Mathematical Analysis I. Test 2. November 9, 2021. *Problem Set 1. Part 1*

*Solutions for each part of the test have to be written on separate sheets! You can use a single sheet for several problems from one part of the test (i.e. you can write solutions 1, 2, 3, 4 on the same sheet). Do not forget to sign **all** the sheets you submit. You can use the sheet with the tasks for solving problems as well.*

I am aware that using any electronic devices, books etc. during the test, as well as communicating with other students, is strictly prohibited. Only one **handwritten** informational sheet is allowed. Any violation of these rules immediately leads to test cancellation. Signature

ATTENTION! You may use the following list of integrals without deriving them on your test. All the other integrals have to be solved using substitutions, transformations of the integrand, etc.

$\int x^\alpha \, dx = \frac{x^{\alpha+1}}{\alpha+1} + C, \ \alpha \neq -1;$

$\int a^x \, dx = \frac{a^x}{\ln a} + C, \ a > 0, \ a \neq 1;$

$\int \frac{dx}{x+a} = \ln |x + a| + C;$

$\int \sin x \, dx = -\cos x + C;$

$\int \cos x \, dx = \sin x + C;$

$\int \frac{dx}{\cos^2 x} = \tan x + C;$

$\int \frac{dx}{\sin^2 x} = -\cot x + C;$

$\int \sinh x \, dx = \cosh x + C;$

$\int \cosh x \, dx = \sinh x + C;$

$\int \frac{dx}{\cosh^2 x} = \tanh x + C;$

$\int \frac{dx}{\sinh^2 x} = -\coth x + C;$

$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \arctan \frac{x}{a} + C, \ a \neq 0;$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin \frac{x}{a} + C, \ a \neq 0;$

$\int \frac{dx}{\sqrt{x^2+a}} = \ln \left| x + \sqrt{x^2+a} \right| + C, \ a \neq 0;$

$\int \sqrt{x^2+a} \, dx = \frac{1}{2} \left(x\sqrt{x^2+a} + a \ln \left| \sqrt{x^2+a} + x \right| \right) + C, \ a \neq 0;$

$\int \sqrt{a^2-x^2} \, dx = \frac{1}{2} \left(x\sqrt{a^2-x^2} + a^2 \arcsin \frac{x}{a} \right) + C, \ a \neq 0.$

1. Find the indefinite integrals:

   (a) [3 points] $\int \dfrac{2x^3 - 12x^2 + 7x - 14}{(x-2)(2x^2 - x + 2)} \, dx;$

   (b) [3 points] $\int \left(x^3 - 4 \right) \arctan(3x) \, dx.$

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2. [2 points] Find the definite integral $\displaystyle\int_{0}^{1} x(3x-5)^{2022}dx$.

3. [2 points] Find the derivative $f'(x)$ if $\displaystyle f(x) = \int_{\cos 3x}^{x^{5x}} \frac{\sin 2t}{t}\, dt$.

4. [2 points] Find the values of t that correspond to inflection points of function $y = y(x)$ given by parametric equations $x = te^{t}$, $y = te^{-2t}$.