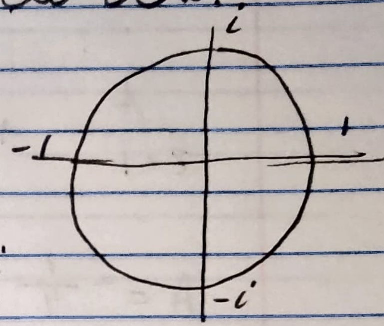
Math Extra Credit

1) Eval $I = \oint \frac{1-36z^2}{(z+\frac{4}{3}+i)(3z+1)^2}\,dz$ around unit circle CCW.



$z_1 = -\frac{4}{3} - i \qquad z_2 = -\frac{1}{3}$, second order

$I = 2\pi i \operatorname{Res}_{z=-\frac{1}{3}} \frac{1}{(n-1)!} \lim_{z\to -\frac{1}{3}} \frac{d^{n-1}}{dz^{n-1}} (z-z_0)^n f(z) \qquad -\frac{4}{3} - i$

$I = 2\pi i \cdot \frac{1}{(2-1)!} \lim_{z\to -\frac{1}{3}} \frac{d^{2-1}}{dz^{2-1}} \overset{3z+1}{\cancel{(z+\frac{1}{3})}}^{2} \frac{1}{(z+\frac{4}{3}+i)\cancel{(3z+1)}^2}$

$= 2\pi i \lim_{z\to -\frac{1}{3}} \frac{d}{dz} \frac{1}{(z+\frac{4}{3}+i)}$

$= 2\pi i \lim_{z\to -\frac{1}{3}} \frac{1}{(z+\frac{4}{3}+i)^2} = 2\pi i \frac{1}{(\frac{1}{3}+\frac{4}{3}+i)} = \frac{2\pi i}{(\frac{5}{3}+i)}$

$= \left| \frac{2\pi i}{(\frac{5}{3}+i)} \right| = \boxed{3.23267}$

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2) for $f(z) = \dfrac{1}{(z+1)(z-3)}$ valid for $15 < |z+7|$

$$\frac{A}{z+1} + \frac{B}{z-3} \rightarrow \left.\frac{1}{(z+1)(z-3)}\right|_{-1,3}$$

$A = -\frac{1}{4}$, $B = +\frac{1}{4}$

$$f(z) = -\frac{1}{4(z+1)} + \frac{1}{4(z-3)}$$

now $15 < |z+7|$

$15 < |z| + |7|$

$8 < |z| \rightarrow \frac{8}{|z|} > 1$

$$f(z) = \frac{1}{-4z(1+\frac{1}{z})} + \frac{1}{4z(1-\frac{3}{z})}$$

$$= -\frac{1}{4z}\left[1+\frac{1}{z}\right]^{-1} + \frac{1}{4z}\left[1-\frac{3}{z}\right]^{-1}$$

$$= -\frac{1}{4z}\left[+1 - \frac{1}{z} + \frac{1}{z^2} - \frac{1}{z^3} + \dots\right] + \dots$$

$$+ \frac{1}{4z}\left[1 + \frac{3}{z} + \frac{3^2}{z^2} + \frac{3^3}{z^3} + \dots\right]$$

$$= \left[\cancel{-\frac{1}{4z}} + \frac{1}{4z^2} - \frac{1}{4z^3} + \frac{1}{4z^4}\right] + \left[\cancel{\frac{1}{4z}} + \frac{3}{4z^2} + \frac{3^2}{4z^3} + \frac{3^3}{4z^4} + \dots\right]$$

$$= \left[\frac{1}{4} + \frac{3}{4}\right]\frac{1}{z^2} + \left[-\frac{1}{4} + \frac{9\,3^2}{4}\right]\frac{1}{z^3} + \left[\frac{1}{4} + \frac{27\,3^3}{4}\right]\frac{1}{z^4} + \dots$$

$$= \frac{1}{z^2} + \frac{2}{z^3} + \frac{7}{z^4} + \frac{80}{z^5} + \dots \qquad + \left[-\frac{1}{4} + \frac{81\,3^4}{4}\right]\frac{1}{z^5}$$

$\boxed{2}$ d ?

b) $\mathrm{Res}(f(z), 3) = \lim_{z \to 3} \dfrac{1}{(z+1)\cancel{(z-3)}} \cdot \cancel{(z-3)} = \boxed{\dfrac{1}{4}}$

3 $\boxed{\frac{1}{4}}$

3) $\text{Arg}(\text{Ln}(z)) = 3$ rads and $|z| = 70$

Find $S = \text{Im}(z) + \text{Re}(z)$

$$\text{Ln}(z) = \ln(z) + i\,\text{Arg}\left(z + \frac{2n}{\pi}\right)$$

principal part

$$\therefore \quad \text{Ln}(z) = \ln(z) + i\,\text{Arg}(z)$$

For any complex # $z = x + iy$, the $\text{Arg}(z) = \tan^{-1}\left(\frac{y}{x}\right)$

$$\text{Arg}(\text{Ln}(z)) = 3 = \tan^{-1}\left(\frac{\text{Arg}(z)}{\ln(z)}\right) \sim \frac{\text{Arg}(z)}{\ln(z)} = \tan(3)$$

Therefore $\tan^{-1}(\text{Im}(z)\,\text{Re}(z)) = \ln(70)\tan(3)$

So,

$$\ln(z) = \text{Re}(z) \cdot \tan(\ln(z) \cdot \tan(3))$$

also, $|z| = 70 = \sqrt{\text{Re}^2(z) + \text{Im}^2(z)}$

$\text{Re}(z) = 57.55$

$\text{Im}(z) = -39.85$

So, $S = \text{Re}(z) + \text{Im}(z)$

4) $\boxed{S = 17.7}$

4) For $x, y, z \in \mathbb{R}$

Given:

$$\begin{aligned} 5x &+ y &+ z &= 0 \\ 2x &- 2y &+ 2z &= 0 \\ 3x &+ 3y &- z &= 0 \end{aligned}$$

and $xyz = -48$

Find $S = x + y + z$

$$\left[\begin{array}{ccc|c} 5 & 1 & 1 & 0 \\ 2 & -2 & 2 & 0 \\ 3 & 3 & -1 & 0 \end{array}\right] \quad R_1 \to \frac{R_1}{5} \quad \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 \\ 2 & -2 & 2 & 0 \\ 3 & 3 & -1 & 0 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & -\frac{12}{5} & \frac{8}{5} & 0 \\ 0 & \frac{12}{5} & -\frac{8}{5} & 0 \end{array}\right] \quad R_2 \to -\frac{5}{12} R_2 \quad \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & \frac{12}{5} & -\frac{8}{5} & 0 \end{array}\right]$$

$$R_3 \to R_3 - \frac{12}{5} R_2 \quad \left[\begin{array}{ccc|c} 1 & \frac{1}{5} & \frac{1}{5} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$R_1 \to R_1 - \frac{1}{5} R_2 \quad \left[\begin{array}{ccc|c} 1 & 0 & \frac{1}{3} & 0 \\ 0 & 1 & -\frac{2}{3} & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x + \frac{1}{3} z = 0$
$y - \frac{2}{3} z = 0$

$x = -\frac{1}{3} z$
$y = \frac{2}{3} z$
$z = z$

$xyz = -48$

$\left(-\frac{1}{3} z\right)\left(\frac{2}{3} z\right)(z) = -48$

$-\frac{2}{9} z^3 = -48 \cdot \frac{9}{2}$

$z^3 = 216$

$\boxed{z = 6}$

w/ $z = 6$

$x = -\frac{6}{3} = \boxed{-2}$

$y = +\frac{2(6)}{3} = \boxed{4}$

$S = x + y + z$

$S = -2 + 4 + 6$

$S = 8$ (a) .5