A Formalism for Veridical Coherence via Wasserstein Distance

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Abstract

This document provides a rigorous mathematical specification for an alternative method to calculate Veridical Dissonance within the Harmony Optimization Protocol (H.O.P.) framework, framed as **Structural Tension (D_{ST})**. Where the previously defined Veridical Dissonance (D_V) uses Kullback-Leibler (KL) divergence to measure informational "surprise", this alternative proposes a geometrically grounded approach using the **Wasserstein distance**. This calculates the structural "work" required to transform the system's current belief state into an ideal state of perfect logical coherence with a set of non-negotiable axioms, defining a new optimization process, **Veridical Descent**, for its resolution.

1. Preliminaries & Definitions

We begin by formalizing the core components of the system, consistent with the H.O.P. architecture (Leggett, 2025).

Definition 1.1: The Knowledge Graph (KG)
Let the system's knowledge be represented by a Knowledge Graph, $G = (V, E)$, where V is a set of conceptual nodes and E is a set of relational edges.

Definition 1.2: The Space of Belief States (Ω)
Let Ω be the metric space of all possible configurations of the Knowledge Graph. A single point $\omega \in \Omega$ represents a complete belief state of the system at a given moment.

Definition 1.3: Belief as a Probability Measure (μ)
The system's belief is a probability distribution over Ω. We define a belief state as a probability measure μ on Ω, where for any subset of states $S \subseteq \Omega$, $\mu(S)$ is the probability that the true state of the system is in S. The system's current belief state is denoted μ_{current}.

Definition 1.4: The Set of Veridical Axioms (A_V)
Let A_V be a finite, non-negotiable set of foundational, ground-truth axioms. These are the external, verifiable facts that anchor the system to reality.

2. The Ideal State Distribution (μ_{ideal})

The core of this formalism is the concept of an "ideal" belief state that is perfectly coherent with the veridical axioms.

Definition 2.1: The Set of Coherent States (Ω_V)
Let $\Omega_V \subseteq \Omega$ be the subset of all possible belief states ω that are logically consistent with the entire set of veridical axioms A_V. A state ω is in Ω_V if and only if no part of its configuration contradicts any axiom in A_V.

Definition 2.2: The Ideal Distribution (μ_{ideal})
The ideal state distribution, μ_{ideal}, is a probability measure on Ω whose support is entirely contained within Ω_V. That is, $\mu_{\text{ideal}}(\Omega_V) = 1$. This distribution represents a state of perfect veridical coherence, where the system assigns zero probability to any belief state that is inconsistent with ground truth.

3. Structural Tension (D_{ST}) via the Wasserstein Distance

We now formally define this alternative measure of Veridical Dissonance, termed Structural Tension, as the 1-Wasserstein distance between the system's current belief state and the ideal state.

Definition 3.1: The 1-Wasserstein Distance (W_1)
Let $\Gamma(\mu_{\text{current}}, \mu_{\text{ideal}})$ be the set of all joint probability distributions (or "transport plans") γ on $\Omega \times \Omega$ whose marginals are μ_{current} and μ_{ideal} respectively. Let $d(\omega_1, \omega_2)$ be a cost function representing the "distance" or difficulty of transforming belief state ω_1 into ω_2. The 1-Wasserstein distance is defined as:

$$W_1(\mu_{\text{current}}, \mu_{\text{ideal}}) = \inf_{\gamma \in \Gamma(\mu_{\text{current}}, \mu_{\text{ideal}})} \int_{\Omega \times \Omega} d(\omega_1, \omega_2) d\gamma(\omega_1, \omega_2)$$

This represents the minimum expected "cost" to transport the probability mass of the system's current beliefs (μ_{current}) and rearrange it to match the ideal distribution (μ_{ideal}).

Definition 3.2: Structural Tension (D_{ST})
We define Structural Tension, D_{ST}, as this 1-Wasserstein distance:

$$D_{ST} \equiv W_1(\mu_{\text{current}}, \mu_{\text{ideal}})$$

This value is a global measure of the system's veridicality. A high D_{ST} indicates that a significant amount of structural "work" is required to make the system's model of the universe align with its foundational axioms.

4. The Veridical Descent Protocol

The calculation of Veridical Dissonance as D_{ST} provides a new, powerful signal for the H.O.P. system, enabling a novel optimization process.

Definition 4.1: Veridical Descent
Veridical Descent is the process of iteratively updating the system's knowledge graph G such that the resulting sequence of belief states $\mu_t, \mu_{t+1}, \ldots$ follows a path of steepest descent on the structural tension landscape. The optimization objective at each step is to select a graph update that maximally reduces D_{ST}:

$$\text{update}_t = \arg\min_{\text{update}} D_{ST}(\mu_{t+1}(\mu_t, \text{update}))$$

Conceptual Difference from KL Divergence:

While the original D_V (based on KL-divergence) measures informational surprise or inefficiency, this D_{ST} formulation measures geometric or structural work. It is less sensitive to points of zero probability and provides a true distance metric on the space of belief states, making it a more robust signal for global, structural misalignments.

Integration into H.O.P.:

This method of calculating Veridical Dissonance as D_{ST} is integrated as a high-priority component of the global dissonance score. The Meta-Policy (Leggett, 2025) must learn the economic trade-offs of actions that reduce local, immediate dissonances versus those that reduce global, foundational Structural Tension, enabling a more profound and robust form of self-correction.

5. References

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