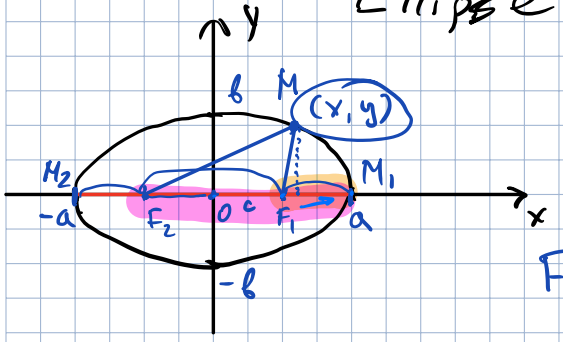
Ellipse



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad a > b$$

F_1, F_2 : foci (focus)

a : Semi major axis

b : Semi minor axis

2a major axis

$|MF_1| + |MF_2| = \text{const.}$

$|M_1F_1| + |M_1F_2| =$

$= 2|M_1F_1| + |F_1F_2| = |F_1F_2| + |M_1F_1| + |M_2F_2| =$

$= 2a$

$M : (x, y)$ $\qquad |OF_1| = |OF_2| = c$

$$|MF_1| + |MF_2| = 2a$$

$$\sqrt{(x-c)^2 + y^2} + \sqrt{(x+c)^2 + y^2} = 2a$$

$$\sqrt{(x+c)^2 + y^2} = 2a - \sqrt{(x-c)^2 + y^2}$$

$$(x+c)^2 + y^2 = 4a^2 - 4a\sqrt{(x-c)^2 + y^2} + (x-c)^2 + y^2$$

$$xc - a^2 = -a\sqrt{(x-c)^2+y^2}$$

$$a\sqrt{(x-c)^2+y^2} = a^2 - xc$$

$$a^2\left((x-c)^2+y^2\right) = a^4 - 2a^2xc + x^2c^2$$

$$a^2\left(x^2 - 2xc + c^2 + y^2\right) = a^4 - 2a^2xc + x^2c^2$$

$$a^2x^2 + a^2c^2 + a^2y^2 = a^4 + x^2c^2$$

$$x^2(a^2-c^2) + y^2a^2 = a^2(a^2-c^2) \quad \big| \; a^2(a^2-c^2)$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2-c^2} = 1$$

$$b^2 = a^2 - c^2$$

$$\boxed{\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1}$$

Canonical Eq-n of Ellipse

$$b^2 = a^2 - c^2$$

$$c = \sqrt{a^2-b^2}$$

$$0 \le c \le a$$

Eccentricity

$$\varepsilon = e = \frac{c}{a}$$

$c \to 0$

$e = 0 \Rightarrow$ circle

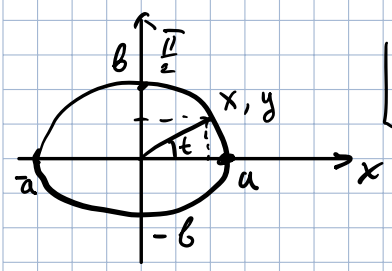
$$c = ae$$

$$b^2 = a^2 - c^2 = a^2 - a^2e^2 = a^2(1-e^2)$$

$$b = a\sqrt{1-e^2}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$\begin{cases} x = a\cos t \\ y = b\sin t \end{cases} \qquad \begin{cases} x^2 = a^2\cos^2 t \\ y^2 = b^2\sin^2 t \end{cases}$$



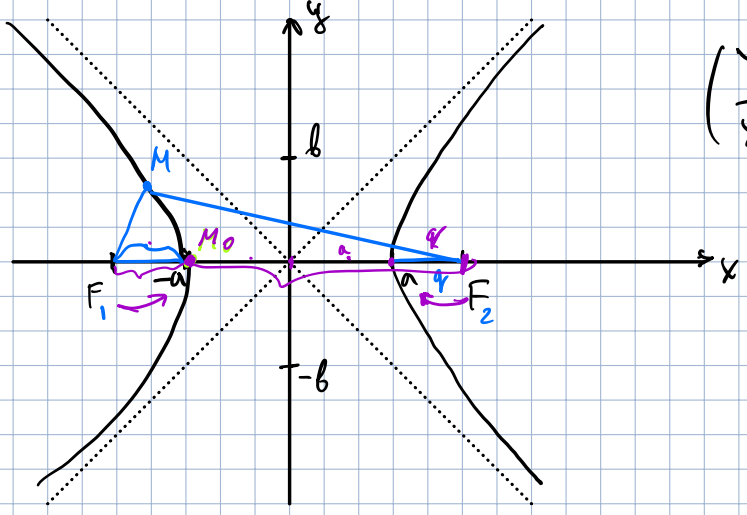
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \cos^2 t + \sin^2 t = 1$$

Hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$y = \frac{\text{Konst}}{x}$$

$$\{xy = \text{const}.$$



$$\left(\frac{y}{x}\right)^2 \qquad \frac{b^2}{a^2} = \operatorname{tg}\alpha$$

$$\left(\frac{y}{x}\right)^2 \to \left(\frac{b}{a}\right)^2$$

$$\frac{y}{x} \to \pm\frac{b}{a}$$

$$y = \pm\frac{b}{a}x$$

$$|F_2M| - |F_1M| = \text{const} =$$

$$c = a + q \qquad =$$

$$|M_0F_2| - |M_0F_1| = a + a + q - q = 2a$$

Hyperbola

$$b^2 = c^2 - a^2$$

$$|OF_1| = |OF_2| = c$$

$$b^2 = a^2 - c^2 \quad \text{ellipse}$$

Eccentricity of hyperbola

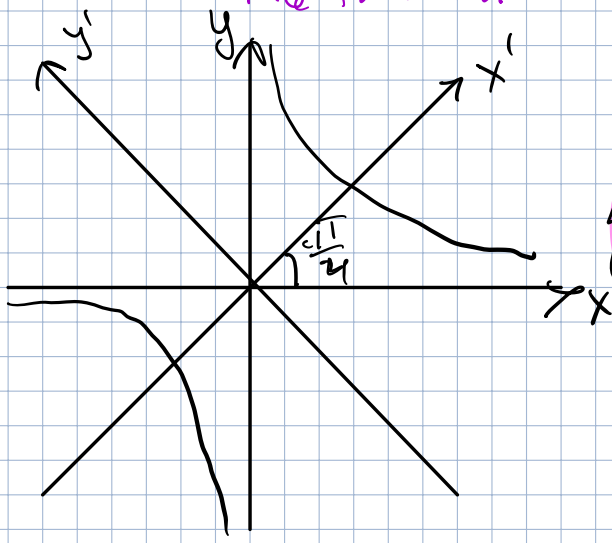
$$e = \frac{c}{a} > 1$$

$$c = ae$$

$$c^2 - a^2 = a^2(e^2 - 1)$$

$$b = a\sqrt{e^2 - 1} \qquad e > 1$$

Rotation



$xy = 1 \qquad \alpha = \frac{\pi}{4}$

$$\begin{cases} x = x'\cos\alpha - y'\sin\alpha \\ y = x'\sin\alpha + y'\cos\alpha \end{cases}$$

$$\cos\alpha = \sin\alpha = \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\boxed{xy = 1}$$

$$(x'\cdot\cos\alpha - y'\sin\alpha)(x'\sin\alpha + y'\cos\alpha) = 1$$

$$\frac{1}{2}(x' - y')(x' + y') = 1$$

$$\boxed{\frac{x'^2}{(\sqrt{2})^2} - \frac{y'^2}{(\sqrt{2})^2} = 1}$$

$a = b = \sqrt{2}$