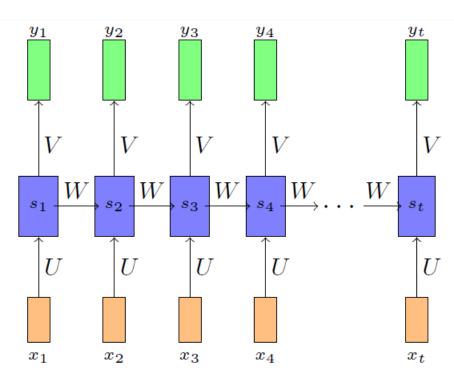
DSE 3121 DEEP LEARNING

Advanced RNNs - LSTM, GRU, BRNN

Dr. Rohini Rao & Dr. Abhilash K Pai

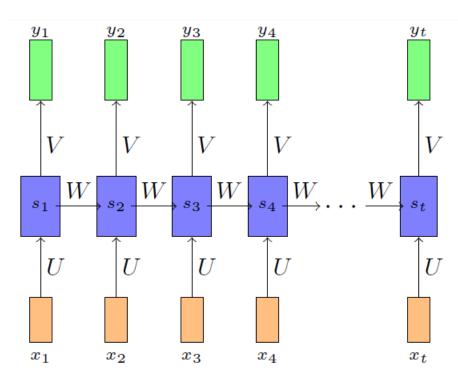
Dept. of Data Science and Computer Applications

MIT Manipal

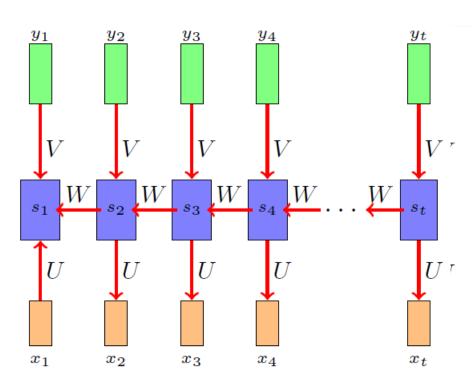


Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

- The state (s_i) of an RNN records information from all previous time steps.
- At each new timestep the old information gets morphed by the current input.
- After 't' steps the information stored at time step tk (for some k < t) gets completely morphed.
- It would be impossible to extract the original information stored at time step t k.

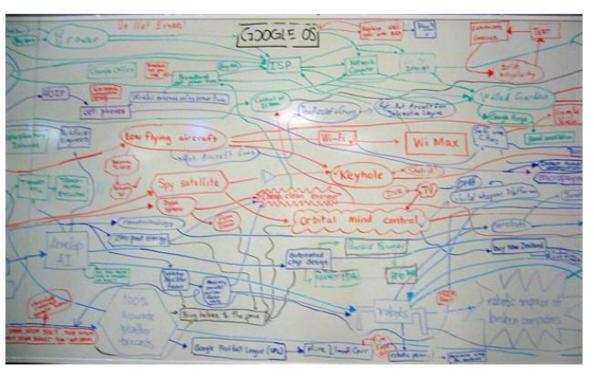


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- Also, there is the Vanishing gradients problem!



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- Also, there is the Vanishing gradients problem!

The white board analogy:



Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

• Consider a scenario where we have to evaluate the expression on a whiteboard:

Evaluate "
$$ac(bd+a) + ad$$
"
given that $a=1$, $b=3$, $c=5$, $d=11$

 Normally, the evaluation in white board would look like:

$$ac = 5$$
 $bd = 33$
 $bd + a = 34$
 $ac(bd + a) = 170$
 $ad = 11$
 $ac(bd + a) + ad = 181$

 Now, if the white board has space to accommodate only 3 steps, the above evaluation cannot fit in the required space and would lead to loss of information.

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A solution is to do the following:

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- A solution is to do the following:
 - Selectively write:

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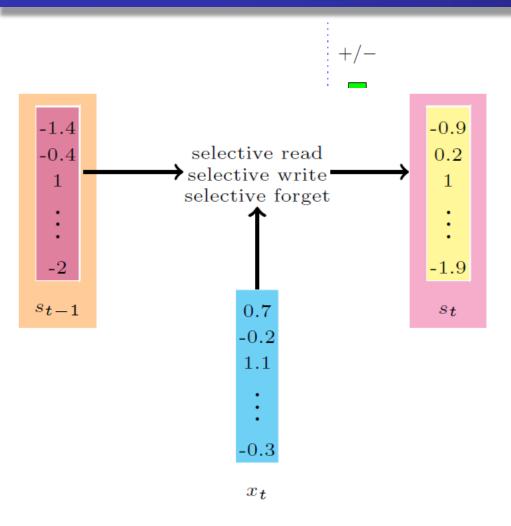
Now the board is full

• So, Selectively forget:

$$ac(bd + a) + ad = 181$$

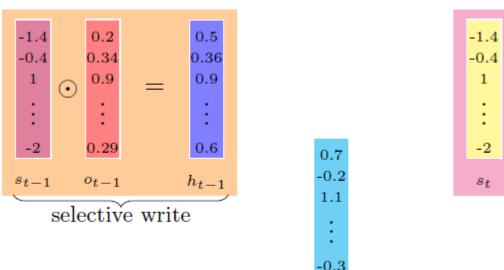
 $ac(bd + a) = 170$
 $ad = 11$

Since the RNN also has a finite state size, we need to figure out a way to allow it to selectively read, write and forget



- RNN reads the document from left to right and after every word updates the state.
- By the time we reach the end of the document the information obtained from the first few words is completely lost.
- In our improvised network, ideally, we would like to:
 - Forget the information added by stop words (a, the, etc.)
 - Selectively read the information added by previous sentiment bearing words (awesome, amazing, etc.)
 - **Selectively write** new information from the current word to the state.

Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras



The RNN has to learn o_{t-1} along with other parameters (W,U,V)

 x_t

$$o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$

$$h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$$

New parameters to be learned are: W_o , U_o , b_o

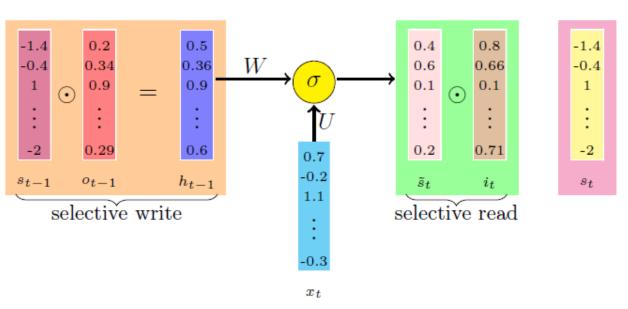
 O_t is called the output gate as it decides how much to pass (write) to the next time step.

Selectively write:

• In an RNN, the state s₁ is defined as follows:

$$s_t = \sigma(Ws_{t-1} + Ux_t)$$
 (ignoring bias)

- Instead of passing s_{t-1} as it is we need to pass (write) only some portions of it.
- To do this, we introduce a vector $\mathbf{o_{t-1}}$ which decides what fraction of each element of $\mathbf{s_{t-1}}$ should be passed to the next state.
- Each element of o_{t-1} (restricted to be between 0 and 1) gets multiplied with s_{t-1}
- How does RNN know what fraction of the state to pass on?



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Selectively read:

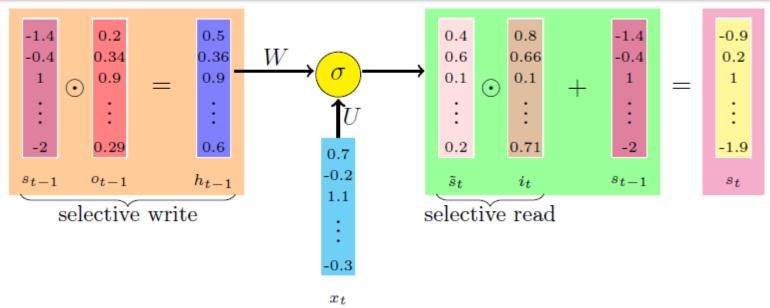
• We will now use h_{t-1} and x_t to compute the new state at the time step t :

$$\tilde{s_t} = \sigma(Wh_{t-1} + Ux_t + b)$$

- Again, to pass only useful information from $\tilde{s_t}$ to s_t , we selectively read from it before constructing the new cell state.
- To do this we introduce another gate called as the input gate:

$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

• And use $i_t \odot \tilde{s_t}$ to selectively read the information.

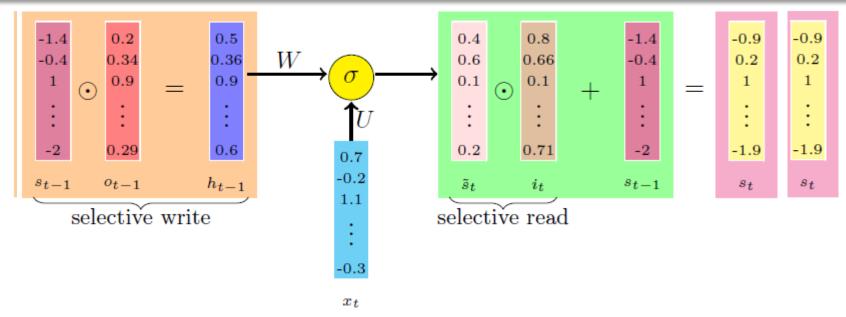


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Selectively forget

• How do we combine s_{t-1} and $\tilde{s_t}$ to get the new state?

$$s_t = s_{t-1} + i_t \odot \tilde{s_t}$$



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Selectively forget

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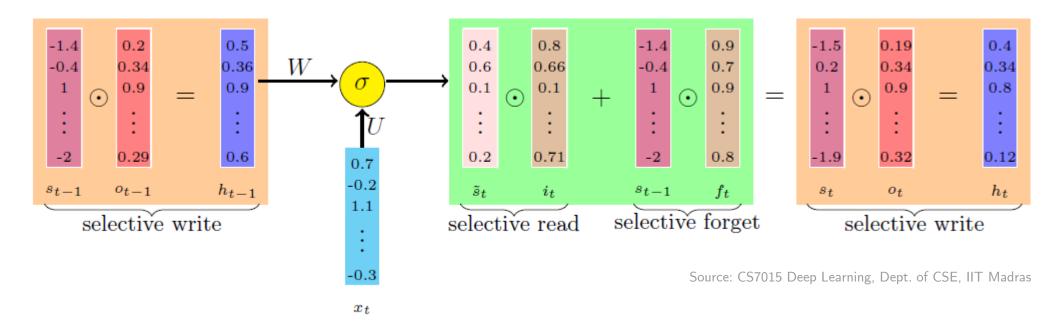
$$s_t = s_{t-1} + i_t \odot \tilde{s_t}$$

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s_t}$$

- But we may not want to use the whole of s_{t-1} but forget some parts of it.
- To do this a forget gate is introduced:

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

LSTM (Long Short-Term Memory)



Gates:

$$o_t = \sigma(W_o h_{t-1} + U_o x_t + b_o)$$

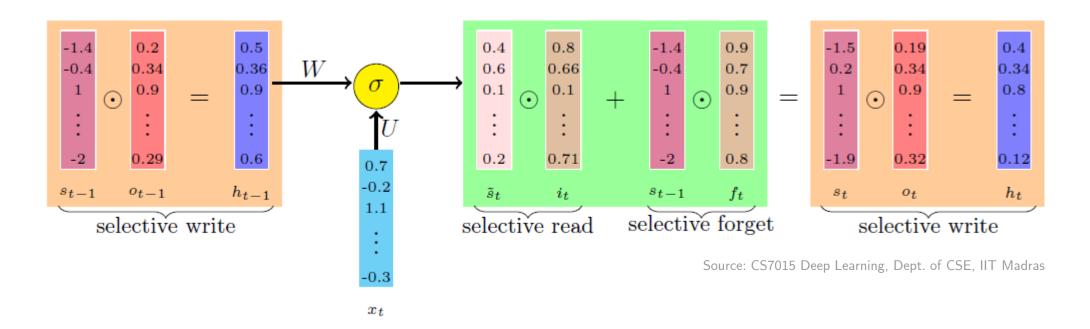
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

States:

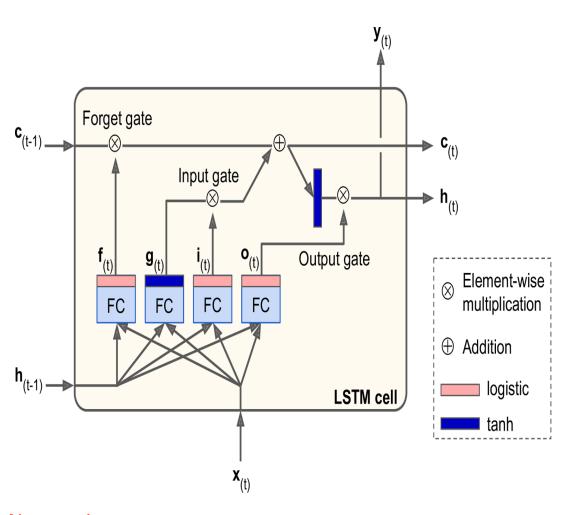
Long-term memory
$$\begin{split} \tilde{s_t} &= \sigma(W h_{t-1} + U x_t + b) \\ & \underbrace{s_t} = f_t \odot s_{t-1} + i_t \odot \tilde{s_t} \\ & \underbrace{h_t} = o_t \odot \sigma(s_t) \text{ and } rnn_{out} \\ & = h_t \end{split}$$
 Short-term memory

LSTM (Long Short-Term Memory)



- LSTM has many variants which include different number of gates and also different arrangement of gates.
- A popular variant of LSTM is the Gated Recurrent Unit (GRU).

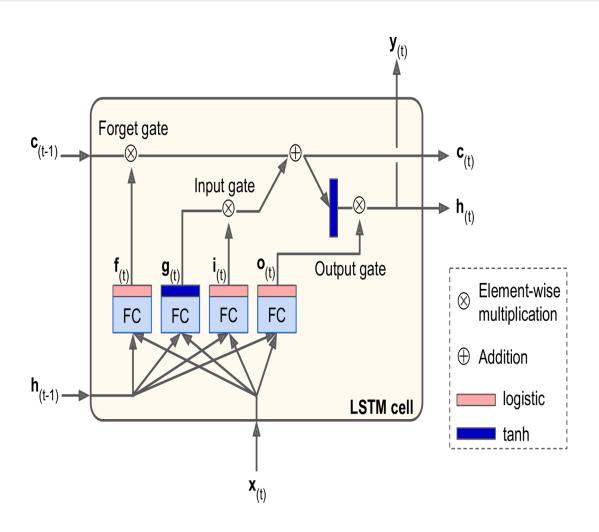
LSTM Cell



- Neuron called a Cell
- FC are fully connected layers
- Long Term state $c_{(t-1)}$ traverses through forget gate forgetting some memories and adding some new memories
- Long term state $c_{(t-1)}$ is passed through tanh and then filtered by an output gate, which produces short term state $h_{(t)}$
- Update gate- $g_{(t)}$ takes current input $x_{(t)}$ and previous short term state $h_{(t-1)}$
- Important parts of output g_(t) goes to long term state

Note: c_t is same as s_t

LSTM Cell



 Gating Mechanism- regulates information that the network stores

Other 3 layers are gate controllers

- Forget gate f_(t) controls which part of the long-term state should be erased
- Input gate $i_{(t)}$ controls which part of $g_{(t)}$ should be added to long term state
- Output gate o_(t) controls which parts of long term state should be read and output at this time state
 - both to h_(t) and to y_(t)

LSTM computations

$$\mathbf{i}_{(t)} = \sigma(\mathbf{W}_{xi}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hi}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{i})$$

$$\mathbf{f}_{(t)} = \sigma(\mathbf{W}_{xf}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hf}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{f})$$

$$\mathbf{o}_{(t)} = \sigma(\mathbf{W}_{xo}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{ho}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{o})$$

$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{g})$$

$$\mathbf{c}_{(t)} = \mathbf{f}_{(t)} \otimes \mathbf{c}_{(t-1)} + \mathbf{i}_{(t)} \otimes \mathbf{g}_{(t)}$$

$$\mathbf{y}_{(t)} = \mathbf{h}_{(t)} = \mathbf{o}_{(t)} \otimes \tanh(\mathbf{c}_{(t)})$$

An LSTM cell can learn to recognize an important input (role of the input gate), store it in long term state, preserve it for as long as possible it is needed (role of forget gate), and extract it whenever it is needed.

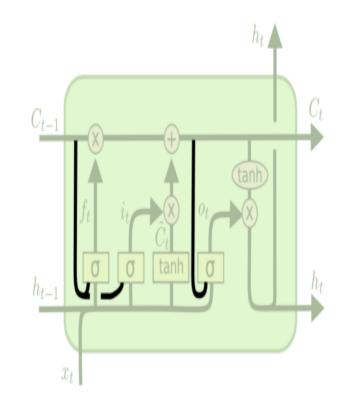
 \mathbf{W}_{xi} , \mathbf{W}_{xf} , \mathbf{W}_{xo} , \mathbf{W}_{xg} are the weight matrices of each of the four layers for their connection to the input vector $\mathbf{x}_{(t)}$.

 \mathbf{W}_{hi} , \mathbf{W}_{ho} , and \mathbf{W}_{hg} are the weight matrices of each of the four layers for their connection to the previous short-term state $\mathbf{h}_{(t-1)}$.

 \mathbf{b}_{i} , \mathbf{b}_{o} , and \mathbf{b}_{g} are the bias terms for each of the four layers.

LSTM with Peephole connections (Felix Gers and Jürgen Schmidhuber in 2000)

- In LSTM cell, the gate controllers get input $\mathbf{x}_{(t)}$ and $\mathbf{h}_{(t-1)}$.
- Can be given more context by letting them peek at the long-term state as well
- LSTM variant with extra connections called *peephole* connections
 - previous long-term state $\mathbf{c}_{(t-1)}$ is added as an input to the controllers of the forget gate and the input gate
 - current long-term state $\mathbf{c}_{(t)}$ is added as input to the controller of the output gate



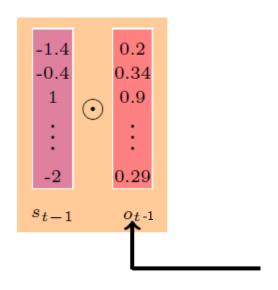
$$f_{t} = \sigma \left(W_{f} \cdot [C_{t-1}, h_{t-1}, x_{t}] + b_{f} \right)$$

$$i_{t} = \sigma \left(W_{i} \cdot [C_{t-1}, h_{t-1}, x_{t}] + b_{i} \right)$$

$$o_{t} = \sigma \left(W_{o} \cdot [C_{t}, h_{t-1}, x_{t}] + b_{o} \right)$$

-1.4 -0.4 1 \vdots -2 s_{t-1}

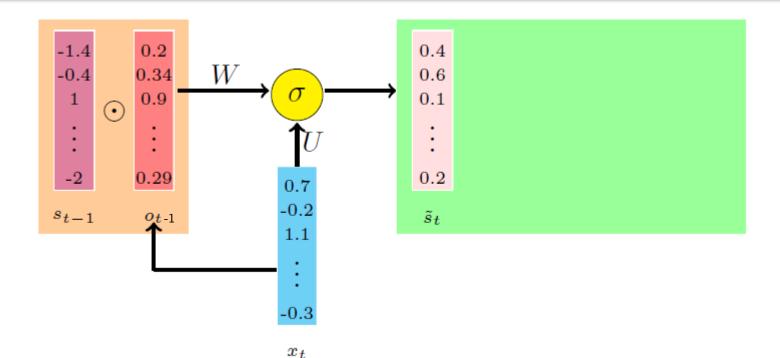
Gates: States:



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

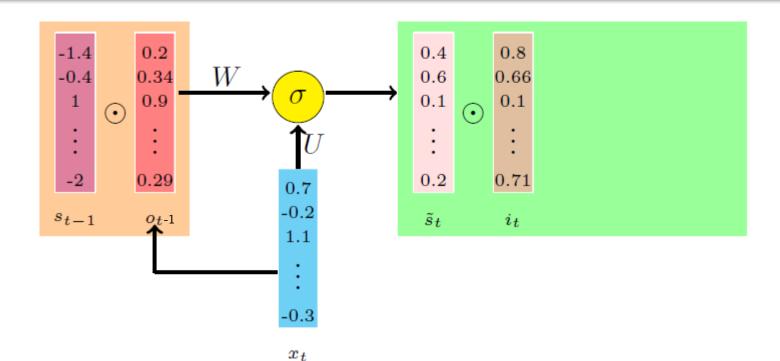


Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$



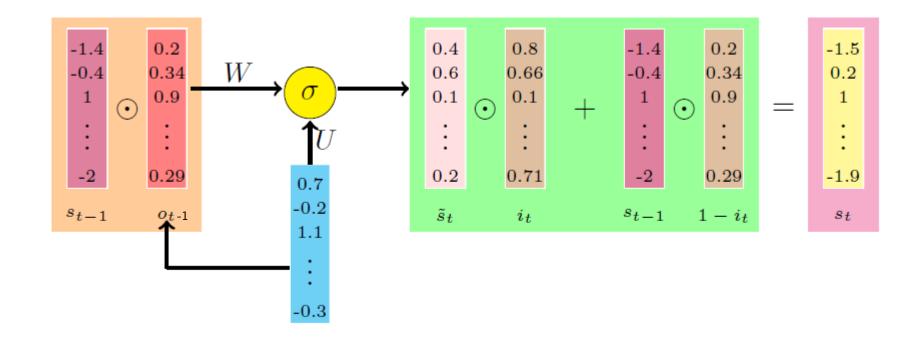
Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$

$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$



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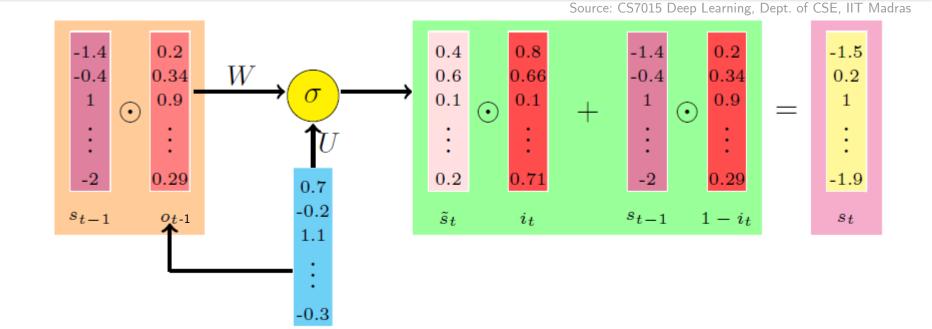
States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

Source: CS7015 Deep Learning, Dept. of CSE, IIT Madras

 x_t



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

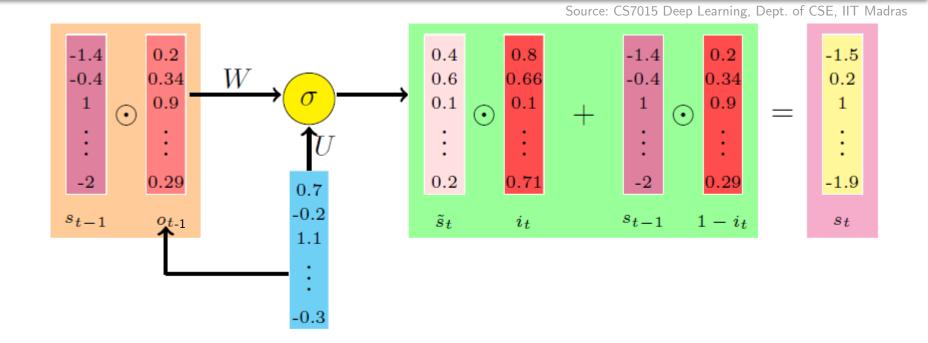
States:

$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

No explicit forget gate (the forget gate and input gates are tied)

 x_t



Gates:

$$o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$$
$$i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$$

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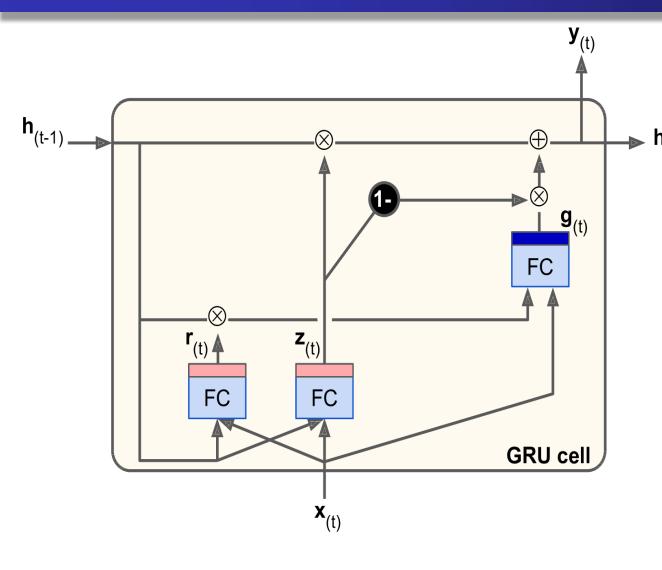
$$\tilde{s_t} = \sigma(W(o_t \odot s_{t-1}) + Ux_t + b)$$

$$s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s_t}$$

The gates depend directly on s_{t-1} and not the intermediate h_{t-1} as in the case of LSTMs

 x_t

Gated Recurrent Unit CELL (Kyunghyun Cho et al, 2014)



The main simplifications of LSTM are:

- Both state vectors (short and long term) are merged into a single vector $\mathbf{h}_{(t)}$.
- Gate controller $z_{(t)}$ controller controls both the forget gate and the input gate.
- If the gate controller outputs
 - 1, the forget gate is open and the input gate is closed.
 - 0, the opposite happens
 - whenever a memory must be written, the location where it will be stored is erased first.
- No output gate, the full state vector is output at every time step.
- Reset gate controller $r_{(t)}$ that controls which part of the previous state will be shown to the main layer $\mathbf{g}_{(t)}$.

LSTM vs GRU computation

$$\mathbf{i}_{(t)} = \sigma(\mathbf{W}_{xi}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hi}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{i})$$

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$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{g})$$

$$\mathbf{c}_{(t)} = \mathbf{f}_{(t)} \otimes \mathbf{c}_{(t-1)} + \mathbf{i}_{(t)} \otimes \mathbf{g}_{(t)}$$

$$\mathbf{y}_{(t)} = \mathbf{h}_{(t)} = \mathbf{o}_{(t)} \otimes \tanh(\mathbf{c}_{(t)})$$

$$\mathbf{z}_{(t)} = \sigma(\mathbf{W}_{xz}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hz}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{z})$$

$$\mathbf{r}_{(t)} = \sigma(\mathbf{W}_{xr}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hr}^{T} \cdot \mathbf{h}_{(t-1)} + \mathbf{b}_{r})$$

$$\mathbf{g}_{(t)} = \tanh(\mathbf{W}_{xg}^{T} \cdot \mathbf{x}_{(t)} + \mathbf{W}_{hg}^{T} \cdot (\mathbf{r}_{(t)} \otimes \mathbf{h}_{(t-1)}) + \mathbf{b}_{g})$$

$$\mathbf{h}_{(t)} = \mathbf{z}_{(t)} \otimes \mathbf{h}_{(t-1)} + (1 - \mathbf{z}_{(t)}) \otimes \mathbf{g}_{(t)}$$

- GRU Performance is good but may have a slight dip in the accuracy
- But lesser number of trainable parameters which makes it advantageous to use

Avoiding vanishing gradients with LSTMs: Intuition

- During forward propagation the gates control the flow of information. They prevent any irrelevant information from being written to the state. Similarly during backward propagation, they control the flow of gradients.
- It is easy to see that during backward pass the gradients will get multiplied by the gate.
- If the state at time t-1 did not contribute much to the state at time t, then during backpropagation the gradients flowing into s_{t-1} will vanish, which is fine. $||f_t|| \to 0$ and $||o_{t-1}|| \to 0$
- The key difference from vanilla RNNs is that the flow of information and gradients is controlled by the gates which ensure that the gradients vanish only when they should.

Avoiding vanishing gradients with LSTMs: Intuition

• Also, we can argue that, in the case of LSTMs there exists at least one path through which the gradients can flow effectively (and hence no vanishing gradients)

LSTM Equations

$$o_k = \sigma(W_o h_{k-1} + U_o x_k + b_o)$$

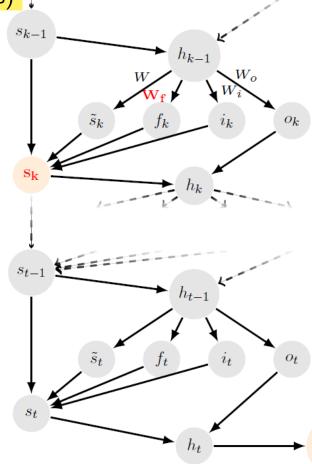
$$i_k = \sigma(W_i h_{k-1} + U_i x_k + b_i)$$

$$f_k = \sigma(W_f h_{k-1} + U_f x_k + b_f)$$

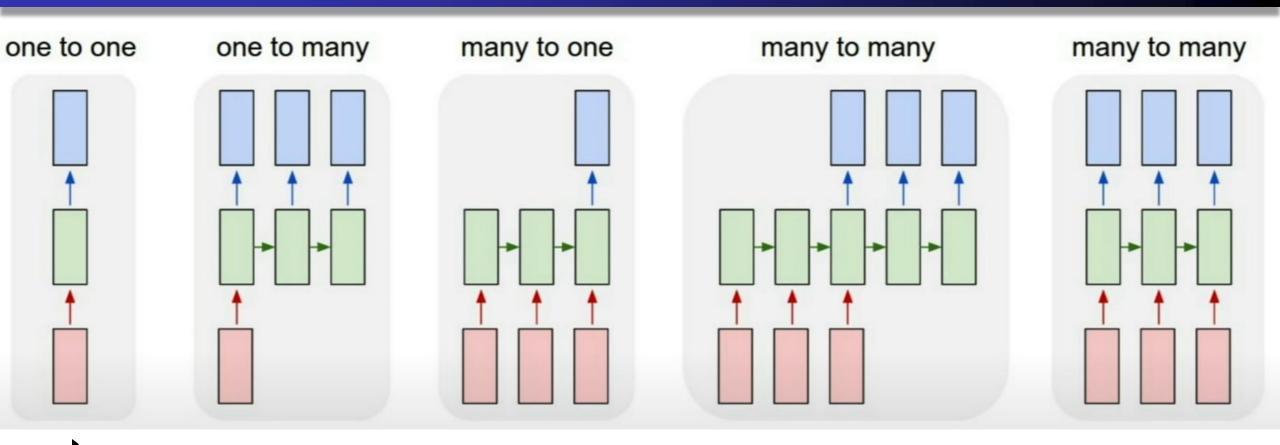
$$\tilde{s}_k = \sigma(W h_{k-1} + U x_k + b)$$

$$s_k = f_k \odot s_{k-1} + i_k \odot \tilde{s}_k$$

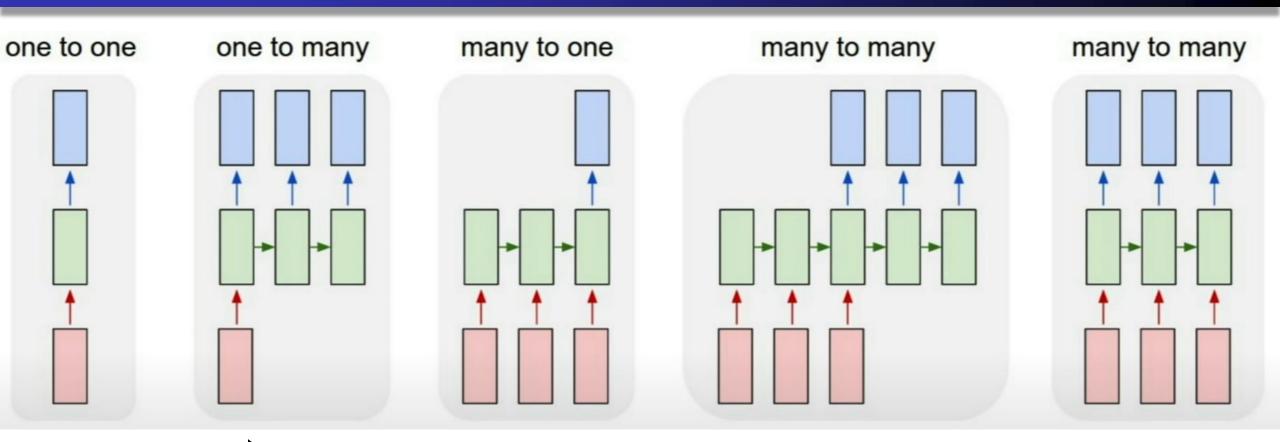
$$h_k = o_k \odot \sigma(s_k)$$



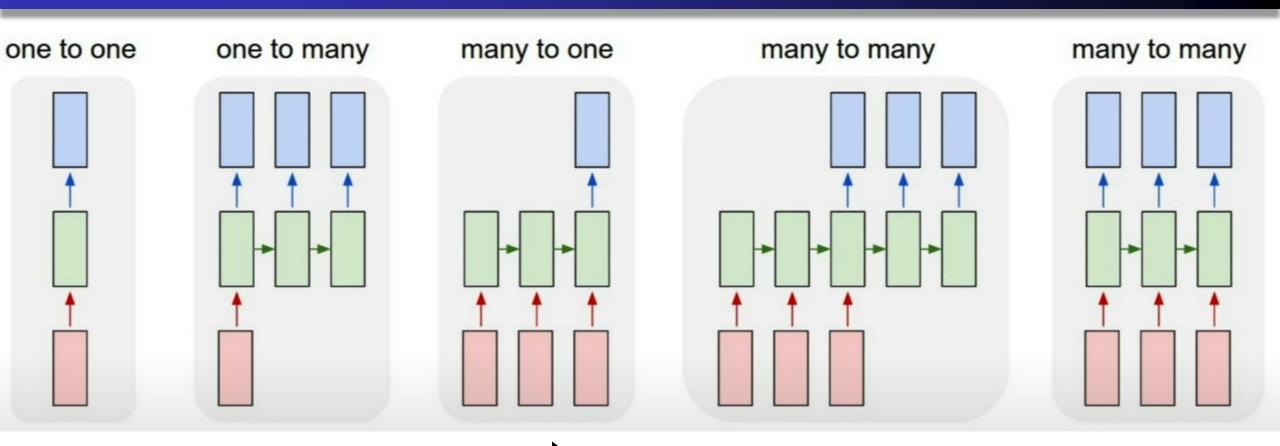
- It is sufficient to show that $\frac{\partial \mathcal{L}_t(\theta)}{\partial s_k}$ does not vanish (because if this does not vanish we can reach W_f through s_k)
- First, we observe that there are multiple paths from $\mathcal{L}_t(\theta)$ to s_k (you just need to reverse the direction of the arrows for backpropagation)
- For example, there is one path through s_{k+1} , another through h_k
- Further, there are multiple paths to reach to h_k itself (as should be obvious from the number of outgoing arrows from h_k)
- So at this point just convince yourself that there are many paths from $\mathcal{L}_t(\theta)$ to s_k



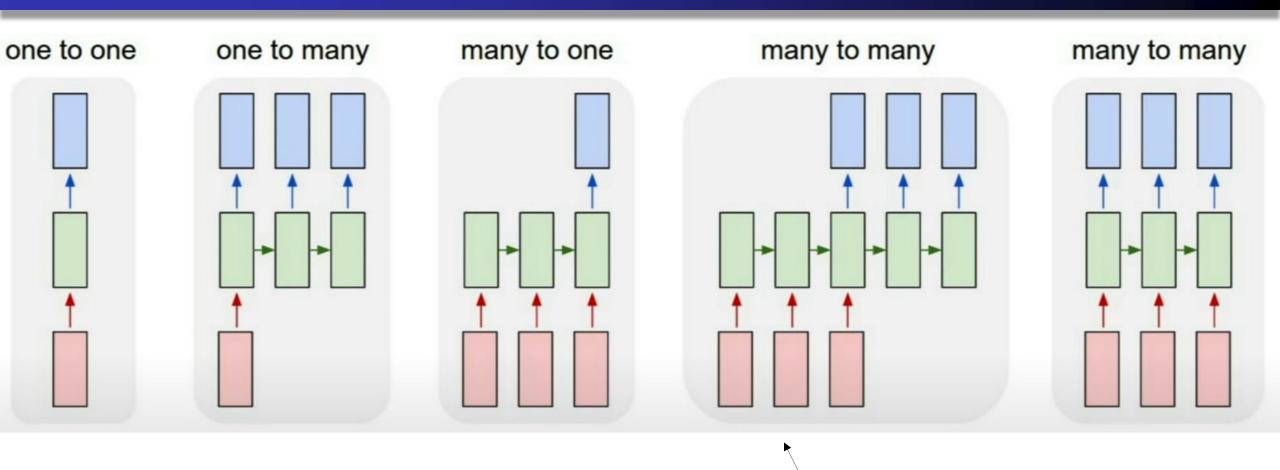
Vanilla RNNs



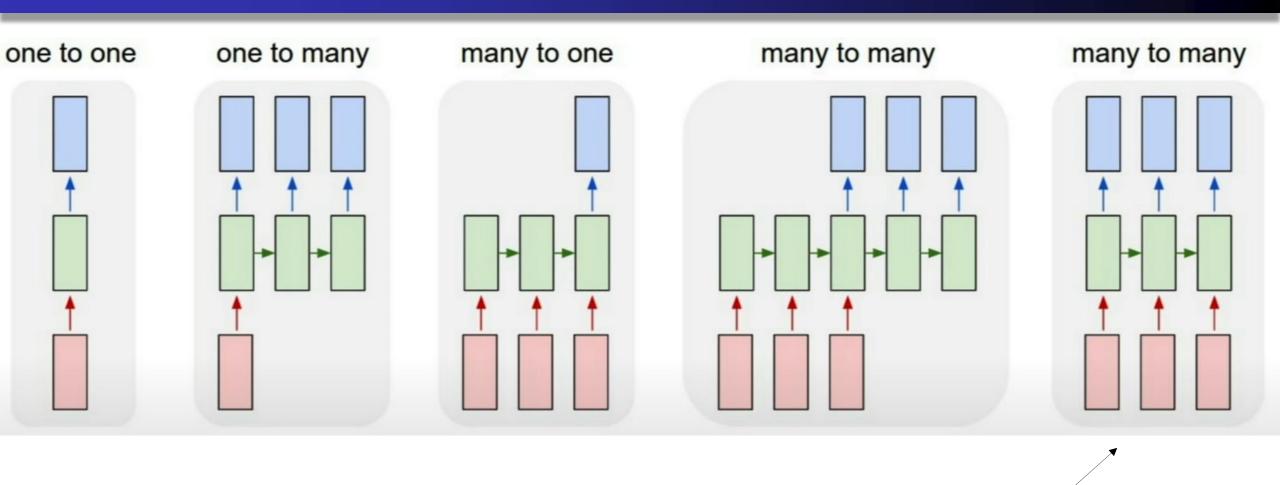
Eg: Image Captioning Image -> Sequence of words



Eg: Sentiment classification
Sequence of words -> Sentiment

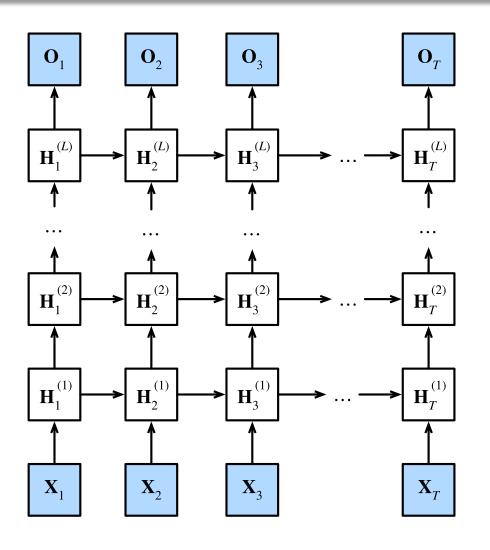


Eg: Machine Translation
Sequence of words -> Sequence of words



Eg: Video Classification on frame level

Deep RNNs



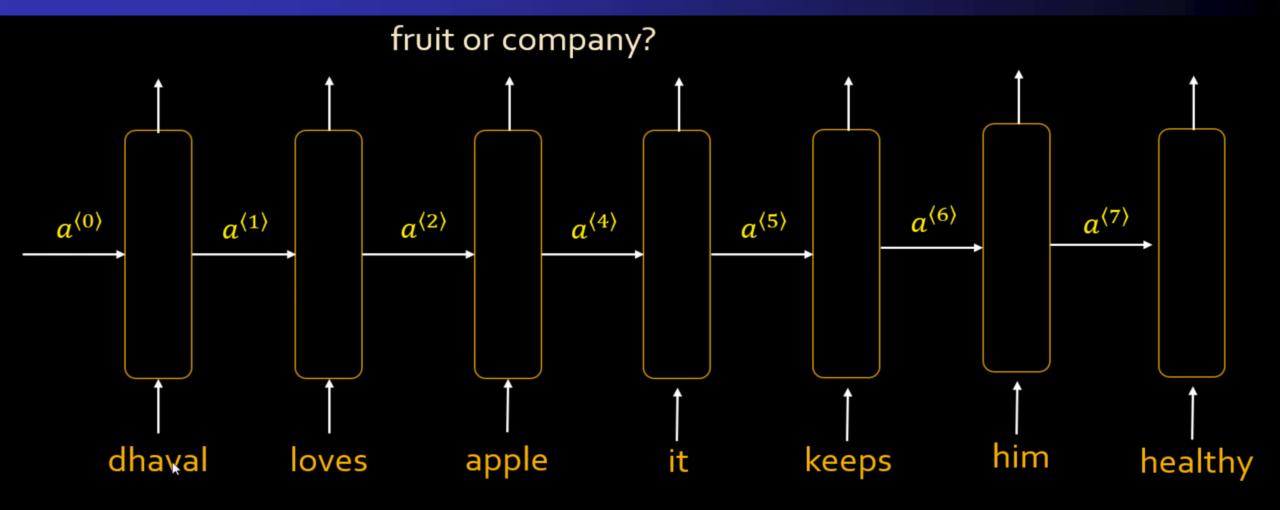
RNNs that are deep not only in the time direction but also in the input-to-output direction.

$$\mathbf{H}_{t}^{(l)} = \phi_{l}(\mathbf{H}_{t}^{(l-1)}\mathbf{W}_{xh}^{(l)} + \mathbf{H}_{t-1}^{(l)}\mathbf{W}_{hh}^{(l)} + \mathbf{b}_{h}^{(l)})$$

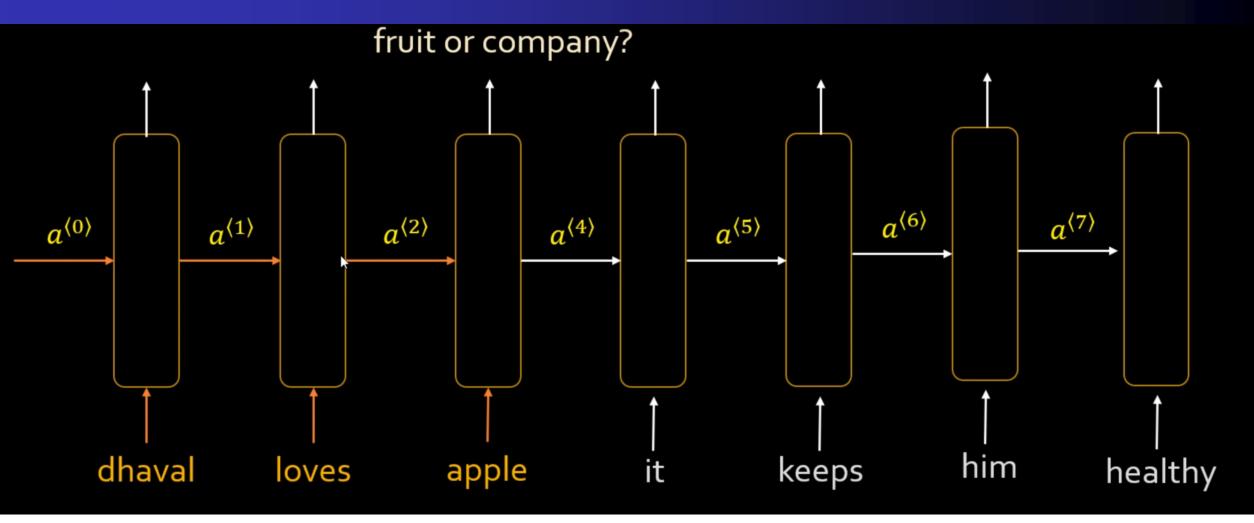
$$\mathbf{O}_t = \mathbf{H}_t^{(L)} \mathbf{W}_{hq} + \mathbf{b}_q$$

Source: Deep Recurrent Neural Networks — Dive into Deep Learning 1.0.0-alpha1.post0 documentation (d2l.ai)

Bi-Directional RNNs: Intuition



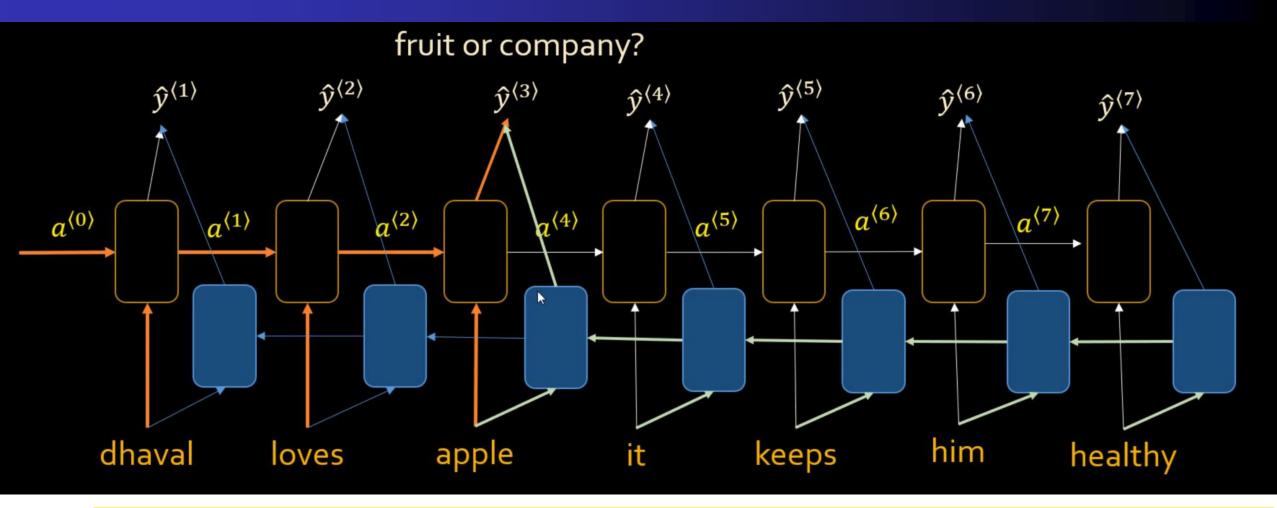
Bi-Directional RNNs: Intuition



• The o/p at the third time step (where input is the string "apple") depends on only previous two i/ps

Source: codebasics - YouTube

Bi-Directional RNNs



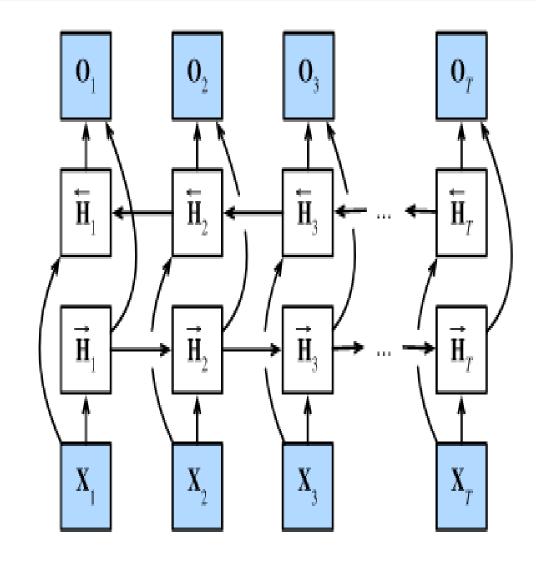
• Adding an additional backward layer with connections as shown above makes the o/p at a time step depend on both previous as well as future i/ps.

Source: codebasics - YouTube

Bi-directional RNNs

Example - speech detection

- I am ____.
- I am _ _ hungry.
- I am ____ hungry, and I can eat half a cake.
- Regular RNNs are causal
 - look at past and present inputs to generate output.
- Use 2 recurrent layers on the same inputs
 - One reading words from left to right
 - Another reading words from right to left
- Combine their outputs at each time step



Source: Bidirectional Recurrent Neural Networks — Dive into Deep Learning 1.0.0-alpha1.post0 documentation (d2l.ai)

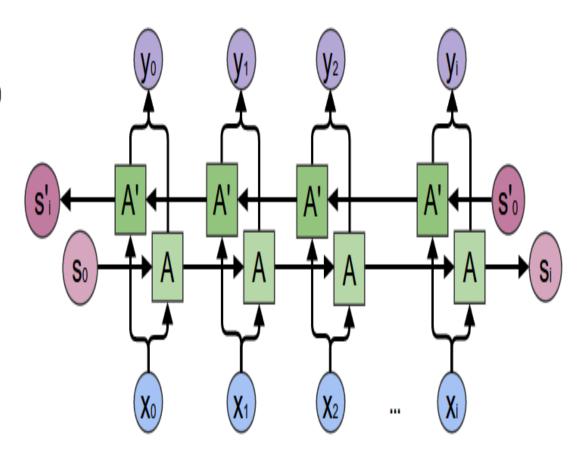
Bi-directional RNN computation

$$A_t(Forward) = \phi(X_t * W_{XA}^{forward} + A_{t-1}(Forward) * W_{AA}^{forward} + b_A^{forward})$$

$$A_t(Backward) = \phi(X_t * W_{XA}^{backward} + A_{t+1}(Backward) * W_{AA}^{backward} + b_A^{backward})$$

- W the weight matrix
- b the bias.
- The hidden state at time t is given by a combination of At(Forward) and At(Backward).
- The output at any given hidden state is:

$$O_t = H_t * W_{AY} + b_Y$$



Bi-directional RNN computation

$$A_t(Forward) = \phi(X_t * W_{XA}^{forward} + A_{t-1}(Forward) * W_{AA}^{forward} + b_A^{forward})$$

$$A_t(Backward) = \phi(X_t * W_{XA}^{backward} + A_{t+1}(Backward) * W_{AA}^{backward} + b_A^{backward})$$

- φ is the activation function
- W the weight matrix
- b the bias.
- The hidden state at time t is given by a combination of At(Forward) and At(Backward).
- The output at any given hidden state is:

$$O_t = H_t * W_{AY} + b_Y$$

