

The closeness centrality of a vertex v is defined as

$$C_C(v_i) = \frac{n-1}{\sum_{i \neq j} d(v_i, v_j)} \text{ where } d(v_i, v_j) \text{ is distance from } v_i \text{ to } v_j.$$

Cycle C_n :

n even

$$\begin{aligned} CC(v) &= \frac{n-1}{2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + \dots + 2 \cdot \left(\frac{n}{2} - 1\right) + \frac{n}{2}} \\ &= \frac{n-1}{2(1 + 2 + \dots + \left(\frac{n}{2} - 1\right)) + \frac{n}{2}} = \frac{4(n-1)}{n^2} \end{aligned}$$

n odd:

$$\begin{aligned} CC(v) &= \frac{n-1}{2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + \dots + 2 \cdot \left(\frac{n-1}{2}\right)} \\ &= \frac{n-1}{2(1+2+\dots+\frac{n-1}{2})} = \frac{4}{n+1} \end{aligned}$$

$K_{p,q}$:

Let $|V_1| = p$, $|V_2| = q$ and $u \in V_1$, $v \in V_2$.

$$\begin{aligned} CC(u) &= \frac{n-1}{1 \cdot q + (p-1)2} = \frac{n-1}{n+p-2} \\ CC(v) &= \frac{n-1}{1 \cdot p + (q-1)2} = \frac{n-1}{n+q-2} \end{aligned}$$

Betweenness centrality $C_B(v) = \sum_{s \neq v \neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}$, where σ_{st} is the total number of shortest paths from vertex s to vertex t and $\sigma_{st}(v)$ is the number of shortest paths from vertex s to vertex t that pass through v .

In C_6 all the vertices will have same $C_B(v) = 2$

In a complete bipartite graph, $K_{1,n-1}$, all the pendant vertices have $C_B(v) = 0$ and the central vertex will have $C_B(v) = \frac{n-1}{2} C_2$.

Obtain closeness centrality of all the vertices in $P_n, K_n \setminus e$.

Obtain betweenness centrality of all the vertices in $P_7, K_n \setminus e, K_{p,q}$.