

Count Sort

Sort array of even & odd

Merge Sort

Stable & In place Sorting

Q. Find **smallest no.** that can be formed by rearranging digits given in an array.

$A = \langle 6, 3, 4, 2, 7, 2, 1 \rangle$
 $\text{ans} \rightarrow \langle 1, 2, 2, 3, 4, 6, 7 \rangle$

$A = \langle 4, 2, 7, 3, 9, 0 \rangle$
 $\text{ans} \rightarrow \langle 0, 2, 3, 4, 7, 9 \rangle$

Approach 1: Sort the array

Arrays.sort()

↓
TC: $O(N \log_2 N)$

Approach 2: Can we use the info that digits are $0 \rightarrow 9$

$\langle -, -, -, -, -, -, -, - \rangle$

0 0 0 ... 1 1 1 ... 2 2 2 ... 3 ... 9

freq of every digit

0 → -
1 → -
2 → -
⋮
9 → -

- ① Count freq of all the digits **int freq[10]**
- ② Use freq array to fill the original array

$A = \langle 6, 3, 4, 2, 7, 2, 1 \rangle$

	0	1	2	3	4	5	6	7	8	9
freq[10]	0	0	0	0	0	0	0	0	0	0
		1	2	1	1		1	1		

$A = \langle \overset{0}{\cancel{6}}, \overset{1}{\cancel{3}}, \overset{2}{\cancel{4}}, \overset{3}{\cancel{2}}, \overset{4}{\cancel{7}}, \overset{5}{\cancel{2}}, \overset{6}{\cancel{1}} \rangle$

Sort based on counting frequency
↓
Count sort

// int A[], int N

int freq[10] = {0}

① for (i = 0; i < N; i++) {
 freq[A[i]] = freq[A[i]] + 1
}

int k = 0

② for (dig = 0; dig ≤ 9; dig++) {
 for (cnt = 1; cnt ≤ freq[dig]; cnt++) {
 A[k] = dig
 k++
 }

return A

dig → cnt
0 → 1
1 → 2
2 → 1

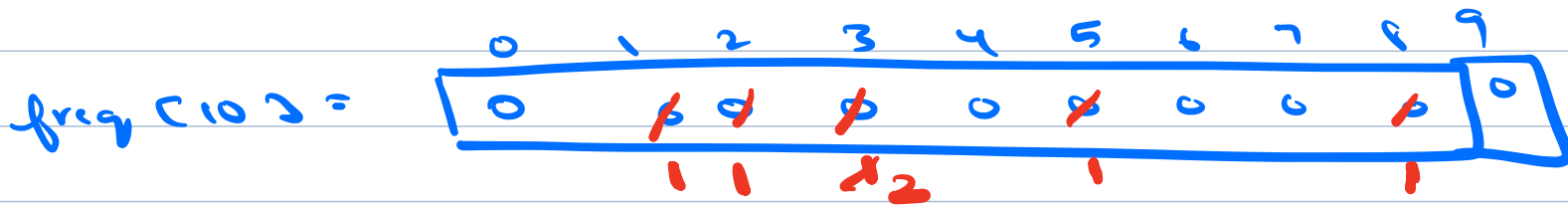
$$TC: O(N + 10 + N) \\ = O(N)$$

3 →
⋮
9
N

$$SC: O(10) = O(1)$$

↓
No. of digits (0-9)

$$A = [1, 3, 8, 2, 3, 5]$$



$$A = [\cancel{1}, \cancel{3}, \cancel{8}, \cancel{2}, \cancel{3}, \cancel{5}]$$

1 2 3 3 5 8

Will Count Sort work if range of A[i] is more than 10^9 ?

$$\langle 10^9, 10^7 + 2, 10^8, 10^9 \rangle$$

$$10^9 \rightarrow 2$$

$$10^7 + 2 \rightarrow 1$$

$$10^8 \rightarrow 1$$

$$\text{data} \rightarrow 0 \text{ to } 10^9$$

$$\text{int freq}[10^9 + 13]$$

$$\text{int} \rightarrow 4 \text{ B}$$

$$10^9 \text{ integers} \rightarrow 4 \text{ B} \times 10^9 = 4 \text{ GB}$$

Count sort works until till 10^6

Range $\rightarrow 0$ to 10^6

int freq[10^6]



$4 \times 10^6 \text{ B} = 4 \text{ MB}$

< 10, 0, 100, 200, 100, 107

range $\rightarrow 0$ to 200

int freq[201]

freq

0	1	2	3	10	100	200
1	-	-	-	2	2	1

Case II $A = [-2, 3, 8, 3, -2, 3]$

Range $\rightarrow -2$ to 8

int freq[11]

No. of ele in range =

$$8 - (-2) + 1 = 11$$

Num	Idx
-2	0
-1	1
0	2
1	3
2	4
3	5
4	6
5	7
6	8
7	9
8	10

$$\text{Num} - (-2) = \text{Idx}$$

$$\text{Num} - \text{Min} = \text{Idx}$$

Ele

$$\text{Num} = \text{Idx} + \text{Min}$$

Ele

$A = [-2 \ 3 \ 8 \ 3 \ -2 \ 3]$

Range $\rightarrow -2$ to 8

int freq[11]

Min
 \downarrow
 -2

Max
 \downarrow
 8

freq[11] =

idx	-2	-1	0	1	2	3	4	5	6	7	8
	0	0	0	0	0	0	0	0	0	0	0
	$\times 2$					$\times 3$					$\times 1$

$A = [-2 \ -2 \ 3 \ 3 \ 3 \ 8]$

① Iterate in array and get min and max $\rightarrow N$

② int freq[max - min + 1] = {0}

for (int i = 0; i < N; i++)
 idx = A[i] - min
 freq[idx]++

int k = 0

for (int i = 0; i < max - min + 1; i++)
 num = i + min

```

    for (cnt = 1 ; cnt ≤ freq[i] ; cnt++)
    {
        A[cnt] = num    k++
    }
}
return A

```

$TC : O(3N + Range)$
 $\approx O(N + Range)$

$SC : O(Range)$

$freq [max - min + 1]$

Gmail → All inboxes

$A1 \rightarrow [9, 8:30, 8:10]$

$A2 \rightarrow [10, 9, \dots]$

Merge 2 sorted arrays.

$A \rightarrow [1, 5, 6, 9]$

$B \rightarrow [2, 4, 8]$

$ans \rightarrow [1, 2, 4, 5, 6, 8, 9]$

Q. Given an integer array where all odd elements are sorted and all even elements are sorted. Sort entire array.

$A = [2, 5, 4, 8, 11, 13, 10, 15, 21]$

Approach : Separate out even and odd
Merge two arrays

odd = [5, 11, 13, 15, 21]

Even = [2, 4, 8, 10]

ans = [2, 4, 5, 8, 10, 11, 13, 15, 21]

```
void Sort (int A[], int N) {
```

```
    list<int> odd, even
    for (i=0 ; i<N ; i++) {
        if (A[i] % 2 == 0)
            even.add(A[i])
        else
            odd.add(A[i])
    }
```

```
    int oddN = odd.size()
```

```
    int evenN = even.size()
```


int o = 0, e = 0, a = 0

while (o < oddN && e < evenN) <

```
    if (odd[o] < even[e]) <
    |   A[a] = odd[o]
    |   a++    o++
    |>
    else <
    |   A[a] = even[e]
    |   a++    e++
    |>
    |>
```

while (o < oddN) <

// when even[] ended

```
| A[a] = odd[o]
| a++    o++
|>
```

while (e < evenN) <

// when odd[] ended

```
| A[a] = even[e]
| a++    e++
|>
```

return A

2N iterations

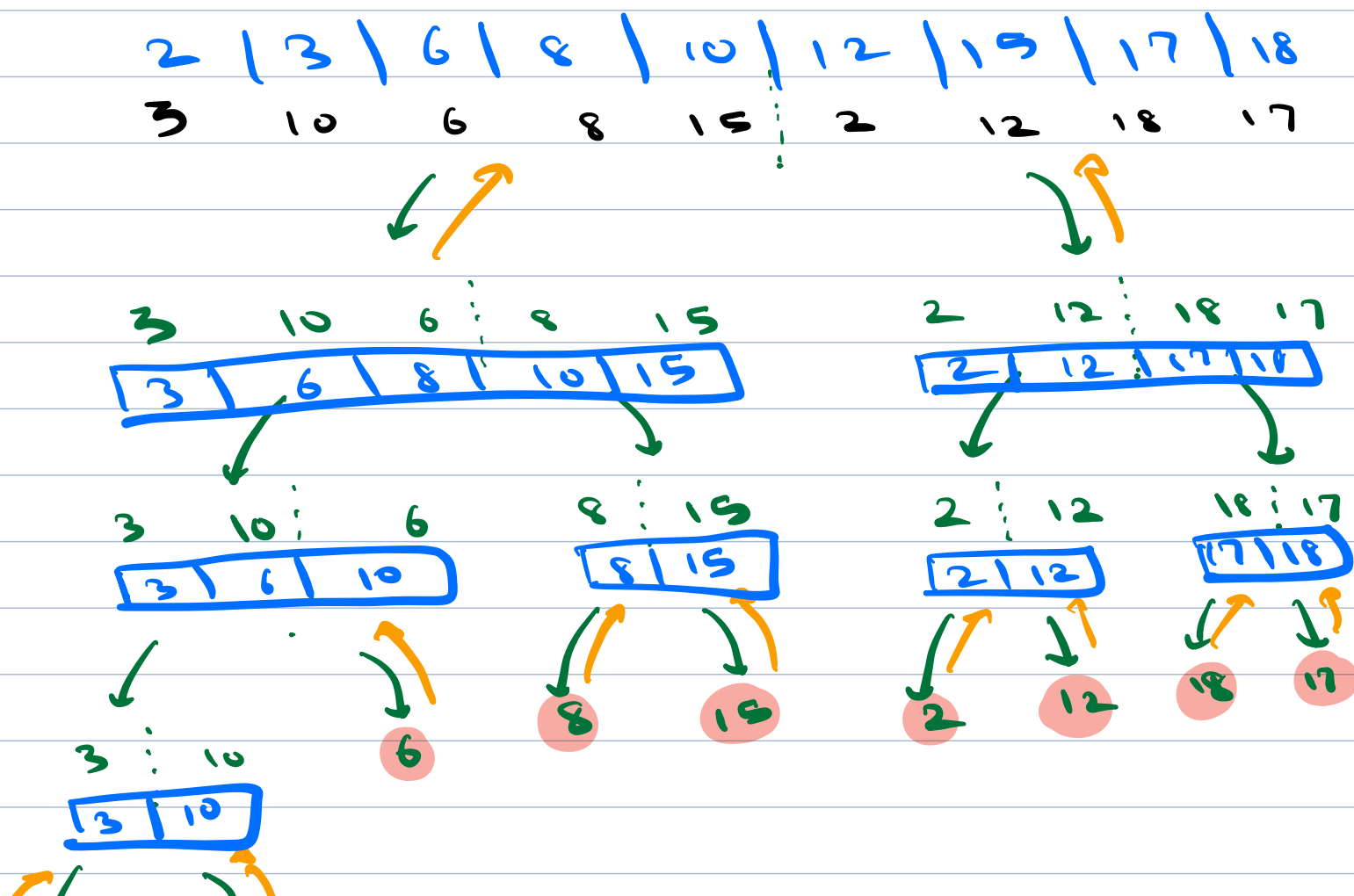
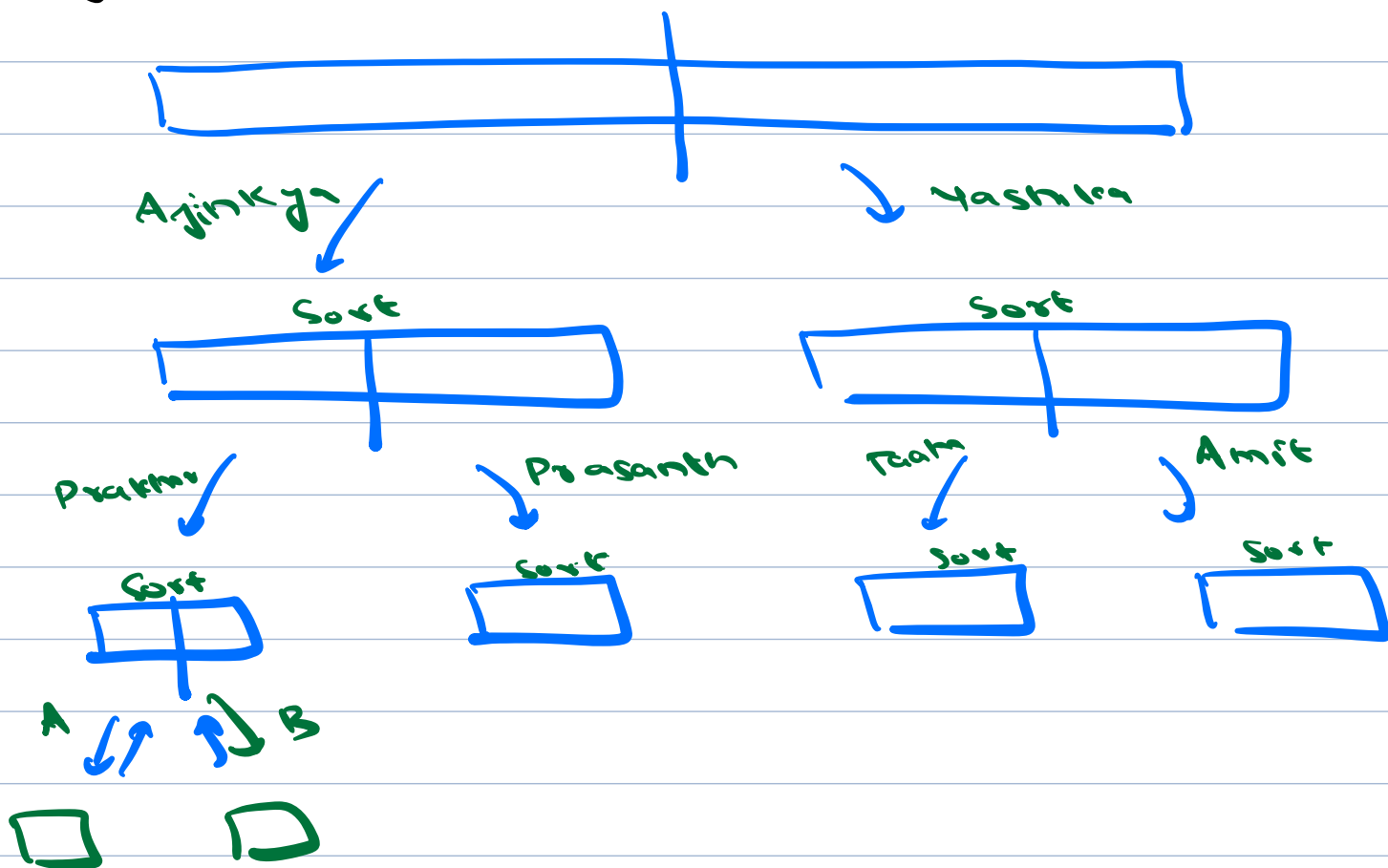
Tc: $O(N)$

Sc: $O(N)$

↓
odd and even list

7

Merge Sort



Sort(A, 0, n-1)

// Given an A[], l and r ; Sort array from index l to r

```
void Sort (int A[], int l, int r) {
```

```
    if (l == r) return
```

```
    int mid =  $\frac{l+r}{2}$ 
```

```
    Sort (A, l, mid)
```

```
    Sort (A, mid+1, r)
```

```
    merge (A, l, mid, r)
```



```
void merge (int A[], int l, int mid, int r) {
```

```
    int n1 = mid - l + 1
```

```
    int n2 = r - (mid + 1) + 1
```

```
    int B[n1], C[n2]
```

```
    int k = 0
```

```
    for (idx = l ; idx <= mid ; idx++)
```

```
        B[k] = A[idx]    k++
```

```
    k = 0
```

```

for (id1 = mid+1; id1 ≤ n1; id1++) <
    C[k] = A[id1]    k++
    >

```

// Merging

```

int b=0, c=0, k=l

```

```

while (b < n1 && c < n2) <

```

```

    if (B[b] ≤ C[c]) <

```

```

        A[k] = B[b]
        k++    b++
    >

```

```

    else <

```

```

        A[k] = C[c]
        k++    c++
    >

```

```

while (b < n1) <

```

```

    A[k] = B[b]
    k++    b++
    >

```

```

while (c < n2) <

```

```

    A[k] = C[c]
    k++    c++
    >

```

```

return

```

Sort (0, 8)

l				mid				r
0	1	2	3	4	5	6	7	8
3	10	6	8	15	2	12	18	17

Sort (0, 4)

l		mid		r
0	1	2	3	4
3	10	6	8	15

Sort (5, 8)

l				r
5	6	7	8	
2	12	18	17	

Sort (0, 2)

l		r
0	1	2
3	10	6

Sort (3, 7)

l	mid	r
3		4
8		15

3 4
18 15

↓ ↓

Sort (3, 3)

l	r
3	8

↗ ↘

Sort (7, 7)

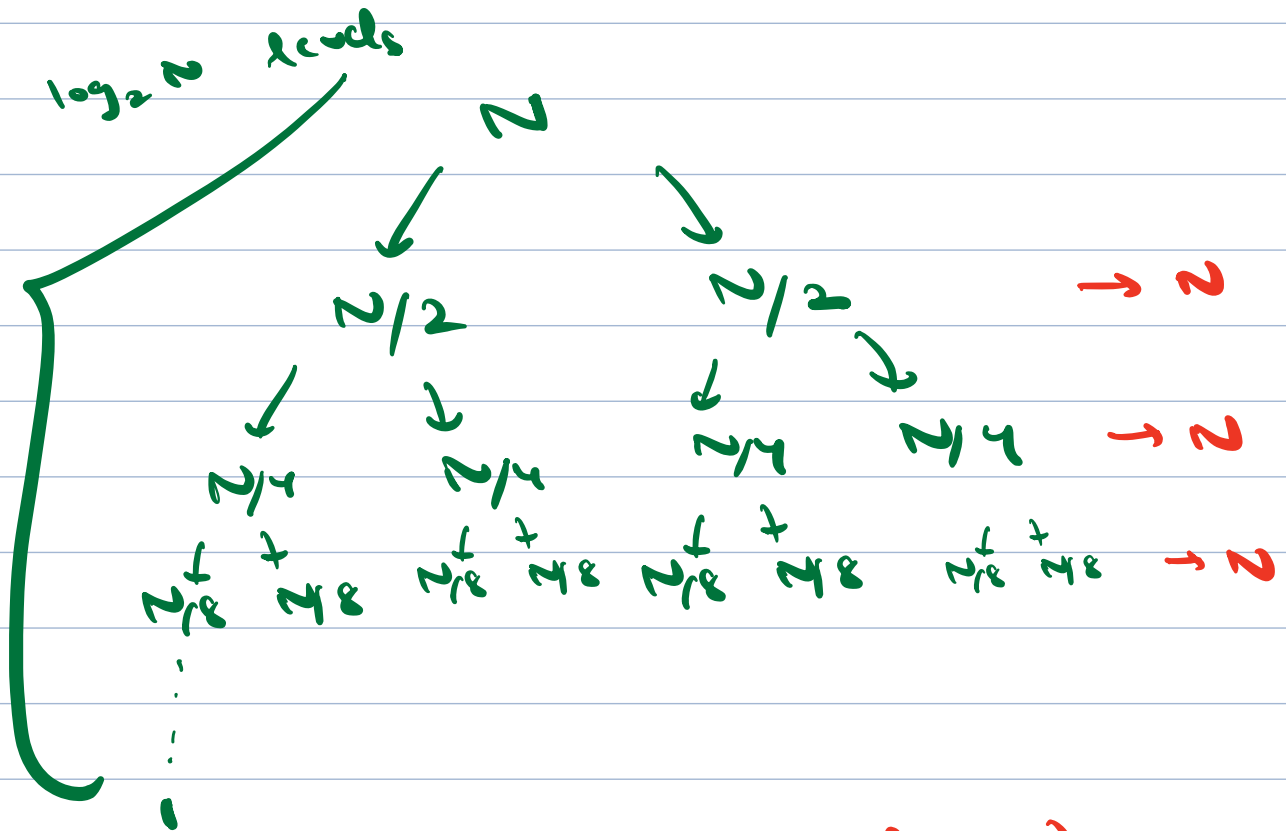
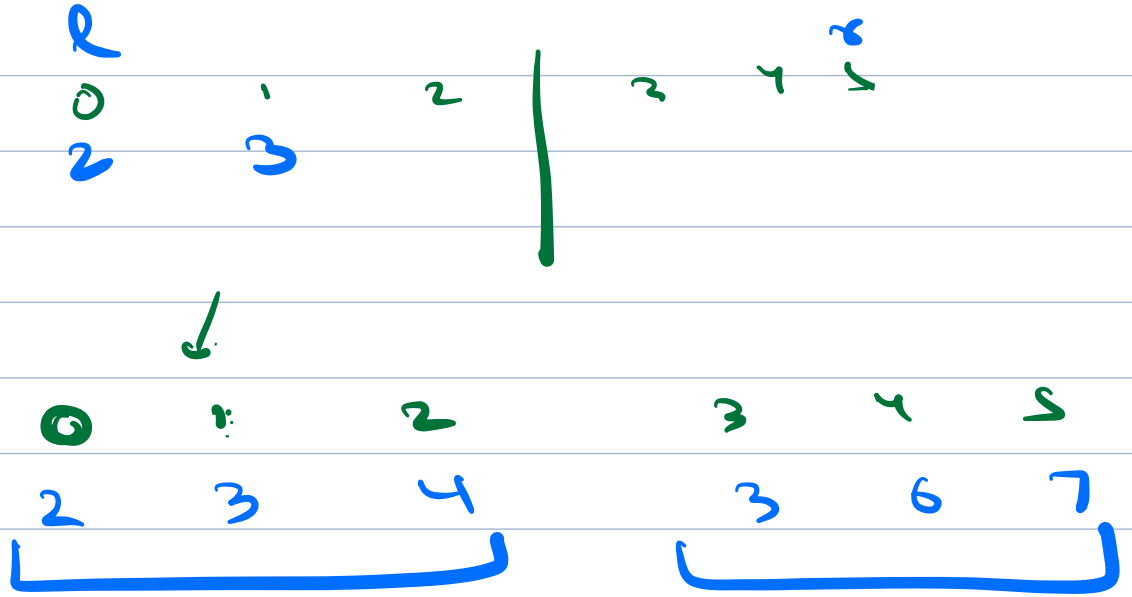
l	r
7	15

Sort (0, 1)

0 1
3 10

Sort (4, 4)

2
6



Inplace \rightarrow No extra space, $SC: O(1)$

Merge sort \times

Insertion, Selection \checkmark

Stable \rightarrow Relative order of equal elements should not change

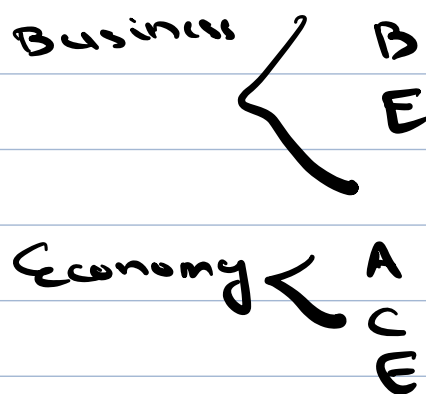
A \rightarrow Economy

B \rightarrow Business

C \rightarrow Economy

D \rightarrow Economy

E \rightarrow Business

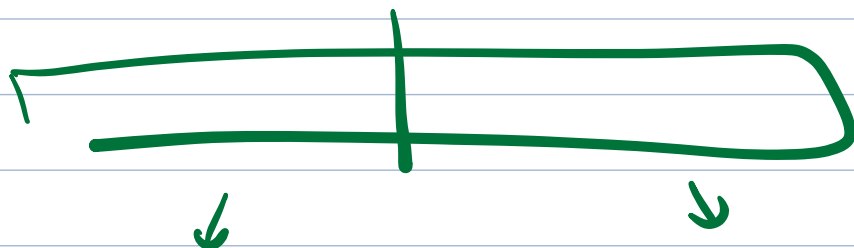


2 4 2 1



1 2 2 4

Stable sort



1 2 4 2 6 8

Merge sort ✓

selection X
