Single Echelon Inventory Management: Safety Inventory

IDS 552

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Snapshot of this lecture

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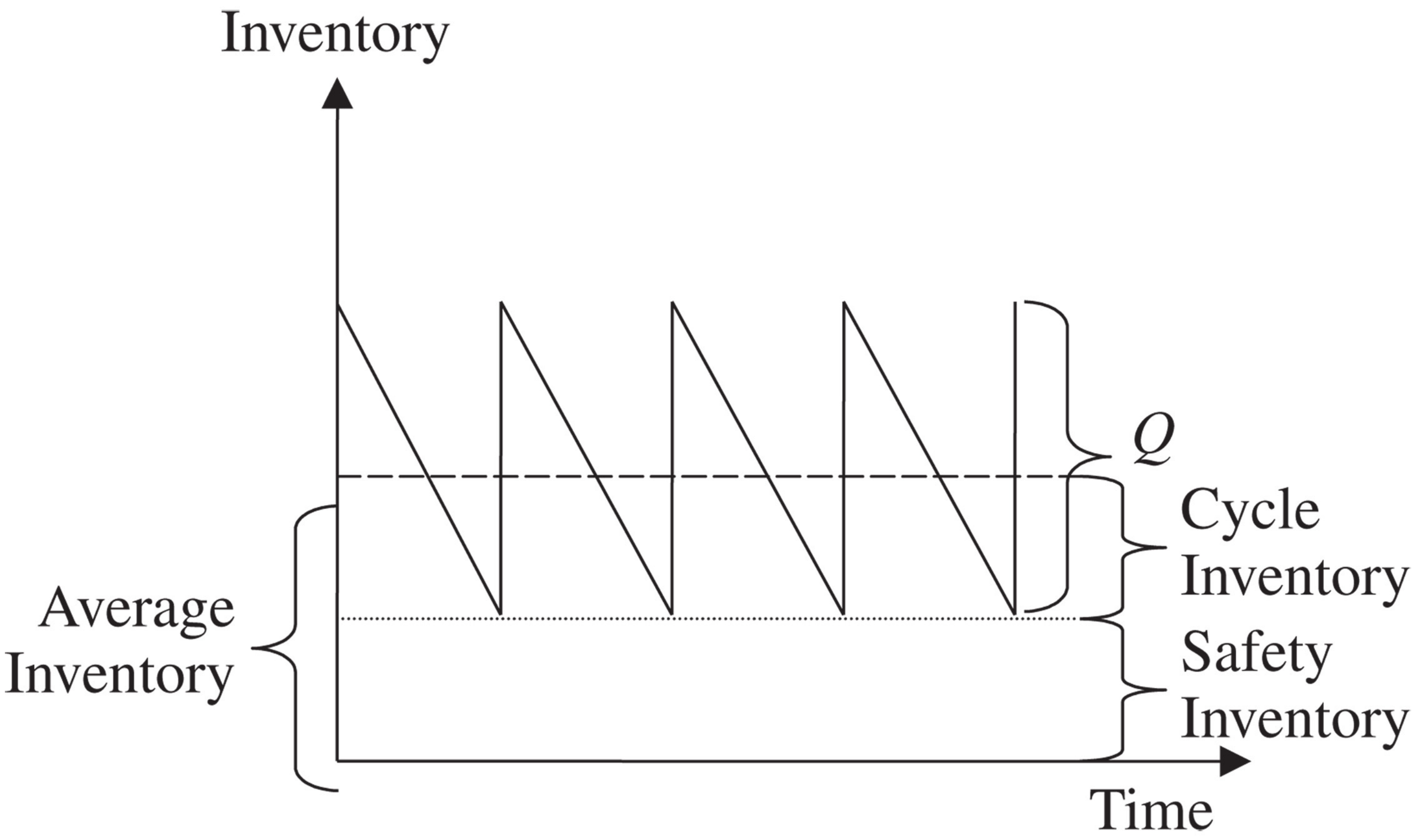

Serves the predictable part of demand

Buffers against uncertainty

How do we set cycle and safety inventory?

Discussion outline

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- **Measures of product availability**
- Continuous review and periodic review inventory replenishment policies
- Levers to improve performance

Modeling demand uncertainty

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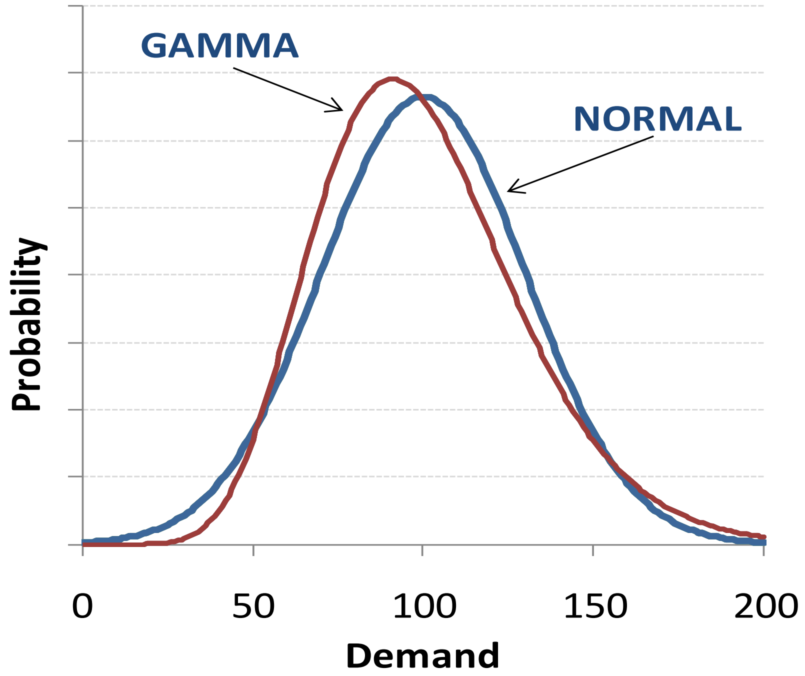
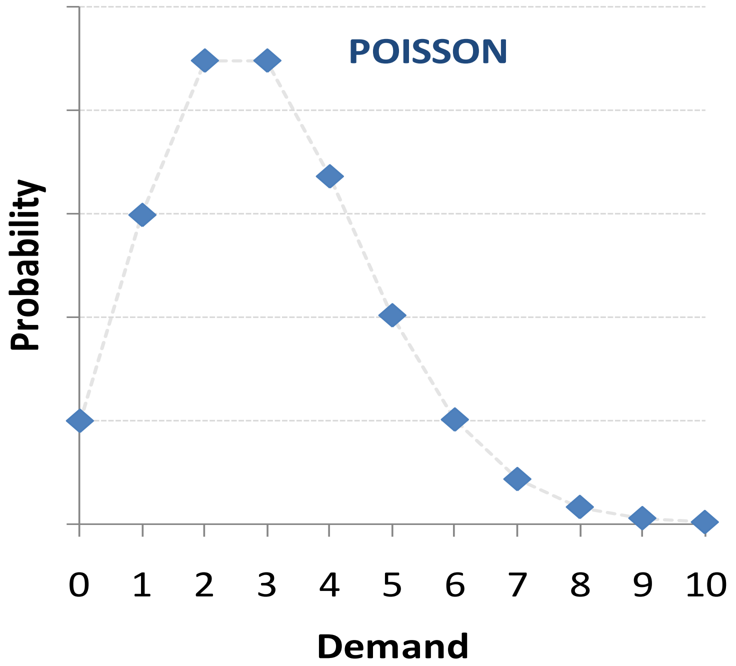
- Our demand model will have two components
  - Mean demand, D – akin to the systematic component in forecasting
  - Standard deviation, σ_D - akin to standard deviation of forecast error (random component)
- Coefficient of variation, $cv = \sigma/D$





Lead time and demand uncertainty

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- Orders have to be placed before actual demand is realized because of processing, manufacturing, and shipping times
- The time between when an order is placed and received is referred to as the **lead time**
- Suppose demand for each period $i = 1, \ldots \mathrm{L}$ is normally distributed with mean D_i and standard deviation σ_i
- Let ρ_{ij} be the correlation coefficient of the demand between periods i and j
- The total demand over the L periods is also normally distributed with mean D_L and standard deviation of σ_L

$$D_L = \sum_{i=1}^{L} D_i \qquad \sigma_L = \sqrt{\sum_{i=1}^{L} \sigma_i^2 + 2 \sum_{i>j} \rho_{ij} \sigma_i \sigma_j}$$

Measuring product availability

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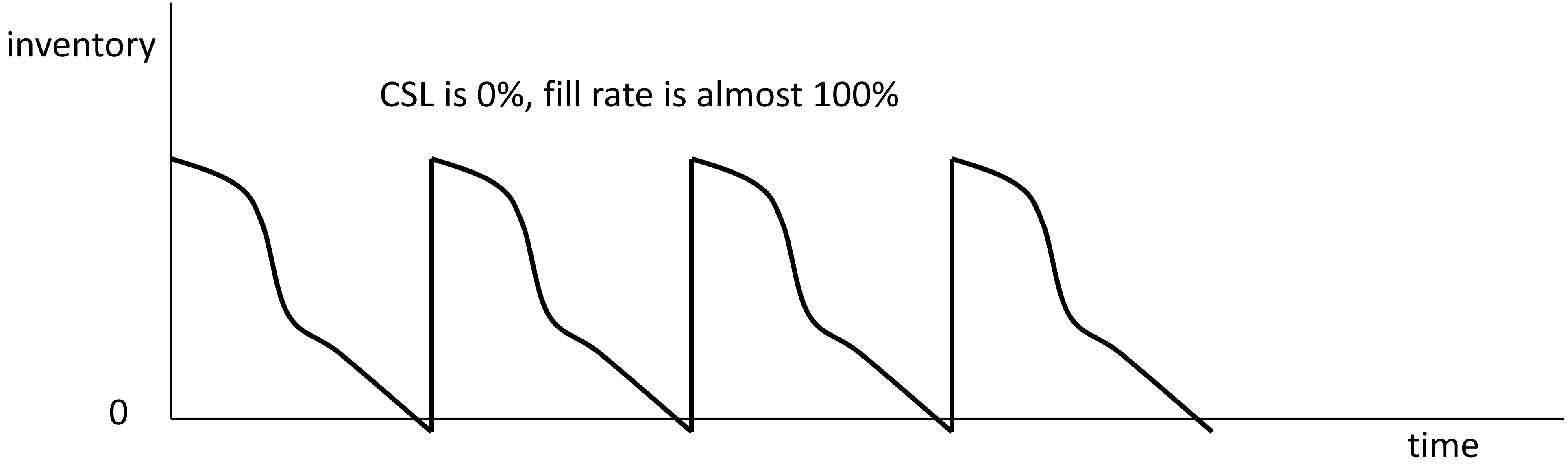
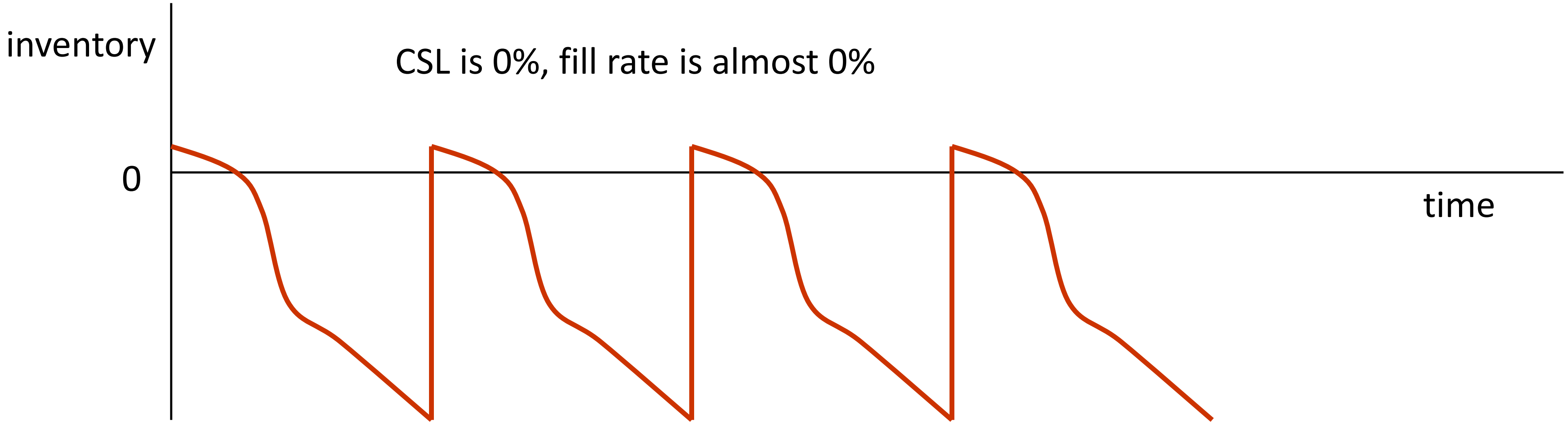
- What happens when a product is out of stock?
  - Backordered (patient customer or unique product or big cost advantage)
  - Lost sale

1. *Product fill rate (fr)*
   - Fraction of demand satisfied from product in inventory
2. *Order fill rate*
   - Fraction of orders filled from available inventory
   - Equivalent to product fill rate when orders contain one product
3. *Cycle service level (CSL)*
   - Fraction of replenishment cycles that end with all customer demand being met

CSL and fr are different





Discussion outline

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- Measures of product availability ✓
  - Fill rate
  - Cycle service level
- **Continuous review and periodic review inventory replenishment policies**
- Levers to improve performance

Continuous review policy

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- When to reorder?
- How much to reorder?

Most often these decisions are related

Continuous Review: Order fixed quantity when total inventory drops below Reorder Point *(ROP)*.

- ROP meets the **demand during the lead time L**.

- One has to figure out the ROP and order quantity, Q

Continuous review policy schematic

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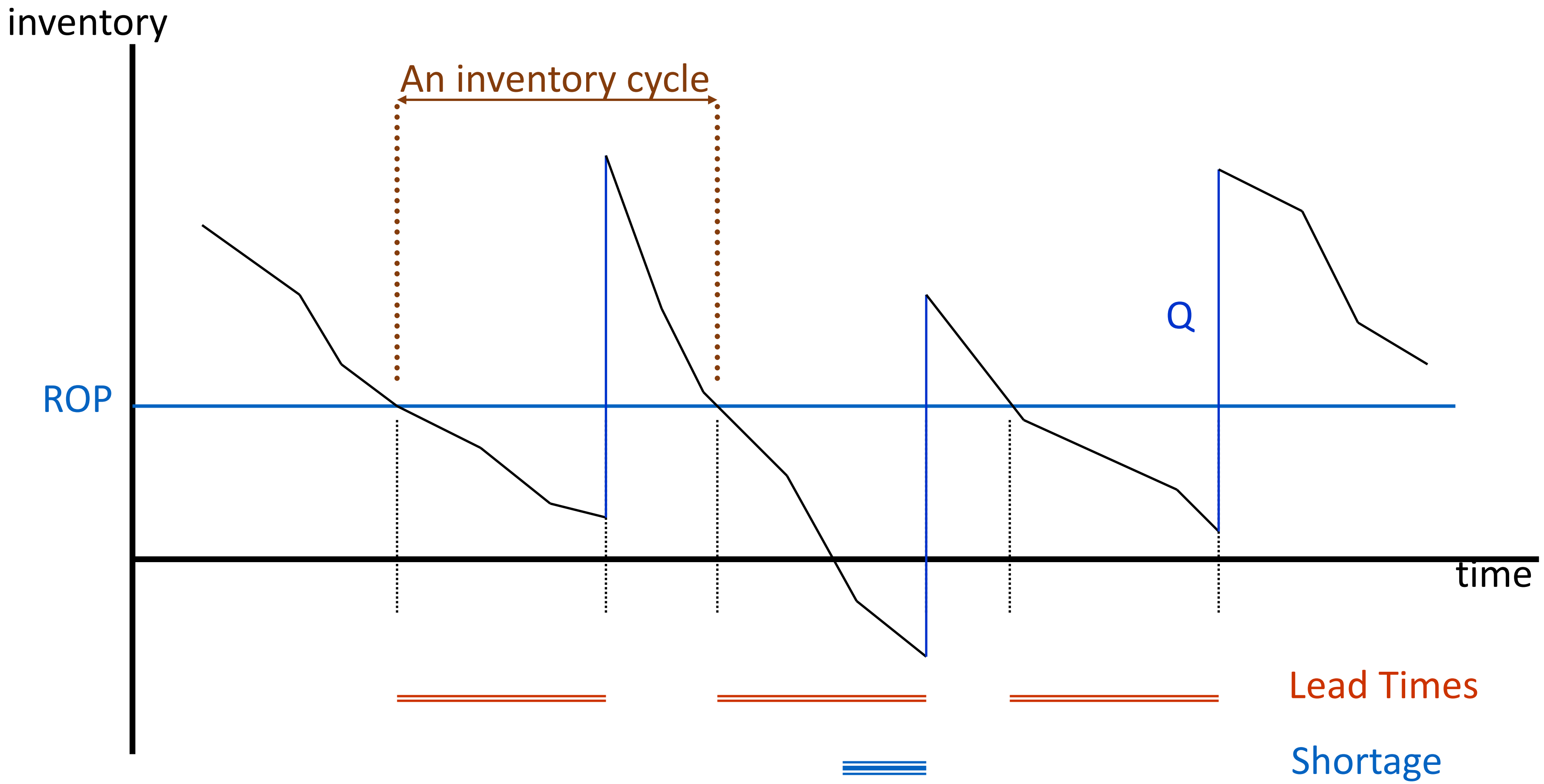

What is average demand in an inventory/replenishment cycle? Q

How do we choose ROP?

How do we choose Q?

Choosing reorder point (ROP)

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- Set to satisfy demand during lead time of L (?)
- Given that demand is random, how can we do this in practice?
- We can choose a cycle service level
  - ➢Fraction of cycles where total lead time demand is less than ROP
  - ➢Or equivalently $\Pr(\text{lead time demand} \leq ROP)$

For example consider 10 cycles:

$$CSL = \frac{1+1+0+1+1+1+0+1+0+1}{10}$$ Write 0 if a cycle has stockout, 1 otherwise

$$CSL = 0.7$$

$$CSL = 0.7 = \text{Probability that a single cycle has sufficient inventory}$$

$$[\text{Sufficient inventory}] = [\text{Demand during lead time} \leq \text{ROP}]$$

Reorder point, safety stock, and CSL

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- Suppose total demand during lead time has mean D_L
- **Can we set ROP equal to mean demand D_L?**
- No!, we may not satisfy the required CSL

- **How much higher should ROP be relative to mean demand D_L?**
- ROP $= D_L +$ safety stock (ss)
- Safety stock is the buffer against uncertainty to meet CSL!

- **Finding ROP and safety stock?**
- $CSL = \Pr(\text{lead time demand} \leq ROP)$
- Given a CSL, choose ROP such that the above equality holds
- ss = ROP - D_L

Computing ROP for Normal distribution

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- Suppose total demand during lead time is normally distributed with mean D_L and standard deviation σ_L, then

$$ROP = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L)$$

$$ss = ROP - D_L = F_S^{-1}(CSL)\sigma_L = NORMSINV(CSL)\sigma_L$$

where

- F and F^{-1} denote the normal cumulative probability distribution and its inverse
- F_S and F_S^{-1} denote the **standard** normal cumulative probability distribution and its inverse

(See the NormalDisitribution spreadsheet in the PoissonAndNormalDistributions excel file on Blackboard)

Computing ROP for Poisson distribution

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- Poisson distribution is a better model for slow moving demand
- The Poisson random variable is a discrete random variable (support: 0,1,2,3,…)
- For a Poisson random variable with mean D_L, the standard deviation is $\sqrt{D_L}$
- Due to the discrete support, we choose the smallest value of ROP so that we meet or exceed the CSL:

$$CSL \leq \Pr(\text{LT demand} \leq ROP)$$

- Once we have ROP, $ss = ROP - D_L$

(See the PoissonDisitribution spreadsheet in the PoissonAndNormalDistributions excel file on Blackboard)

Walmart Example (page 323 of textbook)

Weekly demand for Legos at a Walmart store is normally distributed, with a mean of 2,500 boxes and a standard deviation of 500. The replenishment lead time is 2 weeks. Assuming a continuous review policy, evaluate the ROP for this policy and the safety inventory that the store should carry to achieve a CSL of 90%.

1. Compute D_L and σ_L

$$D_L = D \times L = 2 \times 2500 = 5000 \text{ and } \sigma_L = \sqrt{L}\,\sigma_D = \sqrt{2} \times 500 = 707$$

2. Compute ROP

$$ROP = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L) = 5906$$

3. Compute safety inventory (ss)

$$ss = ROP - D_L = 5906 - 5000 = 906$$

Continuous review policy schematic

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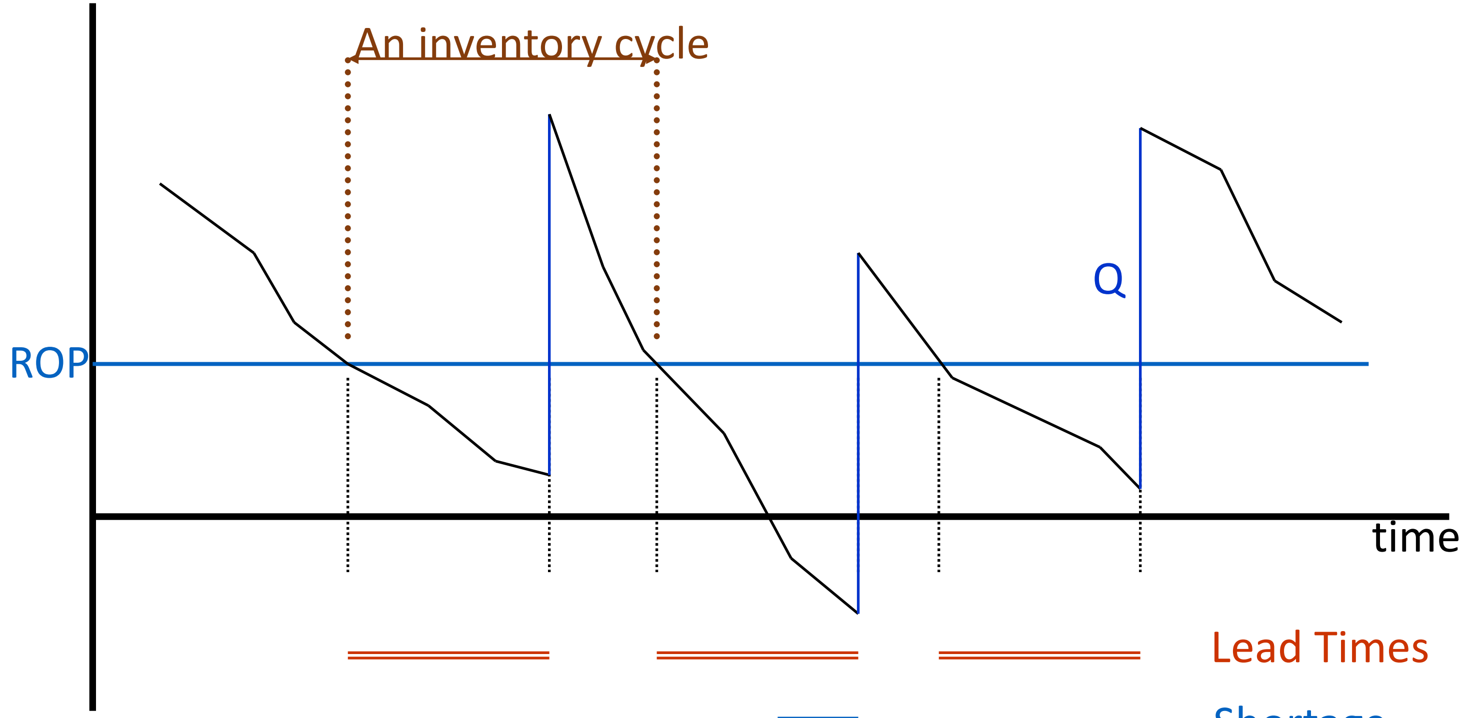

How do we choose ROP?

$\boldsymbol{CSL}$**=**
$\boldsymbol{Pr}(\text{lead time demand} \leq \boldsymbol{ROP})$

How do we choose Q?

Choosing order quantity (Q)

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- **What is the proportion of demand satisfied from available inventory (fill rate)?**
- Fill rate (fr) = 1 – fraction of demand lost
- Fraction of demand lost = $\frac{\text{Expected shortage per cycle}}{\text{Average demand in a cycle}} = \frac{ESC}{Q}$
- ESC is the average number of units of demand not satisfied from inventory in a cycle
- Fraction of demand lost = $\frac{ESC}{Q}$ = 1 - fr
- Given ESC and fr, $Q = \frac{ESC}{1-fr}$
- **How do we find ESC?**

Illustration of no shortage

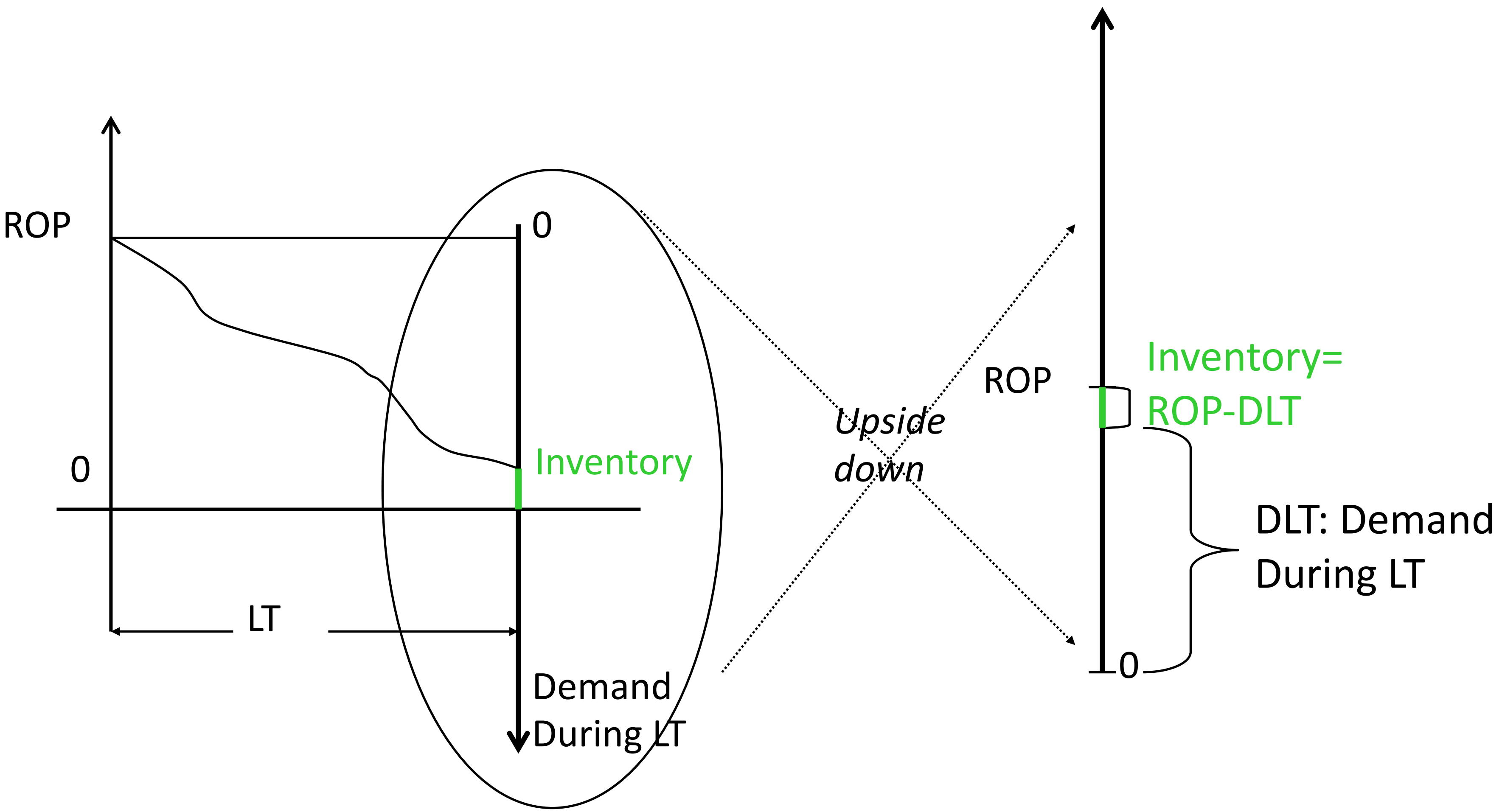
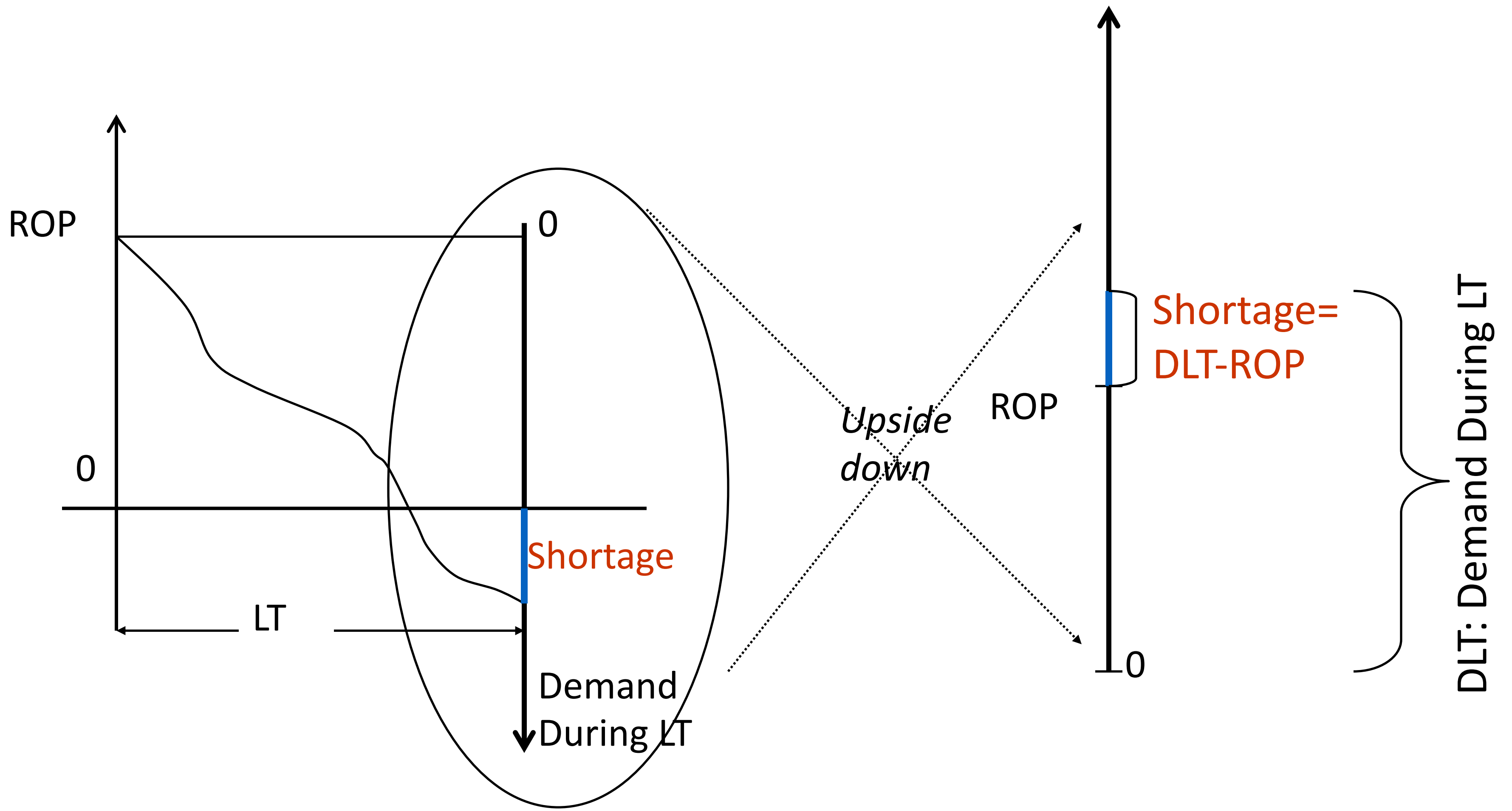


Illustration of shortage



Expected shortage per cycle (ESC)

- ESC is analogous to expected lost sales in the news vendor model!

$$\text{Shortage} = \begin{cases} \text{LT Demand} - \text{ROP}; \text{ if LT Demand} \geq \text{ROP} \\ 0 \qquad\qquad\qquad\quad ; \text{ if LT Demand} < \text{ROP} \end{cases}$$

$$= \max\{\text{LT Demand} - \text{ROP}, 0\}$$

$$ESC = E(\max\{\text{LT Demand} - \text{ROP}, 0\})$$

Simple example of computing ESC

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$$ROP = 10,\ \ D = \begin{Bmatrix} d_1 = 9 \ \text{ with prob } p_1 = 1/4 \\ d_2 = 10 \ \text{ with prob } p_2 = 2/4 \\ d_3 = 11 \ \text{ with prob } p_3 = 1/4 \end{Bmatrix},\ \ \text{Expected Shortage?}$$

$$\text{Expected shortage} = \sum_{\mathrm{i}=1}^{3} \max\{0,(d_i - ROP)\}\, p_i$$

$$= \max\{0,(9\text{-}10)\}\frac{1}{4} + \max\{0,(10\text{-}10)\}\frac{2}{4} + \max\{0,(11\text{-}10)\}\frac{1}{4} = \frac{1}{4}$$

Computing ESC for Normal and Poisson

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- Let demand be normally distributed with mean D_L and standard deviation σ_L
- $ESC = \sigma_L L(z)$, where $z = \frac{ROP - D_L}{\sigma_L}$; $L(z) = NORMSDIST(z) - z\big(1 - NORMSDIST(z)\big)$

- Let demand follow a Poisson distribution with mean D_L and probability mass function $\Pr(k = d)$
- $ESC = \sum_{d=0}^{\infty} \max\{\mathrm{d} - \mathrm{ROP}, 0\} \Pr(D = d)$

(Spread sheets NormalLossFunction and PoissonLossFunction in the PoissonAndNormalDistributions excel file can be used to compute)

Walmart Example (page 323 of textbook)

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Weekly demand for Legos at a Walmart store is normally distributed, with a mean of 2,500 boxes and a standard deviation of 500. The replenishment lead time is 2 weeks.

We computed the ROP (5906) and ss (906) for this example for a CSL of 90%. Now suppose we are told that the desired fill rate is 97.5%. Compute the order quantity Q.

1. Compute z and ESC

$$z = \frac{ROP - D_L}{\sigma_L} = \frac{906}{707} = 1.281 \text{ and } ESC = \sigma_L L(z) = 707 \times L(1.281) = 33.47$$

2. Compute Q using ESC and fr

$$Q = \frac{ESC}{1 - fr} = \frac{33.47}{0.025} = 1339$$

Continuous review policy schematic

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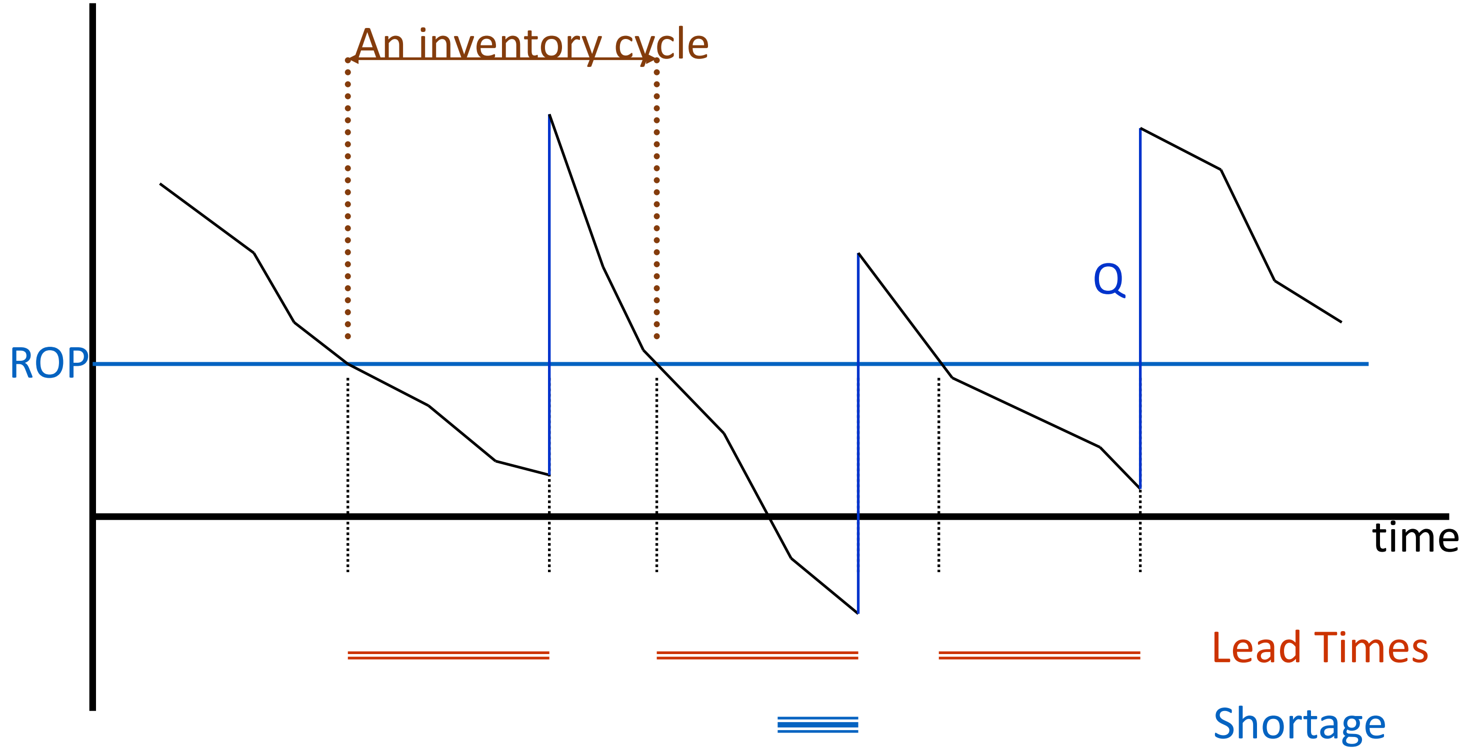


How do we choose ROP?

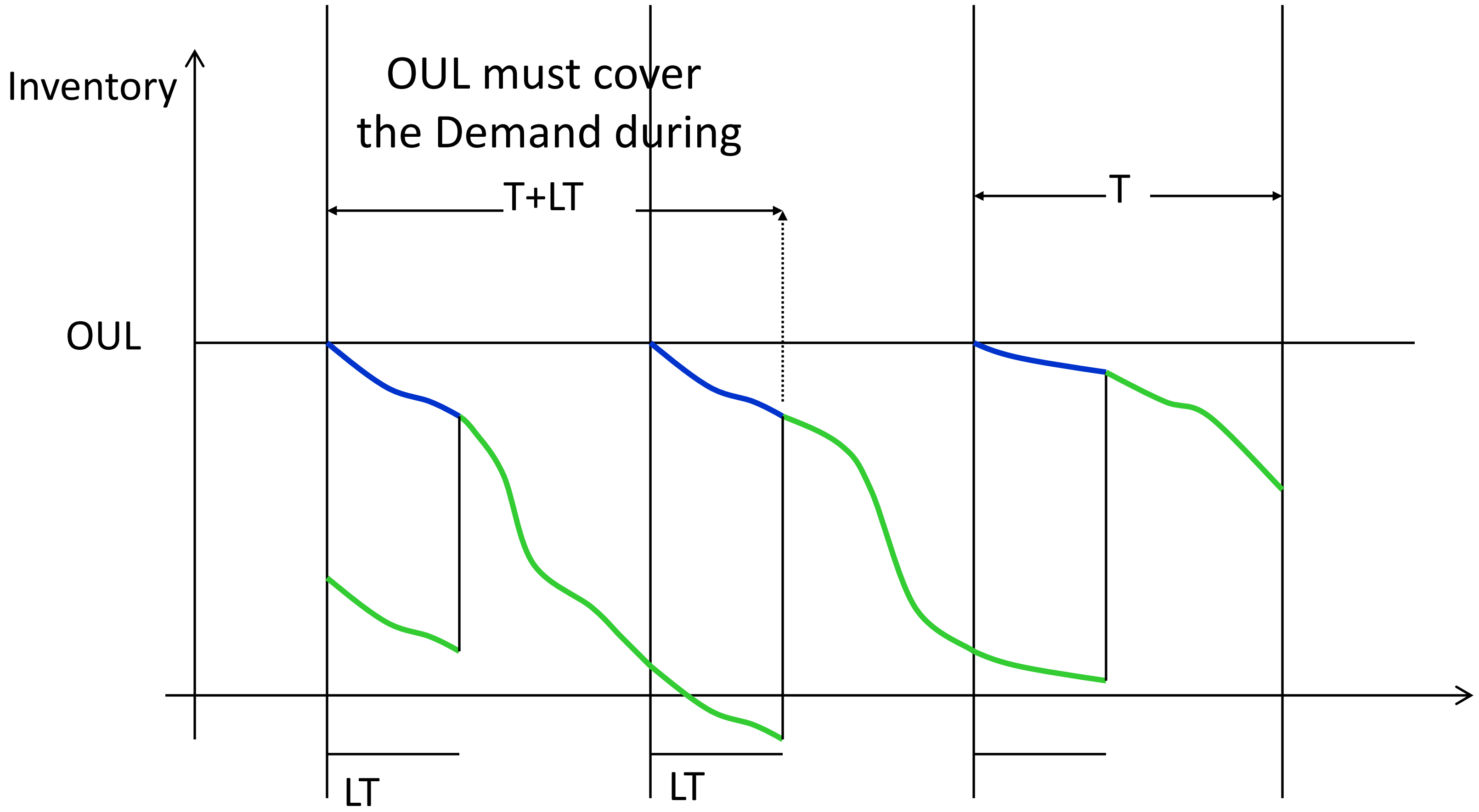
$$CSL = Pr(\text{lead time demand} \leq ROP)$$

How do we choose Q?

$$Q = \frac{ESC}{1 - fr}$$

Periodic review policy

- Order at fixed time intervals (T apart) to raise total inventory to Order up to Level *(OUL)*



Periodic review policy parameters

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- Lot size determined by pre-specified *order-up-to level* (OUL)

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

T: Review interval

CSL: Desired cycle service level

Probability(demand during $L + T \leq OUL$) = CSL

Mean demand during $T + L$ periods, $D_{T+L} = (T + L)D$

Std dev demand during $T + L$ periods, $\sigma_{T+L} = \left(\sqrt{T+L}\right)\sigma_D$

$$OUL = NORMINV(CSL, D_{L+T}, \sigma_{T+L})$$

$$ss = OUL - D_{T+L} = F_S^{-1}(CSL) \times \sigma_{T+L} = NORMSINV(CSL) \times \sigma_{T+L}$$

Walmart Example (page 323 of textbook)

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Weekly demand for Legos at a Walmart store is normally distributed, with a mean of 2,500 boxes and a standard deviation of 500. The replenishment lead time is 2 weeks.

Suppose the store manager decides to implement a periodic review policy with review every **four** weeks. Required CSL is 90%. What is the OUL and safety inventory?

1. Compute D_{T+L} and σ_{T+L}

$$D_{T+L} = D \times (L + T) = (2 + 4) \times 2500 = 15000$$

$$\sigma_L = \sqrt{T + L}\, \sigma_D = \sqrt{2 + 4} \times 500 = 1225$$

2. Compute OUL

$$OUL = F^{-1}(CSL, D_{T+L}, \sigma_{T+L}) = NORMINV(CSL, D_{T+L}, \sigma_{T+L}) = 16570$$

3. Compute safety inventory (ss)

$$ss = OUL - D_{L+T} = 16570 - 15000 = 1570$$

Periodic Vs. continuous review

- Walmart example:
  - Continuous review policy for CSL 90% and fill rate 97.5%: ROP = 5906, ss = 906, and Q = 1339
  - Periodic review policy for CSL 90% and four week review: OUL = 16570, and ss = 1570

- Periodic review ss is larger: Covers the uncertainty over [0,T+L], that is, T periods more than ss in continuous review policy
- Continuous review is harder to implement, but recent advances in IT make this easier
- Continuous review is especially useful for high-sales-value per time products

Discussion outline

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- Measures of product availability ✓
  - Fill rate
  - Cycle service level
- Continuous review and periodic review inventory replenishment policies ✓
- **Levers to improve performance**
  - Factors affecting fill rate
  - Two approaches to reduce safety inventory

Cutting down safety stock: Reduce lead time

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- Reducing lead time by a factor of k decreases safety inventory by a factor $\sqrt{k}$
- Faster transportation
- Better coordination, information exchange, advance retailer demand information to prepare the supplier

Cutting down safety stock: Decrease lead time variability

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- **What if lead time is also uncertain?**
- Assume demand and lead time are normally distributed
  - Average demand per period is D with standard deviation σ_D
  - Average lead time is L with standard deviation s_L
- Demand during lead time is still normally distributed and the prior formulae apply
  - Mean $D_L = D \times L$
  - Standard deviation $\sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$
- Since lead time variability increases demand variability, the safety stock increases

Cutting down safety stock: Reduce demand uncertainty

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- Reducing σ_L by a factor of k decreases safety inventory by a factor of k
- How can we do this?
- Coordinating with supply chain member during forecasting
  - Aggregate demand from multiple partners when making replenishment decisions
  - Place inventory at a single large store instead of k different stores (this is also a form of aggregation)

BMW Example (page 330 of textbook)

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A BMW dealership has **four** retail outlets serving the entire Chicago area (disaggregate option). Weekly demand at each outlet is normally distributed, with a mean of $D = 25$ cars and a standard deviation $\sigma_D = 5$. The lead time for replenishment from the manufacturer is $L = 2$ weeks. The correlation between a pair of locations is ρ.

Assuming $\rho = 0$, we want to evaluate the ROP and safety stock in the following two cases:

1. Decentralized option: Hold inventory at each store
2. Centralized options: Hold inventory at a large single outlet that replaces the four outlets

BMW Example contd…

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Decentralized option

1. Compute σ_L and D_L

$$D_L = D \times L = 2 \times 25 = 50 \text{ and } \sigma_L = \sqrt{L}\,\sigma_D = \sqrt{2} \times 5 = 7.07$$

2. Compute ROP

$$ROP = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L) = 59.06$$

3. Compute safety inventory at one store

$$ss(\text{one store}) = ROP - D_L = 59.06 - 50 = 9.06$$

4. Compute total safety inventory across all four stores

$$ss(\text{total}) = 4 \times ss(\text{one store}) = 4 \times 9.06 = 36.24$$

BMW Example contd…

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Centralized option

1. Compute σ_D^C and D^C

$$D^C = 4 \times D = 4 \times 25 = 100 \text{ and } \sigma_D^C = \sqrt{4}\,\sigma_D = \sqrt{4} \times 5 = 10$$

2. Compute σ_L^C and D_L^C

$$D_L^C = 2 \times D^C = 2 \times 100 = 200 \text{ and } \sigma_L^C = \sqrt{2}\,\sigma_D^C = \sqrt{2} \times 10 = 14.14$$

3. Compute ROP

$$ROP = F^{-1}\left(CSL, D_L^C, \sigma_L^C\right) = NORMINV\left(CSL, D_L^C, \sigma_L^C\right) = 218.12$$

4. Compute safety inventory at large store

$$ss = ROP - D_L^C = 218.12 - 200 = 18.12$$

Safety inventory reduced as a result of aggregation

Component commonality (aggregation?)

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- Dell manufactures 27 PCs with three distinct components: processor, memory, and hard drive. Monthly demand for each of the 27 PCs has mean of 5,000 and a standard deviation of 3,000 (assume normal). The replenishment lead time for each component is one month. Dell is targeting a CSL of 95 percent for component inventory.
- **Disaggregate option:** Dell designs specific components for each PC, resulting in 81 (27 x 3) distinct components.
- **Common-component option:** Dell designs PCs such that three distinct processors, three distinct memory units, and three distinct hard drives can be combined to create 27 PCs. Each component is thus used in nine PCs.
- Evaluate the safety inventory requirements with and without the use of component commonality.

Disaggregate option

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1. Compute σ_L and D_L for a component

$$D_L = D \times L = 5000 \times 1 = 5000 \text{ and } \sigma_L = \sqrt{L}\,\sigma_D = \sqrt{1} \times 3000 = 3000$$

2. Compute ROP

$$ROP = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L) = 9934.56$$

3. Compute safety inventory for one component

$$ss(\text{one component}) = ROP - D_L = 9934.56 - 5000 = 4934.56$$

4. Compute total safety inventory across all 81components

$$ss(\text{total}) = 81 \times ss(\text{one component}) = 81 \times 4934.56 = 399{,}699$$

Common-component option

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1. Compute σ_D^C and D^C for one common component

$$D^C = 9 \times D = 9 \times 5000 = 45000 \text{ and } \sigma_D^C = \sqrt{9}\,\sigma_D = \sqrt{9} \times 3000 = 9000$$

2. Compute σ_L^C and D_L^C

$$D_L^C = D^C = 45000 \text{ and } \sigma_L^C = \sigma_D^C = 9000$$

3. Compute ROP for one common component

$$ROP = F^{-1}\left(CSL, D_L^C, \sigma_L^C\right) = NORMINV\left(CSL, D_L^C, \sigma_L^C\right) = 59803.68$$

4. Compute safety for one common component

$$ss(\text{one common}) = ROP - D_L^C = 59803.68 - 45000 = 14803.68$$

4. Compute safety for all common nine components = 14803.68 x 9 = 133233

Component commonality reduces safety stock as a result of aggregation

Discussion outline

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- Measures of product availability ✓
  - Fill rate
  - Cycle service level
- Continuous review and periodic review inventory replenishment policies ✓
- Levers to improve performance ✓
  - Factors affecting fill rate
  - Two approaches to reduce safety inventory