Least cost Method to find initial basic feasible solution (04 Marks)

11)

12) U-V Method for optimality (03 Marks)

		$V_1=100$	$V_2=96$	$V_3=5$	
		Bharat	Janata	Red Lamp	Supply
$U_1=0$	A	M 100 30 (+)	10 -86	5 60 (-)	90 / 30 / 0
$U_2=-1$	B	12 -87	9 -86	4 50	50 / 0
$U_3=99$	C	7 +	3 80 (-)	11 (+) -93	80 / 0
$U_4=-91$	D	9 40 (-)	5 20 (+)	7 +	60 / 40 / 0
$U_5=-100$	Dummy	0 50	0 +	0 +	50 / 0
	Demand	120	100	110	
		70 / 30 / 0	20 / 0	60 / 0	

Z = 4200; M+N-1 = 7; Solution is non-degenerate

		$V_1=100$	$V_2=-3$	$V_3=5$	
		Bharath	Janatha	Red Lamp	Supply
$U_1=0$	A	M (-) 70	10 +	5 (+) 20	90
$U_2=-1$	B	12 -87	9 +	4 50	50
$U_3=6$	C	7 (+) -99	3 40	11 (-) 40	80
$U_4=8$	D	9 -99	5 60	7 -6	60
$U_5=-100$	Dummy	0 50	0 +	0 +	50
	Demand	120	100	110	

Z = 7830; M+N-1 = 7; Solution is non-degenerate

		$V_1=100$	$V_2=96$	$V_3=5$	
		Bharath	Janatha	Red Lamp	Supply
$U_1=0$	A	M (-) 30	10 -86	5 (+) 60	90
$U_2=-1$	B	12 (+) -87	9 -86	4 (-) 50	50
$U_3=93$	C	7 40	3 40	11 +	80
$U_4=-91$	D	9 0	5 60	7 +	60
$U_5=-100$	Dummy	0 50	0 +	0 +	50
	Demand	120	100	110	

Z = 4000; M+N-1 = 7; Solution is non-degenerate

		$V_1=13$	$V_2=9$	$V_3=5$	
		Bharath	Janatha	Red Lamp	Supply
$U_1=0$	A	M +	10 +	5 90	90
$U_2=-1$	B	12 30	9 +	4 20	50
$U_3=-6$	C	7 (-) 40	3 (+) 40	11 +	80
$U_4=-4$	D	9 (+) 0	5 (-) 60	7 +	60
$U_5=-13$	DUMMY	0 50	0 +	0 +	50
	Demand	120	100	110	

Z = 1590; M+N-1 = 7; Solution is non-degenerate
Solution is optimal

13) Least cost Method :

	A	B	C	D	E	supply
P	5	8	6	6	3 8	8/0
Q	4 4	7	7	6 1	5	5/1/0
R	8	4 4	6 2	6 3	4	9/5/3/0
Dummy	0	0	0 3	0	0	3/0
Demand	4/0	4/0	5/2/0	4/3/0	8/0	25

		$v_1=8$	$v_2=8$	$v_3=10$	$v_4=10$	$v_5=8$	
		A	B	C	D	E	supply
$u_1=0$	P	5 -3	8	6 -4	6 -4	3 8	8
$u_2=-4$	Q	4 4	7 +	7 +	6 1	5 +	5
$u_3=-4$	R	8 +	4 4	6 2	6 3	4 ε	9
$u_4=-10$	Dummy	0 +	0 +	0 3	0	0 +	3
	Demand	4	4	5	4	8	25

Total cost = (3×8) + (4×4) + (6×1)
+ (4×4) + (6×2) + (6×3)
+ (0×3)
= ~~84~~ 92

m+n-1 = no. of allocations

4+5-1 = 8 (non degenerate)

∴ m+n-1 = 7 ; 4+5-1 ≠ 7 (Degenerate)

		$v_1=4$	$v_2=3$	$v_3=5$	$v_4=6$	$v_5=3$	
		A	B	C	D	E	Supply
$u_1=0$	P	5	8	6	6 3	3 5	8
$u_2=0$	Q	4 4	7	7	6 1	5	5
$u_3=1$	R	8	4 4	6 2	6 -1	4 3	9
$u_4=-5$	Dummy	0	0	0 3	0 -1	0	3
	Demand	4	4	5	4	8	25

m+n-1 = 8 ; Non Degenerate

Total cost = (6×3) + (3×5) + (4×4)
+ (6×1) + (4×4) + (6×2)
+ (4×3) + (0×3)
= 95

		$v_1=4$	$v_2=4$	$v_3=6$	$v_4=6$	$v_5=4$	
		A	B	C	D	E	Supply
$u_1=-1$	P	5 +	8 +	6 +	6 +	3 8	8
$u_2=0$	Q	4 4	7 +	7 +	6 1	5 +	5
$u_3=0$	R	8 +	4 4	6 2	6 3	4 ε	9
$u_4=-6$	Dummy	0 +	0 +	0 3	0 +	0 +	3
	Demand	4	4	5	4	8	25

ε = 0

m+n-1 = 7; degenerate

Total cost = (3×8) + (4×4) + (6×1)
+ (4×4) + (6×2) + (6×3)
+ (4×0) + (0×3)
= 92

15)

		9	4	3	0	0	0	0	
Profit/unit	Basis	Q	x_1	x_2	s_1	s_2	s_3	s_4	CR
3	x_2	200	0	1	1	0	-2	0	-100
0	s_2	200	0	0	-1	1	1	0	200
4	x_1	400	1	0	0	0	1	0	400
0	s_4	500	0	0	-1	0	2	1	250
	$[z_j]$	[2200]	4	3	3	0	-2	4	
	$c_j - z_j$		0	0	-3	0	+2	-4	

← Outgoing variable

↑ Incoming Variable

(01 Mark)

		9	4	3	0	0	0	0
Profit/unit	Basis	Q	x_1	x_2	s_1	s_2	s_3	s_4
3	x_2	600	0	1	-1	2	0	0
0	s_3	200	0	0	-1	1	1	0
4	x_1	200	1	0	1	-1	0	0
0	s_4	100	0	0	1	-2	0	1
	$[z_j]$	[2600]	4	3	1	2	0	0
	$c_j - z_j$		0	0	-1	-2	0	0

∴ All $c_j - z_j$ values are either 0 or -ve, the solution is optimal

= (02 Marks)

16) Let x_1, x_2, and x_3 = units of raw material 1, 2, and 3 } (½ Mark)

The objective function:

$$\text{Minimize (Cost)}, Z = 2x_1 + 5x_2 + 3x_3$$

Subject to the constraints:

$$3x_1 + 0x_2 + 1x_3 \geq 10$$
$$5x_1 + 1x_2 + 2x_3 \geq 15$$
$$1x_1 + 2x_2 + 0x_3 \geq 8$$
$$x_1, x_2, x_3 \geq 0$$

(1 Mark)

Introducing Surplus and artificial Variables,

$$\text{Minimize } Z = 2x_1 + 5x_2 + 3x_3 + 0S_1 + 0S_2 + 0S_3 + MA_1 + MA_2 + MA_3$$

(½ Mark)

Subject to:

$$3x_1 + 0x_2 + 1x_3 - S_1 + 0S_2 + 0S_3 + 1A_1 + 0A_2 + 0A_3 = 10$$
$$5x_1 + 1x_2 + 2x_3 + 0S_1 - 1S_2 + 0S_3 + 0A_1 + 1A_2 + 0A_3 = 15$$
$$1x_1 + 2x_2 + 0x_3 + 0S_1 + 0S_2 - 1S_3 + 0A_1 + 0A_2 + 1A_3 = 8$$
$$x_1, x_2, x_3, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$$

(1 Mark)

17) The initial basic feasible solution is

$$A_1 = 10;\ A_2 = 15, \text{ and } A_3 = 8$$

(2 Marks)

3 Mark)

18)

a) Permissible Decrease in M/c hrs

B^*	S_3^*	+ve	-ve
11	$-1/4$	-	-44
8	-2	-	$+16$
2	$1/2$	4	

$LB = 52 - 4 = \underline{48}$ (01 Mark)

Maintenance personnel can take a maximum of 4 hrs to repair the M/C.

b) Permissible decrease in labour hours

B^*	S_2^*	+ve	-ve
11	0	10	-
8	1	8	-
2	0	10	-

$LB = 40 - 8 = 32$

one day off can be given to workers to attend a local festival (01 mark)

19)

Let x_1 and x_2 be the number of units of product A and product B respectively.

(½ Mark)

Objective function:

$$\text{Maximize } Z = 30x_1 + 40x_2$$ (½ Mark)

Subject to Constraints:

Sale requirement $\begin{cases} x_1 \leq 20 \\ x_2 \geq 10 \end{cases}$ (1 Mark)

Labour: $4x_1 + 6x_2 \leq 180$

Material: $x_1 + x_2 \leq 40$

Material hours: $4x_1 + 2x_2 \leq 100$

$x_1, x_2 \geq 0$