Analytical Geometry and Linear Algebra. Lecture 4.

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End of Lecture #3

Review. Lecture 3

- Part 1 (recap). Matrices. Transpose, Addition, Scalar multiplication
- Part 2. Matrix multiplication
- Part 3. Determinants. Scalar Triple Product

Lecture 4. Outline

- Part 1. Change of basis and coordinates
- Part 2. Matrix rank
- Part 3. Matrix inverse

A couple words about linear maps...

Linear map

Definition

Given two vector spaces E and F, a linear map between E and F is a function $f : E \to F$ satisfying the following two conditions:

- $f(a + b) = f(a) + f(b), \forall a, b \in E$
- $f(\lambda a) = \lambda f(a), \forall a \in E, \lambda \in \mathbb{R}$

Example

Let E be the vector space $R[X]^4$ of polynomials of degree at most 4, let F be the vector space $R[X]^3$ of polynomials of degree at most 3, and let the linear map be the **derivative map** $d : E \to F$:

- $d(P + Q) = dP + dQ$
- $d(\lambda P) = \lambda dP, \lambda \in \mathbb{R}$

Example

We choose $(1, x, x^2, x^3, x^4)$ as a basis of E
and $(1, x, x^2, x^3)$ as a basis of F.

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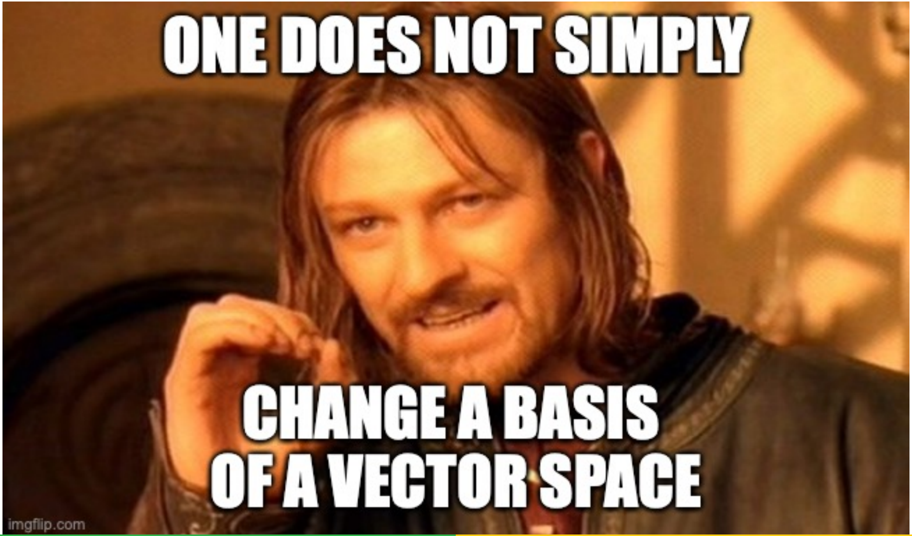
and $(1, x, x^2, x^3)$ as a basis of F.
Then the 4×5 matrix D associated with map d is obtained by expressing the derivative dx^i of each basis vector x^i for $i = 0, 1, 2, 3, 4$ over the basis $(1, x, x^2, x^3)$.

Example

Matrix D =

Change of basis and coordinates


ONE DOES NOT SIMPLY
CHANGE A BASIS
OF A VECTOR SPACE
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Here we are going to derive the formula.