_3
x_3 : c_4
\neg x_3 : x_4 : x_4 : x_4 : c_2, c_4
```

Decide $\neg x_1$

```
Activities Trail: (increment=1): DL1: \neg x_1 : \text{NULL}
\begin{array}{ccc} x_1 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_4 & 0 \end{array}
```

$$c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists: x_1 : c_1, c_3
\neg x_1 : x_2 : c_1, c_2
\neg x_2 : c_3
x_3 : c_4
\neg x_3 : x_4 : x_4 : x_4 : x_5
```

Propagate $\neg x_1$ in

```
Activities Trail: (increment=1): DL1: \neg x_1 : \text{NULL}
\begin{array}{ccc} x_1 & 0 \\ x_2 & 0 \\ x_3 & 0 \\ x_4 & 0 \end{array}
```

```
c_1: \left(\underline{x_1} \vee \underline{x_2} \vee x_4\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)
```

```
Watch lists:
                                      Activities
                                                            Trail:
                                      (increment=1):
                                                                 DL1: \neg x_1: NULL
        x_1: c_1, c_3
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                             x_2 0
       \neg x_2: c_3
                                             x_3 0
       X_3: C_4
                                                   0
                                             X_4
       \neg x_3:
        X<sub>4</sub> :
       \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4)
```

```
c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)
```

```
Watch lists:
                                          Activities
                                                                  Trail:
                                          (increment=1):
                                                                        DL1: \neg x_1: NULL
         x_1: \mathcal{C}_1, \mathcal{C}_3
        \neg x_1:
                                                  x_1 0
         x_2: c_1, c_2
                                                  x_2 0
        \neg x_2: c_3
                                                  x_3 0
        X_3: C_4
                                                       0
                                                  X_{\Delta}
        \neg x_3:
        x_4: c_1
        \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_1: (x_1 \lor x_2 \lor x_4) \rightarrow (x_1 \lor x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(\underline{x_1} \vee \underline{\neg x_2} \vee x_4\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                       Activities
                                                              Trail:
                                        (increment=1):
                                                                    DL1: \neg x_1: NULL
         X_1: \mathcal{C}_1, \mathcal{C}_3
       \neg x_1:
                                               x_1 0
         x_2: c_1, c_2
                                               x_2 0
       \neg x_2: c_3
                                               x_3 0
       X_3: C_4
                                                    0
                                               X_4
       \neg x_3:
        X_4: C_1
       \neg x_4: c_2, c_4
 Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4)
```

$$c_1: \left(x_1 \vee \underline{x_2} \vee \underline{x_4}\right) \wedge c_2: \left(\underline{x_2} \vee \underline{\neg x_4}\right) \wedge c_3: \left(x_1 \vee \underline{\neg x_2} \vee \underline{x_4}\right) \wedge c_4: \left(\underline{x_3} \vee \underline{\neg x_4}\right)$$

```
Watch lists:
                                                                   Trail:
                                           Activities
                                           (increment=1):
                                                                         DL1: \neg x_1: NULL
          x_1: \mathcal{G}_1, \mathcal{G}_3
        \neg x_1:
                                                   x_1 0
         x_2: c_1, c_2
                                                   x_2 0
        \neg x_2: c_3
                                                   x_3 0
        X_3: C_4
                                                        0
                                                   X_{\Delta}
        \neg x_3:
         X_4: C_1, C_3
        \neg x_4: c_2, c_4
  Propagate \neg x_1 in c_3: (x_1 \lor \neg x_2 \lor x_4) \rightarrow (x_1 \lor \neg x_2 \lor x_4)
```

Decide $\neg x_2$

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                    Activities
                                    (increment=1):
        X_1:
      \neg x_1:
                                           x_1 = 0
       x_2: c_1, c_2
                                           x_2 = 0
      \neg x_2: c_3
                                           x_3 0
       X3: C4
                                           X4
                                                0
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
```

```
Trail:

DL1: \neg x_1: NULL

DL2: \neg x_2: NULL
```

Propagate $\neg x_2$ in

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                    Activities
                                     (increment=1):
        X_1:
       \neg x_1:
                                           x_1 = 0
        x_2: c_1, c_2
                                           x_2 = 0
      \neg X_2: C_3
                                           X3
                                                0
       X3 : C4
                                           X4
                                                0
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
```

```
Trail:
1): DL1: \neg x_1 : NULL DL2: \neg x_2 : NULL
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})
```

```
Watch lists:
                                                         Trail:
                                    Activities
                                    (increment=1):
                                                              DL1: \neg x_1: NULL
        X_1:
                                                              DL2: \neg x_2: NULL
       \neg x_1:
                                           x_1 0
        x_2: c_1, c_2
                                                0
                                           X2
      \neg x_2: c_3
                                           x_3 0
       X3 : C4
                                                0
                                           X_4
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_1:(x_1 \lor x_2 \lor x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})
```

```
Watch lists:
                                                          Trail:
                                     Activities
                                     (increment=1):
                                                               DL1: \neg x_1: NULL
        X_1:
                                                               DL2: \neg x_2: NULL
       \neg x_1:
                                            x_1 0
        x_2: c_1, c_2
                                                                           X4 : C1
                                                 0
                                            X2
       \neg x_2: c_3
                                            x_3 0
       X_3: C_4
                                                 0
                                            X_4
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_1: (x_1 \lor x_2 \lor x_4) \rightarrow Assign x_4
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                        Trail:
                                   Activities
                                    (increment=1):
                                                             DL1: \neg x_1: NULL
        X1:
                                                             DL2: \neg x_2: NULL
      \neg x_1:
                                          x_1 0
        x_2: c_1, c_2
                                                                      X4 : C1
                                               0
                                          X2
      \neg x_2: c_3
                                               0
                                          X3
       X3 : C4
                                               0
                                          X_4
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_2:(x_2 \vee \neg x_4)
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                                            Trail:
                                      Activities
                                      (increment=1):
                                                                 DL1: \neg x_1: NULL
         X_1:
                                                                 DL2: \neg x_2: NULL
       \neg x_1:
                                             x_1 0
        x_2: c_1, c_2
                                                                            X4 : C1
                                                  0
                                             X2
       \neg x_2: c_3
                                             x_3 0
       X3 : C4
                                                   0
                                             XΔ
       \neg x_3:
        x_4: c_1, c_3
       \neg x_4: c_2, c_4
 Propagate \neg x_2 in c_2: (x_2 \vee \neg x_4) \rightarrow \text{$\not$$conflict!}
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

Watch lists:

 X_1 : $\neg x_1$:

 x_2 : c_1, c_2

 $\neg x_2$: c_3

x₃: c₄

 $\neg x_3$:

 $x_4: c_1, c_3$

 $\neg x_4$: c_2, c_4

Conflict resolution:

Activities (increment=1):

> x_1 0 0 X2

 x_3 0

0 *X*4

Trail:

DL1: $\neg x_1$: NULL DL2: $\neg x_2$: NULL

 $X_4 : C_1$

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4})$$

```
Watch lists:
                                          Activities
                                                                  Trail:
                                           (increment=1):
                                                                         DL1: \neg x_1: NULL
          X1:
                                                                         DL2: \neg x_2: NULL
        \neg x_1:
                                                  x_1 = 0
         x_2: c_1, c_2
                                                                                    X_4 : C_1
                                                  x_2 = 0
        \neg x_2: c_3
                                                  x_3 0
         X3: C4
                                                  X4
                                                        0
        \neg x_3:
         X_4: C_1, C_3
        \neg x_4: c_2, c_4
 Conflict resolution: \frac{(x_2 \vee \neg x_4) \quad (x_1 \vee x_2 \vee x_4)}{(x_1 \vee x_2)}
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$X_1$$
: C_5

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
: c_3

 $\neg x_3$:

$$x_4: c_1, c_3$$

$$\neg x_4$$
: c_2, c_4

Add conflict clause

Activities (increment=2):

$$x_1$$
 1 x_2 1

$$x_3$$
 0

 x_4 1

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Trail:

DL1: $\neg x_1$: NULL DL2: $\neg x_2$: NULL

 $X_4 : C_1$

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

$$x_1$$
: c_5

$$x_2: c_1, c_2, c_5$$

$$\neg x_2$$
: c_3

$$x_3$$
: c_4

$$\neg x_3$$
:

$$x_4: c_1, c_3$$

$$\neg x_4$$
: c_2, c_4

Backtrack to DL1

Activities Trail: (increment=2):

 x_1 1 x_2 1

*x*₃ 0

 x_4 1

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DL1: $\neg x_1$: NULL

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

Watch lists:

```
x_1 : c_5

\neg x_1 :

x_2 : c_1, c_2, c_5

\neg x_2 : c_3

x_3 : c_4

\neg x_3 :
```

 $x_4: c_1, c_3$ $\neg x_4: c_2, c_4$ Assign x_2 at DL1 by c_5

```
Activities Trail: (increment=2):
```

 x_1 1 x_2 1 x_3 0

 x_4 1

.

X2 : C5

DL1: $\neg x_1$: NULL

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

```
x_1 : c_5

\neg x_1 :

x_2 : c_1, c_2, c_5

\neg x_2 : c_3

x_3 : c_4

\neg x_3 :

x_4 : c_1, c_3
```

 $\neg x_4$: c_2, c_4 Propagate x_2 in

```
Activities Trail: (increment=2):
```

 $\begin{array}{ccc}
 x_1 & 1 \\
 x_2 & 1 \\
 x_3 & 0 \\
 x_4 & 1
 \end{array}$

DL1: $\neg x_1$: NULL

 $x_2 : c_5$

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)
                c_5:(x_1 \vee x_2)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                            Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Trail:
                                                                                                                                                                                                                                                                                                                                                                                              (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL1: \neg x_1: NULL
                                                                                          X1: C5
                                                                          \neg x_1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X2 : C5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                x_1 1
                                                                                      x_2: c_1, c_2, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                              x_2 1
                                                                        \neg X_2: C_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                              x_3 0
                                                                            X_3: C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                x_4 1
                                                                          \neg x_3:
                                                                                x_4: c_1, c_3
                                                                        \neg x_4: c_2, c_4
```

Propagate x_2 in $c_3: (x_1 \vee \neg x_2 \vee x_4)$

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4) \land c_4: (x_3 \lor \neg x_4) \land (x_4 \lor x_2 \lor x_4) \land (x_4 \lor x_4) \lor (x_4 \lor x_4) \land (x_4 \lor x_4) \lor (x_4 \lor x_4)
               c_5:(x_1 \vee x_2)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Trail:
                                                                                                                                                                                                                                                                                                                                                                                   Activities
                                                                                                                                                                                                                                                                                                                                                                                     (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          DL1: \neg x_1: NULL
                                                                                        X1: C5
                                                                        \neg x_1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            X2 : C5
                                                                                                                                                                                                                                                                                                                                                                                                                                                      x_1 1
                                                                                  x_2: c_1, c_2, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              X_4 : C_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                    x_2 1
                                                                        \neg x_2: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                    x_3 0
                                                                          X_3: C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                      x_4 1
                                                                        \neg x_3:
                                                                              x_4: c_1, c_3
                                                                      \neg x_4: c_2, c_4
```

Propagate x_2 in $c_3: (x_1 \vee \neg x_2 \vee x_4) \rightarrow \text{Assign } x_4$

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

$$x_1 : c_5$$

 $\neg x_1 : x_2 : c_1, c_2, c_5$
 $\neg x_2 : c_3$
 $x_3 : c_4$

 x_4 : c_1, c_3 $\neg x_4$: c_2, c_4 Propagate x_4 in

 $\neg x_3$:

```
Activities Trail: (increment=2):  x_1 \quad 1 \\ x_2 \quad 1
```

 x_3 0

 x_4 1

DL1: $\neg x_1 : \text{NULL}$ $x_2 : c_5$ $x_4 : c_3$

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                      X2 : C5
                                          x_1 1
        x_2: c_1, c_2, c_5
                                                                      X4: C3
                                         x_2 1
      \neg x_2: c_3
                                          x_3 0
       X_3: C_4
                                          x_4 1
      \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2:(x_2 \vee \neg x_4)
```

```
c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{\neg x_4}) \wedge c_3: (x_1 \vee \underline{\neg x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{\neg x_4}) \wedge c_5: (x_1 \vee x_2)
```

```
Watch lists:
                                    Activities
                                                         Trail:
                                    (increment=2):
                                                              DL1: \neg x_1: NULL
        X1: C5
       \neg x_1:
                                                                        X2 : C5
                                           x_1 1
        x_2: c_1, c_2, c_5
                                                                        X4: C3
                                           x_2 1
      \neg x_2: c_3
                                           x_3 0
       X_3: C_4
                                           x_4 1
       \neg x_3:
       x_4: c_1, c_3
      \neg x_4: c_2, c_4
 Propagate x_4 in c_2: (x_2 \vee \neg x_4) \rightarrow O.K.
```

 x_4 : c_1, c_3 $\neg x_4$: c_2, c_4

Propagate x_4 in $c_4:(x_3\vee \neg x_4)$

```
c_5:(x_1 \vee x_2)
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg x_1: NULL
        X1: C5
      \neg x_1:
                                                                     X2 : C5
                                         x_1 1
       x_2: c_1, c_2, c_5
                                                                     X4: C3
                                         x_2 1
      \neg x_2: c_3
                                         x_3 0
       X_3: C_4
                                         x_4 1
      \neg x_3:
```

 $c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)$

```
c_1: (x_1 \lor x_2 \lor x_4) \land c_2: (x_2 \lor \neg x_4) \land c_3: (x_1 \lor \neg x_2 \lor x_4) \land c_4: (x_3 \lor \neg x_4)
             c_5:(x_1 \vee x_2)
Watch lists:
                                                                                                                                                                                                                                                                                                                            Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Trail:
                                                                                                                                                                                                                                                                                                                              (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DL1: \neg x_1: NULL
                                                                           X1: C5
                                                             \neg x_1:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X2 : C5
                                                                                                                                                                                                                                                                                                                                                                                     x_1 1
                                                                       x_2: c_1, c_2, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X_4:C_3
                                                                                                                                                                                                                                                                                                                                                                                   x_2 1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 X3 : C4
                                                             \neg x_2: c_3
                                                                                                                                                                                                                                                                                                                                                                                   x_3 0
                                                                 X_3: C_4
                                                                                                                                                                                                                                                                                                                                                                                     x_4 1
                                                             \neg x_3:
                                                                    x_4: c_1, c_3
                                                            \neg x_4: c_2, c_4
             Propagate x_4 in c_4: (x_3 \vee \neg x_4) \rightarrow \text{Assign } x_3
```

$$c_1: (x_1 \vee \underline{x_2} \vee \underline{x_4}) \wedge c_2: (\underline{x_2} \vee \underline{-x_4}) \wedge c_3: (x_1 \vee \underline{-x_2} \vee \underline{x_4}) \wedge c_4: (\underline{x_3} \vee \underline{-x_4}) \wedge c_5: (x_1 \vee x_2)$$

```
Watch lists:
```

```
x_1: c_5

\neg x_1:

x_2: c_1, c_2, c_5

\neg x_2: c_3

x_3: c_4

\neg x_3:
```

 $x_4: c_1, c_3$ $\neg x_4: c_2,$

```
\rightarrow SAT
```

Bonus exercise 9

Assume the following propositional logic formula in CNF:

$$(a \lor b) \land (\neg b \lor c \lor d) \land (\neg b \lor \neg c) \land (c \lor \neg d)$$

Initially, all variables are unassigned. Propagate the first decision a := 0 and resolve the resulting conflict. What is the resulting asserting clause (according to the first UIP)? Note: here, you may identify unit clauses manually, without implementing watched literals.

- **(**)
- (a)
- (¬a)
- **■** (b)
- **■** (¬b)
- \blacksquare $(\neg b \lor c)$

Bonus exercise 9

Assume the following propositional logic formula in CNF:

$$(a \lor b) \land (\neg b \lor c \lor d) \land (\neg b \lor \neg c) \land (c \lor \neg d)$$

Initially, all variables are unassigned. Propagate the first decision a := 0 and resolve the resulting conflict. What is the resulting asserting clause (according to the first UIP)? Note: here, you may identify unit clauses manually, without implementing watched literals.

- **(**)
- (a)
- **■** (¬a)
- **(***b***)**
- **■** (¬*b*)
- \blacksquare $(\neg b \lor c)$

06 SAT solving examples

1 Example 1

SAT solving: Example 2

- We use enumeration and propagation in DPLL-style for the search, and CDCL-style conflict resolution for backtracking.
- We use watched literals (underlined in formulas) to speed up propagation.
- We use VSIDS as variable ordering heuristics and assign the value false to decision variables.
- In VSIDS, to order variables with the same activity value, we use the lexicographic order; in our example this will be a < b < c < d.

Example

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

$$c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_5: (a \lor \neg c \lor d) \land c_6: (\neg c \lor \neg d)$$

Watch lists:

- a :
- $\neg a$:
 - b :
- $\neg b$:
 - **c** :
- $\neg c$:
 - d :
- ¬d:

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (b \vee c) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

Watch lists:

- a: c₁
- $\neg a$:
 - $b: c_1$
- $\neg b$:
 - **c** :
- $\neg c$:
- d :
- $\neg d$:

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee b \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

Watch lists:

 $a: c_1$ $\neg a:$ $b: c_1, c_2$ $\neg b:$ $c: c_2$ $\neg c:$ d: $\neg d:$

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \neg b \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

Watch lists:

```
a: c_1
\neg a: c_3
b: c_1, c_2, c_3
\neg b:
c: c_2
\neg c:
d:
\neg d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)$$

Watch lists:

 $a: c_1, c_4$ $\neg a: c_3$ $b: c_1, c_2, c_3$ $\neg b: c_4$ $c: c_2$ $\neg c:$ d:

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\neg c \vee \neg d)$$

Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5
d:
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

 $a: c_1, c_4, c_5$ $\neg a: c_3$ $b: c_1, c_2, c_3$ $\neg b: c_4$ $c: c_2$ $\neg c: c_5, c_6$ d: $\neg d: c_6$

Example – Activities

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

$$a: c_1, c_4, c_5$$

 $\neg a: c_3$
 $b: c_1, c_2, c_3$
 $\neg b: c_4$
 $c: c_2$

d : ¬*d* : *c*₆

 $\neg c$: c_5, c_6

Activities (increment=1):

a 0b 0c 0d 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

 $a: c_1, c_4, c_5$ $\neg a: c_3$ $b: c_1, c_2, c_3$ $\neg b: c_4$ $c: c_2$

 $\neg c: c_5, c_6$ d:

 $\neg d$: c_6

Activities Trail: (increment=1):

a 0b 0c 0

d = 0

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

Decide $\neg a$

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

```
Activities Trail: (increment=1): DL1: \neg a: NULL a 0 b 0 c 0 d 0
```

$$c_1: (\underline{a} \vee \underline{b} \vee \neg c) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (\underline{a} \vee \underline{\neg b} \vee c) \wedge c_5: (\underline{a} \vee \underline{\neg c} \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})$$

Watch lists:

```
a: c_1, c_4, c_5
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2
\neg c: c_5, c_6
d:
\neg d: c_6
```

Propagate $\neg a$ in

```
Activities Trail: (increment=1):
```

a 0b 0c 0d 0

DL1: ¬a: NULL

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                              Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Trail:
                                                                                                                                                                                                                                                                                                                                                                              (increment=1):
                                                                                      a: c_1, c_4, c_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 DL1: \neg a: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                                                   a
                                                                                    b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                  b 0
                                                                     \neg b: c_4
                                                                               c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                   d = 0
                                                                     \neg c: c_5, c_6
                                                                         d :
                                                                     \neg d: c_6
                 Propagate \neg a in c_1: (a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                   Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          Trail:
                                                                                                                                                                                                                                                                                                                                                                   (increment=1):
                                                                                    a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                   \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                                       a
                                                                                  b: c_1, c_2, c_3
                                                                   \neg b: c_4
                                                                            c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                       d = 0
                                                                   \neg c: c_5, c_6, c_1
                                                                       d :
                                                                   \neg d: c_6
                Propagate \neg a in c_1: (a \lor b \lor \neg c) \rightarrow (a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                  Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                Trail:
                                                                                                                                                                                                                                                                                                                                                                                  (increment=1):
                                                                                       a: \mathcal{C}_1, \mathcal{C}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DL1: \neg a: NULL
                                                                        \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 0
                                                                                                                                                                                                                                                                                                                                                                                                                                                        a
                                                                                     b: c_1, c_2, c_3
                                                                      \neg b: c_4
                                                                               c: c_2
                                                                                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                                      \neg c: c_5, c_6, c_1
                                                                          d :
                                                                      \neg d: c_6
                 Propagate \neg a in c_4: (a \lor \neg b \lor c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                 c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                            Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    Trail:
                                                                                                                                                                                                                                                                                                                                                                                                                            (increment=1):
                                                                                                 a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DL1: \neg a: NULL
                                                                              \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      0
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          a
                                                                                               b: c_1, c_2, c_3
                                                                              \neg b: c_4
                                                                                      C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          d = 0
                                                                              \neg c: c_5, c_6, c_1
                                                                                    d :
                                                                              \neg d: c_6
```

Propagate $\neg a$ in $c_4: (a \lor \neg b \lor c) \to (a \lor \neg b \lor c)$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
               c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                             Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                                             (increment=1):
                                                                                      a: \mathcal{G}_1, \mathcal{G}_4, \mathcal{C}_5
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DL1: \neg a: NULL
                                                                       \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           0
                                                                                                                                                                                                                                                                                                                                                                                                                                                  a
                                                                                      b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                b 0
                                                                     \neg b: c_4
                                                                              C: C_2, C_4
                                                                     \neg c: c_5, c_6, c_1
                                                                           d :
                                                                     \neg d: c_6
                 Propagate \neg a in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                           Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              Trail:
                                                                                                                                                                                                                                                                                                                                                           (increment=1):
                                                                                  a: 91,94,95
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 DL1: \neg a: NULL
                                                                   \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                  0
                                                                                                                                                                                                                                                                                                                                                                                                                             a
                                                                                b: c_1, c_2, c_3
                                                                 \neg b: c_4
                                                                           C: C_2, C_4
                                                                 \neg c: c_5, c_6, c_1
                                                                     d: c<sub>5</sub>
                                                                 \neg d: c_6
                Propagate \neg a in c_5: (a \lor \neg c \lor d) \rightarrow (a \lor \neg c \lor d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6

Decide $\neg b$

```
Activities (increment=1):

a 0
b 0
c 0
d 0
```

```
Trail:

DL1: \neg a: NULL

DL2: \neg b: NULL
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:

a:
\neg a: c_3
b: c_1, c_2, c_3
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Propagate $\neg b$ in

```
Activities (increment=1):

a 0
b 0
c 0
d 0
```

```
Trail:

DL1: \neg a: NULL

DL2: \neg b: NULL
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                       Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         Trail:
                                                                                                                                                                                                                                                                                                                                                       (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL1: \neg a: NULL
                                                                                 a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL2: \neg b: NULL
                                                                 \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                                        a
                                                                             b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                      b 0
                                                                 \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                        c = 0
                                                                          C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                                 \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                                 \neg d: c_6
                Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                           Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     Trail:
                                                                                                                                                                                                                                                                                                                                            (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL1: \neg a: NULL
                                                                              a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    DL2: \neg b: NULL
                                                              \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                              0
                                                                                                                                                                                                                                                                                                                                                                                                         a
                                                                          b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                        b 0
                                                              \neg b: c_4
                                                                       C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                         d = 0
                                                              \neg c: c_5, c_6, c_1
                                                                  d: c_5
                                                              \neg d: c_6
               Propagate \neg b in c_1: (a \lor b \lor \neg c) \rightarrow assign \neg c
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                     Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      Trail:
                                                                                                                                                                                                                                                                                                                                                       (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL1: \neg a: NULL
                                                                                a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      DL2: \neg b: NULL
                                                                  \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                            0
                                                                                                                                                                                                                                                                                                                                                                                                                      a
                                                                             b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                                    b 0
                                                                \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                      c = 0
                                                                         c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                      d = 0
                                                                \neg c: c_5, c_6, c_1
                                                                    d: c_5
                                                                \neg d: c_6
              Propagate \neg b in c_2:(b \lor c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
              c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                               Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                                               (increment=1):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DL1: \neg a: NULL
                                                                               a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DL2: \neg b: NULL
                                                               \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                   0
                                                                                                                                                                                                                                                                                                                                                                                                              a
                                                                           b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                     \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                             b 0
                                                               \neg b: c_4
                                                                        C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                              d = 0
                                                               \neg c: c_5, c_6, c_1
                                                                 d: c_5
                                                               \neg d: c_6
              Propagate \neg b in c_2: (\underline{b} \lor \underline{c}) \to \{ \text{conflict!} \}
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d})
```

```
Watch lists:
a:
\neg a: c_3
```

 $\neg a$: c_3 b: c_1, c_2, c_3 $\neg b$: c_4

 $c: c_2, c_4$ $\neg c: c_5, c_6, c_1$

 $d: c_5$

 $\neg d$: c_6

Conflict resolution:

c = 0

d = 0

DL1: $\neg a$: NULL DL2: $\neg b$: NULL

 $\neg c: c_1$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
                c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d)
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        Trail:
                                                                                                                                                                                                                                                                                                                                                                                                  (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DL1: \neg a: NULL
                                                                                           a :
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               DL2: \neg b: NULL
                                                                          \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       a Ø 1
                                                                                      b: c_1, c_2, c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    \neg c : c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                       b Ø 1
                                                                        \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         c Ø 1
                                                                                      C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                         d 0
                                                                        \neg c: c_5, c_6, c_1
                                                                            d: c_5
```

Conflict resolution: $c_2:(b\lor c) c_1:(a\lor b\lor \neg c)$

 $\neg d$: c_6

c7:(a\b)

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Add conflict clause

Activities Trail: (increment=2):

a 1
b 1
c 1
d 0

DL1: $\neg a$: NULL DL2: $\neg b$: NULL $\neg c$: c_1

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Backtrack to DI 1

```
Activities Trail:
(increment=2):

a 1
b 1
c 1
d 0
```

```
DL1: ¬a:NULL
DL2: ¬b:NULL
¬c:-Gī
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
\neg d: c_6
```

Assign b at DL1 by c_7

```
Activities Trail: (increment=2):
```

b 1c 1d 0

a 1

```
DL1: ¬a: NULL
b: c<sub>7</sub>
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Propagate b in

```
Activities Trail: (increment=2):
```

a 1b 1c 1d 0

DL1: ¬a: NULL b: c₇

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
            c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                                              Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                                                (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              DL1: \neg a: NULL
                                                                               a: c7
                                                               \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        b : c7
                                                                                                                                                                                                                                                                                                                                                                                                              a 1
                                                                               b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                              b 1
                                                               \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                c 1
                                                                      c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                d = 0
                                                               \neg c: c_5, c_6, c_1
                                                                 d: c<sub>5</sub>
                                                               \neg d: c_6
               Propagate b in c_4:(a \vee \neg b \vee c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                              Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   Trail:
                                                                                                                                                                                                                                                                                                                                (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DL1: \neg a: NULL
                                                                           a: c7
                                                            \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         b : C7
                                                                                                                                                                                                                                                                                                                                                                                           a 1
                                                                           b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         C: C_4
                                                                                                                                                                                                                                                                                                                                                                                           b 1
                                                            \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                             c 1
                                                                   C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                             d = 0
                                                            \neg c: c_5, c_6, c_1
                                                              d: c_5
                                                            \neg d: c_6
               Propagate b in c_4: (a \lor \neg b \lor c) \to assign c
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Propagate c in

```
Activities Trail: (increment=2):

a 1
b 1
c 1
```

d = 0

```
DL1: ¬a: NULL
b: c<sub>7</sub>
c: c<sub>4</sub>
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Activities
Watch lists:
                                                         Trail:
                                     (increment=2):
                                                               DL1: \neg a: NULL
        a: c7
       \neg a: c_3
                                                                         b : c<sub>7</sub>
                                            a 1
        b: c_1, c_2, c_3, c_7
                                                                         C: C_4
                                            b 1
       \neg b: c_4
                                            c 1
       c: c_2, c_4
                                            d = 0
       \neg c: c_5, c_6, c_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate c in c_5: (a \lor \neg c \lor d)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b)
                                                                                                                                                                                                                                                                                                                          Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            Trail:
                                                                                                                                                                                                                                                                                                                            (increment=2):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           DL1: \neg a: NULL
                                                                          a: c7
                                                           \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  b : c7
                                                                                                                                                                                                                                                                                                                                                                                      a 1
                                                                         b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                C:C_4
                                                                                                                                                                                                                                                                                                                                                                                      b 1
                                                           \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  d: c_5
                                                                                                                                                                                                                                                                                                                                                                                        c 1
                                                                  c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                        d = 0
                                                           \neg c: c_5, c_6, c_1
                                                             d: c<sub>5</sub>
                                                           \neg d: c_6
              Propagate c in c_5: (a \lor \neg c \lor d) \rightarrow assign d
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Watch lists:
                                   Activities
                                                       Trail:
                                   (increment=2):
                                                            DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                                                      b : c7
                                          a 1
        b: c_1, c_2, c_3, c_7
                                                                      C:C_4
                                          b 1
      \neg b: c_4
                                                                      d: c_5
                                          c 1
       c: c_2, c_4
                                          d = 0
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})
```

```
Activities
Watch lists:
                                                           Trail:
                                      (increment=2):
                                                                 DL1: \neg a: NULL
         a: c<sub>7</sub>
       \neg a: c_3
                                                                           b : c7
                                             a 1
         b: c_1, c_2, c_3, c_7
                                                                           C:C_4
                                             b 1
       \neg b: c_4
                                                                           d: c_5
                                             c 1
        c: c_2, c_4
                                             d = 0
       \neg c: C_5, C_6, C_1
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate c in c_6: (\neg c \lor \neg d) \to \mbox{$f$ conflict!}
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

Activities Trail: (increment=2):

a 1b 1c 1d 0

DL1: $\neg a : \text{NULL}$ $b : c_7$ $c : c_4$ $d : c_5$

 $\neg d$: c_6 Conflict resolution:

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b})$$

```
Watch lists:
                                  Activities
                                                      Trail:
                                  (increment=3):
                                                           DL1: \neg a: NULL
        a: c7
      \neg a: c_3
                                         a 1/3
        b: c_1, c_2, c_3, c_7
                                        b 1/3
      \neg b: c_4
                                         c 1/3
        C: C_2, C_4
                                         d Ø 2
      \neg c: C_5, C_6, C_1
      d: c<sub>5</sub>
      \neg d: c_6
```

b : c7 $C:C_4$ $d: c_5$

Conflict resolution:

$$\frac{c_6{:}(\neg c \vee \neg d)\ c_5{:}(a \vee \neg c \vee d)}{\frac{(a \vee \neg c)\ c_4{:}(a \vee \neg b \vee c)}{(a \vee \neg b)\ c_7{:}(a \vee b)}}{c_8{:}(a)}$$

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Add conflict clause Activities Trail: (increment=3):

a 3 b 3 c 3 d 2 DL1: $\neg a$: NULL b: c_7 c: c_4 d: c_5

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Backtrack to DL0 (increment=3):

a 3
b 3

Activities

a 3b 3c 3d 2

Trail:

DLT: Ja: NULT b: c7 c:e4 d:c5

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (\underline{a})
```

Watch lists:

```
a: c_7
\neg a: c_3
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6 Assign **a** at DL0 by c_8

```
Activities
              Trail:
(increment=3):
                  DL0: a: c_8
     a 3
     b 3
     c 3
     d 2
```

$$c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\underline{\neg a} \vee \underline{b} \vee \neg c) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

```
Watch lists:
```

```
a: c<sub>7</sub>
\neg a: c_3
 b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1
d: c_5
```

 $\neg d$: c_6

Propagate a in

```
Activities
               Trail:
(increment=3):
                   DL0: a: c_8
```

b 3 c 3 d 2

a 3

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```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
                                                                                                                                                                                                                                                                                                                                                                                         Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           Trail:
                                                                                                                                                                                                                                                                                                                                                                                         (increment=3):
                                                                                         a: c<sub>7</sub>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  DL0: a: c_8
                                                                         \neg a: c_3
                                                                                                                                                                                                                                                                                                                                                                                                                                                              a 3
                                                                                         b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                b 3
                                                                       \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                c 3
                                                                             c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                                                d 2
                                                                       \neg c: c_5, c_6, c_1
                                                                         d: c<sub>5</sub>
                                                                       \neg d: c_6
```

Propagate a in $c_3: (\neg a \lor b \lor \neg c)$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                                                                                                                                                                                                                                                                                                                Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                  Trail:
                                                                                                                                                                                                                                                                                                                                (increment=3):
                                                                           a: c<sub>7</sub>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                 DL0: a: c_8
                                                            ¬a: 03
                                                                                                                                                                                                                                                                                                                                                                                           a 3
                                                                           b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                           b 3
                                                            \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                            c 3
                                                                   C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                            d 2
                                                            \neg c: c_5, c_6, c_1, c_3
                                                              d: c<sub>5</sub>
                                                            \neg d: c_6
               Propagate a in c_3: (\neg a \lor b \lor \neg c) \to (\neg a \lor b \lor \neg c)
```

```
c_1: (a \vee \underline{b} \vee \neg \underline{c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \neg \underline{c}) \wedge c_4: (a \vee \neg \underline{b} \vee \underline{c}) \wedge c_5: (a \vee \neg \underline{c} \vee \underline{d}) \wedge c_6: (\neg \underline{c} \vee \neg \underline{d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)
```

```
Watch lists:
```

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6

Decide \neg b
```

```
Activities (increment=3):

a 3
b 3
c 3
d 2
```

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

Trail:

$$c_1: (a \vee \underline{b} \vee \underline{\neg c}) \wedge c_2: (\underline{b} \vee \underline{c}) \wedge c_3: (\neg a \vee \underline{b} \vee \underline{\neg c}) \wedge c_4: (a \vee \underline{\neg b} \vee \underline{c}) \wedge c_5: (a \vee \underline{\neg c} \vee \underline{d}) \wedge c_6: (\underline{\neg c} \vee \underline{\neg d}) \wedge c_7: (\underline{a} \vee \underline{b}) \wedge c_8: (a)$$

Watch lists:

```
a: c_7
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate $\neg b$ in

```
Activities Trail: (increment=3):
```

a 3b 3c 3d 2

```
DL0: a: c_8
DL1: \neg b: \text{NULL}
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
                                                                                                                                                                                                                                                                                                                                 Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       Trail:
                                                                                                                                                                                                                                                                                                                                   (increment=3):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DL0: a:c_8
                                                                            a: c<sub>7</sub>
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       DL1: \neg b: NULL
                                                              \neg a:
                                                                                                                                                                                                                                                                                                                                                                                                a 3
                                                                       b: c_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                b 3
                                                             \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                c 3
                                                                     c: c_2, c_4
                                                                                                                                                                                                                                                                                                                                                                                                d 2
                                                             \neg c: c_5, c_6, c_1, c_3
                                                              d: c<sub>5</sub>
                                                             \neg d: c_6
               Propagate \neg b in c_1:(a \lor b \lor \neg c)
```

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
           c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
Watch lists:
                                                                                                                                                                                                                                                                                                                   Activities
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               Trail:
                                                                                                                                                                                                                                                                                                                   (increment=3):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DL0: a: c_8
                                                                        a: c_7, c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DL1: \neg b: NULL
                                                           ¬a:
                                                                                                                                                                                                                                                                                                                                                                             a 3
                                                                  b: \varphi_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                             b 3
                                                          \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                             c 3
                                                                  C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                             d 2
                                                          \neg c: c_5, c_6, c_1, c_3
                                                             d: c_5
                                                          \neg d: c_6
              Propagate \neg b in c_1: (a \lor b \lor \neg c) \to (a \lor b \lor \neg c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=3):
                                                          DL0: a: c_8
        a: c_7, c_1
                                                          DL1: \neg b: NULL
      \neg a:
                                        a 3
       b: \varphi_1, c_2, c_3, c_7
                                        b 3
      \neg b: c_4
                                        c 3
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg b in c_2:(b \lor c)
```

 $d: c_5$ $\neg d: c_6$

```
c_1: (a \lor b \lor \neg c) \land c_2: (b \lor c) \land c_3: (\neg a \lor b \lor \neg c) \land c_4: (a \lor \neg b \lor c) \land c_4: (a \lor \neg
             c_5: (a \vee \neg c \vee d) \wedge c_6: (\neg c \vee \neg d) \wedge c_7: (a \vee b) \wedge c_8: (a)
                                                                                                                                                                                                                                                                                                                                                                       Activities
Watch lists:
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   Trail:
                                                                                                                                                                                                                                                                                                                                                                         (increment=3):
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL0: a: c_8
                                                                                     a: c_7, c_1
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                         DL1: \neg b: NULL
                                                                      ¬a:
                                                                                                                                                                                                                                                                                                                                                                                                                                              a 3
                                                                               b: \varphi_1, c_2, c_3, c_7
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            C : C2
                                                                                                                                                                                                                                                                                                                                                                                                                                              b 3
                                                                    \neg b: c_4
                                                                                                                                                                                                                                                                                                                                                                                                                                              c 3
                                                                               C: C_2, C_4
                                                                                                                                                                                                                                                                                                                                                                                                                                              d 2
                                                                    \neg c: c_5, c_6, c_1, c_3
```

Propagate $\neg b$ in $c_2:(b \lor c) \to assign c$

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Activities
Watch lists:
                                                         Trail:
                                    (increment=3):
                                                              DL0: a: c_8
        a: c_7, c_1
                                                              DL1: \neg b: NULL
       \neg a:
                                           a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                        C:C_2
                                           b 3
      \neg b: c_4
                                           c 3
       c: c_2, c_4
                                           d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
      \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c)
```

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                      Activities
                                                            Trail:
                                      (increment=3):
                                                                  DL0: a: c_8
         a: c_7, c_1
                                                                  DL1: \neg b: NULL
       \neg a:
                                              a 3
        b: \varphi_1, c_2, c_3, c_7
                                                                            C:C_2
                                              b 3
       \neg b: c_4
                                              c 3
        c: c_2, c_4
                                              d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
 Propagate \neg b in c_3: (\neg a \lor b \lor \neg c) \to \frac{1}{2} \text{ conflict!}
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a)$$

Watch lists:

```
a: c_7, c_1
\neg a:
b: c_1, c_2, c_3, c_7
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Conflict resolution:

Activities Trail: (increment=3):

a 3b 3c 3d 2

DL0: $a: c_8$ DL1: $\neg b: \text{NULL}$ $c: c_2$

```
c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \underline{\neg}\underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a})
```

```
Watch lists:
                                      Activities
                                                           Trail:
                                      (increment=4):
                                                                DL0: a:c_8
         a: c_7, c_1
                                                                DL1: \neg b: NULL
       \neg a:
                                             a 36
        b: \varphi_1, c_2, c_3, c_7
                                            b 36
                                                                           C:C_2
       \neg b: c_4
                                             c 36
        C: C_2, C_4
                                             d 2
       \neg c: c_5, c_6, c_1, c_3
       d: c<sub>5</sub>
       \neg d: c_6
                           c_3:(\neg a \lor b \lor \neg c) c_2:(b \lor c)
 Conflict resolution:
```

 $c_0:(\neg a \lor b)$

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: \varphi_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4):

a 6b 6c 6

d 2

DL0: $a: c_8$ DL1: $\neg b: \text{NULL}$

c : *c*₂

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

```
a: c_7, c_1
\neg a: c_9
b: c_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Backtrack to DL0

Activities (increment=4):

b 6c 6d 2

a 6

DLO: a: c₈
DLT: >b: NULL
c:e₂

Trail:

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land \underline{c_9}: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

Assign b by c_0

```
a: c_7, c_1
\neg a: c_9
b: c_1, c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

```
Activities Trail: (increment=4): DL0: a: c_8
```

a 6b 6c 6d 2

 $b: c_0$

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (a \lor \underline{\neg b} \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

Propagate b in

```
a: c_7, c_1
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b: c_4
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4):

a 6b 6c 6d 2

DL0: a: c₈

b : *c*₉

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \underline{\neg b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Watch lists:
                                  Activities
                                                     Trail:
                                  (increment=4):
                                                          DL0: a: c_8
        a: c_7, c_1
      \neg a: c_0
                                                                   b: c_0
                                        a 6
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b: c_4
                                        c 6
       c: c_2, c_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate b in c_4:(a \vee \neg b \vee c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                         Trail:
                                    (increment=4):
                                                              DL0: a: c_8
        a: c_7, c_1, c_4
                                                                        b : c9
       \neg a: c_0
                                           a 6
        b: c_2, c_3, c_7, c_9
                                           b 6
      \neg b: 64
                                           c = 6
       C: C_2, C_4
                                           d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
 Propagate b in c_4: (a \vee \neg b \vee c) \rightarrow (a \vee \neg b \vee c)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

Decide $\neg c$

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Propagate $\neg c$ in

```
Activities Trail: (increment=4): DL0: a: c_8
a \quad 6 \qquad b: c_9
b \quad 6 \qquad DL1: \neg c: NULL
c \quad 6
d \quad 2
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                     Trail:
                                  (increment=4):
                                                          DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                   b : c9
                                        a 6
                                                          DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                        b 6
      \neg b:
                                        c 6
       C: C_2, C_4
                                        d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg c in c_2:(b \lor c)
```

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                      Trail:
                                   (increment=4):
                                                            DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                     b : c9
                                         a 6
                                                            DL1: \neg c : NULL
        b: c_2, c_3, c_7, c_9
                                         b 6
      \neg b:
                                         c = 6
       C: C_2, C_4
                                         d 2
      \neg c: c_5, c_6, c_1, c_3
       d: c_5
      \neg d: c_6
 Propagate \neg c in c_4: (a \lor \neg b \lor c)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

Decide $\neg d$

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

```
Activities Trail:
(increment=4):

a 6
b 6
c 6
d 2
```

```
DL0: a: c_8

b: c_9

DL1: \neg c: NULL

DL2: \neg d: NULL
```

$$c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})$$

Watch lists:

```
a: c_7, c_1, c_4
\neg a: c_9
b: c_2, c_3, c_7, c_9
\neg b:
c: c_2, c_4
\neg c: c_5, c_6, c_1, c_3
d: c_5
\neg d: c_6
```

Activities Trail: (increment=4):

a 6b 6c 6d 2

DL0: $a: c_8$ $b: c_9$ DL1: $\neg c: NULL$

DL2: $\neg d : NULL$

Propagate $\neg d$ in

```
c_1: (\underline{a} \lor b \lor \underline{\neg c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg a \lor \underline{b} \lor \underline{\neg c}) \land c_4: (\underline{a} \lor \neg b \lor \underline{c}) \land c_5: (a \lor \underline{\neg c} \lor \underline{d}) \land c_6: (\underline{\neg c} \lor \underline{\neg d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (a) \land c_9: (\underline{\neg a} \lor \underline{b})
```

```
Activities
Watch lists:
                                                       Trail:
                                   (increment=4):
                                                            DL0: a: c_8
        a: c_7, c_1, c_4
      \neg a: c_0
                                                                     b : c9
                                          a 6
        b: c_2, c_3, c_7, c_9
                                                            DL1: \neg c : NULL
                                         b 6
      \neg b:
                                                            DI 2. \neg d \cdot NUII
                                          c 6
       C: C_2, C_4
                                          d 2
      \neg c: c_5, c_6, c_1, c_3
      d: c_5
      \neg d: c_6
 Propagate \neg d in c_5: (a \lor \neg c \lor d)
```

$$c_1: (\underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_2: (\underline{b} \lor \underline{c}) \land c_3: (\neg \underline{a} \lor \underline{b} \lor \underline{\neg}\underline{c}) \land c_4: (\underline{a} \lor \neg \underline{b} \lor \underline{c}) \land c_5: (\underline{a} \lor \underline{\neg}\underline{c} \lor \underline{d}) \land c_6: (\underline{\neg}\underline{c} \lor \underline{\neg}\underline{d}) \land c_7: (\underline{a} \lor \underline{b}) \land c_8: (\underline{a}) \land c_9: (\underline{\neg}\underline{a} \lor \underline{b})$$

Watch lists:

$$a: c_7, c_1, c_4$$

$$\neg a: c_9$$

$$b: c_2, c_3, c_7, c_9$$

$$\neg b:$$

$$c: c_2, c_4$$

$$\neg c: c_5, c_6, c_1, c_3$$

$$d: c_5$$

$$\neg d: c_6$$

$$\Rightarrow SAT \quad a \quad b \quad \neg c \quad \neg d$$

Learning target

■ How to compute with the DPLL+CDCL-based SAT solving algorithm?