Theoretical Computer Science

Tutorial Week 13

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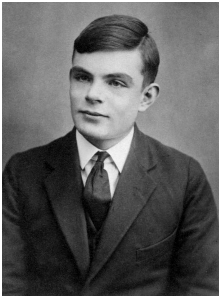
Agenda

Turing Machine

- TIME
- SPACE

Turing Machine



Turing thesis (Church–Turing thesis)

A function on the natural numbers can be calculated by an effective method if and only if it is computable by a Turing machine.

Turing Machine

Definition

A universal Turing machine (UTM) is a Turing machine that simulates an arbitrary Turing machine on arbitrary input.

Fact

A universal TM exists.

HALT-problem

Halting problem

Let T be a TM.
HALT is the set of all strings s such that

$$TIME(T, s) \neq \infty.$$

Fact

HALT is a undecidable problem.

Rice theorem

Rice theorem

If P is a non-trivial property and is recognized by Turing machine M, then the language holding the property $L_p = \{\langle M \rangle \mid L(M) \in P\}$ is undecidable.

Agenda

Turing Machine

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Configuration

Definition

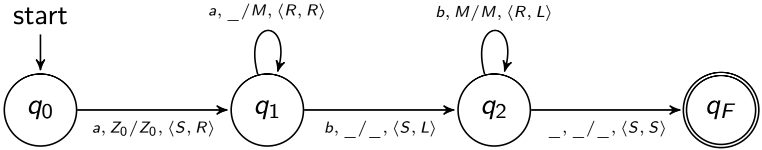
A configuration c of a TM with k-tapes is the following $(k+2)$-tuple:

$$c = \langle q, x{\uparrow}y, \alpha_1{\uparrow}\beta_1, \ldots, \alpha_k{\uparrow}\beta_k \rangle$$

where

- $q \in Q$
- $x \in (\Sigma \cup \{\_\})^*$, $y = y' \cdot \_$ with $y' \in \Sigma^*$
- $\alpha_r \in (\Gamma \cup \{\_\})^*$ and $\beta'_r = \beta'_r \cdot \_$ with $\beta'_r \in \Gamma^*$ and $1 \leq r \leq k$
- $\uparrow \notin \Sigma \cup \Gamma$

Example: Language A^nB^n



$$\langle q_0, \uparrow aabb, \uparrow Z_0\rangle \vdash \langle q_1, \uparrow aabb, Z_0\uparrow\rangle \vdash \langle q_1, a\uparrow abb, Z_0M\uparrow\rangle \vdash$$
$$\vdash \langle q_1, aa\uparrow bb, Z_0MM\uparrow\rangle \vdash \langle q_2, aa\uparrow bb, Z_0M\uparrow M\rangle \vdash$$
$$\vdash \langle q_2, aab\uparrow b, Z_0\uparrow MM\rangle \vdash \langle q_2, aabb\uparrow, \uparrow Z_0MM\rangle \vdash$$
$$\vdash \langle q_F, aabb\uparrow, \uparrow Z_0MM\rangle$$

$$c_0 \vdash c_1 \vdash c_2 \vdash c_3 \vdash c_4 \vdash c_5 \vdash c_6 \vdash c_7 = c_F$$

$$c_0 \vdash^* c_F$$

Acceptance Condition

If $T = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle$ is a TM and $s \in \Sigma^*$,

$$s \text{ is accepted by } T, \text{ if } c_0 \vdash^* c_F$$

where

1. c_0 is an initial configuration defined as $c_0 = \langle q_0, \uparrow s, \uparrow Z_0, \ldots, \uparrow Z_0 \rangle$ where
   - $x = \epsilon$
   - $y = s\_$
   - $\alpha_r = \epsilon$, $\beta_r = Z_0$, for any $1 \leq r \leq k$.
2. c_F is a final configuration defined as $c_F = \langle q, s'\uparrow y, \alpha_1\uparrow\beta_1, \ldots, \alpha_k\uparrow\beta_k \rangle$ where
   - $q \in F$
   - $x = s'$

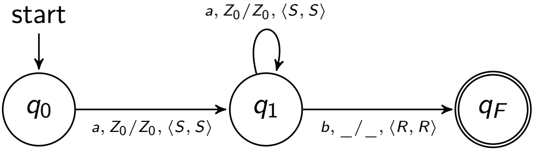
TIME

Definition

Let $T = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle$ be a TM, s be a given string. The length of sequence $c_0 \vdash^* c_F$ is called a **time**-function:

$$TIME(T, s)$$

TIME



Note that if c_F does not exist then $TIME(T, s) = \infty$ (indefinitely).

TIME

Definition

Let f be a function, c be a constant. A TM is called f-time-computable, if

$$TIME(T, s) \leq c \cdot f(|s|)$$

for any s.

TIME

Definition

Let T be a deterministic TM. A TM is called P-time-computable (computable in polynomial time, polynomially computable etc.),

if it is x^n-time-computable for some n.

Definition

Let T be a non-deterministic TM. A TM is called NP-time-computable,

if it is x^n-time-computable for some n.

TIME

Fact

$$P \subseteq NP$$

Problem

$$P = NP?$$

Definition

Let T be a deterministic TM. A TM is called

EXP-time-computable,

if it is 2^{x^n}-time-computable for some n.

Fact

$$P \subseteq NP \subseteq EXP$$

NP-hard problems

Satisfiability problem (SAT)

For Boolean formula, are there variables where the formula is true?

$a \,\&\, (\neg b \vee c)$

NP-hard problems

From graph theory

- Hamiltonian path problem
- Graph coloring problem
- The travelling salesman problem

NP-hard problems

RegExp

Simplifications of RegExp

$$a^* bb^* a((a \mid b)b^* a \mid \epsilon)^*(a \mid b)b^* \mid a^* bb^* =$$
$$= a^* bb^*(a(a \mid b)b^*)^*$$

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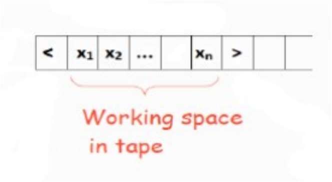
SPACE



Definition

Let $T = \langle Q, \Sigma, \Gamma, \delta, q_0, Z_0, F \rangle$ be a TM, s be a given string. The working space in tape for s is a **SPACE**-function:

$$SPACE(T, s)$$

Definition

Let f be a function, c be a constant. A TM is called f-space-computable, if

$$SPACE(T, s) \leq c \cdot f(|s|)$$

for any s.

SPACE

Definition

Let T be a deterministic TM. A TM is called *PSPACE*-computable,

if it is x^n-space-computable for some n.

Definition

Let T be a non-deterministic TM. A TM is called *NPSPACE*-time-computable,

if it is x^n-space-computable for some n.

SPACE

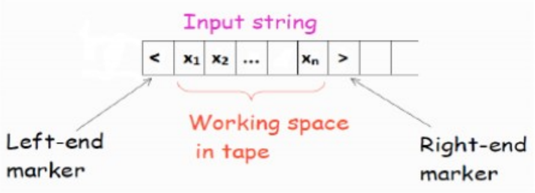
Theorem (Savitch, 1970)

$$PSPACE = NPSPACE$$

Fact

$$P \subseteq NP \subseteq PSPACE \subseteq EXP$$

LBA



Definition

A ND Turing machine that uses only the tape space occupied by the input is called a linear-bounded automaton (LBA).

Grammars, languages and automata

Chomsky hierarchy	Grammars	Languages	Minimal automaton
Type-0	Unrestricted	Recursively enumerable	Turing machine
Type-1	Context-sensitive	Context-sensitive	LBA
Type-2	Context-free	Context-free	NDPDA
Type-3	Regular	Regular	FSA

Thank you for your attention!