

Sometimes the
smallest step in the
right direction
ends up being the
biggest step
of your life



- Naeem Callaway

One Dimensional Array

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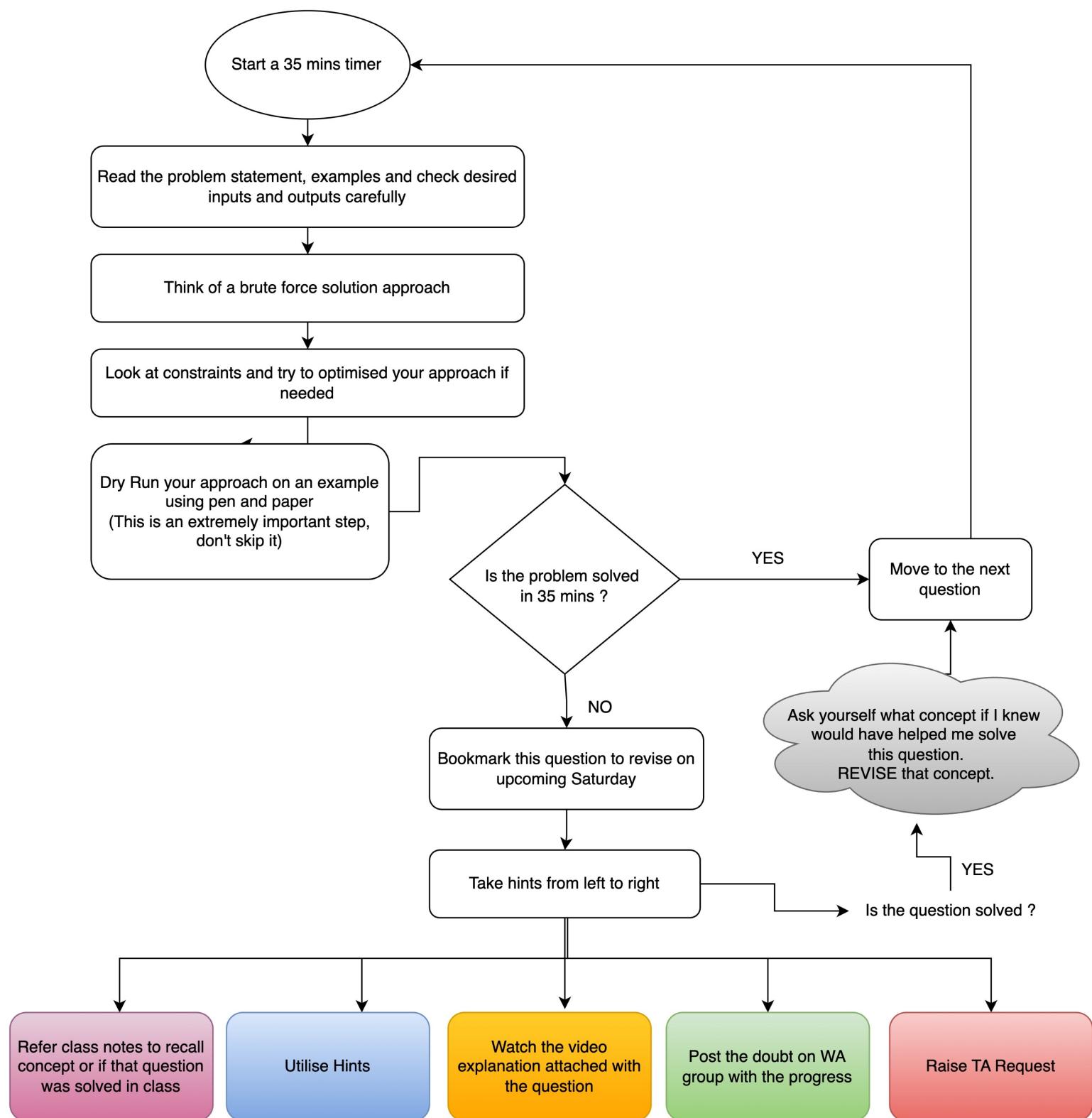
1. Max Subarray Sum
2. Prefix Sum
3. Rain Water Trapped



Notes



How to approach a problem ?





< Question > : Given $\text{arr}[N]$. Find the maximum subarray sum out of all subarrays.

$$1 \leq N \leq 10^5$$

Example 1 : $\text{arr}[] \rightarrow [-3, 2, 4, -1, 3, -4, 3]$

The array elements are indexed from 0 to 6 below them. The subarray $[2, 4]$ is highlighted in green and has a blue arrow pointing to the number 8 above it, indicating its sum.

Example 2 : $\text{arr}[] \rightarrow [4, 5, 2, 1, 6]$
The array elements are indexed from 0 to 4 below them. The subarray $[4, 5, 2, 1]$ is highlighted in green and has a blue arrow pointing to the number 18 below it, indicating its sum.

Example 3 : $\text{arr}[] \rightarrow [-4, -3, -6, -9, -2]$
The array elements are indexed from 0 to 4 below them. The subarray $[-2]$ is highlighted in green and has a blue arrow pointing to the number -2 below it, indicating its sum.

Idea -1

Brute Force

(1) Find all the subarrays. Sum
(2) Compare them and take the largest.

T.C: $O(N^3)$

Carry forward T.C. $O(N^2)$



Observations

Case 1

When all elements are positive

2	4	5	1	3
---	---	---	---	---

⇒ sum of all the elements.

Case 2

When all elements are negative

-7	-3	-11	-8	-15
----	----	-----	----	-----

⇒ largest number

Case 3

	+ve	
--	-----	--

⇒ 1 2 3 4 -2 5 3.

1 2 3 4 -20 5 3

at any given point
if sum becomes
0 or negative,
leave and start
fresh,

Kadane's Algorithm

***Dry-Run**

Minimum sum
 → if all -ve add & work
 → if all +ve smallest number.
 → if -ve and +ve

$\text{arr[]} \rightarrow [5, 6, 7, -3, 2, -10, -12, 8, 12, -4, 7, -2]$

	0	1	2	3	4	5	6	7	8	9	10	11
0 Cur-sum \Rightarrow	5	11	18	15	17	7	0	8	20	16	23	21
0 max-sum \Rightarrow	5	11	18	18	18	18	18	18	20	20	23	25

$$\text{ans} = 25$$

$A = []$	$\left[-2, 3, 4, -1, 5, -10, 7 \right]$	
$0 \begin{cases} \text{run-sum} \\ -\infty \end{cases}$	$0 \quad \quad 5 \quad \quad 7 \quad \quad 6 \quad \quad 11 \quad \quad 1 \quad \quad 8$	
$0 \begin{cases} \text{max-sum} \\ -\infty \end{cases}$	$-2 \quad \quad 3 \quad \quad 7 \quad \quad 7 \quad \quad 11 \quad \quad 11 \quad \quad 11$	$\text{ans} = \underline{\underline{11}}$

$\text{function maximum Subarray (arr[], n)}$

{

$\text{max-sum} = -\infty \quad // -\infty - 1$

$\left[-2, -1, -5 \right]$

$\text{run-sum} = 0$

$\text{for (} i=0; i < n; i++ \text{)}$

{

$\text{run-sum} += \text{arr}[i];$

$\text{if (run-sum } > \text{max-sum) }$

{

$\text{max-sum} = \text{run-sum};$

{

$\text{if (run-sum } < 0 \text{)}$

$\text{run-sum} = 0$

{ return max-sum;

T.C: $O(N)$

S.C: $O(1)$



Continuous Sum Query

< Question > : Initially all elements of an arr[N] are 0. Then you are given Q queries.

Every query contains i-idx and value. Increment elements from ith.idx to last idx by value. Return final state of arr[].

$$\text{arr[]} \rightarrow [\underset{0}{0}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{0}, \underset{5}{0}, \underset{6}{0}]$$

$1 \leq N \leq 10^5$

$1 \leq Q \leq 10^5$

Queries → 3

idx	val	$[\underset{0}{0}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{0}, \underset{5}{0}, \underset{6}{0}]$
3	4	$+4 +4 +4 +4$

1	3	$+3 +3 +3 +3 +3 +3$
4	-2	$-2 -2 -2$

result = $[0, 3, 3, 7, 5, 5, 5]$

BF Idea

$$A = [\underset{0}{0}, \underset{1}{0}, \underset{2}{0}, \underset{3}{0}, \underset{4}{0}, \underset{5}{0}]$$

1, 3	$+3 +3 +3 +3$
0, 2	$+2 +2 +2 +3$
4, 1	$+1$

$$[2, 5, 5, 5, 6].$$

1) iterate over every query. ↗ Q

↳ add the value from i^{th} to n^{th} index. n

T.C = O(Q * n)



Observation

arr →

a_0	a_1	a_2	a_3	a_4
0	1	2	3	4

$1, 3$ +3
 $2, -1$ -1
 $4, 2$ 2
 $1, 5$

$Q = [$
 $[1, 3],$
 $[2, -1],$
 $[1, 5],$
 $]$

</> Code $Q[i][\text{idx value}]$

for ($i = 0$; $i < Q$; $i++$)

{ idx = $Q[i][0]$

val = $Q[i][1]$

arr[idx] += val;

// processing query

T.C: $O(N+Q)$

arr =

0	1	2	3	4
0	8	-1	0	0

psum. 0 8 1 7 1 7 1 7

arr = {0}

T.C: $O(N+Q)$

S.C: $O(1)$.

// calculate prefix sum

for ($i \rightarrow 1$ to N)

{ arr[i] += arr[i-1];

s



< Question > : Initially all elements of an arr[N] are 0. Given Q queries.

Every query contains [s, e, val]. Increment elements from s to e by val.

Return the final state of arr[].

arr[10] → [0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0]
 0 1 2 3 4 5 6 7 8 9

$1 \leq N \leq 10^5$

Queries → 3

$1 \leq Q \leq 10^5$

s	e	val	0	1	2	3	4	5	6	7	8	9
3	6	3				+3	+3	+3	+3			
2	7	-3				-3	-3	-3	-3	-3	-3	
1	9	4				4	4	4	4	4	4	4

0 4 4 4 4 4 4 1 4 4 .

[0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0 , 0]
 +3 -3 +3
 4

→ [0 , 4 , -3 , 3 , 0 , 0 , 0 , -3 , 3 , 0]
 Prefix sum ↗.

**Quiz :**

$\text{arr}[8] \rightarrow [0, 0, 0, 0, 0, 0, 0, 0]$

0 1 2 3 4 5 6 7

Queries $\rightarrow 4$

si	ei	val	$[0, 0, 0, 0, 0, 0, 0, 0]$							
1	4	3		3					-3	
0	5	-1		-1					1	
2	2	4			4	-4				
4	6	3					3		-3.	
$= [-1, 3, 4, -4, 3, -3, 1, -3]$										
\downarrow prefix sum										

H.W
 \equiv

Write optimized code
for this.





< / > **Code**

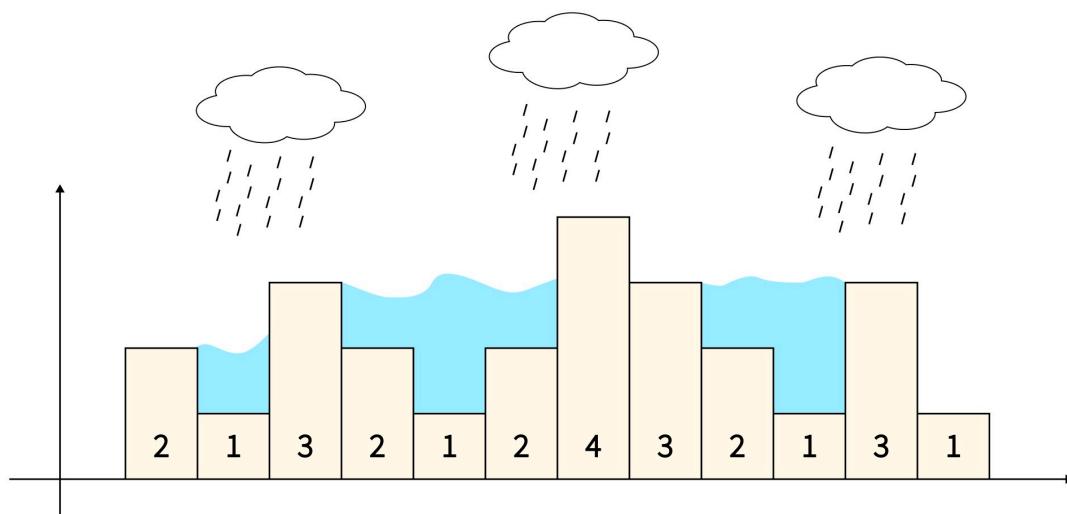


< Question > : Given $\text{arr}[N]$, where $\text{arr}[i] \rightarrow$ height of building.

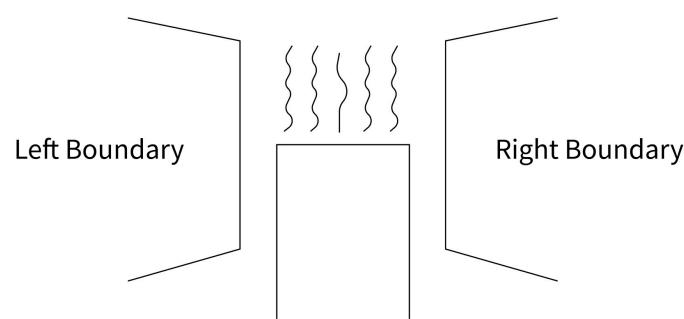
Return amount of water trapped on all the buildings.

$\text{arr} [2 \ 1 \ 3 \ 2 \ 1 \ 2 \ 4 \ 3 \ 2 \ 1 \ 3 \ 1]$

$1 \leq N \leq 10^5$

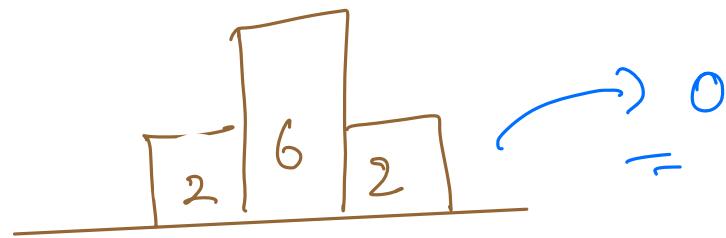
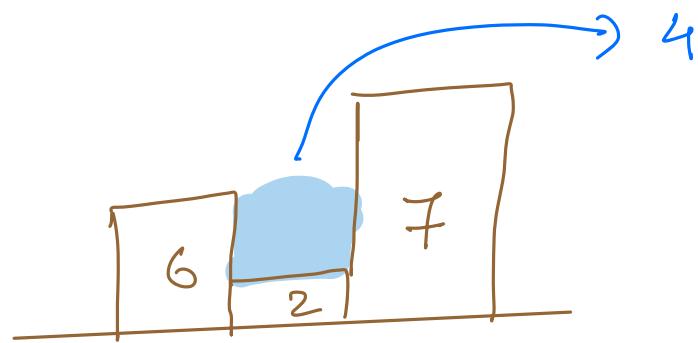
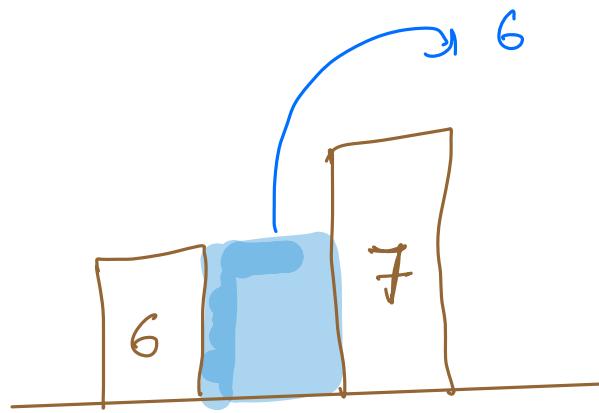


Idea -1 Find the amount of water trapped on every building.

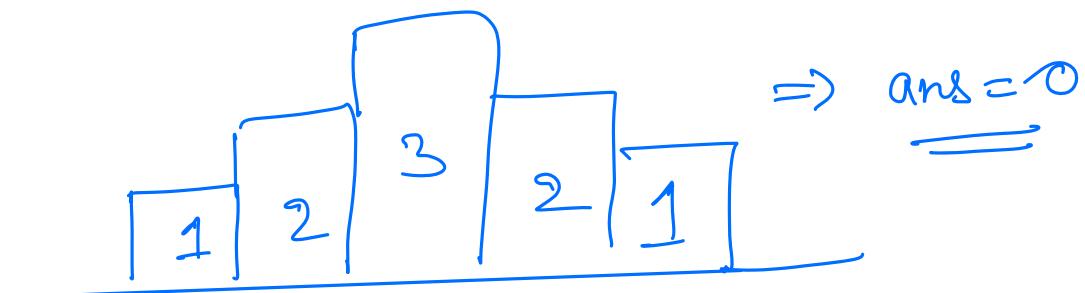




Observation



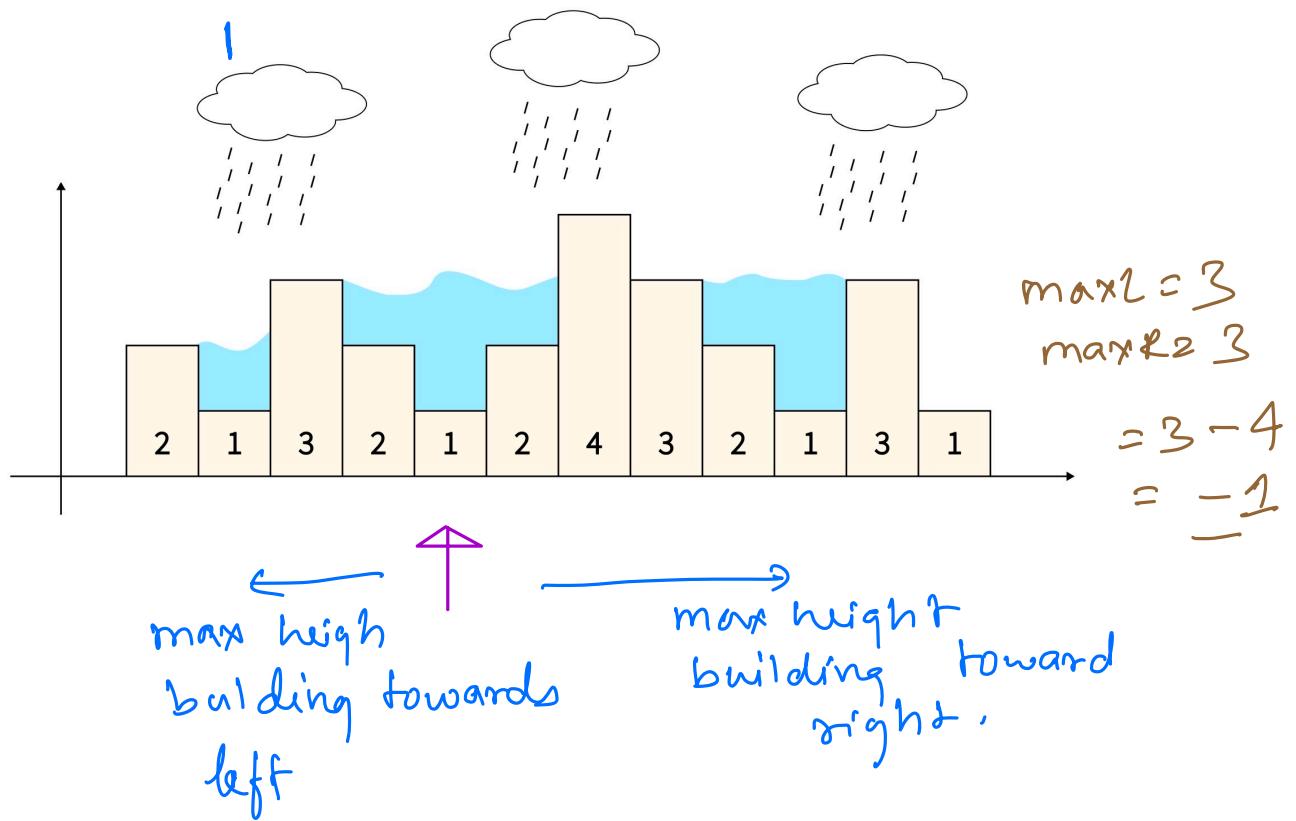
Quiz





</> Code

Brute force



$$\Rightarrow \min(\max(\text{left}), \max(\text{right})) - A[i]$$

Brute force

$$\text{ans} = 0$$

for ($i = 1$ to $N-2$) $O(N)$

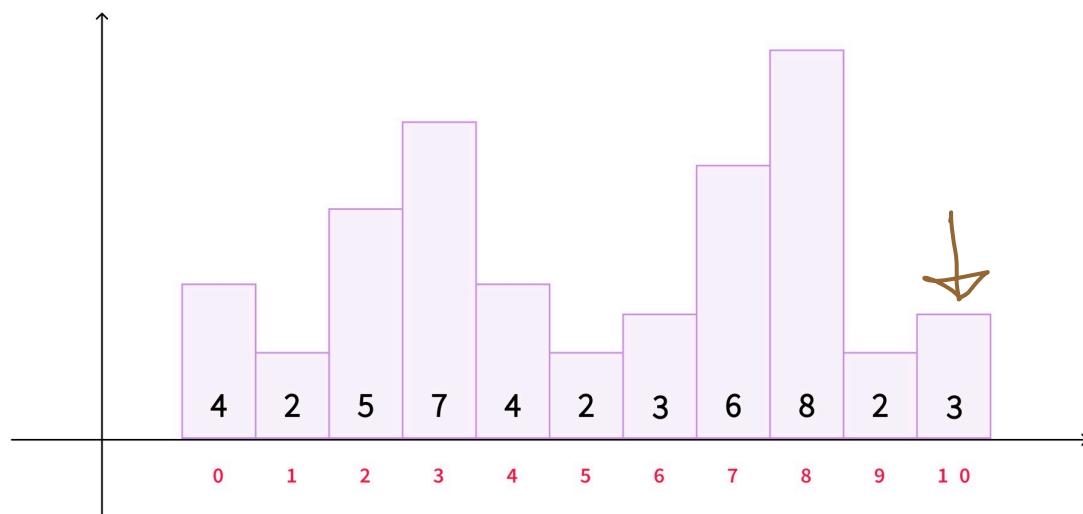
$$\exists \quad \begin{aligned} \text{maxL} &= \max(0, i-1); O(n) \\ \text{maxR} &= \max(i+1, n-1); O(n) \end{aligned}$$

$$\text{water} = \min(\text{maxL}, \text{maxR}) - A[i]$$

if ($\text{water} \geq 0$) $\text{ans} += \text{water};$

5.

T.C: $O(N^2)$

***Dry-Run** $\text{lmax}[] \rightarrow [4 \ 4 \ 5 \ 7 \ 7 \ 7 \ 7 \ 7 \ 8 \ 8 \ 8]$ $\text{rmax}[] \rightarrow [8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 8 \ 3 \ 3]$

$\Rightarrow \min(\text{lmax}[i], \text{rmax}[i]) - A[i]$

T.C: $O(N)$
S.C: $O(N)$



</> Code

Optimised approach

$$\text{lmax} \leftarrow \{ 0 \}$$

for (i=1 to N-1)

$$\text{lmax}[i] = \max(\text{lmax}[i-1], A[i-1]).$$

$$\text{rmax} \leftarrow \{ 0 \}$$

for (i=N-2 to 0)

$$\text{rmax}[i] = \max(\text{rmax}[i+1], A[i+1]).$$

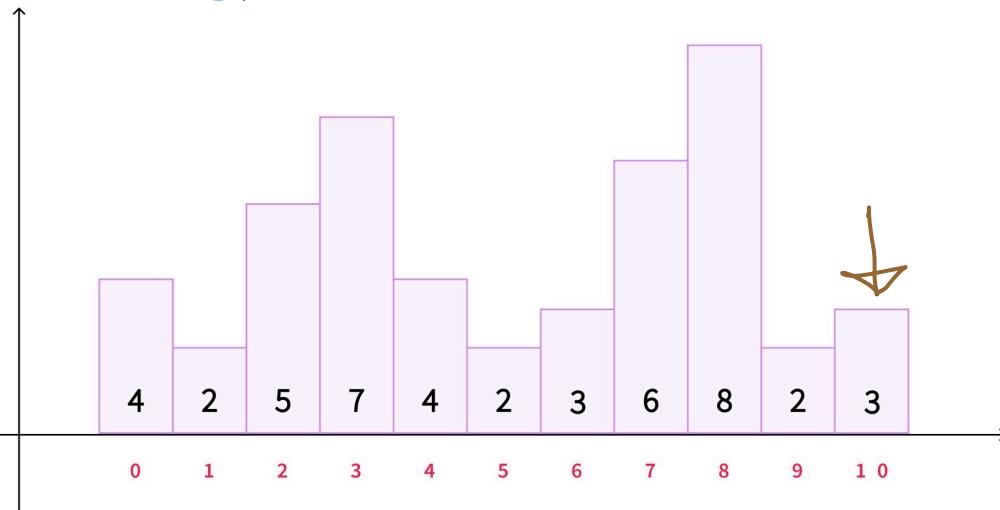
for (i=1 to N-2) O(N)

{

$$\text{water} = \min(\text{lmax}[i], \text{rmax}[i]) - A[i]$$

if (water >= 0) ans += water;

5.



$$\text{lmax}[] \rightarrow [4 4 5 7 7 7 7 8 8 8]$$

$$\text{rmax}[] \rightarrow [8 8 8 8 8 8 8 8 3 3]$$