

# Welcome 😊

Agenda: Bit manipulation  
Properties  
2-3 questions.

## Bitwise Operators.

$\&$     $|$     $\wedge$     $\sim$   
AND   OR   XOR   NOT

a	b	$a \& b$	$a   b$	$a \wedge b$	$\sim a$
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

same puppy  
same shame

## Basic AND Properties

1. Even / Odd number.

$$A \& 1 = 1 \quad \underline{\underline{\text{ODD}}}$$

$$A \& 1 = 0 \quad \text{EVEN}$$

eg:  $0111 = 7$   
 $0001$   
 $\& \underline{0001} = 1$

2.  $A \& 0 = 0$

3.  $A \& A = A$

## BASIC OR Properties

$$A \mid 0 = A$$

$$A \mid A = A$$

$$A \mid 1 = A \rightarrow \text{if } A \text{ is odd.}$$

$$A+1 \rightarrow \text{if } A \text{ is even}$$

$$\begin{array}{r} 0111 \\ 0001 \end{array}$$

## BASIC XOR Properties

$$A \wedge 0 = A$$

$$A \wedge A = 0$$

$$A \wedge 1 = A-1 \rightarrow \text{if } A \text{ is odd.}$$

$$A+1 \rightarrow \text{if } A \text{ is even}$$

$$\begin{array}{r} 0111 \\ 0001 \\ \hline 0110 \end{array} \wedge \begin{array}{r} 0110 \\ 0001 \\ \hline 0111 \end{array}$$

## Commutative Property

$\Rightarrow$  Order of operands does not affect result of bitwise opera<sup>n</sup>.

$$A \& B = B \& A$$

$$A \mid B = B \mid A$$

$$A \wedge B = B \wedge A$$

## Associative Property

→ grouping of operands does not affect the result.

$$(A \& B) \& C == A \& (B \& C)$$

$$(A | B) | C == A | (B | C)$$

$$(A \wedge B) \wedge C == A \wedge (B \wedge C)$$

Quiz

$$a \wedge b \wedge a \wedge d \wedge b$$

$$\Rightarrow a \wedge a \wedge b \wedge b \wedge d$$

// commutative prop.

$$\Rightarrow (a \wedge a) \wedge (b \wedge b) \wedge d$$

$$\Rightarrow 0 \wedge 0 \wedge d$$

$$\Rightarrow 0 \wedge d$$

$$\Rightarrow \underline{d}$$

Quiz

$$\cancel{1} \wedge \cancel{3} \wedge \cancel{5} \wedge \cancel{3} \wedge 2 \wedge \cancel{1} \wedge \cancel{5}$$

$$\Rightarrow \underline{2}$$

## Left Shift Operator ( $\ll$ )

$\Rightarrow$  Shifts the bits of a number to the left by a specified number of positions.

$\Rightarrow$  It can be used to multiply a number by 2 raised to the power of specified no. of pos<sup>n</sup> bit number

$$\underline{a = 10} \Rightarrow 0000\ 1010$$

7 6 5 4 3 2 1 0

$a \ll 0$	$=$	$0000\ 1010$	$=$	10	$\times 2$
$a \ll 1$	$=$	$0001\ 0100$	$=$	20	$\times 2$
$a \ll 2$	$=$	$0010\ 1000$	$=$	40	$\times 2$
$a \ll 3$	$=$	$0101\ 0000$	$=$	80	$\times 2$
$a \ll 4$	$=$	$1010\ 0000$	$=$	160	$\times 2$
$a \ll 5$	$=$	$0100\ 0000$	$=$	<del>320</del>	<del><math>\times 2</math></del>

Overflow

$\Rightarrow$  Left shift a number beyond bit capacity of its datatype can lead to overflow.

$$\Rightarrow \boxed{\begin{aligned} a \ll n &= a * 2^n \\ 1 \ll n &= 2^n \end{aligned}}$$

## Right Shift Operator.

- ⇒ Shifts the bits of a number to the right by a specified number of positions.
- ⇒ Right shift operator divides the number by 2.
- ⇒ Overflow does not happen in right shifts.

	<u><u>a = 20</u></u>		
		7 6 5 4 3 2 1 0	
a >> 0	0 0 0 1 0 1 0 0	⇒ 20	÷ 2
a >> 1	0 0 0 0 1 0 1 0	⇒ 10	÷ 2
a >> 2	0 0 0 0 0 1 0 1	⇒ 5	÷ 2
a >> 3	0 0 0 0 0 0 1 0	⇒ 2	÷ 2
a >> 4	0 0 0 0 0 0 0 1	⇒ 1	÷ 2
a >> 5	0 0 0 0 0 0 0 0	⇒ 0	

$$a \gg n = a / 2^n$$

$$1 \gg n = 1 / 2^n$$

---

## Power of Left Shift

i) SET  $i^{\text{th}}$  bit

$$N = 45 \rightarrow 101101$$

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	0	0	1	0	0
$1 < i < 2$	<hr/>					
	1	0	1	1	0	1
	<hr/>					

$\rightarrow 45$

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	1	0	0	0	0
$1 < i < 4$	<hr/>					
	1	1	1	1	0	1
	<hr/>					

$N | (1 < i)$  —  $\rightarrow N$  if  $i^{\text{th}}$  bit is already set  
 $\rightarrow N + (1 < i)$  if  $i^{\text{th}}$  bit is unset

2) Toggle / FLIP  $i^{\text{th}}$  bit

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	0	0	1	0	0
$1 < i < 2$	<hr/>					
	1	0	1	0	0	1
	<hr/>					

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	1	0	0	0	0
$1 < i < 4$	<hr/>					
	1	1	1	1	0	1
	<hr/>					

$N \wedge (1 < i) \rightarrow$  Flip  $i^{\text{th}}$  bit

3) Unset a bit

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	0	0	1	0	0
$1 < i < 2$	<hr/>					
	1	0	1	0	0	1
	<hr/>					

	5	4	3	2	1	0
45	1	0	1	1	0	1
OR	0	1	0	0	0	0
$1 < i < 4$	<hr/>					
	1	1	1	1	0	1
	<hr/>					

if (checkBit(N, i))  $\rightarrow$  check if bit is set.  
 {  
 $N = N \wedge (1 < i)$  // Unset a set bit  
 }

Q1 Check if  $i^{\text{th}}$  bit is set or not

$$0 \& 1 = 0$$

$$1 \& 1 = 1$$

<div style="display: flex; align-items: center;"><div style="margin-right: 10px;">45 AND <math>1 &lt; 2</math></div><div style="text-align: center;"><div style="display: flex; justify-content: space-around; margin-bottom: 5px;"><span>5</span><span>4</span><span>3</span><span>2</span><span>1</span><span>0</span></div><div style="display: flex; justify-content: space-around;"><div style="border: 2px solid red; padding: 2px;">1</div>0</div><div style="display: flex; justify-content: space-around;"><div style="border: 2px solid red; padding: 2px;">1</div>0</div><div style="display: flex; justify-content: space-around;"><div style="border: 2px solid red; padding: 2px;">0</div>0</div></div><div style="border-top: 1px solid black; margin-top: 5px; display: flex; justify-content: space-around;"><div style="border: 2px solid red; padding: 2px;">0</div>0</div><div style="border-top: 1px solid black; margin-top: 5px; display: flex; justify-content: space-around;"><div style="border: 2px solid red; padding: 2px;">0</div>0</div></div> <div style="margin-left: 10px; align-self: center;"><math>\neq 0</math></div>
---

1. Shift 1 to the  $i^{\text{th}}$  bit ( $1 < i$ )

2.  $X = (N \& (1 < i))$

if ( $X > 0$ )  $\rightarrow$   $i^{\text{th}}$  bit is set  
else  $\rightarrow$   $i^{\text{th}}$  bit is unset.

Q2 Given an integer  $N$ , count total no. of set bits in  $N$ .

eg  $N = 12$

$$\underline{\underline{11}}00 \Rightarrow 2$$

App Iterate over all bits of integer (max. 32) and check whether it is set or not. If set, then increment ans by 1

func count Bit ( N )

{

ans = 0

for ( i = 0 ; i < 32 ; i++ )

{

if ( checkBit ( N , i ) )  $\Rightarrow$  checks  $i^{\text{th}}$  bit  
is set or not  
 $\downarrow$   $\downarrow$   
True False

{

ans = ans + 1

}

}

return ans

}

T.C  $\Rightarrow O(32) \approx O(1)$

App 2

We can use right shift operator.

code

func count Bit ( N )

{

ans = 0

while ( N > 0 )

{

if ( N & 1 )

{

ans = ans + 1

}

N = ( N >> 1 )

}

return ans

}

T.C =  $O(\log N)$   
 $\approx O(32)$   
for larger  
numbers

$\Rightarrow$  App 2 takes lesser time than App 1



Q

Given  $A, B, C$ , create a pattern.

Pattern require  $A$  0's followed by  $B$  1's followed by  $C$  0's.

Write a func<sup>n</sup> to return decimal value of this number.

$$0 \leq A, B, C \leq 20$$

eg:

$$A = 4$$

$$B = 3$$

$$C = 2$$

$$\begin{array}{cccccccc} 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{array}$$

$$000011100 = \underline{\underline{28}}$$

long ans = 0

for ( $i \rightarrow C$  to  $B+C-1$ )

{  
    ans = ans | ( $1 \ll i$ )

}

return ans

App-2

$$\rightarrow 2^B - 1$$

$$1000 \rightarrow 0\underline{\underline{111}}$$

$$\rightarrow (2^B - 1) \ll C \rightarrow \text{shift to left } C \text{ times}$$