Advanced Algorithms and Datastructures - Exam Notes

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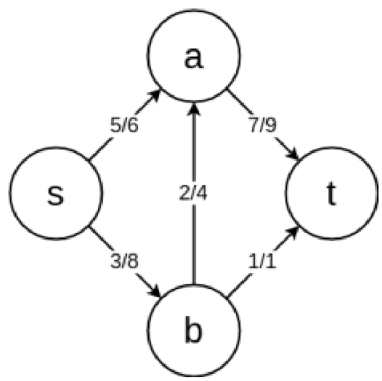
2021

Max Flow

Disposition

1. Introduction
2. Example of running the Edmonds-Karp Algorithm
3. Proof of the *Max-flow min-cut theorem*



Presentation

Linear Programming and Optimization

Disposition

1. Introduction
2. Preparing and running SIMPLEX on example
3. Proof of weak duality

Presentation

Randomized Algorithms

Disposition

1. Introduction
2. Example on running randomized quicksort + motivation behind randomness
3. Analysis of expected runtime of randomized quicksort
4. Example on running the min-cut algorithm
5. Las Vegas Algorithms vs Monte Carlo Algorithms

Presentation

Hashing

Disposition

Presentation

Van Emde Boas Trees

Disposition

Presentation

NP-Completeness

Disposition

Presentation

Exact Exponential Algorithms and Parameterized Complexity

Disposition

Presentation

Approximation Algorithms

Disposition

1. Introduction
2. Definition of the *approximation ratio*, a $\rho(n)$*-approximation algorithm* and a *randomized* $\rho(n)$*-approximation algorithm*.
3. The Vertex-cover problem
   (a) Introduction
   (b) Proof that `APPROX-VERTEX-COVER` is a 2-approximation algorithm
4. MAX-3-CNF
   (a) Introduction
   (b) Proof that the randomized algorithm for `MAX-3-CNF` is a randomized 8/7-approximation algorithm

Presentation

Definition of *approximation ratio*
We say that an algorithm for a problem has an ***approximation ratio*** of $\rho(n)$ if, for any input of size n, the cost C of the solution produced by the algorithm is within a factor of $\rho(n)$ of the cost C^* of an optimal solution

$$\max\left(\frac{C}{C^*}, \frac{C^*}{C}\right) \leq \rho(n).$$

Definition of $\rho(n)$-*approximation algorithm*
If an algorithm achieves an approximation ratio of $\rho(n)$, we call it a $\rho(n)$-***approximation algorithm***.

Definition of *randomized* $\rho(n)$-*approximation algorithm*
We say that a randomized algorithm for a problem has an ***approximation ratio*** of $\rho(n)$ if, for any input of size n, the expected cost C of the solution procuded by the randomized algorithm is within a factor of $\rho(n)$ of the cost C^* of an optimal solution:

$$\max\left(\frac{C}{C^*}, \frac{C^*}{C}\right) \leq \rho(n).$$

We call a randomized algorithm that achieves an approximation ratio of $\rho(n)$ a ***randomized*** $\rho(n)$-***approximation algorithm***

Introduction to *vertex cover*
A ***vertex cover*** of an undirected graph $G = (V, E)$ is a subset $V' \subseteq V$ such that if (u, v) is an edge of G, then either $u \in V'$ or $v \in V'$ (or both). The size of a vertex cover is the number of vertices in it. The ***vertex-cover problem*** is to find a vertex cover of minimum size in a given undirected graph. We call such a vertex cover an ***optimal vertex cover***.

The set C of vertices that is returned by `APPROX-VERTEX-COVER` is a vertex cover, since the laogrithm loops until every edge in $G.E$ has been covered by some vertex in C.

Algorithm 1 APPROX-VERTEX-COVER

Require: Undirected graph G
1: $C = \emptyset$
2: $E' = G.E$
3: **while** $E' \neq \emptyset$ **do**
4:     let (u, v) be an arbitrary edge of E'
5:     $C = C \cup \{u, v\}$
6:     remove from E' edge (u, v) and every edge incident on either u or v
7: **return** C

Proof that `APPROX-VERTEX-COVER` is a 2-approximation algorithm
Let A denote the set of edges that line 4 picked. Not two edges in A share

an endpoint. Thus no two edges in A are covered by the same vertex from an optimal cover C^*, and we have the lower bound

$$|C^*| \geq |A| \tag{1}$$

on the size of an optimal vertex cover. Since A consists of the edges between two vertices in C (and since all of the elements in C are unique), we have the (exact) upper bound on the size of the vertex cover returned

$$|C| = 2|A| \tag{2}$$

Combining equation (1) and (2), we obtain

$$|C| = 2|A| \leq 2|C^*|$$

Polygon Triangulation

Disposition

1. Introduction
2. The 3-coloring approach
   (a) Example on running the algorithm
   (b) Proving that the 3-coloring approach is optimal in worst case
3. Example on partitioning a polygon into monotone pieces + runtime analysis
4. Example on triangulating a monotone polygon + runtime analysis

Presentation