Analytical Geometry and Linear Algebra. Lecture 6.

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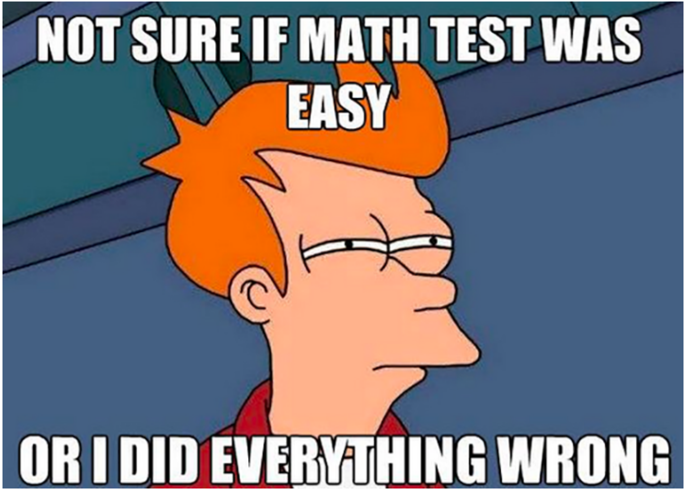
NOT SURE IF MATH TEST WAS EASY
OR I DID EVERYTHING WRONG

Lecture 6. Outline

- Part 1. Straight line in plane
- Part 2. Equations of a line

Locus

Definition

"When a point moves so as to satisfy some geometrical condition or conditions, the path traced out by the point is called the **locus** of the point."

From: P. R. Vittal. "Analytical Geometry: 2D and 3D".

Locus: Example

Suppose a point $P(x, y)$ moves such that its distances from two fixed points $A(2, 3)$ and $B(5, -3)$ are equal. Then the geometrical law is $PA = PB \Rightarrow PA^2 = PB^2$

$$(x-2)^2 + (y-3)^2 = (x-5)^2 + (y+3)^2 \Rightarrow$$

$$2x - 4y - 7 = 0$$

(locus is a straight line)

Locus: other examples

$$x^2 + y^2 = r^2$$

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The parabola is defined as the locus of a point which moves so that it is always the same distance from a fixed point (called the focus) and a given line (called the directrix)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

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Part 1. Straight line in plane

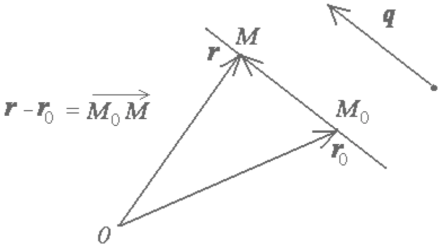
Definition

Given a point M_0 and a vector $\mathbf{a}$, set of all points M for which:

$$\overline{M_0M} = t\mathbf{a}$$

$t \in \mathbb{R}$

Parametric Vector Equation



Equation:

$$\mathbf{r} - \mathbf{r_0} = t\mathbf{q}$$

where t is a parameter ($t \in \mathbb{R}$).

Parametric Equation in 3D

In **rectangular Cartesian coordinate system**

Equation of a line: $\begin{cases} x = x_0 + q_x t \\ y = y_0 + q_y t \\ z = z_0 + q_z t \end{cases}$

$$\mathbf{r} - \mathbf{r_0} = [x - x_0, y - y_0, z - z_0]^\top$$

$$\mathbf{q} = [q_x, q_y, q_z]^\top$$

Canonical equation of a line

Eliminating t from the system:

$$\begin{cases} x = x_0 + q_x t \\ y = y_0 + q_y t \\ z = z_0 + q_z t \end{cases}$$

we get the Canonical equation

$$\frac{x - x_0}{q_x} = \frac{y - y_0}{q_y} = \frac{z - z_0}{q_z}$$

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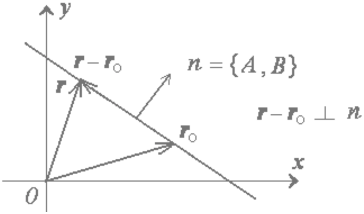
$$\frac{x - x_0}{q_x} = \frac{y - y_0}{q_y} = \frac{z - z_0}{q_z}$$

Give two points: M_0 and M_1:

$$\frac{x - x_0}{x_1 - x_0} = \frac{y - y_0}{y_1 - y_0} = \frac{z - z_0}{z_1 - z_0}$$

2D case



$$Ax + By + C = 0$$

for point M_0 on a line:

$$Ax_0 + By_0 + C = 0$$

$$(\mathbf{r} - \mathbf{r_0}) = [x - x_0, y - y_0]^\top ; \mathbf{n} = [A, B]^\top$$

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{r_0}) = 0$$

Angle Between Two Lines

1. The angle between two lines is the angle between direction vectors of the lines.

$$\cos\theta = \frac{\mathbf{p}\cdot\mathbf{q}}{\|\mathbf{p}\|\|\mathbf{q}\|}$$

Where $\mathbf{p}$ and $\mathbf{q}$ are direction vectors of lines.

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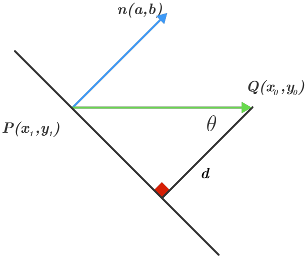
2. The angle between two lines is the angle between normal vectors of the lines.

$$\cos\theta = \frac{\mathbf{n_1}\cdot\mathbf{n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|}$$

Where $\mathbf{n_1}$ and $\mathbf{n_2}$ are normal vectors of lines.

Calculation of distances

Distance From a Point to a Line



$d = ?$

$$d = \frac{|\mathbf{n} \cdot \overline{PQ}|}{\|\mathbf{n}\|} = \dots$$

What if the point Q is on one side of a line, but the normal vector $(\mathbf{n})$ points to the opposite side?

Distance between point and line

Find the perpendicular distance from the point (5, 6) to the line $-2x + 3y + 4 = 0$,

Useful links

- https://www.geogebra.org
- https://youtu.be/fNk_zzaMoSs
- http://immersivemath.com/ila