

Essentials of Analytical Geometry and Linear Algebra 1 Syllabus

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1 Introduction

Section 1. Vector algebra

Topics covered in this section:

- Vector spaces
- Basic operations on vectors (summation, multiplication by scalar, dot product)
- Linear dependency and in-dependency of the vectors
- Basis in vector spaces

Questions covered in this section:

1. How to perform the shift of the vector?
2. What is the geometrical interpretation of the dot product?
3. How to determine whether the vectors are linearly dependent?
4. What is a vector basis?

Concepts:

1. Vector spaces. General concepts.
2. Dot product as an operation on vectors.
3. Basis in vector spaces. Its properties.

Section 2. Introduction to matrices and determinants

Topics covered in this section:

- Relationship between Linear Algebra and Analytical Geometry
- Matrices 2x2, 3x3
- Determinants 2x2, 3x3
- Operations on matrices and determinants
- The rank of a matrix
- Inverse matrix
- Systems of linear equations
- Changing basis and coordinates

Questions covered in this section:

1. What is the difference between matrices and determinants?
2. Matrices A and C have dimensions of $m \times n$ and $p \times q$ respectively, and it is known that the product ABC exists. What are possible dimensions of B and ABC ?
3. How to determine the rank of a matrix?
4. What is the meaning of the inverse matrix?
5. How to restate a system of linear equations in the matrix form?

Concepts:

1. Operations on matrices and determinants.
2. Inverse matrix.
3. Systems of linear equations and their solution in matrix form.
4. Changing basis and coordinates.

Section 3. Lines in the plane and in the space

Topics covered in this section:

- General equation of a line in the plane
- General parametric equation of a line in the space
- Line as intersection between planes
- Vector equation of a line
- Distance from a point to a line
- Distance between lines
- Inter-positioning of lines

Questions covered in this section:

1. How to represent a line in the vector form?
2. What is the result of intersection of two planes in vector form?
3. How to derive the formula for the distance from a point to a line?
4. How to interpret geometrically the distance between lines?
5. List all possible inter-positions of lines in the space.

Concepts:

1. Lines in the plane and in the space. Equations of lines.
2. Distance from a point to a line.
3. Distance between two parallel lines.
4. Distance between two skew lines.

Section 4. Planes in the space

Topics covered in this section:

- General equation of a plane
- Normalized linear equation of a plane
- Vector equation of a plane
- Parametric equation a plane
- Distance from a point to a plane
- Projection of a vector on the plane
- Inter-positioning of lines and planes
- Cross Product of two vectors
- Triple Scalar Product

Questions covered in this section:

1. What is the difference between general and normalized forms of equations of a plane?
2. How to rewrite the equation of a plane in a vector form?
3. What is the normal to a plane?
4. How to interpret the cross products of two vectors?
5. What is the meaning of scalar triple product of three vectors?

Concepts:

1. Planes in the space. Equations of planes.
2. Distance from a point to a plane, from a line to a plane.
3. Projection of a vector on the plane.
4. Cross product, its properties and geometrical interpretation.
5. Scalar triple product, its properties and geometrical interpretation.

Section 5. Quadratic curves

Topics covered in this section:

- Circle
- Ellipse
- Hyperbola
- Parabola
- Canonical equations
- Shifting of coordinate system
- Rotating of coordinate system
- Parametrization

Questions covered in this section:

1. Formulate the canonical equation of the given quadratic curve.
2. Which orthogonal transformations of coordinates do you know?
3. How to perform a transformation of the coordinate system?
4. How to represent a curve in the space?

Concepts:

1. Determine the type of a given curve with the use of the method of invariant.
2. Compose the canonical equation of a given curve.
3. Determine the canonical coordinate system for a given curve.

Section 6. Quadric surfaces

Topics covered in this section:

- General equation of the quadric surfaces
- Canonical equation of a sphere and ellipsoid
- Canonical equation of a hyperboloid and paraboloid
- Surfaces of revolution
- Canonical equation of a cone and cylinder
- Vector equations of some quadric surfaces

Questions covered in this section:

1. What is the type of a quadric surface given by a certain equation?
2. How to compose the equation of a surface of revolution?
3. What is the difference between a directrix and generatrix?
4. How to represent a quadric surface in the vector form?

Concepts:

1. Determine the type of a quadric surface given by a certain equation.
2. Compose the equation of a surface of revolution with the given directrix and generatrix.
3. Represent a given equation of a quadric surface in the vector form.