PAGE EDGE

DATE: / /

# Activity - 3

Q1. Prove by PMI that $1+2+3+ \dots + n = \frac{n(n+1)}{2}$

Let $P(n)$ be the statement that $1+2+3+ \dots + n = \frac{n(n+1)}{2}$

Step (1) put $n=1$

$L.H.S = 1$

$R.H.S = \frac{1(1+1)}{2}$

$\therefore L.H.S = R.H.S$

$\therefore P(1)$ is true

step(ii) Assume that $P(n)$ is true for $n=k$

Let $P(k)$ is true

$1+2+3+ \dots + k = \frac{k(k+1)}{2}$

step(iii) Now, we have to prove that $f(n)$ is true for $n=(k+1)$ L.H.S of $P(k+1)$

$1+2+3 \dots + k + k - 1 = \frac{(k+1)(3k+2)}{2}$

Now,

$1+2+3+ \dots + k + k + 1$

$= \frac{k(k+1)}{2} + (k+1$

$= (k+1)\left[\frac{k}{2}+1\right]$

$= \frac{(k+1)(k+2)}{2}$ R.H.S

$\therefore P(n)$ is true for $n \in N$

$\therefore$ By PMI $P(A)$ is true $\forall n \in N$

Proved

Scanned by CamScanner

Q2. Prove by PMI that $1^2+2^2+3^3+ \dots +n^3 = \frac{n(n+1)(2n+1)}{6}$

Let $P(n)$ be the statement that $1^2+2^2+3^2+ \dots +n^2 = \frac{n(n+1)(2n+1)}{6}$

step 1) Inductive base

Put $n=1$

$L.H.S = 1^2 = 1$

$R.H.S = \frac{1(1+1)(2\times1+1)}{6}$

$= \frac{1\times2\times3}{6} = \frac{6}{6} = 1$

$\therefore$ L.H.S = RHS

$\therefore$ $P(1)$ is true

step(ii) Inductive hypothesis

Assume that $P(n)$ is true for $n=k$

$1^2+2^2+3^2+ \dots + k^2 = \frac{k(k+1)(2k+1)}{6}$

$\therefore$ $P(k)$ is true

Step (iii) Inductive step

we have to prove that $P(n)$ is true for $n=(k+1)$

$1^2+2^2+3^2+ \dots +(k+1)^2 = \frac{k+1)(k+2)(2k+3)}{6}$

L.H.S

$= 1^2+2^2+3^2+ \dots +k^2+(k+1)^2$

$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2$ {from step(ii)}

$= (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$

$= (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$

$= (k+1)\left[\frac{2k^2+4k+3k+6}{6}\right]$

$= (k+1)\left[\frac{(k+2)(2k+3)}{6}\right] = R \cdot H \cdot S$

$\therefore$ L.H.S = R.H.S

$\therefore$ P(n) is true for $n \in N$ Proved

Q3. Prove by PMI that $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots \frac{1}{n(n+1)} = \frac{n}{n+1}$

let P(n) be the statement that $\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{n(n+1)} = \frac{n}{n+1}$

Step (i) Inductive base

Put n = 1

L.H.S $= \frac{1}{1\cdot 2} = \frac{1}{2}$

R.H.S $= \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ L.H.S = R.H.S

$\therefore$ P(1) is true

Step (ii) Inductive hypothesis

Assume that P(n) is true for n = k

$\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

$\therefore$ P(k) is true

step (iii) Inductive test

we have to prove that $P(n)$ is true for $n = (k+1)$

$$1^3 + 2^3 + 3^3 + \cdots + (k+1)^3$$

L.H.S

$$1^3 + 2^3 + 3^3 + \cdots + k^3 + (k+1)^2$$

$$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3 \text{ from step ii}$$

$$= \frac{k^2(k+1)^2}{2} + (k+1)^3$$

$$= (k+1)^2 \left[\frac{k^2}{2} + k + 1\right]$$

$$= \left\{(k+1)^2 \left(\frac{k(k+1)}{2}\right)\right\}^2$$

R.H.S

$\therefore$ $P(n)$ is true for $\forall$ $n \in N$ proved

Q4. prove by PMI that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{1}\right]^2$

let $P(n)$ be the statement that $1^3 + 2^3 + 3^3 + \cdots n^3$

$$= \left[\frac{n(n+1)}{2}\right]^2$$

step (i) Inductive base

Put $n = 1$

$$L.H.S = 1^3 = 1$$

$$R.H.S = \left[\frac{1(1+1)}{2}\right]^2 = 1$$

$\therefore$ L.H.S = R.H.S

$\therefore$ $P(1)$ is true.

step (ii) Inductive hypothesis

Assume that P(n) is true for n = k

$1^3 + 2^3 + 3^3 + --- + k^3 \left(\frac{k(k+1)}{2}\right)^2$

$\therefore$ P(k) is true

step (iii) Inductive test

we have to prove that P(n) is true for n = (k+1)

$1^3 + 2^3 + 3^3 + --- + (k+1)^3$

L.H.S

$1^3 + 2^3 + 3^3 + - - + k^3 + (k+1)^2$

$= \left[\frac{k(k+1)}{2}\right]^2 + (k+1)^3$ from step ii]

$= \frac{k^2(k+1)^2}{2} + (k+1)^3$

$= (k+1)^2 \left[\frac{k^2}{2} + k + 1\right]$

$= (k+1)^2 \left[\frac{k^2 + 2k + 2}{2}\right]$

$= (k+1)^2 \left[\frac{k(k+1)}{2}\right]^2$ R.H.S

$\therefore$ P(n) is true for $\forall$ $n \in N$

= Proved

Q5' counting principle

In a Survey of 60 people it was found that 25 read News week magazine 26 read time, 26 read fortune 9 read both, News week & fortune 11 read both NewWeek & Time 8 read both time & fortune 3 read all three magazine

a) Find the no. of people who read at least one of the 3 magazine

Given $|N| = 25$  $|N \cap F| = 9$  $U = 60$

$|T| = 26$  $|N \cap T| = 11$  $|N \cap T \cap F| = 3$

$|F| = 26$  $|T \cap F| = 8$

a) Atleast one of 3 magazine

N or T or F

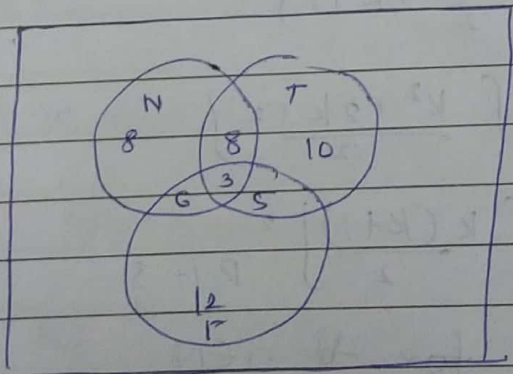
$|N \cup T \cup F| = ?$

$|N \cup T \cup F| = |N| + |T| + |F| - |N \cap T| - |N \cap F| - |T \cap F| + |N \cap T \cap F|$

$= 25 + 26 + 26 - 9 - 11 - 8 + 3$

$= 52$

b) Fill the venn diagram with correct value in eight regions




$U = 60$

$U \rightarrow |N \cup T \cup F| = 60 - 52 = 8$

$|N \cap T \cap F| = 3$

$|N \cap F| = 9 - 3 = 6$

$|N \cap T| = 11 - 3 = 8$

$|T \cap F| = 8 - 3 = 5$

$|N| = 25 - 8 - 6 - 3 = 8$

$|T| = 28 - 8 - 3 - 5 = 10$

$|F| = 26 - 6 - 5 - 3 = 12$

Scanned by CamScanner

c) Find the no. of people read exactly one magazine

$N+T+E = 8+10+12$
$= \underline{30}$

Q6 combinatione

A farmer buys 3 cow, 2 Pigs & 4 hens from a man who has 6 cow, 5 pig and 8 hens. How may choices does the farmer have

${}^6C_3$ = cows
${}^3C_2$ = Pigs
${}^4C_8$ = hens

According to principle of counting

$${}^6C_3 \times {}^3C_2 \times {}^4C_8 = \frac{6!}{3!3!} \times \frac{5!}{2!3!} \times \frac{8!}{4!\times 4!}$$

$$= \frac{6\times5\times4}{3\times2} \times \frac{5\times4}{1\times2} \times \frac{8\times7\times6\times5}{4\times3\times2}$$

$$= 20\times10\times70$$

$$= 1900$$

Q7 6 men, 5 women To form the committee of 5 members you should have 2 womens.

Given

Total = 11

members = 5

According to combination

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

$$^3C_2 \times {}^6C_3 = \frac{5!}{2!3!} \times \frac{6!}{3!3!}$$

$$= \frac{5\times4\times3\times\cancel{2}\times\cancel{1}}{2\times1\times3\times\cancel{2}\times1} \times \frac{6\times5\times4\times3\times\cancel{2}\times1}{3\times2\times1\ 3\times2\times1}$$

$$= \frac{5\times4}{2\times1} \times \frac{6\times5\times4}{3\times2\times1}$$

$$= \frac{20}{2} \times \frac{120}{6}$$

$$= 10\times20$$

$$= 200$$

Q8. Inclusion - Exclusion principle

A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for applications programming of these hired 15 are expected to performs jobs of both types How many programmers must be hired

Given

$|A| = 20$

$|B| = 30$

$|A\cap B| = 5$

$|A\cup B| = ?$

$$\therefore |A\cup B| = |A| + |B| - |A\cap B|$$

$$= 20+30-5$$

$$= 50-5$$

$$= 45$$

Q9. Solve the reccurence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ given that $a_r = 0$, $a = 3$

Given recorrence relation is

$$a_r - 7a_{r-1} + 10_{r-2} = 0$$

and given that $a_0 = 0$, $a_1 = 3$

This is second order reccorrence relation

The characteristic equation is

$$m^2 - 7m + 10 = 0$$

$$\Rightarrow (m-2)(m-5) = 0$$

$$m = 2, 5$$

The general solution is

$$a_r = c_1 (2)^r + c_2 (5)^r \rightarrow 2$$

Putting in eq $a_0 = 0$ i.e $a_r = 0$ and $a = 0$

$$c_1 (2)^0 + c_2 (5)^0 = 0$$

$$\Rightarrow c_1 + c_2 = 0 \rightarrow 3$$

Again Putting in equation 2 $a_1 = 3$ ie $a_r = 3$ and $r = 1$

$$c_1 (2)^1 + c_2 (5)^1 = 3$$

$$2c_1 + 5c_2 = 3 \rightarrow 4$$

Solving eq 3 and 4 we get

$$c_1 = -1 \text{ and } c_2 = 1$$

The required general solution is put in eq 2

$$\boxed{a_r = 5^r - 2^r}$$