Øvelser - Divide-and-Conquer

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4.3 - 1

Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$:
Vi gætter på, at $T(n) = O(n^2)$ og skal derfor vise, at $T(n) \leq cn^2$:

$$T(n) = T(n-1) + n \leq c(n-1)^2 + n$$

$$= C(n^2 + (-1)^2 - 2n) + n = cn^2 - 2cn + c + n$$

$$= cn^2 + c + n(-2c+1) \leq cn^2), \text{for } c > \frac{1}{2}$$

4.3 - 2

Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(\lg n)$:
Vi gætter på, at $T(n) = O(\lg n)$ og skal derfor vise $T(n) \leq c \lg n$:

$$T(n/2) + 1 \leq c(\lg(n/2)) + 1$$

$$= c(\lg n - \lg 2) + 1 = c(\lg n - 1) + 1$$

$$= c \lg n - c + 1 \leq c \lg n \text{ for } c \geq 1$$

4.3 - 6

Show that the solution to $T(n) = 2T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \lg n)$:
Vi gætter på, at $T(n) \leq c \cdot (n-d) \lg(n-d)$.

$$T(n) = 2T(\lfloor n/2 \rfloor + 17) + n \leq 2T(n/2+17) + n \leq 2(c \cdot (n/2+17-d) \lg(n/2+17-d))$$

$$\leq 2(c(n/2) \lg(n/2)) = cn \lg n/2 = cn(\lg n - \lg 2)$$

$$= cn(\lg n - 1) \leq cn \lg n$$

4.4 - 2

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T(n/2) + n^2$. Use the substitution method to verify your answer
Vi har, at summen over omkostningerne af hele træet, kan findes ved

$$\sum_{i=0}^{\lg n} \frac{n^2}{2^i} = O(n^2)$$

Herefter gætter vi på, at $T(n) \leq cn^2$:

$$T(n) = T(n/2) + n^2$$

$$
\begin{aligned}
&\leq c(n/2)^2 + n^2 \\
&= c(n^2/4) + n^2 \\
&= n^2 c/4 + n^2 \\
&= n^2(c/4 + 1) \leq cn^2, \text{ for } c \geq 4/3
\end{aligned}
$$

4.4 - 4

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$. Use the substitution method to verify your answer

Træet er et fuldt binært træ med højden n og omkostning 1 i hver knude. Herved har vi $O(2^n)$. Vi gætter på $T(n) \leq 2^n - 1$:

$$T(n) = 2T(n-1) + 1 \leq 2c(n^{n-1} - 1) + 1 = c2^n - 1 \leq c2^n$$