VARIANT 1

| Full name: | Group: |
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| | |

| Task: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Total |
|---|---|---|---|---|---|---|---|---|---|---|
| Score: | | | | | | | | | | |

1. (1 point) For each of the following statements mark it as True or False. Justify each answer.
   (a) $\det \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix} = 0$ True / False
   (b) The result of Dot product operation is a vector. True / False
   (c) Inverse matrix ($A^{-1}$) is always exists. True / False
   (d) It is always possible to change one basis to any other basis of the same space. True / False
   (e) Multiplication vector by a scalar operation is always applicable. True / False

2. (2 points)
   (a) Find the determinant of the following matrix: $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & 0 & 2 & 4 \\ 2 & 1 & 3 & 1 \end{bmatrix}$
   (b) Let $A$ be a square matrix. Prove that its left and right inverses are the same matrix.

3. (2 points) Find angles between vectors $\mathbf{a}$ and $\mathbf{b}$, $\mathbf{b}$ and $\mathbf{c}$, $\mathbf{a}$ and $\mathbf{c}$.
   $\mathbf{a} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{c} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$

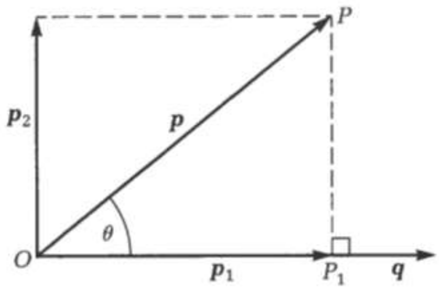
4. (2 points) For which values $x$, vectors $\mathbf{a}$ and $\mathbf{b}$ are basis of some space? Explain your answers.
   $\mathbf{a} = \begin{bmatrix} x \\ 1-x \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} x \\ 2 \end{bmatrix}$

5. (2 points) Show that the result of a cross product $a \times b$ will not change if one adds to one of the vectors some vector $\mathbf{x}$ which is collinear to another vector of the cross product.

6. (2 points) Let $\mathbf{b}_1 = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$, $\mathbf{c}_1 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}$, $\mathbf{c}_2 = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$, and consider the bases for $\mathbb{R}^2$ given by $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$. Find the change of coordinate matrix from $\mathcal{B}$ to $\mathcal{C}$.

7. (2 points) Find all face areas of a parallelepiped, if its edges are:
$\begin{bmatrix} 2 \\ 7 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$

8. (2 points) Decompose the vector $\mathbf{p} = (1, 2, 3)$ into components parallel and perpendicular to the vector $\mathbf{q} = (1, -2, 2)$.




9. (Extra. 3 points) Point $M$ is the intersection of medians of the equilateral triangle $ABC$. The old coordinate system is given by origin $A$ and two basis vectors $\overline{AB}, \overline{AC}$ and the new coordinate system is given by origin $M$ and two basis vectors $\overline{MB}, \overline{MC}$. Find the coordinates of a point in the old coordinate system given its coordinates $x', y'$ in the new one.

End of Test 1