Øvelser - Divide-and-Conquer

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4.3 - 1

Show that the solution of $T(n) = T(n-1) + n$ **is** $O(n^2)$:
Vi gætter på, at $T(n) = O(n^2)$ og skal derfor vise, at $T(n) \leq cn^2$:

$$T(n) = T(n-1) + n \leq c(n-1)^2 + n$$

$$= C(n^2 + (-1)^2 - 2n) + n = cn^2 - 2cn + c + n$$

$$= cn^2 + c + n(-2c + 1) \leq cn^2), \text{for } c > \frac{1}{2}$$

4.3 - 2

Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ **is** $O(\lg n)$:
Vi gætter på, at $T(n) = O(\lg n)$ og skal derfor vise $T(n) \leq c \lg n$:

$$T(n/2) + 1 \leq c(\lg(n/2)) + 1$$

$$= c(\lg n - \lg 2) + 1 = c(\lg n - 1) + 1$$

$$= c \lg n - c + 1 \leq c \lg n \text{ for } c \geq 1$$