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Demand Forecasting

IDS 552

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Discussion Outline

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- Forecasting methods
  - Moving average forecasting methods
  - Winter’s method
- Forecasting error and value analysis
  - Measures of forecast error
  - Is your forecasting process adding any value?

Introduction

Sephora

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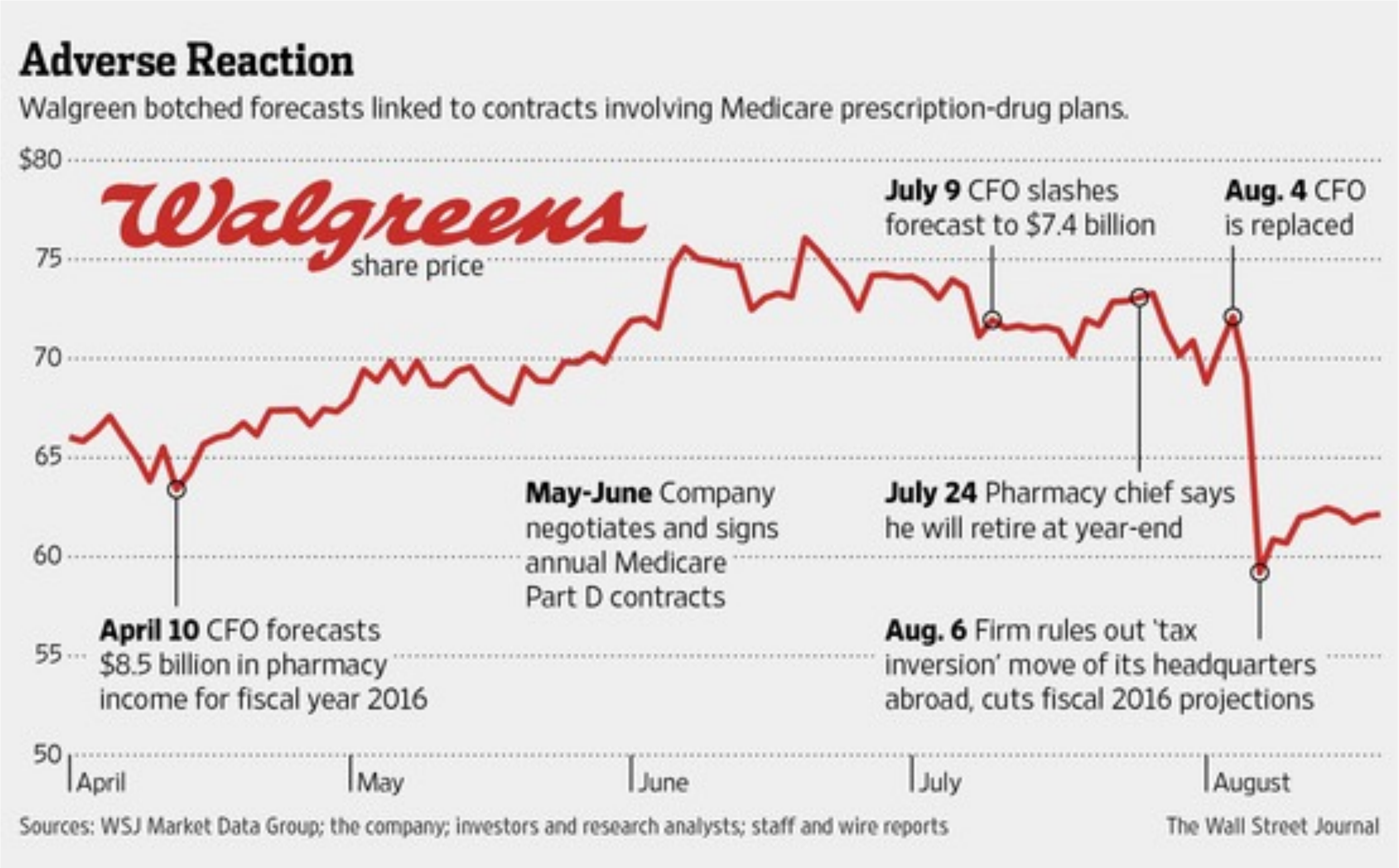
Sephora has deployed TXT Retail TXTPlanning software to forecast and replenish its stores on a global scale…..

…. The solution supports four key objectives initially set out by Sephora: basing sales forecasts on sell-through rates of stores and being able to drill down through data by week, item and store level; managing all products with TXTPlanning, including slow-movers; shifting demand from stores to its distribution centers and suppliers for greater forecast accuracy; and forecasting demand while taking into account special events and promotional activity, as well as the seasonality of products.

Source: Sephora rejuvenates forecasting and replenishment, December 22 2015, Chain Store Age.

Walgreens billion dollar blunder



Forecasting methods

Characteristics of forecasts

1. Forecasts are always inaccurate and should thus include both the expected value of the forecast and a measure of forecast error
2. Long-term forecasts are usually less accurate than short-term forecasts
3. Aggregate forecasts are usually more accurate than disaggregate forecasts
4. In general, the farther up the supply chain a company is, the greater is the distortion of information it receives

Types of forecasting methods

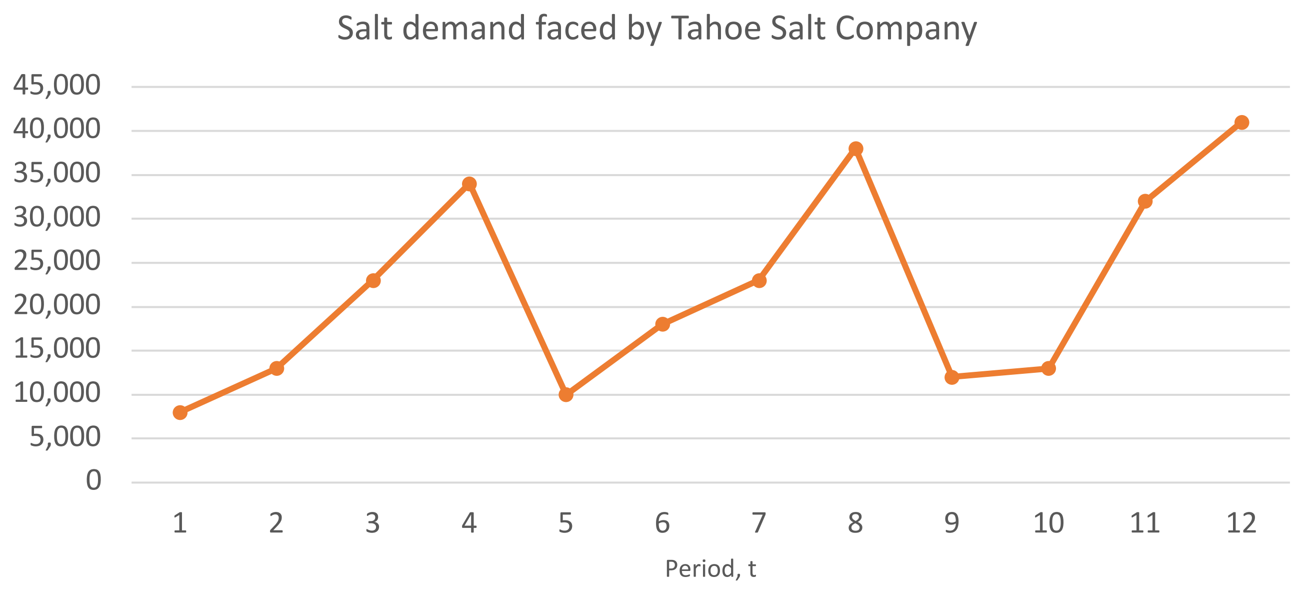
1. Qualitative
   - Primarily subjective
   - Rely on judgment
2. Time Series
   - Use historical demand only
   - Best with stable demand
3. Causal
   - Relationship between demand and some other factor
4. Simulation
   - Imitate consumer choices that give rise to demand

We will be focusing on time series forecasting methods

Example time series

- Time series? Just a plot of a numeric quantity over time
- We will focus on demand but the quantity could be sales, price, temperature, ….

Year	Quarter	Period, t	Demand, Dt
1	2	1	8,000
1	3	2	13,000
1	4	3	23,000
2	1	4	34,000
2	2	5	10,000
2	3	6	18,000
2	4	7	23,000
3	1	8	38,000
3	2	9	12,000
3	3	10	13,000
3	4	11	32,000
4	1	12	41,000



Tahoe salt example

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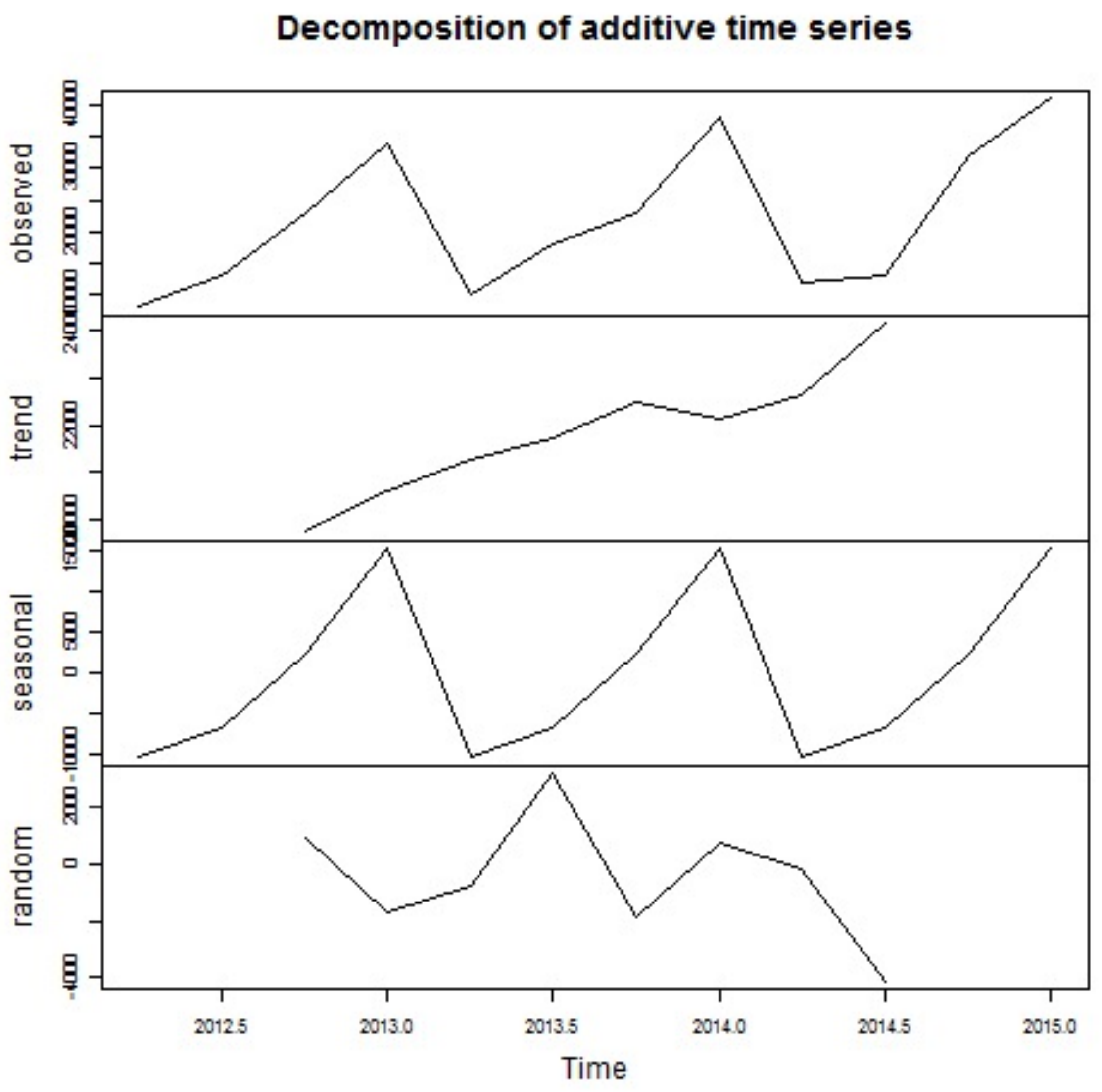
- Lets load data using the R template....

Components of a demand observation

Observed demand (D) = systematic component (S) + random component (R)

- Systematic component – predictable part
  - ***Level*** (current deseasonalized demand)
  - ***Trend*** (growth or decline in demand)
  - ***Seasonality*** (predictable seasonal fluctuation)
- Random component – deviation of observed demand from systematic component

Times series components



Time series forecast models

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1. Use historical data to estimate level, trend, and seasonality terms L_t, T_t, and S_t, respectively

2. Construct forecast (note that level + trend is sometimes just called trend)
   - Additive model
   $$F_t = L_t + T_t + S_t$$
   - Multiplicative model
   $$F_t = L_t \times T_t \times S_t$$
   - Mixed model
   $$F_t = (L_t + T_t) \times S_t$$
     - Note that this is called multiplicative in many places, especially in the R package we use.

3. Forecasts relate to actual demand (D_t) through an error term E_t

$$D_t = F_t + E_t$$

Forecasting systematic component: Time series methods

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- Static method
  - Mixed systematic component = (level + trend) x seasonal factor
  - Regression to estimate level, trend, and seasonality
- Adaptive methods
  - Moving average
  - Exponential smoothing
  - Winter's method for estimating a trend with seasonality

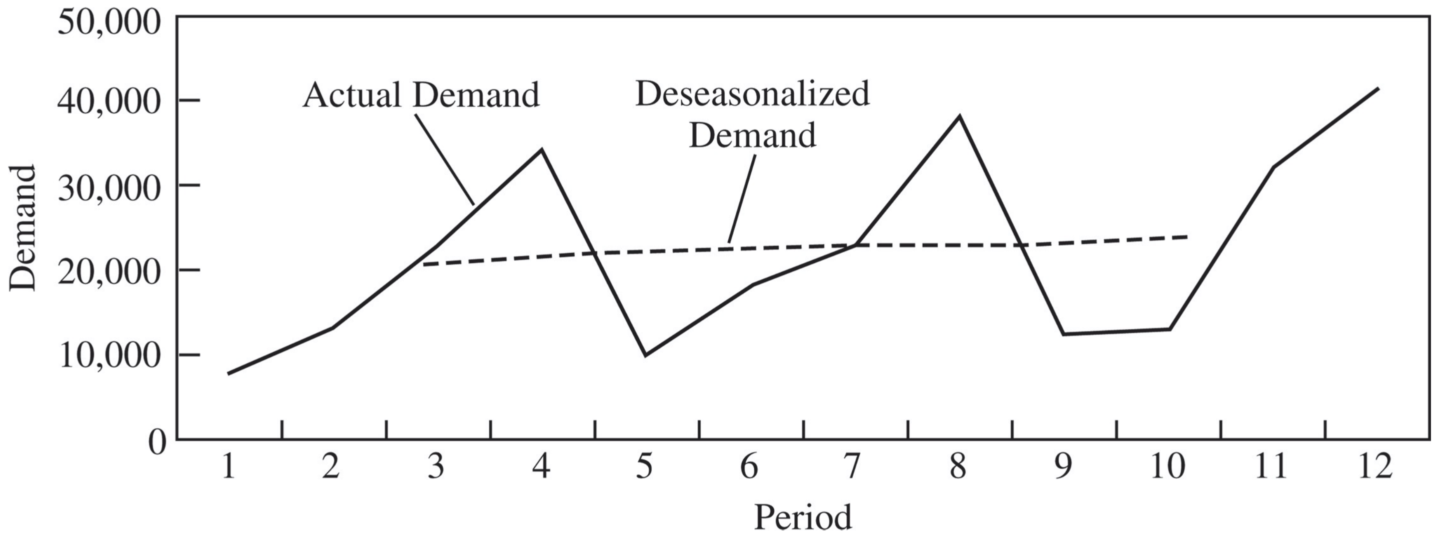
Static method

$$F_{t+l} = [L + (t + l)T]S'_{t+l}$$

1. Compute de-seasonalized demand
2. Use regression on de-seasonalized demand to estimate
   - level, L (intercept)
   - trend, T (slope)
3. Use ratios of demand and de-seasonalized demand to compute seasonal factors, S'

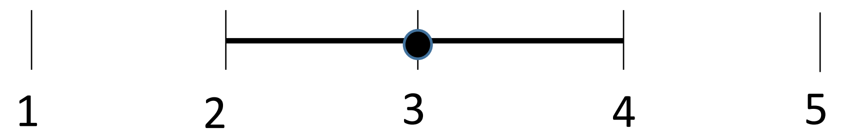
Tahoe salt quarterly demand example

Periodicity, p = 4

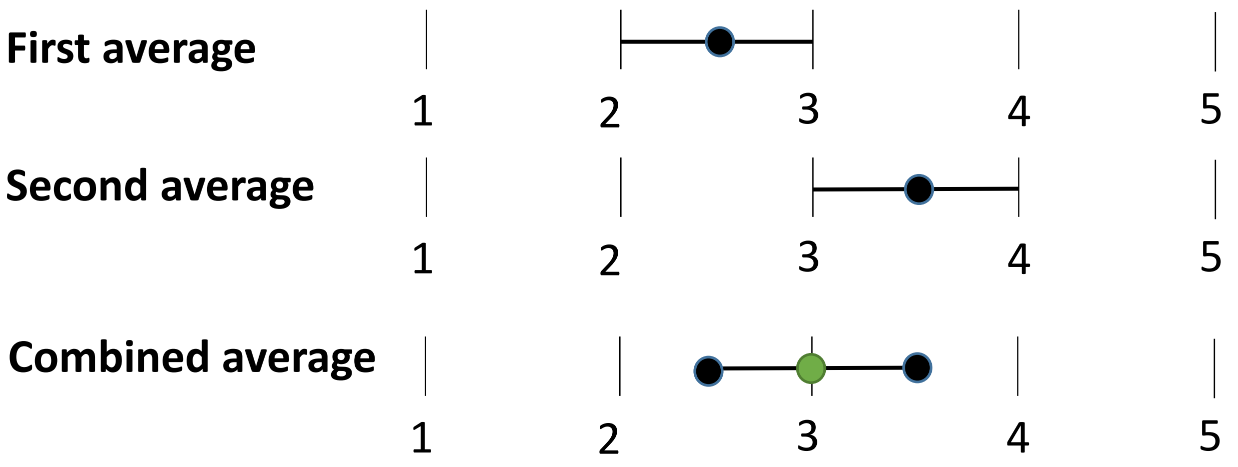


Computing de-seasonalized demand

- Idea: average demand over p consecutive periods
- If p is odd (say p =3), then the deasonalized demand at t = 3 averages the following periods



- If p is even (say p =2), then the deasonalized demand at t = 3 combines two averages



De-seasonalized demand equations

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$$\bar{D}_t = \begin{cases} \left[D_{t-(p/2)} + D_{t+(p/2)} + \sum_{i=t+1-(p/2)}^{t-1+(p/2)} 2D_i \right] / (2p) \text{ for } p \text{ even} \\ \sum_{i=t-[(p-1)/2]}^{t+[(p-1)/2]} D_i / p \text{ for } p \text{ odd} \end{cases}$$

Periodicity p = 4, t = 3

$$\bar{D}_t = \left[D_{t-(p/2)} + D_{t+(p/2)} + \sum_{i=t+1-(p/2)}^{t-1+(p/2)} 2D_i \right] / (2p)$$

$$\bar{D}_3 = \left[D_1 + D_5 + \sum_{i=2}^{4} 2D_i \right] / 8$$

Computing de-seasonalized demand

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	A	B	C
1	*Period* *t*	*Demand* D_t	*Deseasonalized Demand*
2	1	8,000	
3	2	13,000	
4	3	23,000	19,750
5	4	34,000	20,625
6	5	10,000	21,250
7	6	18,000	21,750
8	7	23,000	22,500
9	8	38,000	22,125
10	9	12,000	22,625
11	10	13,000	24,125
12	11	32,000	
13	12	41,000	

Cell	Cell Formula
C4	=(B2+B6+2*SUM(B3:B5))/8

Figure 7-2 Excel Workbook with Deseasonalized Demand for Tahoe Salt

Linear regression of de-seasonalized demand

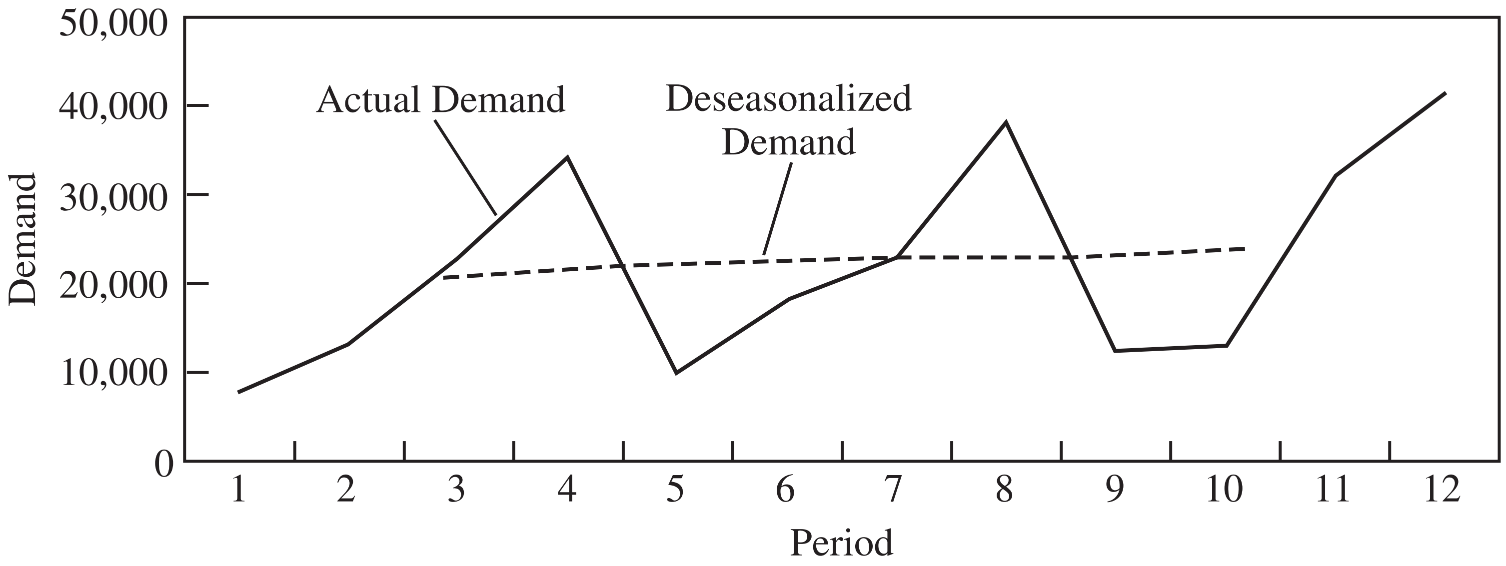

Figure 7-3 Deseasonalized Demand for Tahoe Salt

A linear relationship exists between the deseasonalized demand and time based on the change in demand over time, by linear regression

$$\overline{D_t} = 18439 + t * 524$$

Estimating seasonal factors

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	A	B	C	D
1	*Period* t	*Demand* D_t	*Deseasonalized Demand (Eqn 7.4)* $\bar{D}_t$	*Seasonal Factor (Eqn 7.5)* $\bar{S}_t$
2	1	8,000	18,963	0.42
3	2	13,000	19,487	0.67
4	3	23,000	20,011	1.15
5	4	34,000	20,535	1.66
6	5	10,000	21,059	0.47
7	6	18,000	21,583	0.83
8	7	23,000	22,107	1.04
9	8	38,000	22,631	1.68
10	9	12,000	23,155	0.52
11	10	13,000	23,679	0.55
12	11	32,000	24,203	1.32
13	12	41,000	24,727	1.66

Cell	Cell Formula
C2	=18439+A2*524
D2	=B2/C2

$$\bar{S}_t = \frac{D_t}{\bar{D}_t}$$

Figure 7-4 Deseasonalized Demand and Seasonal Factors for Tahoe Salt

Estimating seasonal factors

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- Seasonal factor for period t, $\bar{S}_t = \frac{D_t}{\bar{D}_t}$
- Number of cycles in data (r) = number of periods/ periodicity
- For the Tahoe salt example, we have 12 periods and p = 4, which gives r = 3
- We set the seasonal factors by averaging the period t seasonal factors with the same position in each cycle

$$S_i = \frac{\sum_{j=0}^{r-1} \bar{S}_{jp+i}}{r}, \text{for } i = 1,2,\ldots,p$$

Estimating seasonal factors

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$$S_i = \frac{\sum_{j=0}^{r-1} \bar{S}_{jp+i}}{r}$$

$$S_1 = \frac{(\bar{S}_1 + \bar{S}_5 + \bar{S}_9)}{3} = \frac{(0.42 + 0.47 + 0.52)}{3} = 0.47$$

$$S_2 = \frac{(\bar{S}_2 + \bar{S}_6 + \bar{S}_{10})}{3} = \frac{(0.67 + 0.83 + 0.55)}{3} = 0.68$$

$$S_3 = \frac{(\bar{S}_3 + \bar{S}_7 + \bar{S}_{11})}{3} = \frac{(1.15 + 1.04 + 1.32)}{3} = 1.17$$

$$S_4 = \frac{(\bar{S}_4 + \bar{S}_8 + \bar{S}_{12})}{3} = \frac{(1.66 + 1.68 + 1.66)}{3} = 1.67$$

Estimating seasonal factors

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$$F_{13} = (L + 13T)S_{13} = (18{,}439 + 13 \times 524)0.47 = 11{,}868$$
$$F_{14} = (L + 14T)S_{14} = (18{,}439 + 14 \times 524)0.68 = 17{,}527$$
$$F_{15} = (L + 15T)S_{15} = (18{,}439 + 15 \times 524)1.17 = 30{,}770$$
$$F_{16} = (L + 16T)S_{16} = (18{,}439 + 16 \times 524)1.67 = 44{,}794$$

Static forecast

- We can use the level, trend, and seasonal factors to use the static forecasting formula

$$F_{t+l} = [L + (t + l)T]S'_{t+l}$$

- This formula is static because the L, T, and S' are not dependent on time

Tahoe salt example

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- Go to R template and decompose time series ….

Moving average

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- Used when demand has no observable trend or seasonality

Systematic component of demand = level

- The level in period t is the average demand over the last N periods

$$L_t = (D_t + D_{t-1} + \ldots + D_{t-N+1}) \,/\, \mathrm{N}$$

$$F_{t+1} = L_t \quad \text{and} \quad F_{t+n} = L_t$$

- After observing the demand for period t + 1, revise the estimates

$$L_{t+1} = (D_{t+1} + D_t + \ldots + D_{t-N+2}) \,/\, N, \quad F_{t+2} = L_{t+1}$$

Simple moving average - Example

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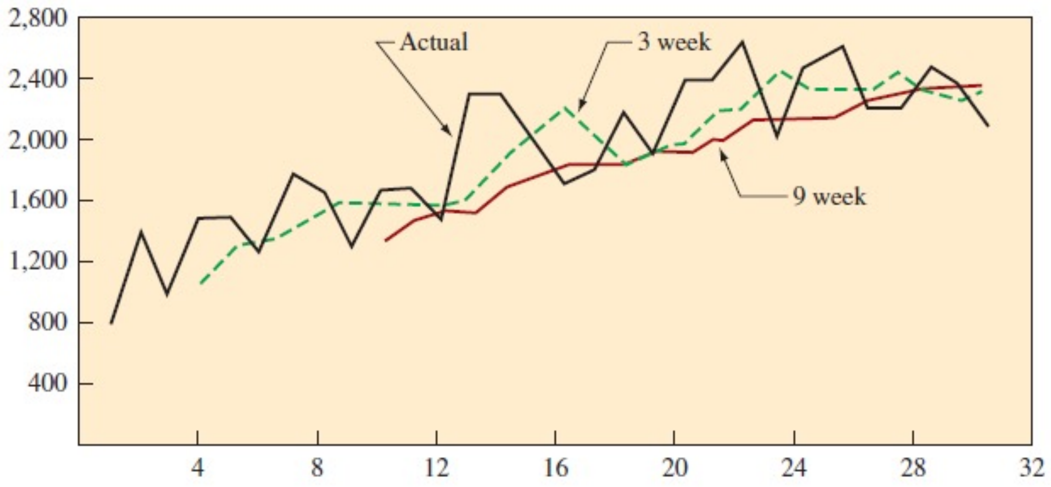
WEEK	DEMAND	3 WEEK	9 WEEK	WEEK	DEMAND	3 WEEK	9 WEEK
1	800			16	1,700	2,200	1,811
2	1,400			17	1,800	2,000	1,800
3	1,000			18	2,200	1,833	1,811
4	1,500	1,067		19	1,900	1,900	1,911
5	1,500	1,300		20	2,400	1,967	1,933
6	1,300	1,333		21	2,400	2,167	2,011
7	1,800	1,433		22	2,600	2,233	2,111
8	1,700	1,533		23	2,000	2,467	2,144
9	1,300	1,600		24	2,500	2,333	2,111
10	1,700	1,600	1,367	25	2,600	2,367	2,167
11	1,700	1,567	1,467	26	2,200	2,367	2,267
12	1,500	1,567	1,500	27	2,200	2,433	2,311
13	2,300	1,633	1,556	28	2,500	2,333	2,311
14	2,300	1,833	1,644	29	2,400	2,300	2,378
15	2,000	2,033	1,733	30	2,100	2,367	2,378



Simple Exponential Smoothing

- Used when demand has no observable trend or seasonality

  Systematic component of demand = level
- Initial estimate of level, L_0, assumed to be the average of all historical data

Simple exponential smoothing

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Given data for Periods 1 to *n*

$$L_0 = \frac{1}{n}\sum_{i=1}^{n} D_i$$

Revised forecast using smoothing constant $0 < \alpha < 1$

$$L_{t+1} = \alpha D_{t+1} + (1-\alpha)L_t$$

Thus

$$L_{t+1} = \sum_{n=0}^{t-1} \alpha(1-\alpha)^n D_{t+1-n} + (1-\alpha)^t D_1$$

Note that exponential smoothing is a weighted moving average

Current forecast

$$F_{t+1} = L_t \quad \text{and} \quad F_{t+n} = L_t$$

Winter's method

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- Appropriate when the systematic component of demand is assumed to have a level, trend, and seasonal factor

Systematic component = (level + trend) x seasonal factor

$$F_{t+1} = (L_t + T_t)S_{t+1} \quad \text{and} \quad F_{t+l} = (L_t + lT_t)S_{t+l}$$

- Note that the systematic component has a similar form to what we used for the static method but the level, trend, and seasonal factors are time dependent

Winter's method steps

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- Initial estimates of L_0, T_0, and $S_1, \dots, S_p$ obtained using the static approach

- After observing demand for period $t + 1$, revise estimates for level, trend, and seasonal factors

$$L_{t+1} = \alpha(D_{t+1}/S_{t+1}) + (1 - \alpha)(L_t + T_t)$$

$$T_{t+1} = \beta(L_{t+1} - L_t) + (1 - \beta)T_t$$

$$S_{t+p+1} = \gamma(D_{t+1}/L_{t+1}) + (1 - \gamma)S_{t+1}$$

α = smoothing constant for level

β = smoothing constant for trend

γ = smoothing constant for seasonal factor

Tahoe salt example

- Go to R template and forecast!

Forecast error and value analysis

Measures of Forecast Error

- Forecast errors contain valuable information and must be analyzed for two reasons:
    - Managers use error analysis to determine whether the current forecasting method is predicting the systematic component of demand accurately
    - All contingency plans must account for forecast error

Measuring forecast error

- Mean squares error and mean absolute deviation estimate a distribution for the random component (recall that forecasts have systematic and random components)

- Forecast error: $E_t = F_t - D_t$
- Mean squared error: $MSE_n = \frac{1}{n}\sum_{t=1}^{n} E_t^2$
- MSE intuition: penalizes large errors much higher than small errors

- Absolute deviation: $A_t = |E_t|$
- Mean absolute deviation: $MAD_n = \frac{1}{n}\sum_{t=1}^{n} A_t$
- MAD intuition: penalizes errors proportional to their size

Measuring forecast error contd…

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- Mean absolute percentage error: $MAPE_n = \frac{\sum_{t=1}^{n} \left|\frac{E_t}{D_t}\right| 100}{n}$
- MAPE measures forecast error relative to demand

- Bias: $bias_n = \sum_{t=1}^{n} E_t$
- Bias fluctuates around zero if the error is truly random

- Tracking signal: $TS_t = \frac{bias_t}{MAD_t}$
- Tracking signals outside +6 and -6 are signs of over-forecasting and under-forecasting, respectively

Tahoe salt example

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- Go to R template and compute forecast errors

Forecast value analysis (FVA)

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- Approach to identify steps in the forecasting process that add value and which steps do not by comparing against a baseline forecast
- Examples of situations where FVA may be useful:
  1. Planners A, B, and C forecast demand for three different product lines A, B, and C. As their manager, you want to determine who is adding value to the company
  2. Your company is considering the purchase of a million dollar forecasting system to replace the existing forecasting approach. You want to determine if this will actually improve forecasting accuracy

A naive (non-FVA) approach for situation 1

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- Compute mean absolute percentage error (MAPE) for each planner's forecast and rank them

	MAPE
Planner A – Product A	25%
Planner B – Product B	30%
Planner C – Product C	35%

- Is it fair to say give planner A a higher bonus based on MAPE?
- What are we missing?

A idealized FVA approach

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- Compare each planner's forecast against a baseline forecast which is simply the mean of the actual demand

	MAPE
Mean – Product A	20%
Mean – Product B	30%
Mean – Product C	40%

- Conclusions:
- Actual demand for product C is more variable than product A and B, that is, it is the hardest to forecast
- Planner A's forecast is worse than using the mean, Planner B's forecast adds no value, but Planner C adds value
- Takeaway: A naïve application of forecast error measures can be deceptive!

Practical baseline forecasts

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- Unfortunately, the mean of actual demand is unknown but there are a number of other baseline forecasts that can be used
- Last period actuals: Use the last periods actual demand as a forecast for the actual demand
- Moving average (e.g. Winter's method): Use a simple method such as Winter's method which is easy to implement as the baseline forecast

Challenge when applying FVA

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- Applying FVA requires that you have access to data before and after each step of the forecasting process. Companies may not track this data.
- Takeaway: It is important to gather data before and after each step of a business process so that objective performance measures can be tracked.
- With data storage and analytics booming, gathering data is fairly inexpensive