Engineering Economics

Decision making
① Define the problem
② Decision criteron
③ Allocate weight to criterons.
④ Develop alternatives.
⑤ Analyze (comparing
⑥ Selection / Implementation
⑦ Feedback [Review the decision making process]

Classification of economics

1) Micro economics : study of economics in smaller scale (Profit loss, growth, production rate) Ex: industry, business.

2) Macro economics : Larger scale (GDP, unemployment)

* Macro influences micro economics.

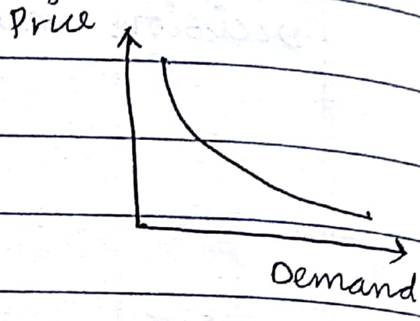
Factors of production

① Land - industry space and raw material.
② Labour
③ Capital < Real capital (assests (building)) / Financial capital (money)
④ Enterpreneurship.

Demand : Need + purchasing power

Law of demand:

$$\text{demand} \propto \frac{1}{\text{Price}}$$



Determinants of demand:

① Cost of the product : cost $\propto$ 1/demand.

② Income of consumer : Income $\propto$ demand.

③ Prices of related goods:

Related goods — substitutes (m$^{\text{r}}$ products available)
— complements (accessories with the products)

④ Taste and preferences

⑤ Advertisements.

Exceptions to the law of demand:

① Emergency / Necessary good (Medicine, petrol/diesel, oil)
demand ↑, price ↑.

② Status symbol (Gold, luxury cars, land)

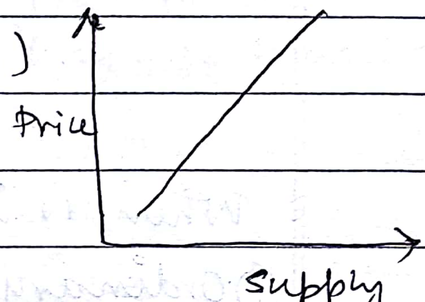
③ Expectation — Price of product (If consumer expects price to ↑, demand ↑)
└ Income (If consumer expects his income to ↑, demand may ↑ or ↓)

④ Gefens goods / Inferior goods.
When high priced goods demand ↑s, which is the demand of other goods (inferior goods).

Supply: Production & transportation market.

Law of supply:

Price ∝ supply (↑ profit)



Determinants of supply:

1. Cost of production
2. Resources available
3. Technology used.

* Dynamics of demand and supply.

Time value of money

Money has time value → it can earn money over time (earning power)
→ its purchasing power changes over time (inflation)

* Time value is measured in terms of interest rate.
* Interest is cost of money - a cost to the borrower & an earning to the lender.
* Relationship between interest & time leads to concept of time value.

Interest: Rental amount charged by financial institutions for use of money.

Simple Interest : Charging an interest rate only to an principal amount.

$$SI = PiN/100$$

$$F = P(1+iN)$$

When N is not a full year, SI is calculated.

1. Ordinary SI : Year is divided into 12 months having 30 days and year is 360 (12×30) days.
2. Exact SI : Exactly calendar no, of days N → fraction of days the loan with its effect in that year.

1. Future sum of money to be paid, if P=₹1000, N= 2 months, i = 10%. (31, 28 days)

Use both methods -

$$SI = PNi/100$$

$$= 1000 \times 10 \times \left(\frac{30+30}{360}\right) / 100$$

$$= ₹16.66$$

$$F = P + SI = ₹1016.67$$

Exact SI : $F = P(1+iN)$

$$= 1000\left(1 + 0.1\left(\frac{31+28}{365}\right)\right)$$

$$F = ₹1016.16$$

* For loan → exact
For deposit → ordinary.

Compound Interest

Charging interest rate to initial sum & to any previously accumulated interest that has been withdrawn.

* Principal changes every year.

$$F = P(1+i)^n$$

Year	Principal (P)	Interest (i%)	Compound Amount
1	P	$P \times i$	$P + P \times i$
2	$P(i+1)$	$P(i+1) \times i$	$P(1+i)^2$
n	$P(1+i)^{n-1}$	$P(1+i)^{n-1} \times i$	$P(1+i)^n$.

(1) Find compound amount if ₹5000 is charged $i = 18\%$ after 5 yrs.

$F = P(1+i)^n = 5000(1+0.18)^5 = ₹ 11438.78$.

Time value equivalence

* Equal deposit, deposit < total money to be withdrawn
* Loan $\cancel{D}$

Cash flow diagram:

Pictorial representation of receipt & payments associated with economic situations.

Steps to construct cash flow diagram:

(1) Define time frame over which cash flows occur.
(2) Establish a horizontal scale which is divided into

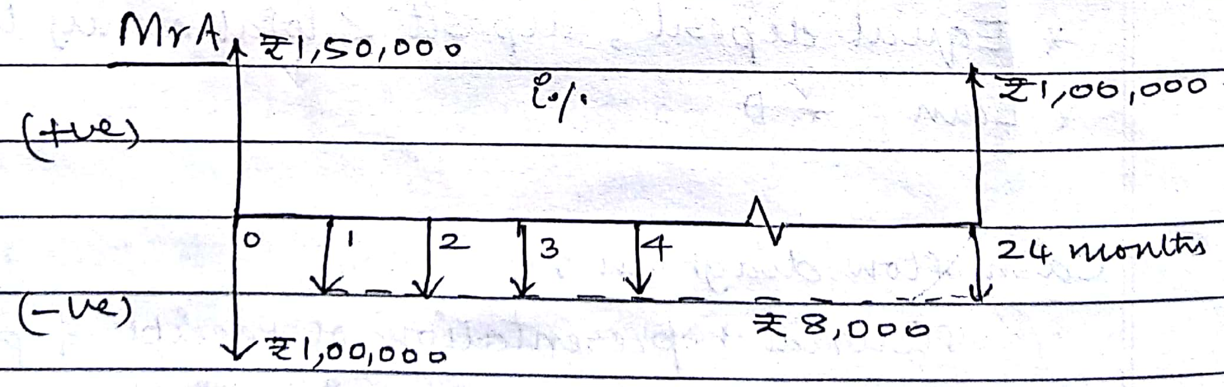
periods usually years.

(3) Receipts & payments are then located on the time frame as follows

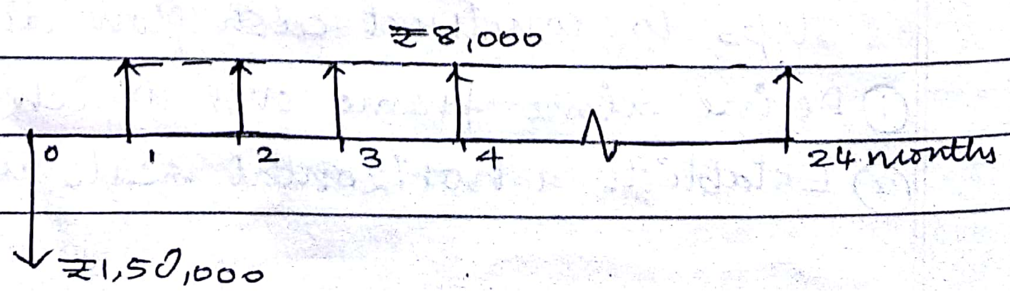
* All receipts are represented on +ve side.
* All payments are represented on -ve side.
* Magnitude of receipts & payments are represented by varying the heights of lines.

(1) Mr A buys a car by making a down payment of ₹1,00,000 & taking a loan of ₹1,50,000 from a bank. He makes equal monthly repayments of ₹8,000 to the bank to clear the loan in full for a period of 2yrs. After making the last payment he sells the car for ₹1,00,000. Draw 2 cashflow diagram, one for Mr A & another for bank using above cashflows.

Mr A



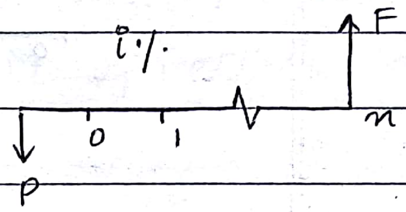
Bank:



① Single payment series Compound Amount

Ex: F

Cashflow diagram



CP is known, F is unknown

$$F = P(1+i)^n$$

$$F = P \times \boxed{(F/P, i, n)} \rightarrow \text{Factor}$$

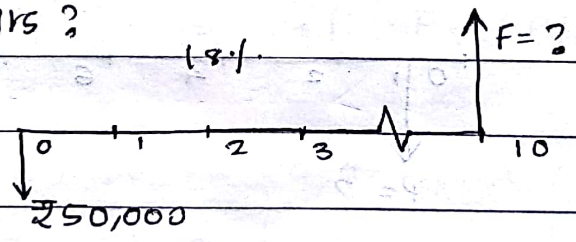
Known
objective
Interest rate

If an amount P is invested now and earns at the rate of i% / yr, the total amount accumulated after n yrs,

$$F = P(1+i)^n$$

$$F = P \times (F/P, i, n)$$

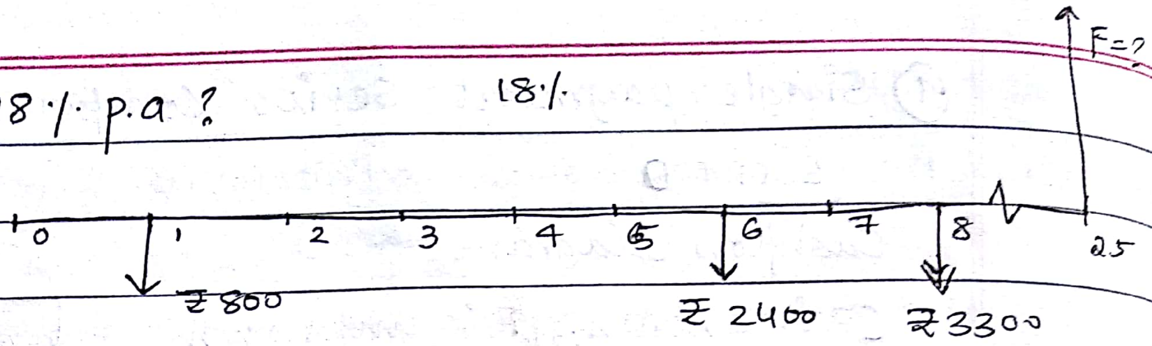
① A person deposits a sum of ₹50,000 at an interest rate of 18% p.a for a period of 10 yrs. Find maturity value after 10 yrs?



$$F = P(F/P, i, n)$$

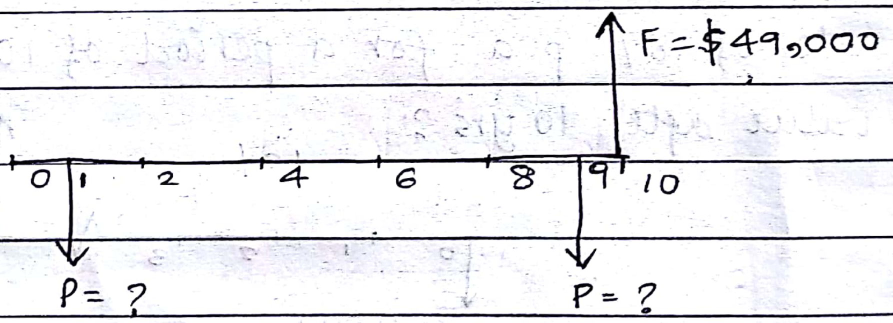
$$= 50000 \times 5.234 = \underline{₹2,61,700}.$$

② How much money will be accumulated in 25 yrs if ₹800 is deposited one year from now ₹2400 6yrs from now & ₹3300 8yrs from now all at an interest rate of 18% p.a ?



$$F = 800 \times (F/P, i, 24) + 2400 (F/P, 18, 19,$$
$$+ 3300 (F/P, 18, 17)$$
$$= 800 \times 53.109 + 2400 \times 23.214 + 3300 \times 1667$$
$$F = ₹153218.4.$$

③ A company is planning to make 2 equal deposits such that 10 yrs from now the company will have \$49,000 to make an investment. If 1st deposit is made at 1 yr from now & 2nd 9 yr from now, how much should be deposited each time at 15% p.a ?



$$F = P (F/P, 15, 9) + P (F/P, 15, 1)$$
$$49000 = P \times 3.518 + P \times 1.150$$
$$P = \$10497.$$

④ IDBI came out with an issue of deep discount bond in the year 1998. The bonds were offered at a deep discounted price of ₹ 12750. Maturity period for the bond was 30yrs with maturity value of ₹ 5,00,000. Determine the rate of return on the investment.

$F = ₹ 5,00,000$

$P = ₹ 12750$

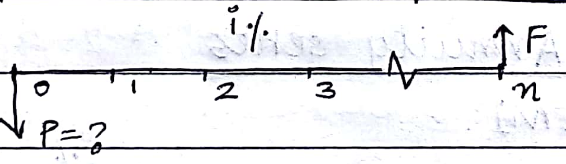
$n = 30yrs.$

$F = P(1+i)^n$

$50000 = ₹ 12750 (1+i)^{30}$

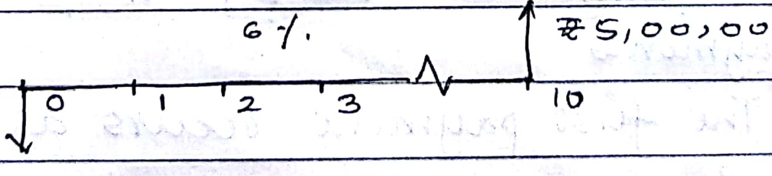
$i = 13\%$

② Single payment series present worth



$P = F/(1+i)^n$ $\quad$ $P = F \times (P/F, i, n)$

① A person wishes to have a sum of ₹5,00,000 10 yrs from now to make a small investment. If he plans to deposit a lumpsome amount which will fetch an $i = 6\%$. Determine the sum.

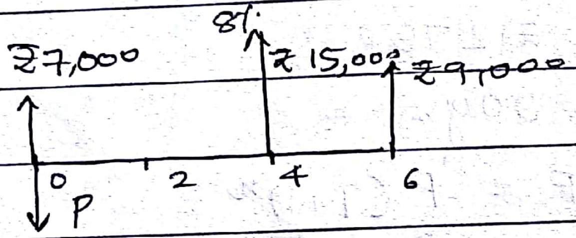


$P = F(P/F, 6\%, 10)$

$= 5,00,000 \times 0.5584 = ₹ 2,79,200$

* Count the number of spaces to calculate present worth & future worth.

② What is the present worth of ₹7,000 now, ₹15,000 4 yrs from now & ₹9,000 6yrs from now all at 8% p.a?



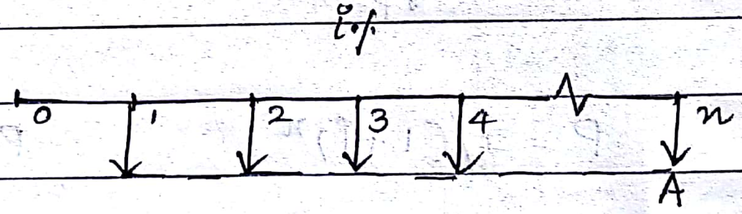
$$P = 7000 + 15000 \times (P/F, 8\%, 4) + 9000 \times (P/F, 8\%, 6)$$

$$= 7000 + 15000 \times 0.7350 + 9000 \times 0.6302$$

$$P = ₹\ 23,698.8$$

Annuity series

Ex: RD, EMI.



Characteristics of an annuity series:

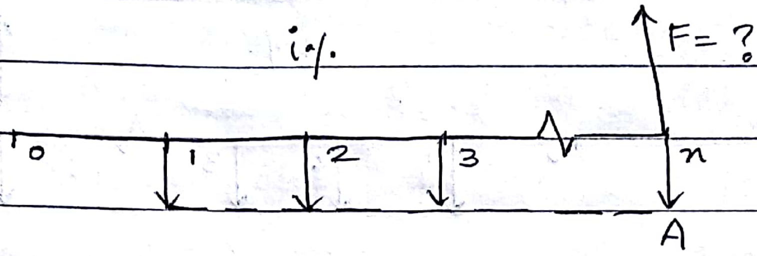
① Magnitude of the payments remain the same & is represented by A.

② There will be equal periods b/w 2 successive payments.

③ The first payment occurs at the end of first period.

(Uniform)

③ Equal payment series compound interest



$$F = A(1+i)^{N-1} + A(1+i)^{N-2} + \dots$$

$$F = A\left[(1+i)^{N-1} + (1+i)^{N-2} + \dots\right] \quad \text{①}$$

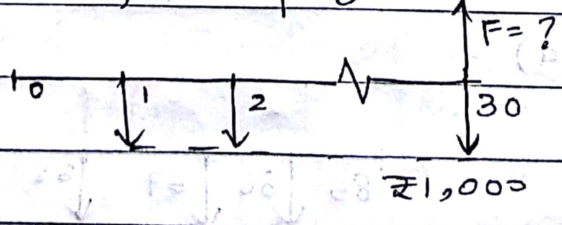
Multiply ① by $(i+i)$ on both sides,

$$F(i+1) = A\left[(1+i)^{N} + (1+i)^{N-1} + \dots\right] \quad \text{②}$$

$$\boxed{F = A\frac{\left[(1+i)^{N} - 1\right]}{i}}$$

$$F = A(F/A, i, N)$$

① If annual deposits of ₹1,000 are made into its saving accounts for 30 years begining 1 year from now. How much will be in fund immediately after last payment, if fund pays an interest of 19.25% p.a?

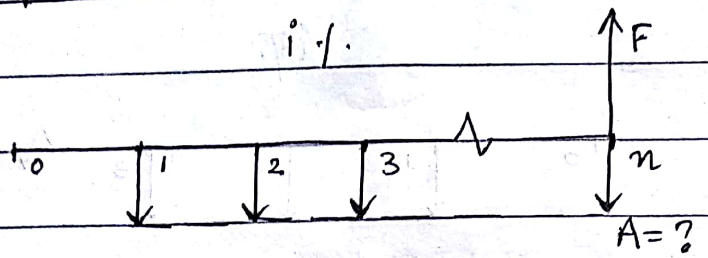


$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = 1000 \times \left[\frac{(0.1925+1)^{30} - 1}{0.1925}\right]$$

$$F = ₹\,10,16,499.32$$

* To arrive at n in annuity, count the no. of transactions.

(4) Equal payment series sinking fund

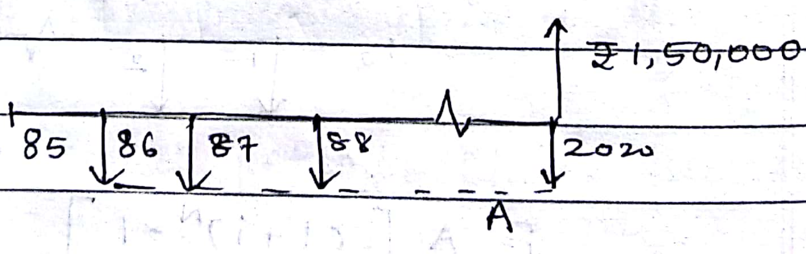


$$A = F\left[\frac{i}{(1+i)^n - 1}\right]$$

$$A = F\,[A/F, i, N]$$

* ~~To arr~~ This factor is used to determine a series of equal payment A occurs at the end of each ~~of~~ n interest periods that are equivalent to known single future amount F.

(1) How much money Mr A must deposit into ~~A~~ savings account each year starting from 1986 if he wants to have \$1,50,000 in the year 2020? (16 % p.a)



$$A = F \times \left[\frac{(i)}{(1+i)^n - 1}\right] = 150000\left[\frac{0.16}{(1+0.16)^{35} - 1}\right]$$

$$A = \$133.84$$

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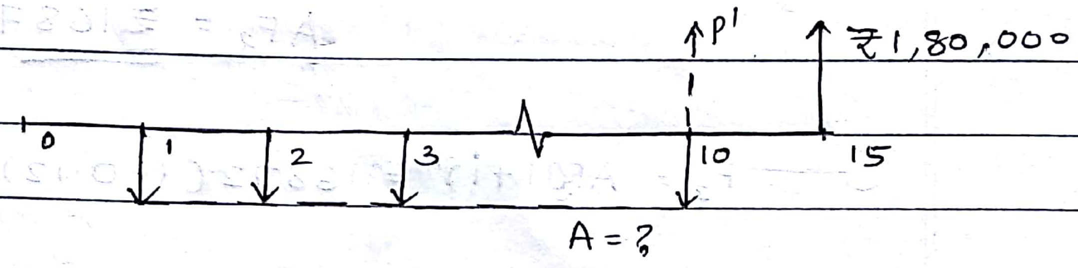
(2) Determine the equal amount ~~of~~ that would have to be deposited at the end of each year for 10 years starting from 1st yr into a fund in order to have ₹1,80,000 at the end of 15th year at 14% p.a ?



$$P' = F / (i+1)^n$$

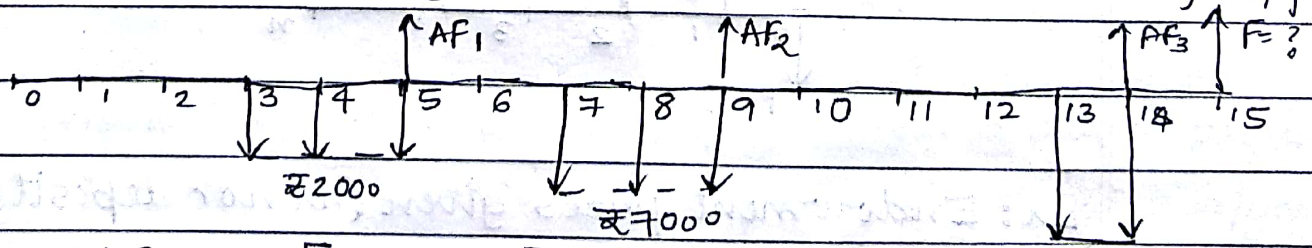
$$= 1,80000 / (1+0.14)^5 = ₹\ 93486.35.$$

$$A = F \times \left[\frac{i}{(1+i)^n - 1} \right]$$

$$= 93486.35 \times \left[\frac{0.14}{(0.14+1)^{10} - 1} \right]$$

$$A = ₹\ 4834.5.$$

(3) Find total amount accumulated in the year in the account at end of 15 year at an interest rate of 12% p.a?



$$AF_1 = A \left[\frac{(1+i)^n - 1}{i} \right] = 2000 \times \left[\frac{(1+0.12)^3 - 1}{0.12} \right]$$

$$AF_1 = ₹6748.8$$

$$F_1 = AF(1+i)^n = 6748.8\,(1+0.12)^{10} = ₹20960$$

$$AF_2 = A\left[\frac{(1+i)^n - 1}{i}\right] = 5000\left[\frac{(1+0.12)^3 - 1}{0.12}\right]$$

$$AF_2 = ₹16872.$$

$$F_2 = AF(1+i)^n = 16872\,(1+0.12)^6 = ₹33302.33$$

$$AF_3 = A\left[\frac{(1+i)^n - 1}{i}\right] = 10000\left[\frac{(1+0.12)^2 - 1}{0.12}\right]$$

$$AF_3 = ₹21200.$$

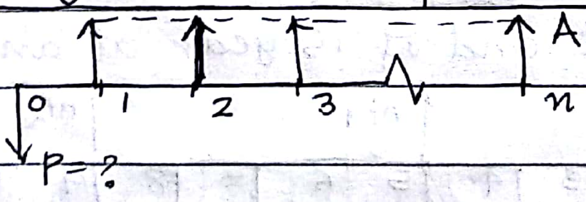
$$F_3 = AF_3\,(1+i)^n = 21200\,(1+0.12)^1$$

$$F_3 = ₹23744.$$

$$F = F_1 + F_2 + F_3 = 20960 + 33302.33 + 23744$$

$$F = ₹78007.08$$

⑤ Equal payment series present worth.



Ex: Endowment prizes given (Donor deposits amount & equal amount is taken every year).

$$F = A\left[\frac{(i+1)^n - 1}{i}\right]$$

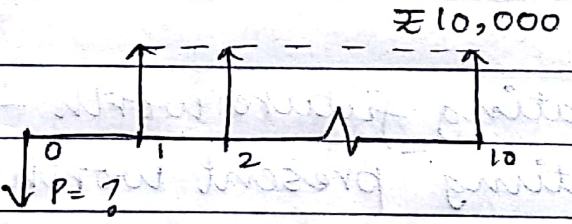
$$P(1+i)^n = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$$P = A = \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

or

$$P = A\,(P/A, i, N)$$

① A firm is available for sale for ₹65,000 now. The firm yields a cash inflow of ₹10,000 / year for next 10 years. If money is worth 12%, can the firm be bought ?

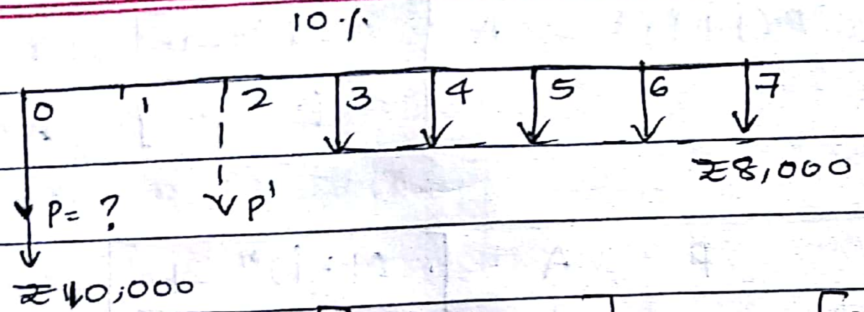


$$P = 10000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 10000 \times \left[\frac{(1+0.12)^{10} - 1}{0.12\,(0.12+1)^{10}}\right]$$

$$P = \underline{₹56,502}$$

The firm cannot be bought.

② A person buys a machine by making a down payment of ₹10,000 & the balance in payments of ₹8,000/year for 5 years starting 3 years from now at 10% p.a. What is cost of the machine ?



$$P' = 8000 \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 8000\left[\frac{(1+0.1)^5 - 1}{0.1(0.1+1)^5}\right]$$

$$P' = ₹\,30{,}326$$

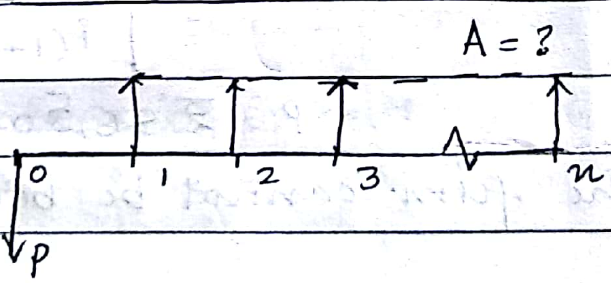
$$P^* = P'/(1+i)^n$$
$$= 30{,}326/(1+0.1)^2 = ₹\,25{,}063.$$

$$\therefore P'' = 10000 + 25063 = ₹35063$$

* Calculating future worth – compounding
Calculating present worth – discounting

⑥ Equal payment capital recovery factor

Ex: Loan



$$A = P\left[\frac{i(i+1)^n}{(i+1)^n - 1}\right]$$

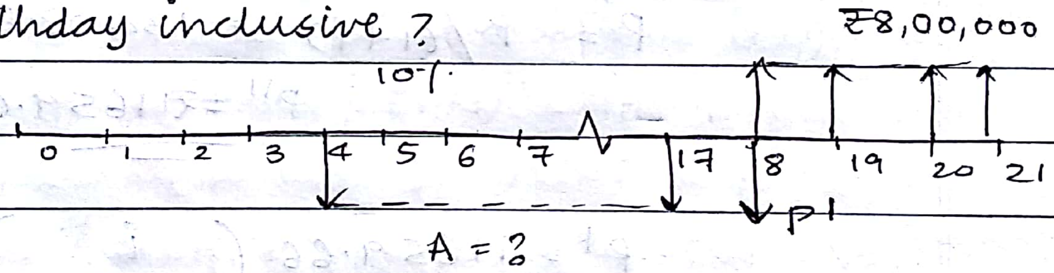
$$A = P\,[A/P, i, n]$$

① Mr A borrows ₹45,000 with a promise to repay it in 10 equal annual installment, starting 1 year from now, how much would his payments be if $i = 20\%$ p.a?

$$A = P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = 45000\left[\frac{0.2\,(0.2+1)^{10}}{(0.2+1)^{10} - 1}\right]$$

$$A = ₹10733.5$$

② A person decides to make advance plans to finance his 3 year old son's education. Money has to be deposited at 10% p.a. What annual deposit on each birthday from 4 to 17th year inclusive must be made to provide ₹8,00,000 on each birthday from 18th to 21st birthday inclusive?



$$p' = 8{,}00{,}000\left[\frac{(1+i)^n\, i}{(1+i)^n - 1}\right]$$

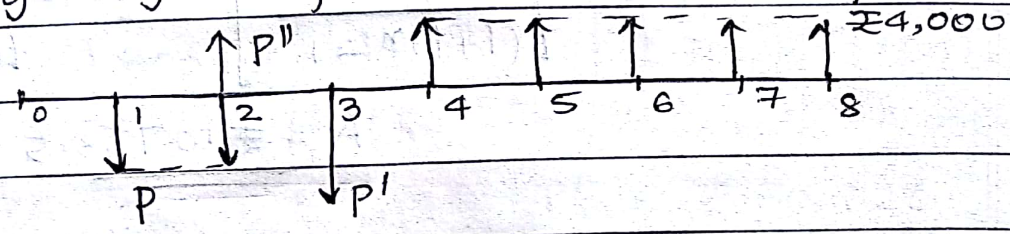
$$= 8{,}00{,}000\left[\frac{(1+0.1)^4 \times 0.1}{(1+0.1)^4 - 1}\right]$$

$$p' = ₹2535892$$

$$A = p'\left[\frac{i}{(i+1)^n - 1}\right] = 2535892\left[\frac{0.1}{(1+0.1)^{14} - 1}\right]$$

$$A = ₹90{,}648.56$$

(3) Determine equal amounts P that should be deposited into an account during 1st & 2 years from now in order to withdraw ₹4000/year for 5 years starting 4 years from now at 15% p.a ?



$$P' = 4000\left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 4000\left[\frac{(0.15+1)^5 \times 0.15}{0.15(0.15+1)^5}\right]$$

$$P' = ₹13408.62$$

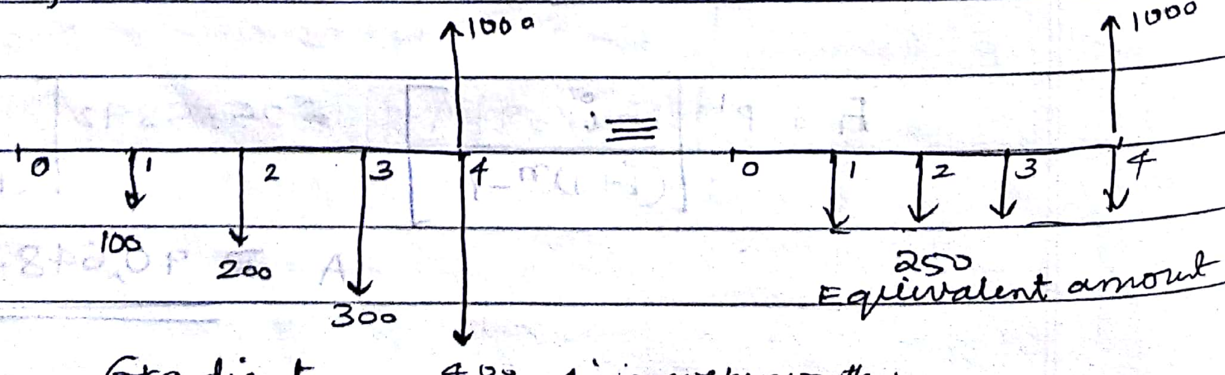
$$P'' = P/(1+i) = 13408.62/(1+0.15)$$

$$P'' = ₹11659.66$$

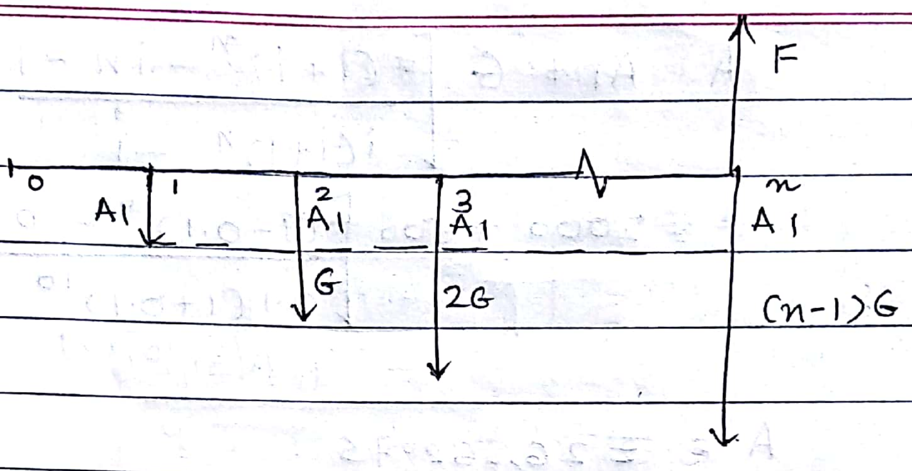
$$P' = 11659.66\left(\frac{i}{(1+i)^n - 1}\right)$$

$$= 11659.66\left(\frac{0.15}{(1+0.15)^2 - 1}\right) = ₹5423$$

(7) Uniform gradient series annual equivalent amount.



Gradient amount (↑ing every month)



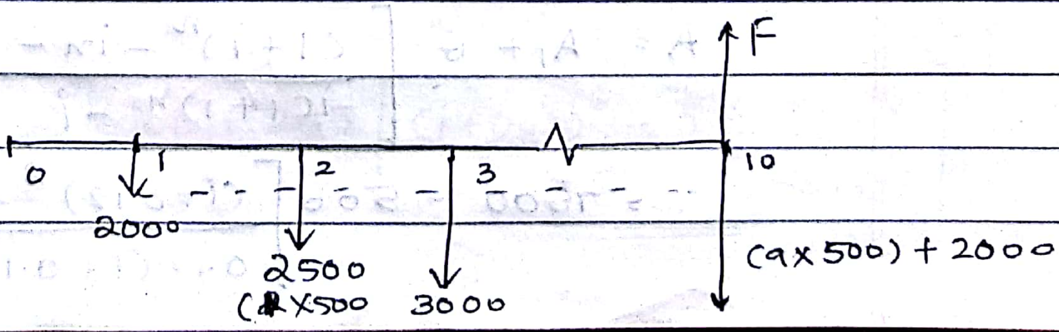
An uniformly ↑ing / ↓ing series can be evaluated by calculating F or P for each individual payment & then summing the collection.
Calculation time can be reduced by converting this gradient series into an equivalent annuity of equal payments A.

The formula for translation is given by

$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$A = A_1 + G\,(A/G, i, n)$$

① Mr A has 10 yrs of service before he ~~retaires~~ retires. He now plans to deposit ₹25,000 in the 1st year & ↑s his deposit by ₹500 for the remaining 9 years. If the $i = 10\%$ p.a, find amount accumulated at the end of 10th year?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 2000 + 500\left[\frac{(1+0.1)^{10} - 0.1\times10 - 1}{0.1(1+0.1)^{10} - 0.1}\right]$$

$(A/G, 10, 10)$

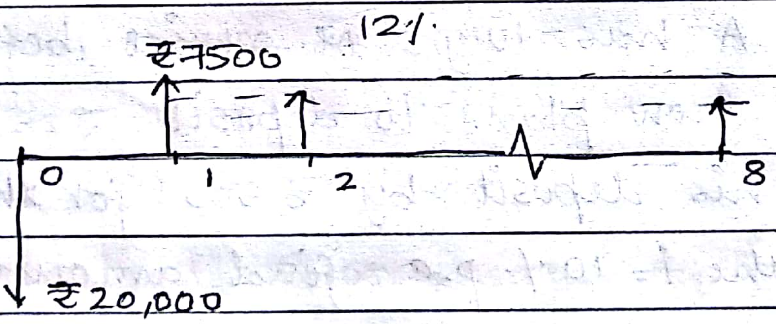
$A = ₹\,26{,}862.73$

$$F = A \times F(F/A, i, n) = A\left[\frac{(1+i)^n - 1}{i}\right]$$

$= 26862.73 \times 15.937$

$F = ₹\,4{,}28{,}111.32$

② A new piece of material handling equipment costs ₹20,000 and is expected to save ₹7,500 in the 1 year of operation. The savings would reduced by ₹500 each following year until the equipment is worn out at the end of 8 years, determine net present worth of machine at interest rate of 12%.



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 7500 - 500\left[\frac{(1+0.12)^8 - 0.12\times8 - 1}{0.12(1+0.12)^8 - 0.12}\right]$$

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$A = ₹6043.42$

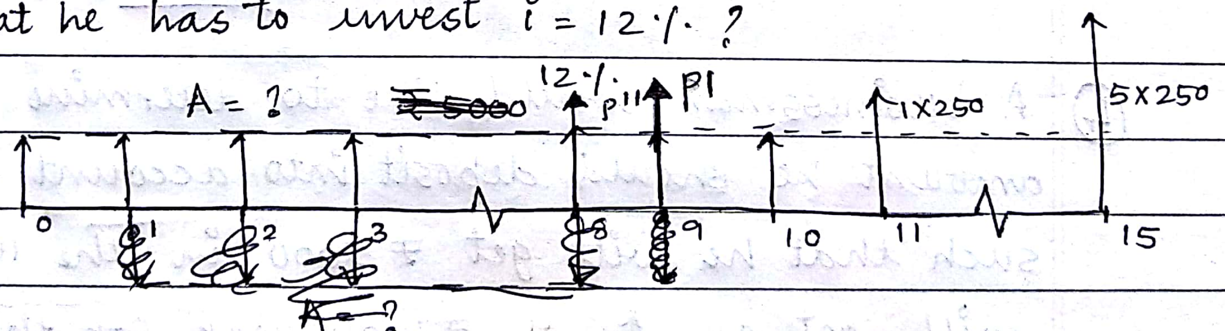
$$P = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right] = 6043.42 \left[\frac{(1+0.12)^8 - 1}{0.12(0.12+1)^8}\right]$$

$P = ₹\,30021.63$.

Net Present Value (NPV) = Present worth of revenue − Present worth cost

= 30022 − 20000 ₹10022

③ A person is planning to withdraw ₹5000 ~~in the~~ in the 10 years from now & then onwards he ↑s his withdrawal amount by ₹250 upto 15 years. To make these withdrawals he is planning to invest an equal amount for 8 years starting from 1st year. Find this equal amount what he has to invest $i = 12\%$?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 5000 + 250\left[\frac{(1+0.12)^6 - 0.12\times 6 - 1}{0.12(1+0.12)^6 - 0.12}\right]$$

$A = ₹\,5543.68$

P' is present worth of uniform gradient series,

$$P' = A \times \left[\frac{(1+i)^n - 1}{i(1+i)^n}\right]$$

$$= 5543 \left[\frac{(1+0.12)^6 - 1}{0.12(1+0.12)^6}\right]$$

$$P' = ₹\ 22789.$$

P'' is future worth of annuity series,

$$P'' = P'/(1+i)^n$$

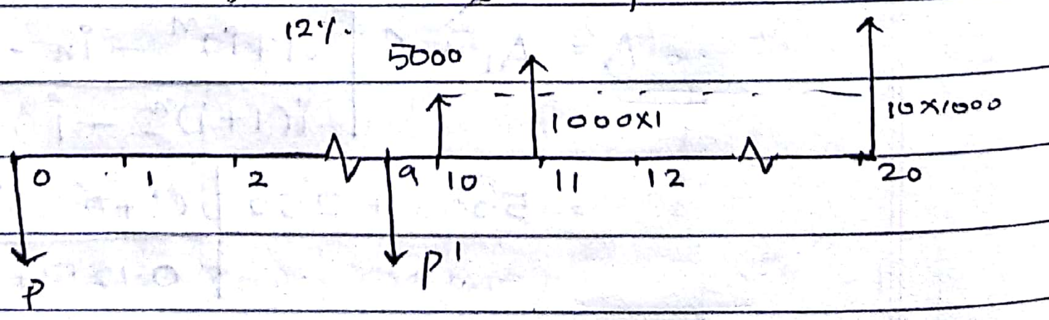
$$P'' = 22789/(1+0.12)^1 = ₹\ 20347$$

To find equal amount A,

$$A = P\left(\frac{i}{(1+i)^n - 1}\right) = 20347 \times \left(\frac{0.12}{(1+0.12)^8 - 1}\right)$$

$$A = ₹\ 1663.21$$

(4) A businessman would like to determine what amount he should deposit into account 12% p.a such that he will get ₹ 5000 in the 10th year & will get an ↑ of ₹ 1000/year for the next 10 yrs?



$$A = A_1 + G\left[\frac{(1+i)^n - in - 1}{i(1+i)^n - i}\right]$$

$$= 5000 + 1000\left[\frac{(1+0.12)^{11} - 11\times0.12 - 1}{0.12(1+0.12)^{11} - 0.12}\right]$$

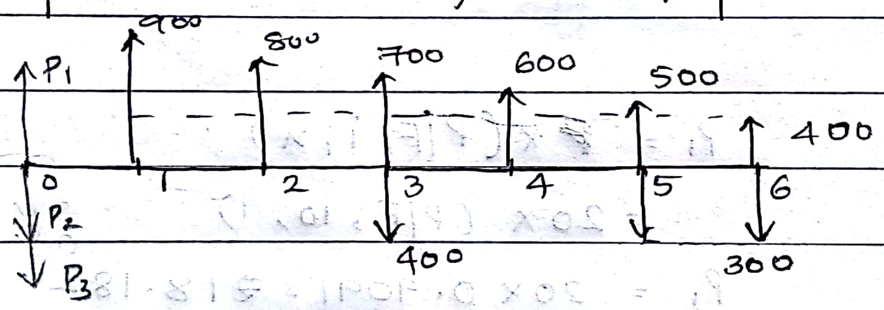
$$A = ₹\,8895.24$$

$$P' = A \times (P/A, i, n \to 11)$$

$$= 8895 \times 5.938 = ₹\,52820$$

$$P = P'/(1+i)^n = 52820/(1+0.12)^9 = ₹\,19047$$

④ Calculate present worth if $i = 12\%$ p.a.



$$A = A + G(A/G, i, N)$$

$$= 900 - 100 \times (2.172)$$

$$A = ₹\,682.8$$

$$P_1 = A \times (P/A, i, N)$$

$$= 682.8 \times 4.111 = ₹\,2806.99.$$

$$P_2 = 400 \times (P/F, i, n \to 3)$$

$$= 400 \times 0.718 = ₹\,284.72.$$

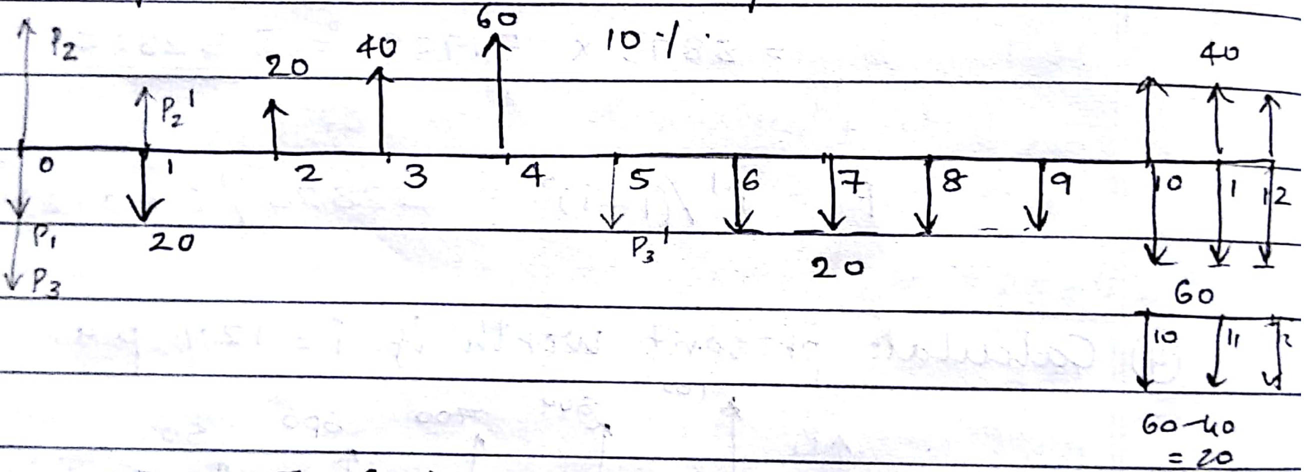
$$P_3 = 300 \times (F/A, 12, 2) \times (P/F, 12, 6)$$

$$= 300 \times 2.120 \times 0.5066$$

$$= ₹\,322$$

$$P = P_1 - P_2 - P_3 = 2200$$

(5) Find present worth at 10% p.a.



$$P_1 = F \times (P/F, i, n)$$

$$= 20 \times (P/F, 10, 1)$$

$$P_1 = 20 \times 0.9091 = ₹\,18.182$$

$$A = A_1 + G\,(A/G, i, n \to 3)$$

$$A = 20 + 20(0.937) = ₹\,38.74.$$

$$P_2' = A \times (P/A, i, 3)$$

$$= 38.74 \times 2.487 = ₹\,96.34$$

$$P_2 = P_2' \times (F/P, i, 1)$$

$$= 96.34 \times 0.9091 = ₹\,87.58$$

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$P_3' = A \times (P/A, i, 7)$

$= 20 \times 4.868 = ₹ 97.36$.

$P_3 = P_3' \times (P/F, i, 5) = 97.36 \times 0.6209$

$= ₹ 60.45$.

$P = P_2 - P_1 - P_3 = 87.58 - 18.482 - 60.45$

$= ₹ 8.948$.

$$20\underbrace{(P/F,10,1)}_{0.9091} + 20\underbrace{(P/A,10,4)}_{3.170}\underbrace{(P/F,10,5)}_{0.6209} + 20\underbrace{(P/A,10,3)}_{2.487}$$

$$\times \underbrace{(P/F,10,9)}_{0.4241} - [20 + 20\underbrace{(A/G,10,3)}_{\cancel{1.38}\ 0.937}] \times \underbrace{(P/A,10,3)}_{2.487}\underbrace{(P/F,10,1)}_{0.9091}$$

$P = -8.946$

Chapter - 2

Nominal and effective interest rate

Interest rate are usually calculated on an annually basis. However it may be compounded several times in a yr. i.e quarterly, monthly, half yearly etc. But in all the cases, interest charges is represented annually.

For example, if one year is divided into quarters & each quarter is charged with an interest rate of 2% then this is stated as, 8% p.a compounded quarterly. Stated in this fashion the interest is known as nominal interest rate and is given,

$$\text{Nominal interest} = \frac{r}{n}$$

r - rate of interest / yr.

n - No. of times in a yr interest is compounded

The future worth at the end of one year of ₹1000 earning an interest rate of 8% compounded quarterly is,

$$F_{Q_1} = 1000(1 + 0.02)^1 \qquad i = 8/4 = 0.02$$

$$= 1020.$$

$$F_{Q_2} = 1020\,(1+0.02) = 1040.4$$

llly $F_{Q_4} = ₹1082.43$ —— (1)

If the interest rate is 8% compounded annually,

$$F = 1000(1+0.08) = ₹1080 \quad \text{——— (2)}$$

* Comparing (1) & (2) it is evident that nominal interest gives higher returns.

Effective interest rate: is defined as ratio of interest charged for one year to the principal:

$$i_{eff} = \frac{F-P}{P} \times 100$$

For example, for a 1 year loan of ₹1000 at a nominal interest rate of 8% compounded quarterly, we have effective interest rate,

$$i_{eff} = \frac{1082.43-1000}{1000} \times 100 = 8.234\%$$

* Therefore effective interest rate is the equivalent interest rate which gives the same final amount when compounded annually for the interest which is being compounded more than once in a year:

* 8% compounded quarterly ≡ 8.243% compounded annually.

$$i_{eff} = \frac{F-P}{P} \times 100$$

$$= \frac{P(i+1)-P}{P} \times 100 = (1+i)^n - 1$$

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$$\boxed{i_{eff} = (1 + r/n)^n - 1}$$

$(r/n) \rightarrow$ represent rate of interest applicable for each compounding period.

(power) $n \rightarrow$ no. of time in a year / payment period interest is calculated.

① A finance company charges interest at the rate of 18% p.a compounded monthly. Calculate effective interest rate.

$$r/n = 18/12 = 1.5\%$$

$$i_{eff} = (1 + 18/12)^{12} - 1$$

$$i_{eff} = 19.56\%$$

② Calculate effective interest rate if rate of interest 6% is compounded ① yearly ② semi-annually, ③ quarterly ④ Monthly ⑤ Daily.

① $i_{eff} = (1 + r/n)^1 - 1 = 6\%$.

② $i_{eff} = (1 + 0.06/2)^2 - 1 \approx 6.09\%$.

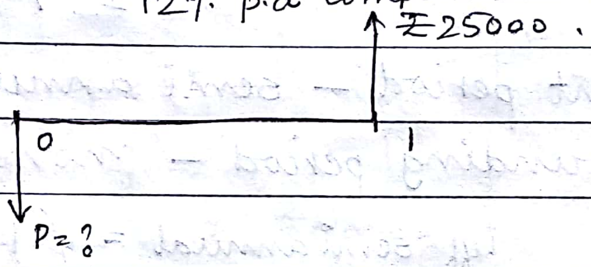
③ $i_{eff} = (1 + 0.06/4)^4 - 1 = 6.13\%$.

(4) $i_{eff} = \left(1 + \frac{0.06}{12}\right)^{12} - 1 = 6.16\%$

(5) $i_{eff} = \left(1 + \frac{0.06}{365}\right)^{365} - 1 = 6.18\%$

(3) If a person wants to have ₹25000 today, How much he had to deposit 1yr ago, when the investment earned an interest at the rate of 12% compounded monthly ?

12% p.a compounded mothly



$r/n = 12/12 = 1\%$

$F = i_{eff} = \left[1 + .12/12\right]^{12} - 1 = 12.68\%$

$P = \frac{F}{(1+i)^n} = \frac{25000}{(1+0.1268)^1} = ₹\,22186.7$

$= \frac{25000}{\left(1 + .12/12\right)^{12}} =$

(4) A credit card company charges interest at a rate of 2% per month on the unpaid balance. Calculate effective interest rate if the person is

* Calculation is easier when payment period & compounding period is same.

clearing the dues every semi annual period ?

$$i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$$

* When payment period & compounding period, i_{eff} is calculated.

Solution :

$$\cancel{i_{eff}} = \left(\cancel{1 + \frac{0.02}{2}}\right)$$

Payment period – semi annual period
Compounding period – monthly.

$$i_{eff} \text{ semi annual} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.24}{12}\right)^6 - 1 =$$

2% per month ≡ 24% p.a compounded monthly

$$i_{eff} \text{ semi annual} = 0.126 = \underline{\underline{12.6\%}}$$

(24% p.a compounded monthly & 12.6% compounded annually will give same amount at the end of half ~~an~~ year).

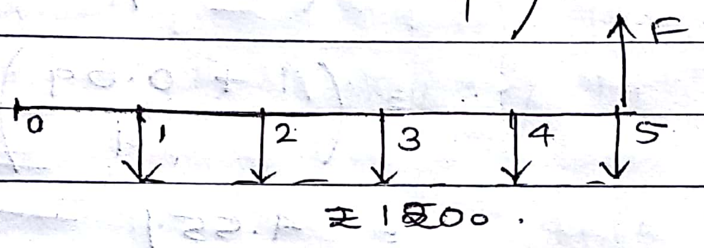
(2) An amount of ₹1200/year is to be paid ~~to an~~ into an account starting one year from now for the next 5 years. Determine total amount accumulated in the account at the end of 5th yr

If interest rate is 12% p.a compounded quarterly?

Payment period = ~~\$~~ yearly.
compounding period = quarterly.
No. of times in 1yr ~~payment~~ interest is calculated — 4.

$$i_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$$

$$= \left(1 + \frac{0.12}{4}\right)^{4} - 1 \approx \cancel{15.9\%}\ 12.55\%$$



$$\cancel{A=₹1} \quad F = A\left[\frac{(1+i)^{n} - 1}{i}\right]$$

$$= 1200 \times \left[\frac{(1+0.1255)^{5} - 1}{0.1255}\right]$$

$$F = ₹7707.$$

(3) Alpha is a vendor for Ford Motor company. An engineer is on Alpha's committee to evaluate bids for a new generatn machine. The vendor's bid include the interest rate as stated below. Alpha will make payments only on semi annual payments, help the engineers to (1) determine

the best bid.

BID 1 : 9% p.a compounded quarterly

BID 2 : 3% p. quarter,, " = 12% p.a compounded quarterly

BID 3 : 8.8 p.a " monthly.

(2) Calculate effective interest rate for annual payments.

→ (1) Bid (1) $i_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$

$= \left(1 + \frac{0.09}{4}\right)^{2} - 1$

$= 4.55\%$

Bid (2) $i_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$

$= \left(1 + \frac{0.12}{4}\right)^{2} - 1$

$= \cancel{1.5\%}\ \cancel{0.0}\ 6.09\%$

Bid (3) $i_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1$

$= \left(1 + \frac{0.088}{12}\right)^{6} - 1$

$= 4.48\%$

Best bid : BID (3) (Least from buyers POV)

(2) Bid (1) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1$

$= \left(1 + \frac{0.09}{4}\right)^4 - 1$

$= \underline{9.3\%}$

Bid (2) $i_{eff} = \left(1 + \frac{0.02}{4}\right)^4 - 1$

$= \cancel{3.03\%}$ 2.55%.

Bid (3) $i_{eff} = \left(1 + \frac{0.088}{12}\right)^{12} - 1$

$= \underline{9.16\%}$.

(4) A company is planning to invest ₹6000 every 6 months & savings will be cont, for 5 years. Determine amount accumulated in the account at the end of 5th year in the following case

(1) When i = 12% compounded semi annually

(2) " i = 12% " annually

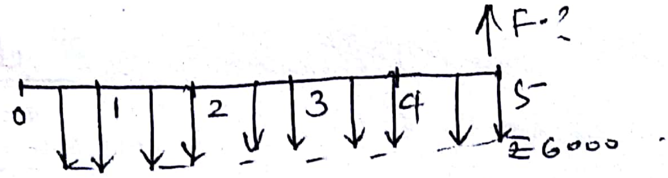
(3) " i = 12% " quarterly.

(1) $i_{eff} = \left(1 + \frac{r}{n}\right)^n - 1 = \left(1 + \frac{.12}{2}\right)^1 - 1$

$\left[\frac{(1+i)^n - 1}{i}\right]$ $= \underline{6\%}$

$F = A \times \cancel{(1+i)^n} = \cancel{6000 \times (1+0.06)^5}$

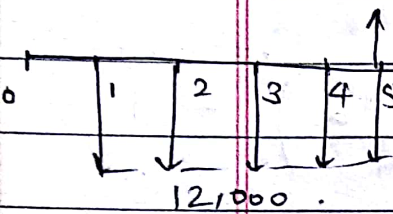
$= \cancel{₹8029.35}$



$6000 \times (F/A, 6\%, 10)$

$$F = 6000 \left[\frac{(1+0.06)^{10} - 1}{0.06} \right] = ₹\,79084.76$$

(2) $i = 12\%$ compounded annually $= i_{eff}$. [₹6000 for semi year will be ₹12000]



$$F = 12000 \times \left[\frac{(1+0.12)^{5} - 1}{0.12} \right] = ₹\,76234.16$$

~~₹105292.41~~

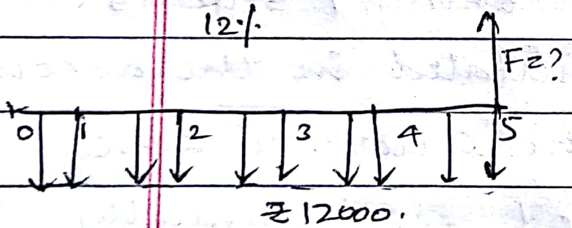
(3) $$i_{eff} = \left(1 + \frac{r}{n}\right)^{n} - 1 = \left(1 + \frac{12}{4}\right)^{2} - 1$$

$$= 6.09\%$$

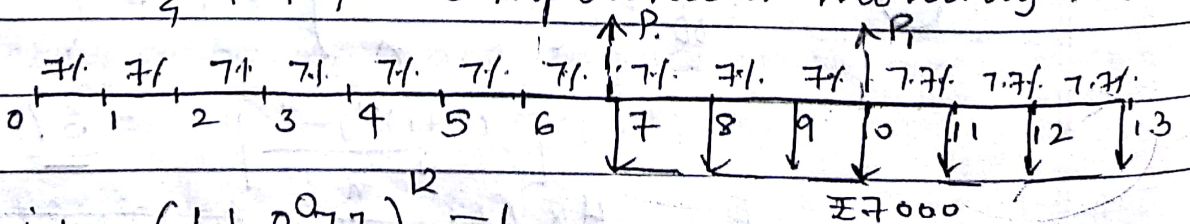
$$F = 6000 \times \left[\frac{(1+0.0609)^{10} - 1}{0.0609} \right] = ~~₹\,121822.309~~$$

$$= ₹\,79419.82$$

12%



(5) A seven payment annuity of ₹7000 begins 7 years from now. Determine the worth of this annuity in the 7th yr if interest rate is 7% p.a for 1st 10yrs & 7.7% compounded monthly thereafter



$$i_{eff} = \left(1 + \frac{0.077}{12}\right)^{12} - 1$$

$$= 7.97\%$$

Present worth for 11th, 12th, 13th year, (at 10th yr)

$$P_1 = A\left((1+i)^n - 1 / i(1+i)^n\right) = 7000\left(\frac{(1+0.0797)^3 - 1}{0.0797(0.0797+1)^3}\right)$$

$= ₹\ 18049.44.$

Present worth of ₹18049 at 7th year (single payment 7% p.a.)

$$P_2 = 18049/(1+0.07)^3 = ₹14733.36$$

Present worth for 7th, 8th, 9th, 10th, (at 6th yr)

$$P_3 = 7000\left((1+0.07)^4 - 1 / 0.07(0.07+1)^4\right) = ₹23710$$

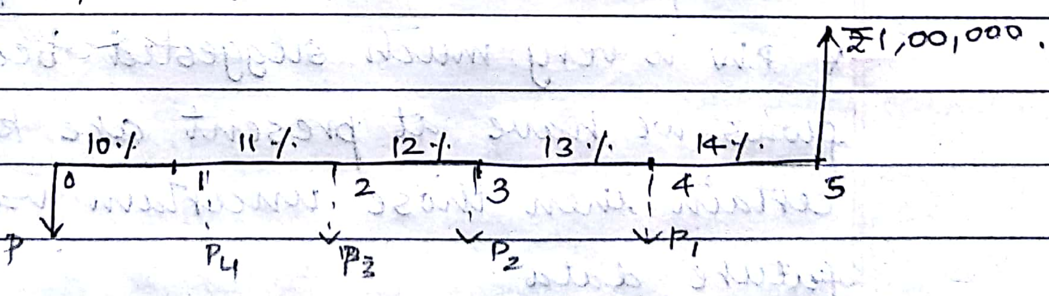
Future worth of ₹23710 at 7th yr

$= 23710 \times (1+0.07) = ₹25369.7$

Total present worth $= 14733.36 + 25369.7$
$= ₹40103$

Varying Interest Rate: (due to inflation)

① A company wants to have ₹1,00,000 available in the 5th year to make an investment. Determine the lumpsome amount the company should deposit now if the interest rate in the yrs 1, 2, 3, 4, 5 are 10%, 11%, 12%, 13% & 14%.



$$P = F/(1+i)^n = 100000/(1+0.14)(1+0.13)(1+0.12)(1+0.11)(1+0.1)$$

$$P = F \times (F/P, 14, 1) \times (F/A, 13, 1) \times (F/P, 12, 1) \times (F/P, 11, 1) \times (F/P, 10, 1)$$

$P = ₹56765$

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Chapter - 3

EVALUATION OF ALTERNATIVES

Objectives: * Select best alternative economically.
* Understand various bases for comparison of alternatives.

Assumptions:

1) Cash flows are known (investment & forecasted revenue of project are known)
2) Cash flows are in constant-value dollars. (Neglecting inflation)
3) The interest rate is known [rate of return] tax
4) Comparisons are made with before tax cash flows.

COMPARISON OF ALTERNATIVE WITH EQUAL LIFE:

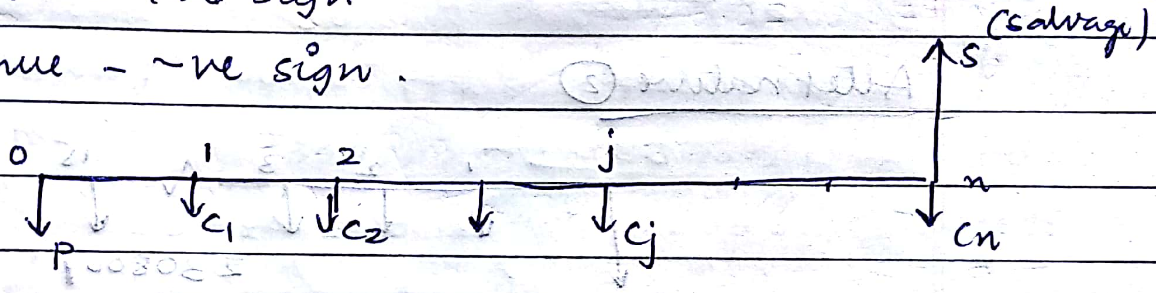
I PRESENT WORTH METHOD

* Equivalent value at present of the asset based on the time value of money.
* Discounted value of future sums i.e all future cash flows are discounted to present & compared.
* PW is very much suggested because any cash flows we have at present are known & are certain then those uncertain values at some future data.
* PW comparison are made only b/w co terminated (ending at same time) proposals to assure equivalent outcomes.

Two methods in calculating PW:
* Cost dominated cash flow diagram.
* Revenue dominated " " "

Cost dominated cash flow diagram:

salvage value → Return from deposit^n of assests.
* cost — +ve sign
Revenue — -ve sign.

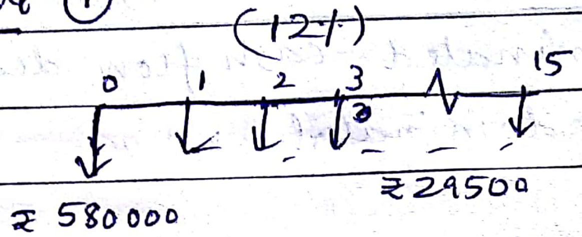


* Alternative with min PW should be selected.

① A construction company receives two bids for an elevator to be installed in their newly constructed apartment, the details of which are given in the table. Determine best bid based on present worth method @ 12% interest rate:

Alternative	Initial cost (Rs)	Service life 15	Annual operat^g cost.
1	580000	15	29500
2	640000	15	30800.

Alternative ①

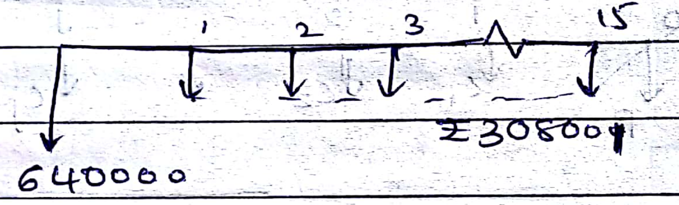


$PW_A = 580000 + 29500 \times (P/A, 12, 15)$

$= 580000 + 29500 \times 6.811$

$= ₹7,80,924.5$

Alternative ②



$PW_2 = 640000 + 30800 \times 6.811$

$= 8,49,778.8$

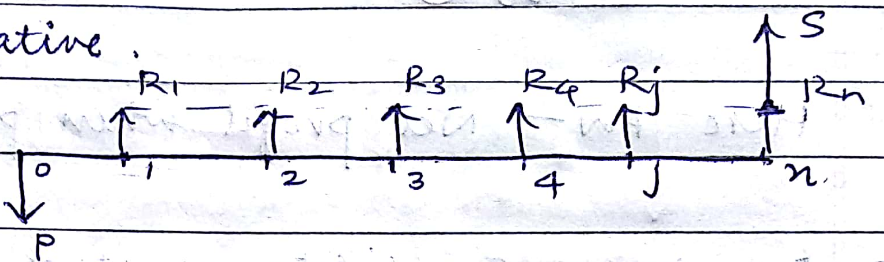
∴ Alternative ① is best bid.

* PW – represent total cost reqd to implement the project & run it for 15 yrs.

Revenue dominated cash flow diagram

* Revenue – +ve
* Cost – -ve sign.

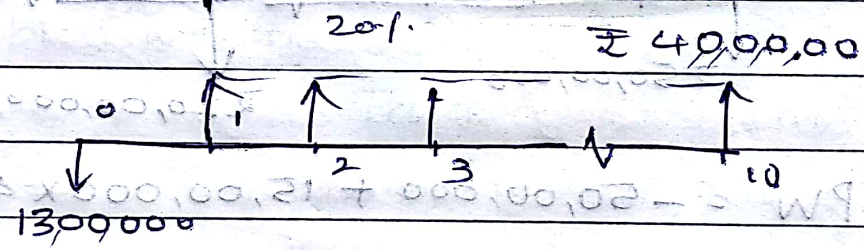
* Alternative with maximum PW of revenue is best alternative.



① Initial outlay & annual revenues of product firm. Find best alternative if $i = 20\%$ compounded annually.

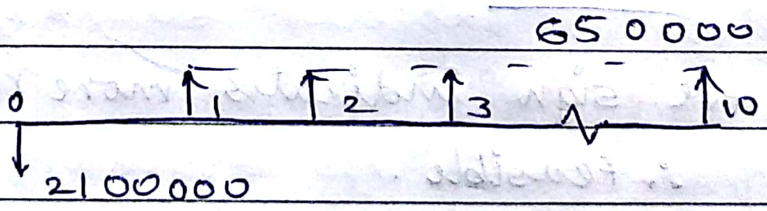
Technology	Initial cost (₹)	Annual Revenue (₹)	Life (yrs)
1	1300000	400000	10
2	2100000	650000	10
3	2300000	860000	10.

→

① 

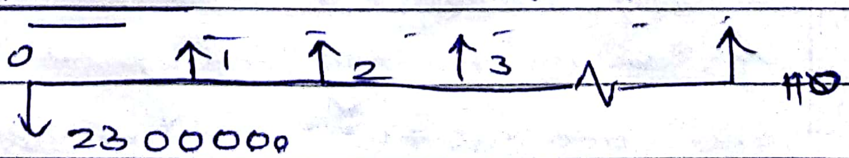
$PW_1 = -1300000 + 400000 \times (P/A, 20, 10)$

$= ₹\,3,76,800$.

② 

$PW_2 = -2100000 + 650000 \times 4.192$

$= ₹\,6,24,800$

③ 

$PW_3 = -2300000 + 860000 \times 4.192 = ₹\,13,05,120$

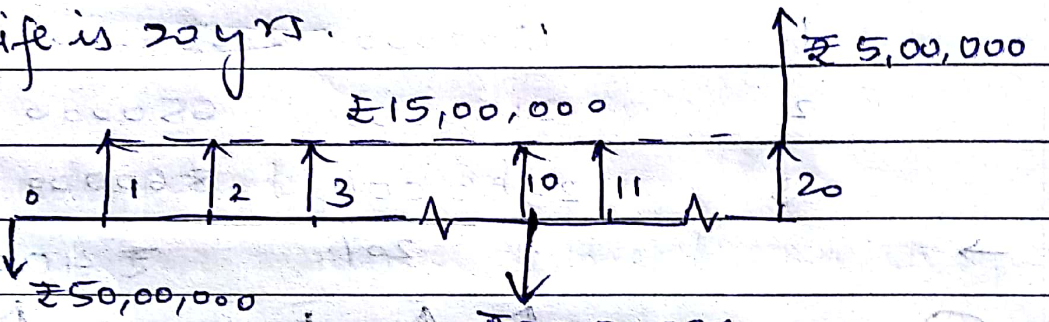
Technology ③ is best bid.

Here PW — Net profit when project runs for 10yrs.

③ Details of feasibility report of a project are as shown below. Check feasibility of project based on present worth method if i = 20%.

Initial outlay	Annual revenue	Modernizing cost at end of 10yrs	Salvage value
₹ 50,00,000	₹ 15,00,000	₹ 20,00,000	₹ 5,00,000

* Life is 20 yrs.



$$PW = -50{,}00{,}000 + 15{,}00{,}000 \times \underset{(P/A,\,20,\,20)}{4.870}$$

$$- 20{,}00{,}000 \times \cancel{0.026} + \underset{(P/F,\,20,\,10)}{0.161}$$

$$+ 5{,}00{,}000 \times \underset{(P/F,\,20,\,20)}{0.0261}$$

$$= +₹\ 1995050$$

(+ve sign indicates more revenue than investment)

∴ Feasible.

COMPARISON OF ALTERNATIVES ~~W/O~~ WITH UNEQUAL LIFE:

① LCM Method:

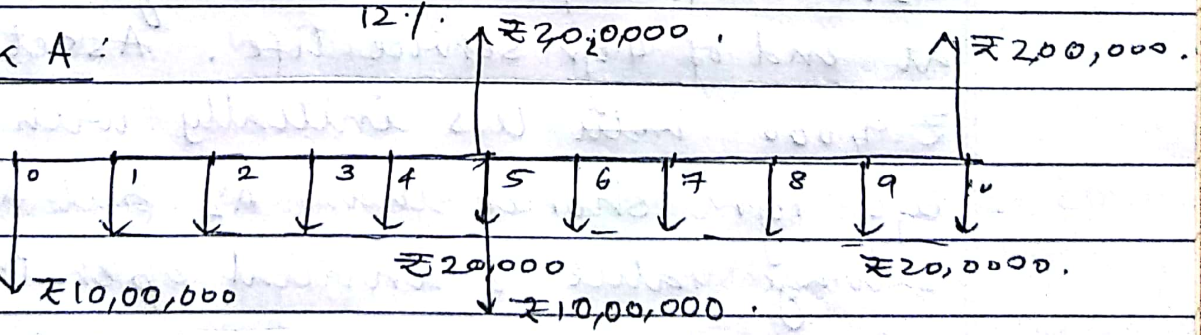
In this method, the alternatives are co-terminated by selecting an analysis period which is the least common multiples of the life's of involved assests. The assumptn here is that the assests will be repeatedly replaced by successor having idintical cost characteristics.

① Two types of trucks are available for mining applicatn. Assume i=12% & determine best truck based on present worth method.

Particulars	Truck A	Truck B
Initial ~~Est~~ cost	₹ 10,00,000	₹ 15,00,000
Annual operatg cost	₹ 20,000	₹ 15,000.
Salvage value	₹ 2,00,000	₹ 5,00,000.
Service life	5 years	10 years.

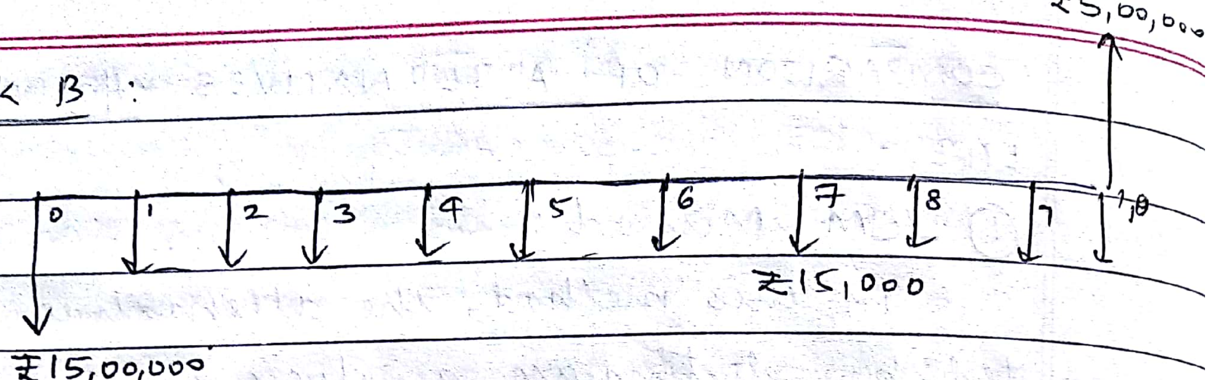
→ LCM = 10 yrs.

Truck A:



$PW = +10,00,000 + 10,00,000 \times (P/F, 12, 5) + 20,000 \times (P/A, 12, 10)$
$- 2,00,000 \times (P/F, 12, 5) - 2,00,000 \times (P/F, 12, 10)$
$= +10,00,000 + 10,00,000 \times 0.5674 + 20,000 \times 5.650$
$- 2,00,000 \times 0.5674 - 2,00,000 \times 0.3220 = ₹ +1400,820$

Truck B :



$PW = +1500000 + 15000 \times (P/A, 12, 10) - 5,00,000 \times (P/F, 12, 10)$

$= +15,00,000 + 15000 \times 5.650 - 5,00,000 \times 0.3220$

$= +1423750$

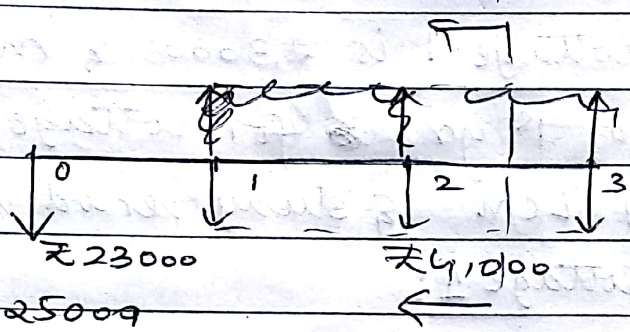
② Study period method

Here the analysis is based on a specified durat^n^ corresponding to the length of the project/ time period during which the assests are in service.

① Assets A_1 & A_2 have the capability of satisfactorily performing a reqd func^n^. Initial cost of asset A_2 ₹32000 & expected salvage value ₹4,000 at end of 4 yr service life. Asset A_1 costs ₹9,000 ~~initi~~ less initially with economic life 1 yr shorter than A_2 but A_1 has no salvage value & annual operating cost exceed those of A_2 by ₹2,500. If $i = 15\%$, select the best alternative when comparison is by 2 year study period method.

	A_1	A_2	
~~Asset A_1~~ :	23000	32000	Initial cost
	—	4000	Salvage
	2500	—	Operating cost
	3	4	Life

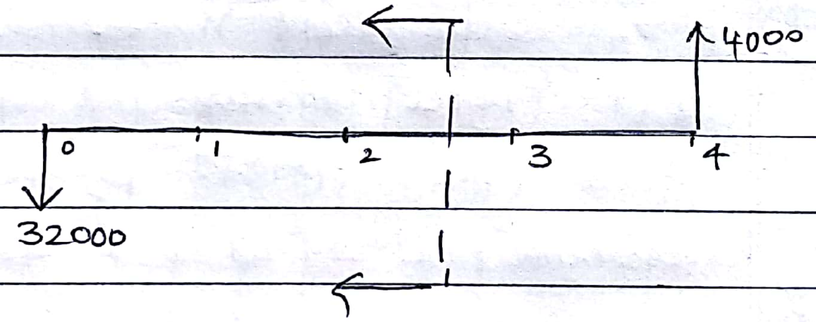
Asset A_1:



$PW = 23000 + \cancel{4,000}\ 2500\ (P/A, 15, 2)$

$= 23000 + \cancel{4000} \times 1.626 = \cancel{29504}\ ₹27065$

Asset A_2:



$PW = 32000$

II FUTURE WORTH METHOD

① Two holiday cottages are under considerath. Compare them based on future worth method when $i = 12\%$. when neither of a cottage has a rett realisable salvage value.

Particulars	Cottage 1	Cottage 2
First cost	₹4500	₹10,000.
Estimated life	3yrs	4yrs.
Annual maintanance cost	₹1,000	₹720.

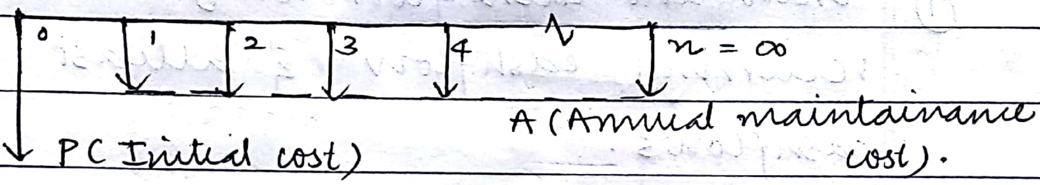
Additinal data: Market value at the end of 2nd yr for cottage 1 is ₹3000 & market value at the end of 1st yr ~~₹~~ for cottage 2 is ₹8500. Solve using LCM & study period method (5yrs)

→ Cottage 1:

0 1 2 3

III CAPITALISED COST METHOD

* It is used for comparing assexts ~~for~~ with infinite life.
* Evaluatn of projects such as dams, railway line, are done using capitalized cost method.
* Capitalized cost is present worth of an alter--native that will last forever.



$$PW = P + A(P/A, i, n \to \infty)$$

$$= P + A\left[\frac{(1+i)^n - 1}{i(i+1)^n}\right] \quad \text{(Divide \& multiply by } (1+i)^n\text{.)}$$

$$= P + A\left[\frac{(i+1)^n\left[(i+1)^n - 1\right]}{i(i+1)^n\,(1+i)^n}\right]$$

$$PW = P + A\left[\frac{1}{i}\right] \quad \left[\lim_{n \to \infty}\right].$$

$$\boxed{PW = P + \frac{A}{i}} \qquad \boxed{\text{capitalized cost } CC = P + \frac{A}{i}}$$

Cash flows in a project with infinite life can be of two types:

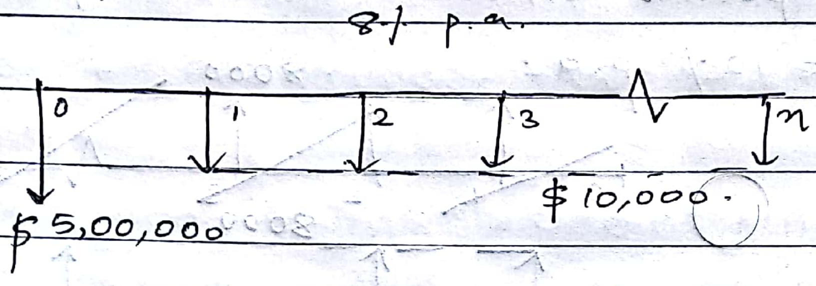
① Recurring cost ② Non-recurring cost.

* Annual operating cost of \$50,000 & rework cost \$40,000 every 12 years are examples of recurring cashflows.
* Examples for non-recurring cash flows are initial investment & one time cashflow estimate in future.

Procedure to solve capitalized cost problem:

1. Draw the cash flow diagram showing all non recurring cashflow & atleast 2 cycles of recurring cashflows.
2. Find the present worth of all non-recurring cashflow using single payment present worth relat^n ship.
3. Find equivalent uniform annual amount for one cycle of all recurring cashflows & divide the amount by interest rate to get capitalized cost of all recurring cashflows.
4. Divide all the uniform cashflows occuring from year 1 to infinity by the interest rate to get capitalized cost of those uniform cash flows.
5. Add the values obtained in ~~ab~~ above steps to get total capitalized cost of given investment.

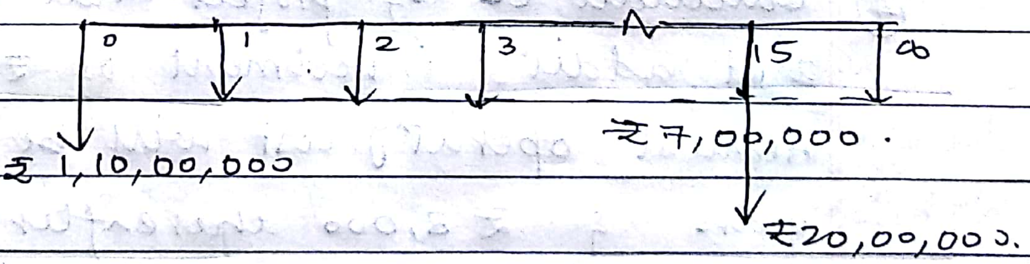
① Calculate CC of project having initial cost \$5,00,000 & annual maintainance cost \$10,000 at 8% p.a.



$$CC = PW = P + A/i = 5,00,000 + \frac{10000}{0.08}$$

$$\cancel{PW}\ CC = \$6,25,000$$

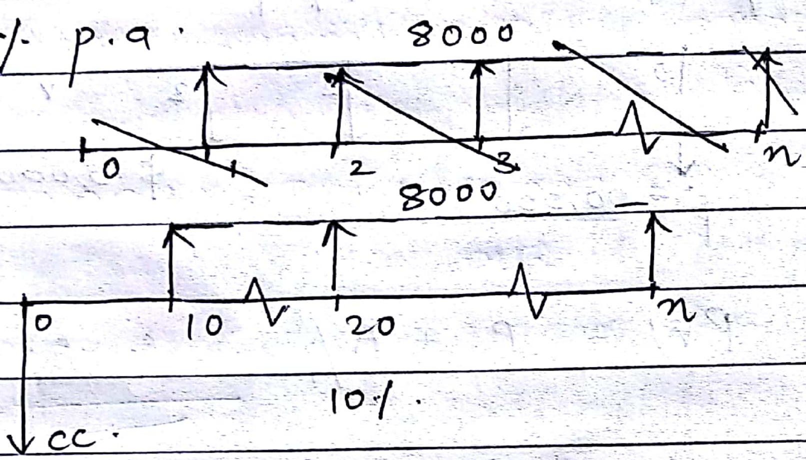
② A public project has an initial cost of ₹ ~~1,10,00,000~~ ₹ 1,10,00,000 & annual operating cost ₹ 7,00,000. Further the project will have one time major repair work of ₹ 20,00,000 in the 15th yr, Find CC if $i = 12\%$.



$$CC = 1,10,00,000 + \frac{20,00,000}{(1+0.12)^{15}} + \frac{7,00,000}{0.12}$$

$$= ₹\ 1,71,98,725.86$$

(3) ₹ 8,000 is to be withdrawn from a savings account at the end of every 10 year. Calculate lumpsome amount to be deposited now if $i = 10\%$ p.a.



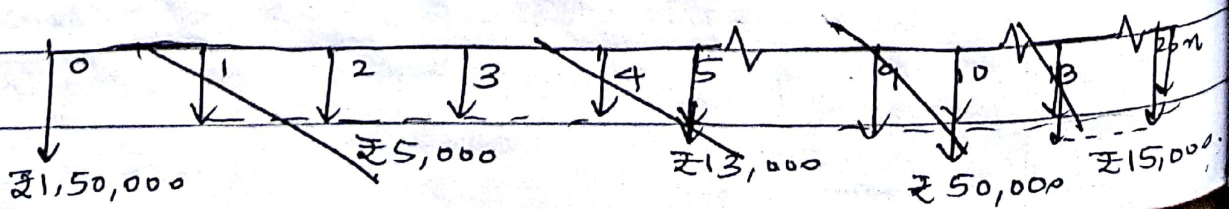
Finding equivalent uniform amount for one cycle of recurring amount,

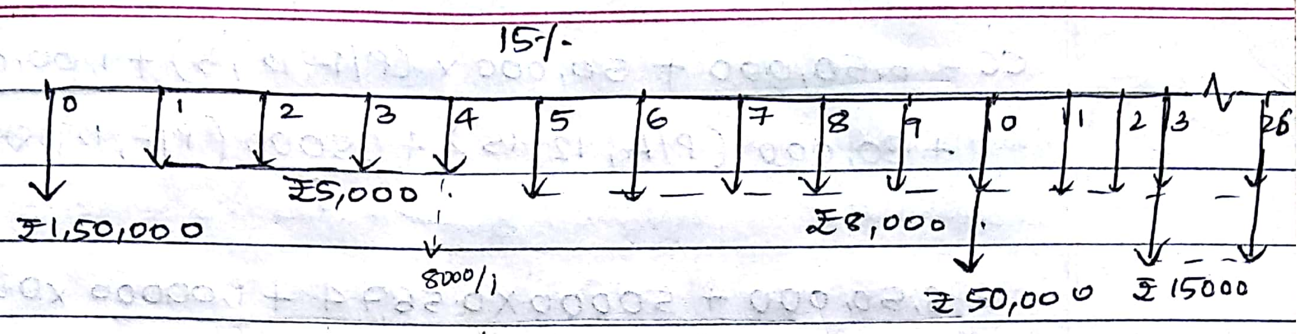
$$CC = 8000 \times (A/F, 10, 10) / 0.1$$

$$= 8000 \times 0.0627 / 0.1$$

$$CC = 5016$$

(4) Calculate CC of project that initial cost ₹1,50,000 and additnal investment of ₹50,000 after 10yrs. Annual operatng cost will be ₹5,000 for 1st 4 years & ₹8,000 thereafter. In additn there is expected to be a recurring major rework costng ₹15,000 every 13 years. Assume $i = 15\%$.





$CC = 1,50,000 + 5,0000(P/F, 15, 10) + 5000 \times (P/A, 15, 4)$

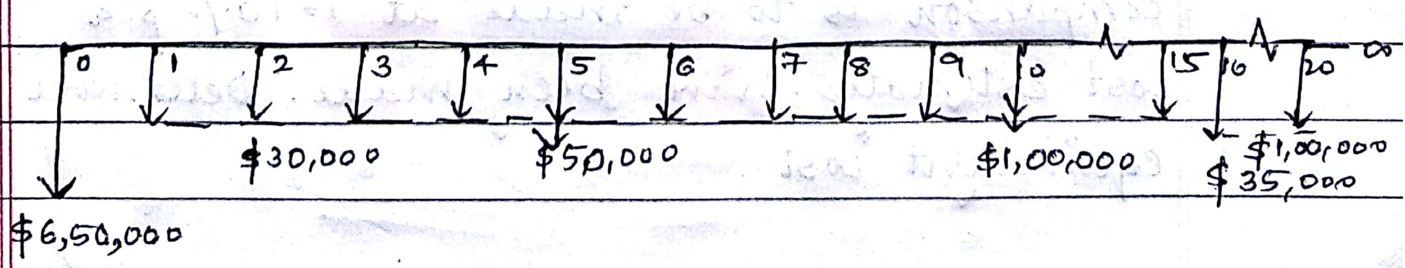
$+ \frac{8000}{i} \times (P/F, 15, 4) + 13000\,(\cancel{P/A}, A/F, 15, 13)/i$

$= 1,50,000 + 50,000 \times 0.2472 + 5000 \times \cancel{5.019}\ 2.855$

$+ \frac{8000}{0.15} \times \cancel{0.4972}\ 0.5718 + \frac{13000 \times 0.0291}{0.15}$

$= ₹\ 2,10,041$

(5) The expenditures involved in building & maintaining a bridge are shown. Calculate capitalized cost if i = 12%. Cost of design & building bridge is \$6,50,000. It is estimated that bridge will have to be renovated every 10 yrs at a cost of \$1,00,000. In additn \$50,000 has to spent in 5th yr to relay the road on the bridge. Annual operatng cost are expected to be \$30,000 for 1st 15 years & \$35,000 therafter.



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$$CC = 6{,}50{,}000 + 50{,}000 \times (P/F, 12, 5) + \underline{1{,}00{,}000\,(A/F, 12, 10)}$$

$$+ 30{,}000\,(P/A, 12, 15) + \frac{35000}{i}(P/F, 12, 15)^{0.12}$$

$$= 6{,}50{,}000 + 50000 \times 0.5674 + \frac{1{,}00000 \times 0.0570}{0.12}$$

$$+ 30{,}000 \times 6.8111 + \frac{35000}{0.12}(0.1827)$$

$$CC = \$983487.5$$

IV ANNUAL WORTH METHOD (Gives working capital reqd every yr)

1. In an annual worth method, all the receipts & payment occuring over a period of time are converted to an equivalent uniform yearly amount.
2. Annual worth method is widely used to because of the inclinatn to view a year's gains & losses as a yardstick of progress.
3. Major tools used in annual worth calculatn are capital recovery factor & sinking fund factor.

1) Two types of power converters α & β are ainder consideratn for a particular applicatn. An economic comparison is to be made at $i = 10\%$ p.a. Following cost estimates have been made. Determine annual equivalent cost.

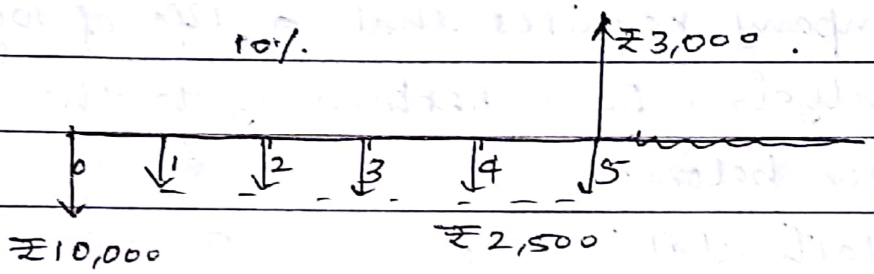
EUAC / EUAW

Equivalent uniform annual cost – Cost dominant

" " worth – Revenue "

Particulars	α	β
Purchase price	₹10,000	₹25,000.
Annual operating cost	₹2,500	₹1,200.
Salvage value	₹3,000	₹5000.
Life	5 yrs	9 years.

α

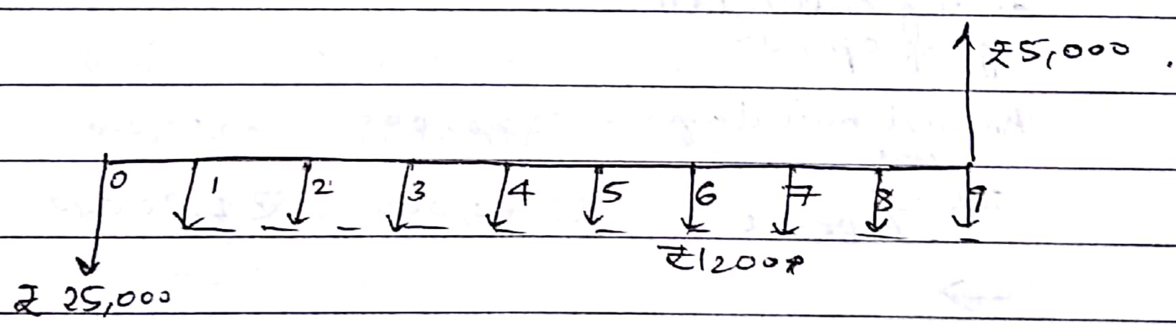


$$EUAC_{\alpha} = 10000\,(A/P, 10, 5) + 2500 - 3000\,(A/F, 10, 5)$$

$$= 10000 \times 0.2638 + 2500 - 3000\,(0.1638)$$

$$= ₹\ 4646.6$$

β :



$$EUAC_{\beta} = 25000 \times (A/P, 10, 9) + 1200 - 5000\,(A/F, 10, 9)$$

$$= 25000 \times 0.1736 + 1200 - 5000 \times 0.0736$$

$$= ₹5172.$$

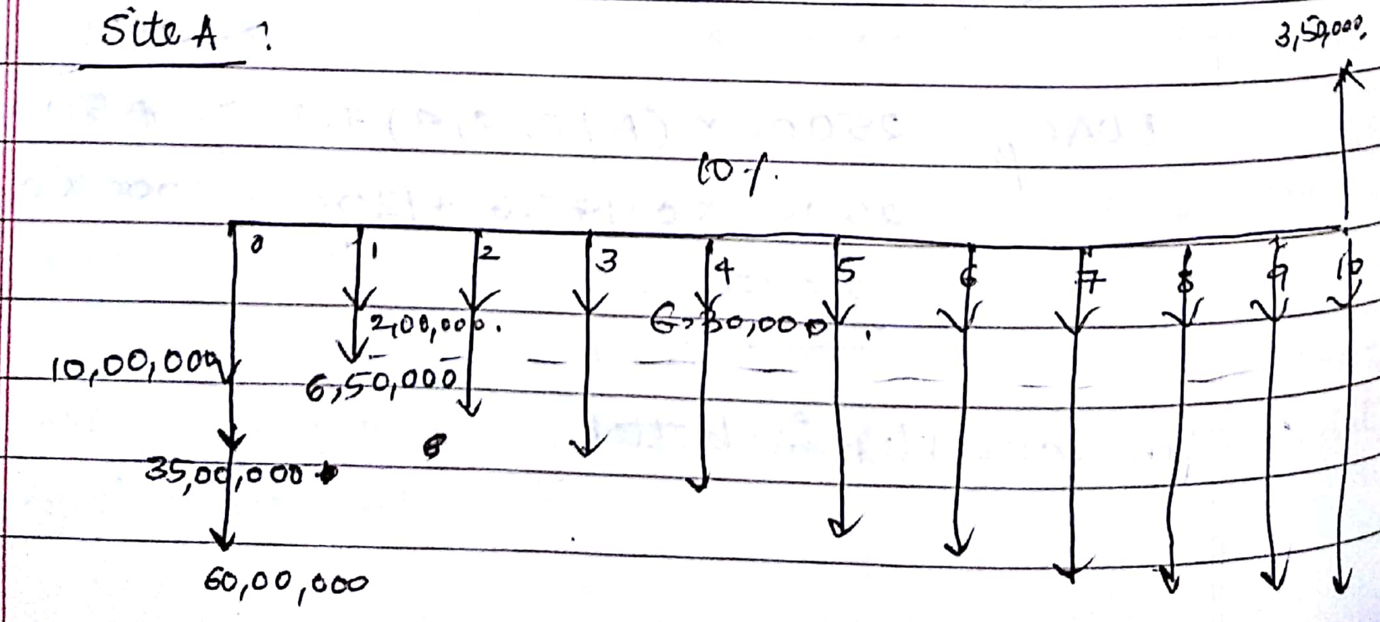
∴ α converter is better..

(2) A beverage company is planning expansn of its cold storage. 3 alternative sites are being considerd that uses a MARR of 10%. Sites A & B require an expenditure of ₹35,00,00 on land whereas site C requires ₹45,00,000. Estimated income ↑ due to new sites available is annualized at ₹24,80,00. Company requires that a life of 10yrs be used for analysis. Data pertaining to the project are as given below:

Particular	A	B	C
Building & machi-nery	₹60,00,000	₹70,00,000	₹80,00,000
cost of compressors.	₹10,00,000	₹13,50,000	₹8,50,000
Energy cost during 1st yr of operatn	₹6,50,000	₹4,80,000	₹6,50,000.
↑ in energy cost during each additnl yr of operatn	₹30,000	₹20,000	₹35,000.
Annual maintainance cost	₹2,00,000	₹1,50,000	₹5,00,000.
Salvage value	₹3,50,000	₹4,30,000	₹1,80,000.

→

Site A :



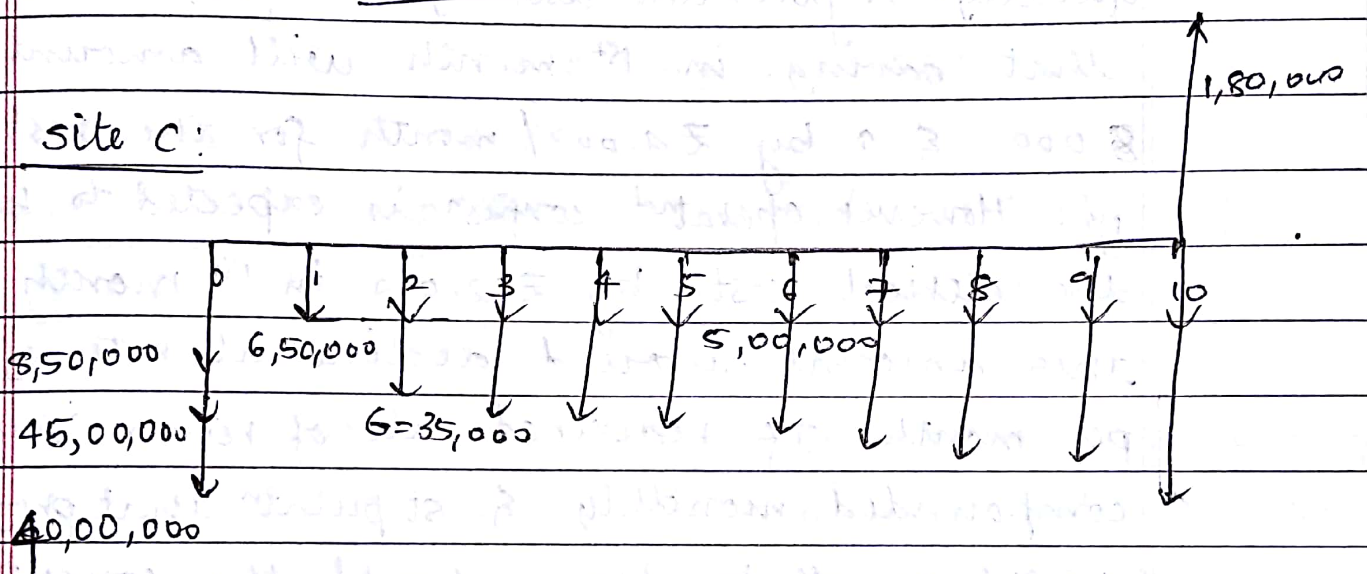
$$EUAC_A = \overset{10,500000}{\cancel{60,00000}}(A/P, 10, 10) + \cancel{35,00,000}(A/P,10,10)$$

$$+ 200,000 + 6,50,000 \overset{+30,000}{} (A/G, 10, 10)$$

$$- 3,50,000 \times (A/F, 10, 10)$$

$$= 2648629.57$$

site C:



$$EUAC_A = 9350000 \times (A/P, 10, 10) + 5,00,000$$

$$+ 650,000 + 35000 (A/G, 10, 10)$$

$$- 180000 (A/F, 10, 10)$$

$$= 27,90,7666$$

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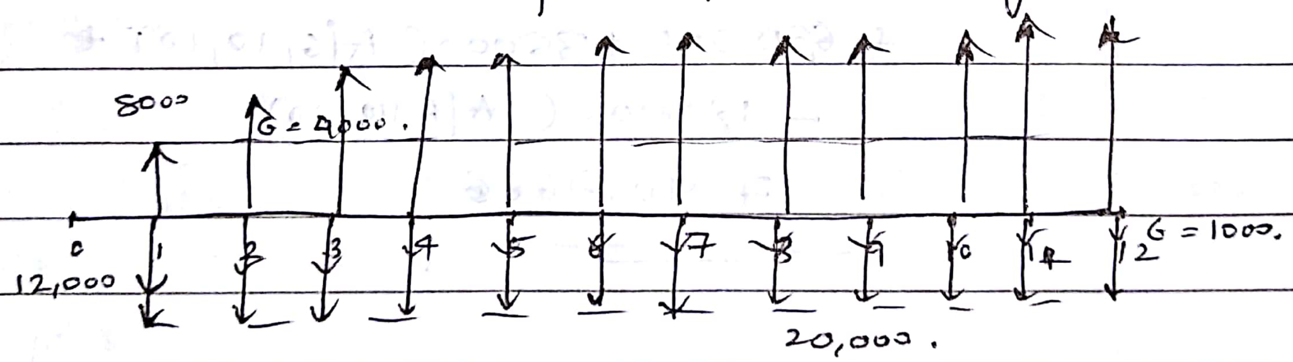
(3) A consulting firm proposes to provide a self inspectn training for clerks who work with insurance claim. The program last for 1yr & cost 20,000 / month. & professor to improve service quality. A potential user of the program estimates that saving in 1st month will amount to 8,000 & ↑ by ₹4,000/ month for the rest of yr. However operatnl confusn is expected to boost the clerical cost by ₹12,000 in 1st month but this amount, would decline at rate of ₹1,000 per month. If required rate of return is 12%. compounded monthly & stipulatn that program should pay off in 1yr, should the consultant be hired?

i = 12% p.a compounded monthly.



PP – monthly.

CP – monthly.

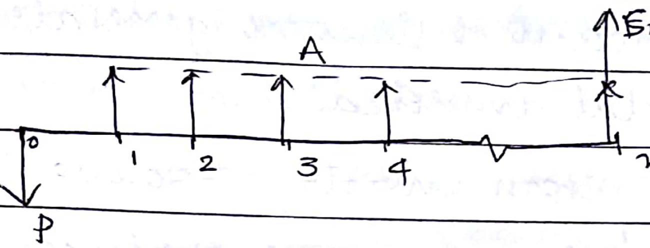
$i = \left(\cancel{CP} + \frac{r}{n}\right) = \frac{0.12}{12} = 0.01\%.$

$EAVC_\alpha = 8000 + 4000\,(A/G, 1\%, 12) - \left[12000 - 1000\,(A/G, 1, 12)\right]$

$- 20{,}000.$

$= -\cancel{16914} \;\; \cancel{-16195} \;\; +2905$ ∴ should be hired.

VI CAPITAL RECOVERY WITH RETURN



P = Co

S_n = salvage

A - annuity.

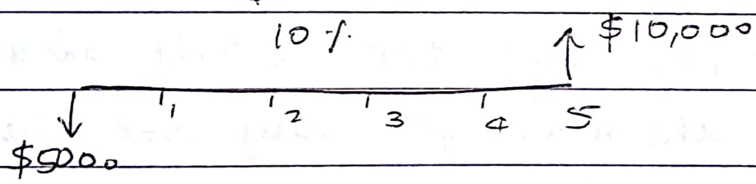
$$CR(i) = P(A/P, i, n) - S_n(A/F, i, n)$$

$$(A/F, i, n) = (A/P, i, n) - i.$$

$$CR(i) = P(A/P, i, n) - S_n[(A/P, i, n) - i]$$

$$\boxed{CR(i) = (P - S_n)(A/P, i, n) + S_n i}$$

1. An asset with a 1st cost of \$5,000 has an estimated service life of 5 yrs & estimated salvage value of \$10,00~~0~~. For an $i = 10\%$, what is capital recovery with return?



$$CR(i) = (5000 - 1000\cancel{0})(A/P, 10, 5) + 1000\cancel{0} \times 0.1$$

$$= + \overset{4000}{\cancel{5000}} \times 0.2638 + 1000\cancel{0} \times 0.1$$

$$CR(i) = \$1155.2$$

VI RATE OF RETURN METHOD (ROR) / RETURN ON INVESTMENT (ROI)

It is amount of income generated in an year in the capital invested.

For ex, is a person invests ₹50,000 to start a company & after one yr the company produces a profit of 2500 then ROR is 2500/50,000 = 5%.

Types of ROR:

① MARR (Minimum Acceptable/Attractive Rate of Return)

Lowest level of return at which the investment is acceptable & is set by the top management.

Ex: Suppose that a manager knows that investg in an alternative such as bond/FD yields a known ROR (6% say) then while analyzing a new project, the manager may use the return of investg in bond as MARR (6%). The manager will only implement the project if its ancipated ~~result~~ return is atleast equal or more than return from investg in bond.

② IRR (Internal Rate of Return)

The IRR for a project is defined as the ROR at which the present value of cash inflow and cash outflows are equal or the rate at which the

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investment breaks even.

IRR allows manager to rank project easily. The project with highest IRR is preferred.

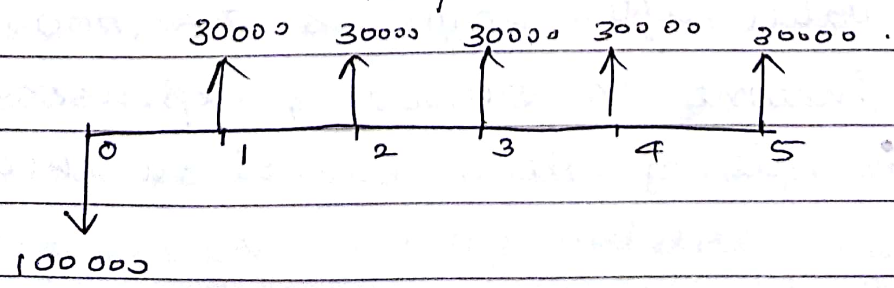
Easy of comparison makes IRR attractive.

③ ERR (External Rate of Return)

It is the possible ROR for an investment under the given economic conditn.

① A person is planning a new business. The initial outlay & cashflow pattern for new business are as listed below. The expected life of business is 5yr. Calculate IRR.

Period	Cashflow
0	− 100000
1	30000
2	30000
3	30000
4	30000
5	30000.



$$-100000 + 30000 \times (P/A, i, 5) = 0$$

$$3.33 = (P/A, i, 5)$$

$i = 15\%$

$$\frac{+100000}{30{,}000} = \frac{(i+1)^5 - 1}{i(i+1)^5}$$

$$3.33 = \frac{(i+1)^5 - 1}{i(i+1)^5}$$

$i = 10\%$

$3.33 \neq 3.79$

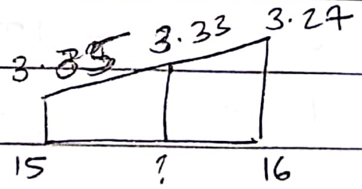
$i = 15\%$

~~3.33 = 3.35~~

$i = 16\%$

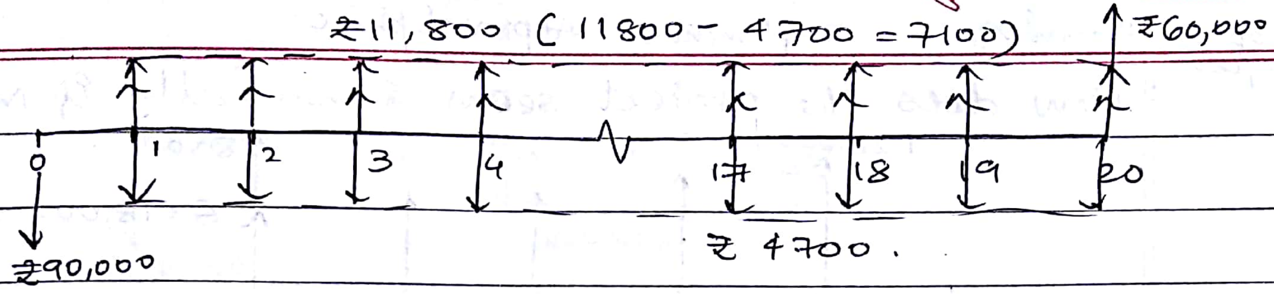
$3.33 \neq 3.27$

16	—	3.27
?	—	3.33
15	—	3.35



$i = 15.5\%$

② A farm house can be purchased for ₹90,000 & resale value after 20 yrs is ₹60,000. If annual rental income is ₹11,800 & expenses ₹4,700. What is rate of return earned on this farmhouse.



$(P/F, i, 20)$

$$-90{,}000 + 7100(P/A, i, 20) + 60000 = 0$$

~~7100 (P/A, i, 20) = 4.22.~~

$\cancel{\dfrac{(i+1)^5 - 1}{i(i+1)^5} = 4.22}$.

~~At 10%. i = 3.79~~ ~~RHS = 3.79~~

~~At 8%.~~ ~~RHS = 3.99~~

~~At 6%.~~ ~~RHS = 4.21~~

~~At 5%.~~ ~~RHS = 4.32~~.

~~RHS = 4.22~~

~~i = 5%.~~ ~~4.32~~

? ~~4.22~~

~~i = 10%.~~ ~~4.21~~

$i = 7.5\%$.

$i = 5\%$. 21095

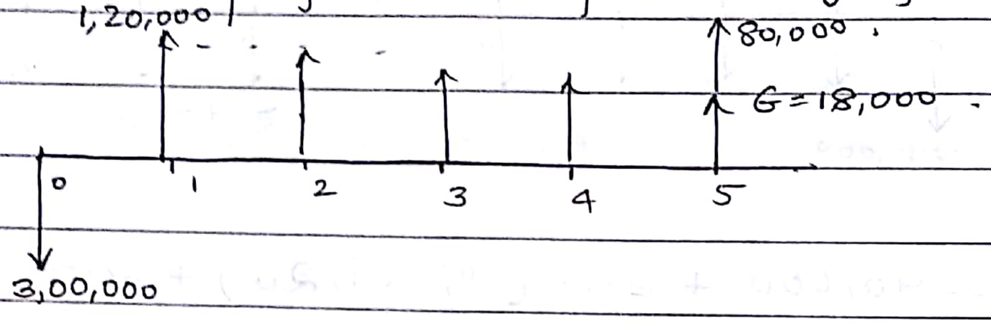
$i = ?$

$i = 10$ −20635.

(3) A oil company has purchased an oil well by paying 3,00,000. Field engineers estimates that net receipt will be \$1,20,000 during 1st yr with reductn of \$18,000 in following yrs. It plans to sell the well after 5 yrs for \$80,000.

	PW/FW/CC	AW	IRR
Interpretation	Expenditure	Working capital	—
Level of management	Lower level/ML	LL/ML	Top management
Phase of project	Planning	Implementation	Approval phase.

How does the project seem financially if MARR = 20% ?



$$-3,00,000 + \left[120,000 \overset{18000}{} (A/G, i, 5) \right] \overset{P/A}{(\cancel{A/P}, i, 5)}$$
$$+ 80,000 \times (P/F, i, 5) = 0 .$$

$i = 10\%$

~~263834.604~~. 81081.22

(MARR) $i = 20\%$.

2723

$i = \cancel{30}\%$. 0

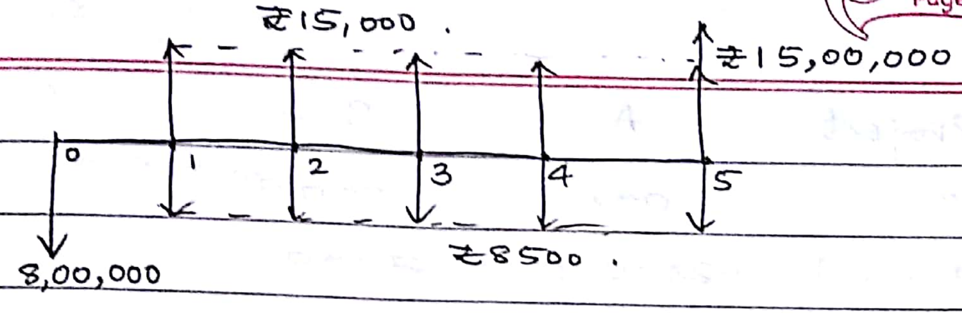
-51469

IRR = 20.5%.

IRR > MARR.

∴ Project is feasible.

(4) A patch of land is likely ↑ in value. Cost of land now is ₹8,00,000 & expected to be ₹15,00,000 within 5 yrs. During this period, it can be rented for small scale industry at ₹15,000 /yr. Annual taxes are ₹8,500 & are likely remain constant. What ROR will be earned on the investment if estimates are accurate ?



$$-8,00,000 - 8500(\cancel{A/P}\ P/A, i, 5) + 15000(\cancel{A/P}\ P/A, i, 5) + 15,00,000\,(P/F, i, 5) = 0.$$

$i = 10\%$. ~~220438.5~~ 155991.5

$i = 15\%$. -32412

IRR = 14.1%

Limitations of IRR method:

Case I: ₹ 90,000 MARR = 16% , ~~(Retained earning, investment pool)~~

Project	A	B
Investment	-₹50000	-₹85,000.
IRR	35%	29%

$$\text{Overall ROR}^{(A)} = \frac{50000 \times 0.35 + 85000 \times 0.29}{90000} = 26.56\%$$

$$\text{Overall ROR}^{(B)} = 28.37\%.$$

By inspectn project A seems better because of higher IRR but overall ROR (B) is greater. ∴ IRR can't be used. (It is due to different investment on 2 projects).

Case 2: ₹ 5000 MARR = 10%.

Project	A	B
0	−1000	−5000
1	2000	7000
PW	₹818	₹1364.

$PW(A) = \cancel{-5000 + 2000/(0.1)^1}$

$= -5000 + [2000 + 4400](P/F, 10, 1)$

$= -1000 + 2000/1.1 = 818$

$PW(B) = -5000 + \left[\frac{7000}{1.1}\right] = 1364$

$IRR(A) = 100\%$.

$IRR(B) = 40\%$.

Even though IRR of A ~~of~~ is better than B. But decisⁿ should be taken on present worth ∴ Project is preferred.

Since IRR method ~~is~~ doesnot give proper results for above 2 cases. Incremental analysis is implemented.

Incremental analysis

A project C with investment ₹4000 & return 5000.

~~A~~. C

−4000

+5000

IRR = 25%.

↑ 5000 (~~7000~~−2000−

↓ 4000 (5000−1000)

$-4000 + \frac{5000}{(1+i)^1} = 0$

$\boxed{i = 25\%}$

$\Delta_{IRR} = 25\%$.

Instead of the investing amount at MARR = 10%, it is invested at 25%, making project B feasible.

* If $\Delta_{IRR} >$ MARR, Project B. (project with higher initial investment)

Decision rules:

① If $\Delta_{IRR} >$ MARR, select a project with ↑ initial project.

② If $\Delta_{IRR} <$ MARR, select a project with lower initial project.

③ If $\Delta_{IRR} =$ MARR, select any project.

① A college student wants to start a small scale painting business during his off school hours. To economise his startup, he purchases used painting items. He has 2 options

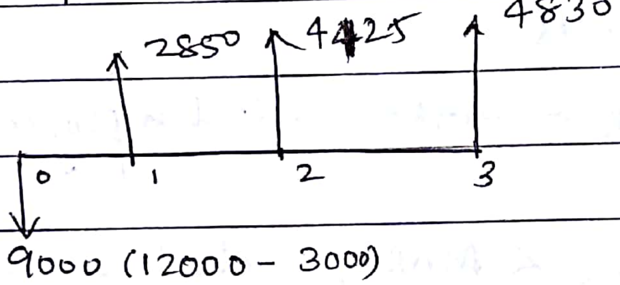
option 1: Do most of the painting by himself by limiting his business to residential jobs.

option 2: Purchase some painting equipment & hire some helpers to do some residential & commercial jobs. In either case, he expects to fold up business in 3 yrs.

cashflows are given below. MARR = 10%.

n	B_1	B_2
0	−3000	−12000
1	1350	4200
2	1800	6225
3	1500	6330
IRR	25%	17.43%

* Unequal initial investment & IRR, use incremental analysis.
* If project IRR is more than MARR, adopt incremental analysis.



PW eqⁿ.

$$-9000 + 2850 \times (P/F, i, 1) + 4425 (P/F, i, 2) + 4830 (P/F, i, 3) = 0.$$

$i = 15\%$

$\Delta IRR > MARR$, Select project with higher investment B_2.

(2) Consider the following 3 sets of mutually exclusive alternative. Which project would you select based on ROR on incremental investment (MARR = 15%)

n	D_1	D_2	D_3
0	−2000	−1000	−3000
1	1500	800	1500
2	1000	500	2000
3	800	500	1000

* Investing extra in next highest alternative is feasible is checked.

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IRR 34.47% 40.76% 24.81%

∴ Compare D_1 & D_2 (2nd & 3rd highest investment)

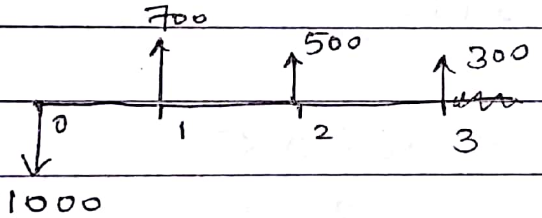
$D_1 - D_2$

-1000

700

500

300



$$-1000 + \frac{700}{(i+1)^1} + \frac{500}{(1+i)^2} + \frac{300}{(1+i)^3} = 0.$$

$i = 10\%$ 274.98.

$i = 20\%$.

$i = 25\%$ 33.6

$i = 27\%$ 7.

$i = 28\%$ -4.89.

$i = 27$ 7

$i = ?$ 0

$i = 28$ -4.89

$$\frac{27-28}{28-x} = \frac{7+4.89}{-4.89-0}$$

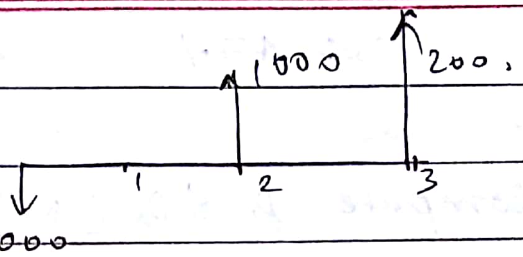
$$28 - x = \frac{1 \times (+4.89)}{7+4.89}$$

$$28 - x = +0.411$$

$\Delta IRR_{D_1 - D_2}$ $x = 27.6\% > 15\%$.

Compare 1 & 3 $\rightarrow$ ↑ investment.

$D_3 - D_1$



1000

0

1000

200

$$-1000 + \frac{1000}{(i+1)^2} + \frac{200}{(1+i)^3} = 0$$

10 % = −23.

8 % = 16.105

8	—	16.105
x	—	?
10	—	−23.

$$\frac{10-8}{x-8} = \frac{16.105+23}{0-16.105}$$

$$\frac{2}{x-8} = \frac{39.105}{16.105}$$

$$x - 8 = +0.82$$

$$x = 8.82$$

$\Delta IRR_{D_1 - D_3} = 8.82 < 15\%$

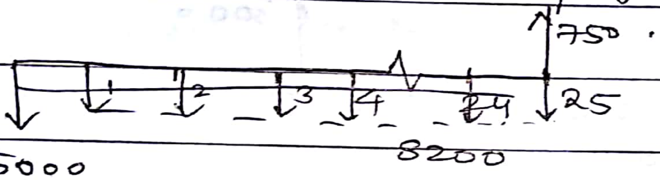
$\therefore$ D_1 is selected.

(3) Select the best alternative using incremental ROR

MARR = 10%.

Year	Alt A	Alt B
0	−15000	−21000
1−25	−8200	−7000
25	+750	+1050

~~Calculate IRR for each project~~,



~~$-15000 - 8200(P/A, i, 25) + 750(P/F, i, 25) = 0.$~~

~~$i = 15\%$~~ ~~-67982~~

~~$i = 5\%$~~

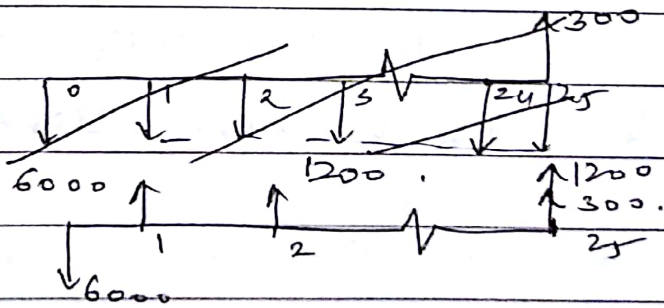
IRR cannot be calculated because no return.

A − B.

−6000

+1200

300



$-6000 + 1200(P/A, i, 25) + 300(P/F, i, 25) = 0.$

$i = 10\%$ ~~-16864.47~~ 4920.09

$i = 15\%$ 1765.92

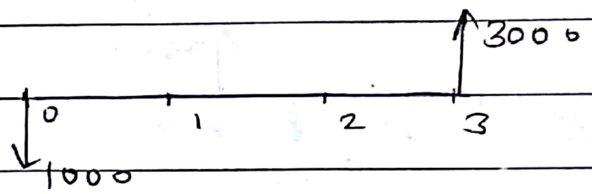
$i = 20\%$ -59.25 $x :$ $15 + \frac{(20 - 15) \times (1765.92)}{(59.25 + 1765.92)}$

$x = 19.82\% > 10\%$.

∴ Alt B.

④ Select best alternative. if MARR = 10%.

n	E_1	E_2	E_2 (LCM method)	$E_2 - E_1$.
0	-2000	-3000	-3000	-1000 ~~-100~~
1	1000	4000	4000 - 3000	0
2	1000		4000 - 3000	0
3	1000.		4000.	3000



$$-1000 + \frac{3000}{(i+1)^3} = 0.$$

~~i = 10%~~.

$$3 = (1+i)^3$$

$$(i+1) = \sqrt[3]{3}$$

$$i = 44.22\%$$

E_2 is best alternative.

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Replacement analysis

Replace an existing equipment

To determine when asset (in service) should be replaced by a more economical alternative

1) Loss
2) Quatblity degrades
3) Production may come to a halt
4) Maintainance cost ↑s (annuity)

* Application of annual worth method.

Reasons for replacement

* More efficiency * New technology * ↓ in quality
* ↑ maintainance * outdated technology

① Deterioration (wear & tear, high manufacturing cost, ↑ maintainance, higher rejection rates, degrading quality).

② Obsolescence (new development, order asset moves ways accompanying objectives, less appealing)

③ Inadequacy (when current operatn conditions change, an order asset occasionally lacks capacity to meet new requirements)

Basic Terminologies

① Physical life of an asset: Time period upto which we are prepared to keep the obsolite asset in service (may not be efficient working life) of vintage case.

② Economic life of an asset.

Time period spent to use machine in less. Time after which we save money by replacing the asset

③ Sunk cost

Cost incurred by past actions & not relevant to decisions because they cannot be changed.

₹ 35000 + ₹5000 to keep car working → ₹32,000.

Sunk cost : unforce cost, but doesnt affect decisions

Sunk cost = Present book value − Present Market value

actual value

= 35000 − 32000 = ₹3000

General Nature

1. Defender : Existing ~~Ad~~ asset to be replaced.
2. Challenger : Asset proposed to be replaced.

Methods

1. Outsider's point of view (the defender is sold for market value)
2. Cash flow approach (the ~~de~~ cost of defender is only considered ~~on~~ ~~no~~ no initial cost considered)
3. Economic life. * here the defender is sold & used for challenger)

Outsider's Point of view

(1) A company purchased machine X a year ago for ₹ 8500 with following:

Estimated life = 6 yrs.

Salvage value = ₹ 1000

operating expenses = ₹ 8000 / year

At end of 1st year, a salesman offers machine Y for ₹ 11500 which has:

Estimated life = 5 yrs

Salvage value = ₹ 1500.

Operating cost = ₹ 5500/year due to improvement.

salesman offers ₹3500 for machine X if machine Y is purchased.

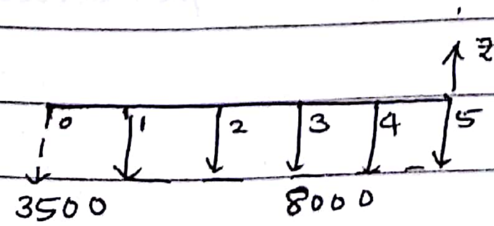
This appears ~~less~~ low to the company. but best offer received new here is only ₹ 3000.

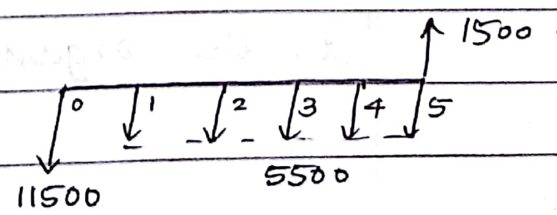
Assume i = 8%. Determine best course of action by taking outsider's point of view.

→ *Outsider's pov can be used when remaining life of defender is equal / unequal to the life of the challenger.

Analyzing for 5 years

Defender MKX 8% Challenger MKY





Management chooses we cant go with X (no additional investment) ∴ Outsider's Point of view is reqd.

X → no initial cost ∵ 1 year ago it was bought

So ~~use~~ we take market value, ₹ 8500 isnot applicable.

↳ ₹3500 / ₹3000 (to maximize gain)

From 3rd person point of view, evaluatn of alternative

① EUAC = 8000 + 3500 (A/P, 8, 5) − 1000 (A/F, 8, 5)

= ₹870 6.25

② EUAC = 5500 + 11500 (A/P, 8, 5) − 1500 (A/F, 8, 5)

= ₹ 8125 (least).

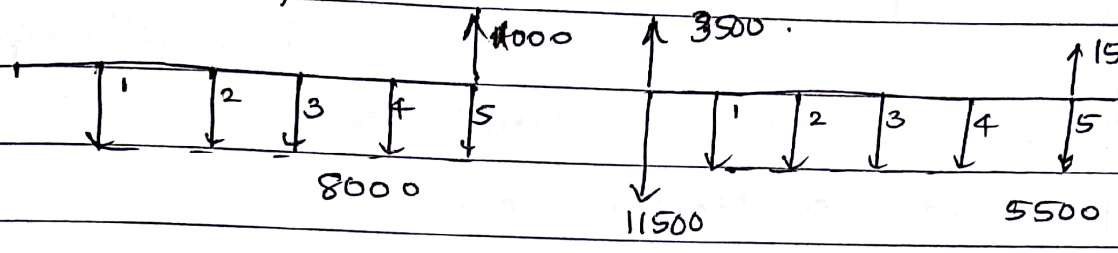
Saving = ₹581.25

Same decision we be conveyed

Cash Flow Approach / Insider's Point of view

This method is applicable only when the remaining life of defender = life of the challenger. It is based on the fact that if challenger is selected, the defender's market value is a cash inflow for the challenger and if defender is selected there is no actual expenditure of cash to the organisation.

Here, the defender's first cost is taken as zero & the market value of defender is subtracted from challenger's first cost



① $EAUC = 8000 - 1000\,(A|F, 8, 5) = ₹7829.5$

② $EAUC = 5500 + 11500\,(A/P, 8, 5) - 3500\,(A/P, 8, 5) - 1500\,(A/P, 8, 5)$

$= ₹7248.25$ → least.

saving $= ₹581.25$

Decision doesn't change w.r.t method.

Economic life of an asset :

The cost of owning & operating an assest can be divided into 2 categories: (1) Capital cost (2) operating cost.

Capital cost have 2 components : Initial investment & salvage value at the time of investment. The annual equivalent of capital cost called capital recovery with return over a period of n yrs can be calculated,

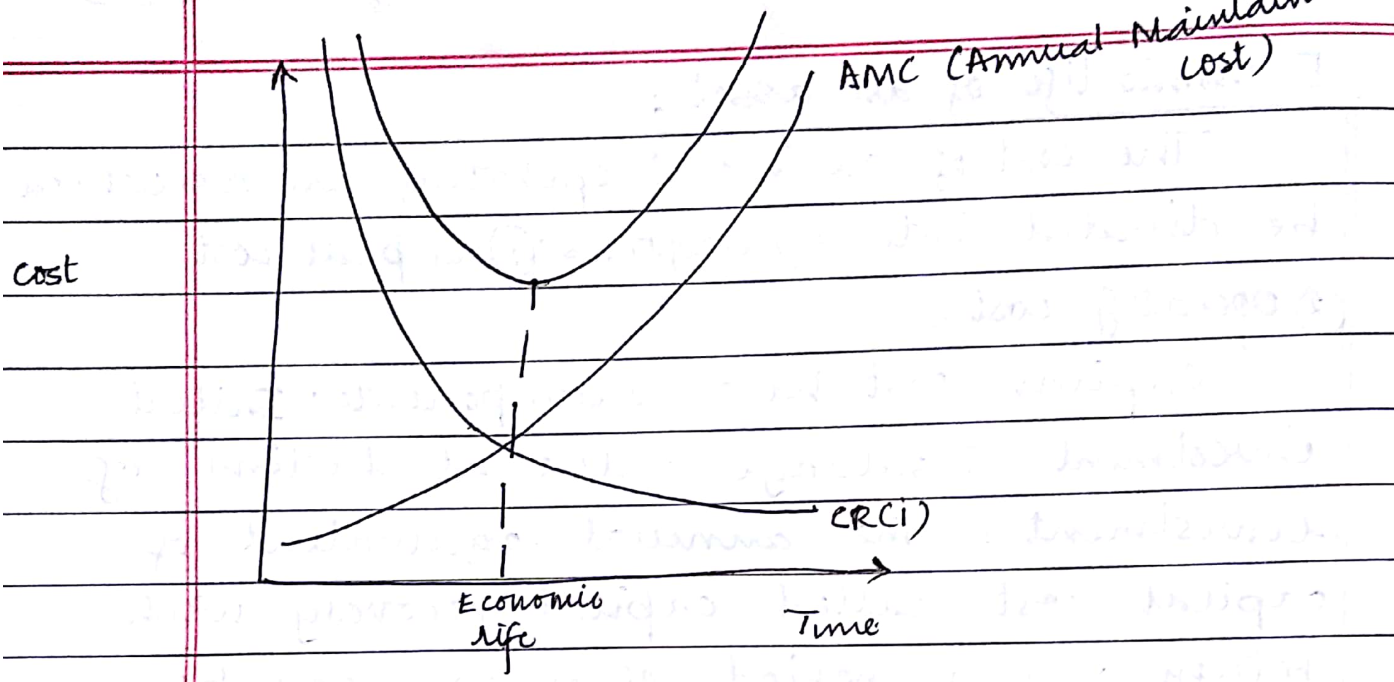
$$CR(i) = (P - S_n)\,(A/P, i, n) + S_n i \quad \text{--- (1)}$$

The operating cost of an asset include the maintainance cost, labour cost, material cost etc For the same equipment, the labour & material cost are constant for a given level of productn. However the maintainance cost tend to ↑ as the asset ages. Because of the ↑ing trend of maintainance cost, the total operating cost of the asset ↑s, as it ages. The operating cost can be expressed as,

$$OC(i) = \sum_{n=1}^{N} OC_n\,(P/F, i, n)\,(A/P, i, n) \quad \text{--- (2)}$$

The total annual cost of owning & operating the asset is the summatn of capital recovery cost & the operating expenses.

Cost ↓s over many yrs' ∵ More oppurtunities to repay the debt.



The economic life of an asset is the period of useful life that minimizes the total annual equivalent cost of owning & operat^ng the asset.

① A machine has a 1st cost of ₹1,30,000. The anticipated salvage value & annual operatg cost for next 5 yrs are as given. Determine economic life if i=10%.

n	Salvage value	AOC
1	90000	25000
2	80000	27000
3	60000	30000
4	20000	35000
5	0	45000

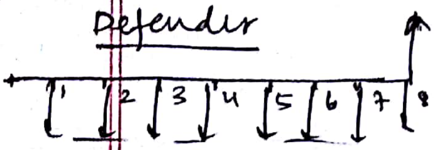
n	$CR(i) = (P - S_n)(A/P,i,n) + S_n i$	$OC(i) = \sum_{n=1}^{N} \left(OC_n (P/F,i,n)\right)(A/P,i,n)$	$TC(i) = CR(i) + OC(i)$
1	$(130000 - 90000) \times (A/P,10,1) + 90000 \times 0.1$ $= 53000$	25000	78000
2	$(130000 - 80000)(A/P,10,2) + 80000 \times 0.1$ $= 36810$	$(25000 \times (P/F,i,n) + 27000 \times (P/F,i,n)] \times 0.5762 \rightarrow (A/P,i,2)$ $= 25952.22$	62762
3	34147	27173.63	61320 ✓
4	36705	28863	65567
5	34294	~~26153~~ 31504	65798

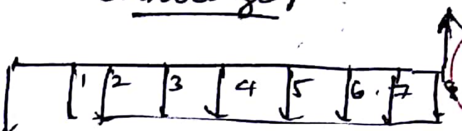
Economic life = 3 yrs.

EWh

Defender

Challenger





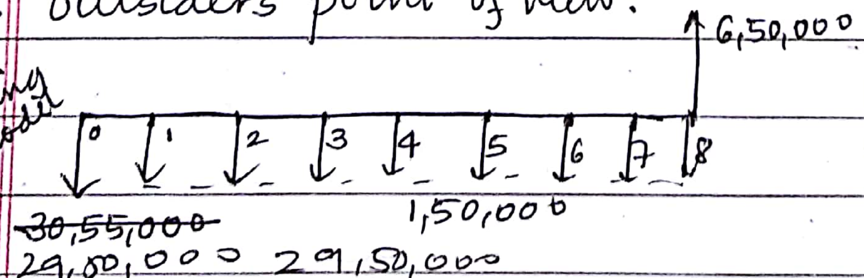
When outsider POV, it is evaluatn of alternatives, so the market value of ₹29,50,000 is considered.

② A constructn company had purchased a piece of constructn equipment 3 yrs ago at a cost of ₹ 40,00,000. Estimated life & salvage value at the time of purchase were 12 yrs & 8,50,000. Operatng & maintainance cost was ₹ 1,50,000. The constructn company is now considering replacing of existng model by new model. Due to depreciatn, the present book value of existng equipment is ₹ 30,55,000. Current market value of existing equipment is ₹ 29,50,000. Revised estimate of salvage value & remaining life are ₹ 6,50,000 & 8 yrs. Annual operatng & maintainance cost are same.

Initial cost of new model = ₹35,00,000, estimated life & salvage value, annual operatng cost are 8yrs, ₹ 9,00,000, ₹ 1,25,000. If $i = 10\%$ p.a, should the company retain old equipment or buy new model. Use outsiders POV & cash flow method.

→ Outsider's point of view:

Existing model



Market value is considered in outsider's ~~POV (Outsider's is buying for market value)~~

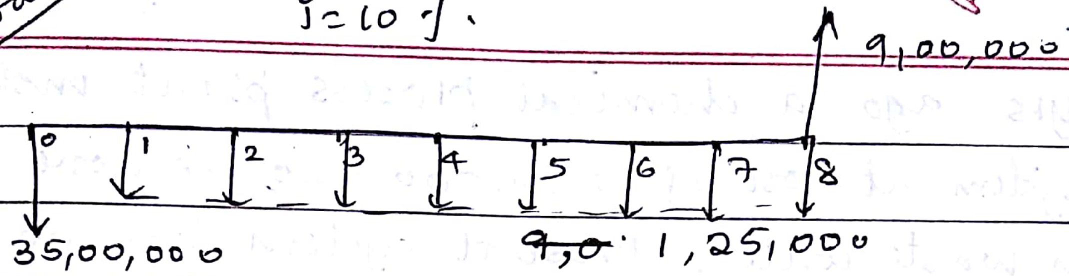
$$EUAC = \cancel{30,55,000}\ (A/P, 10, 8) + 1,50,000 - 6,50,000\ (A/F, 10, 8)$$

$$= \cancel{30,55,000}\ 29,50,000 \times 0.1874 + 1,50,000 - 6,50000 \times 0.0874$$

$$= ₹\ \cancel{665697}\ 646020$$

New model

$i = 10\%$



$EAUC = 35,00,000\,(A/P, 10, 8) + 1,25,000 - 9,00,000\,(A/F, 10, 8)$

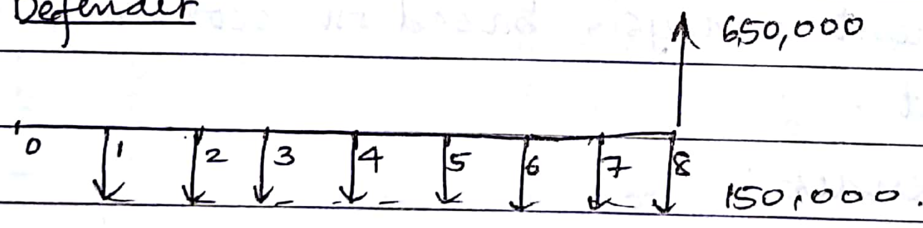
$= 35,00,000 \times 0.1874 + 125000 - 9,00000 \times 0.0874$

$= \underline{702,240}$

$\therefore$ Existing

<u>Cash flow approach</u>

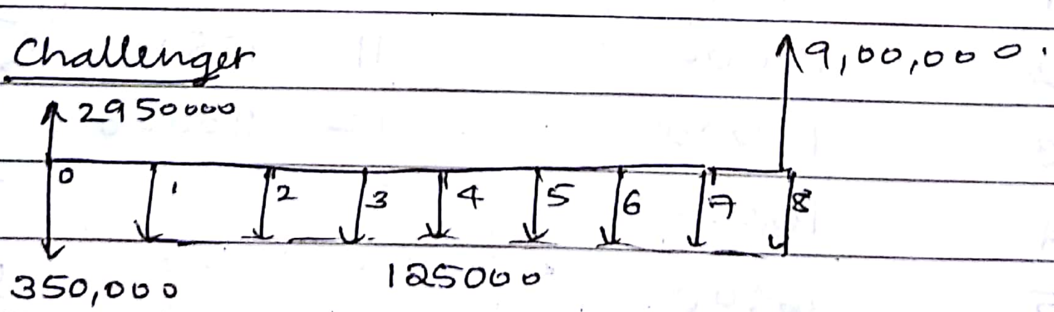
<u>Defender</u>



$EAUC = 150,000 - 650000\,(A/F, 10, 8)$

$= 93140$

<u>Challenger</u>



$EAUC = 550000\,(A/P, 10, 8) + 125000 - 900,00\cdot(A/F, 10, 8)$

$= \underline{56220}$

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(3) 3 yrs ago a chemical process plant installed a system at cost of \$ 20,000 to remove pollutants from waste water. Present system has no present salvage value & will cost \$ 14,500 to operate in the next yr with operating cost expected to ↑ @ rate of \$500/yr therafter.

The new system designed to replace existing system at a cost of \$10,000. New system is expected to have 1st yr. operating cost of \$9,000 with these cost ↑s @ rate of \$1000/yr. The new system is estimated to work for 12 yrs before break down. Salvage value of the system at any future time is zero. If i = 12% p.a, conduct replacement analysis based on economic life of the asset.

→ Defender :

n	AOC	n	
1	14500	10	19000
2	15000	11	19500
3	15500	12	20000
4	16000		
5	16500		
6	17000		
7	17500		
8	18000		
9	18500		

P = 0 (Sunk cost)

Defender

n	$CR(i) = (P - S_n)(A/P, i, n) + S_n i$	$OC(i) = \sum_{1}^{n} OC(P/F, i, n)(A/P, i, n)$	TC
1	—	$= 14500$ ~~$\times (P/F, 12, 1)(A/P, 12,$~~	14500
2	—	$= 14500 (P/F, 12, 1) + 15000 (P/F, 12, 2)$ $(A/P, 12, 2)$ $= [14500 \times 0.8929 + 15000 \times 0.7972] \times 0.5917 =$	14736

Challenger

1	$10000 \times$ ~~0.5917~~ 1.1200 ~~$= 5917$~~ 11200	9000	~~9000~~ ~~14917~~ 20200
2	$=$ ~~5917~~ 10000×0.5917 $=$ ~~0.5917~~ 5917	$[9000 \times 0.8929 + 10000 \times 0.7972] \times 0.5917$ 9417.99	15336.99
3	10000×0.4163 $= 4163$	$[9000 \times 0.8929 + 10000 \times 0.7972 + 11000 \times 0.7118] \times 0.4163$	
5	1000×0.2774 2774	10774.24	13548.24

Economic life of defender = 1 yrs. Cost @ 1yr = 14500

" " " challenger = 5 yr Cost @ 5yr = 13548.24

DEPRECIATION

Reduction in value of an asset over a period of time.

Types of depreciation:

"physical impairment"

① Physical depreciatn: ↓ in cost due to wear & tear.

② Functnal " : ↓ in cost due to better technology (obsolescence)

received as part of business expenses - that

* Depreciatn is ~~xxxxxx~~ used for tax. reduce tax.

Depreciatn is taken in account when tax amount is determine (Revenue - (cost + depreciatn)

= Net profit → determines tax to be paid]

* Salvage value is calulated from book value (invoice value) after deducting depreciatn.

Book value for current yr, $B_1 = B_{t-1} - D_t$

D_t → Depreciatn. cost for current yr.

B_{t-1} ↳ book value for previous yr.

If $B_{t-1} = ₹3,00,000$.

$D_t = 10\%$ of ₹3,00,000.

$B_t = 300000 - 30000 = ₹2,70,000$

Three methods to determine depreciatn:

① Straight line method

② Declining balance method

③ Double declining balance method.

Straight line method:

Depreciatn amount is constant.

Straight line depreciation assumes that value of an asset decreases by constant amount as the asset ages. Annual depreciation is given by

$$\boxed{\frac{\text{Purchase price} - \text{Salvage value}}{\text{Years of useful life}} = \text{Annual depreciatn } (D_t)}$$

① A person started a small business by purchasing a machine. First cost \$25000. Useful life is 5yrs. Salvage value is \$5000. Make a straight line depreciatn schedule for D_t every yr.

$\cancel{D_1 = \frac{25000-5000}{1} = \$20000.}$

$\cancel{D_2 = \frac{25000-5000}{2} =}$

$\cancel{D_3 = \frac{25000-5000}{3}} =$ $D_t = \frac{25000 - 5000}{5} = \underline{\underline{4000}}$

$\cancel{D_4 = \frac{25000-5000}{4} =}$

$\cancel{D_5 = \frac{25000-5000}{5} =}$

n	Value before Depreciatn	Depreciatn amount	Value after depreciatn
1	25000	4000	21000.
2	21000	4000	17000
3	17000	4000	13000
4	13000	4000	9000
5	9000	4000	5000 [Salvage value]

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(2) A person started a small business. He bought some machinery worth \$25,000 for the new business. Useful life of this asset is 7yrs & salvage value is \$4000. Make a straight line depreciatn schedule showing book value & depreciatn value every yr.

$D_t = 3000$

Value before depreciatn	D_t	After depreciatn
25000	3000	22000
22000	3000	
19000		
16000		
13000		
10000		
7000.		4000.
~~4000~~		

DECLINING BALANCE METHOD

1) Method assumes that an asset ↓s in value at a faster rate in early portn of the service life than in the later portn of its life.

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End of yr	Depreciatn charge during year t	Book value at end of yr, t
0	—	$P = B_0$
1	$R \times B_0 = R \times P$	$B_0 - R \times B_0 = (1-R) B_0 = (1-R)P = B_1$
2	$R \times B_1 = R(1-R)P$	$(1-R) B_1 = (1-R)^2 P = B_2$
3	$R \times B_2 = R(1-R)^2 P$	$(1-R) B_2 = (1-R)^3 P = B_3$
⋮		
t	$R \times B_{t-1} = R(1-R)^{t-1} P$	$(1-R) B_{t-1} = (1-R)^t P = B_t$
⋮		
n	$R \times B_{n-1} = R(1-R)^{n-1} P$	$(1-R) B_{n-1} = (1-R)^n P = B_n$

$$B_n = (1-R)^n P$$

$$\frac{B_n}{P} = (1-R)^n$$

$$\left(\frac{B_n}{P}\right)^{1/n} = 1 - R$$

$$\boxed{R = 1 - \left(\frac{B_n}{P}\right)^{1/n}}$$

(1) An asset costs ₹5000 now & salvage value is 1000 at end of its service life & R = 30% per yr. Determine depreciatn charges for 3 yrs & its book value at end of each year.

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$P = 5000$ $S_n = 1000$ $R = 30\%$

Year	Depreciation charge during t	Book value at end of yr $B_0 = (1-R)^n P$
0	—	5000 (B_0)
1	~~5000~~ 1500 ($B_0 \times R$)	~~3500~~ $B_1 =$ 3500 ($(1-R) \times B_0$)
2	~~3500~~ 1050 ($B_1 \times R$)	2450 (B_2) ($(1-R) B_1$)
3	735 (0.3×2450)	$(1-0.3) \times 2450 = 1715$ (B_3)

(2) An asset purchased for ₹2,50,000. It has an expected life of 10 yrs & $S_n = 50000$ at end of 10^{th} yr. What will undepreciated amount of capital remaining in asset at end of 6th yrs. If asset is depreciated according to declining balance method. Also calculated the depreciation charge for 6th yr.

$$R = 1 - \left(\frac{B_n}{P}\right)^{1/n}$$

$$= 1 - \left(\frac{50000}{250000}\right)^{1/10} = 14.86\%$$

B_t (for 6 yrs)

DOUBLE DECLINING BALANCE DEPRECIATⁿ.

Annual rate of decrease = 2/years of useful life.

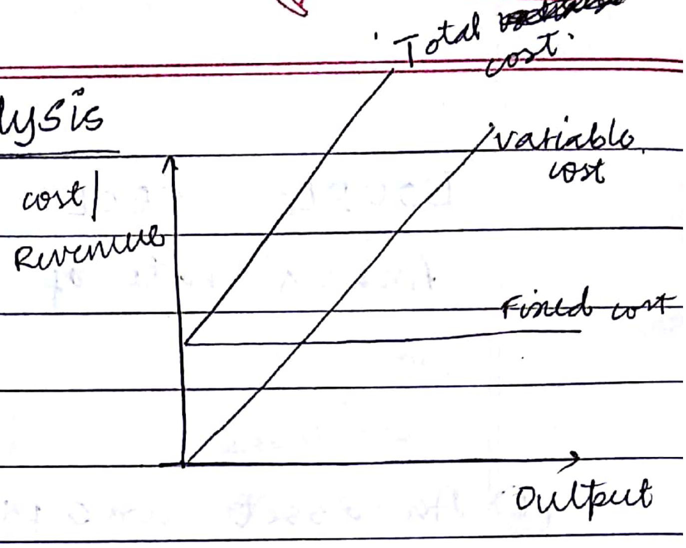
(1) An asset was purchased 10 yrs ago for ₹500,000. It is depreciated according to DDB for estimated life of 20 yrs. Salvage value is 50,000. Calculate current book value.

$$R = 2/20 = 10\%$$

$$B = (1-R)^{10} P = ₹17433.92$$

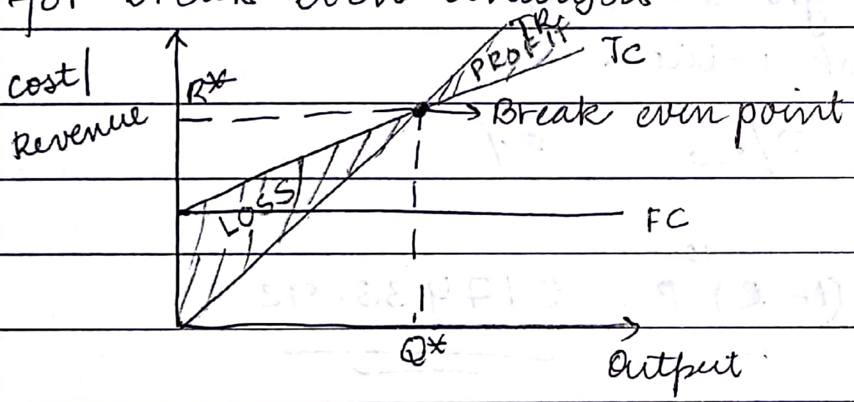
Break-even analysis

Cost → ① Fixed cost (FC)
② Variable cost (VC)



Total cost $TC = FC + Q * VC$.

* The variable cost line is not shown in the graph for break even analysis.



TR → Total Revenue

Break even point : Total cost = Total revenue.
Value of x-intercept of break even pt → Q*.
Q* → For this output, no revenue.
Output > Q*, profit.
Output < Q*, loss.

Value of y-intercept → R*.
R* – total cost generated to generate break-even.

Break even point is that point of activity where total sales revenue & total expenses are equal. It is the point of zero profit. It is a pt where losses cease to occur while profits have not yet begun.

In case the firm produces & sells less than the break even pt, it would incur losses, while if it produces & sells more than level of break even point it makes profit.

It reveals the relatnship b/w the volume & cost of productn on one hand and revenue & profit obtaind from the sales on the other hand. It captures the relatnship b/w FC, VC & TR.

Terminology

Let Q be the units of output or actual output.

Q_B → break even quantity.

P → selling price or price per unit.

TR → Total Revenue. $TR = P \times Q$.

TFC → Total fixed Cost,

TC → Total cost. $TC = FC + Q \times VC$..

According to Martz, Curry & Frank,

break even pt is the level of activity where total cost = total revenue.

$$TR = TC$$

$$P \times Q = FC + Q \times VC$$

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$(P - VC)\,Q = FC$

Break even quantity, $\boxed{Q^* = \frac{FC}{(P - VC)}}$

① A ship ~~company~~ can carry a max of 1,00,000 passengers /month at a fee ₹850. Variable cost / passenger is ₹100 while fixed cost are ₹75,00,000 / month. Calculate ① Break even quantity.

② Break even sales value.

$$Q^* = \frac{FC}{(P - VC)} = \frac{75,00,000}{(850 - 100)} = 10,000$$

Break even sales value $R^* = 10,000 \times 850$

$= ₹85,00,000$

② A manufacturer sells his product at ₹5 each. The variable cost are ₹2/ unit & fixed cost amount to ₹60,000. ① Calculate break even point ② What would be the profit if firm sells 30,000 units ③ what would be break even pt if firm spends ₹3,000 on advertising

$$Q^* = \frac{FC}{(P - VC)} = \frac{60,000}{(5 - 2)} = 20000$$

$10000 \times \cancel{3}\,(5-2) = ₹30000$

Profit = ~~10,000 × 5 = 50,000~~

$~~30000~~ = \frac{60000}{(P-3)}$

$~~2\ P-3~~$

$~~P=5~~$

$$Q^* = \frac{60000 + 30000}{(5-2)} = \underline{21000}$$

<u>Contribution</u>

To determine health of firm.

Diff, b/w SP & VC.

<u>Break even analysis from Engineering Economics Perspective</u>

Often, we have a choice b/w 2 alternatives where one of them may seem economical under one set of conditⁿ & other under another set. By altering the values of some variables & holding others constant, it is often possible to find value for variable which makes 2 alternatives equally economical. This values may be described as break even pt.

Mathematically,

$$TC_1 = TC_2.$$

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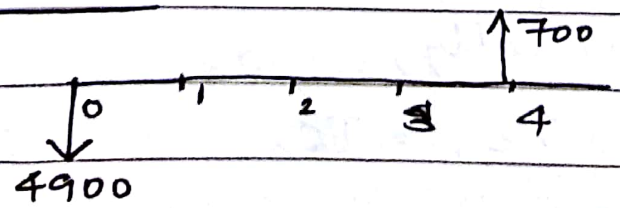
① A 50 HP motor is reqd to drive a pump to remove water from a tunnel. The unit will be needed for a period of 3yrs. 2 alternatives are under considerdtn. A calls for constructn of power line & purchase of electric motor at TC of \$4,900. Salvage value of this equipment after 4 yrs is estimated to be \$700. Cost of power for per hour operation is estimated to be \$2.9 & maintainonce is estimated as \$420/yr. Alternative B calls for purchase of a dieselpump set at a cost of \$1925 & it will have no salvage at the end of 4yr. Cost of diesel per hour of Operatn is estimated as \$1.47 maintainance is estimated at \$0.53/hr of operatn. Cost of wages chargable when the engine runs is \$2.8/hr.

① How many hours per year the 2 machines have to run so that the 2 alternative incur equal cost? ② If no, of hours of operatn is estimated at 100/yr, which alternative is econo-mical. Consider rate of interest 10%.

→

Alternative A



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$AW_1 = (4900 - 700)(A/P, 10, 4) + 700 \times 0.1$
$+ 2.94 \times x + 420$

$AW_1 = 1815.1 + 2.94x.$

$AW_2 = 1925 \times (A/P, 10\%, 4) + 1.47 \times x$
$+ 0.53 \times x$
$+ 2.8x$

$= 607.3375 + 4.8x.$

$AW_1 = AW_2.$ $4.8x$

$1815.1 - 607.3375 = -2.94x$

$x = 635.66$ hrs.

(2)

$AW_1 = 1815.1 + 2.94x$ $\xrightarrow{} 100$

$= \$2109.1$

$AW_2 = 607.335 + 4.8x$ $\xrightarrow{} 100$

$= \$1087.335$ ✓