Electromagnetics 2 Mini-Project

EECE.4610 – Engineering Capstone Proposal

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Waveguide is a metal box, open at both sides, and can be either air filled (hollow), or filled with a dielectric. They are used to transmit an EM wave from one place to another without any power loss. However, as wave guides operate as high pass filters, only frequencies above the guides cut off frequency will be able to transmit down the box. The length and width of the opening as well as the properties of the internal material are parameters to determine the wave guides cutoff frequency.

When finding the values of the cutoff frequency, one must consider the different "modes" of operation, meaning the different cutoff frequencies, f_c, and if it's a TE or TM wave propagating through the guide. Ideally, the wave frequencies that will be traveling down the guide will be between the two lowest cutoff frequency modes. This is because when multiple modes are at play, there would be a superposition between them, possibly causing interference and loss.

Anticipated Cutoff Frequencies for Their Respective Waveguide Modes

$$f_{c_{1,0}} = \frac{3.0 * 10^8}{2}\sqrt{\left(\frac{1}{2.286 * 10^{-2}}\right)^2 + \left(\frac{0}{1.016 * 10^{-2}}\right)^2} = 6.561\ GHz$$

$$f_{c_{2,0}} = \frac{3.0 * 10^8}{2}\sqrt{\left(\frac{2}{2.286 * 10^{-2}}\right)^2 + \left(\frac{0}{1.016 * 10^{-2}}\right)^2} = 13.123\ GHz$$

$$f_{c_{0,1}} = \frac{3.0 * 10^8}{2}\sqrt{\left(\frac{0}{2.286 * 10^{-2}}\right)^2 + \left(\frac{1}{1.016 * 10^{-2}}\right)^2} = 14.764\ GHz$$

The list of cut off frequencies above were solved for using equation 6 and are the expected values that the simulations would determine. Being a cutoff frequency, it is expected that there will be no propagation below the set value for the specific waveguide mode. Meaning, there will be a gap in lines on the plots as the frequencies are neared or passed.

Figure 1. Propagation Constant (β) vs. Frequency Plot

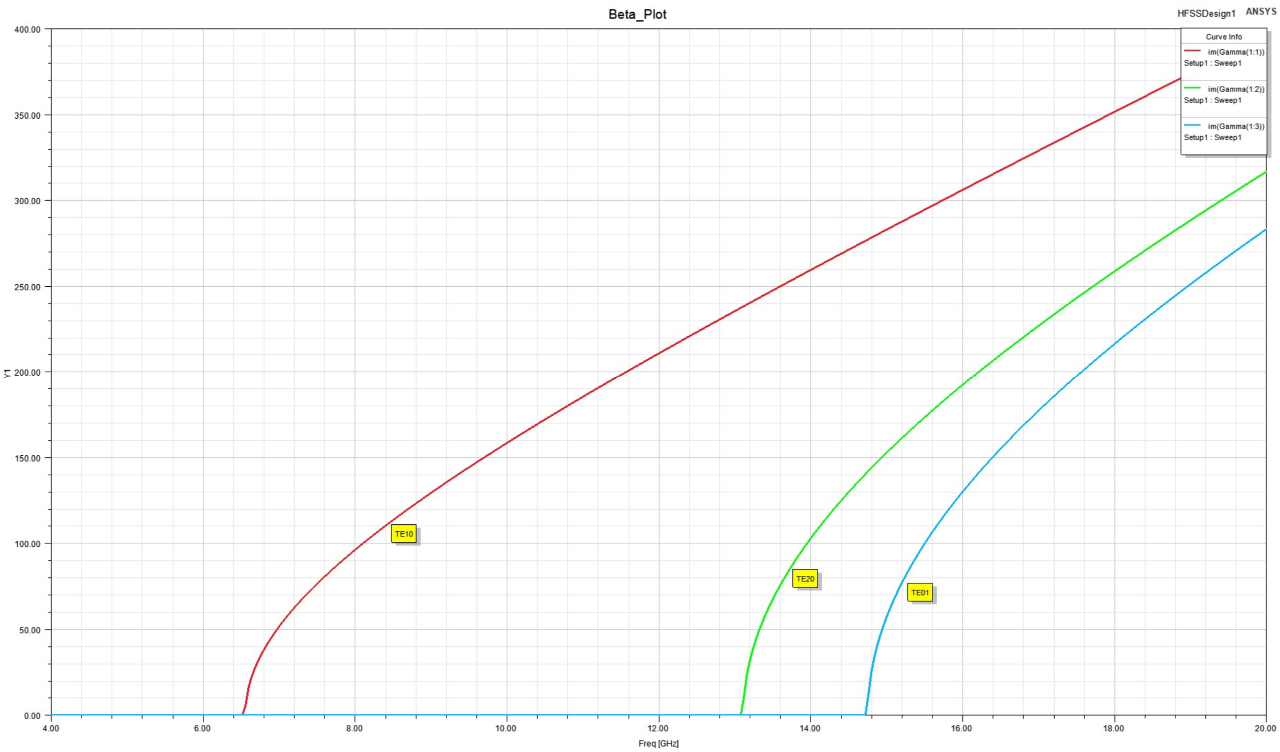


Figure 1 describes how β changes as the propagation frequency does. Due to how each wave guide operating mode has its own cutoff frequency, they create a phase shift in the wave as it travels down the guide. To solve β at a give point, ϕ (phee, phase angle) and z (the point of the shift) need to be known. Firstly, using equation 1, phee can be converted to radians from degrees, then the result of the conversion can be plugged into ϕ in equation 2 and divided by the point of the shift. Because the phase shift will be dependant on the wave guide modes, three β graphs are found for each mode.

$$\frac{\pi}{180} * \phi = phee\ in\ radians \qquad \text{Equation 1}$$

$$\beta = \frac{\phi}{z} \qquad \text{Equation 2}$$

Figure 2. Wavelength (λ) vs. Frequency Plot

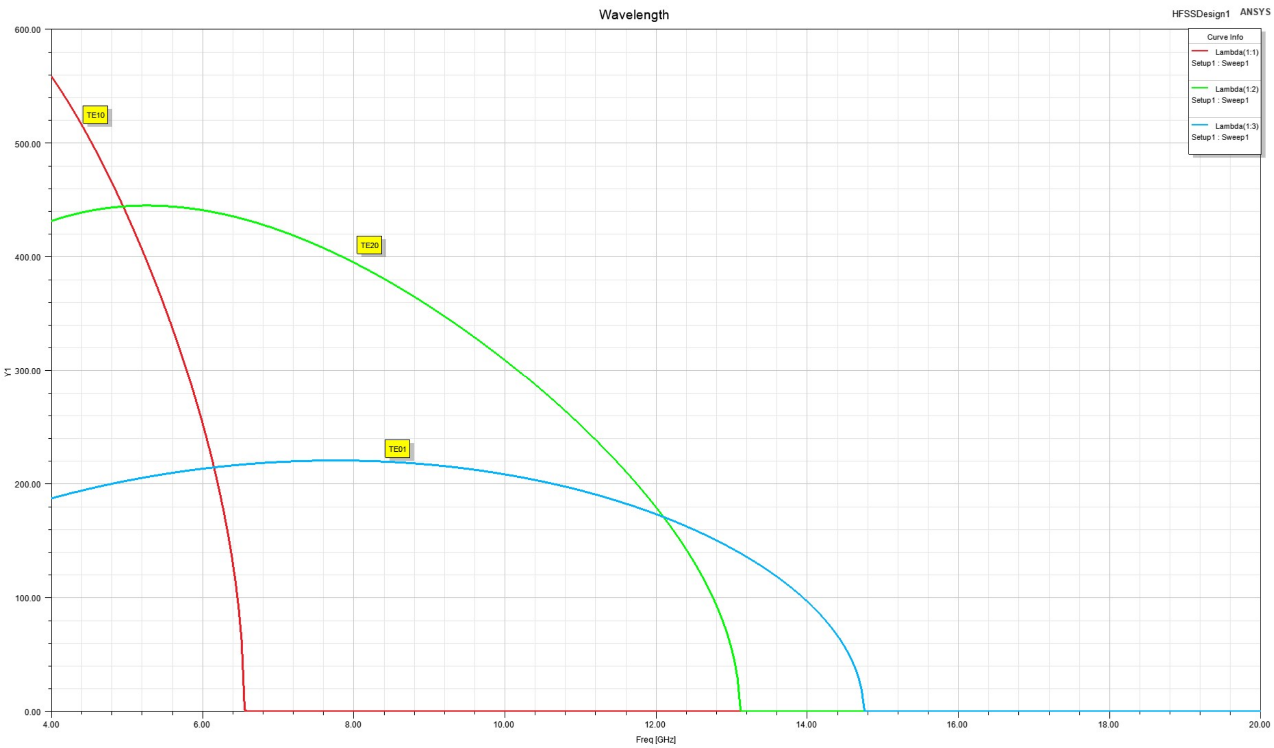


Figure 2 shows how λ is also dependent on the different guide modes. Equation 3 shows how the wavelength in free space changes with the different cut off frequencies. The general formula for λ can be seen in equation 4. The equation for finding the wave speed is equation 5. The notation of equation 5 is "u_{p0}" is the wave speed, and "c" is the speed of light. The other two variables, μ_r *and* ε_r are typically given constants, but for the waveguide being filled with air, both are equal to one. Using equation 6, cut off frequencies can be found for the different modes, and then λ_{cutoff} for the respective frequencies.

$$\lambda_{guide} = \frac{\lambda_{freespace}}{\sqrt{1-\frac{(\lambda_{freespace})^2}{(\lambda_{cutoff})^2}}} \qquad \text{Equation 3}$$

$$\lambda = \frac{u_0}{f} \qquad \text{Equation 4}$$

$$u_{p_0} = \frac{c}{\sqrt{\mu_r * \varepsilon_r}} \qquad \text{Equation 5}$$

$$f_{c_{m,n}} = \frac{u_{p_0}}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \qquad \text{Equation 6}$$

Figure 3. Intrinsic Impedance of the Wave (η) vs. Frequency Plot

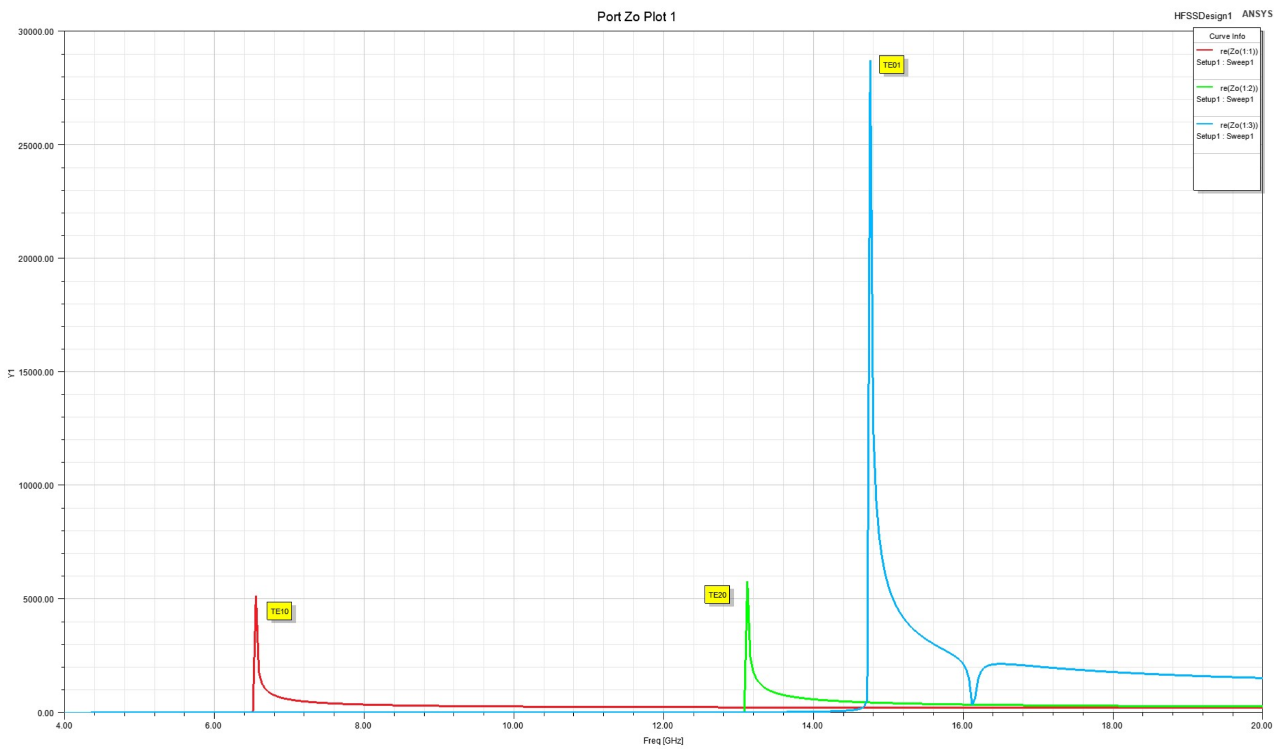
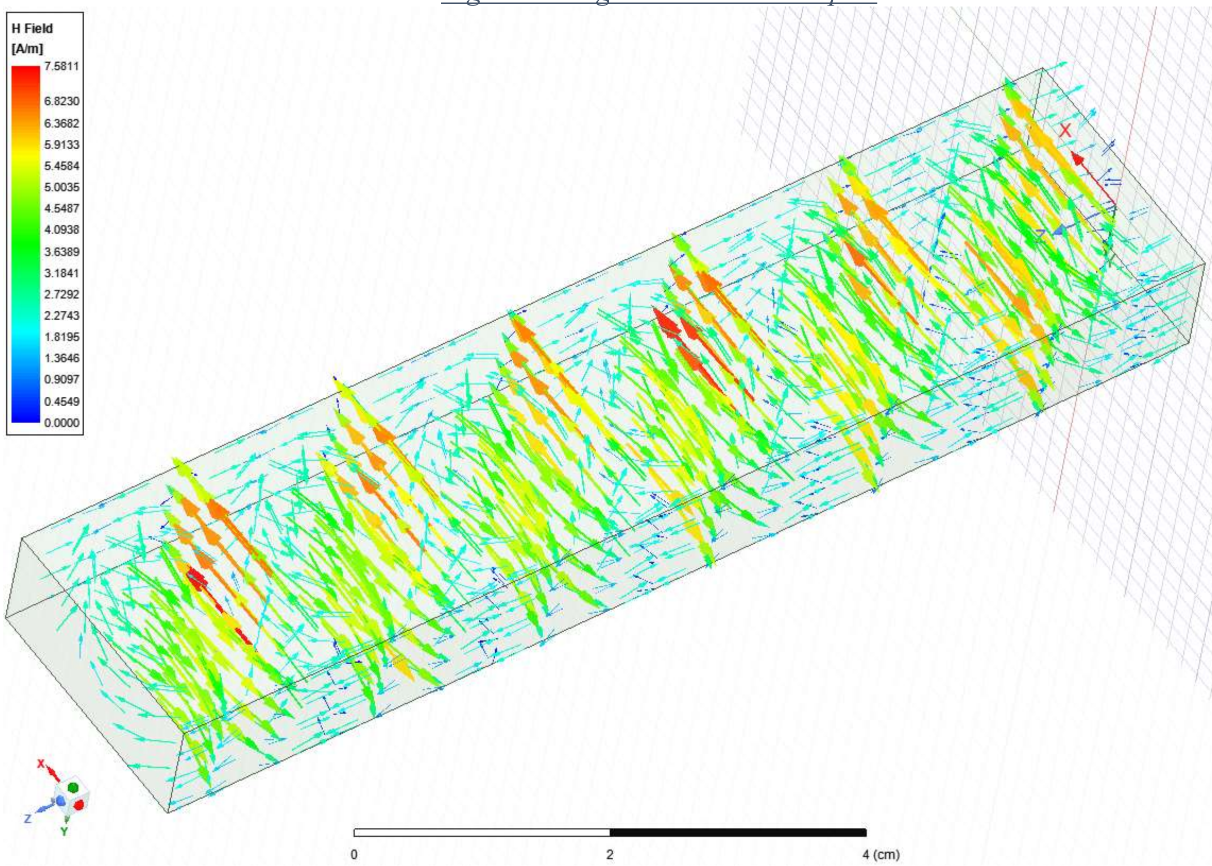


Figure 3 shows how the impedance of the wave is changed as it goes though a waveguide. Like many of the formula for wave properties in a waveguide, impedance relies on the wave's unbounded intrinsic impedance (η_c) and frequency (f), and the cut off frequency for the given mode ($f_{c_{m,n}}$) as seen in equation 7 below.

$$Z_{m,n}(f) = \eta_c \sqrt{1 - \left(\frac{f_{c_{m,n}}}{f}\right)^2} \qquad \text{Equation 7}$$

Figure 4. Magnetic Field Vector plot



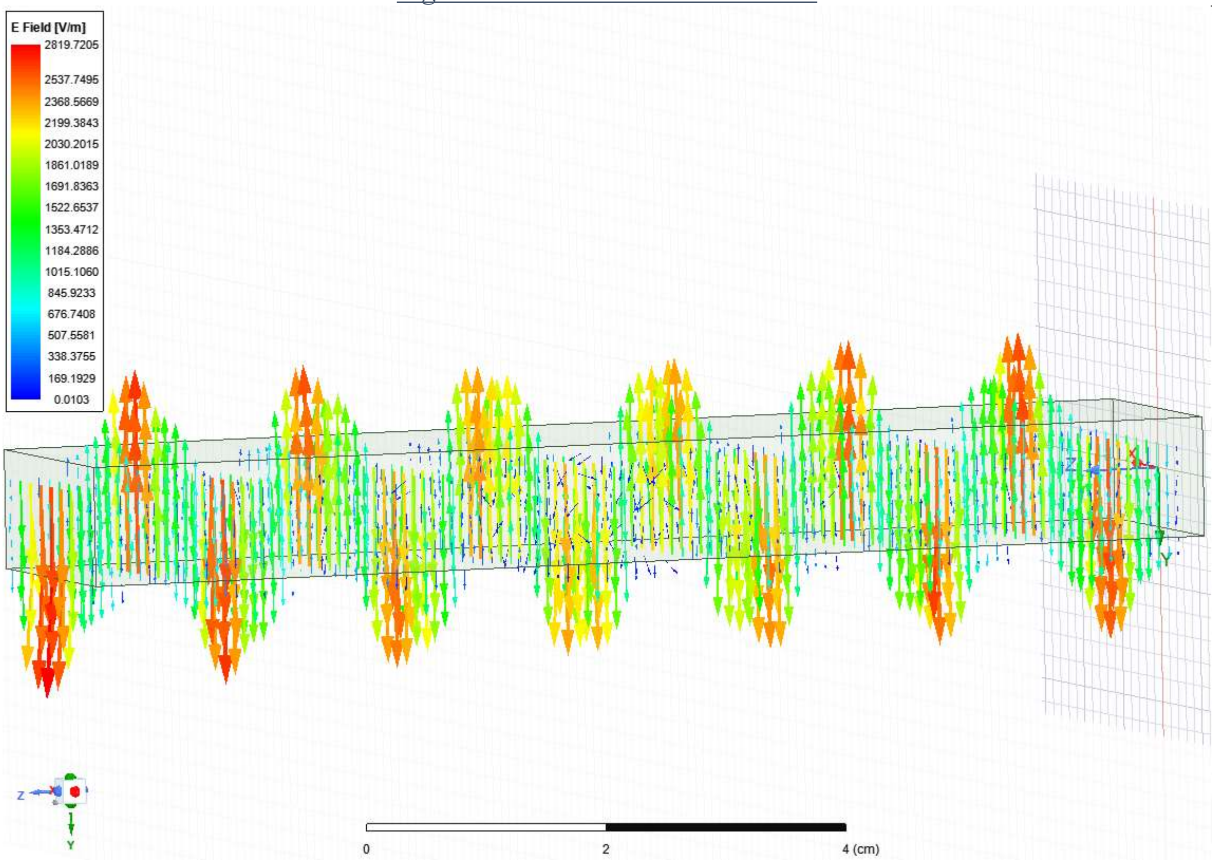
In figure 4, it can be seen how the Magnetic wave "bounces" off the side walls of the waveguide. In contrast, figure 5 shows the E-field "bouncing" off the top and bottom walls of the guide. Because of this, there can be a wave pointing in the direction of propagation. The equations for the x, y, and z components of the H-field are found with equations 8, 9 and 10 respectively.

$$\widetilde{\mathrm{H}_x} = \frac{j\beta}{k_c^2}\left(\frac{m\pi}{a}\right)\mathrm{H}_0 \sin\left(\frac{m\pi}{a}\right)\cos\left(\frac{n\pi}{b}\right)e^{-j\beta z} \quad \text{Equation 8}$$

$$\widetilde{\mathrm{H}_y} = \frac{j\beta}{k_c^2}\left(\frac{n\pi}{b}\right)\mathrm{H}_0 \cos\left(\frac{m\pi}{a}\right)\sin\left(\frac{n\pi}{b}\right)e^{-j\beta z} \quad \text{Equation 9}$$

$$\widetilde{\mathrm{H}_z} = \mathrm{H}_0 \cos\left(\frac{m\pi}{a}\right)\cos\left(\frac{n\pi}{b}\right)e^{-j\beta z} \quad \text{Equation 10}$$

Figure 5. Electric Field Vector Plot



Here, in figure 5, the propagation of an electric field can be seen. Due to the waveguide mode being TE, there cannot be an E_z, because "T" means transverse, and there cannot be an E-field component in the direction of propagation. Equations 11 and 12 describe how to find the E-field's x and y components inside of the waveguide.

$$\widetilde{E_x} = \frac{j\omega\mu}{k_c^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi}{a}x\right)\sin\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \qquad \text{Equation 11}$$

$$\widetilde{E_y} = \frac{-j\omega\mu}{k_c^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi}{a}x\right)\cos\left(\frac{n\pi}{b}y\right)e^{-j\beta z} \qquad \text{Equation 12}$$

Analysis on Adjusting a and b Such That TE_{10} or TE_{01} will Attenuate the Wave

My approach to solving for when the wave would attenuate was more along the line of visual inspection and a quick calculation for confirmation. The larger the waveguide becomes, the smaller the number in the radical equates to. Thus, decreasing the size of the waveguide openings, the larger the radical becomes and in turn, the cut off frequency. So, by decreasing the size of the waveguide, the cut off frequency will increase.

An easier way to achieve the same results, would be to fill the waveguide with a dielectric with its μ_r and ε_r values set smaller than 1. For example, changing those values to 0.1, , will generate a cutoff frequency of 65.6 GHz. This works because the wave speed in free space, equation 5, is increased where the radical of equation 12 will remain the same, causing f_c to increase.

$$f_{c_{1,0}} = \frac{\frac{3*10^8}{\sqrt{\mu_r * \varepsilon_r}}}{2}\sqrt{\left(\frac{1}{0.002286}\right)^2 + \left(\frac{0}{0.001016}\right)^2} \qquad \text{Equation 12}$$