

**A Proof of Convergence for Non-Heuristic Systems: Fault Prediction and Localization In A
Spacecraft EPS**

by

Keerthana Srinivasan

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Aldershot Secondary School

Introduction

Advancements in deep space exploration have significantly influenced the development of space systems, particularly electronic subsystems. Space organizations increasingly demand electronics with reduced size, enhanced capacitance, and integration of artificial intelligence (AI) and machine learning (ML). This shift from single-core to embedded or multi-core systems has improved computational capabilities but has also introduced vulnerabilities. Embedded systems, essential for modern spacecraft, often fail in the harsh conditions of space, with 41% experiencing temporary or permanent malfunctions. In Earth orbit, redundancy strategies or limited mission lifespans can mitigate these failures. However, these solutions are inadequate for deep space missions, where greater autonomy and reliability are critical. Electronic reliability assessments currently rely on the Functional Methodology for Electronic Assessment (FMEA), a manual process with only a 55% success rate. This approach is impractical for future missions, where astronauts must independently assess and resolve faults without real-time support from Earth. The increased frequency of failures, combined with communication delays, makes traditional recovery options, such as redundancy, impractical for deep space exploration. To address this challenge, an automated system capable of rapidly and accurately detecting and localizing faults is imperative. The Extended Kalman Filter (EKF) has been successfully used in prior studies to identify current or voltage-based faults in electrical power systems (EPS). However, while the EKF is effective at fault detection, it lacks the capability to localize faults—a critical requirement for efficient resolution. Recent advancements in quantum computing present an opportunity to enhance fault localization in EPS. However, many proposed algorithms in the literature rely on adiabatic quantum computing (AQC), which employs heuristic approaches. Heuristic methods, while efficient for certain applications, cannot present definitive solutions, making them unsuitable for precise fault localization aerospace frameworks. This proposal outlines an algorithm combining the strengths of the EKF, bifurcation modeling, and Quantum Markov Chain Monte Carlo (QMCMC) to detect and localize faults in spacecraft EPS. By leveraging QMCMC's ability to explore the entire state space and efficiently converge on a target distribution, this approach aims to provide a robust, non-heuristic solution for fault localization. This integration will enhance the reliability and autonomy of electronic systems in deep space missions, paving the way for future exploration.

Mathematical Framework

This section will aim to explain how an EKF, bifurcation model, and QMCMC can interact with one another mathematically.

Integration of EKF, Bifurcation, and QMCMC

To begin, a bifurcation model is implemented to observe the system dynamics of the spacecraft EPS, in addition to being able to detect potential for faults through Jacobian eigenvalues. To observe the system dynamics of the EPS, particularly regarding voltage, current, capacitance, resistance, and inductance, equations (1) and (2) were implemented. Any spacecraft EPS is governed by system equations (1) and (2), where equation (1) is the derivative of bus current values and equation (2) is the derivative of bus voltage values.

$$\frac{d_{i_s}}{dt} = \frac{1}{L_{eq}} (V_{ref} - v_{bus} - R_{eq} i_s) = f(i_s, v_{bus}) \quad (1)$$

$$\frac{d_{v_{bus}}}{dt} = \frac{1}{C} (i_s - \frac{v_{bus}}{R} - \frac{P}{v_{bus}}) = g(i_s, v_{bus}) \quad (2)$$

By using the 4th Order Runge-Kutta method, as shown in equations (3a-f), the bus current and bus voltage can be calculated and iterated upon in time steps of 0.1 seconds.

$$k_1^i = f(i_s, v_{bus}), k_1^v = g(i_s, v_{bus}) \quad (3a)$$

$$k_2^i = f(i_s + \frac{\Delta t}{2} \cdot k_1^i, v_{bus} + \frac{\Delta t}{2} \cdot k_1^v), k_2^v = g(i_s + \frac{\Delta t}{2} \cdot k_1^i, v_{bus} + \frac{\Delta t}{2} \cdot k_1^v) \quad (3b)$$

$$k_3^i = f(i_s + \frac{\Delta t}{2} \cdot k_2^i, v_{bus} + \frac{\Delta t}{2} \cdot k_2^v), k_3^v = g(i_s + \frac{\Delta t}{2} \cdot k_2^i, v_{bus} + \frac{\Delta t}{2} \cdot k_2^v) \quad (3c)$$

$$k_4^i = f(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v), k_4^v = g(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v) \quad (3d)$$

$$i_s = i_s + \frac{\Delta t}{6} (k_1^i + 2k_2^i + 2k_3^i + k_4^i) \quad (3e)$$

$$v_{bus} = v_{bus} + \frac{\Delta t}{6} (k_1^v + 2k_2^v + 2k_3^v + k_4^v) \quad (3f)$$

The following step after each Runge-Kutta computation is to analyze system stability through an eigenvalue analysis. These eigenvalues represent the trajectory of instability in the spacecraft EPS over time. Given a Jacobian J , negative eigenvalues would indicate instability is decreasing or approaching 0. In contrast, if $J > 0$, the EPS is approaching instability. J is represented in equation (4), where L_{eq} , R_{eq} , and C are the equivalent inductance, equivalent resistance, and capacitance, respectively.

$$J = \begin{bmatrix} -\frac{R_{eq}}{L_{eq}} & -\frac{1}{L_{eq}} \\ \frac{1}{C} & \frac{1}{C} \left(\frac{P}{v_{bus}^2} - \frac{1}{R} \right) \end{bmatrix} \quad (4)$$

Equivalent inductance and resistance are used to simplify the EPS being mathematically modelled, acting as a summation of all inductors and resistors on the microgrid. L_{eq} and R_{eq} are calculated using equations (5) and (6). In this scenario, both inductors and resistors are assumed to be in parallel.

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots \frac{1}{L_{NL}}, \quad NL = 15 \quad (5)$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots \frac{1}{R_{NR}}, \quad NR = 25 \quad (6)$$

The eigenvalues of J are solutions of equation (7a), which is a function of det , which can become the quadratic formula depicted in (7b) when solving for eigenvalues λ .

$$det(J - \lambda I) = 0 \quad (7a)$$

$$\lambda_{1,2} = \frac{-\frac{R_{eq}}{L_{eq}} + \frac{1}{C}(\frac{P}{v_{bus}^2} - \frac{1}{R})}{2} \pm \frac{1}{2} \sqrt{\left(-\frac{R_{eq}}{L_{eq}} + \frac{1}{C}(\frac{P}{v_{bus}^2} - \frac{1}{R})\right)^2 - 4detJ} \quad (7b)$$

If both λ_1 and λ_2 are negative real numbers, the system is stable or approaching stability. However, if any or both roots have positive real numbers, the system is approaching instability.

Moreover, as i_s and v_{bus} are calculated through Runge-Kutta, the Jacobian eigenvalues can be reevaluated to ensure stability. Instability can occur at three points: if P exceeds the maximum threshold, v_{bus} becomes too small, or R_{eq} becomes infinitely small. P is directly dependent on v_{bus} because more intuitively, $P = v_{bus} i_s$, and $P = P_L - P_{PPV}$.

In addition to i_s and v_{bus} being updated in time steps, P_{PPV} can also be updated in timesteps if a spacecraft is orbiting a celestial body. However, please note that evaluating P_{PPV} is optional, and if a spacecraft is travelling through interstellar space, may not be necessary. For the purposes of this proposal, sunlight $S = 1370 \text{ W/m}^2$, while $P_m = 368 \text{ W}$. The efficiency of a single solar cell η can be calculated using equation (8).

$$\eta = \frac{P_m^2}{S} \quad (8)$$

The solar flux or irradiance ϕ measures the amount of sunlight received one astronomical unit away from the Sun. This is calculated as a product of the solar constant S_{sol} and the cosine of the incidence angle θ , as shown in equation (9).

$$\phi = S_{sol} \cos(\theta) \quad (9)$$

The power produced by a single solar cell P_{cell} is shown in equation (10), where A_{sc} is the area of the solar cell and l represents losses. Finally, PPV can be calculated using equation (11).

$$P_{cell} = \phi \cdot \eta \cdot A_{sc} (1 - l) \quad (10)$$

$$P_{PPV} = n \cdot P_{cell} \quad (11)$$

In time steps, P_{PPV} would be updated based on change in the angle of incidence over time.

In the event where a fault occurs, and is sensed by the bifurcation model, the EKF can detect the type of fault that is occurring. This is done through calculating the residual—the difference between the directly measured data and the data predicted by the EKF. This is usually done through measuring and calculating the predicted state of i_s and v_{bus} . The variance in the residual over time—measured through whiteness, mean, and covariance tests—are an indicator of whether or not there is a fault present. A fault present (e.g., high impedance fault) would cause the residual to vary over time t . The calculation for the predicted state vector $\hat{x}_{i|i-1}$ is represented in equation (12) and the calculation for the measurement state vector z_i is represented in equation (13).

$$\hat{x}_{i|i-1} = \Theta_i x_i + G_i u_i + \Gamma_i w_i \quad (12)$$

$$z_i = H_i x_i + v_i \quad (13)$$

Where Θ is the state transition matrix, or the fundamental dynamics influencing x_i without external influence, G is the input distribution matrix, or changes in external influence that may impact Θ , and Γ represents noise distribution or unexpected behaviours to account for (e.g., solar flare). H_i is the output matrix. u_i is a control input that remains constant, but is scaled by G_i , and w_i is assumed unpredictability in the state transition equation, typically modelled as $w_i \sim \mathcal{N}(0, 1)$. Γ works to scale w_i and weigh the impact of w_i on different variables. v_i is a vector of random measurement errors.

The true predicted state $z_{i|i-1}$ is essentially a version of $\hat{x}_{i|i-1}$ laid out in the format of matrix H_i , as shown in equation (14). Residual V_i can be calculated as the difference between z_i and $z_{i|i-1}$ in equation (15).

$$z_{i|i-1} = H_i \hat{x}_{i|i-1} \quad (14)$$

$$V_i = z_i - z_{i|i-1} \quad (15)$$

Furthermore, the Kalman gain—from a more intuitive perspective—is the ratio between the level of uncertainty present in the predicted state versus the measured state. To calculate the Kalman gain represented in equation (17), the predicted error variance should first be estimated, as shown in equation (16).

$$P_{i|i} = \Phi_i P_{i-1} \Phi_i^T + Q_i \quad (16)$$

$$K_i = P_i - H_i^T \cdot (H_i P_i - H_i^T + R_i)^{-1} \quad (17)$$

Finally, the ultimate predicted state x_i can be found through adding the product of the residual and Kalman gain to z_i .

$$x_{i|i} = z_i + K_i V_i \quad (18)$$

Estimating the variance of the residual V_i was done through whiteness, mean, and covariance tests. While these tests will not be covered in this section, they will be explained briefly. Whiteness tests intend to assess if a situation carries heteroscedasticity, or change in variance due to exogenous variables. Mean tests observe if the mean of the residual variance is 0 or close to 0. Finally, covariance will address the variance between different residual values over time. Overall, each of these tests aim to observe if the variance is constant, or not. If the variance in the residual is not constant, that could indicate abnormal noise and disturbances caused by a fault.

If the variance of V_i is changing over time, the immediate next step would be to enter the QMCMC framework. QMCMC will allow for fault localization through assessing countless permutations of nodes in the EPS before reaching convergence. The benefit of using a quantum algorithm in this case is superposition, or the ability to observe all nodes at once. There is the added benefit of entanglement, or the ability to observe correlations between different nodes in order to assess fault propagation.

QMCMC is built upon the classical Ising model, which is where there are various spin states s present, where $s = (s_1, s_2, \dots, s_n)$, containing a group of n spin states where $s_j = \pm 1$. These spin states can be representative of faulty (-1) or healthy (+1) nodes dependent on internal (e.g., difference in predicted and actual current) and external (e.g., solar irradiation) components pertaining to the spacecraft's environment. Through the energy function $E(s)$, states within s can be combined to form a singular environmental variable.

$$E(s) = \sum_{j>k=1}^n J_{jk} s_j s_k - \sum_{j=1}^n h_j s_j \quad (19)$$

J_{jk} and h_j make up the problem specification. J_{jk} is representative of the relationship between s_j and s_k . When $J_{jk} > 0$, spins s_j and s_k will prefer to align at +1, while when $J_{jk} < 0$, s_j and s_k will prefer to align at -1. Meanwhile, h_j indicates the bias of s_j . If $h_j > 0$, s_j will bias towards +1, while if $h_j < 0$, s_j will bias towards -1.

In order to converge on the correct Boltzmann distribution $\mu(s)$, QMCMC—similar to classical Markov chains—will traverse random points on $E(s)$ to observe various Boltzmann distributions. The goal is to do this at relatively low temperatures $T \rightarrow 0$, in order to preserve computational power. However, to do this at a speed akin to classical computation, only the local minima of $E(s)$ will be explored. To ensure the Boltzmann distribution is normalized for (0,1), the partition function Z is employed, where

$$Z = \sum_s \exp(-E(s)/T) \quad (20)$$

Moreover, selecting the transition probability A applies a well-known formula that has been applied in classical computing through Boltzmann Machine Training and Interference.

$$A(s'|s) = \min(1, \frac{\mu(s')}{\mu(s)}) \quad (21)$$

Essentially, the next state will be s' with probability A and s with probability $1 - A$. Depending on which is greater, the QMCMC model will stay in its current position along $E(s)$ or transfer to another state. $\frac{\mu(s')}{\mu(s)}$ can be mathematically simplified as $\exp(-\Delta E/T)$. Transition between states can be done in a state of superstition, such that it is done almost instantaneously.

Eventually, the Boltzmann distribution will reach convergence, signifying a final probability distribution that can be used to determine the probability of which a fault is occurring at a certain node s_j . Moreover, the correlation between nodes J_{jk} can be implemented as a way for observing fault propagation. Both of these values, when shown to an astronaut, can be used to figure out where a fault needs to be resolved.