Mathematical Analysis I. Final Examination 2021. *Problem Set 1. Part 1*

Name _____ **Group Number** _____

Solutions for each part of the examination have to be written on separate sheets! You can use a single sheet for several problems from one part of the examination (i.e. you can write solutions 1, 2, 3 on the same sheet). Do not forget to sign ***all*** *the sheets you submit. You can use the sheet with the tasks for solving problems as well.*

I am aware that using any electronic devices, books etc. during the examination, as well as communicating with other students, is strictly prohibited. Only one **handwritten** informational sheet is allowed. Any violation of these rules immediately leads to test cancellation. Signature

1. [2 points] Find all values of parameter β such that the series $\sum\limits_{k=1}^{\infty}\left(\cosh\frac{\pi}{k}-\cos\frac{\pi k}{k^2+1}\right)^{\beta}$ is convergent.

2. [3 points] Let us consider a functional sequence $h_n(x)=2^{\frac{x^2-nx+1}{n}}$, $x\in(0;+\infty)$.

   (a) [1 point] Find the limit function of the sequence.

   (b) [1 point] Find out if the sequence converges uniformly on $\Delta_1=(0;10)$.

   (c) [1 point] Find out if the sequence converges uniformly on $\Delta_2=(5;+\infty)$.

3. [3 points] Find all values of parameter γ such that the integral $\int\limits_0^{+\infty}\frac{(\cosh x-1)\,dx}{(e^x-1)\left(\sqrt{x}+\sqrt[4]{x}\right)^{\gamma}}$ converges.

Mathematical Analysis I. Final Examination 2021. *Problem Set 1. Part 2*

Name _____ **Group Number** _____

Solutions for each part of the examination have to be written on separate sheets! You can use a single sheet for several problems from one part of the examination (i.e. you can write solutions 4, 5 on the same sheet). Do not forget to sign ***all*** *the sheets you submit. You can use the sheet with the tasks for solving problems as well.*

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ATTENTION! You may use the following list of integrals without deriving them on your test. All the other integrals have to be solved using substitutions, transformations of the integrand, etc.

$\int x^\alpha\,dx = \frac{x^{\alpha+1}}{\alpha+1} + C$, $\alpha \neq -1$;
$\int a^x\,dx = \frac{a^x}{\ln a} + C$, $a > 0$, $a \neq 1$;
$\int \frac{dx}{x+a} = \ln|x+a| + C$;
$\int \sin x\,dx = -\cos x + C$;
$\int \cos x\,dx = \sin x + C$;
$\int \frac{dx}{\cos^2 x} = \tan x + C$;
$\int \frac{dx}{\sin^2 x} = -\cot x + C$;
$\int \sinh x\,dx = \cosh x + C$;
$\int \cosh x\,dx = \sinh x + C$;
$\int \frac{dx}{\cosh^2 x} = \tanh x + C$;
$\int \frac{dx}{\sinh^2 x} = -\coth x + C$;
$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\arctan\frac{x}{a} + C$, $a \neq 0$;
$\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C$, $a \neq 0$;
$\int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x + \sqrt{x^2+a}\right| + C$, $a \neq 0$;
$\int \sqrt{x^2+a}\,dx = \frac{1}{2}\left(x\sqrt{x^2+a} + a\ln\left|\sqrt{x^2+a}+x\right|\right) + C$, $a \neq 0$;
$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}\left(x\sqrt{a^2-x^2} + a^2\arcsin\frac{x}{a}\right) + C$, $a \neq 0$.

4. [2 points] Find the area of the surface of revolution obtained by revolving the curve $x^2 + 25y^2 = 9$ around y-axis.

5. [3 points] Find the indefinite integral $\displaystyle\int \frac{9x^2+10x+25}{(x^2+5x+8)(1-3x)}dx$.

Mathematical Analysis I. Final Examination 2021. *Problem Set 1. Part 3*

Name _____ **Group Number** _____

Solutions for each part of the examination have to be written on separate sheets! You can use a single sheet for several problems from one part of the examination (i.e. you can write solutions 6, 7, 8 on the same sheet). Do not forget to sign ***all*** *the sheets you submit. You can use the sheet with the tasks for solving problems as well.*

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6. [2 points] Find $y^{(k)}(x)$ if $y(x) = (2x^2 - 5)\sin^4 x$, $k > 2$ (i.e. you have to get a formula without sums or dots, you have to find all the derivatives in the formula, but you need not expand the expression obtained and combine like terms in it).

7. [4 points] Let us consider function $g(x) = \frac{x^4}{(x-2)^3}$.

   (a) Find the asymptotes of the graph $y = g(x)$.

   (b) Find points of local extrema of $g(x)$.

   (c) Find inflection points of $g(x)$.

   (d) Draw the graph of $y = g(x)$ taking into account everything obtained above.

   HINT: you might need that $g'(x) = \frac{x^3(x-8)}{(x-2)^4}$, $g''(x) = \frac{48x^2}{(x-2)^5}$.

8. [3 points] Find the limit $\lim\limits_{x\to 0}\left(\dfrac{1}{(1-x)^2} - \ln\left(2x + \sqrt{1+10x^2}\right)\right)^{\frac{1}{1\sqrt{1+8x^3}}}$.

   HINT: decompose the expression in parentheses using Taylor's formula with $o(x^3)$.