Linear Programming Problem (LPP)

Sensitivity Analysis (or post-optimality analysis)

- In an LP model, the coefficients (parameters) are assumed to be constant and known with certainty during a planning period.
- These input parameters value may change due to dynamic nature of the business environment.
- These parameters may raise doubt on the validity of the optimal solution of the given LP model.
- A decision-maker, in such situations, would like to know how changes in these parameters may affect the optimal solution and the range within which the optimal solution will remain unchanged.

Linear Programming Problem (LPP)

Sensitivity Analysis

- Sensitivity analysis and parametric linear programming are the two techniques that are used to evaluate the effect on an optimal solution of any LP problem due to changes in its parameters.
  - **Sensitivity analysis** determines the sensitivity range (both lower and upper limit) within which the LP model parameters can vary (one at a time) without affecting the optimality of the current solution.
  - **Parametric analysis** is the study of measuring the effect on the optimal solution of the LP model due to changes at a time in more than one input parameter value outside the sensitivity range.
- It provides the sensitive ranges (both lower and upper limits) within which the LP model parameters can vary without changing the optimality of the current optimal solution.

Linear Programming Problem (LPP)

Sensitivity Analysis: RHS Sensitivity

- It does not begin until the optimal solution to the given LP model has been obtained.
- The RHS sensitivity involves examining how changes in the Right hand side coefficients of the constraints affect the optimal solution.
- Since slack variables are associated with the constraints, changes in the RHS affect the values of the slack variables and consequently the feasibility of the solution.

Linear Programming Problem (LPP)

Sensitivity Analysis: RHS Sensitivity

- It does not begin until the optimal solution to the given LP model has been obtained.

Steps

- Identify the slack and surplus variables that represent each resources.
- Prepare a table extracted from final simplex table showing the quantity column, slack variable column, positive and negative ration calculation column.
- Perform calculation:
  - Lower bound = original resource value – smallest number in positive ratio column
  - Upper bound = original resource value – largest number in negative ratio column
- If the positive and negative ratio column has no value then lower bound is $-\infty$ and upper bound is $+\infty$

Linear Programming Problem (LPP)

Sensitivity Analysis: RHS Sensitivity

Maximize $Z = 25x_1+42x_2+30x_3$

Subject to:

$3x_1+4x_2+2x_3 \leq 60$

$2x_1+1x_2+2x_3 \leq 36$

$1x_1+3x_2+2x_3 \leq 62$

$X_1, x_2, x_3, \geq 0$

Optimal Solution table

	C_j	25	42	30	0	0	0	
C_b	**Basis**	x_1	x_2	x_3	s_1	s_2	s_3	**RHS (Q)**
42	x_2	1/3	1	0	1/3	-1/3	0	8
30	x_3	5/6	0	1	-1/6	2/3	0	14
0	s_3	-5/3	0	0	-2/3	-1/3	1	10
	Z_j	39	42	30	9	6	0	
$C_j - Z_j$		-14	0	0	-9	-6	0	

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of non basic variables

Maximize $Z = 25x_1+42x_2+30x_3$

Subject to:

$3x_1+4x_2+2x_3 \leq 60$

$2x_1+1x_2+2x_3 \leq 36$

$1x_1+3x_2+2x_3 \leq 62$

$X_1, x_2, x_3, \geq 0$

Optimal Solution table

	C_j	25	42	30	0	0	0	
C_b	**Basis**	x_1	x_2	x_3	s_1	s_2	s_3	**RHS (Q)**
42	x_2	1/3	1	0	1/3	-1/3	0	8
30	x_3	5/6	0	1	-1/6	2/3	0	14
0	s_3	-5/3	0	0	-2/3	-1/3	1	10
	$C_j - Z_j$	-14	0	0	-9	-6	0	

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of non basic variables

Consider the non-basic variable x_1

Let Δ be the permissible change in the profit coefficient

C_j	$25+ \Delta$
Basis	x_1
x_2	1/3
x_3	5/6
s_3	-5/3
Z_j	39
$C_j - Z_j$	$-14+ \Delta$

The current solution remains optimal as long as $C_j – Z_j$ values are either 0 or negative.

$$-14+ \Delta \leq 0$$
$$\Delta \leq 14$$

$$25 - \infty \leq C_j \leq 25 + 14$$

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of basic variables

Consider the basic variable x_2

Its profit coefficient is 42

Let Δ be the permissible change in the profit coefficient

Optimal Solution table

	C_j	25	$42+\Delta$	30	0	0	0	
C_b	**Basis**	x_1	x_2	x_3	s_1	s_2	s_3	**RHS (Q)**
$42+\Delta$	x_2	1/3	1	0	1/3	-1/3	0	8
30	x_3	5/6	0	1	-1/6	2/3	0	14
0	s_3	-5/3	0	0	-2/3	-1/3	1	10
	Z_j	39	$42+\Delta$	30	$9+(\frac{1}{3}\Delta)$	$34+(\frac{2}{3}\Delta)$	0	
$C_j - Z_j$		-14	0	0	$-9-(\frac{1}{3}\Delta)$	$-(34+(\frac{2}{3}\Delta)$	0	

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of basic variables for **x_2 variable**

Absolute value of C_j - Z_j	Corresponding number in x_2 row	Positive ratio	Negative ratio
14	1/3	42	
9	1/3	27	
6	-1/3		-18

Upper bound = original value – (smallest number in the positive ratio column)
= 42 – 27
= 15
Lower bound = original value – (largest number in the negative ratio column)
= 42 – (-18)
= 60

Range: 15 → 60

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of basic variables

Consider the basic variable x_3

Its profit coefficient is 30

Let Δ be the permissible change in the profit coefficient

Optimal Solution table

	C_j	25	42	30+Δ	0	0	0	
C_b	**Basis**	x_1	x_2	x_3	s_1	s_2	s_3	**RHS (Q)**
42	x_2	1/3	1	0	1/3	-1/3	0	8
30+Δ	x_3	5/6	0	1	-1/6	2/3	0	14
0	s_3	-5/3	0	0	-2/3	-1/3	1	10
	Z_j	39	42+Δ	30+Δ	$9-(\frac{1}{6}\Delta)$	$6-(\frac{1}{3}\Delta)$	0	
$C_j - Z_j$		-14	0	0	$-(9-(\frac{1}{6}\Delta))$		0	

Linear Programming Problem (LPP)

Sensitivity Analysis: Profit Coefficients of basic variables for **x_3 variable**

Absolute value of C_j - Z_j	**Corresponding number in x_3 row**	**Positive ratio**	**Negative ratio**
14	5/6	16.8	
9	-1/6		-54
6	2/3	9	

Upper bound = original value – (smallest number in the positive ratio column)
= 30 – 9
= 21
Lower bound = original value – (largest number in the negative ratio column)
= 30 – (-54)
= 84

Range: 21 → 84

Reference Books:

- **Vohra, N. D.** (2006) Quantitative Techniques in Management, Third Edition. New Delhi: Tata McGraw Hill Publishing Company Limited.
- **Sharma, J. K.** (2009) Operations Research – Theory and Applications, Fourth Edition. New Delhi: MacMillan Publishers India Ltd.
- **Rao, S. S.** (2009) Engineering Optimization – Theory and Practice. Fourth Edition. New Jersey: John Wiley & Sons, Inc.
- **Taha, H. A.** (2010) Operations Research: An Introduction, Ninth Edition. New Jersey: Prentice Hall.