Question Paper

Exam Date & Time: 29-Sep-2023 (10:45 AM - 12:45 PM)

MANIPAL ACADEMY OF HIGHER EDUCATION
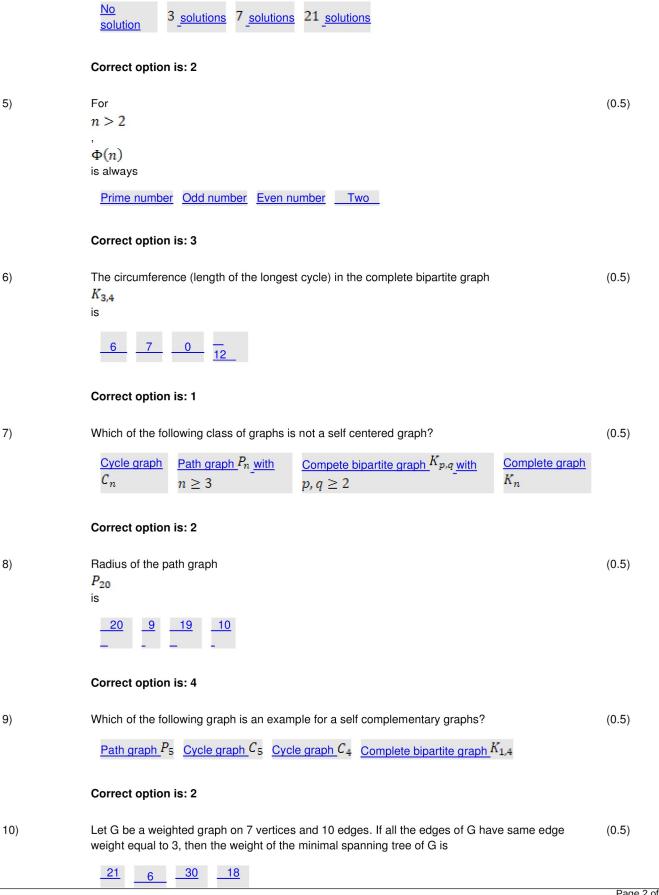
B.Tech Vth Semester Midterm Examination September 2023

MATHEMATICAL FOUNDATIONS FOR DATA SCIENCE-III [MAT 3151]

Marks: 30   **Duration: 120 mins.**

MCQ

Answer all the questions.   Section Duration: 20 mins

1)   Let a and b be two integers, not both zero. Then $\gcd(a, b) = 1$ if and only if   (0.5)

$1 = ax + by$, for some integers x and y	$1 = ax + by$, for all integers x and y	$1 = ax + by$, for unique integers x and y	$0 = ax + by$, for some integers x and y

Correct option is: 1

2)   If p is a prime and $p \mid ab,$ then   (0.5)

$p \mid a$ or $p \mid b$	$p \mid a$ and $p \mid b$	$p \mid a$, always	$p \mid 1$

Correct option is: 1

3)   If $ca \equiv cb \,(mod\,n)$ and $\gcd(c, n) = d$, then   (0.5)

$a \equiv b \,(mod\,n)$	$a \equiv b \,(mod\,d/n)$	$a \equiv b \,(mod\,d)$	$a \equiv b \,(mod\,n/d)$

Correct option is: 4

4)   The linear congruence   (0.5)

$9x \equiv 21 \,(mod\,30)$

has how many incongruent solutions?

No solution   3 solutions   7 solutions   21 solutions

Correct option is: 2

5) For
$n > 2$
,
$\Phi(n)$
is always   (0.5)

Prime number   Odd number   Even number   Two

Correct option is: 3

6) The circumference (length of the longest cycle) in the complete bipartite graph $K_{3,4}$ is   (0.5)

6   7   0   12

Correct option is: 1

7) Which of the following class of graphs is not a self centered graph?   (0.5)

Cycle graph C_n Path graph P_n with $n \geq 3$ Compete bipartite graph $K_{p,q}$ with $p, q \geq 2$ Complete graph K_n

Correct option is: 2

8) Radius of the path graph P_{20} is   (0.5)

20   9   19   10

Correct option is: 4

9) Which of the following graph is an example for a self complementary graphs?   (0.5)

Path graph P_5 Cycle graph C_5 Cycle graph C_4 Complete bipartite graph $K_{1,4}$

Correct option is: 2

10) Let G be a weighted graph on 7 vertices and 10 edges. If all the edges of G have same edge weight equal to 3, then the weight of the minimal spanning tree of G is   (0.5)

21   6   30   18

Correct option is: 4

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DESCRIPTIVE

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Answer all the questions.

11)   Solve the simultaneous congruences using Chinese Remainder Theorem   (4)

$$x \equiv 6 \ (mod \ 11)$$

$$x \equiv 13 \ (mod \ 16)$$

$$x \equiv 9 \ (mod \ 21)$$

$$x \equiv 19 \ (mod \ 25)$$

12)   If $diameter(G) \geq 3$, then show that $diameter(\bar{G}) \leq 3$. Hence, show that every non trivial   (4)
        self complementary graph has diameter 2 or 3.

13)   If m and n are relatively prime integers, then   (3)

        $m^{\Phi(n)} + n^{\Phi(m)} \equiv 1 \ (mod \ mn)$, where Φ denotes the Euler's Phi function.

14)   A) Find the residue of 128^{129} divided by 17.   (3)

        B) Find $29^{25} \ (mod \ 11)$.

15)   Find the value of the following: [One can use the fact that for any odd number   (3)
        , $\left(\frac{2}{q}\right) = (-1)^{(q^2-1)/8}$]

        A. Legendre symbol $\left(\frac{29}{541}\right)$

        B. Jacobi symbol $\left(\frac{109}{385}\right)$

16)   Prove that a tree with n vertices has n-1 edges.   (3)

17)   Let G be a connected graph with at least 3 vertices. Show that G is bipartite if and only if it has no   (3)
        odd cycles.

18)   Prove that if a graph G has n vertices and minimum degree $\delta(G) \geq \frac{n-1}{2}$, then G is connected.   (2)

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