VIth SEMESTER
OPERATIONS RESEARCH
2. THE TRANSPORTATION MODEL

Introduction – Transportation Model

- Simplex algorithm can be used to solve any LPP for which the solution exists. As the number of variables and constraints increases the computation by this method becomes laborious.
- Methods requiring simplified calculation is the distribution model or transportation model.
- It can be used in scheduling, production, investment, plant locations, etc.
- It is a special kind of LPP in which goods are transported from a set of sources to a set of destinations subjected to supply and demand of the sources and destination respectively. The total cost of transportation is minimized.

Introduction – Transportation Model

Assumptions:

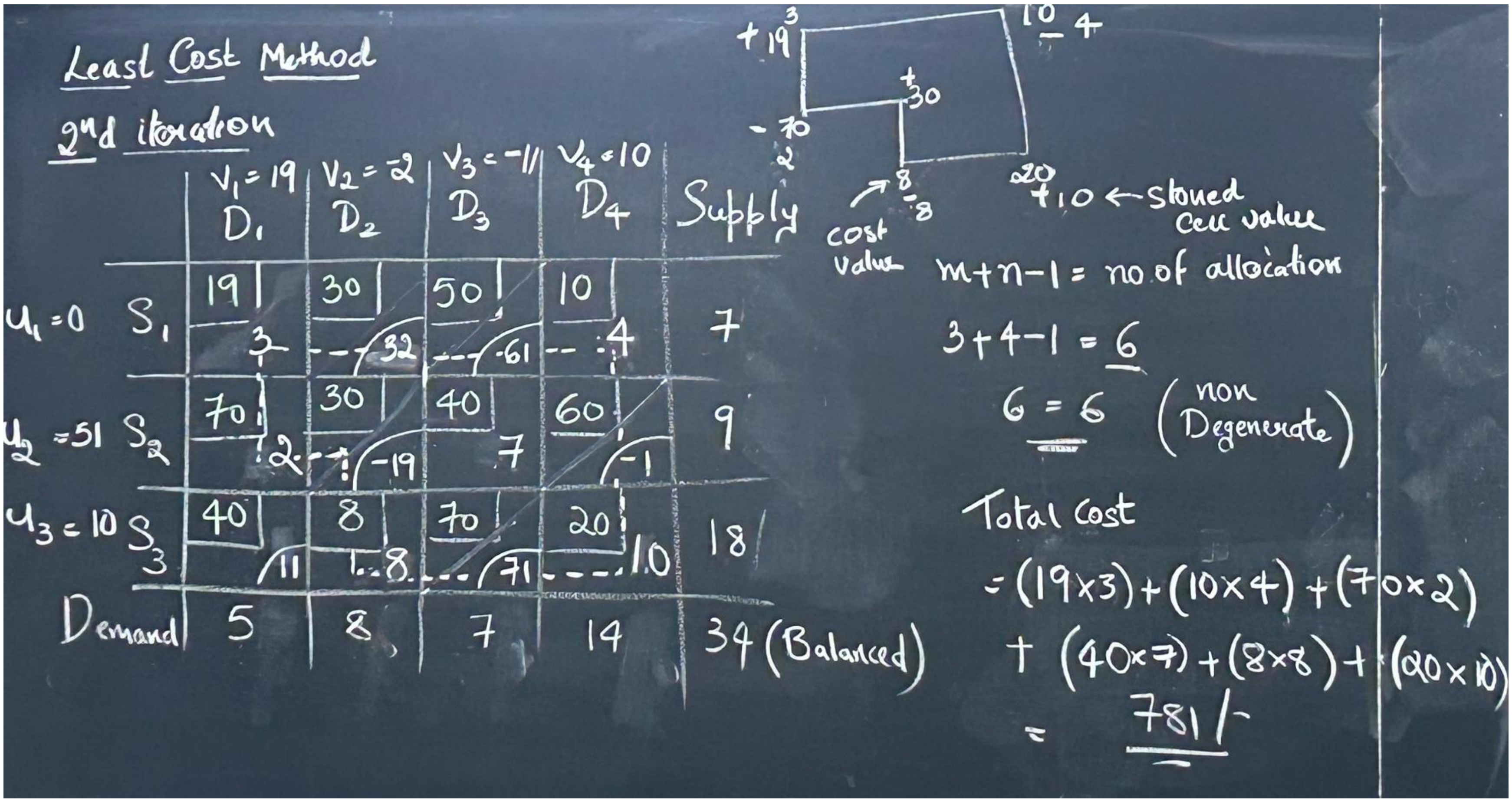
- Total quantity of item available at different sources is equal to total requirement at different destinations.
- Items can be transported conveniently from all sources to destinations.
- The unit transportation cost of the item from all the sources to destinations is certainly and precisely known.
- The transportation cost on a given route is directly proportional to the number of item shipped on to that route.
- The objective is to minimize the total transportation cost for the organisation as a whole and not for individual supply and distribution centres.

Introduction – Transportation Model

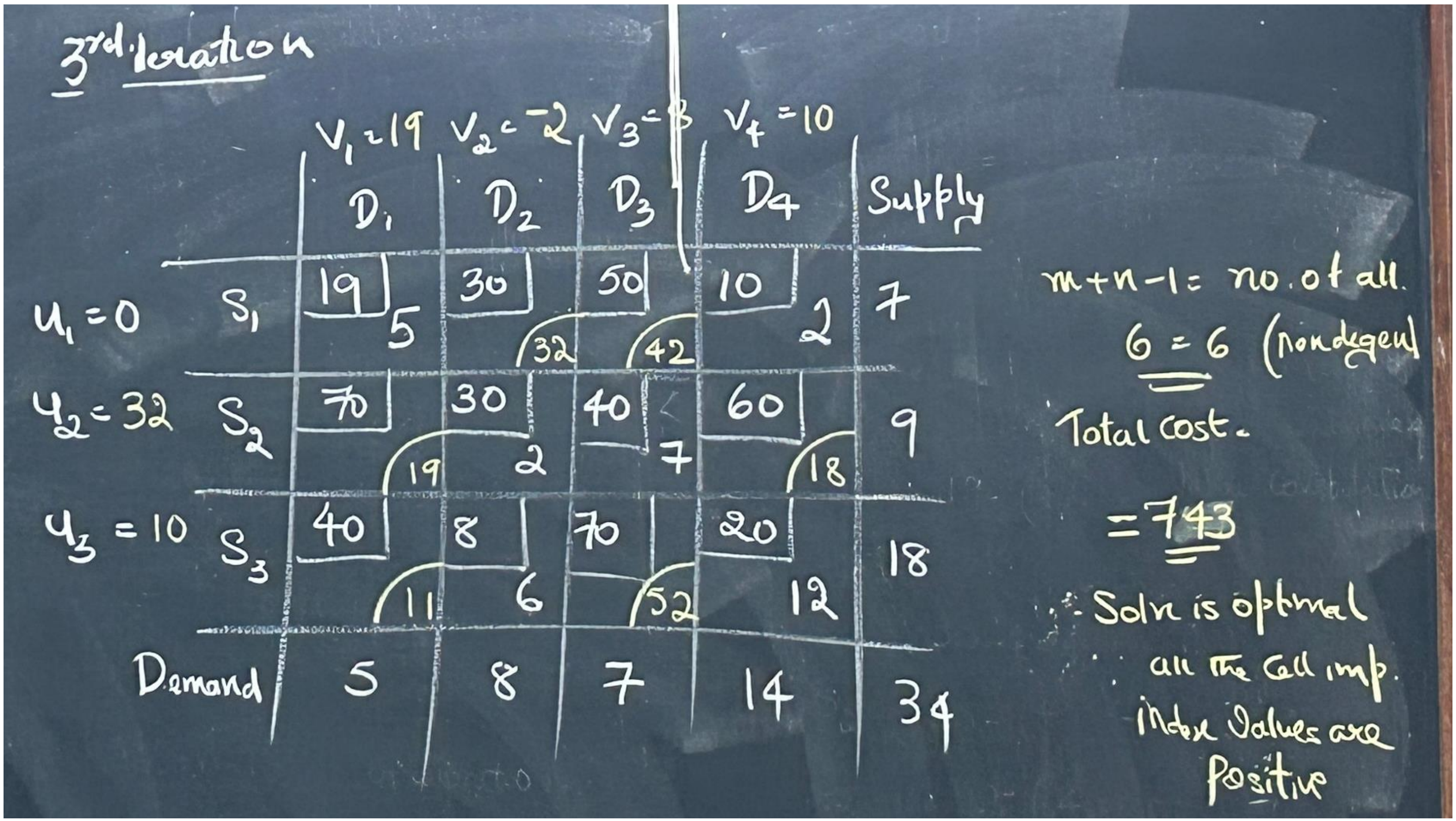
- The structure of transportation problem involves a large number of shipping routes from several supply centres to several demand centres.
- The objective is to determine shipping routes between supply centres and demand centres in order to satisfy the required quantity of goods or services at each destination centre, with available quantity of goods or services at each supply centre at the minimum transportation cost and/ or time.

Introduction – Transportation Model

Modified Distribution Method [U-V Method]



Modified Distribution Method [U-V Method]



Transportation Model

Special Cases: Degeneracy

- When the number of allocated (occupied) cell is less than m+n-1, then the solution is degenerate.
- In such cases, the current solution cannot be improved further because it is not possible to draw closed paths. The dual variables U_i and V_j for performing optimality test cannot be computed.

Transportation Model

Special Cases: Degeneracy

- To resolve, allocated a very small quantity close to the zero to one or more (if required) unoccupied cell to get m+n-1 equals to number of allocations.

- It is denoted by Δ or €. This quantity would neither affect the total cost nor the supply and demand values.

Transportation Model

Special Cases: Degeneracy

- Steps after initial basic solution
  1. Check whether m+n-1 = number of allocated cells (not satisfied, go to step 2, otherwise step 3)
  2. Convert the necessary number of unallocated cells into allocated cells to satisfy the above condition. To do this:
     - Start from the least cost value of the unallocated cell.
     - Check the loop formation one by one.
     - There should be no closed loop formation.
     - Select that cell as a new allocated cell and assign Δ or €

Transportation Model

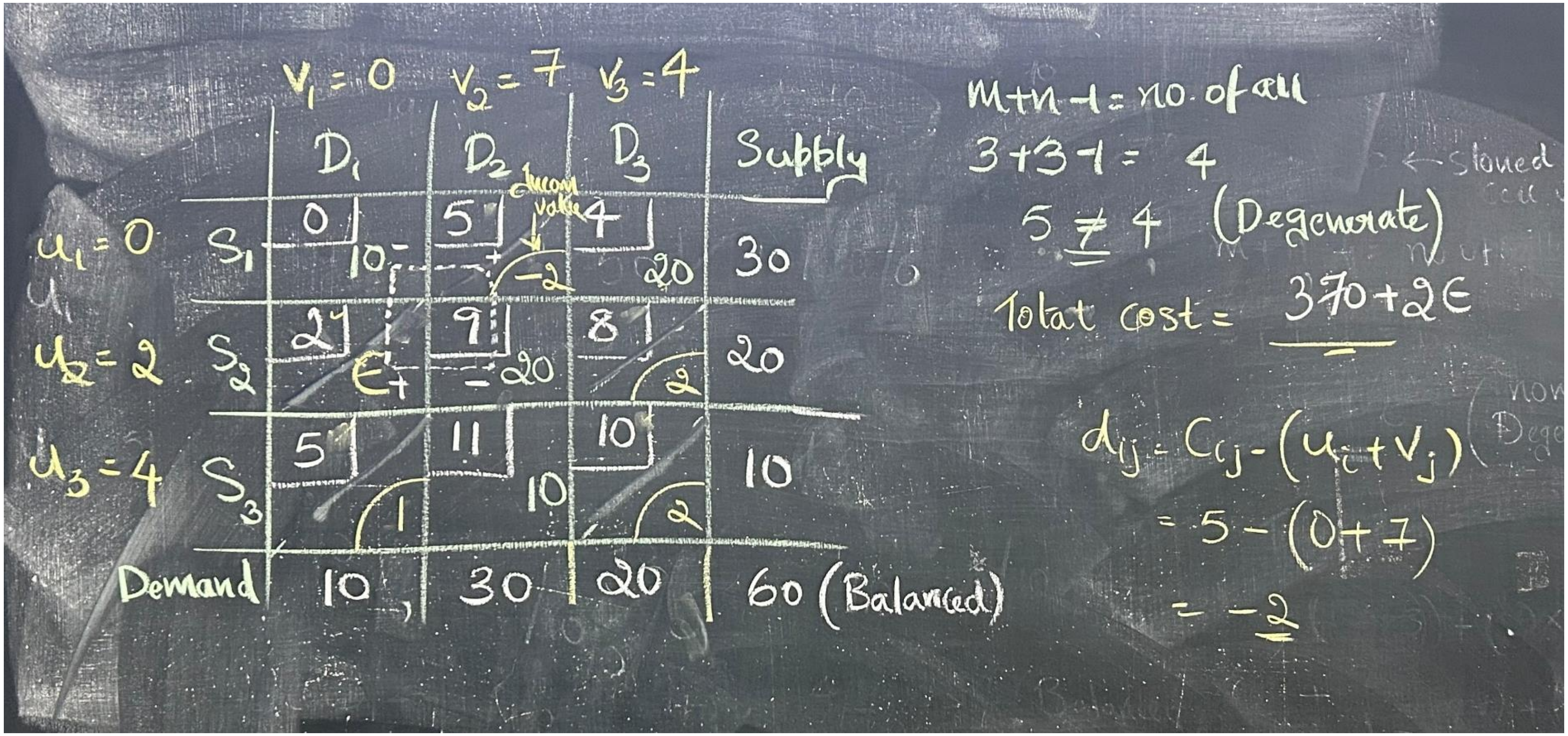
Special Cases: Degeneracy

3. Calculate the value using stepping stone method or Modified Distribution Method (MODI) [(UV) method] for finding the optimal solution.

- Closed Loop formation
  - ✓ Draw the lines horizontally and vertically towards the allocated cell from one of the empty cell.
  - ✓ The turning points of the closed loop should be only at the allocated cell.
  - ✓ Finally, the loop should complete at the cell from where we have started.

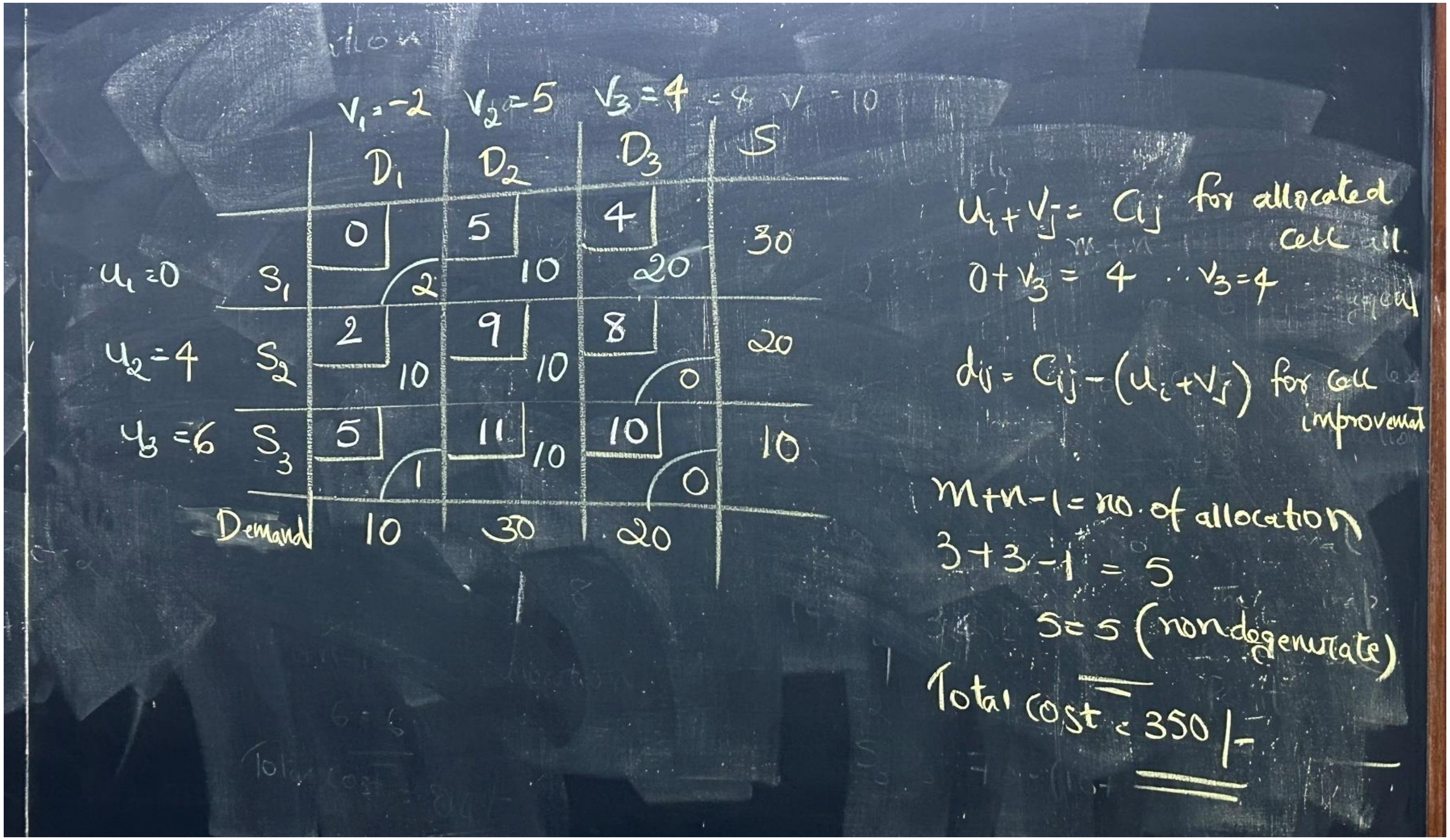
Transportation Model

Special Cases: Degeneracy



Transportation Model

Special Cases: Degeneracy



Transportation Model

Special Cases: When Supply is greater than demand

- Powerco has three electric plants that supply the needs of three cities. Each power plant can supply the following numbers of kilowatt-hours of electricity: plant-1 25 million, plant-2 35 million, plant- 3 30 million. The peak power demands in these cities at a certain time of the day are as follows: city-1 30 million, city-2 20 million, and city-3 20 million. The costs of sending 1 million kWh of electricity from the plant to the city is give in the table below.

  Solve the given transportation model using least cost method and test the optimality using MODI method.

	City 1	**City 2**	**City 3**	**Supply**
Plant 1	1	2	1	25
Plant 2	0	4	5	35
Plant 3	2	3	3	30
Demand	30	20	20	

Optimal Solution - Total Cost = $75

Transportation Model

Special Cases: When Supply is lesser than demand

- Three reservoirs are available to supply the water needs of three cities. The reservoir 1 can supply up to 5 million gallons of water per day, reservoir 2 supplies 4 million gallons per day, and reservoir 3 supplies 6 million gallons per day. Also, city 1 would receive 3 million gallons per day, city 2 receives 9 million gallons per day and city 3 receives 7 million gallon per day. The cost of transporting 1 million gallons of water from each reservoir to each city is shown in the table.

  Solve the given transportation model using least cost method and test the optimality using MODI method.

	City 1	City 2	City 3	Supply
Reservoir 1	3	8	6	5
Reservoir 2	4	5	2	4
Reservoir 3	1	10	9	6
Demand	3	9	7	

Optimal Solution -
Total Cost = $75

Transportation Model

Special Cases: Maximization of Profits

Solve the given transportation model using North West Corner method and test the optimality using stepping stone method.

	City 1	City 2	City 3	Supply
Reservoir 1	20	15	13	24
Reservoir 2	19	12	21	11
Reservoir 3	17	13	18	15
Demand	16	20	14	

P_{ij} – Profit or payoff associated with each route

- Among the cell improvement value, the most positive value is taken as the incoming value. In the closed path, less negative sign value is considered for allocating the cell in the next iteration.
- In the optimal solution, all the cell improvement value will be zero and negative.

Reference Books:

- **Vohra, N. D.** (2006) Quantitative Techniques in Management, Third Edition. New Delhi: Tata McGraw Hill Publishing Company Limited.
- **Sharma, J. K.** (2009) Operations Research – Theory and Applications, Fourth Edition. New Delhi: MacMillan Publishers India Ltd.
- **Rao, S. S.** (2009) Engineering Optimization – Theory and Practice. Fourth Edition. New Jersey: John Wiley & Sons, Inc.
- **Taha, H. A.** (2010) Operations Research: An Introduction, Ninth Edition. New Jersey: Prentice Hall.

Any Questions