Activity-2

Q1) Let A be the set of all triangles in a plane and let R be a relation if it is reflexive Symmetric and transitive show that R is an equilence relation in A

The relation satisfies the following propesties

(i) Reflexivity

let Δ be arbitary triangle A then

$\Delta \cong \Delta \Rightarrow (\Delta, \Delta) \in R$ for all values of Δ in A.

$\therefore$ R is Reflexive

(ii) Symmetry

let $\Delta_1, \Delta_2 \in A$ such that $(\Delta_1, \Delta_2) \in R$ then

$(\Delta_1, \Delta_2) \in R \Rightarrow \Delta_1 \cong \Delta_2$

$\Rightarrow \Delta_2 \cong \Delta_1$

$\Rightarrow (\Delta_2, \Delta_1) \in R$

$\therefore$ R is symmetric

(iii) Transitivity

let $\Delta_1, \Delta_2, \Delta_3 \in A$ such that (Δ_1, Δ_2) and $(\Delta_2, \Delta_1) \in R$

the $(\Delta_1, \Delta_2) \in R$ and $(\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1 \cong \Delta_2 \quad (\Delta_2, \Delta_3) \in R$

$\Rightarrow \Delta_1 \cong \Delta_3$

$\Rightarrow (\Delta_1, \Delta_3) \in R$ $\therefore$ R is transitive

Thus R is reflexive symmetric and transitive

Hence, R is an equivalence relation

Q2 let A be the set of all line in x-y plane and let R be a relation in A, defined by

$R = \{(L_1, L_2) : L_1 \parallel L_2\}$

Show that R is an equivalence relation in A $y = 3x + 5$

The given relation satisfies the following properties

(i) Reflexivity

let be an arbitary line in A, then

$L \parallel A \Rightarrow (L, L) \in R \ L \in A$

Thus R is Reflexive

(ii) Symmetry

let $L_1, L_2 \in A$ such that $(L_1, L_2) \in R$ then

$(L_1, L_2) \in R \Rightarrow L_1 \parallel L_2$

$\Rightarrow L_2 \parallel L_1$

$\Rightarrow (L_2, L_1) \in R$

$\therefore$ R is symmetric

iii) Transitivity

let $L_1, L_2, L_3 \in A$ such that $(L_1, L_2) \in R$ and $(L_2, L_3) \in R$ then (L_1, L_2) and $(L_2, L_3) \in R$

$\Rightarrow L_1 \parallel L_2$ and (L_2, L_3)

$\Rightarrow L_1 \parallel L_3$

$\Rightarrow (L_1, L_3) \in R$

$\therefore$ R is transitive

Thus R is reflexive, symmetric and transitive there equivalence relation

Q3. Let S be the set of all real number and let R be a relation S defined by $R = \{(a,b) : a \le b^2\}$

① Non reflexivity

clearly, $1/2$ is a real number and $1/2 \le (\frac{1}{2})^2$ is not true

$\therefore (1/2, 1/2) \notin R$

Hence, R is not reflexive

ii) Non symmetry

consider the real numbers $1/2$ and 1

clearly $1/2 \le 1^2 \Rightarrow (1/2, 1) \in R$

But $1 \le 1/2$ is not true and so $(1, 1/2) \notin R$

Thus $(1/2, 1) \in R$ but $(1, 1/2) \notin R$

Hence R is not symmetric

iii) Non transitivity

consider the real numbers 2, -2 and 1, clearly $2 \le (-2)^2$

& $-2 < (1)^2$ but $2 \le 1^2$ is not true, Thus $(2, -2) \in R$

and $(-2, 1) \in R$ but $(2, 1) \notin R$

Hence R is not transitive

$\Rightarrow$ Equivalence class and partitions

Q4 Which of these collection of subset are partitions of

$S = \{-3, -2, -1, 0, 1, 2, 3\}$

(a) $\{-3, -1, 1, 3\}, \{-2, 0, 2\}$

(b) $\{-3, -2, -1, 0\} \{0, 1, 2, 3\}$

(c) $\{-3, 3\}, \{-2, 2\}, \{-1, 1\}, \{0\}$

(d) $\{-3, -2, 2, 3\}, \{-1, 1\}$

Sol:- $S_1 = \{-3, -1, 1, 3\}$ and $S_2 = \{-2, 0, 2\}$

$S_1 \cap S_2 = \phi$ and $S_1 \cup S_2 = \{-3, -1, 1, 3\} - 2, 0, 2\} = S$

Also $S_1 \neq \phi$ and S_2

By this defination of Pastition, the given collection of subset is a pastition

$\{-3, -1, 1, 3\} \cap \{-2, 0, 2\} = \phi$ (yes)

(b) $S_3 = \{-3, -2, -1, 0\}$ and $S_4 = \{0, 1, 2, 3\}$

$S_3 \cap S_4 \neq \phi$

Therefore, the given collection of subset is not a Pastition

(c) $S_5 = \{-3, -2\}$, $S_6 = \{-2, 2\}$, $S_7 = \{-1, 1\}$, $S_8 = \{0\}$

$S_5 \cap S_6 \cap S_7 \cap S_8 = \phi$ (yes)

$S_5 \cup S_6 \cup S_7 \cup S_8 = \{-3, 3, -2, 2, -1, 1, 0\} = S$

Also $S_5 \neq 0$, $S_6 \neq 0$, $S_7 \neq \phi$, $S_8 \neq \phi$

By the definition of pastition the given collection of subset is a pastition

(d) $S_9 = \{-3, -2, 2, 3\}$ and $S_{10} = \{-1, 1\}$

$S_9 \cap S_{10} = \phi$ and $S_9 \cup S_{10} = \{-3, -2, 2, 3, -1, 1 \neq S$

the given collection of subset is not a pastition

Q5. Show that the relation R on the set of all bit strings such that $\leq$ if S and t contain the Same numbes of 1s is an equivalence relation.

A = set of all bit string

$R = \{(s, t) \mid$ s and t have the same number of is$\}$

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(i) Reflexivity $\therefore \forall s \in A \ (s,s) \in R$
$(s,s) \in R$ means s & s have same number of 1s.
$\therefore$ R is Reflexive

ii) symmetry $\forall s\ t \in A\ [(s,t) \in R \rightarrow (t,s) \in R]$
$(s,t) \in R$ means s & t have same number of 1s
$(t,s) \in R$ means t and s have same number of 1s
$\therefore$ R is Symmetric

(iii) Transitivity $\forall s, t, t \in A\ ((s,t) \in R \wedge (t,u) \in R) \rightarrow$
$(s,u) \in R$ if $(s,t) \in R \wedge (t,v) \in R$ then $(s,v) \in R$
because
s and t have the same number of 1s t and u
have the same number of 1s $\therefore$ then it is
abvious that s and u have the same number of
1s

$\therefore$ R is transitive.

Q6) Show that the function $f: N \rightarrow N$ defined by
$f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$
is one-one and onto

Sol:- suppose $f(x_1) = f(x_2)$
case 1:- when x is odd and x_2 is even
in this case $f(x_1) = f(x_1) \Rightarrow x_1 + 1 = x_2 - 1$
$\Rightarrow x_2 - x_1 = 2$
This a a controdiction, since the difference b/w
an odd integer and an even integer can never be
2
In this case $f(x_1) \neq x_2)$

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Similarly when x_1 is even and x_2 is odd then
$f(x_1) \neq x_2$

Case 2:- when x_1 and x_2 are odd
In this case $f(x_1) = f(x_2) \Rightarrow x+1 = x_2 - 1$
$\Rightarrow x_1 = x_2$

$\therefore$ f is one - one

Case 3:- when x_1 and x_2 are both eve
in this case $f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$
$\Rightarrow x_1 = x_2$

$\therefore$ f is one - one

in order to show that f is onto, let $y \in N$:

case 1:- when y is odd
in this case $(y+1)$ is even
$\therefore f(y+1) = (y+1) - 1 = y$

case 2:- when y is even
in this case $(y-1)$ is even
$f(y-1) = y-1+1 = y$

Thus each $\in N$ (co-domain of f) has it's pre-image in dom (f)
$\therefore$ f is onto

Q7: Show that $f: N \to N$ defined by
$f(x) \begin{cases} \frac{n+1}{2}, & f\ n \text{ is odd} \\ n/2 & f\ n \text{ is even} \end{cases}$

Many - one onto function

We have

$f(1) = \frac{1+1}{2} = 2/2 = 1$ and
$f(2) = 2/2 = 1$

Thus $f(1) = f(2)$ while $1 \neq 2$

$\therefore$ f is Many one

in order to show that if is onto, consider an arbitary element $n \in N$

if n is odd then 2n is even and $f(2n) = \frac{2n}{2} = n$

Thus, for each $n \in N$ (whether even(or) odd) there exists its Pre-image in N

$\therefore$ f is onto

Hence, f is many-one onto

Q8. let $A = R - \{3\}$ and $B = R - \{1\}$

let $f: A \to B$: $f(x) = \frac{x-2}{x-3}$ for all values of $x \in A$, show that f is one-one and onto

f is one-one since

$f(x_1) = f(x_2) \Rightarrow \frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_3 - 3}$

$\Rightarrow (x_1 - x_2)(x_3 - 3) = (x_1 - 3)(x_2 - 2)$

$\Rightarrow x_1 x_2 - 3x_1 - 2x_2 + 6 = x_1 x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow x_1 = x_2$

let $y \in R$ such that $y = \frac{x-2}{x-3}$

then $(x-3)y = (x-2) \Rightarrow x = \left(\frac{3y-2}{y-1}\right)$

clearly x is defined when $y \neq 1$

Also $x = 3$ will given as $1 = 0$, which is false

$x \neq 3$

And, $f(x) = \frac{\left(\frac{3y-1}{y-1} - 2\right)}{\left(\frac{3y-2}{y-1} - 3\right)} = y$

Thus for each $y \in B$, there exists $x \in A$ such that
$f(x) = y$ $\therefore f$ is onto
Hence f is onto one-one

Q9 Let A and B be two non-empty set that
$f(x) = y$ functions $f = (A \times B) \to (B \times A) : f(b, a)$
is a bijective function

f is one-one since
$f(a_1, b_1) = f(a_2, b_2) \Rightarrow (b_1, a_1) = (b_2, a_2)$
$\Rightarrow (a_1 = a_2)$ and $(b_1 = b_2)$
$\Rightarrow (a_1, b_1) = (a_2, b_2)$

in order to show that f is onto, let (b, a) be an orbitary element of $(B \times A)$
Then $(b, a) \in (B \times A)$
$\Rightarrow b \in A$ and $a \in A$
$\Rightarrow (a, b) \in (A \times B)$
Thus, for each $(b, a) \in (B \times A)$, there exists $(a, b) \in A \times B$ such that
$f(a, b) = (b, a)$
$\therefore f$ is onto
Thus f is one-one onto and and hence bijective

Q10: consider function $f : x \to y$ and define a relation R in x by $R = \{(a, b) : f(a) = f(b)$ that R is an equivalence Relation
(1) Reflexivity
Let $a \in x$ then
$f(a) = f(a) \Rightarrow (a, a) \in R$
R is reflexive

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(ii) Symmetry

Let $(a,b) \in R$ then

$(a,b) \in R \Rightarrow f(a) = f(b) \Rightarrow f(b) = f(a)$

$\Rightarrow (b,a) \in R$

(iii) Transitivity

Let $(a,b) \in R$ and $(b,c) \in R$ then

$(a,b) \in R$, $(b,c) \in R$

$f(a) = f(b)$ & $f(b) = f(c)$

$\Rightarrow (a,c) \in R$

R is transitive