

Computer Architecture
Tutorial 3

Combinational Logic Circuits

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Terminology

Terminology

A Boolean function

(introduced by George Boole)

A function taking one or multiple binary inputs and producing exactly one binary output

Sample boolean variables:

$$A, B, C, D, F \in \{0,1\}$$

Expressions in one notation:

$$F(A, B) = A * B$$

$$F(A, B, C) = A * B + C$$

$$F(A, B, C, D) = A * (B + C) + D$$

Another notation:

$$F(A, B) = \mathbf{AND}(A, B)$$

$$F(A, B, C) = \mathbf{OR}(\mathbf{AND}(A, B), C)$$

$$F(A, B, C, D) = \mathbf{OR}(\mathbf{AND}(A, \mathbf{OR}(B, C)), D)$$

Other notations are used as well

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Truth table	A table specifying the output of a Boolean function for every combination of input values

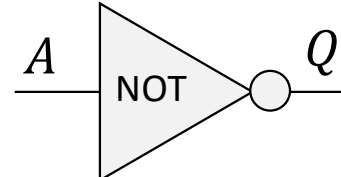
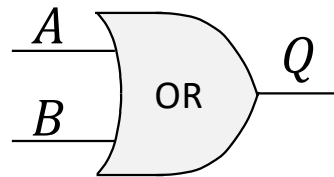
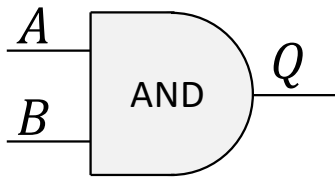
Truth table for function $F = A * B$:

<i>A</i>	<i>B</i>	<i>F = A * B</i>
0	0	0
1	0	0
0	1	0
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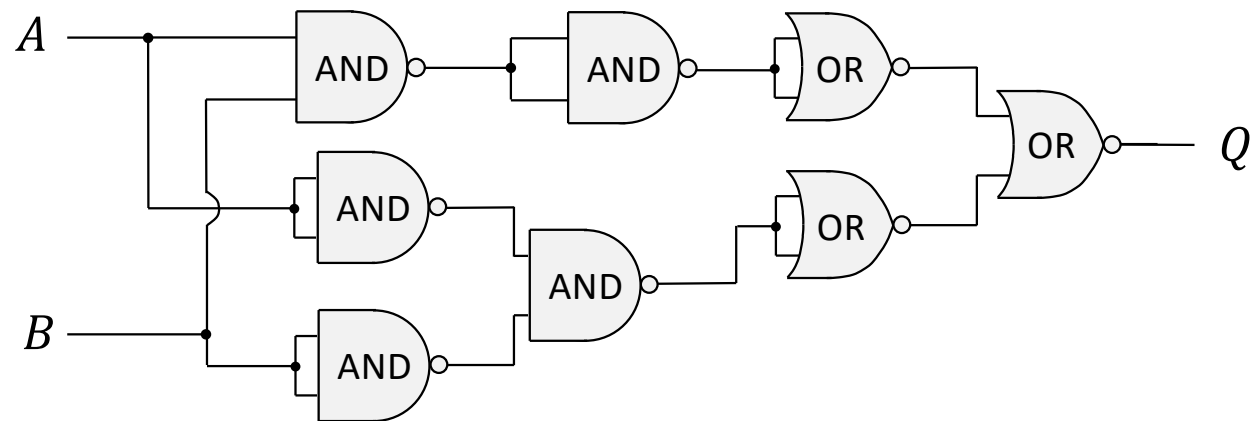
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Sample logic circuit:

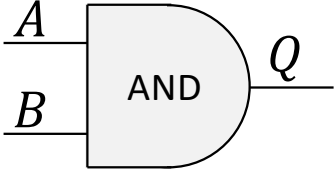


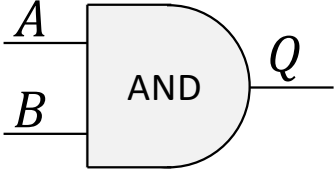
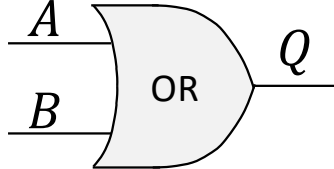
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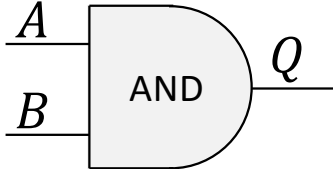
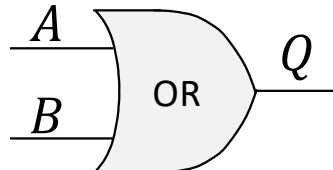
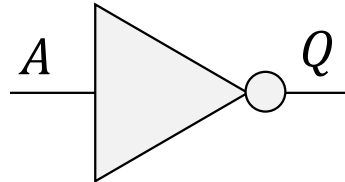
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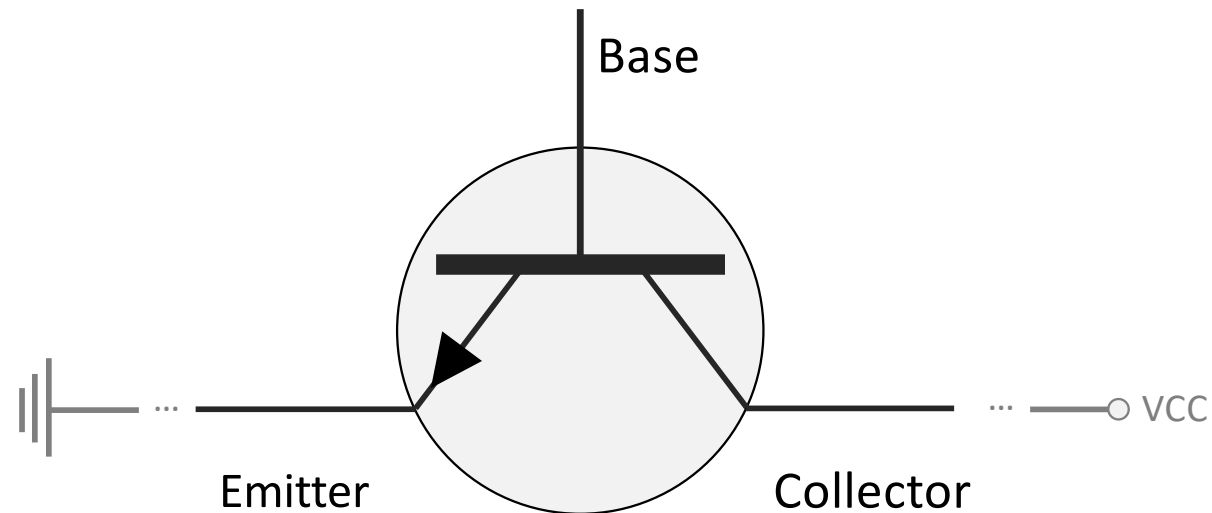
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An Integrated Circuit (informally: chip, or microchip)	<p>The antonym for a “discrete electrical circuit”, that is a set of discrete electrical devices, which can be assembled and reassembled again;</p> <p>Key advantages over discrete circuits:</p> <ul style="list-style-type: none">- Compact size;- Performance (e.g. the propagation delay – to be discussed next);- Lower manufacturing price

Logic Gate	Symbolic Representation	Truth Table															
AND		<table><tr><th><i>A</i></th><th><i>B</i></th><th><i>Q=AB</i></th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table>	<i>A</i>	<i>B</i>	<i>Q=AB</i>	0	0	0	1	0	0	0	1	0	1	1	1
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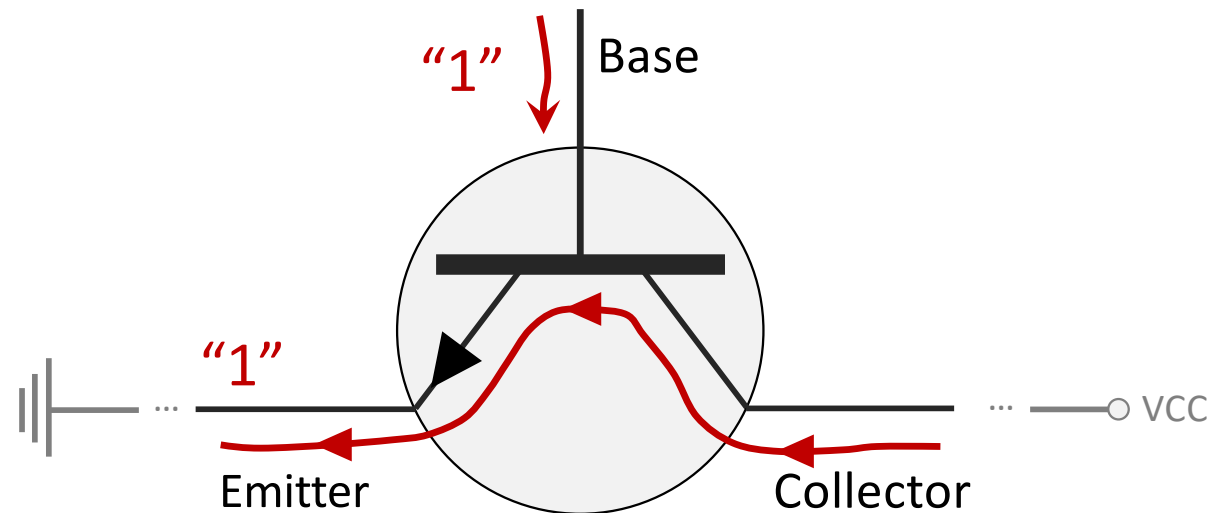
Transistor: building block for logic gates



Transistor is a fast switch:

If there is voltage at the Base terminal (Base = "1"), then current flows between Emitter and Collector;
Otherwise (Base = "0"), there is no current

Transistor: building block for logic gates

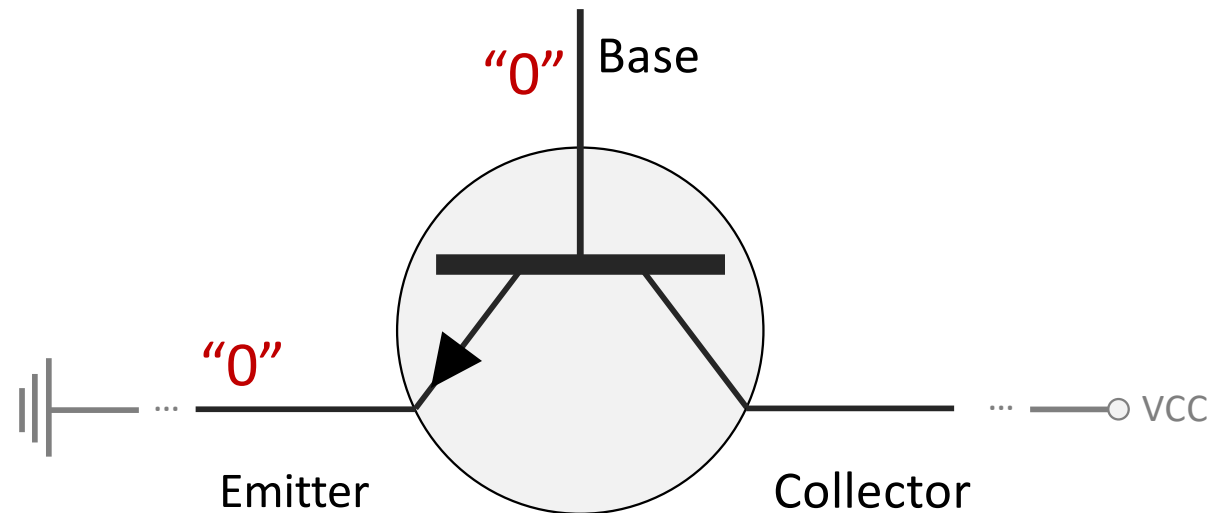


Note: Current direction depends on the transistor type (e.g. NPN or PNP)

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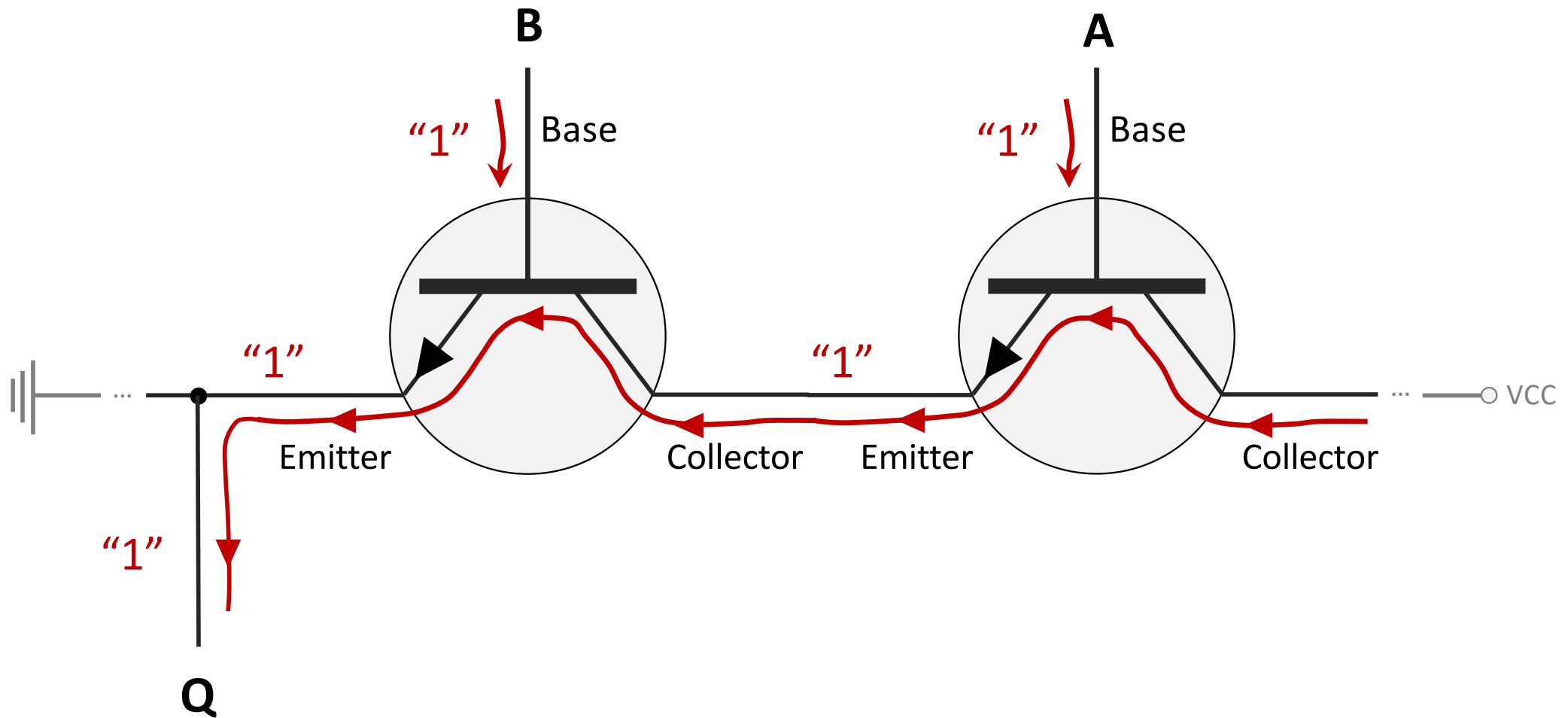
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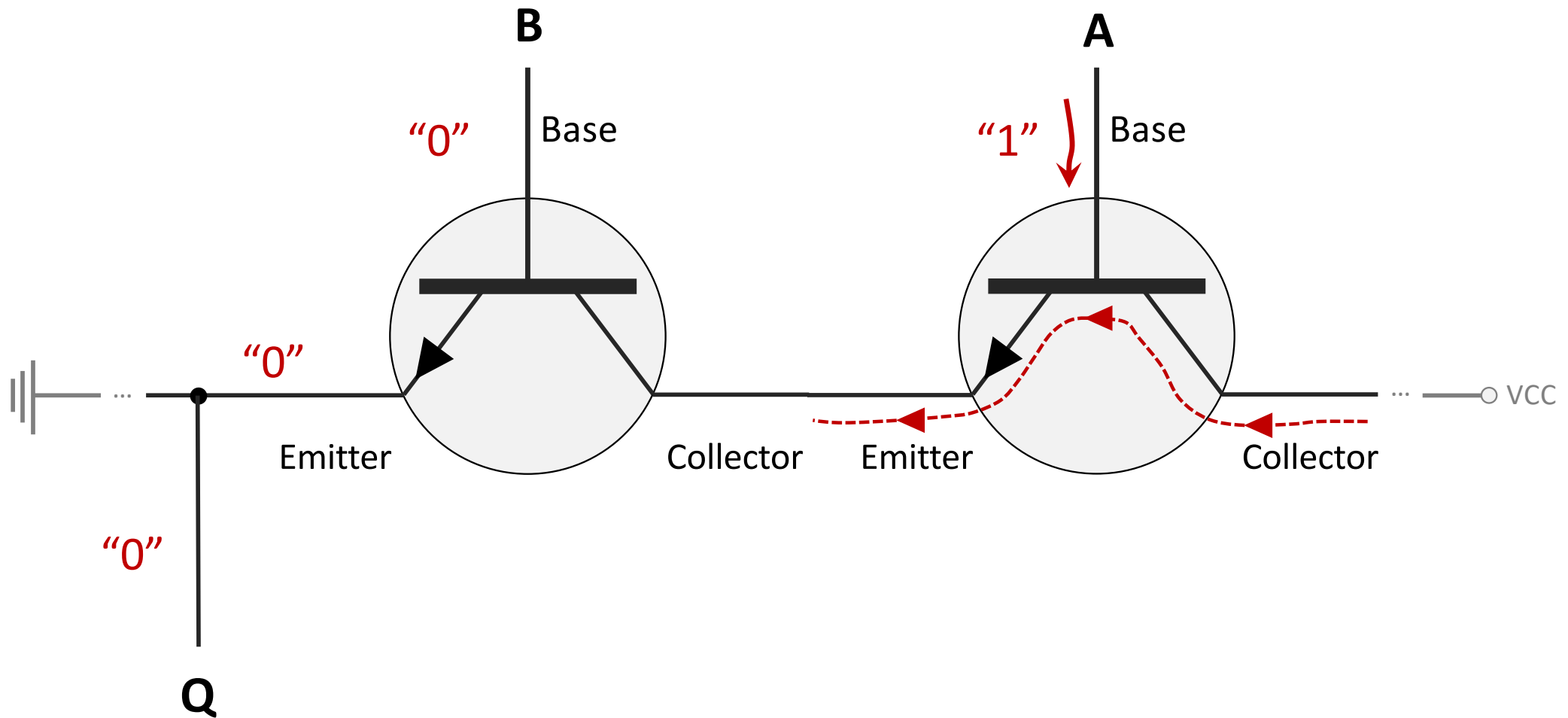
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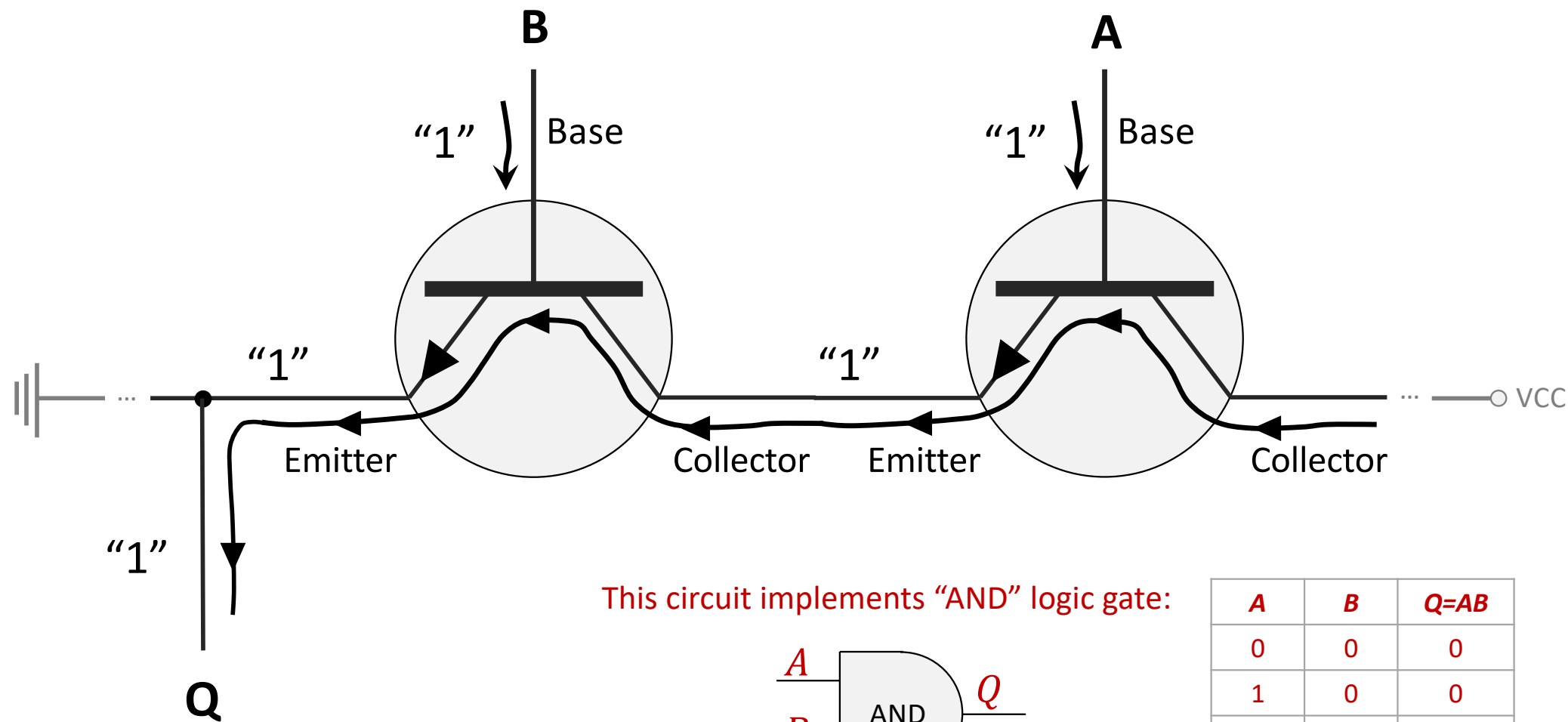
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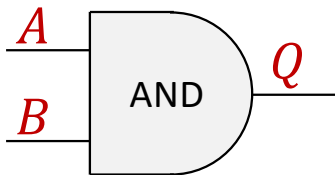
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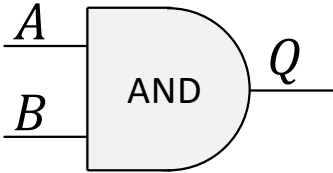
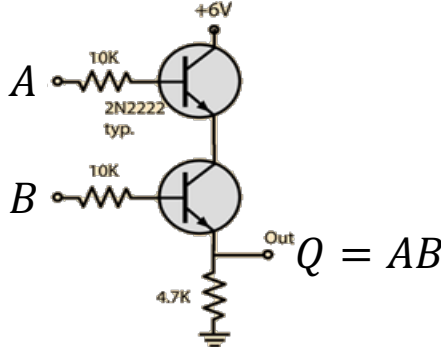
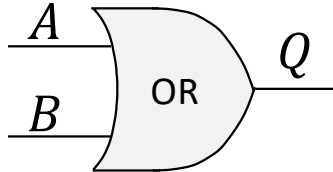
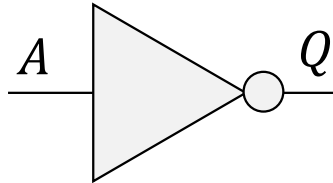
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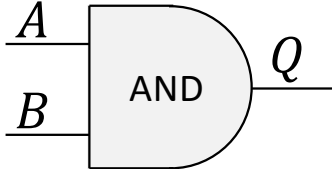
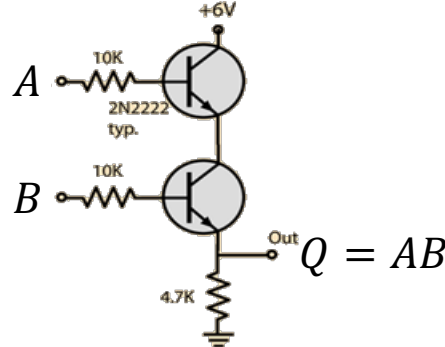
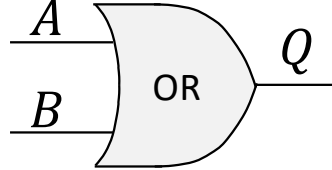
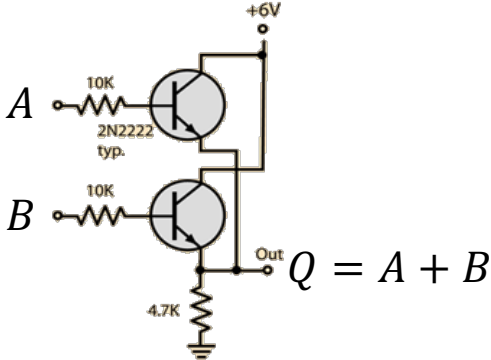
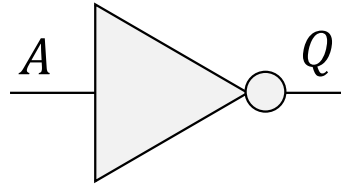
This circuit implements "AND" logic gate:



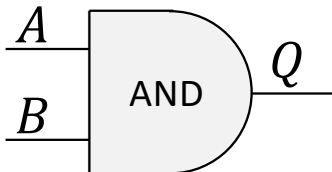
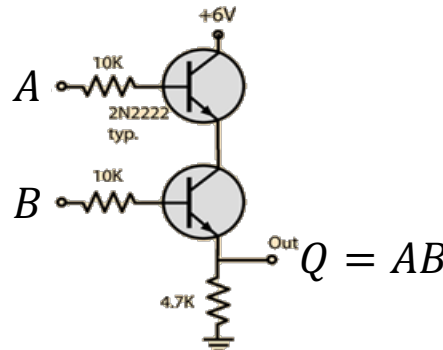
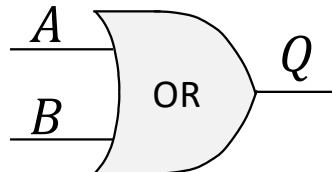
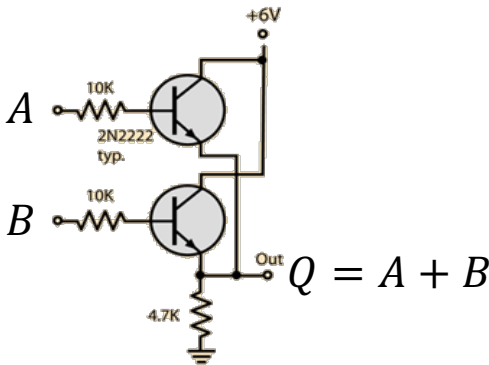
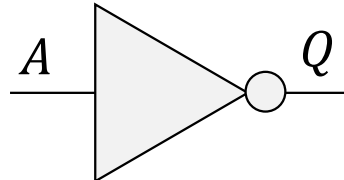
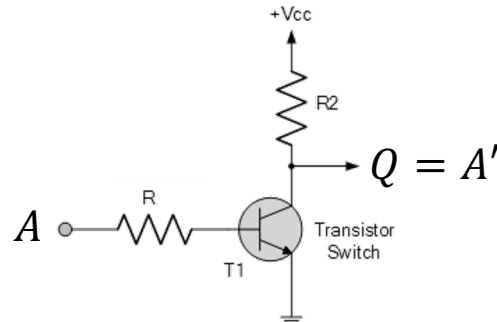
A	B	Q=AB
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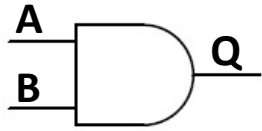
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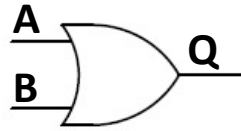
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Basic Logic Gates



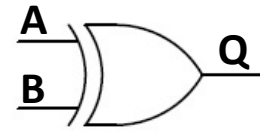
AND

A	B	Output
0	0	0
0	1	0
1	0	0
1	1	1



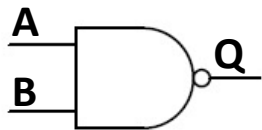
OR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1



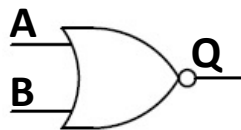
XOR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0



NAND

A	B	Output
0	0	1
0	1	1
1	0	1
1	1	0



NOR

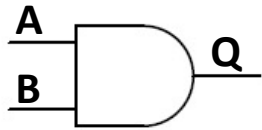
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1	1	0



XNOR

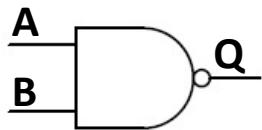
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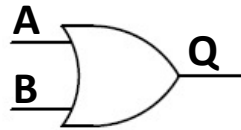
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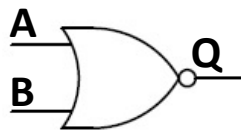
NAND

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1	0	1
1	1	0



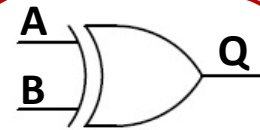
OR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	1



NOR

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	0



XOR

A	B	Output
0	0	0
0	1	1
1	0	1
1	1	0



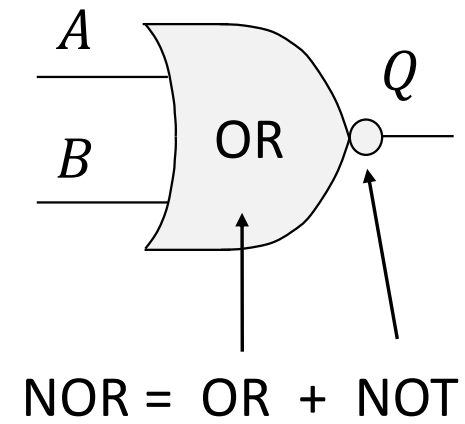
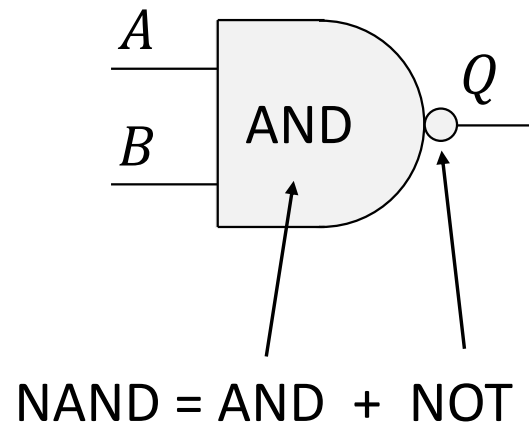
XNOR

A	B	Output
0	0	1
0	1	0
1	0	0
1	1	1

XOR = eXceptional OR:

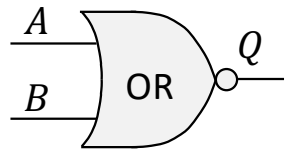
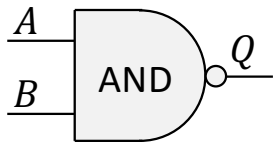
unlike OR, if both inputs are "1"s,
the output is "0"

NAND and NOR Logic Gates



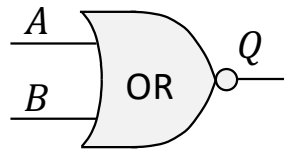
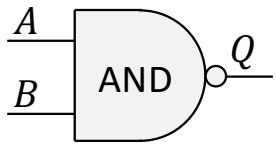
Universal Logic Gates: NAND and NOR

We can implement any other gate, by using one of them

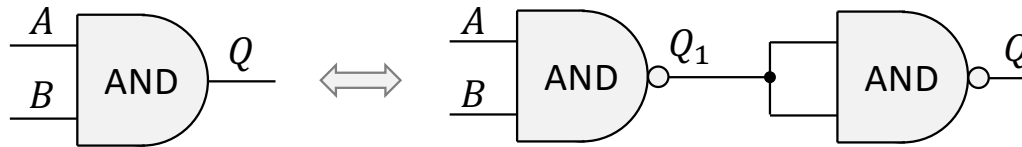


Universal Logic Gates: NAND and NOR

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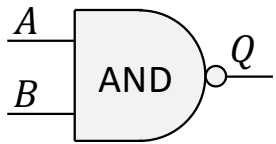


Example: AND gate through NAND

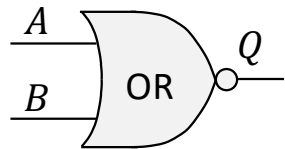
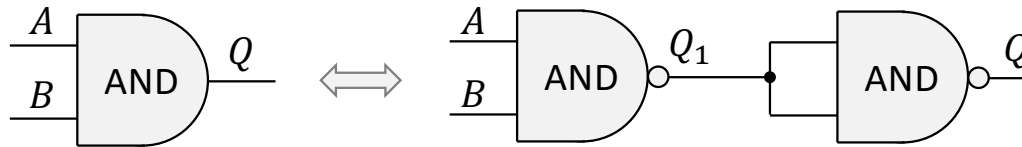


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Example: AND gate through NAND



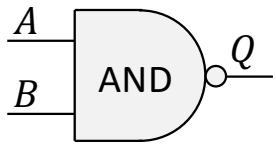
To proof circuits equivalence, we compare truth tables:

A	B	$Q=AB$
0	0	0
1	0	0
0	1	0
1	1	1

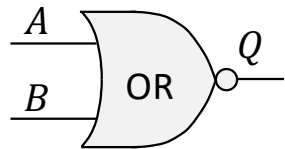
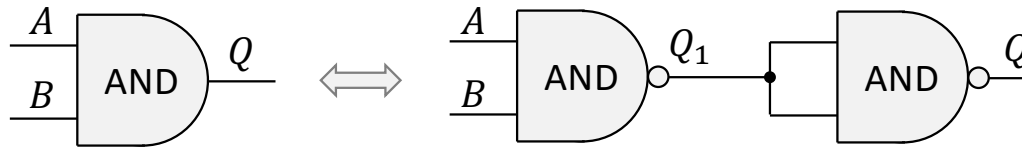
A	B	Q_1	Q_1	Q
0	0	1	1	0
1	0	1	1	0
0	1	1	1	0
1	1	0	0	1

Universal Logic Gates: NAND and NOR

We can implement any other gate, by using one of them



Example: AND gate through NAND



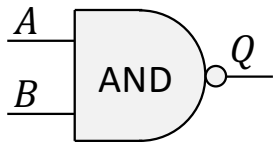
To proof circuits equivalence, we compare truth tables:

A	B	$Q=AB$
0	0	0
1	0	0
0	1	0
1	1	1

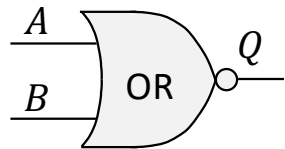
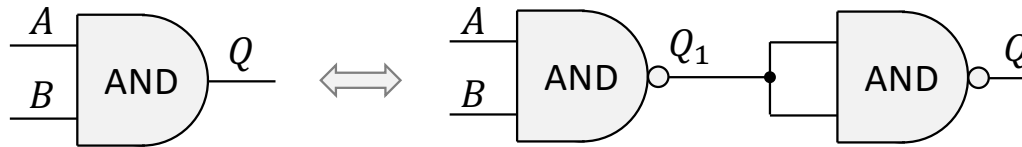
A	B	Q_1	Q_1	Q
0	0	1	1	0
1	0	1	1	0
0	1	1	1	0
1	1	0	0	1

Universal Logic Gates: NAND and NOR

We can implement any other gate, by using one of them



Example: AND gate through NAND



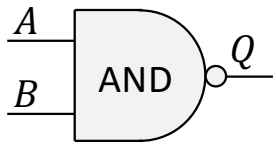
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A	B	$Q=AB$
0	0	0
1	0	0
0	1	0
1	1	1

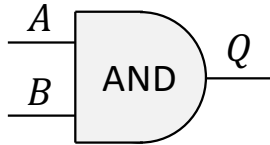
A	B	Q_1	Q_1	Q
0	0	1	1	0
1	0	1	1	0
0	1	1	1	0
1	1	0	0	1

Universal Logic Gates: NAND and NOR

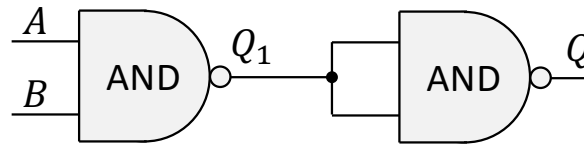
We can implement any other gate, by using one of them



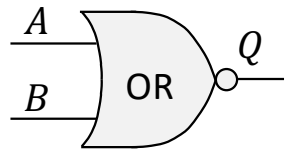
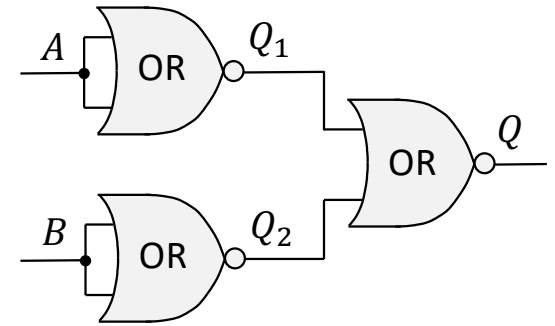
Original gate:



NAND impl.:



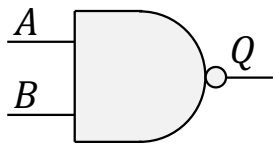
NOR impl.:



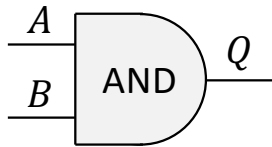
Universal Logic Gates: NAND and NOR

We can implement any other gate, by using one of them

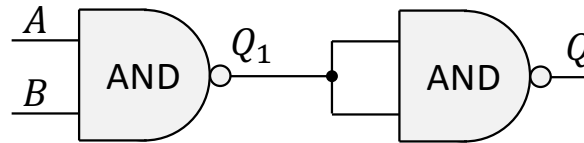
NAND



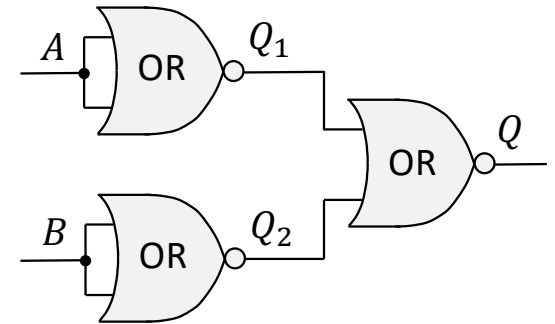
Original gate:



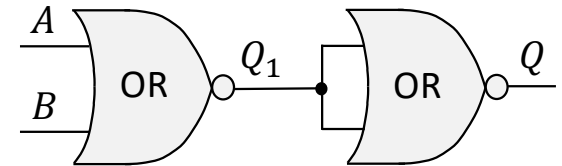
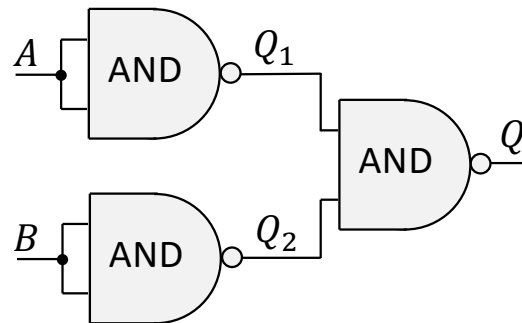
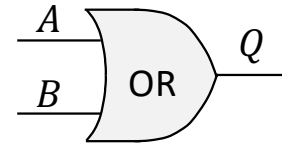
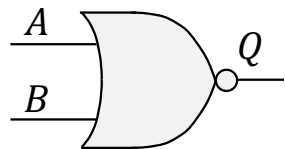
NAND impl.:



NOR impl.:



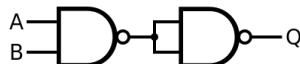
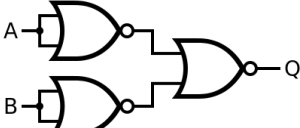

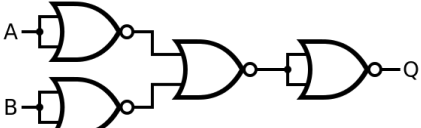
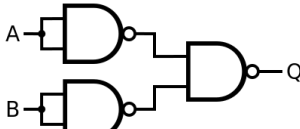
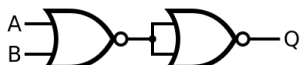
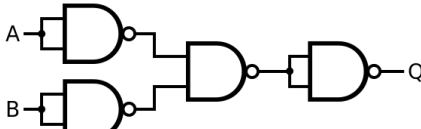

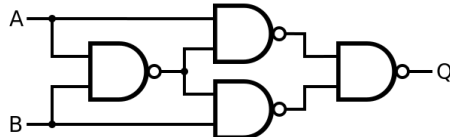
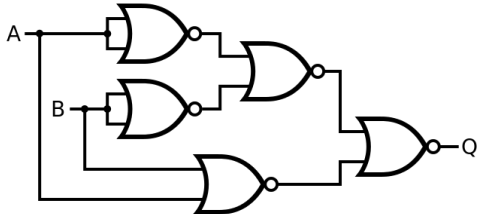
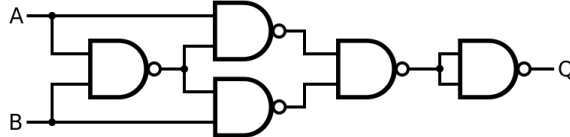
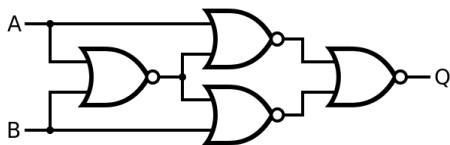


NOR



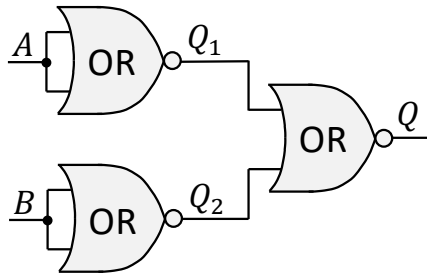
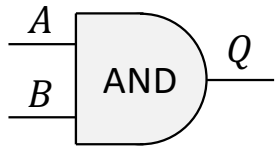
Universal Logic Gates: NAND and NOR

We can implement any other gate, by using one of them

type	NAND construction	NOR construction
NOT		
AND		
NAND		
OR		
NOR		
XOR		
XNOR		

How to Prove the Equivalence of Logic Circuits?

Way 1: Truth Tables

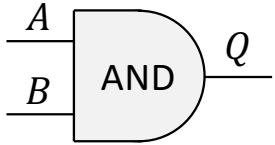


A	B	$Q=AB$
0	0	0
1	0	0
0	1	0
1	1	1

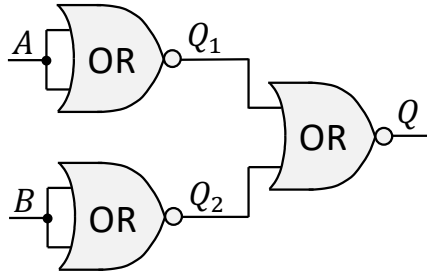
A	B	Q_1	Q_2	Q
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

How to Prove the Equivalence of Logic Circuits?

Way 1: Truth Tables



A	B	Q=AB
0	0	0
1	0	0
0	1	0
1	1	1

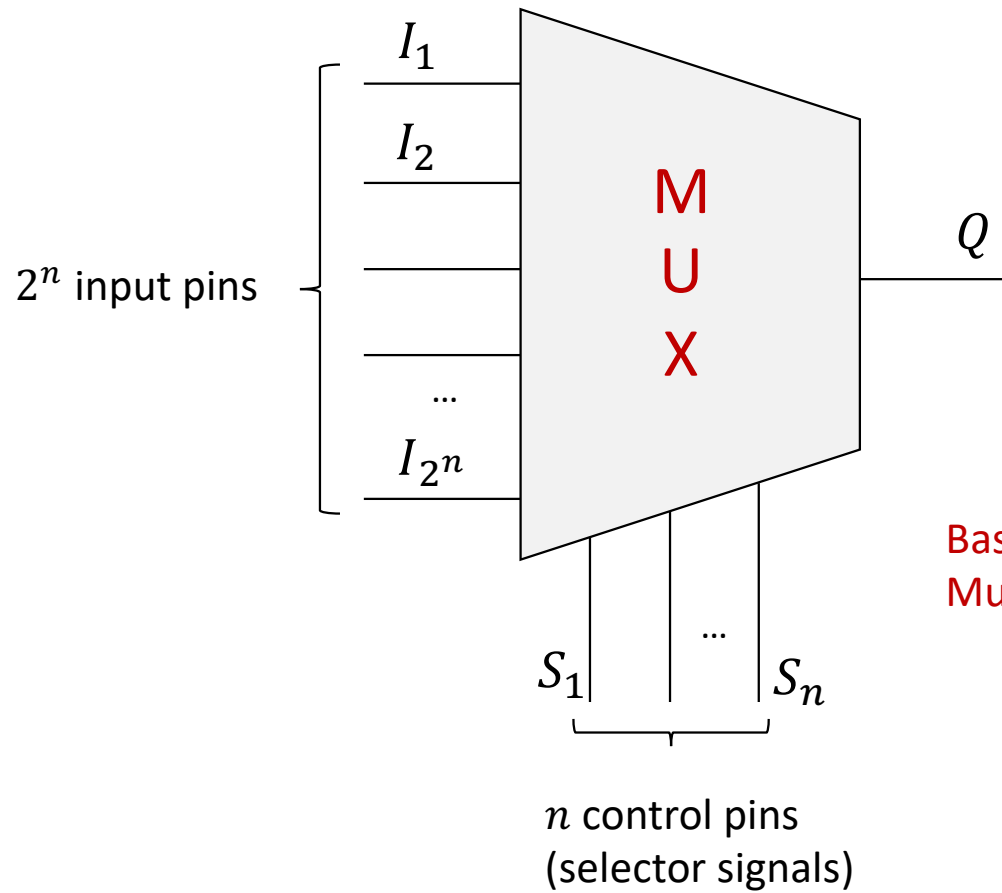


A	B	Q₁	Q₂	Q
0	0	1	1	0
1	0	0	1	0
0	1	1	0	0
1	1	0	0	1

Way 2: Boolean Algebra Laws

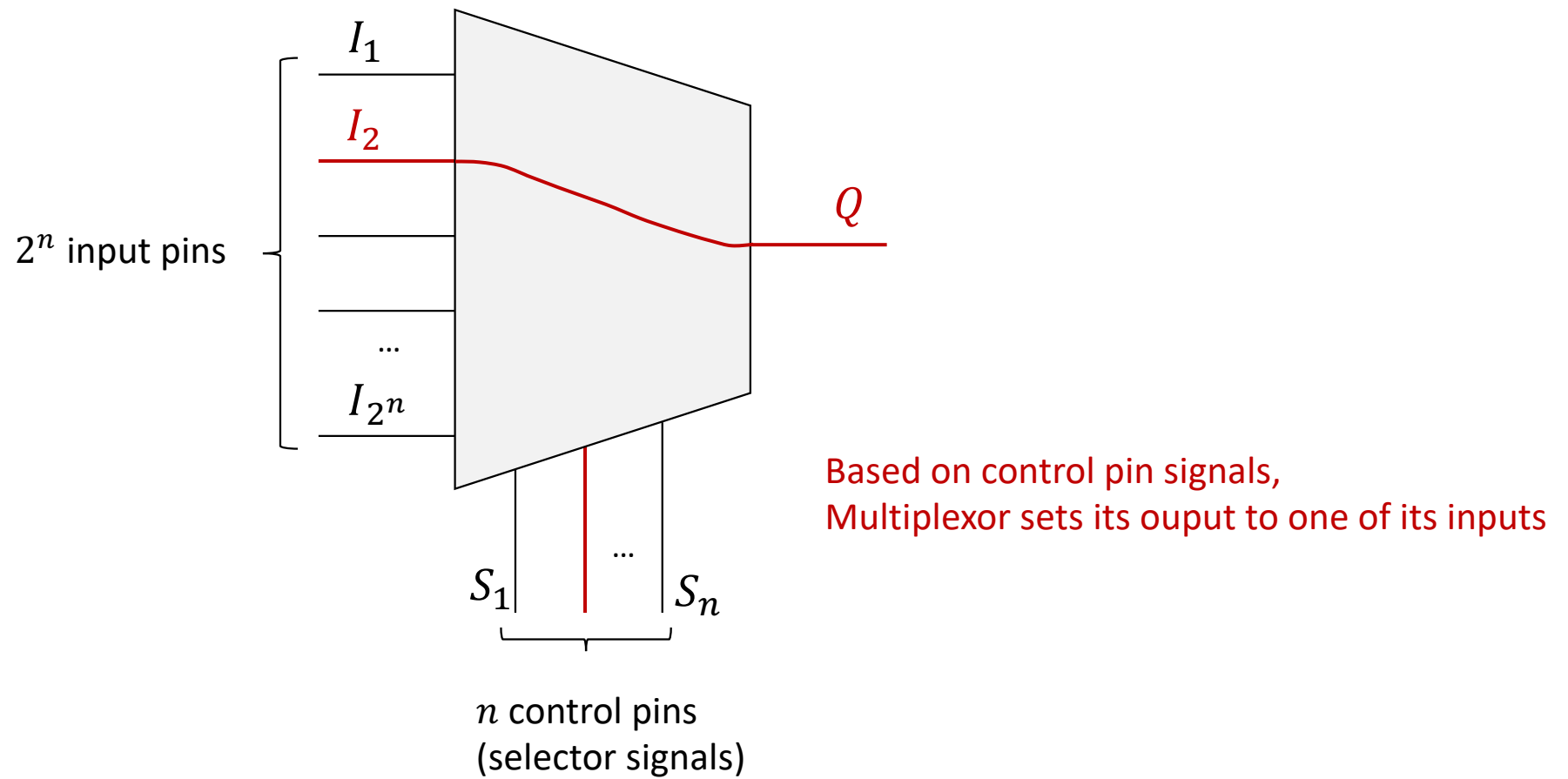
Name	AND Form	OR Form
Identity Law	$1A = A$	$0 + A = A$
Null Law	$0A = 0$	$1 + A = 1$
Idempotent Law	$AA = A$	$A + A = A$
Inverse Law	$AA' = 0$	$A + A' = 1$
Commutative Law	$AB = BA$	$A + B = B + A$
Associative Law	$(AB)C = A(BC)$	$(A + B) + C = A + (B + C)$
Distributive Law	$A+BC = (A + B) (A + C)$	$A(B + C) = AB + AC$
Absorption Law	$A(A + B) = A$	$A + AB = A$
De Morgan's Law	$\overline{AB} = \overline{A} + \overline{B}$	$\overline{A + B} = \overline{A} \overline{B}$

Multiplexer (or Selecting circuit) Example of a Combinational Logic Circuit

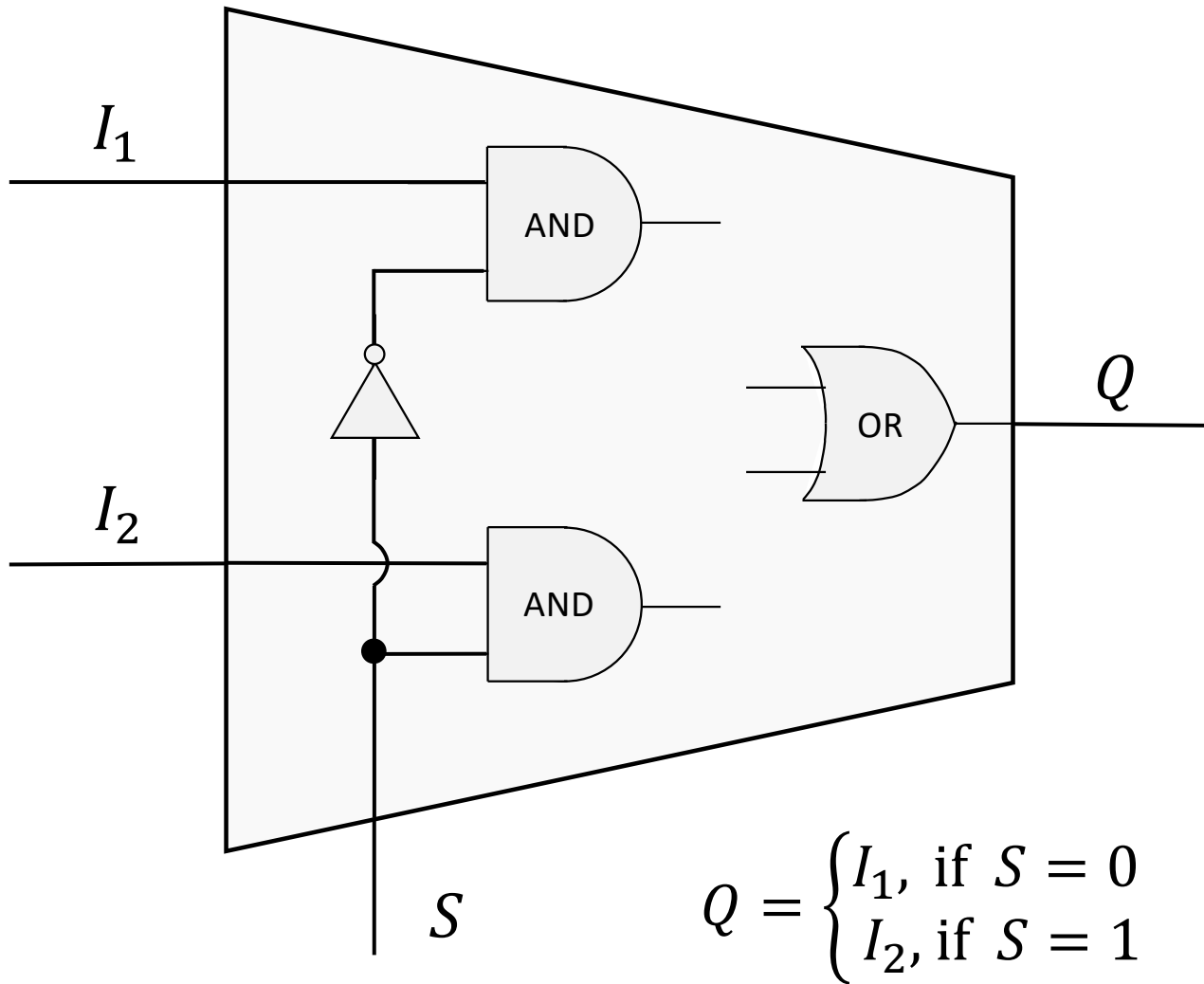


Based on control pin signals,
Multiplexor sets its output to one of its inputs

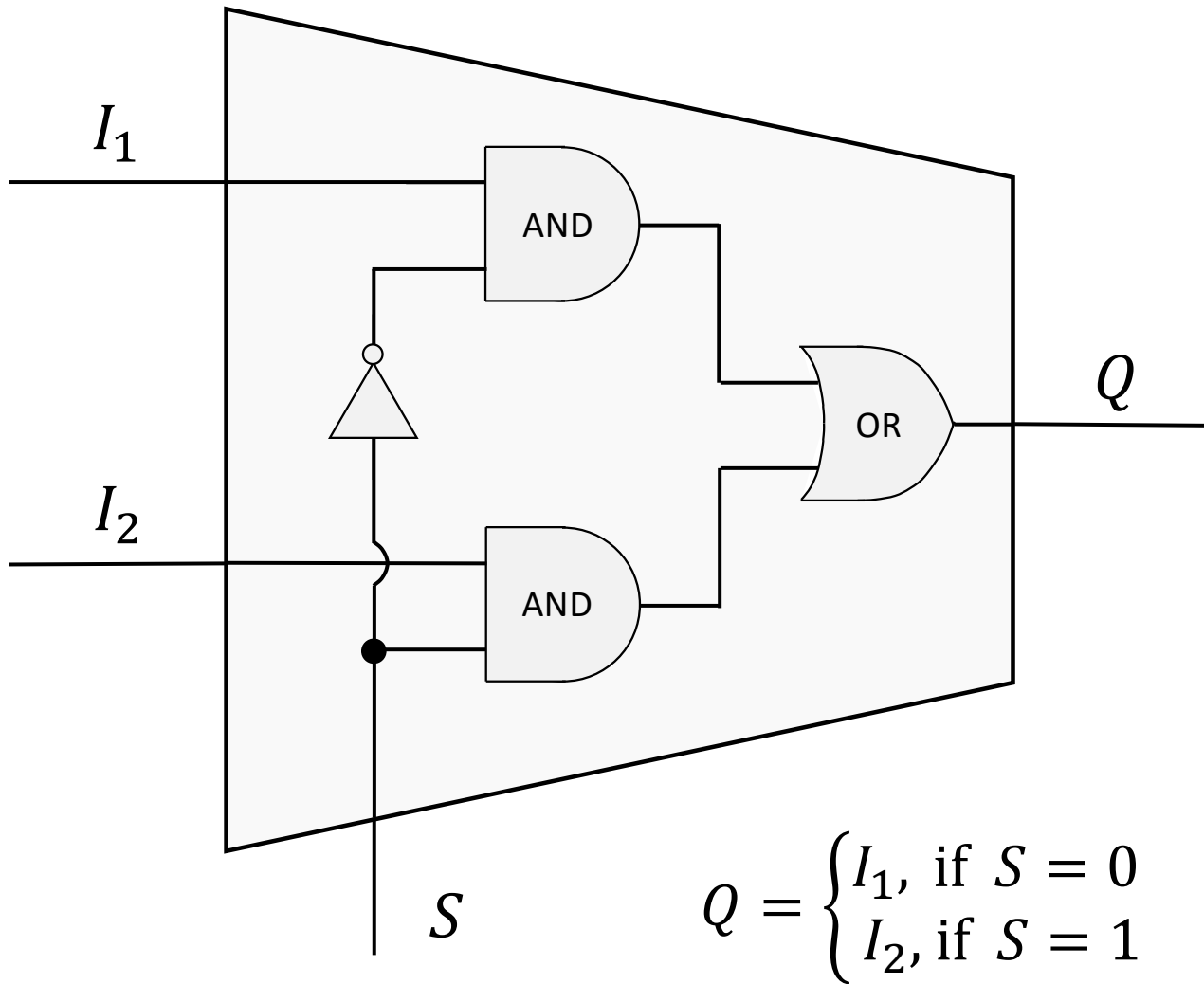
Multiplexor (or Selector): an Example of a Combinational Logic Circuit



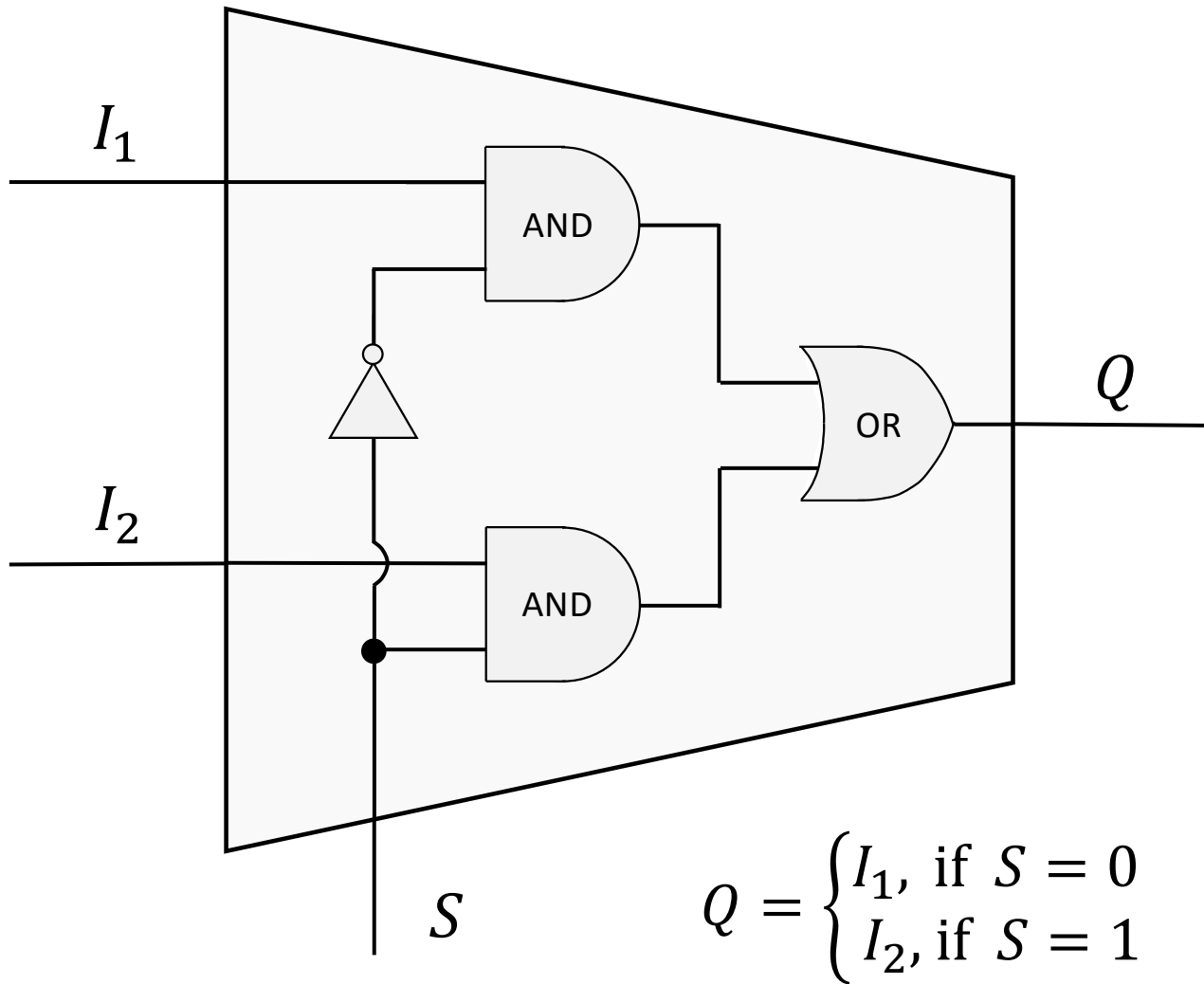
Sample Implementation of a 2-to-1 Multiplexor



Sample Implementation of a 2-to-1 Multiplexor



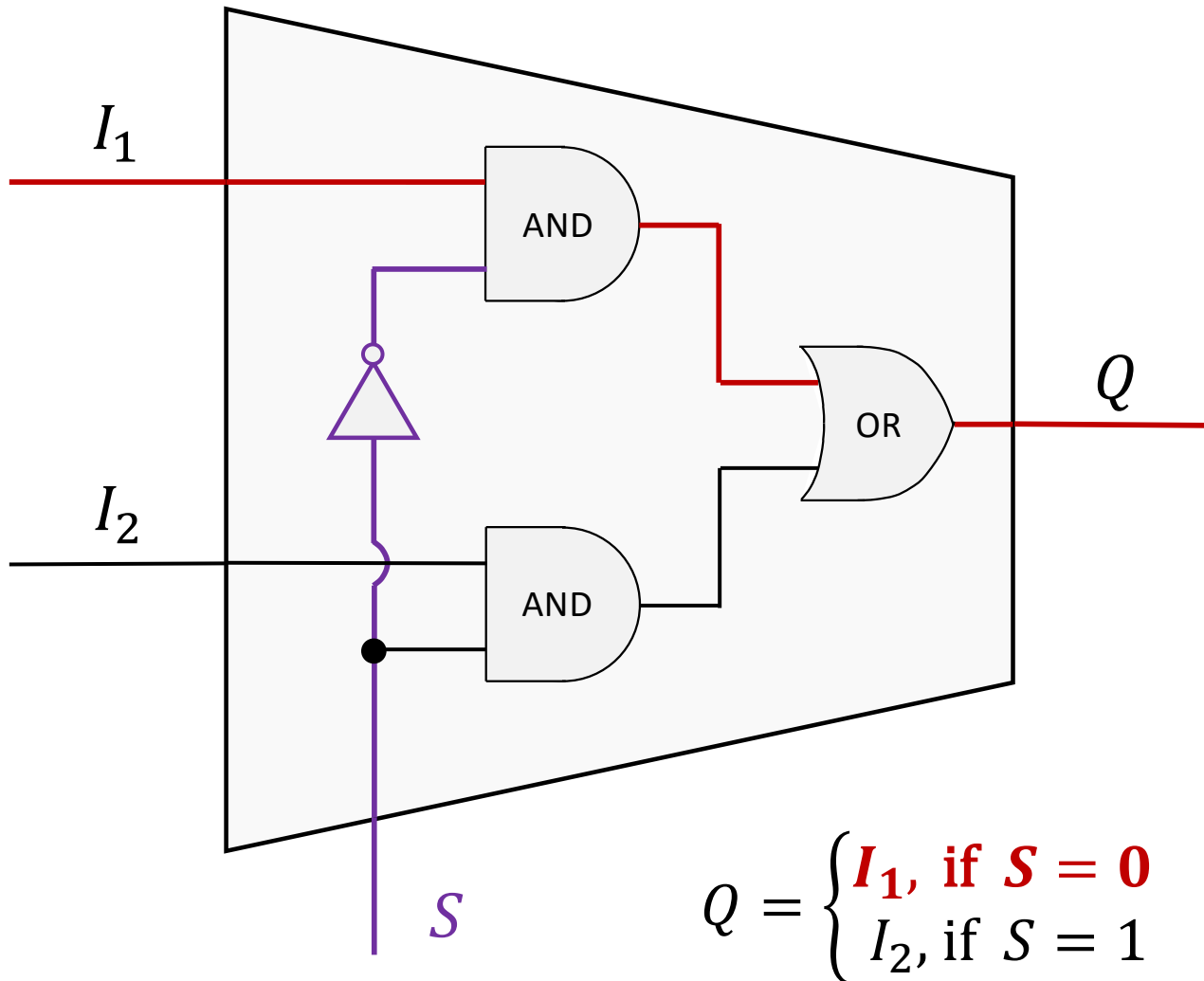
Sample Implementation of a 2-to-1 Multiplexor



Verification of correctness:

I_1	I_2	S	Q
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	0
1	0	1	0
0	1	1	1
1	1	1	1

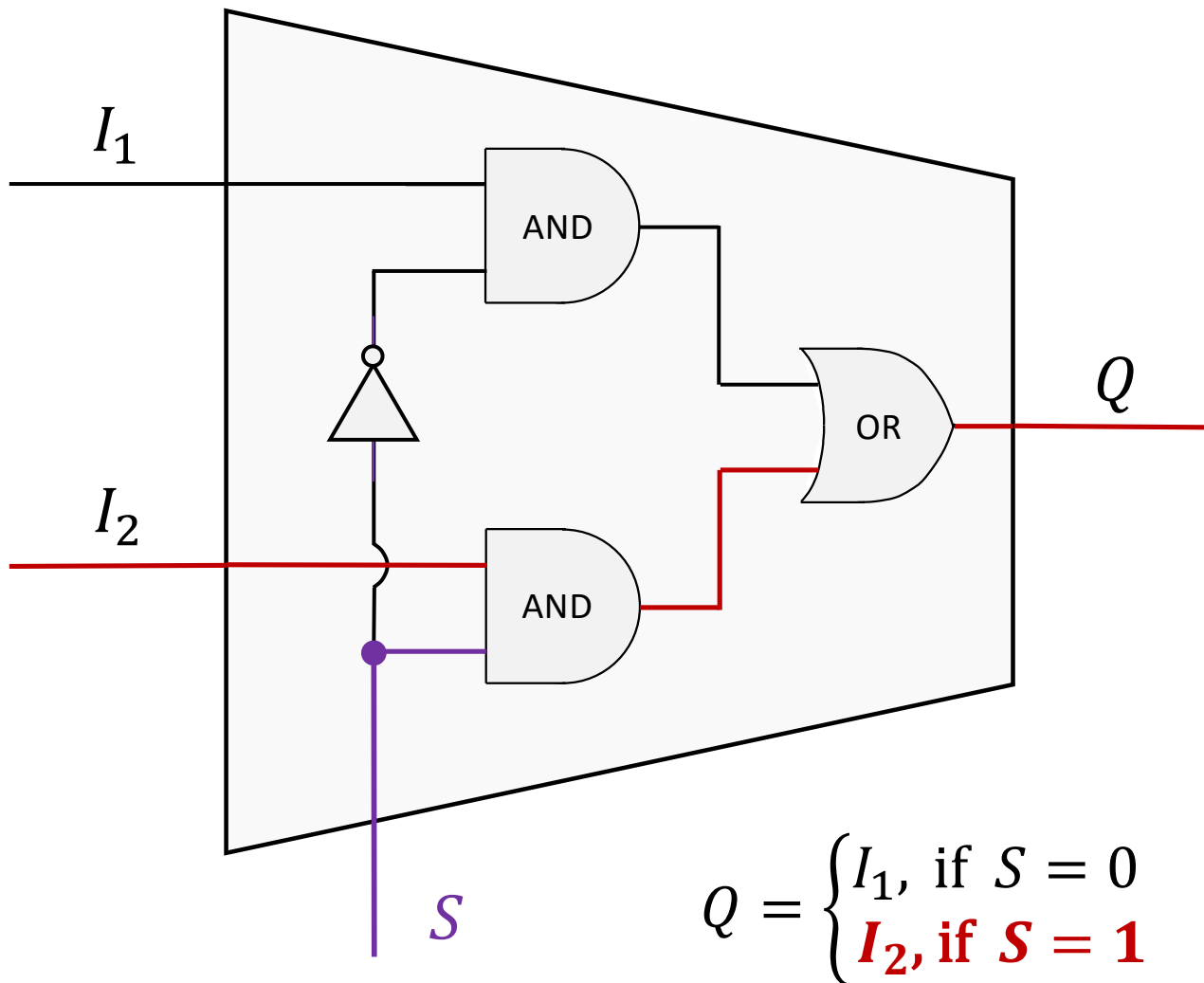
Sample Implementation of a 2-to-1 Multiplexor



Verification of correctness:

I_1	I_2	S	Q
0	0	0	0
1	0	0	1
0	1	0	0
1	1	0	1
0	0	1	0
1	0	1	0
0	1	1	1
1	1	1	1

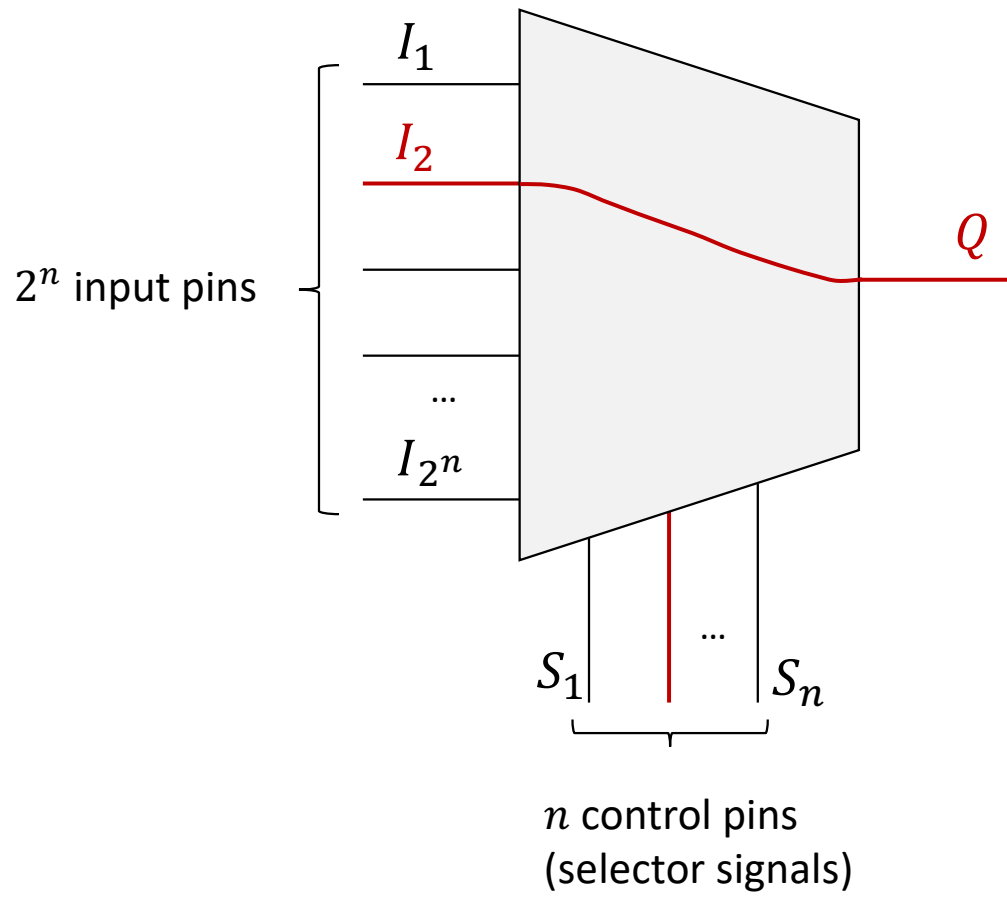
Sample Implementation of a 2-to-1 Multiplexor



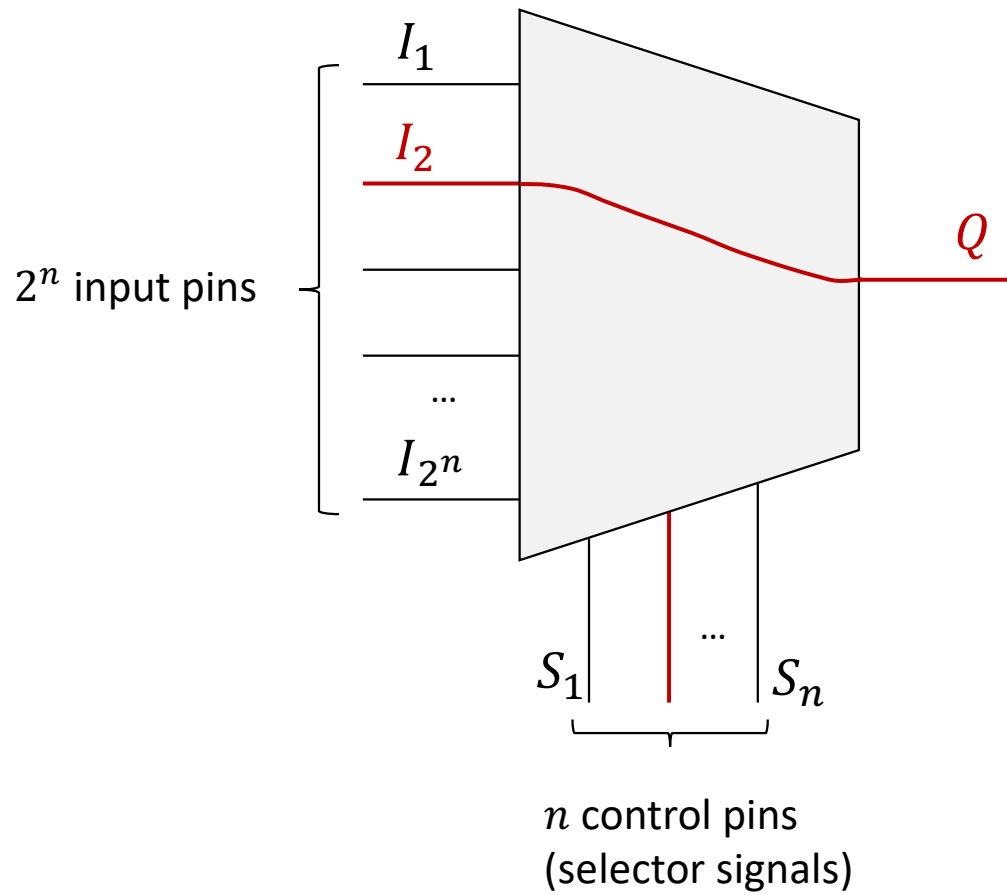
Verification of correctness:

I_1	I_2	S	Q
0	0	0	0
1	0	0	1
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0	0	1	0
1	0	1	0
0	1	1	1
1	1	1	1

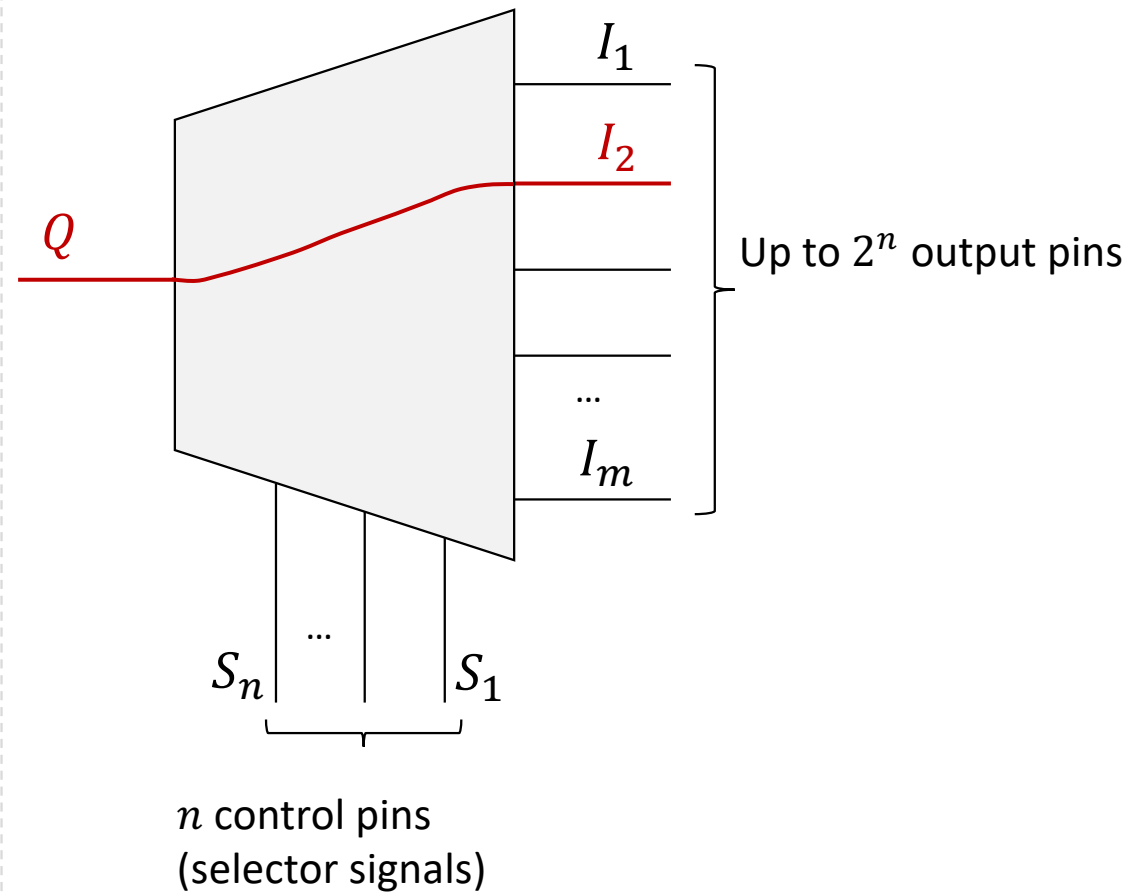
Multiplexer



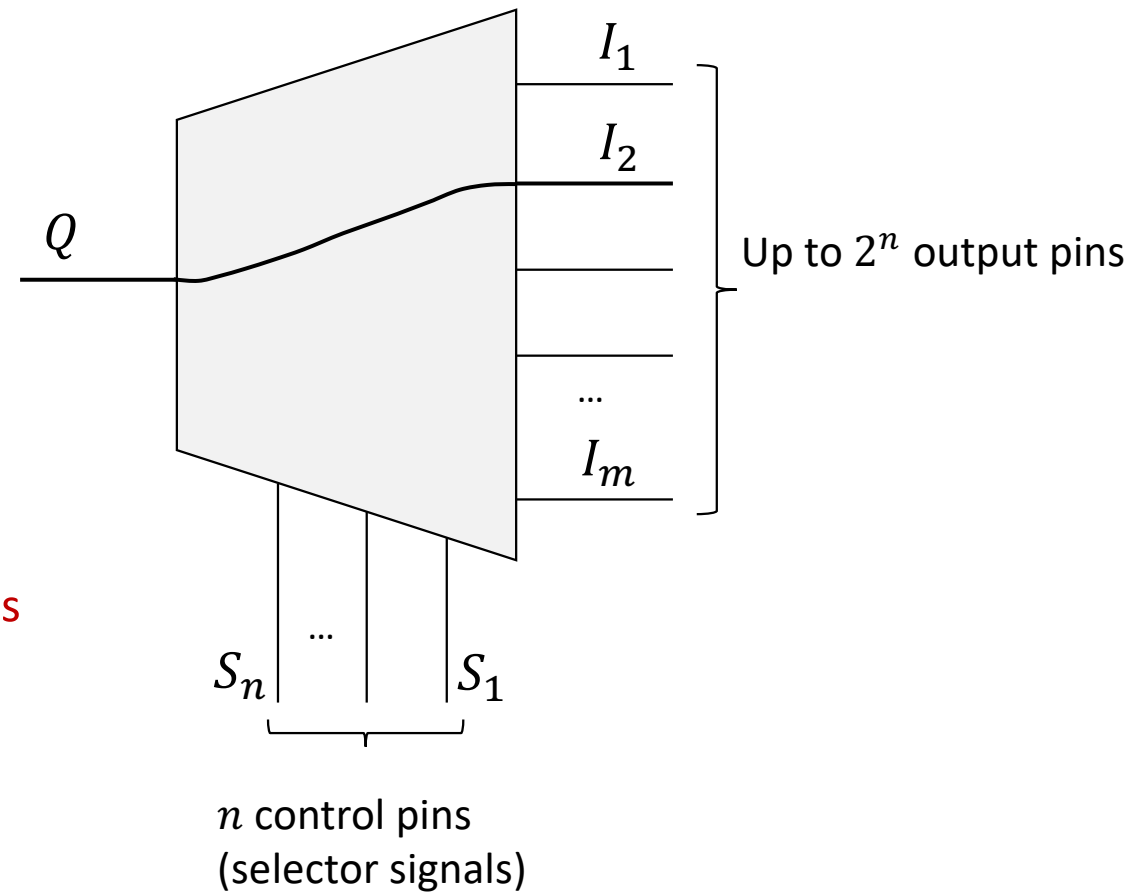
Multiplexer



Demultiplexer

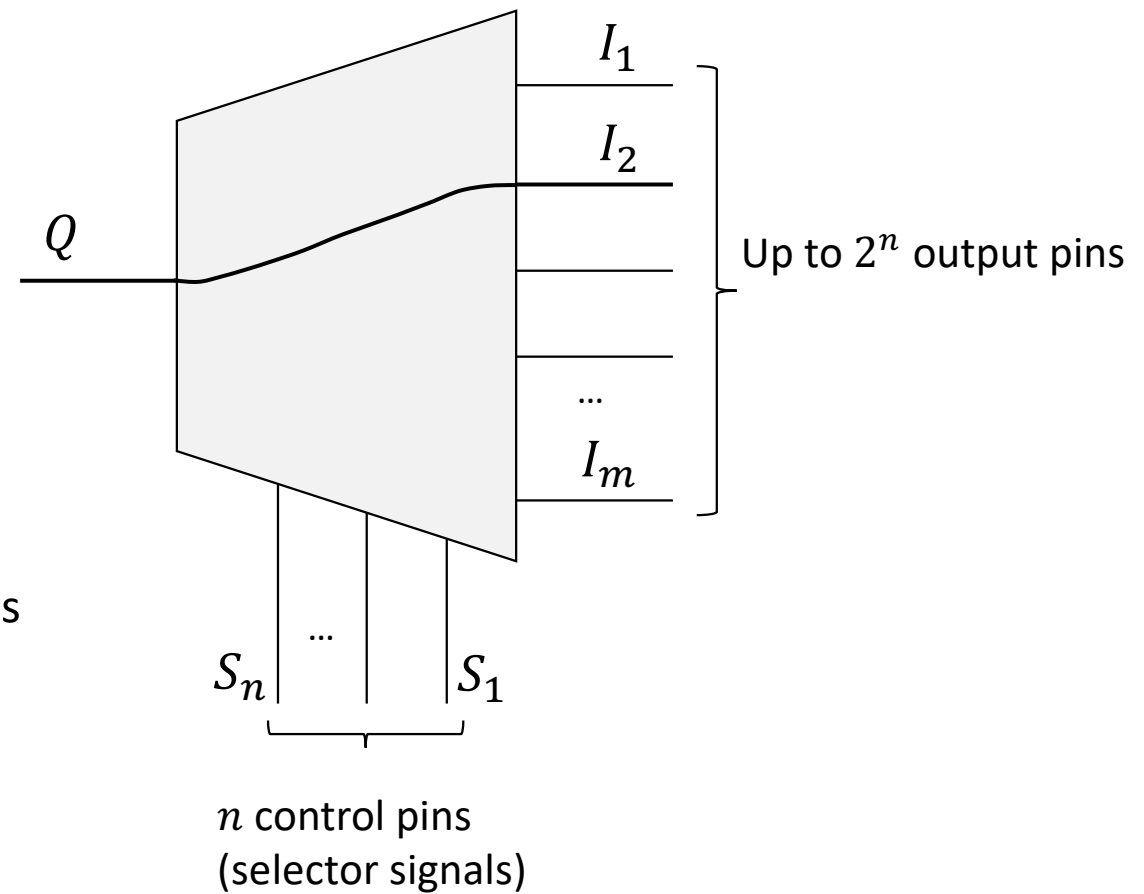


Demultiplexer



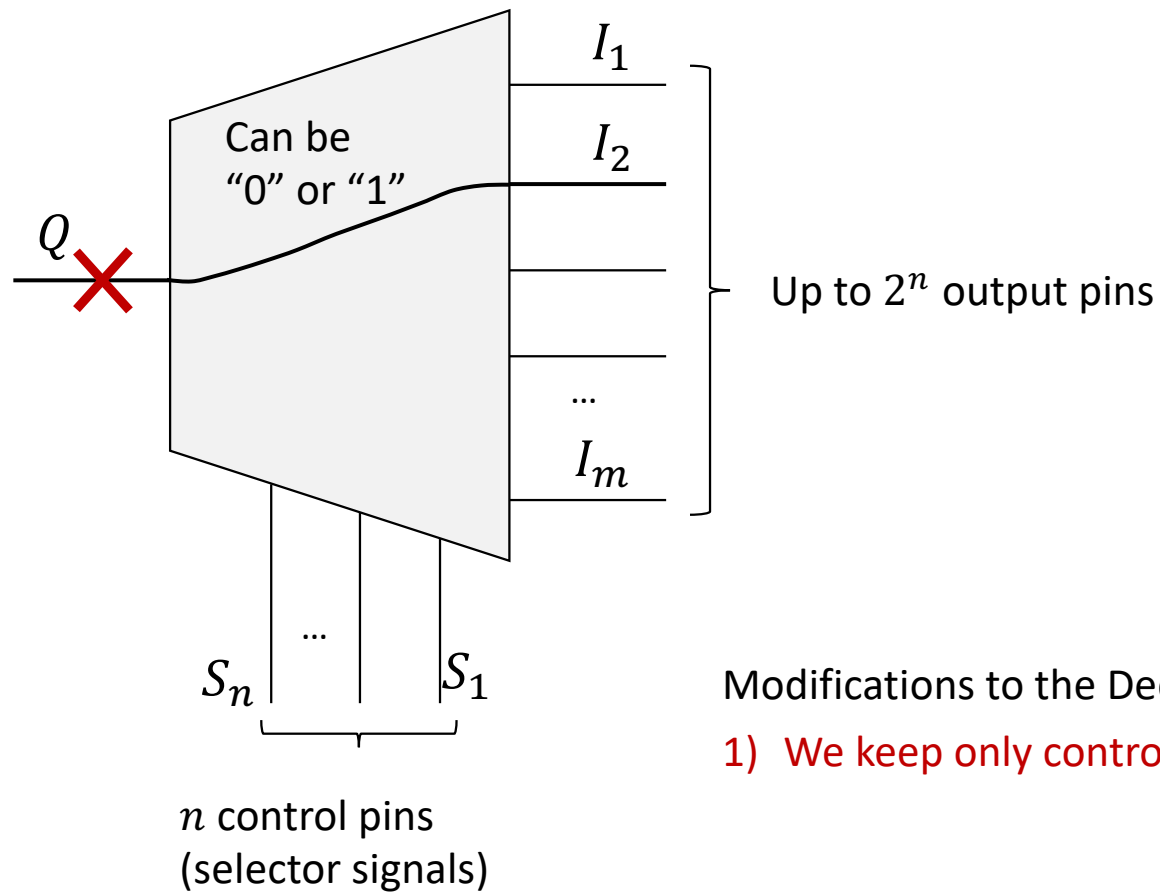
Based on control pin signals,
demultiplexor sends its input to one of its outputs

Demultiplexer



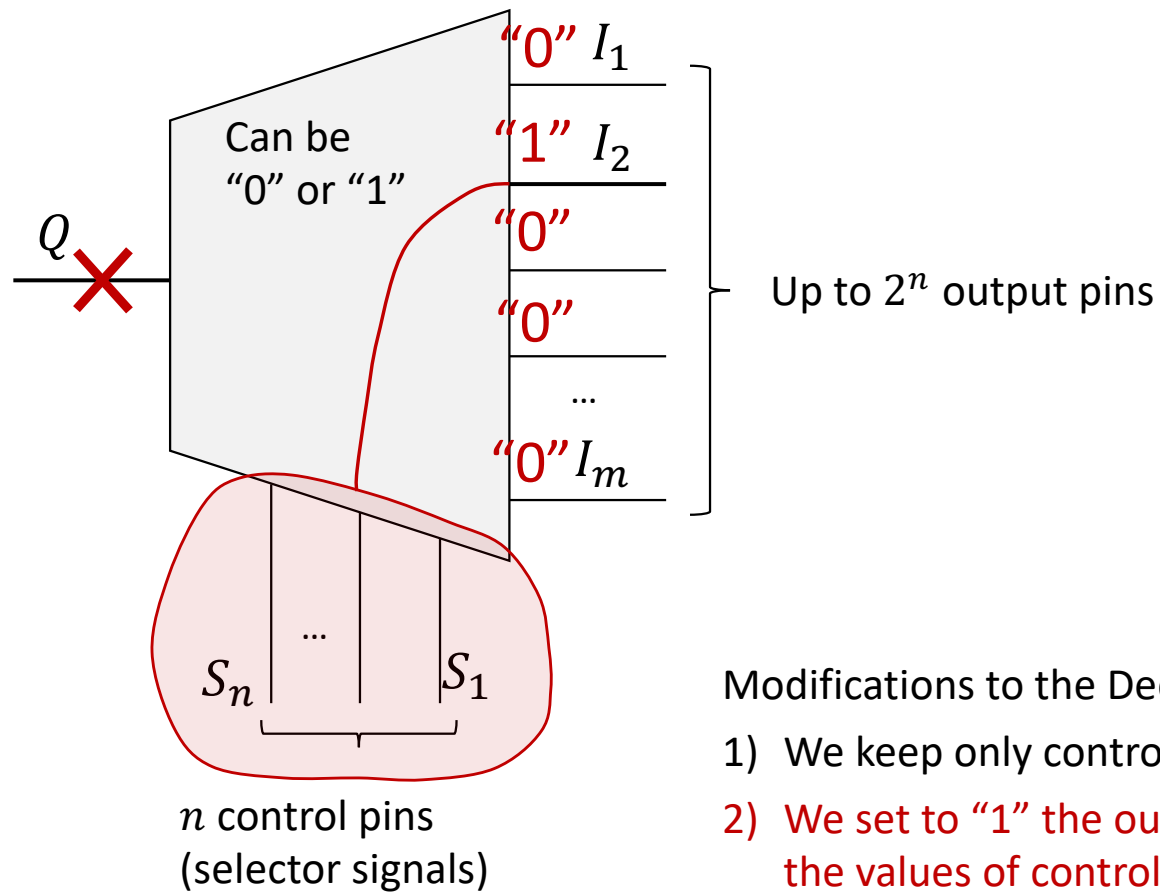
Based on control pin signals,
demultiplexor sends its input to one of its outputs

All other outputs are set to some
default value (e.g. 0)



Modifications to the Decoder:

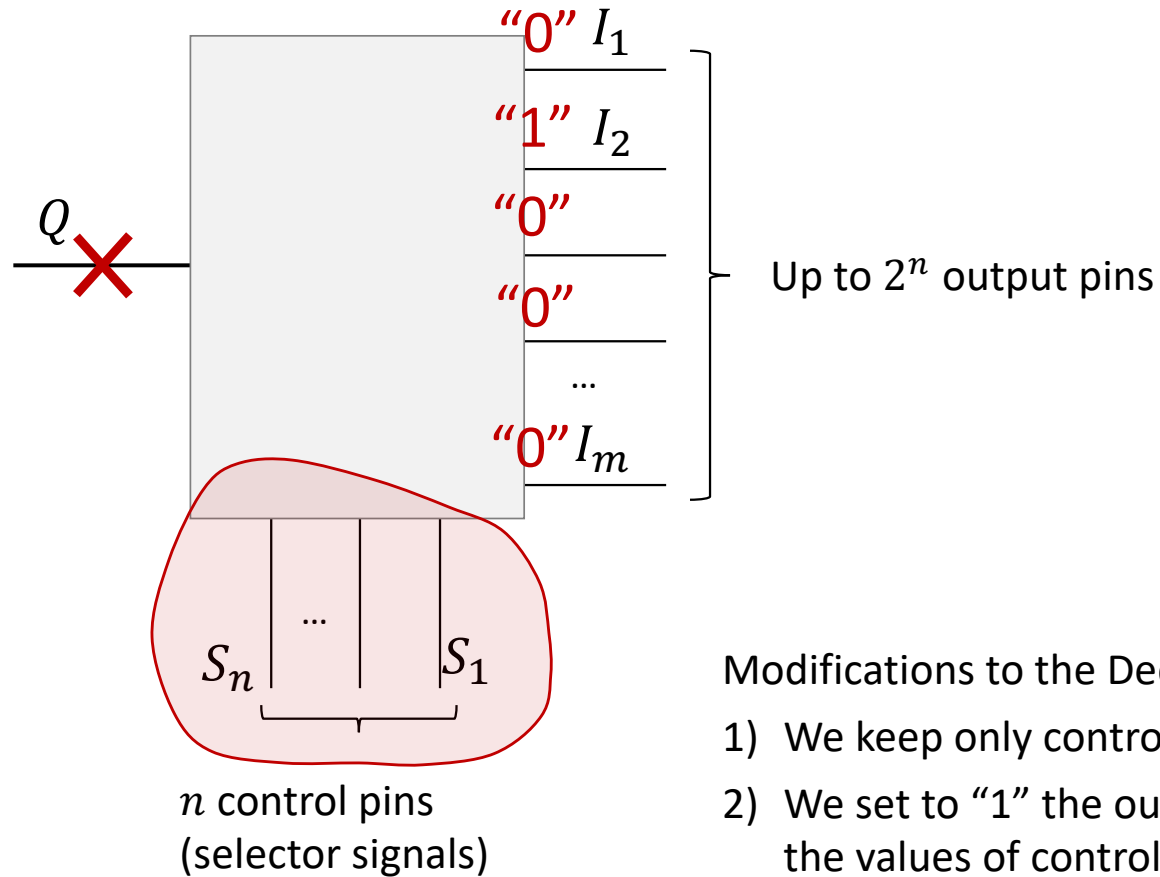
1) We keep only control pins, and remove input pin Q ;



Modifications to the Decoder:

- 1) We keep only control pins, and remove input pin Q ;
- 2) We set to "1" the output pin, which corresponds to the values of control pins;

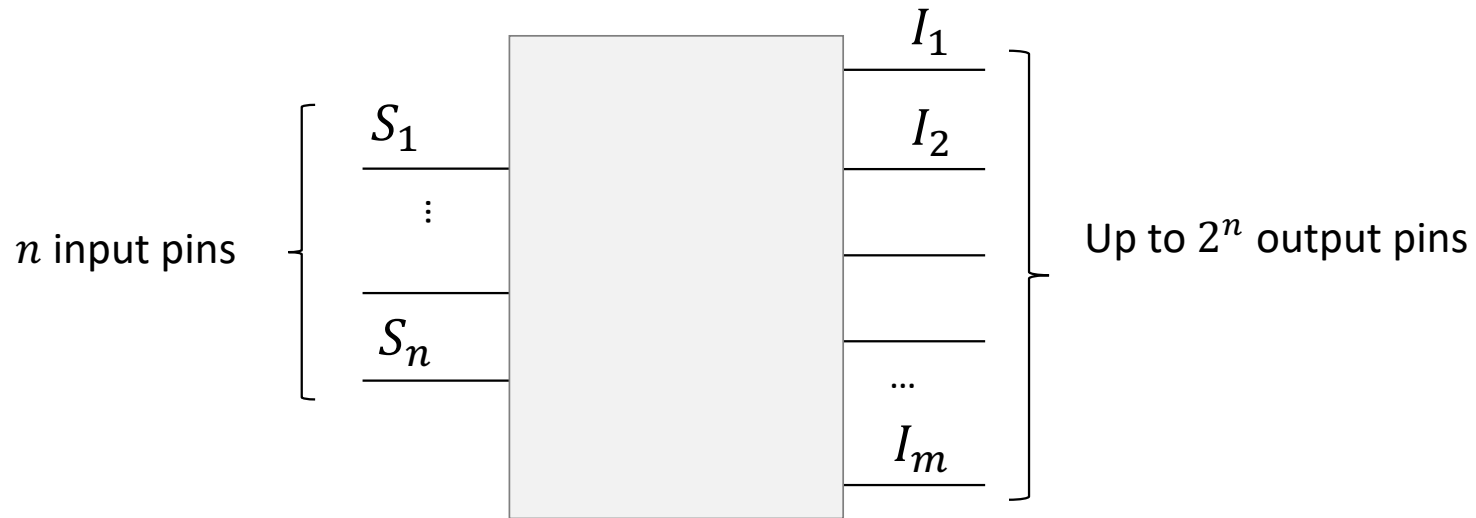
Decoder



Modifications to the Decoder:

- 1) We keep only control pins, and remove input pin Q ;
- 2) We set to "1" the output pin, which corresponds to the values of control pins;
- 3) We change the shape to a rectangle

Decoder

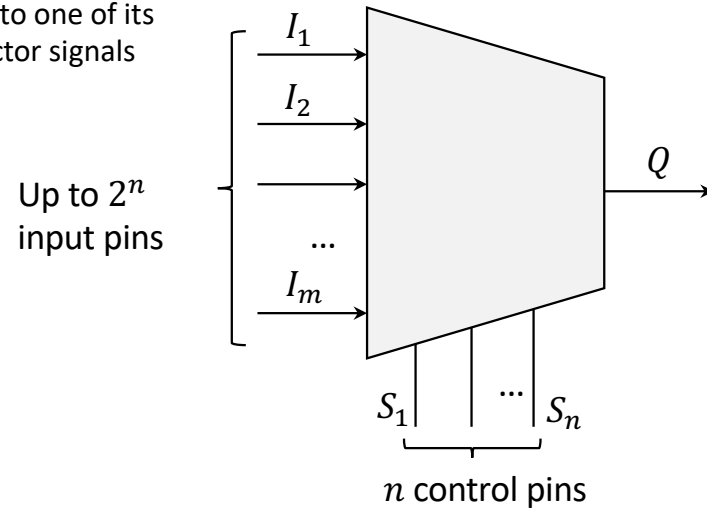


Modifications to the Decoder:

- 1) We keep only control pins, and remove input pin Q;
- 2) We set to "1" the output pin, which corresponds to the values of control pins;
- 3) We change the shape to a rectangle

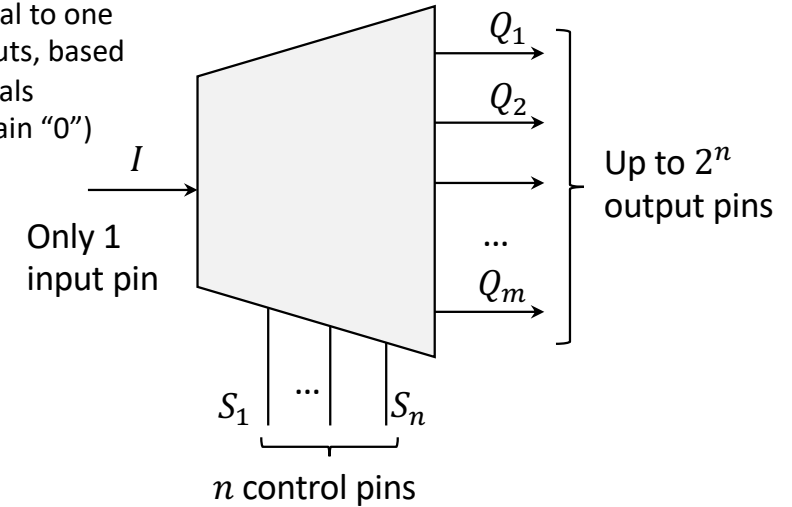
Multiplexor

Sets a specific output to one of its inputs, based on selector signals



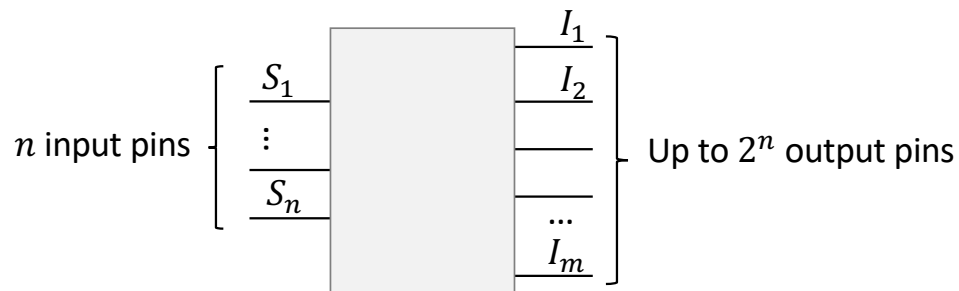
Demultiplexor

Forwards input signal to one of its multiple outputs, based on the selector signals (other outputs remain "0")



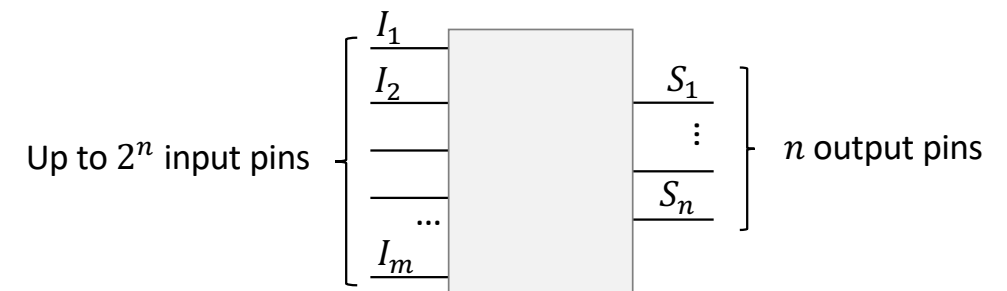
Decoder

Sets to "1" exactly one output pin, which corresponds to the signals of input pins; All other pins are set to "0"



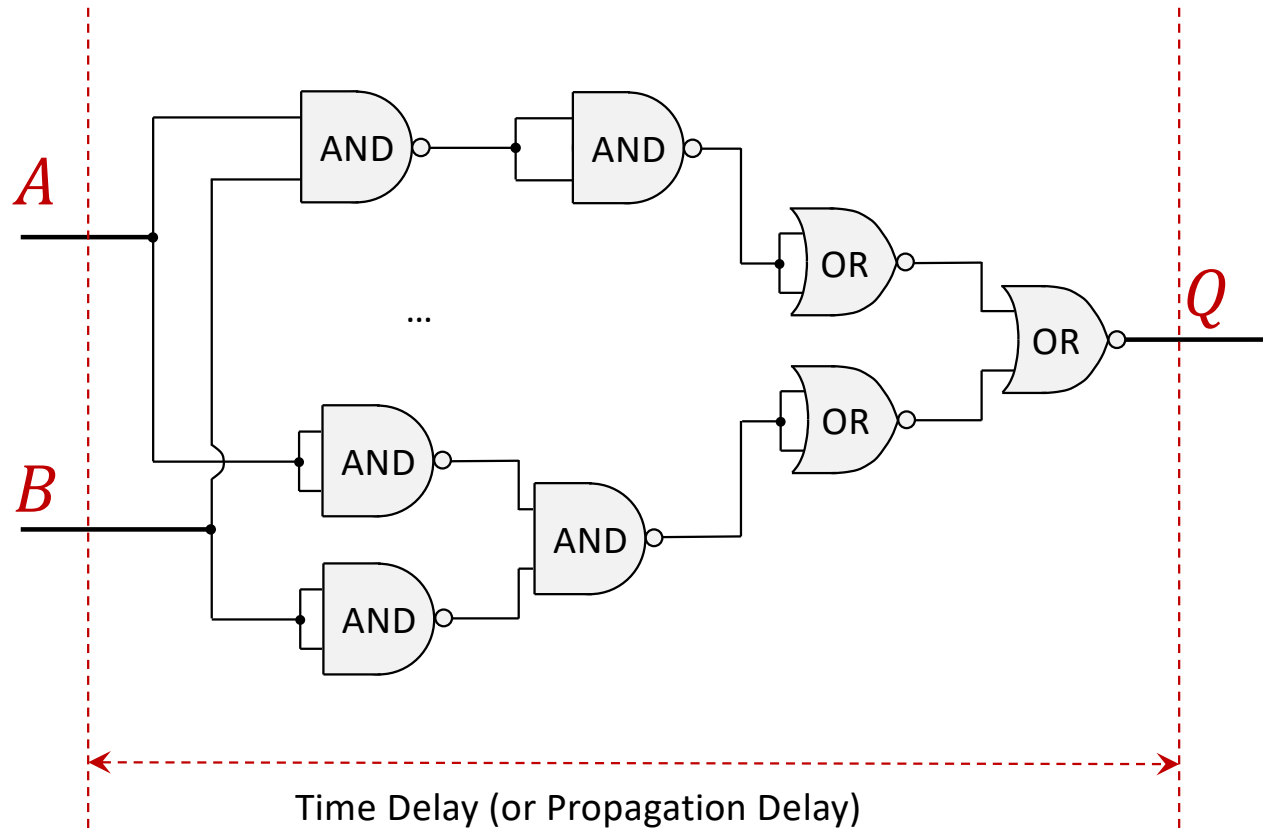
Encoder

The opposite function of a decoder; Only one input pin can be set to "1", while all others – "0"



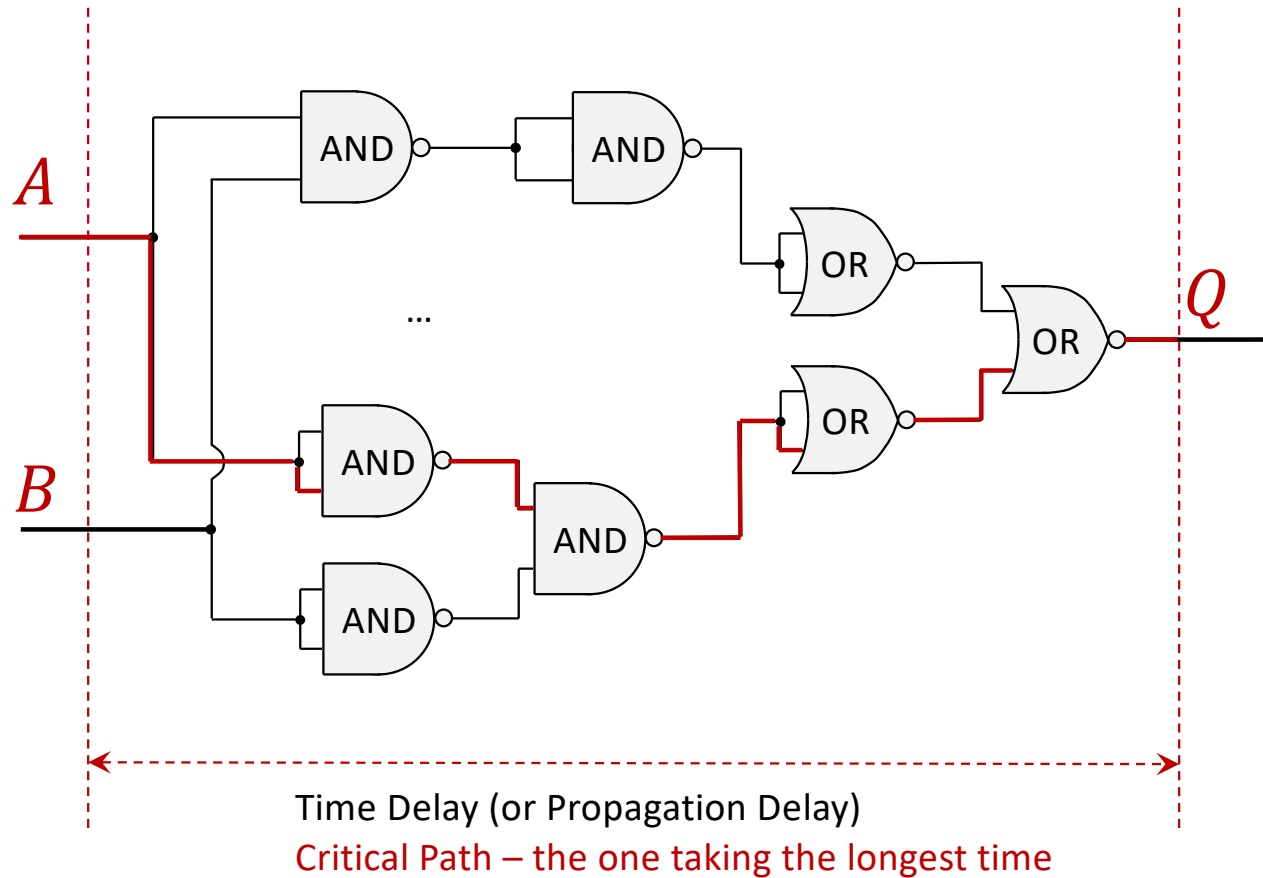
Propagation Delay

There is ALWAYS a time delay between the change of input signals, and the update of an output signal



Critical Path and Propagation Delay

There is ALWAYS a time delay between the change of input signals, and the update of an output signal

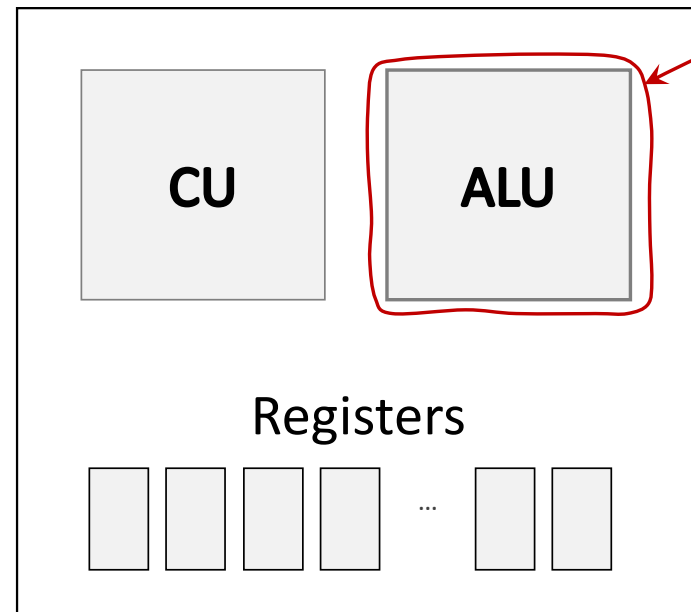


Combinational Logic Circuit

- Combines input signals and outputs the result;
- Implemented by using logic gates, such as AND, OR, NOT, NAND, etc.
- Does not use memory elements (registers);
- Does not use clock signal (typically)

Typical use: arithmetic operations

Types of Logic Circuits for CPU Components



Combinational logic circuit
for arithmetical and logic
operations

Types of Logic Circuits for CPU Components

