Summer Bootcamp 2021

Introduction to Computer Science

Lecture 3 (Part III)

Introduction to the Traveling Salesman Problem (TSP)

Artem Burmyakov

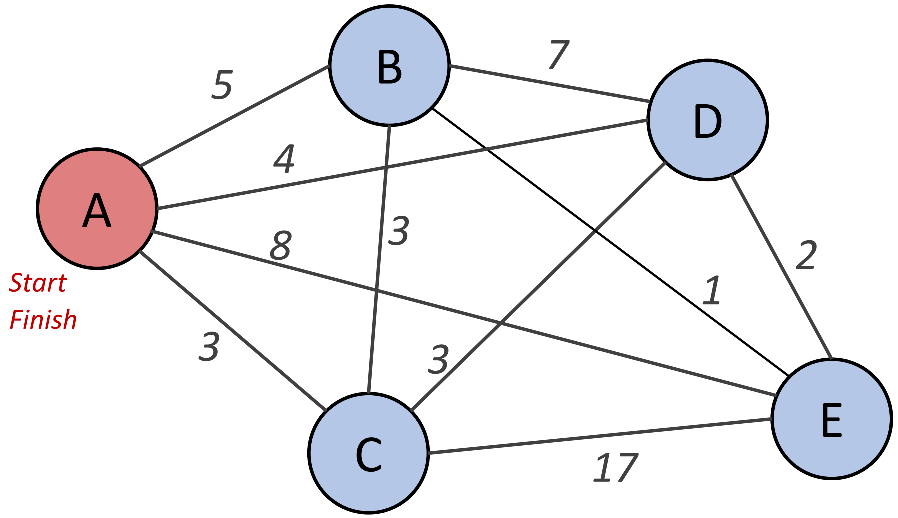
August 04, 2021

Innopolis University

Introduction: the Traveling Salesman Problem (TSP)

Find the shortest route through all cities, starting and ending at city A,

such that no city is visited twice.



Graph representing the distances (or travel times) between cities

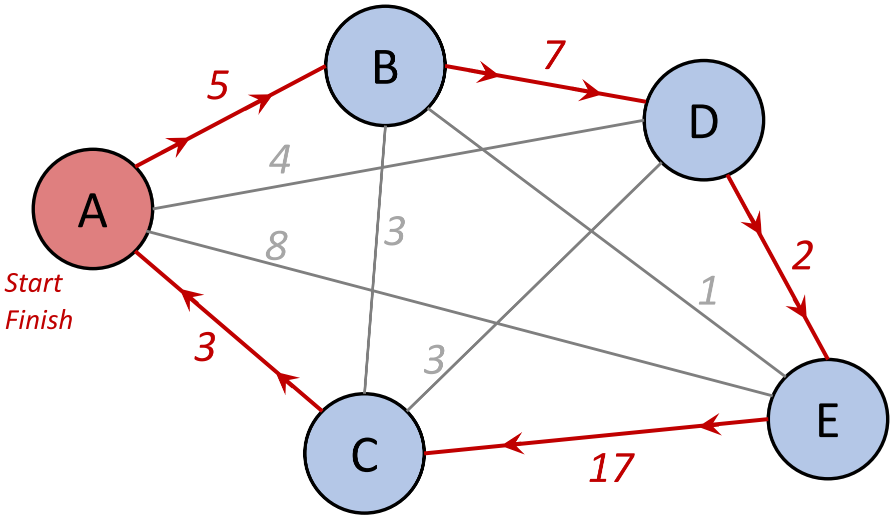
Assumptions:

- Any two cities are directly connected;
- Any city except A is visited once

Introduction: the Traveling Salesman Problem (TSP)

Find the shortest route through all cities, starting and ending at city A,

such that no city is visited twice.



Graph representing the distances (or travel times) between cities

Route length: 5+7+2+17+3 = 34

One possible route (selected in red)

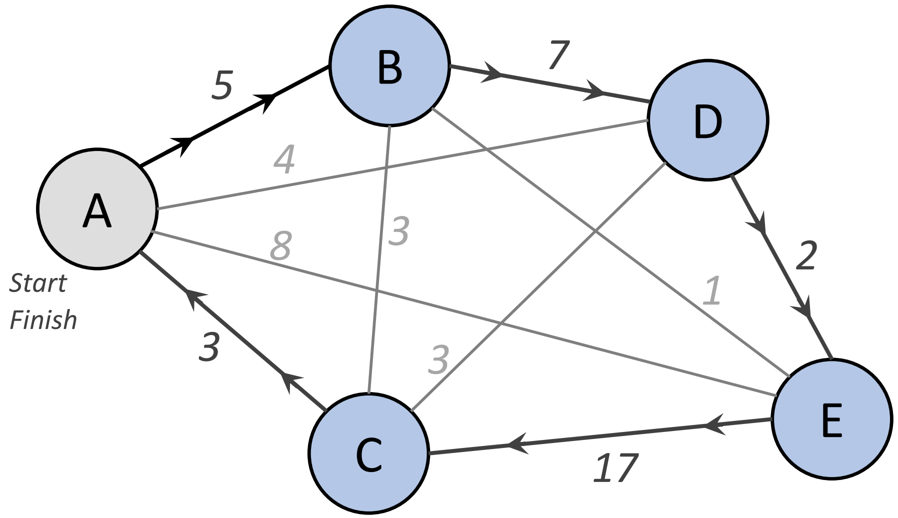
$A \longrightarrow B \longrightarrow$D $\longrightarrow$E $\longrightarrow$C $\longrightarrow$A

has length 5+7+2+17+3 = 34

Introduction: the Traveling Salesman Problem (TSP)

Find the shortest route through all cities, starting and ending at city A,

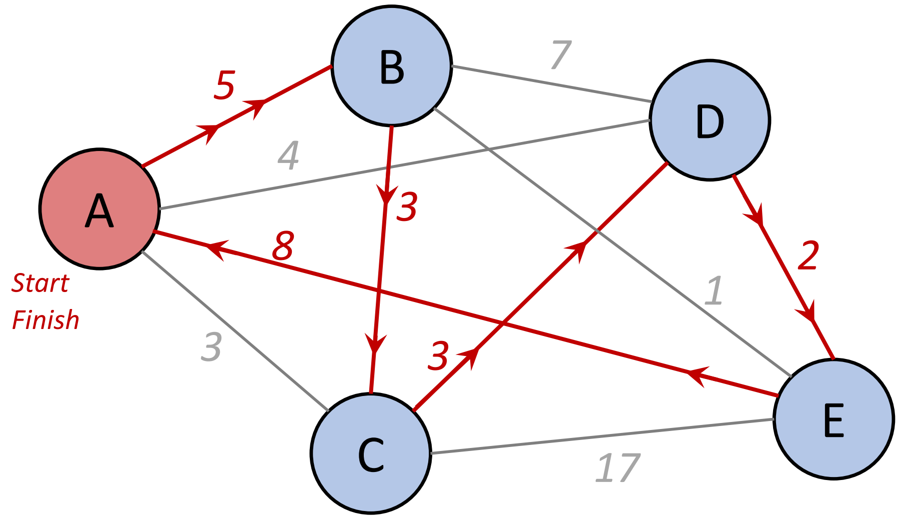
such that no city is visited twice.



Graph representing the distances (or travel times) between cities

Route length: 5+7+2+17+3 = 34

A different shorter route:



Graph representing the distances (or travel times) between cities

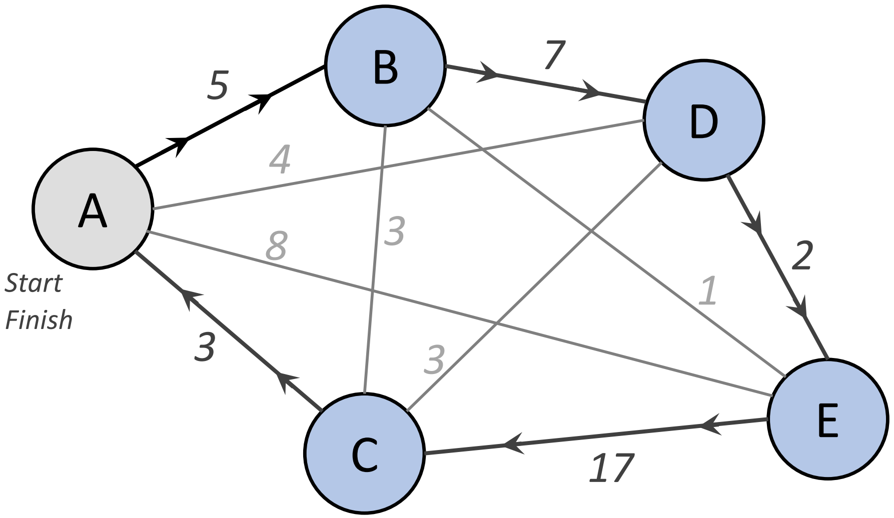
Route length: 5+3+3+2+8 = 21

(But is it the shortest (optimal) among all possible ones?!)

Introduction: the Traveling Salesman Problem (TSP)

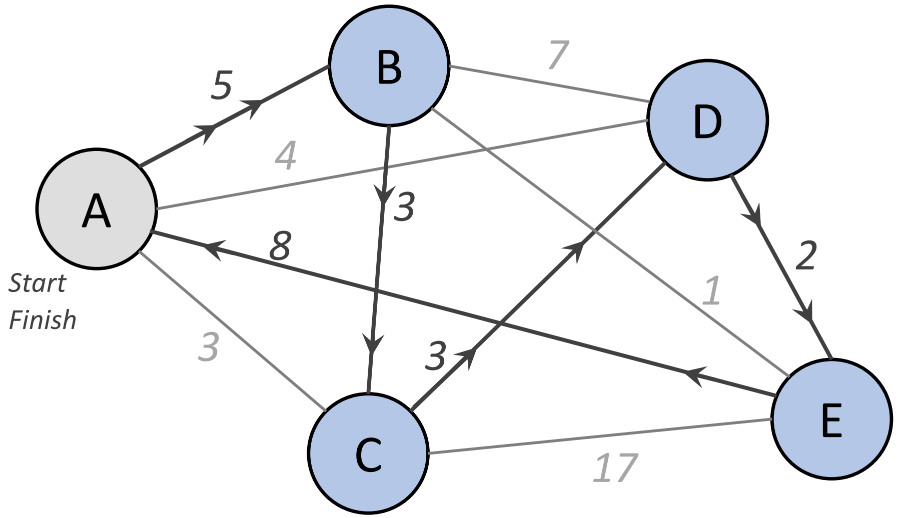
Find the shortest route through all cities, starting and ending at city A,

such that no city is visited twice.



Graph representing the distances (or travel times) between cities

Route length: 5+7+2+17+3 = 34

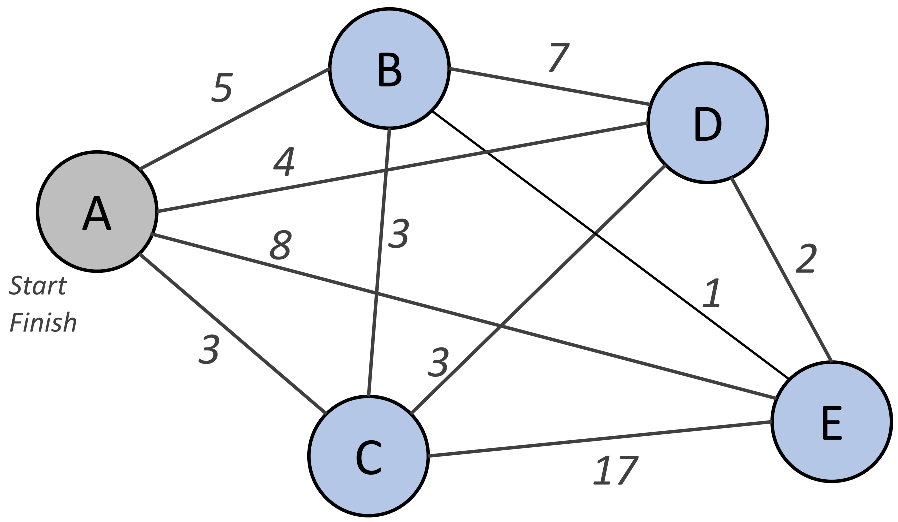


Graph representing the distances (or travel times) between cities

Route length: 5+3+3+2+8 = 21

For the case of 4 cities (plus A), 24 routes must be examined (by the brute-force search algorithm)

How many feasible routes are?

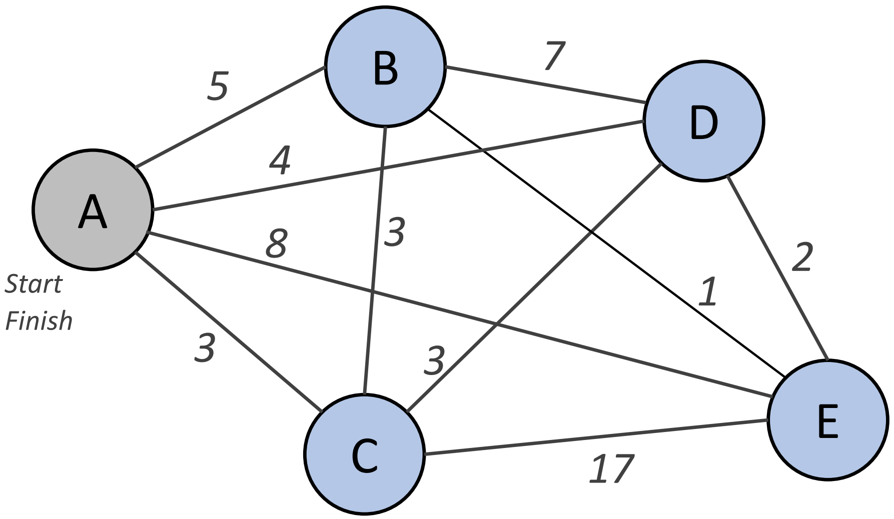


Graph representing the distances (or travel times) between cities

Assumptions:

- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are?



Graph representing the distances (or travel times) between cities

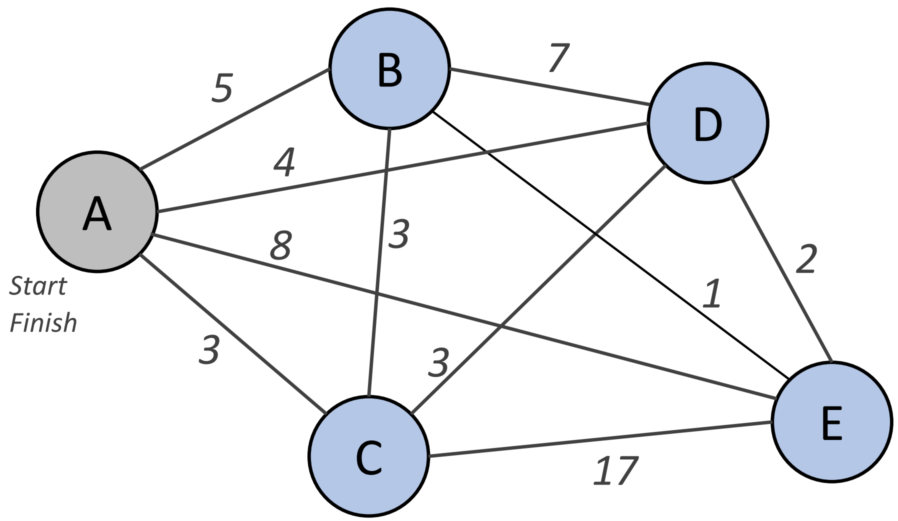
Route passes through all cities, starting and ending at A:

A	?	?	?	?	A

Assumptions:

- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are?



Graph representing the distances (or travel times) between cities

Route passes through all cities, starting and ending at A:

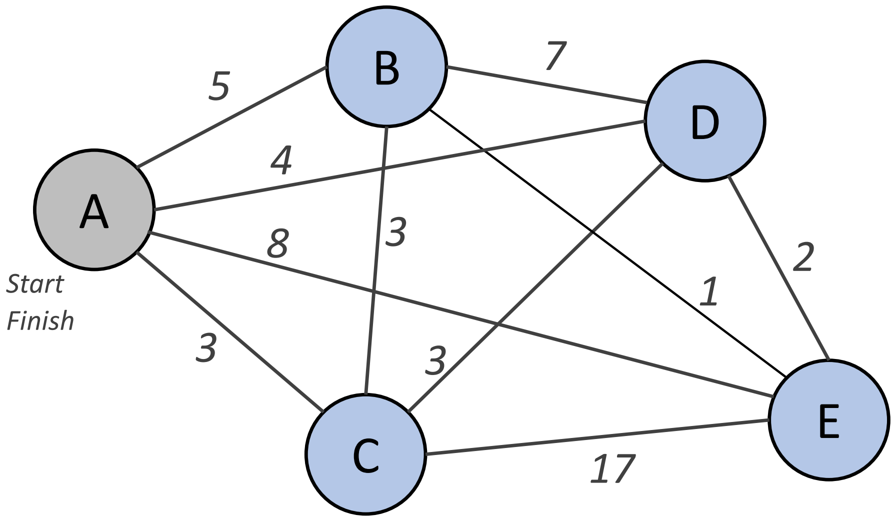
A	?	?	?	?	A

4 possible cases:
B, C, D, or E

Assumptions:

- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are?



Graph representing the distances (or travel times) between cities

Route passes through all cities, starting and ending at A:

A	?	?	?	?	A

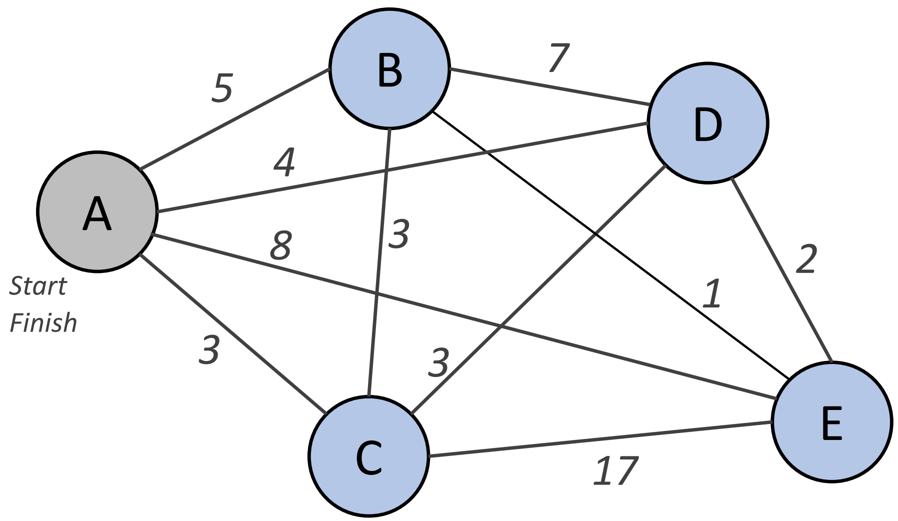
4 cases
(B, C, D, or E)

3 cases:
all cities (B, C, D, or E),
except the one visited before

Assumptions:

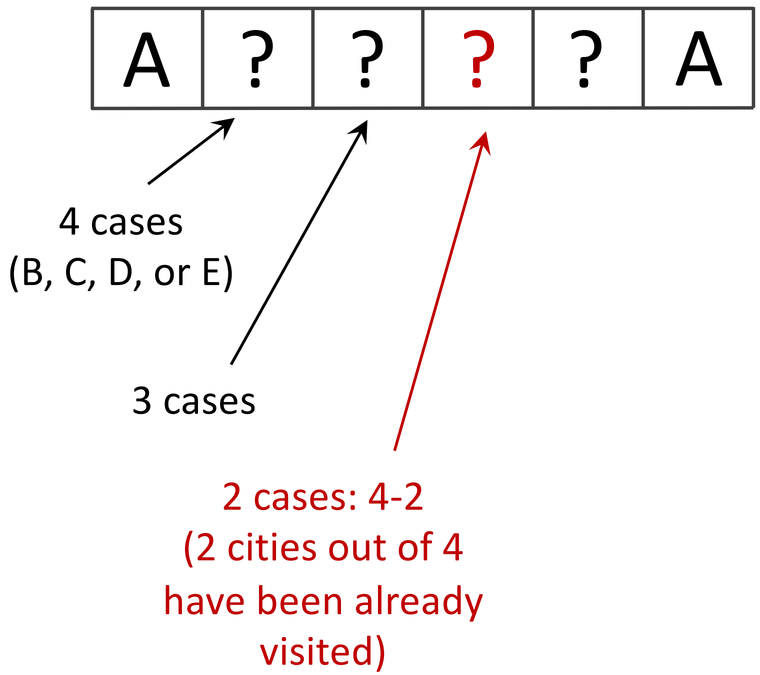
- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are?



Graph representing the distances (or travel times) between cities

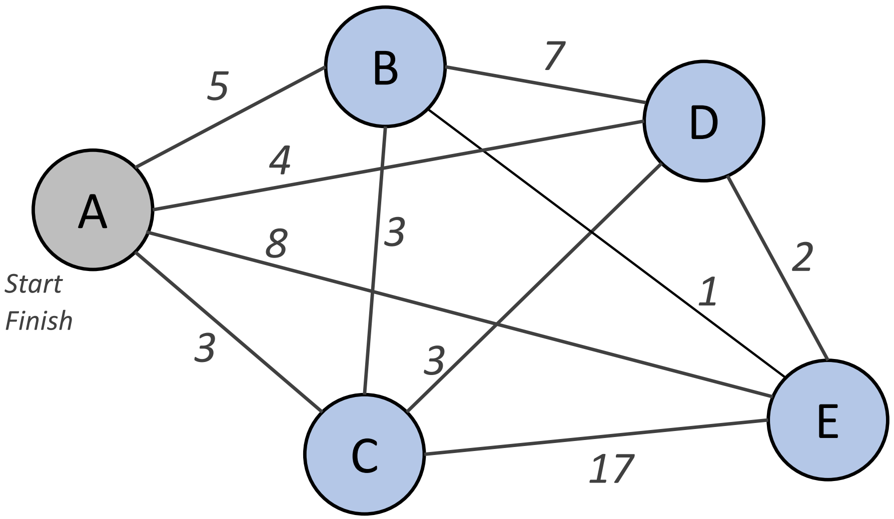
Route passes through all cities, starting and ending at A:



Assumptions:

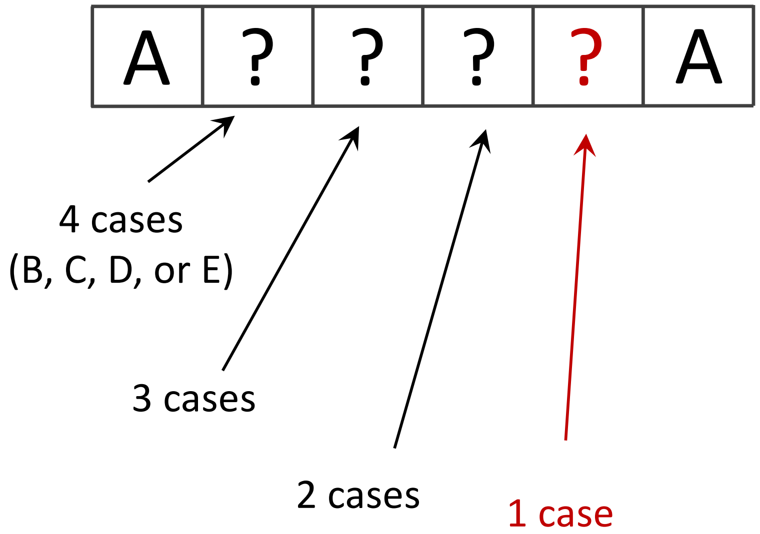
- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are?



Graph representing the distances (or travel times) between cities

Route passes through all cities, starting and ending at A:



Assumptions:

- Any two cities are directly connected;
- Any city except A is visited once

How many feasible routes are there in the graph?

There are 4×3×2 ×1 = 24 feasible routes to be examined for our graph, e.g.:

$A \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow E \longrightarrow A$
$A \longrightarrow B \longrightarrow C \longrightarrow E \longrightarrow D \longrightarrow A$
$A \longrightarrow B \longrightarrow D \longrightarrow C \longrightarrow E \longrightarrow A$
$A \longrightarrow B \longrightarrow D \longrightarrow E \longrightarrow C \longrightarrow A$
$A \longrightarrow B \longrightarrow E \longrightarrow C \longrightarrow D \longrightarrow A$
$A \longrightarrow B \longrightarrow E \longrightarrow D \longrightarrow C \longrightarrow A$

$A \longrightarrow C \longrightarrow B \longrightarrow D \longrightarrow E \longrightarrow A$
$A \longrightarrow C \longrightarrow B \longrightarrow E \longrightarrow D \longrightarrow A$
$A \longrightarrow C \longrightarrow D \longrightarrow B \longrightarrow E \longrightarrow A$
$A \longrightarrow C \longrightarrow D \longrightarrow E \longrightarrow B \longrightarrow A$
$A \longrightarrow C \longrightarrow E \longrightarrow B \longrightarrow D \longrightarrow A$
$A \longrightarrow C \longrightarrow E \longrightarrow D \longrightarrow B \longrightarrow A$

$A \longrightarrow D \longrightarrow B \longrightarrow C \longrightarrow E \longrightarrow A$
$A \longrightarrow D \longrightarrow B \longrightarrow E \longrightarrow C \longrightarrow A$
$A \longrightarrow D \longrightarrow C \longrightarrow B \longrightarrow E \longrightarrow A$
$A \longrightarrow D \longrightarrow C \longrightarrow E \longrightarrow B \longrightarrow A$
$A \longrightarrow D \longrightarrow E \longrightarrow B \longrightarrow C \longrightarrow A$
$A \longrightarrow D \longrightarrow E \longrightarrow C \longrightarrow B \longrightarrow A$

$A \longrightarrow E \longrightarrow B \longrightarrow C \longrightarrow D \longrightarrow A$
$A \longrightarrow E \longrightarrow B \longrightarrow D \longrightarrow C \longrightarrow A$
$A \longrightarrow E \longrightarrow C \longrightarrow B \longrightarrow D \longrightarrow A$
$A \longrightarrow E \longrightarrow C \longrightarrow D \longrightarrow B \longrightarrow A$
$A \longrightarrow E \longrightarrow D \longrightarrow B \longrightarrow C \longrightarrow A$
$A \longrightarrow E \longrightarrow D \longrightarrow C \longrightarrow B \longrightarrow A$

Route passes through all cities, starting and ending at A:

A	?	?	?	?	A
	4 cases	3 cases	2 cases	1 case	

4×3×2 ×1 = 24 cases in total

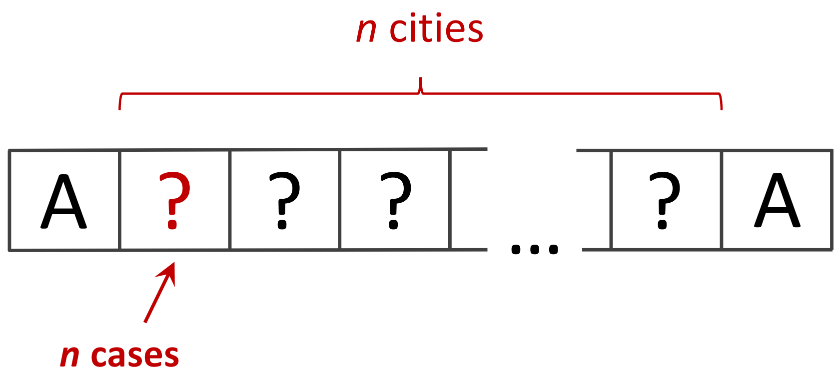
How many feasible routes are there in the graph?

General case for *n* cities:

$$\boxed{A}\overbrace{\boxed{?}\boxed{?}\boxed{?}\cdots\boxed{?}}^{n\ \text{cities}}\boxed{A}$$

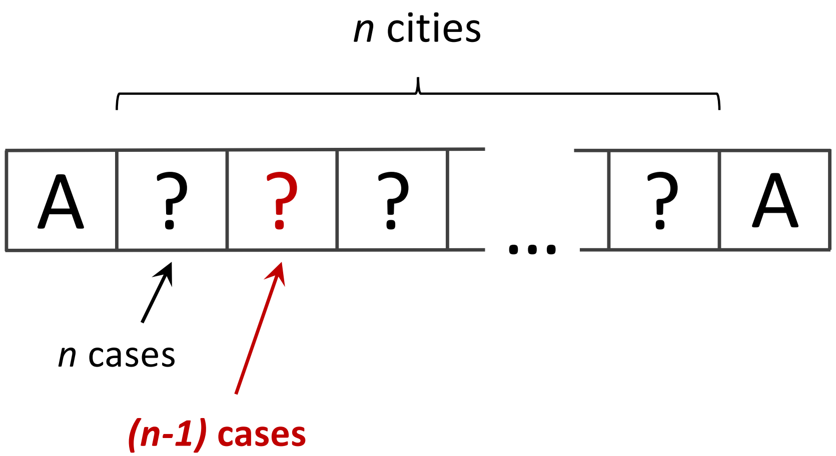
How many feasible routes are there in the graph?

General case for *n* cities:



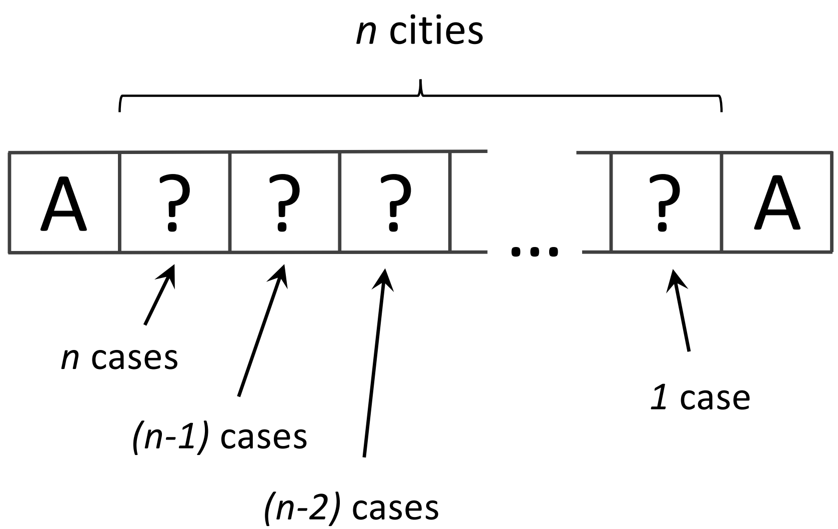
How many feasible routes are there in the graph?

General case for *n* cities:



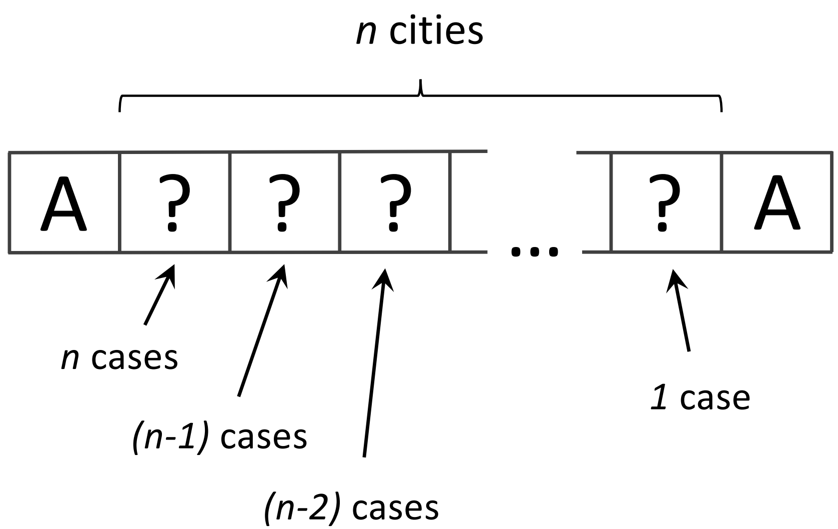
How many feasible routes are there in the graph?

General case for *n* cities:



How many feasible routes are there in the graph?

General case for *n* cities:



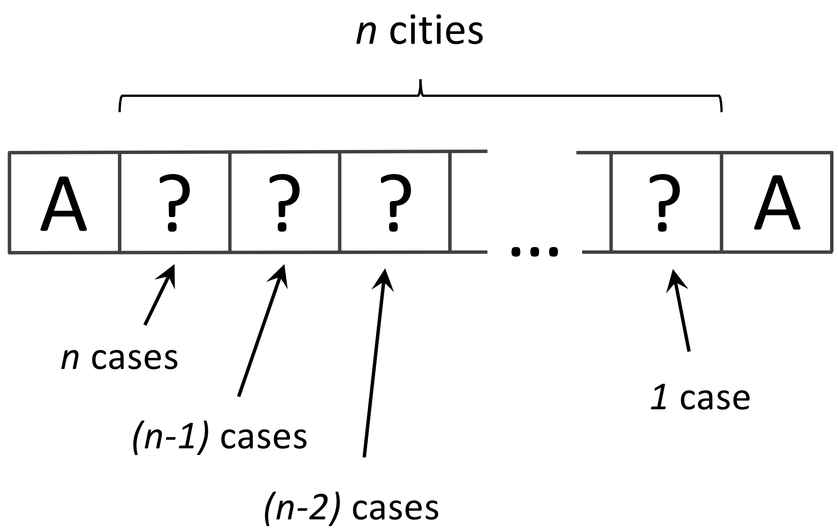
The number of feasible routes to be examined equals to:

$$n! = n\times(n-1)\times(n-2)\times\cdots\times3\times2\times1$$

("n factorial")

How many feasible routes are there in the graph?

General case for n cities:



The number of feasible routes to be examined equals to:

$$n! = n\times(n-1)\times(n-2)\times\cdots\times3\times2\times1$$

Brute-force search or an exhaustive search

- an algorithm, that enumerates all feasible scenarios, in order to determine an optimal solution.