

In the final exam, you are allowed to use a pen, paper, a single A4-sized one-sided hand-written cheat sheet, and a calculator. Any communication with other students during the exam is considered cheating and will result in a zero-point grade.

VARIANT 1

Full name	Group	Signature

Task #:	1	2	3	4	5	6	7	8	9	10	Total	Extra
Max pts.:	2	2	2	2	3	3	3	4	4	5	30	1
Score:												

- (2 point) List at least **three** applications for **each** of the following:
 - matrices;
 - quadratic surfaces.
- (2 points) Describe geometrically the subspaces of \mathbb{R}^3 **spanned** by the following sets of vectors.
 - $\{(1, 0, 0), (2, 0, 0)\}$;
 - $\{(1, 0, 0), (0, 1, 0)\}$;
 - $\{(1, 0, 0), (0, 1, 0), (1, 1, 0)\}$;
 - $\{(1, 0, 0), (0, 1, 0), (1, 1, 1)\}$.
- (2 points) Suppose A, B and C are 3x3 matrices, and $|A| = 2, |B| = 3, |C| = 5$. Find the following quantities:
 - $|AB|$;
 - $|A^{-1}B|$;
 - $|2C|$;
 - $|AB| - |BA|$.
- (2 points) What is the polar form of the equation of the line $x = 1$?
- (3 points) Find the angle between the planes:
 $2x - y + z = 6, x + y + 2z = 3$.
- (3 points) Find a vector that is orthogonal to both $v_1 = (1, 0, 1)$ and $v_2 = (1, 3, 0)$ and which dot product with vector $v_3 = (1, 1, 0)$ equals to 8.
- (3 points) What are the coordinates of the center of the hyperbola:
 $4x^2 - 9y^2 - 40x + 54y - 17 = 0$?
- (4 points) Let the line l_1 to be the intersection of the planes $2x - y - z = 3$ and $x + 2y + z = 2$. Let the line l_2 to be the line with parametric equation $x = 1 + 2t, y = -2 - t, z = -1 - t$. Find the equation of the plane that contains the line l_1 and is parallel to the line l_2 .
- (4 points) Given the bases A and B, find the transition matrices from basis A to a Cartesian coordinate system (with the standard basis $S = \{(1, 0)^T, (0, 1)^T\}$); from Cartesian coordinate system to the basis B and from the basis B to the basis A. Given that
 - $A = \{(1, 1)^T, (-1, 2)^T\}$ and $B = \{(-2, 1)^T, (-2, -1)^T\}$
- (5 points) Find the radius of the circle defined by the two following equations:
 $x^2 + y^2 + z^2 - 8x + 4y + 8z - 45 = 0$ and $x - 2y + 2z = 3$.

Extra: (1 point) How to multiply 2 matrices in Numpy?

End of Final Exam

In the final exam, you are allowed to use a pen, paper, a single A4-sized one-sided hand-written cheat sheet, and a calculator. Any communication with other students during the exam is considered cheating and will result in a zero-point grade.

VARIANT 2

Full name	Group	Signature

Task #:	1	2	3	4	5	6	7	8	9	10	Total	Extra
Max pts.:	2	2	2	2	3	3	3	4	4	5	30	1
Score:												

- (2 point) List at least **three** applications for **each** of the following:
 - systems of linear equations;
 - quadratic surfaces.
- (2 points) Describe geometrically the subspaces of \mathbb{R}^3 **spanned** by the following sets of vectors.
 - $\{(1, 1, 0), (-2, 1, 0)\}$;
 - $\{(-1, 0, 0), (-2, 0, 0)\}$;
 - $\{(1, -1, 0), (0, 2, 0), (-1, -1, 1)\}$;
 - $\{(-1, 0, 0), (0, 10, 0), (1, -1, 0)\}$.
- (2 points) Suppose A , B and C are 3×3 matrices, and $|A| = 4$, $|B| = 6$, $|C| = 7$. Find the following quantities:
 - $|AB|$;
 - $|A^{-1}B|$;
 - $|3C|$;
 - $|AB| + |BA|$.
- (2 points) What is the polar form of the equation of the line $y = 1$?
- (3 points) Find the angle between the planes:
 $4x - y + 2z = 2$, $2x + 2y + z = 4$.
- (3 points) Find a vector that is orthogonal to both $v_1 = (4, 0, 2)$ and $v_2 = (3, 1, 0)$ and which dot product with vector $v_3 = (2, 4, 0)$ equals to 16.
- (3 points) What are the coordinates of the center of the hyperbola:
 $2x^2 - 3y^2 - 12x + 42y - 161 = 0$?
- (4 points) Let the line l_1 to be the intersection of the planes $x + 2y + z = 1$ and $2x - y - z = 2$. Let the line l_2 to be the line with parametric equation $x = 1 + 2t$, $y = -2 - t$, $z = -1 - t$. Find the equation of the plane that contains the line l_1 and is parallel to the line l_2 .
- (4 points) Given the bases A and B , find the transition matrices from basis A to a Cartesian coordinate system (with the standard basis $S = \{(1, 0)^T, (0, 1)^T\}$); from Cartesian coordinate system to the basis B and from the basis B to the basis A . Given that
 - $A = \{(-2, 1)^T, (-2, -1)^T\}$ and $B = \{(1, 1)^T, (-1, 2)^T\}$
- (5 points) Find the radius of the circle defined by the two following equations:
 $x^2 + y^2 + z^2 - 4x + 8y + 4z - 45 = 0$ and $x - 2y + 2z = 3$.

Extra: (1 point) How to multiply 2 matrices in Numpy?

End of Final Exam

In the final exam, you are allowed to use a pen, paper, a single A4-sized one-sided hand-written cheat sheet, and a calculator. Any communication with other students during the exam is considered cheating and will result in a zero-point grade.

VARIANT 3

Full name	Group	Signature

Task #:	1	2	3	4	5	6	7	8	9	10	Total	Extra
Max pts.:	2	2	2	2	3	3	3	4	4	5	30	1
Score:												

- (2 point) List at least **three** applications for **each** of the following:
 - vectors and vector spaces;
 - conic sections.
- (2 points) Describe geometrically the subspaces of \mathbb{R}^3 **spanned** by the following sets of vectors.
 - $\{(0, 0, 1), (0, -1, 0)\}$;
 - $\{(0, 0, 2), (0, 0, -1)\}$;
 - $\{(0, 0, 1), (0, -10, 0), (2, 4, 0)\}$;
 - $\{(2, 0, 0), (0, 4, 0), (8, 8, 8)\}$.
- (2 points) Suppose A, B and C are 3x3 matrices, and $|A| = 0, |B| = 5, |C| = 2$. Find the following quantities:
 - $|AB|$;
 - $|A^{-1}B|$;
 - $|4C|$;
 - $|AB| - |(AB)^T|$.
- (2 points) What is the polar form of the equation of the line $x = y$?
- (3 points) Find the angle between the planes:
 $x - 2y + 2z = 3, 2x + 2y + z = 6$.
- (3 points) Find a vector that is orthogonal to both $v_1 = (2, 0, 2)$ and $v_2 = (3, 1, 0)$ and which dot product with vector $v_3 = (3, 2, 0)$ equals to 36.
- (3 points) What are the coordinates of the center of the hyperbola:
 $8x^2 - 2y^2 - 32x + 20y - 50 = 0$
- (4 points) Let the line l_1 to be the intersection of the planes $2x + y + z = 1$ and $x - 2y - z = 2$. Let the line l_2 to be the line with parametric equation $x = 1 + 2t, y = -2 - t, z = -1 - t$. Find the equation of the plane that contains the line l_1 and is parallel to the line l_2 .
- (4 points) Given the bases A and B, find the transition matrices from basis A to a Cartesian coordinate system (with the standard basis $S = \{(1, 0)^T, (0, 1)^T\}$); from Cartesian coordinate system to the basis B and from the basis B to the basis A. Given that
 - $A = \{(-1, 3)^T, (1, -1)^T\}$ and $B = \{(1, 2)^T, (1, -3)^T\}$
- (5 points) Find the radius of the circle defined by the two following equations:
 $x^2 + y^2 + z^2 + 4x - 8y - 4z - 45 = 0$ and $x - 2y + 2z = 3$.

Extra: (1 point) How to multiply 2 matrices in Numpy?

End of Final Exam