VARIANT 1

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (3 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x + y + z = 1$ and $2x - y - 5z = 1$.

2. (4 points)

   (a) Solve the system $\mathbf{Aw} = \mathbf{b}$, where $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

   $$\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

   (b) Draw relative positions of the planes that correspond to equations.

3. (3 points) Find the distance between the parallel planes $2x-y+2z+2 = 0$, $6x-3y+6z-4 = 0$.

4. (5 points) Two sides of a rhombus ABCD are parallel to the lines $y = -4x+5$ and $y = -\frac{1}{4}x+2$. Side length is $\sqrt{68}$. Vertex A has coordinates $(-2, 8)$. Find equations of diagonals of the rhombus and the point (E) of their intersection.

   Hint: Assume that vertex B has coordinates $(0, 0)$.

5. (5 points) Given the standard basis $E = \{1, x, x^2\}$, Find the transition matrix from the standard basis E to the basis $V = \{1, 1 - x, 1 + x - x^2\}$. Represent vector $p(x) = 3 + 2(1 - x) - 3(1 + x - x^2)$ in the standard basis E.

6. (5 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answer.

   $$\begin{bmatrix} 1 & 3 & 3 & \alpha \\ \alpha & 6 & 6 & 3 \\ 1 & \alpha & 3 & 1 \end{bmatrix}$$

7. (5 points) Find the equations of the lines parallel to $3x - 2y = 8$ and having distance $\sqrt{52}$ from point F(-2,-4).

8. (5 points) Compose the equations of lines passing through point $A(3, 2)$ and forming angles of 45° with the line $x - 2y = 3$

VARIANT 2

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (3 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x - y + z = 1$ and $2x - y - 5z = 1$.

2. (4 points)

   (a) Solve the system $\mathbf{Aw} = \mathbf{b}$, where $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

   $$\mathbf{A} = \begin{bmatrix} 3 & 1 & 1 \\ 5 & -1 & -1 \\ 1 & -1 & 5 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -2 \\ 10 \\ 12 \end{bmatrix}$$

   (b) Draw relative positions of the planes that correspond to equations.

3. (3 points) Find the distance between the parallel planes $2x-2y+z+3 = 0$, $4x-4y+2z+5 = 0$

4. (5 points) Two sides of a rhombus ABCD are parallel to the lines $5x - 2y + 1 = 0$ and $2x + 5y - 100 = 0$. Side length is $\sqrt{29}$. Vertex A has coordinates $(2, 5)$. Find equations of diagonals of the rhombus and the point (E) of their intersection.

   Hint: Assume that vertex B has coordinates $(0, 0)$.

5. (5 points) Given the standard basis $E = \{1, x, x^2\}$, Find the transition matrix from the standard basis E to the basis $V = \{1, 1 + x, 1 - x + x^2\}$. Represent vector $p(x) = 1 + (1 + x) - 2(1 - x + x^2)$ in the standard basis E.

6. (5 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answer.

   $$\begin{bmatrix} 1 & 2 & 2 & \alpha \\ \alpha & 4 & 4 & 2 \\ 1 & \alpha & 2 & 1 \end{bmatrix}$$

7. (5 points) Find the equations of the lines parallel to $3x + y = 8$ and having distance $\sqrt{10}$ from point F(-2,-4).

8. (5 points) Compose the equations of lines passing through point $A(3, 2)$ and forming angles of 45° with the line $x - 2y = 0$

VARIANT 3

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (3 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x - y - z = 1$ and $2x - y - 5z = 1$.

2. (4 points)

   (a) Solve the system $\mathbf{Aw} = \mathbf{b}$, where $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

   $$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 1 & -1 & -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$$

   (b) Draw relative positions of the planes that correspond to equations.

3. (3 points) Find the distance between the parallel planes $-12x + 6y - 12z - 24 = 0$, $2x - y + 2z + 2 = 0$

4. (5 points) Two sides of a rhombus ABCD are parallel to the lines $\frac{x}{3} + \frac{y}{2} = 1$ and $-2x+3y+1 = 0$. Side length is $\sqrt{52}$. Vertex A has coordinates $(-6, -4)$. Find equations of diagonals of the rhombus and the point (E) of their intersection.

   Hint: Assume that vertex B has coordinates $(0, 0)$.

5. (5 points) Given the standard basis $E = \{1, x, x^2\}$, Find the transition matrix from the standard basis E to the basis $V = \{1, 1 + x, 1 - x - x^2\}$. Represent vector $p(x) = 2 + 1(1 + x) - (1 - x - x^2)$ in the standard basis E.

6. (5 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answer.

   $$\begin{bmatrix} 2 & \alpha & 1 & 4 \\ \alpha & 3 & 3 & 2 \\ 2 & 1 & \alpha & 4 \end{bmatrix}$$

7. (5 points) Find the equations of the lines parallel to $2x + 3y = 8$ and having distance $\sqrt{52}$ from point F(-2,-4).

8. (5 points) Compose the equations of lines passing through point $A(3, 2)$ and forming angles of 45° with the line $x - 2y = 6$

VARIANT 4

Full name:	Group:

Task:	1	2	3	4	5	6	7	8	Total
Score:									

1. (3 points) Find the distance from the point $(1, 1, -1)$ to the line of intersection of the planes $x + y + z = -1$ and $2x - y - 5z = 1$.

2. (4 points)

   (a) Solve the system $\mathbf{Aw} = \mathbf{b}$, where $\mathbf{w} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$.

   $$\mathbf{A} = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 3 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} -3 \\ -1 \\ 3 \end{bmatrix}$$

   (b) Draw relative positions of the planes that correspond to equations.

3. (3 points) Find the distance between the parallel planes $x - 2y + z = 3$, $4x - 4y + 2z + 5 = 0$

4. (5 points) Two sides of a rhombus ABCD are parallel to the lines $2x-5y = 0$ and $5x-2y+2 = 0$. Side length is $\sqrt{116}$. Vertex A has coordinates $(-10, -4)$. Find equations of diagonals of the rhombus and the point (E) of their intersection.

   Hint: Assume that vertex B has coordinates $(0, 0)$.

5. (5 points) Given the standard basis $E = \{1, x, x^2\}$, Find the transition matrix from the standard basis E to the basis $V = \{1, 1 + x, 1 + x + x^2\}$. Represent vector $p(x) = 1 + 2(1 + x) - (1 + x + x^2)$ in the standard basis E.

6. (5 points) Find rank of the following matrix for all possible values of parameter α, $\alpha \in R$. Explain your answer.

   $$\begin{bmatrix} 1 & \alpha & 2 & 1 \\ \alpha & 4 & 3 & 4 \\ 1 & 1 & \alpha & 2 \end{bmatrix}$$

7. (5 points) Find the equations of the lines parallel to $3x - y = 8$ and having distance $\sqrt{10}$ from point F(-2,-4).

8. (5 points) Compose the equations of lines passing through point $A(3, 2)$ and forming angles of 45° with the line $x - 2y = 1$