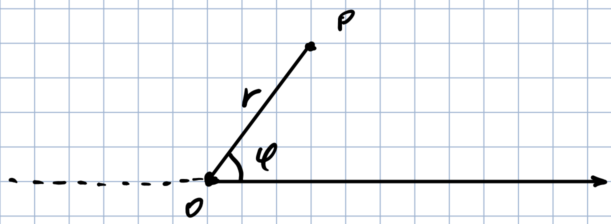
Polar Coordinates



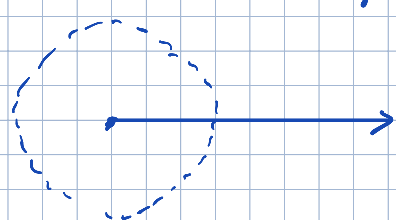
$P(r, \varphi)$

Definition

$$\begin{cases} x = r\cos\varphi \\ y = r\sin\varphi \end{cases}$$

Circle:

$r = r_0 = \ell$ (const)



$r = \ell$ – eq-n of a circle.

Line:

$\varphi =$ const.

General form of a Line:

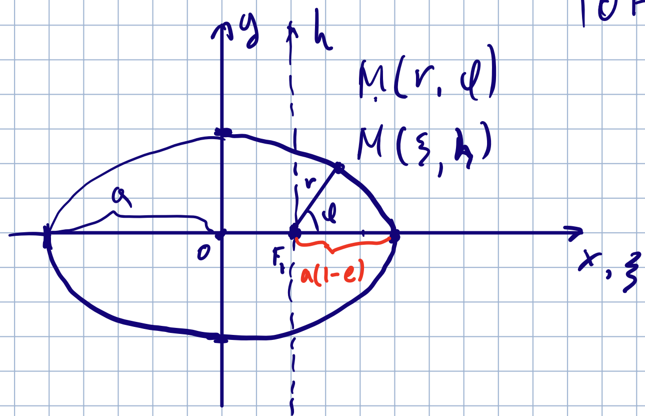
$$Ax + By + C = 0$$

$$A\cos\varphi + B\sin\varphi = \frac{\ell}{r}$$

eq-n of a line.

Conic sections in Polar coord-s.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{ellipse.}$$



$$|OF_1| = ae$$

$$\begin{cases} \xi = x - ae \\ h = y \end{cases}$$

$$\begin{cases} x = \underbrace{\xi + ae} \\ y = \underbrace{h} \end{cases}$$

$$\begin{cases} \xi = r \cdot \cos\varphi \\ h = r \cdot \sin\varphi \end{cases} \Rightarrow \begin{cases} x = r \cdot \cos\varphi + ae \\ y = r \cdot \sin\varphi \end{cases}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\to \frac{(r\cos\varphi + ae)^2}{a^2} + \frac{r^2\sin^2\varphi}{b^2} = 1 \qquad \Big| \cdot \underbrace{\left(a^2(1-e^2)\right)}$$

$$b^2 = a^2(1-e^2)$$

$$(r^2\cos^2\varphi + 2rae\cos\varphi + a^2e^2)(1-e^2) + r^2\sin^2\varphi =$$

$$= a^2(1-e^2)$$

$$r^2\cos^2\varphi\,(1-e^2)+2rae\cos\varphi\,(1-e^2)+$$

$$+\,a^2e^2(1-e^2)+r^2\sin^2\varphi=a^2(1-e^2)$$

$$r^2-e^2r^2\cos^2\varphi+2rae\cos\varphi+a^2e^2-a^2e^4-$$

$$-\,2rae^3\cos\varphi=a^2(1-e^2)$$

$$p=a(1-e^2)$$

$$r^2-e^2r^2\cos^2\varphi+2aer(1-e^2)\cos\varphi=a^2(1-e^2)^2$$

$$r^2(1-e^2\cos^2\varphi)+2rae\cos\varphi\,(1-e^2)-a^2(1-e^2)^2=0$$

$$p=a(1-e^2)$$

$$r^2(1-e^2\cos^2\varphi)+2rpe\cos\varphi-p^2=0$$

$$r_{1,2}=\frac{1}{1-e^2\cos^2\varphi}\underbrace{(-ep\cos\varphi\pm p)}=$$

$$= \frac{p(-e\cos\varphi \pm 1)}{1-e^2\cos^2\varphi} = \pm \frac{p}{1+e\cos\varphi}$$

$$\boxed{r = \frac{p}{1+e\cos\varphi}}$$

$$r = \frac{1}{1-e^2\cos^2\varphi}(-ep\cos\varphi - p) =$$

$$= -\frac{p}{(1-e\cos\varphi)} \quad (r<0)$$

$$\boxed{r = \frac{p}{1+e\cos\varphi}}$$

eq-n of ellipse in polar coord.

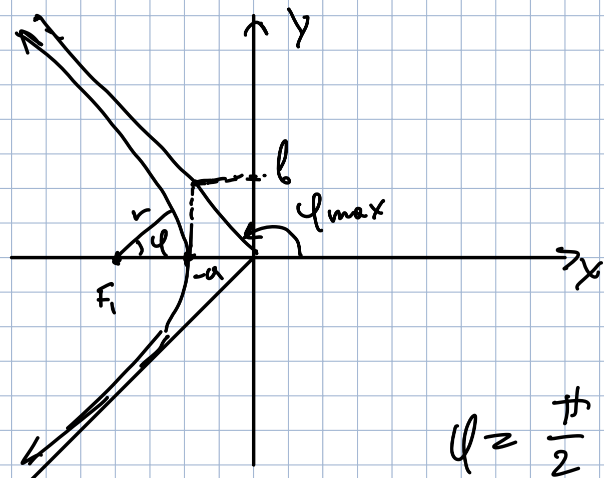
$$\varphi = 0. \quad r(0) = \frac{p}{1+e} = \frac{a(1-e^2)}{1+e} =$$

$$= a(1-e)$$

$$\varphi = \pi : r(\pi) = a(1+e$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \boxed{r(\varphi) = \frac{-p}{1 + e\cos\varphi}}$$



$$\varphi = \frac{\pi}{2} \qquad r = -p$$

$$p = a(1 - e^2)$$

$$e > 1$$

$$p' = a(e^2 - 1)$$

$$r = \frac{p'}{1 + e\cos\varphi}$$

We consider eq-n of hyperbola

$$r = \frac{p}{\underbrace{1 + e\cos\varphi}}$$

What could be φ_{max} $\qquad p = a(e^2 - 1)$

$$1 + e\cos\varphi_{max} = 0$$

$$e\cos\varphi_{max} = -1$$

$$e^2\cos^2\varphi_{max} = 1$$

$$\cos^2\varphi_{max} = \frac{1}{e^2} \qquad\qquad \frac{1}{\cos^2\alpha} = tg^2\alpha + 1$$

$$\frac{1}{\cos^2\varphi_{max}} = e^2$$

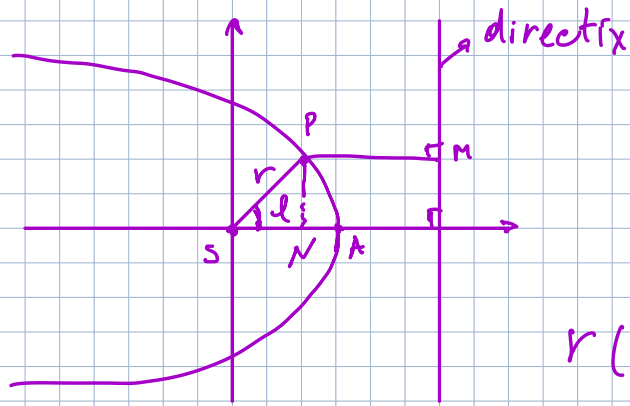
$$tg^2\alpha = \frac{1}{\cos^2\alpha} - 1$$

$$tg^2\varphi_{max} = e^2 - 1 \qquad\qquad b^2 = a^2(e^2 - 1)$$

$$e^2 - 1 = \frac{b^2}{a^2}$$

$$tg^2\varphi_{max} = \frac{b^2}{a^2} \quad \longleftarrow$$

$$\varphi_{max} = \arctan\frac{b}{a}$$



Home work

$$\frac{SP}{PM} = e$$

$$r(\theta) = ?$$