VIth SEMESTER
OPERATIONS RESEARCH
1. INTRODUCTION

Introduction – Operation Research (OR)

- Operation research is defined as "a scientific approach to decision making, that seeks to determine the best way to design and operate a system under conditions that necessitate the allocation of scarce resources".
- Provides a set of algorithm that act as tools for effective problem solving and decision making.
- Extensive applications in engineering, business, and public systems.
- Used extensively by manufacturing and service industries in decision making.

Introduction – Operation Research (OR)

History of OR:

- Origin during World War II when the British military asked scientists to analyze military problems.
- The application of mathematics and scientific method to military applications is called operation research.
- Today it is called management science.
- One important thing about operation research is “resources are scarce and need to be used effectively”.

Introduction – Operation Research (OR)

Main phases of the OR:

- A company manufactures three types of clothing: Kurta, Suits, and Sarees. They have three different types of wool qualities i.e., red, green, and blue. The types of wool are required to produce the clothing types. The quantity of wool however is limited. The company now need to maximize the profits with the best wool available. The demand qualities of wool availability, and the profits on the types of clothing are given.

Quality of wool	Type of clothing			Quantity
	Suit	Kurta	Saree	
Red	2	3	0	5
Green	0	4	6	10
Blue	3	1	8	12
Income per unit length of cloth (Rs.)	3	5	8	

Introduction – Operation Research (OR)

Main phases of the OR:

- Definition of the problem –
  - It includes objective of the study, description of decision alternatives, and specification of the limits or boundary or restrictions.
- Construction of mathematical model
- Obtaining the input data
- Identification of alternative feasible solution
- Solution to the model using mathematical tools.
- Validation and Implementation of solution.

Pre-Modeling Phase	Modeling Phase	Post-Modeling / Implementation Phase

Introduction – Operation Research (OR)

Main phases of the OR:

- Pre-Modeling Phase
  - Identification of the problem
  - Quantification of the problem
- Modeling Phase
  - Data collection
  - Formulate a mathematical model of the problem
  - Identification of possible alternative solutions
- Post-Modeling / Implementation Phase
  - Interpretation of the solution
  - Model validation
  - Monitor and Control

Introduction – Operation Research (OR)

Scope of OR:

- Industry
- Agriculture
- Defense

Limitations of OR:

- Lack of qualitative factors and psychological issues
- Money and time costs
- Magnitude of computations
- Implementations

Linear Programming Problem (LPP)

- Linear programming was first conceived by Dantzig at the end of second world war.
- Dantzig's first paper was "programming in linear structures".
- The term linear programming is coined by Koopmans.
- Resources are limited, to optimize the use of this resources, programming and planning activities are formulated to maximize or minimize a linear form of profit or cost functions whose variables are restricted to values satisfying a system of linear constraints.
- Linear programming is a technique of determining an optimum schedule of independent activities in view of available resources.

Linear Programming Problem (LPP)

Mathematical Formulation of Linear Programming

- Identify the decision variables in the problem
- Formulate the objective function to be optimized as a linear function of decision variables
- Formulate the constraints of the problem such as resource limitations, market conditions as linear equations/inequalities in terms of decision variables.
- Add non-negativity constraint as that negative values of the decision variable do not have any physical interpretation.

Linear Programming Problem (LPP)

- Model – an idealistic representation or the replica of a problem under consideration.
- Mathematical models should be quantifiable for making analysis.
- Three basic elements of mathematical model:
  - Decision variables and parameters
  - Objective functions
  - Constraints or Restrictions

Linear Programming Problem (LPP)

Mathematical Formulation of Linear Programming

- Consider a small manufacturer making two products A and B. Two resources R1 and R2 are required to make these products. Each units of product A requires 1 unit of R1 and 3 units of R2. Each unit of product B requires 1 units of R1 and 2 units of R2. The manufacturer has 5 units of R1 and 12 units of R2 available. The manufacturer also makes a profit of Rs. 6 per unit of product A and Rs. 5 per unit of product B sold. How much units of products A and B are to be manufactured by the company for maximizing the profits.

Linear Programming Problem (LPP)

- **Decision Variables and Parameters:** these are nothing but input or dominant variables of the system.
  - while solving the system we obtain the best value of the decision variables which gives the optimal solution.
  - X and Y are the decision variables and represent output (refer: example)
  - The variables that we want to solve are called decision variables

Linear Programming Problem (LPP)

- **Objective Function:** A function which represent "what we are solving this problem for".
  - It represent the total profit or cost to carrying out a set of activity at some levels.
  - Maximisation type or minimisation type
  - The profit function is the objective function (refer: example)
  - Is a function of decision variables which is to be optimized to obtain the best solution to the problem under consideration.
- **Objective Function Coefficient:** Constant representing cost/unit or profit/unit of carrying out an activity.

Linear Programming Problem (LPP)

- **Constraints:** are the functions of the decision variables that accounts to the physical limitation of the system.
  - The conditions which actually represent the resources that are available and the resources that have to be used efficiently.
  - These constraints restrict the decision variables from taking higher or unlimited values.
- **Technology Coefficient:** Various technologies are required to complete an activity.
- **Resource Availability:** During planning period, the resources available to complete the task.

Linear Programming Problem (LPP)

- **Non-negativity Condition**
  - Some additional condition may exist like $x \geq 0$ is called the non-negativity condition.
  - For all LPP, we have to explicitly state that the decision variable should take non-negative values.

Linear Programming Problem (LPP)

- Mathematical programming problem with the objective function and constraints are linear functions of the decision variables.
- The mathematical structure is

$$\text{Max/min} \qquad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2 \\ \vdots \\ a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m \end{cases}$$

x_j = decision variables
b_i = constraint levels
c_j = objective function coefficients
a_{ij} = constraint coefficients

Linear Programming Problem (LPP)

Mathematical Formulation of Linear Programming

- A company manufactures two products X and Y, by using the three machines A, B, and C. Each unit of X takes 1 hour on machine A, 3 hours on machine B and 10 hours on machine C. Similarly, product Y takes one hour, 8 hours and 7 hours on Machine A, B, and C respectively. In the coming planning period, 40 hours of machine A, 240 hours of machine B and 350 hours of machine C is available for production. Each unit of X brings a profit of Rs 5/- and Y brings Rs. 7 per unit. How much of X and Y are to be manufactured by the company for maximizing the profit?.

Linear Programming Problem (LPP)

Allocation Problem

- Chip mont co. manufactures silicon wafer circuits for the use in microprocessors. Computer makers presently buy Chip Mont's entire production of the following types of silicon chips: central processing unit (CPU), integrated circuit (IC), and memory. The following data apply:

	CPU	IC	Memory	Max. Available
Silicon	0.05	0.02	0.01	10000 sheets
Labor	0.2	0.5	0.1	200000 min
Chemical wash	0.1	0.4	0.15	400000 hours
Profit per wafer ($)	0.25	0.4	0.15	

Formulate the linear programming model for the problem.

Linear Programming Problem (LPP)

Blending Problem

- A firm produces an alloy having following specifications:

  Specific gravity ≤ 0.98

  Chromium ≥ 8

  Melting point ≥ 450 degree Celsius

Property	A	B	C
Specific gravity	0.92	0.97	1.04
Chromium (%)	7	13	16
Melting point (Degree Celsius)	440	490	480
Cost of raw material per unit (Rs.)	90	280	40

Formulate the linear programming model for the problem.

Linear Programming Problem (LPP)

Diet Problem

- A person wants to decide the constituents of a diet which will fulfill his daily requirements of proteins, fats and carbohydrates at the minimum cost. The choice is to be made from four different types of foods. The yields per unit of these foods are given in table below.

Food Type	Proteins (yield per unit)	Fats (yield per unit)	Carbohydrates (yield per unit)	Cost per unit (in Rupees)
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum Requirement	800	200	700	

Formulate the linear programming model for the problem.

Portfolio and Media Selection Problem

- Consider the case of ABC mutual fund. It has developed the investment alternatives given in the table. The return on investment (ROI) is expressed as the annual rate of return on the capital. The risk is the subjective assessment of the safety of the investment on a scale of 0 to 10 made by the portfolio manager. The term of investment is the average length of the time required to realize the rate of return.

  The ABC's objective is to maximize ROI. The guidelines for selecting the portfolio are (i) The average risk should not be exceed 2.5

  (ii) The average term of investment should not exceed six years

  (iii) At least 15% of the funds should be retained in the form of cash.

Alternative	ROI	Risk	Term
Stocks	12%	2	4
Bonds	10%	1	8
Growth Stock	15%	3	2
Speculations	25%	4	10
Cash	0	0	0

Formulate the linear programming model for the problem.

Product Mix Problem

- A firm manufactures three products A,B and C. Time to manufacture product A is twice that for B and thrice that for C and if the entire labour is engaged in making product A,1600 units of this product can be produced. These products are to be produced in the ratio 3:4:5. There is demand for at least 300,250 and 200 units of products A,B and C and the profit earned per unit is Rs.90, Rs40 and Rs.30 respectively.

Raw Material	**Requirement per unit product (Kg)**		
	A	B	C
P	6	5	9
Q	4	7	8

Formulate the problem as a linear programming problem and find all the constraints for the above product mix problem.

Linear Programming Problem (LPP)

Blending Problem

A firm produces an alloy having following specifications

specific gravity ≤ 0.98

Chromium ≥ 8

Melting point $\geq 450^\circ c$

Raw materials A, B, & C having the properties as shown can be used to make the alloy.

Property	Properties of Raw materials		
	A	B	C
specific gravity	0.92	0.97	1.04
Chromium	7%	13%	16%
Melting Point	440°C	490°c	480°C
Cost of raw material Per unit	Rs 90	Rs 280	Rs 40

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Linear Programming Problem (LPP)

Blending Problem

Find the proportion in which A, B & C be used to obtain alloy of desired properties while the cost of raw materials is minimum.

Soln:-

Let x_1, x_2, and x_3 be the percentage of raw materials A, B, & C

Minimize $Z = 90x_1 + 280x_2 + 40x_3$

Subjected to:

$$0.92x_1 + 0.97x_2 + 1.04x_3 \leq 0.98$$

$$7x_1 + 13x_2 + 16x_3 \geq 8$$

$$440x_1 + 490x_2 + 480x_3 \geq 450$$

$$x_1 + x_2 + x_3 = 100$$

$$x_1, x_2, x_3 \geq 0$$

Linear Programming Problem (LPP)

Diet Problem

A person wants to decide the (constraints)|constituents of a diet which will fulfill his daily requirements of protiens, fats & carbohydrates at the minimum cost. The choice is to be made from four different types of food. Formulate the LPP.

Food type	Proteins	Facts	Carbohydrates	Cost/unit
1	3	2	6	45
2	4	2	4	40
3	8	7	7	85
4	6	5	4	65
Minimum requirement	800	200	700	

Linear Programming Problem (LPP)

Diet Problem

Soln:-

Let x_1, x_2, x_3, & x_4 be the units of food type 1, 2, 3, & 4

Minimize, $z = 45x_1 + 40x_2 + 85x_3 + 65x_4$

Subject to:

$$3x_1 + 4x_2 + 8\cancel{4}x_3 + 6x_4 \geq 800$$

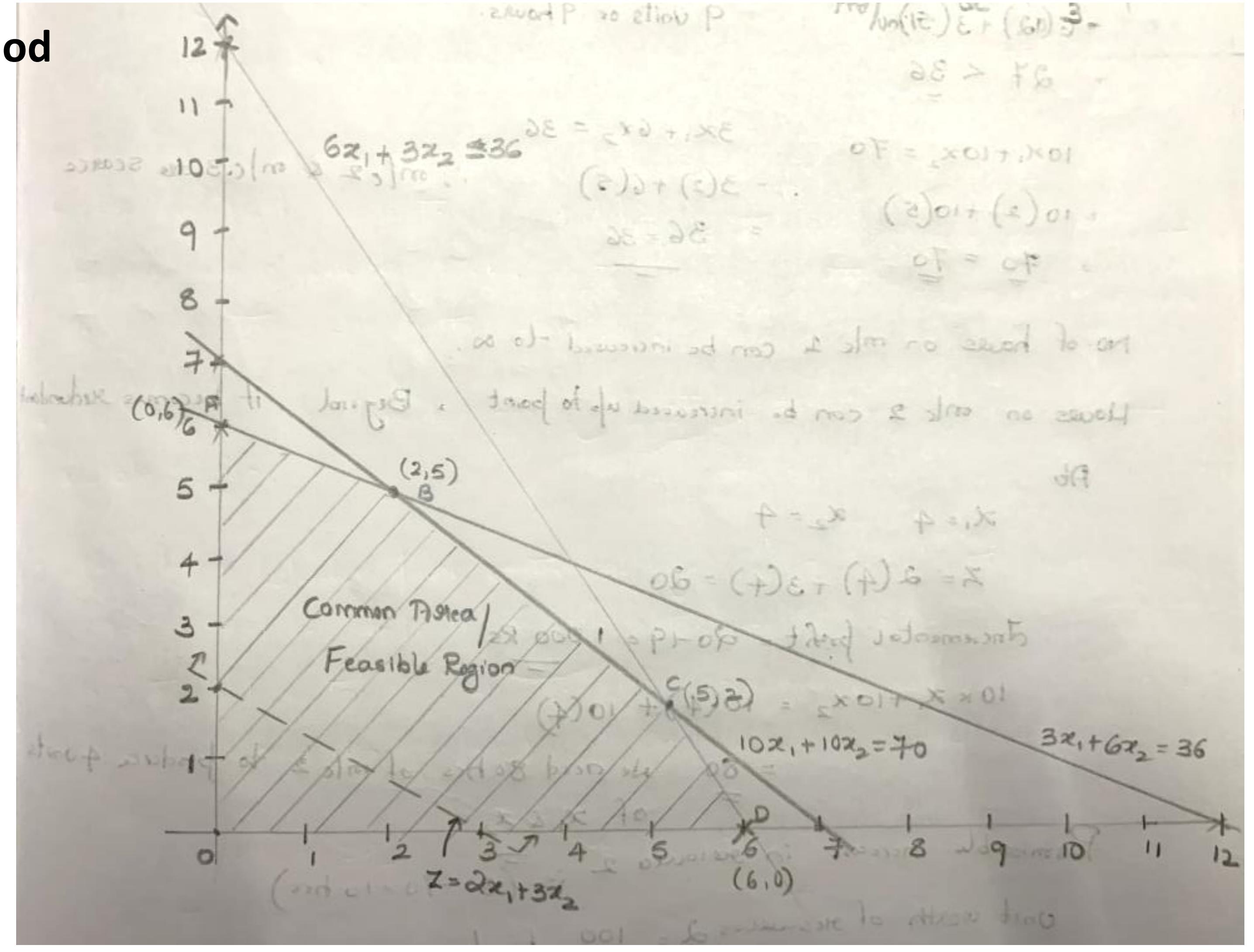
$$2x_1 + 2x_2 + 7x_3 + 5x_4 \geq 200$$

$$6x_1 + 4x_2 + 7x_3 + 4x_4 \geq 700$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Linear Programming Problem (LPP)

Graphical Method



Linear Programming Problem (LPP)

Graphical Method:

- Is a method to calculate the optimum solution.
- Different methods are available in LPP (i.e., Simplex method, Dual Simplex method, Big M method, etc.) to calculate optimal solutions.
- Graphical methods are used when there are only two variables.

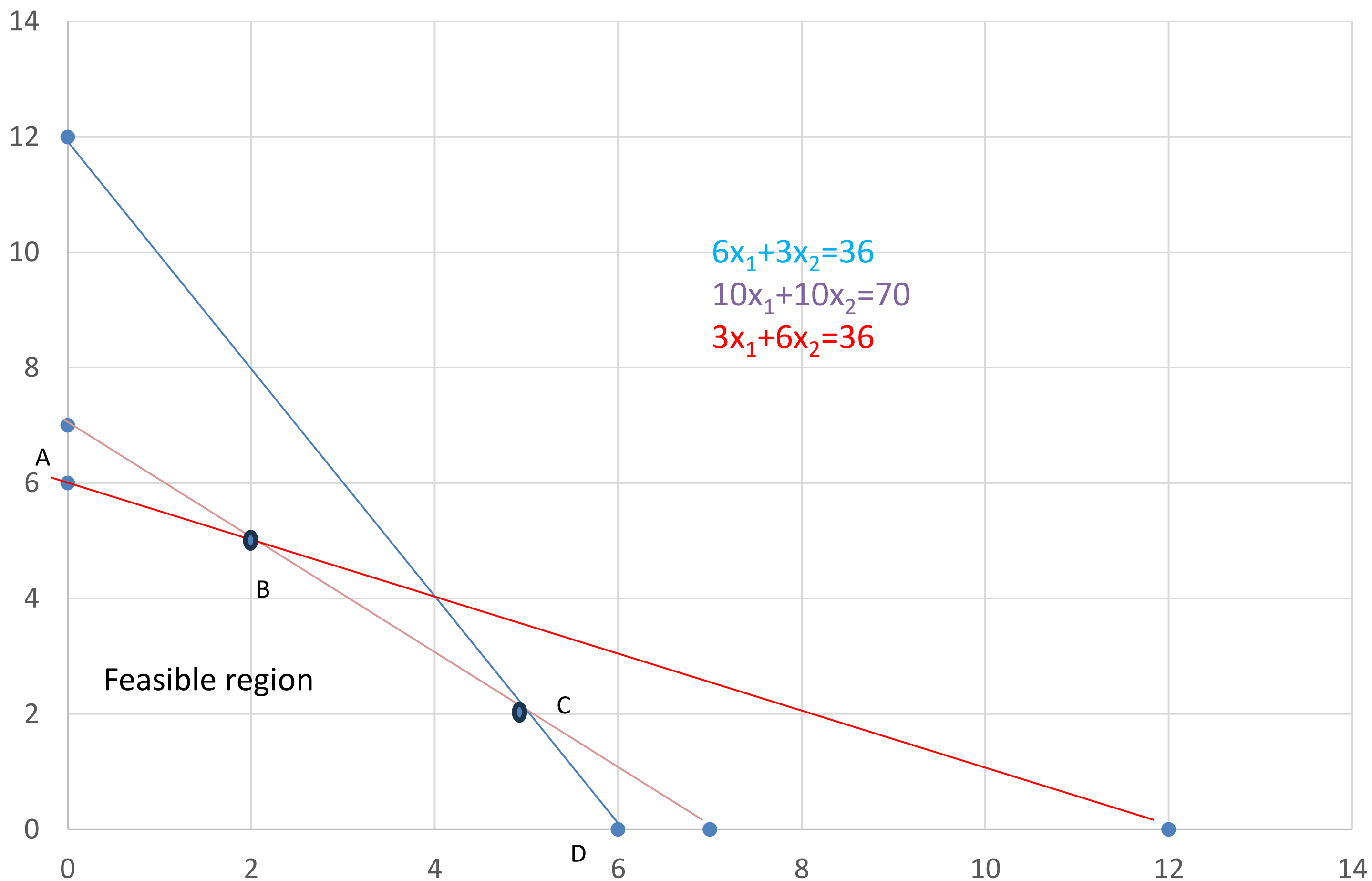
Linear Programming Problem (LPP)

Terminologies:

- **Feasible region:** The common region which satisfies all the constraints. Every point inside the feasible regions are feasible solution.
- **Optimum solution:** is a feasible solution where the objective function reaches its maximum (or minimum) value.

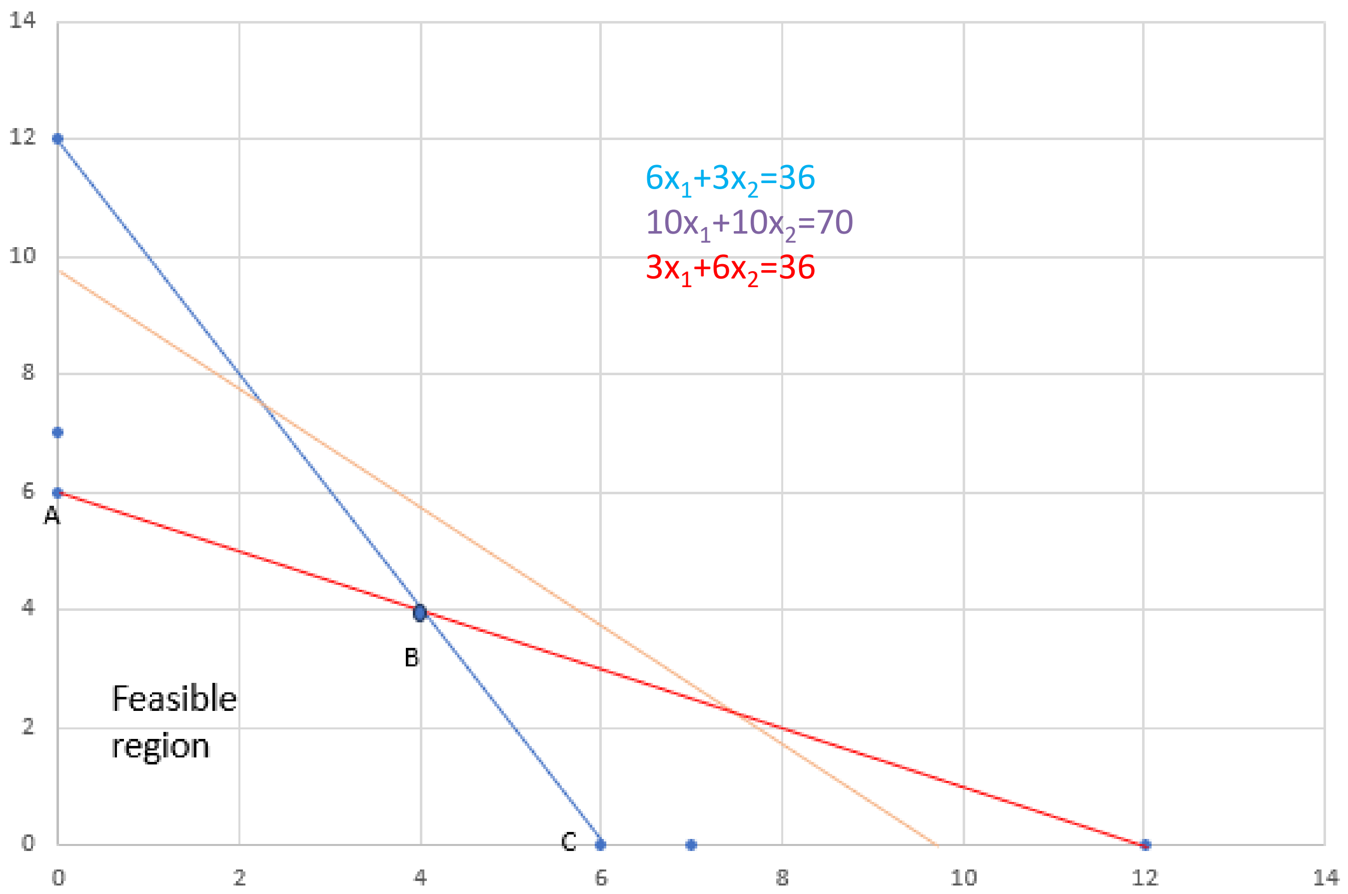
Linear Programming Problem (LPP)

Sensitivity Analysis:



Linear Programming Problem (LPP)

Sensitivity Analysis:



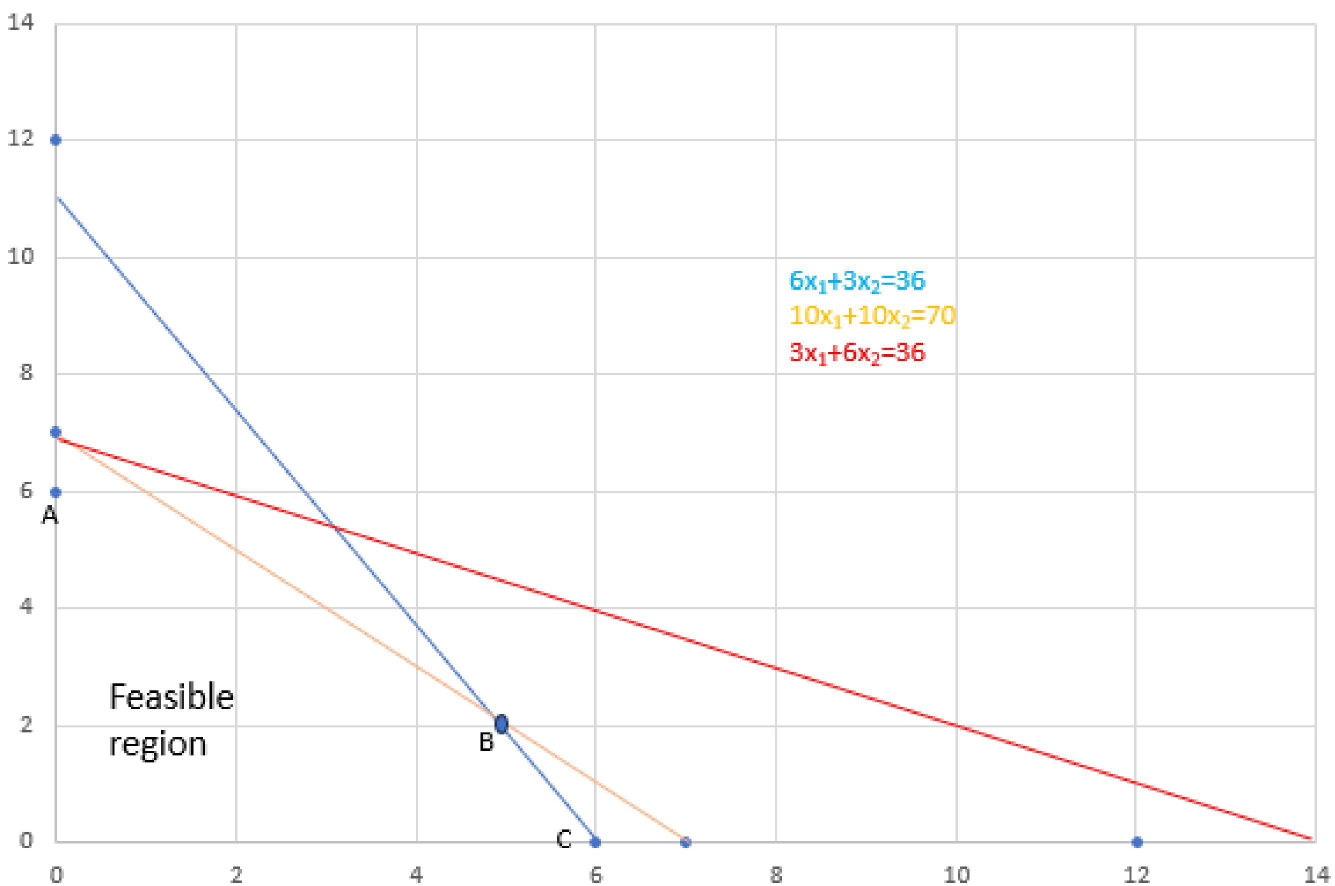
At X_1=4 X_2 = 4
Z = 20000
Incremental Profits = 20 -19 = 1000

The capacity of machine 2 can be increased to:
$10x_1+10x_2=70$
40 + 40 = 80 hours

Permissible increase in resources 2:
80 - 70 = 10 hours

Linear Programming Problem (LPP)

Sensitivity Analysis:



At X_1=0 X_2 = 7
Z = 21000
Incremental Profits = 21 -19 = 2000

The capacity of machine 3 can be increased to:
$3x_1+6x_2=36$
0 + 42 = 42 hours

Permissible increase in resources 3:
42 - 36 = 6 hours

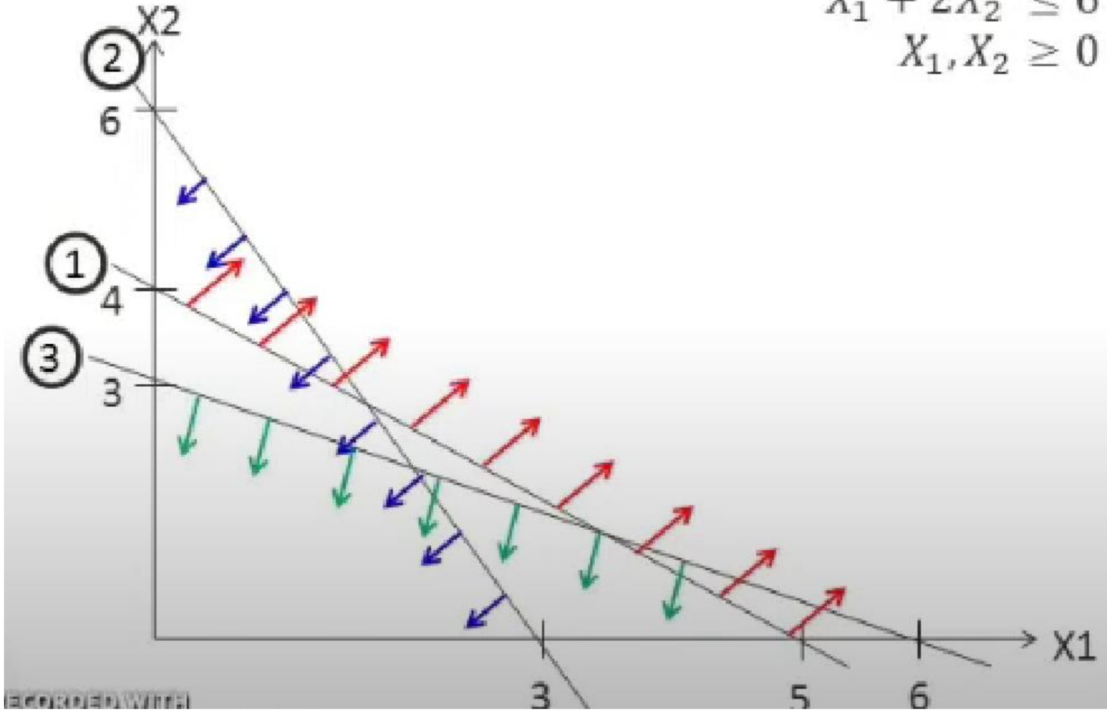
Linear Programming Problem (LPP)

Special Cases: No Feasible Solution

Maximize : $Z = 5X_1 + 4X_2$

Subject to :

$$
\begin{aligned}
4X_1 + 5X_2 &\geq 20 \\
6X_1 + 3X_2 &\leq 18 \\
X_1 + 2X_2 &\leq 6 \\
X_1, X_2 &\geq 0
\end{aligned}
$$



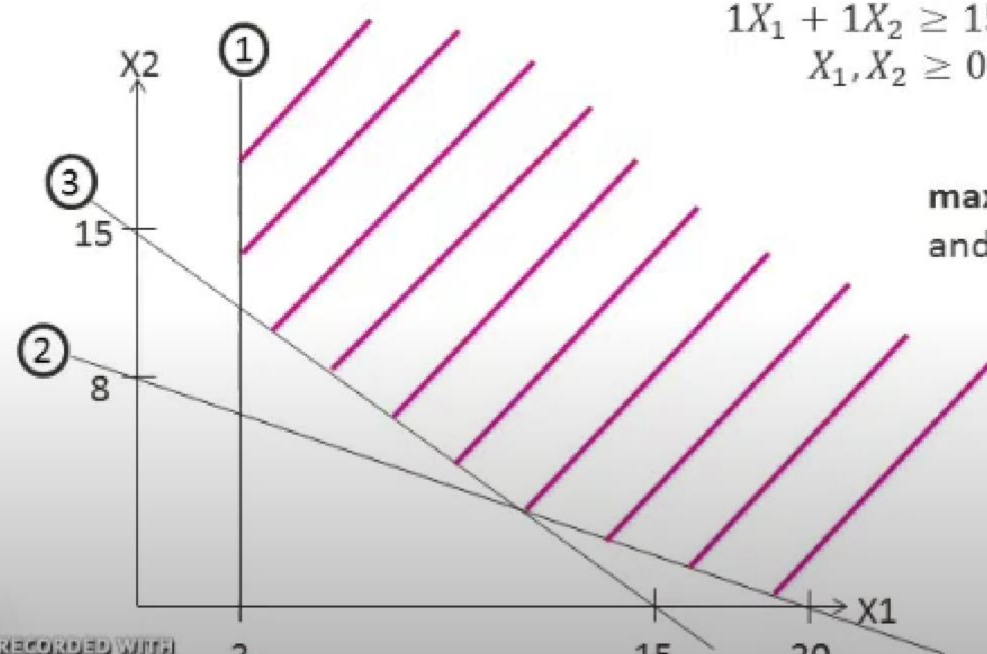
Linear Programming Problem (LPP)

Special Cases: Unbounded Solution

Maximize : $Z = 5X_1 + 20X_2$

Subject to :

$$\begin{aligned} 4X_1 \quad\quad\quad &\geq 12 \\ 2X_1 + 5X_2 &\geq 40 \\ 1X_1 + 1X_2 &\geq 15 \\ X_1, X_2 &\geq 0 \end{aligned}$$



This model is a **maximization** problem and the feasible region is **open**.

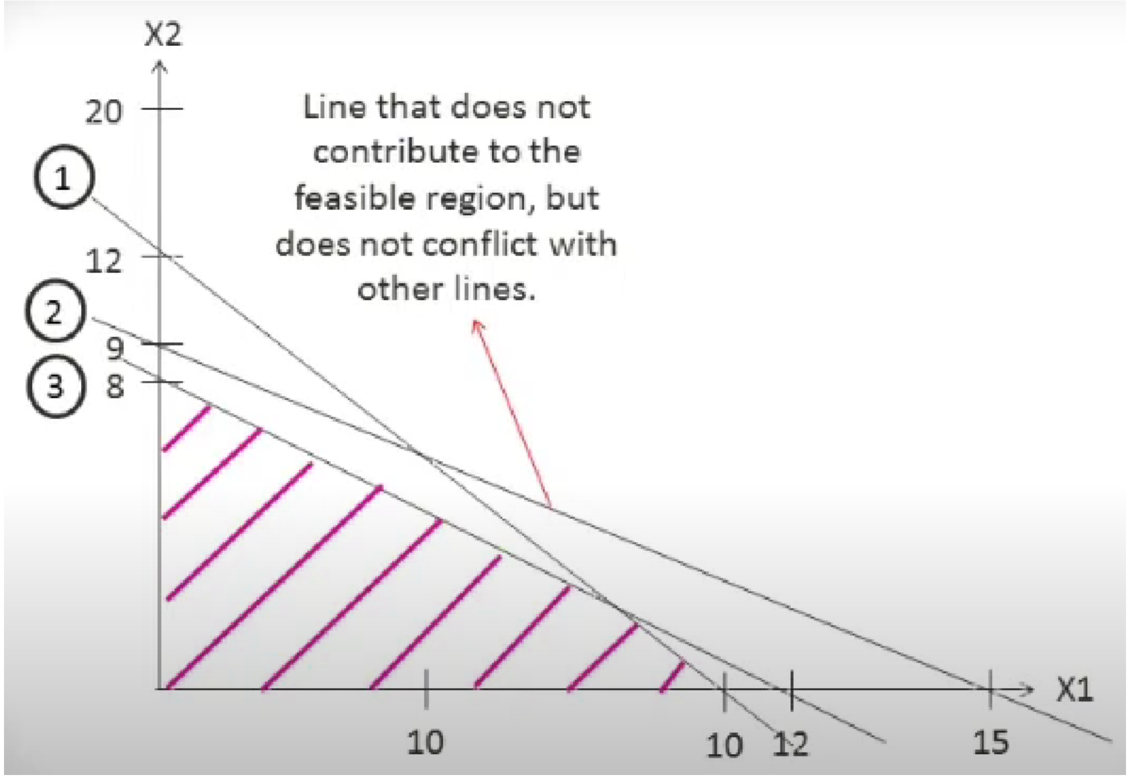
Linear Programming Problem (LPP)

Special Cases: Redundancy

Maximize : $Z = 2X_1 + 5X_2$

Subject to :

$$6X_1 + 5X_2 \leq 60$$
$$2X_1 + 3X_2 \leq 24$$
$$3X_1 + 5X_2 \leq 45$$
$$X_1, X_2 \geq 0$$



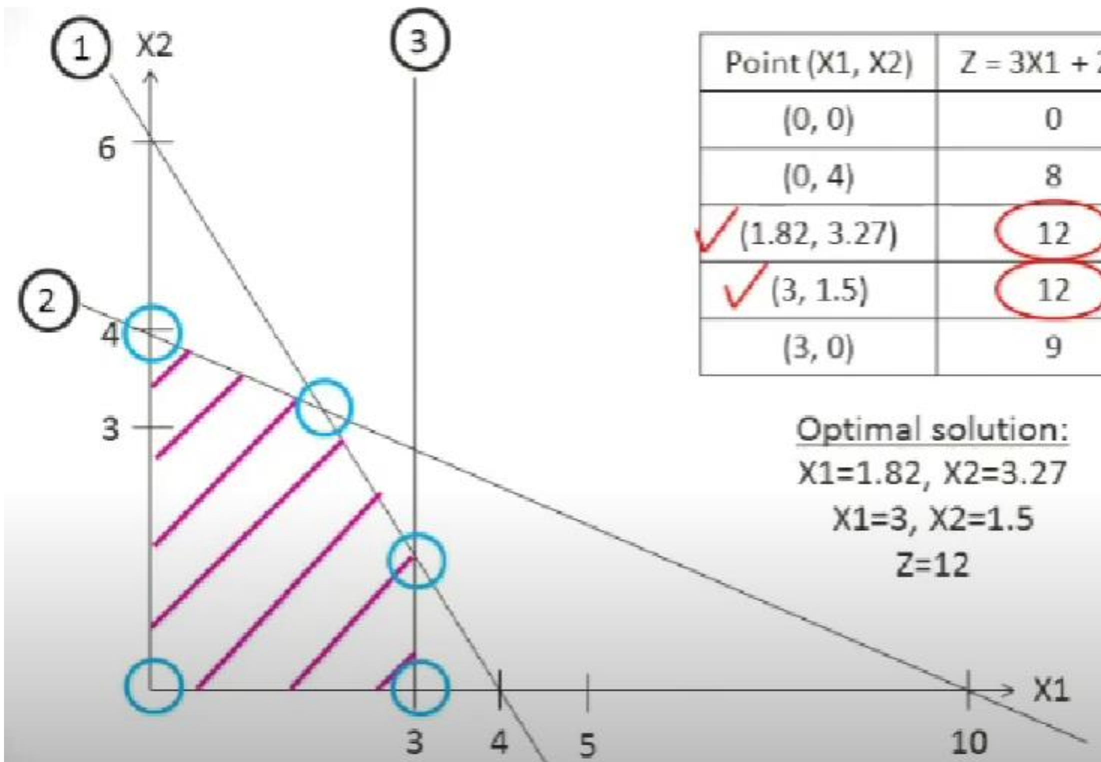
Linear Programming Problem (LPP)

Special Cases: Multiple Optimal Solution

Maximize : $Z = 3X_1 + 2X_2$

Subject to :

$$6X_1 + 4X_2 \leq 24$$
$$2X_1 + 5X_2 \leq 20$$
$$X_1 \leq 3$$
$$X_1, X_2 \geq 0$$



Point (X1, X2)	Z = 3X1 + 2X2
(0, 0)	0
(0, 4)	8
✓ (1.82, 3.27)	12
✓ (3, 1.5)	12
(3, 0)	9

Optimal solution:
X1=1.82, X2=3.27
X1=3, X2=1.5
Z=12

Linear Programming Problem (LPP)

Types of representation of LPP

1. **Canonical form:** The objective function is of maximization type; all constraints are inequalities of ≤ type, and all variables should be non-negative.
2. **Standard form:** The objective function is of maximization or minimization type, RHS of constraints equations should be non-negative, and all variables should be non-negative.

Linear Programming Problem (LPP)

- The general form of LPP

$$\text{Max/min} \quad z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

subject to:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \ (\leq, =, \geq)\ b_1 \\ a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \ (\leq, =, \geq)\ b_2 \\ \quad\vdots \\ a_{m1}x1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ (\leq, =, \geq)\ b_m \end{cases}$$

x_j = decision variables
b_i = constraint levels
c_j = objective function coefficients
a_{ij} = constraint coefficients

Linear Programming Problem (LPP)

Any form of LPP can be reduced to standard form using the following transformation:

- A constraint of inequality ≤ can be reduced to an equation by adding a non-negative variable called the **slack variable (S)** to the LHS of the inequality.
- A constraint inequality of the form ≥ can be reduced to an equivalence by subtracting a non-negative variable called the **surplus variable (S)** from LHS of the inequality and also an **artificial variable (A)** is added to the equation.

Linear Programming Problem (LPP)

- **The general form of LPP**

Consider the general linear programming problem involving n variables and m constraints $(m \leq n)$:

Determine the values of variables $x_1, x_2, \ldots, x_n$ which

$$
\begin{aligned}
\text{maximize} \quad & Z = c_1x_1 + c_2x_2 + \ldots + c_n x_n, \\
\text{subject to} \quad & a_{11} x_1 + a_{12} x_2 + \ldots + a_{1n} x_n \leq b_1, \\
& a_{21} x_1 + a_{22} x_2 + \ldots + a_{2n} x_n \leq b_2, \\
& \vdots \qquad\quad \vdots \\
& a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn} x_n \leq b_m,
\end{aligned}
$$

where $\quad x_1, x_2, \ldots, x_n \geq 0.$

Linear Programming Problem (LPP)

- **The standard form of LPP**

maximize $\quad Z = c_1x_1 + c_2x_2 + \ldots + c_n x_n,$

subject to

$$\left.\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+1} &= b_1, \\
a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n + x_{n+2} &= b_2, \\
\vdots \qquad\qquad\qquad\qquad\qquad\qquad &\;\;\vdots \\
a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n + x_{n+m} &= b_m,
\end{aligned}\right\}\ \textbf{Constraints}$$

where $\quad x_1, x_2, \ldots, x_n, x_{n+1}, x_{n+2}, \ldots, x_{n+m} \geq 0.$

slack variables $x_{n+1}, x_{n+2}, \ldots, x_{n+m}$ in the constraints

Linear Programming Problem (LPP)

Terminologies

- **Solution:** A set of variables x_1, x_2, ….. x_{n+m} is a solution if it satisfies the constraints.
- **Feasible Solution:** A set of variables x_1, x_2, ….. x_{n+m} is a feasible solution if it satisfies the constraints and non-negativity restrictions.
- **Basic Solution:** A solution obtained by setting any n variables (among m+n variables) equal to zero and solving the remaining m variables.
  - The m variables are called **basic variables**
  - All the n variables that have been put equal to zero each is **non-basic variables**

Linear Programming Problem (LPP)

Terminologies

- **Basic Feasible Solution:** A basic solution that also satisfies the non-negativity restrictions.
  - All variables in a basic feasible solution are ≥ 0.
  - Every basic feasible solution of a problem is an extreme point of the convex set of feasible solutions and every extreme point is a basic feasible solution of the set of constraints.

Linear Programming Problem (LPP)

Simplex Method

- The computation method is based on fundamental property "*that the optimal solution to an LP problem, if it exists, occurs only at one of the corner points of the feasible region*".
- Starts with initial basic feasible solution i.e., origin which is one of the corner points of the feasible region. This solution is tested i.e., it is ascertained whether improvements in the value of objective function is possible by moving to the corner points of the feasible region.

Linear Programming Problem (LPP)

Simplex Method: Steps

1. Express the problem in standard form
2. Find initial basic feasible solution
3. Perform optimality test
4. Iterate towards an optimal solution
5. Perform optimality test for second feasible solution

Linear Programming Problem (LPP)

Simplex Method: Steps

1. *Express the problem in standard form*
   - If the decision variables are non-negative, RHS of the constraints are non-negative, and the constraints are expressed as equations.
   - If the equations are of ≤ type, add slack variables to LHS to convert it into =.
   - Slack variables represent idle resources and contribute zero to the objective function.

Linear Programming Problem (LPP)

Simplex Method: Steps

2. *Find initial basic feasible solution*

- A start is made with a feasible solution by assuming profit earned is zero (here $x_1 = x_2 = 0$).
- The variables with zero values are non-basic variables and the values we get for s1 and s2 (basic variables) on substitution is initial basic feasible solution.

Linear Programming Problem (LPP)

Simplex Method: Steps

2. *Find initial basic feasible solution*

Example Numerical

	Contribution/unit c_j Basis (Basic variables)	3	4	0	0	
		Body matrix		*Identity matrix*		
c_B		x_1	x_2	s_1	s_2	b
0	s_1	1	1	1	0	450
0	s_2	2	1	0	1	600

Linear Programming Problem (LPP)

Simplex Method: Steps

2. *Find initial basic feasible solution*

Example Numerical

	Contribution/unit c_j Basis (Basic variables)	3	4	0	0	
		Body matrix		Identity matrix		
c_B		x_1	x_2	s_1	s_2	b
0	s_1	1	1	1	0	450
0	s_2	2	1	0	1	600

- ***C_B*** **:** Coefficient of the current basic variable in the objective function.
- **Basis (product mix column):** Basic variables (i.e., slack variables) of the current solution.
- **The body matrix:** Resource coefficients i.e., resource required to make a unit of product.

Linear Programming Problem (LPP)

Simplex Method: Steps

2. *Find initial basic feasible solution*

Example Numerical

	Contribution/unit c_j Basis (Basic variables)	3	4	0	0	
		Body matrix		Identity matrix		
c_B		x_1	x_2	s_1	s_2	b
0	s_1	1	1	1	0	450
0	s_2	2	1	0	1	600

- **Identity matrix:** Coefficients of the slack variables in the constraints.
- **b-column (quantity column):** Quantities of available resources or RHS values of the constraints or values of the basic variables in the basic feasible solution found earlier.
- Variables not entered under basis column are non-basic and value is 0.

Linear Programming Problem (LPP)

Simplex Method: Steps

3. *Perform optimality test*

- To check whether initial basic feasible solution can be improved or not.
- This solution involves zero profit; an improved solution should result in profits greater than 0.

Example Numerical

	c_j	3	4	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	1	(1)	1	0	450	450 ← (*key row*)
0	s_2	2	1	0	1	600	600
	Z_j	0	0	0	0	0	(Profit lost/unit)
(N.E.R.)	c_j-Z_j	3	4	0	0		(Net profit/unit)
			↑ K				

Linear Programming Problem (LPP)

Simplex Method: Steps

3. *Perform optimality test*

Example Numerical

	c_j	3	4	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	1	(1)	1	0	450	450 ← (*key row*)
0	s_2	2	1	0	1	600	600
	Z_j	0	0	0	0	0	(Profit lost/unit)
(N.E.R.)	c_j-Z_j	3	4	0	0		(Net profit/unit)
			↑ K				

- **Z_j value** under b-column represent the current profit.
  - Z_j value represent the contribution lost per unit of variables.
- **$C_j – Z_j$ (Index row or net evaluation row):** This row determines the optimal solution.

Linear Programming Problem (LPP)

Simplex Method: Steps

4. *Iterate towards an optimal solution*

- At each iteration, the simplex method moves the current basic feasible solution to an improved basic feasible solution.
- Selection of the entering variable
  - Maximum positive $\mathbf{c_j - z_j}$ value
- Selection of the leaving variable
  - Elements under quantity column is divided by the corresponding element of the key column and the row containing the minimum non-negative ratio is marked.
  - The critical ratio (last column) indicates the number of units of a variable that can be produced by trading all the current levels of basic variables.

Linear Programming Problem (LPP)

Simplex Method: Steps

4. *Iterate towards an optimal solution*

Example Numerical

	c_j	3	4	0	0		
c_B	Basis	x_1	x_2	s_1	s_2	b	θ
0	s_1	1	(1)	1	0	450	450 ← (*key row*)
0	s_2	2	1	0	1	600	600
	Z_j	0	0	0	0	0	(Profit lost/unit)
(N.E.R.)	c_j-Z_j	3	4	0	0		(Net profit/unit)
			↑ K				

Critical ratio column

– In critical ratio column, zero is considered as non-negative and negative ratios are discarded.

– All ratios are negative or infinity the solution is unbounded.

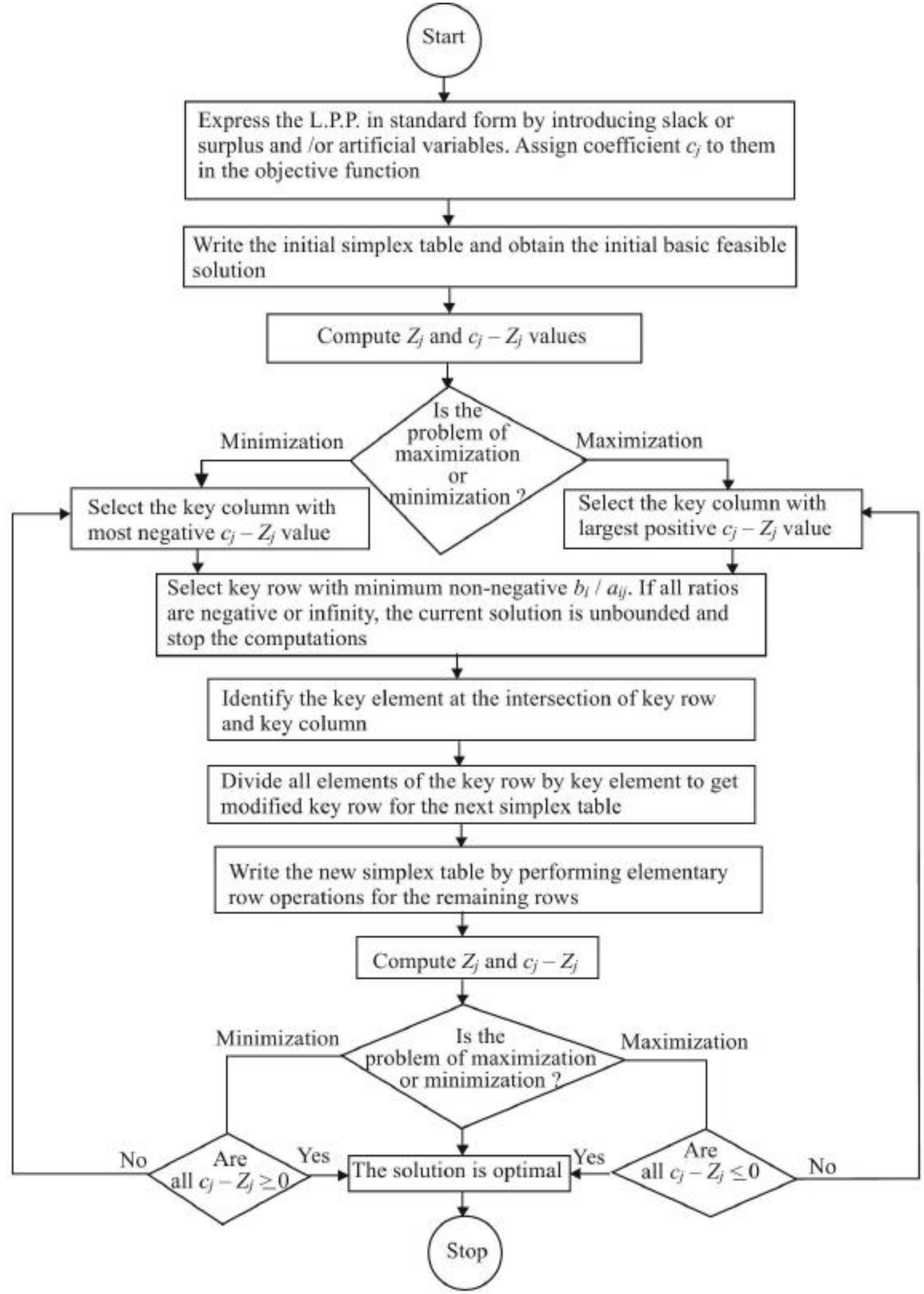
Linear Programming Problem (LPP)

Simplex Method: Steps

5. Perform optimality test for second feasible solution

- **Cj – Zj (Index row or net evaluation row):** All elements should be 0 or negative, the solution is optimal.
- The computational procedure comes to an end.
- Values of the basic variables (basis column) can be seen in the quantity column and **Zj value** under critical ratio column gives the total contribution (profits).

Flowchart for the Simplex Algorithm



Linear Programming Problem (LPP)

Simplex Method: Remarks

- The optimal solution satisfies the non-negativity constraints, the constraints and gives maximum possible value of Z.
- The availability of values for the slack variables in the optimal solution indicates that the resources are underutilized.
- Elements in Cj – Zj row under slack variables indicates shadow prices (or accounting values) of the resources.

Linear Programming Problem (LPP)

Artificial Variable Techniques

- The constraints are of (≤) type with non-negativity RHS and the introduction of slack variables provided the initial basic feasible solution.
- In many LPP, slack variables cannot provide such a solution.
- In these problems, at least one of the constraints is of (≥) or (=) type and artificial variables will be introduced.
- *Artificial variables are fictious and no physical meaning. They assume the role of slack variables in the first iteration, only to be replaced at a later iteration. Thus, they are merely a device to get the starting basic feasible solution so that simple algorithm can be applied as usual to get the optimal solution.*

Linear Programming Problem (LPP)

Any form of LPP can be reduced to standard form using the following transformation:

- A constraint of inequality ≤ can be reduced to an equation by adding a non-negative variable called the **slack variable (S)** to the LHS of the inequality.
- A constraint inequality of the form ≥ can be reduced to equivalence by subtracting a non-negative variable called the **surplus variable (S)** from LHS of the inequality and also an **artificial variable (A)** is added to the equation.

Linear Programming Problem (LPP)

Artificial Variable Techniques

- There are two (closely related) techniques available to solve the problems:
  1. The big M- method or M-technique or method of penalties
  2. The two-phase method

Linear Programming Problem (LPP)

The big M- method: Steps

1. Express the LPP in standard form by introducing slack variables.
   - Add it to LHS of the constraints when (≤) type
   - Subtract it from LHS of the constraints when (≥) type
2. Add non-negativity variables to the LHS of all the constraints of initially (≥) or (=) type. These variables are called artificial variables.
   - Artificial variables should not appear in the final solution.
     - These variables are assigned a very large per unit penalty in the objective function.
     - The penalty is designated by –M for maximation and M for minimization problems, where M >0.
     - Value of M is larger or much higher than the cost coefficients of other.

Linear Programming Problem (LPP)

Sensitivity Analysis

- In an LP model, the coefficients (parameters) are assumed to be constant and known with certainty during a planning period.
- These input parameters value may change due to dynamic nature of the business environment.
- These parameters may raise doubt on the validity of the optimal solution of the given LP model.
- A decision-maker, in such situations, would like to know how changes in these parameters may affect the optimal solution and the range within which the optimal solution will remain unchanged.

Linear Programming Problem (LPP)

Sensitivity Analysis

- Sensitivity analysis and parametric linear programming are the two techniques that are used to evaluate the effect on an optimal solution of any LP problem due to changes in its parameters.
  - **Sensitivity analysis** determines the sensitivity range (both lower and upper limit) within which the LP model parameters can vary (one at a time) without affecting the optimality of the current solution.
  - **Parametric analysis** is the study of measuring the effect on the optimal solution of the LP model due to changes at a time in more than one input parameter value outside the sensitivity range.
- It provides the sensitive ranges (both lower and upper limits) within which the LP model parameters can vary without changing the optimality of the current optimal solution.

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Any Questions