CS215 assignment – II

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Given, X and Y are two independent random variables.   
f\_X(.) and f\_Y(.) are their PDFs respectively.   
  
Consider a random variable Z = XY.   
To derive,   
PDF of Z in terms of X and Y, f\_X and f\_Y.   
  
Cumulative Distribution Function (CDF) of Z is F\_Z(z).   
We know that,   
  
F\_Z(z) = P(Z ≤ z)   
  
F\_Z(z) = P(XY ≤ z) [Given XY = z]   
  
Since, X and Y are independent their PDFs is   
f\_X(x) f\_Y(y)   
  
⇒ That means,   
F\_Z(z) =

Question 2

As the inequality xy ≤ z differently acts for positive and negative set of values of x,   
we’ll split into two cases.   
  
Case 1: x > 0   
 y <= z/x   
  
  
 y ∈ (-∞, z/x)

Case 2: x < 0   
 y >= z/x   
  
 y ∈ (z/x,∞)

F\_Z(z)=

F\_Z(z)= +

As and

F\_Z(z)=1-F\_y(z/x)]f\_X(x)dx + F\_Y(z/x)fX(x)dx

For PDF of Z we have to derivate F\_Z(z)

Means, f\_Z(z)=

f\_Z(z)= - f\_Y(z/x)f\_X(x)dx+.

f\_Z(z)=

## Question 3)

We are considering random variable X.

Pdf is fX(.)

x1,x2,…xn are independent samples from this pdf.

E[x]=

E[X]=

Since xi are independent, E[xi]=E[x]

E[ ]=(1/n).

=(1/n).n.E[X]=E[X]

E[X]=mean of these samples

So, is estimate for E[X]

For 2nd case:

Why , is wrong,

estimate=

Because it is mean of {} where i=1,2,…n

Which estimates to E[ and not E[X].

## Question 5:

To prove,  
P(X ≥ x) ≤ φx (t) for t > 0 …eqn 1  
P(X ≥ x) ≤ φx(t) for t < 0 …eqn2

Given here φx(t) is MGF.

means, φx(t) = E(

To prove eqn 1,

let us use Markov’s inequality

P(X ≥ a) ≤ E[X] / a, X ≥ 0, a > 0

(If we replace X with then P(≥ a) ≤ E[] / a)

for case X ≥ x

if we apply function f(y) = e^(ty)

we get ≥   
when t > 0, the two events are same

Hence P(X ≥ x) = P(≥ )

so, P(X ≥ x) ≤ E[/

≤ .E[]=. φx(t)

P(X ≥ x) ≤ φx (t) for t > 0

Hence proved.

To prove eqn2,

take t < 0

then, P(X ≤ x) = P( ≥ )

(as t < 0, if X large → will be small)

applying markov’s inequality,

P(X ≤ x) ≤ E[] / = φx(t)

Consider X =

where each Xi are independent Bernoulli random variables,

which means, Xi ∈ {0,1}

with E[Xi] = pi

μ = = E[X]

MGF of X is,

φx(t) = ]

=

By using Inequality  
1 + x ≤

we can write,

1 - pi + pi.= 1 + pi ( - 1) ≤

φx(t) ≤

=  
=

To show P(X > (1+δ)μ) ≤ / e^(t(1+δ)μ) where t>=0, δ>0

from eqn 1,

P(X > (1+δ)μ) ≤ φx(t) / e^(t(1+δ)μ)

≤/ e^(t(1+δ)μ), where t > 0, δ > 0

Tightening the bound by choosing optimal value of t,

that is to make bound small, we want to choose t that minimizes the function

f(t) = - t(1+δ)μ

derivate w.r.t ‘t’,

f ‘(t)= (1+δ))

f′′(t)=μ

putting f ′(t)=0 gives:

μ−μ(1+δ)=0⇒et=1+δ

Since f′′(t)= μ>0 for all t, this critical point is a minimum.

Putting this t in in original inequality gives:

P(X > (1+δ)μ) ≤ / =

This is upper tail for chernoff bound.

Question 6).

Given,   
  
n independent coin tosses with head probability p.   
T be the trail number, at which we get the first head.   
  
To Derive,   
E(T) in terms of p and n. (T)   
  
As E(T) T is the trail number at which we get our first head, then T can belongs to range from 1 to n.   
that is T can be is varied from 1 to n.   
  
consider T = 1.   
then our first coin toss should result in head,   
so, P(T = 1) = p (∵ probability of getting head is p)   
  
consider T = 2.   
then we should definetly get tails in the first toss and definetly get head in second toss.   
Hence, P(T = 2) = (1-p).p ( P(H) = p so, P(T) = 1-p )   
  
similarly for T = k (1 ≤ k ≤ n)   
we must get tails in all the tosses from 1 to k-1 and definetly get a head on kth toss.   
Hence, P(T = k) = (1-p)^(k-1) \* p --------> (i)   
  
  
We know,   
E(X) = Σ (xi \* P(X = xi)) , i = 1 to m   
  
Hence,   
E(T) = Σ (ti \* P(T = ti)), i = 1 to n   
  
E(T) = 1\*p + 2\*(1-p)\*p + ... + k\*(1-p)^(k-1)\*p + ... + n\*(1-p)^(n-1)\*p   
  
Hence,   
E(T) = Σ [i \* (1-p)^(i-1) \* p], i = 1 to n

## Question 1)a)

i) given total n subjects,

Testing each subject → needs n tests n subjects divided into ½ disjoint subsets each with s subject.

Probability that a person haves disease = p.

To find expected total no of tests,

first consider one pool with s subjects.

Round 1:

All pools have to be tested.so, n/s is the expected no. of tests for round1.

Round 2:

Probability that pool tests negative is, (1-p)^s

[because prob that one person won’t have disease is (1-p) and all 's' persons not having disease is (1-p) . (1-p) ... s times]

Probability that pool tests positive is, 1 - (1-p)^s

expected no. of pools testing positive in round 2 is, n (1 - (1-p)^s)/s (as n/s is number of pools)

for each pool s subjects have to be tested.

So, s . n/s(1 - (1-p)^s) = n (1 - (1-p)^s)

Hence,expectedtotal no.of tests is

= n/s + n (1 - (1-p)^s)

ii) we got T(S) = n/s+ n (1 - (1-p)^s)

w.k.t.,

(1-x)^n = 1 - n x + n C₂ n² x² + ... + n Cₙ xⁿ

if n is very less then xⁿ, n², ... nⁿ tends to zero in that case, (1-x)^n ≈ 1 - n x

given p is very less,

Hence, TCS) =n/s + n (1 - (1-s p))

T(S) =n/s + n (sp)

Hence proved.

To find the value of s for which expected number is least is,

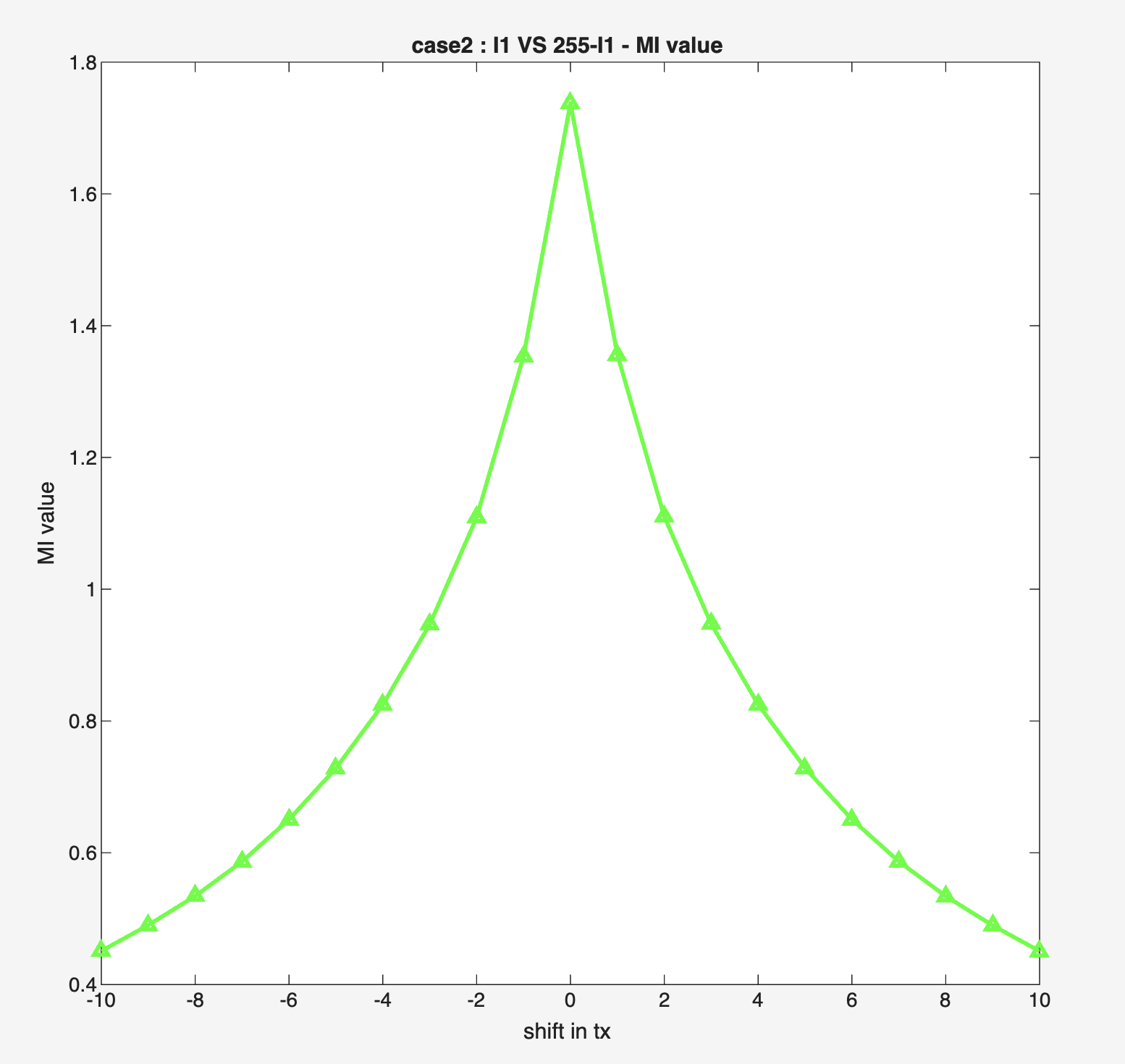
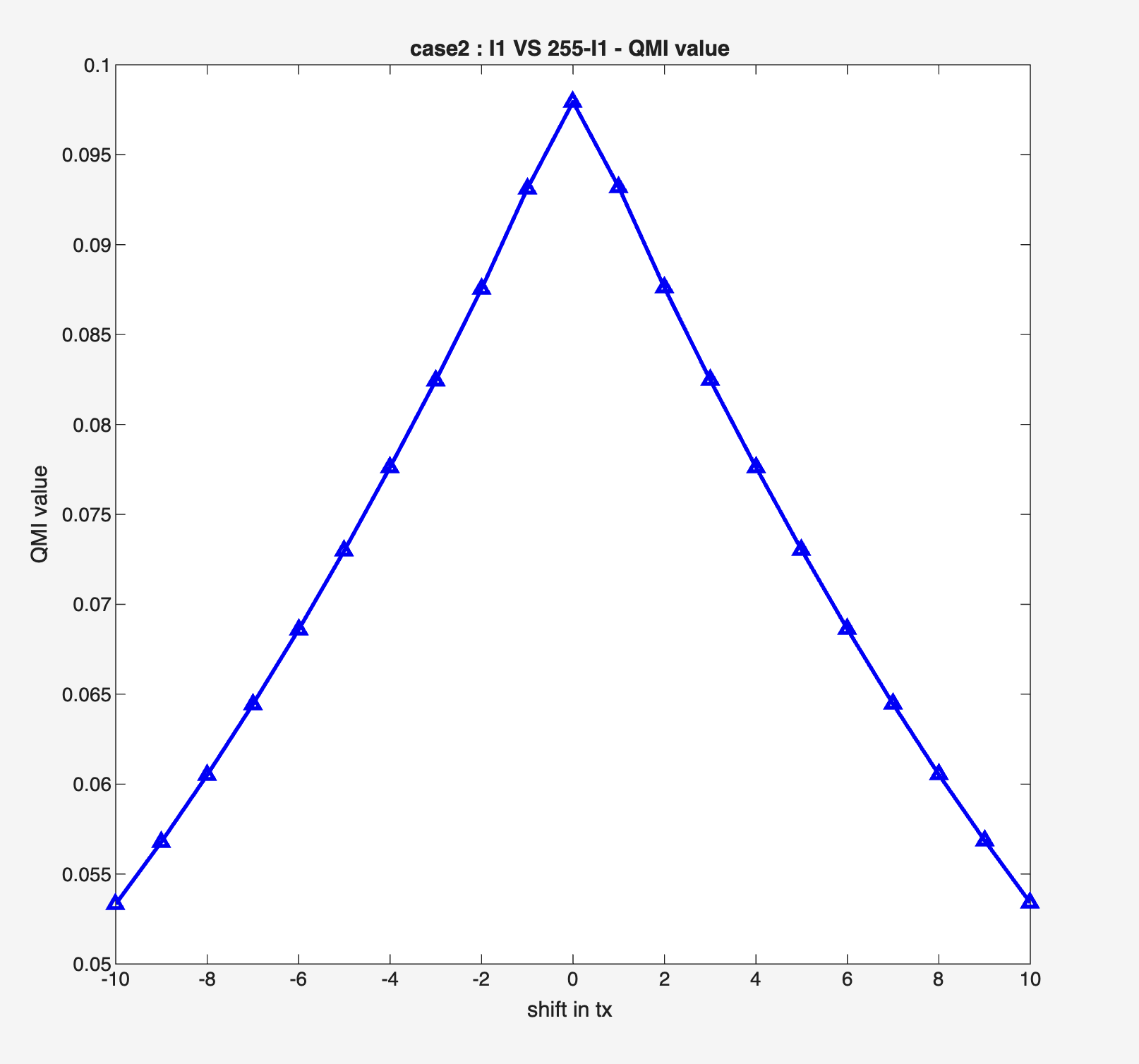
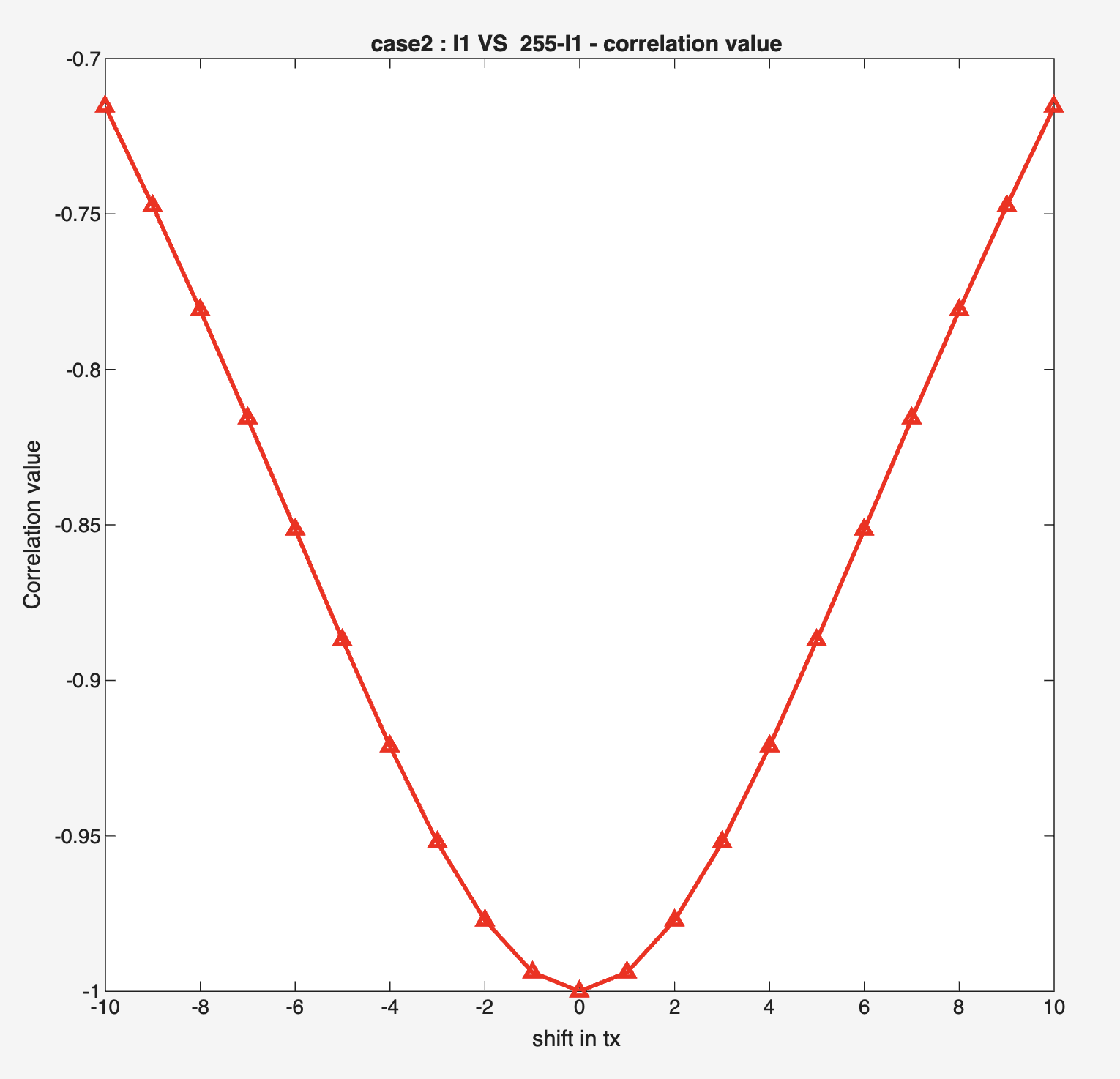
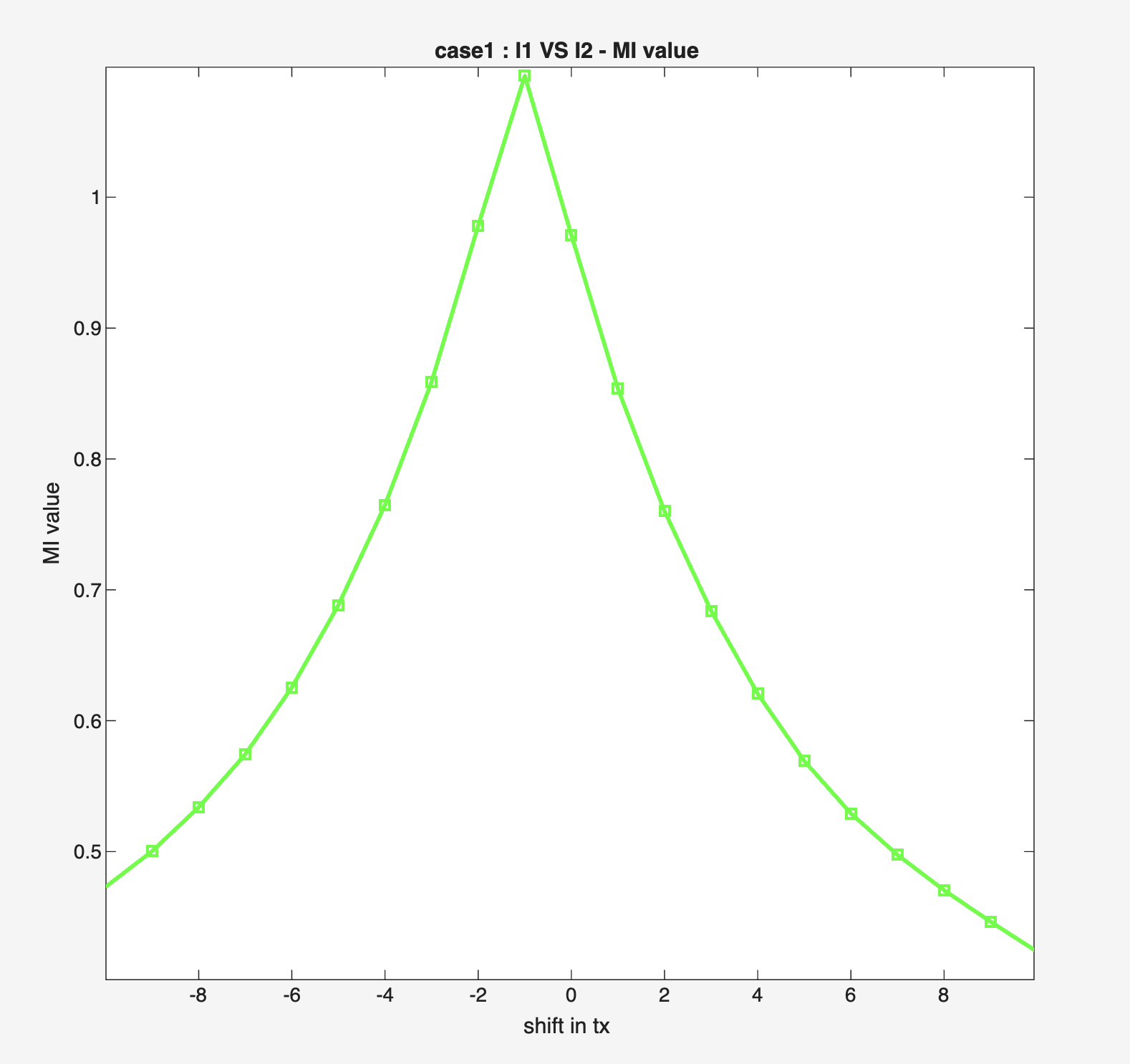
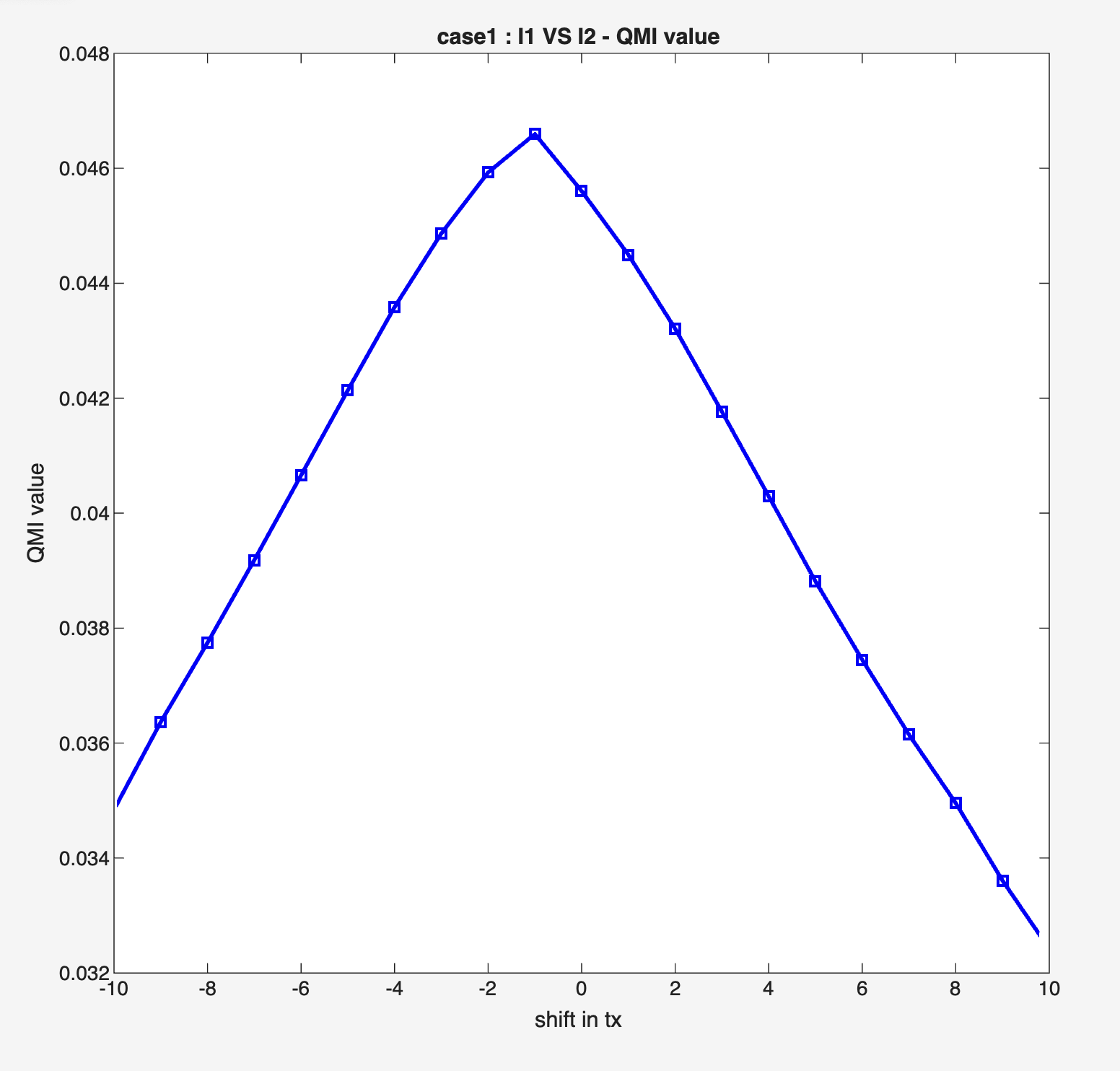
The value of s for which d T(S)/ds =0 and d..>0

# Question 4 : for case 1 (I1 VS I2 ) g6.png

Graph 1 – Correlation vs Shift

i)This correlation graph mostly seems to be flat but it takes a dip to the lowest point which is the zero shift .

ii) in the curve the alignment matters , as the lowest is at the zero shift position



Graph 2 – QMI vs Shift (Case 1)

The graph for this shows a balanced peak at zero

as the peak is at zero shift point , the curve tends to fall on either side of it

This proves that QMI effectively captures the strongest statistical dependence when the images are perfectly aligned, even without any intensity changes.

Graph 3 – M1 VS Shift

\*As the above two graphs (similar to them ) , it has its peak at the zero shift and it decays evenly on both the sides .

\* But the slight difference is that here the peak is sharper than QMI, which proves the MI’s higher sensitivity to alignment changes.

\* By which we can conclude that MI is a very reliable measure for original image alignment.

Graph 4 – Correlation vs Shift (Case 2)

\* As observed in the graph , Correlation remains negative for all the shifts because of the inverse intensity relationship.

\* the minimum point is at the zero shift , that means that it is the indication of alignment

CASE 2

. Graph 5 – QMI vs Shift (Case 2)

\* QMI shows a clear maximum at zero shift just as in case 1 , totally unaffected by the negative transformation.

\* The curve is smooth and symmetric, which confirms QMI’s robustness against intensity inversion.

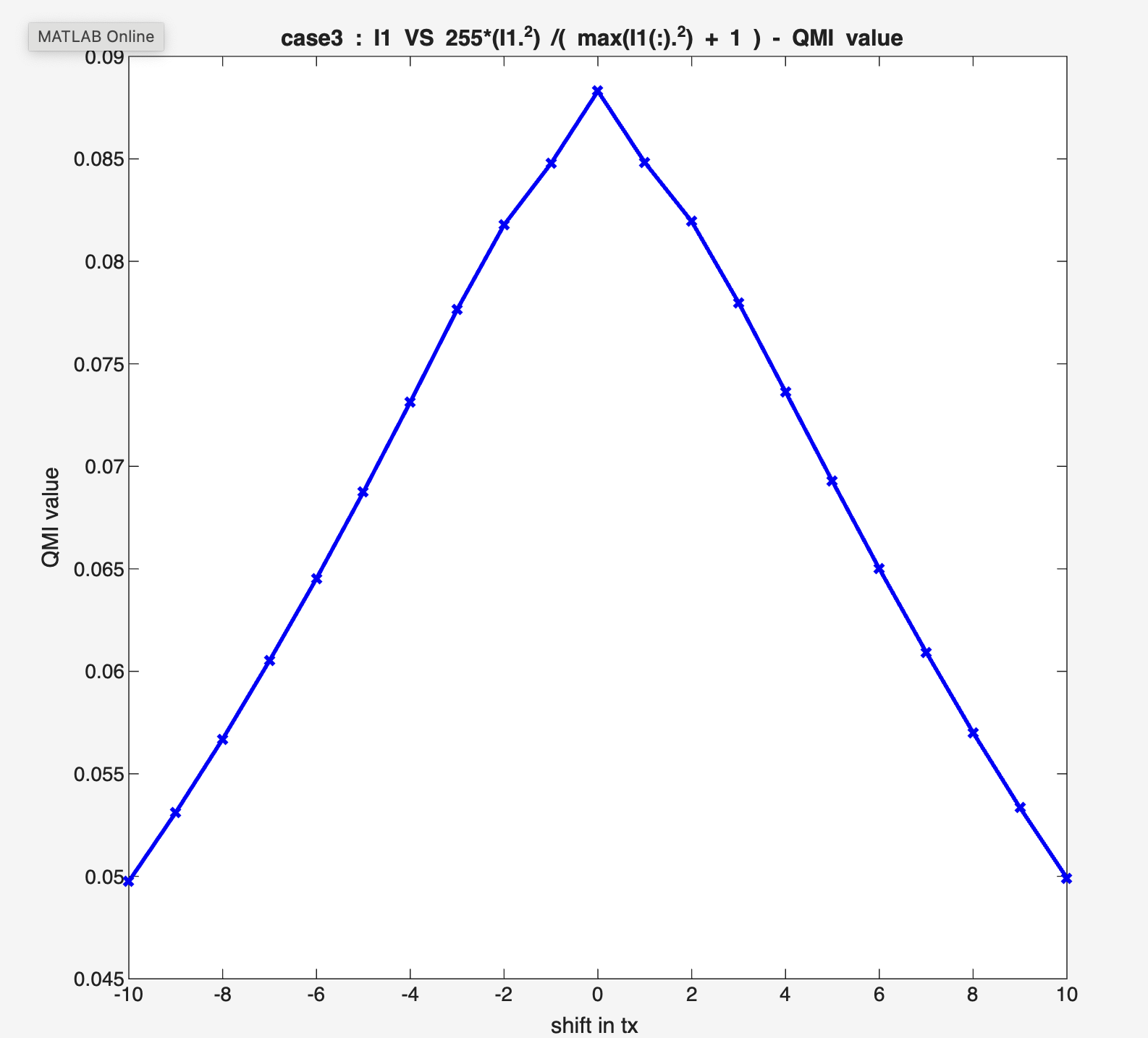
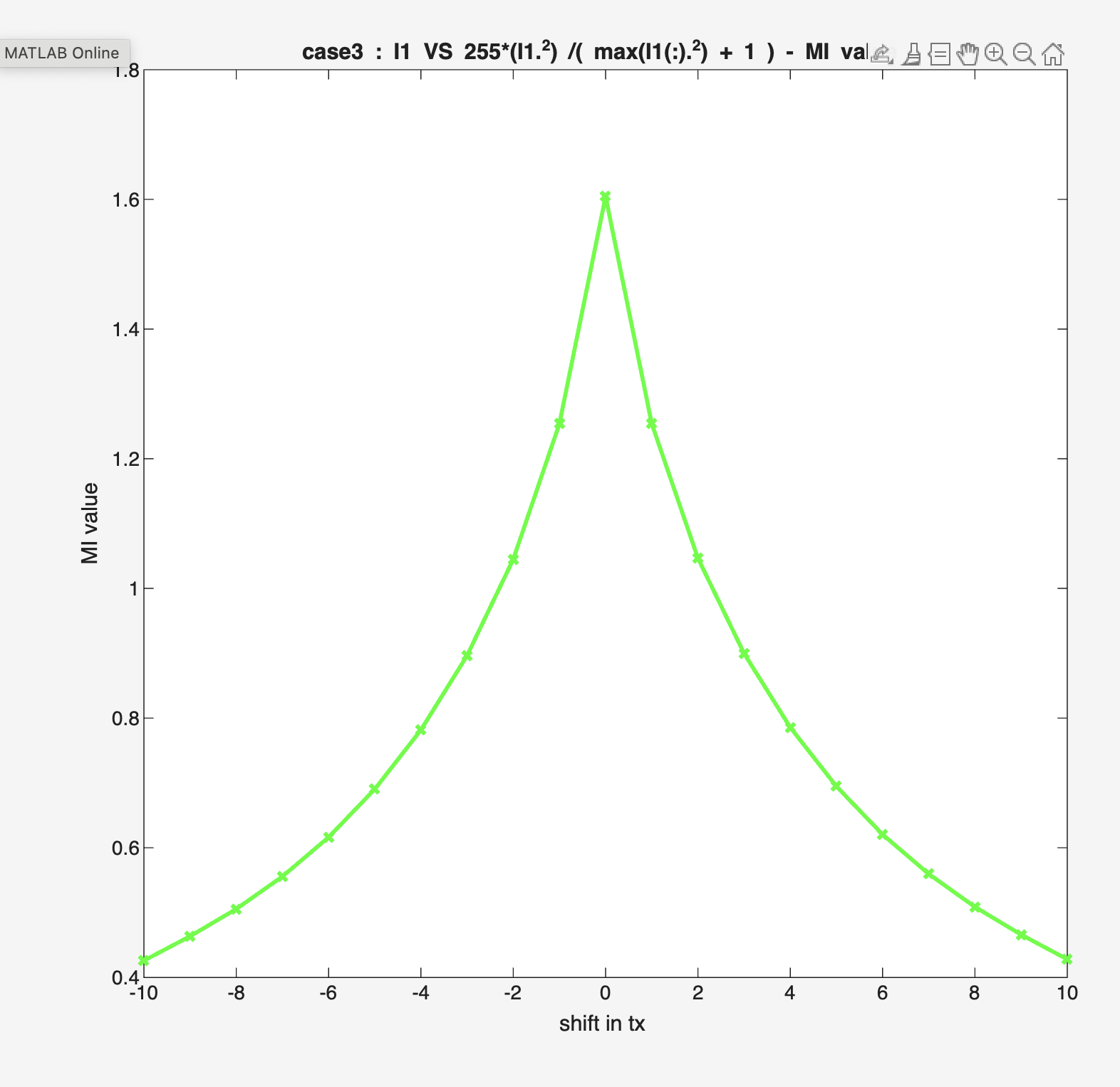
\* By this it can be concluded that QMI are more reliable than correlation graph for transformed images

. Graph 6 – MI vs Shift (Case 2)

\* MI also peaks at zero shift, similar to that of QMI, but it has a sharper peak

\* It performs reliably even when intensity values are inverted.

\* When intensity changes are involved , MI are more to be used .



Graph 7 – MI vs Shift (Case 3)

\*Similar to the graphs of case 1 and case 2 , it has the most prominent peak at the zero shift position

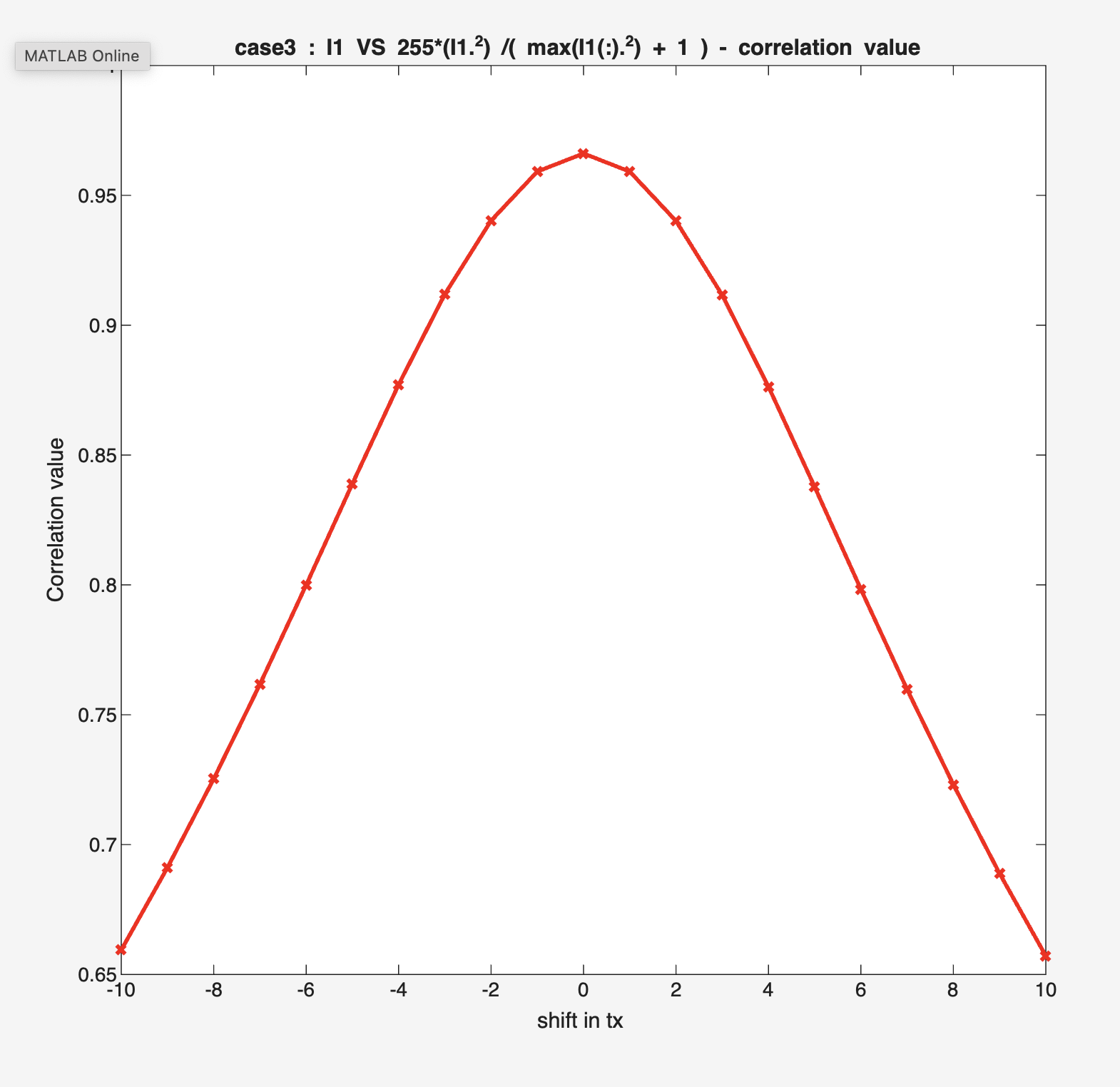
CASE 3

Graph 8 – QMI vs Shift (Case 3)

QMI has smooth peak at the zero shift similar to that of case 1 and case 2

\* The peak value is slightly lower .

\* this concludes that QMI adapts well to non-linear intensity changes



Graph 9 – Correlation vs Shift (Case 3)

\* In this graph Correlation has a positive peak at zero shift, , but the peak is still less than 1