AN EXPLORATION OF THE EFFECTIVENESS AND CONVERGENCE FOR A NON-HEURISTIC QUANTUM FAULT LOCALIZATION ALGORITHM APPLIED ON A SPACECRAFT EPS

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ABSTRACT. We prove that instability of a fault can be detected using the bifurcation method. Additionally, we prove that classification of various faults can be completed using the variance of the residual in an Extended Kalman Filter over 100 iterations. Finally, we prove the irreducibility and aperiodicity of a Quantum Markov Chain with a transition matrix P for a problem specification j_{jk} and h_{jk}.

Throughout this exploration, we will be solving for a high impedance fault in a EPS with 10 nodes.

0.1. Bifurcation to Detect Instability.

- $L_{eq} = 5$
- $R_{eq} = 5$ or $40\,\Omega$
- $V_{ref} = 12\,\mathrm{V}$
- $v_{bus} = 12\,\mathrm{V}$
- $i_s = 2\,\mathrm{A}$
- $C = 238\,\mu\mathrm{F}$
- $P = 5.8\,\mathrm{W}$
- $R = 106\,\Omega$
- $\Delta t = 0.1\,\mathrm{s}$

Definition 1. *Let $f(i_s, v_{bus})$ be i_s' and $g(i_s, v_{bus})$ be v_{bus}', where*
$f(i_s, v_{bus}) = \frac{1}{L_{eq}}(V_{ref} - v_{bus} - R_{eq} i_s)$
and
$g(i_s, v_{bus}) = \frac{1}{C}(i_s - \frac{v_{bus}}{R} - \frac{P}{v_{bus}})$

Therefore, we can use the fourth order Runge Kutta method to integrate i_s and v_{bus} over a time step of 0.1s. This will allow us to observe instability calculated when R_{eq} increases from 5 to 40 Ω, causing a high impedance fault. Please note that an exploration by Carbone et al. (2016) has covered this particular topic before.

Let us first calculate i_s and v_{bus} for when a system is stable. Here, assume $R_{eq} = 5\Omega$

$k_1^i = f(i_s, v_{bus}) = 2$

$k_2^i = f\left(i_s + \frac{\Delta t}{2} k_1^i, v_{bus} + \frac{\Delta t}{2} k_1^v\right) = -1.0005897$

$k_3^i = f\left(i_s + \frac{\Delta t}{2} k_2^i, v_{bus} + \frac{\Delta t}{2} k_2^v\right) = -1.49987475$

$k_4^i = f\left(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v\right) = -0.5009$

$k_1^v = g(i_s, v_{bus}) = 0.005897$

$k_2^v = g\left(i_s + \frac{\Delta t}{2} k_1^i, v_{bus} + \frac{\Delta t}{2} k_1^v\right) = 0.001696$

$k_3^v = g\left(i_s + \frac{\Delta t}{2} k_2^i, v_{bus} + \frac{\Delta t}{2} k_2^v\right) = 0.003795$

$k_4^v = g\left(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v\right) = -4.04616 \times 10^{-4}$

$$i_s = i_s + \frac{\Delta t}{6}\left(k_1^i + 2k_2^i + 2k_3^i + k_4^i\right) = 0.7497$$
$$v_{bus} = v_{bus} + \frac{\Delta t}{6}\left(k_1^v + 2k_2^v + 2k_3^v + k_4^v\right) = 12.002746$$

Now, consider the Jacobian $J = \begin{bmatrix} -\frac{R_{eq}}{L_{eq}} & -\frac{1}{L_{eq}} \\ \frac{1}{C} & \frac{1}{C}(\frac{P}{v_{bus}^2} - \frac{1}{R}) \end{bmatrix}$.

Definition 2. *Carbone et al. (2016), through an empirical study, found that the trace and determinant of J are crucial in assessing voltage stability of a dynamic or parametric EPS, where:*

$$\mathrm{tr}(J) = a + d$$
$$\det(J) = |J| = ad - bc$$

In the case of system is considered to have voltage stability if $\det(J) > 0$ *and* $\mathrm{tr}(J) < 0$. *If either of these conditions are not met, the system exhibits instability.*

$$\begin{aligned}
\mathrm{tr}(J) &= a + d \\
&= -\frac{5}{5} + \frac{1}{238}\left(\frac{5.8}{144.0659115} - \frac{1}{106}\right) \\
&= -1 + \frac{1}{238}\left(\frac{5.8}{144.0659115} - \frac{1}{106}\right) \\
&\approx -0.9999
\end{aligned}$$

$$\begin{aligned}
\det(J) &= ad - bc \\
&= \left(-\frac{5}{5}\right)\left(\frac{1}{238}\left(\frac{5.8}{144.0659115} - \frac{1}{106}\right)\right) - \left(-\frac{1}{5}\right)\left(\frac{1}{238}\right) \\
&\approx 7.1082 \times 10^{-4}
\end{aligned}$$

$$\boxed{\because\ \mathrm{tr}(J) < 0 \text{ and } \det(J) > 0, \therefore \text{ the system is stable, and there is no fault.}}$$

Now, let us explore the same scenario, but in the event of a high impedance fault, where $R_{eq} = 40\Omega$.

$$k_1^i = f(i_s, v_{bus}) = -16 \qquad k_1^v = g(i_s, v_{bus}) = 0.005897$$
$$k_2^i = f\left(i_s + \tfrac{\Delta t}{2}k_1^i, v_{bus} + \tfrac{\Delta t}{2}k_1^v\right) = 47.9994 \qquad k_2^v = g\left(i_s + \tfrac{\Delta t}{2}k_1^i, v_{bus} + \tfrac{\Delta t}{2}k_1^v\right) = -0.02772$$
$$k_3^i = f\left(i_s + \tfrac{\Delta t}{2}k_2^i, v_{bus} + \tfrac{\Delta t}{2}k_2^v\right) = -207.9948 \qquad k_3^v = g\left(i_s + \tfrac{\Delta t}{2}k_2^i, v_{bus} + \tfrac{\Delta t}{2}k_2^v\right) = 0.10673$$
$$k_4^i = f\left(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v\right) = 1647.9371 \qquad k_4^v = g\left(i_s + \Delta t \cdot k_3^i, v_{bus} + \Delta t \cdot k_3^v\right) = -0.86802 \times 10^{-4}$$
$$i_s = i_s + \tfrac{\Delta t}{6}(k_1^i + 2k_2^i + 2k_3^i + k_4^i) = 220.6578$$
$$v_{bus} = v_{bus} + \tfrac{\Delta t}{6}(k_1^v + 2k_2^v + 2k_3^v + k_4^v) = 11.8826$$

$$\begin{aligned}\det(J) &= ad - bc \\ &= \left(-\frac{40}{5}\right)\left(\frac{1}{238}\left(\frac{5.8}{141.1962} - \frac{1}{106}\right)\right) - \left(-\frac{1}{5}\right)\left(\frac{1}{238}\right) \\ &\approx -2.2332 \times 10^{-4}\end{aligned}$$

$$\boxed{\because\ \det(J) < 0,\ \therefore\ \text{the system is instable, and there is a fault present.}}$$

These results suggest that the bifurcation method can serve as a useful tool for detecting instability in spacecraft EPS, particularly during deep space operations.

0.2. **Extended Kalman Filter to Classify Faults.** Through the same givens as in 0.1, our goal here is to not just detect, but classify a high impedance fault as high impedance. The following content will show how the variance of the residual found in an Extended Kalman Filter (EKF) can be used to calculate faults.

Let us show that every fault-involving a new range of variable changes-will lead to a different residual variance. To begin, let us observe how the residual variance changes due to a unique input matrix G.

G is the input distribution matrix used in calculating the predicted state vector $\hat{x}_{i|i-1}$.

$$\hat{x}_{i|i-1} = \Theta x_i + G u_i$$

Where Θ is the state transition matrix, G is the input distribution matrix considering factors not covered through Θ, and u_i is a control input that remains constant. Here Θ is a 2×2 identity matrix, G is a 2×8 matrix observing how each variable involved in our system equations impact $f(i_s, v_{bus})$ and $g(i_s, v_{bus})$. This can be done through calculating the covariance between each variable and $f(i_s, v_{bus})$ and $g(i_s, v_{bus})$. We can perform these calculations easily using `extract_matrix_values.py`, yielding the following matrix G:

$$G = \begin{bmatrix} 0.0779 & -476.7329 & 0.0645 & -2.1031 & -2.2038 & -0.0233 & 0.0007 & -0.0042 \\ -0.0029 & 0.6928 & 0.0089 & 0.0117 & 0.0093 & 0.1123 & -0.0037 & 0.0208 \end{bmatrix}$$

Corollary 1. *Given that the state transition matrix Θ is an identity matrix and the control input u_i is constant, the input distribution matrix G is the sole determinant of variation in the predicted state $\hat{x}_{i|i-1}$, and consequently in the residual. Specifically:*

$$|\{G_{ij} \mid i = 1, \ldots, m, j = 1, \ldots, n\}| = m \cdot n$$

This means that every potential entry in G is unique. Thus, the variation introduced by G directly influences the residual, and different fault types will result in distinct residuals and, consequently, distinct residual variances.

Moreover, we can explore the consistency of residual variances when classifying for high impedance faults, showing potential for residual variance to be used in fault classification in the future. Since high impedance faults in this scenario are classified by unusually high R_{eq} in a range of instability, we can calculate residual variance over multiple trials from the range of 20-60Ω. Note that every R_{eq} was selected at random.

R_{eq}	Residual Variance
37	0.1277
25	0.1720
40	0.0929
56	0.1076
45	0.1197

Here, there is a sample standard deviation of only 0.02988, meaning that residual variances for high impedance faults are placed in the 95% confidence range.

$\boxed{\therefore \text{ the variance of the residual can be used in classifying high impedance faults.}}$