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Aggregate Planning: Part 2

Optimization

IDS 552

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Discussion outline

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- What is mathematical programming?
- Optimal aggregate planning formulation
- Sensitivity analysis

Aggregate planning using math programming

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- Math programming is a field in optimization that focuses on finding solutions to problems involving the allocation of scarce resources to minimize/maximize an objective (e.g. cost)

Minimize Costs

Subject to: Demand, capacity, initial inventory requirements

Elements of a math program:

- Parameters: Data that is fixed in the formulation (e.g., production capacity of a machine)
- Decision variables: Represent the decisions that you want to take (e.g., production quantity)
- Objective: Maximize or minimize some function of decision variables (e.g. minimize production cost)
- Constraints: Business rules that must be satisfied by decision variables

$$\text{(function of decision variables)} \begin{array}{c} \leq \\ = \\ \geq \end{array} \text{(function of parameters)}$$

A simple example

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- A farmer owns 60 acres of land. She may plant either wheat or corn. Planting wheat and corn lead to profits of $200 and $300, respectively.
- Each planed acre uses fertilizer and labor as follows:

	Wheat	Corn
Labor (workers)	3	2
Fertilizer (tons)	2	4

- Overall, 100 workers and 120 tons of fertilizer are available
- How many acres of wheat and corn should the farmer plant to maximize profit?

Linear programming formulation

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- Decision variables?
  - (i) x – acres planted with wheat; (ii) y – acres planted with corn
- Objective function?
  - maximize profit = $\max 200x + 300y$
- Constraints?

$$3x + 2y \leq 100 \text{ (labor)} \qquad x + y \leq 60 \text{ (land)}$$

$$2x + 4y \leq 120 \text{ (fertilizer)} \qquad x, y \geq 0$$

Graphical solution

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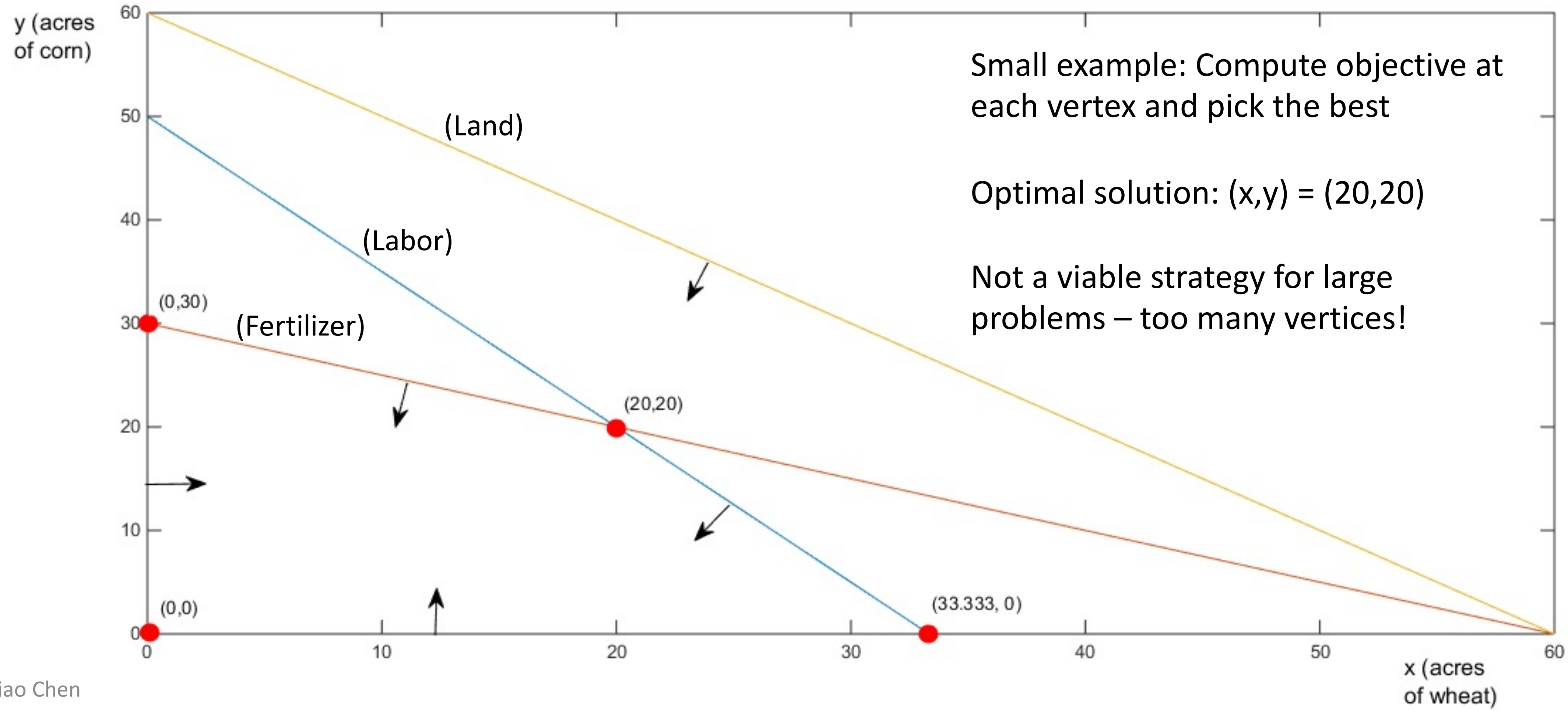
- Constraints define the feasible solutions, that is, the values of (x,y) that satisfy our constraints



Small example: Compute objective at each vertex and pick the best

Optimal solution: (x,y) = (20,20)

Not a viable strategy for large problems – too many vertices!

Classification of math programs

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- Linear programs: all decision variables can take continuous values
- Integer linear programs: all decision variables must take integer values
- Mixed integer linear programs: some decision variables must take integer values
- These three types of math programs arise widely in supply chain management and are solved on a daily basis in practice

Solving math programs in the real world

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- Solving large scale math programs in practice requires sophisticated optimization techniques
- Fortunately, there are a number of commercial and open source solvers for solving math programs
- The most popular commercial solvers are IBM ILOG CPLEX and GUROBI
- The most popular open source solver belongs to the COIN OR project
- All these solvers can be called from the Julia language
- An excel plugin called OpenSolver can be used to call the COIN OR solver

Commercial solvers have advanced substantially

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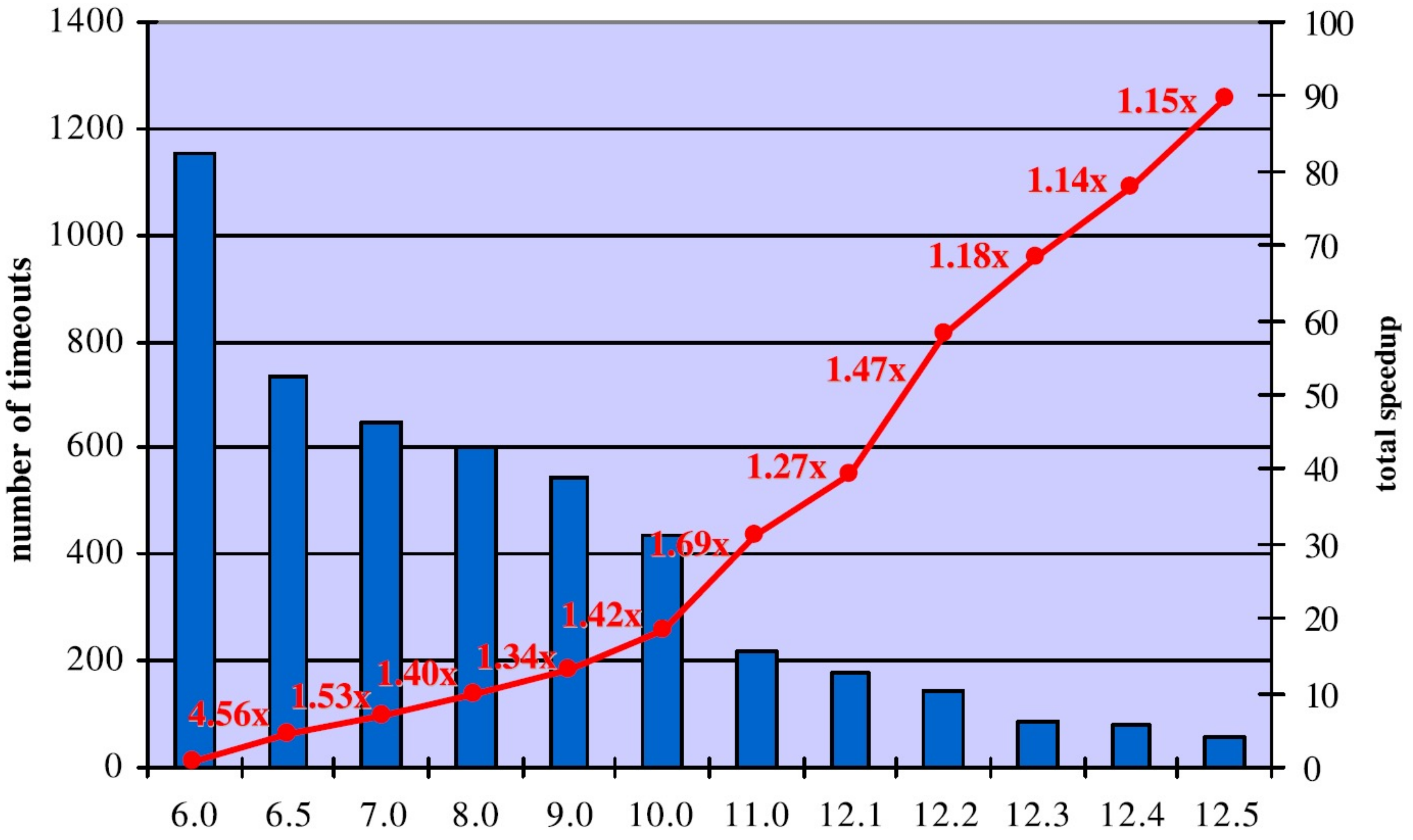
Improvements in commercial solver Cplex from version 6.0 to 12.5 (1998 to 2013):

- Unsolved instances reduced from 1152 to 55
- Speedup factor of roughly 90

Source: MIP: Analyzing 12 years of progress, Facets of combinatorial optimization, T. Achterberg and R. Wunderling



Red Tomato planning parameters

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Item	*Value*
Materials	$10/unit
Inventory holding cost	$2/unit/month
Marginal cost of a backorder	$5/unit/month
Hiring and training costs	$300/worker
Layoff cost	$500/worker
Regular time cost	$4/hour
Over time cost	$6/hour
Cost of subcontracting	$30/unit
Revenue	$40/unit
Labor hours required per unit	4 hours/unit
Regular time per day	8 hours/day
Max overtime hrs per employee per month	10 hours/month
Working days per month	20 days
Starting inventory	1000 units
Ending inventory	500 units
Starting workforce size	80 workers

Aggregate demand

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Month	*Demand Forecast*
January	1,600
February	3,000
March	3,200
April	3,800
May	2,200
June	2,200
Total	16,000

Aggregate planning decision variables for Red Tomato

W_t = Number of employees in month t, $t = 1, \ldots, 6$

H_t = Number of employees hired at the beginning of month t, $t = 1, \ldots, 6$

L_t = Number of employees laid off at the beginning of month t, $t = 1, \ldots, 6$

P_t = Production in units of shovels in month t, $t = 1, \ldots, 6$

I_t = Inventory at the end of month t, $t = 1, \ldots, 6$

S_t = Number of units backordered at the end of month t, $t = 1, \ldots, 6$

C_t = Number of units subcontracted for month t, $t = 1, \ldots, 6$

O_t = Number of overtime hours worked in month t, $t = 1, \ldots, 6$

Aggregate planning objective

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- Minimize total cost
- Total cost components:
  1. Regular time labor
  2. Hiring costs
  3. Layoff costs
  4. Overtime labor
  5. Inventory holding
  6. Stockout
  7. Materials costs
  8. Subcontracting cost

$$Min \sum_{t=1}^{6} 4\times 8\times 20\times W_t + \sum_{t=1}^{6} 300\, H_t + \sum_{t=1}^{6} 500\, L_t + \sum_{t=1}^{6} 6\, O_t$$

$$+ \sum_{t=1}^{6} 2\, I_t + \sum_{t=1}^{6} 5\, S_t + \sum_{t=1}^{6} 10\, P_t + \sum_{t=1}^{6} 30\, C_t$$

Aggregate planning constraints

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- Workforce size for each month is based on hiring and layoffs

$$W_t = W_{t-1} + H_t - L_t, \quad or$$
$$W_t - W_{t-1} - H_t + L_t = 0 \quad for \quad t = 1,\ldots,6, \quad where \quad W_0 = 80.$$

- Production (in hours) for each month cannot exceed capacity (in hours)

$$4P_t \le 8 \times 20 W_t + O_t \quad \text{or}$$
$$40 W_t + O_t/4 - P_t \ge 0, \quad for \quad t = 1,\ldots,6.$$

- Overtime for each month

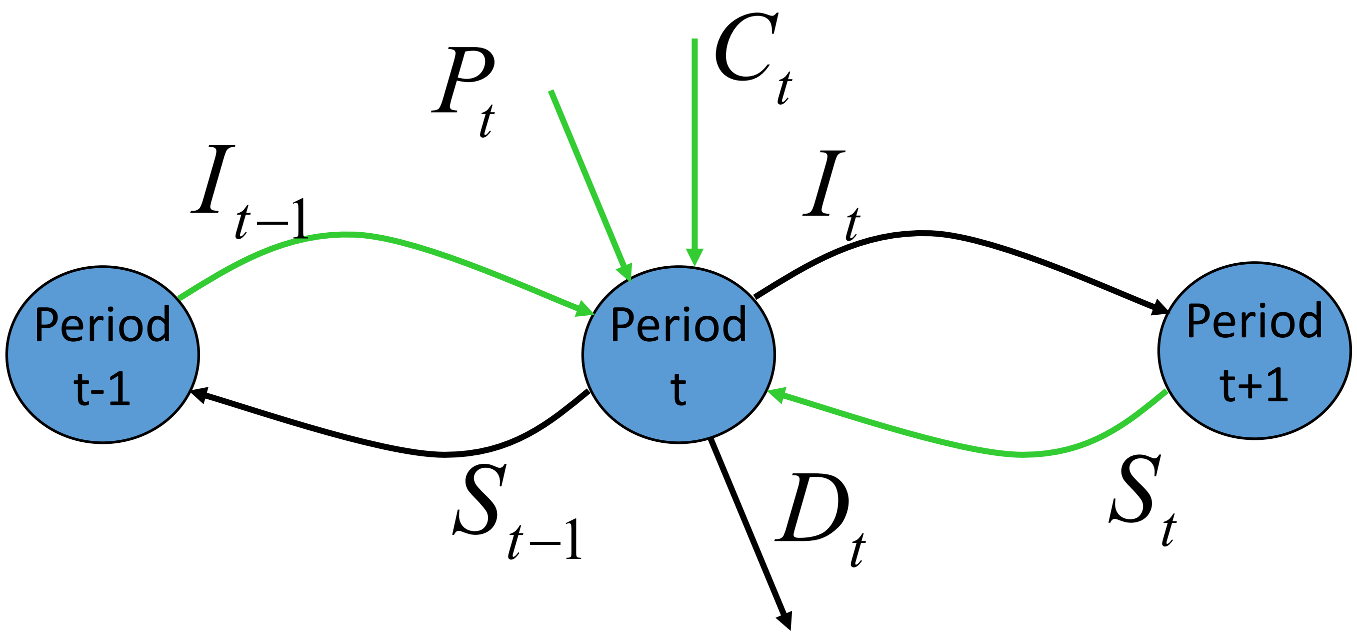
$$\mathrm{O_t} \le 10\,\mathrm{W_t} \quad \text{or}$$
$$10\,\mathrm{W_t} - \mathrm{O_t} \ge 0 \quad \text{for} \quad \mathrm{t} = 1,\ldots,6.$$

Aggregate planning constraints

- Inventory balance for each month

$$I_{t-1} + P_t + C_t + S_t = D_t + S_{t-1} + I_t,$$
$$I_{t-1} + P_t + C_t - D_t - S_{t-1} - I_t + S_t = 0,$$
for $t = 1,\ldots,6$, where $I_0 = 1{,}000$, $S_0 = 0$, $S_6 = 0$, and $I_6 \geq 500$.

D_t denotes demand at time t



Solving the aggregate planning linear program

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- Solve the formulation
  - Total cost=$422,275
  - Total revenue=$640,000
  - Total profit = $217,725

- Apply the first month of the plan
- Delay applying the remaining part of the plan until the next month
- Rerun the model with new data next month

- This is called rolling horizon execution

Solving the aggregate planning mixed integer program

- Enforce condition that # of employees, # hired, and # laid off are integer
- Solve the formulation
  - Total cost=$422,660
  - Total revenue=$640,000
  - Total profit = $217,340

Increase in demand fluctuation

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Month	*Demand Forecast*
January	1,000
February	3,000
March	3,800
April	4,800
May	2,000
June	1,400
Total	16,000

- Solve the optimization model with revised data
  - Total cost=$433,080 (cost increased by $10,420 relative to less variable forecast)
  - Total revenue=$640,000
  - Total profit = $206,840

Sensitivity analysis

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- One of the advantages of having an easy to solve optimization model is the ability to study the impact of different scenarios
- We already looked at the impact of increase in demand fluctuation
- We could have changed cost parameters and capacities
- It is possible to consider the impact of new constraints (e.g.: company policy requiring that the workforce size must remain the same for at least three consecutive periods)