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Linear Algebra Assignment

i) $2x - 3y + 7z = 5,\ 3x + y - 3z = 13,\ 2x + 19y - 47z = 32$

$$A = \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ 13 \\ 27 \end{bmatrix}$$

$$R_2 \to R_2 - \frac{3}{2} R_1$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & \frac{11}{2} & \frac{-22}{2} \\ 0 & 22 & -54 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ \frac{11}{2} \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 0 & \frac{11}{2} & -\frac{27}{2} \\ 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 5 \\ \frac{11}{2} \\ 5 \end{bmatrix}$$

Rank of $A = 2$ rank of $A:B = 3$

$\rho(A) \neq \rho(A:B)$ Incosistency

ii) $2x - y + 3z = 8,\ -x + 2y + z = 4,\ 3x + y - 4z = 0$

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 2 & 1 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 8 \\ 4 \\ 0 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

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$$A = \begin{bmatrix} -1 & 2 & 1 \\ 2 & -1 & 3 \\ 3 & 1 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 8 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 7 & -1 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$$A = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 3 & 5 \\ 0 & 0 & -\frac{38}{5} \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 16 \\ 12 \end{bmatrix}$$

$\varrho(A) = \varrho(A:B)$ consistency

2 iii) $4x - y = 12, \; -x + 5y - 2z = 0, \; -2x + 4z = -8$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -2 & 0 & 4 \end{bmatrix} \quad X = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -8 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ -4 & 0 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -16 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -34 \end{bmatrix}$$

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$$A = \begin{bmatrix} 4 & -1 & 0 \\ -4 & 20 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 0 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & +19 & -8 \\ 0 & -1 & 8 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ -4 \end{bmatrix}$$

$$A = \begin{bmatrix} 4 & -1 & 0 \\ 0 & 19 & -8 \\ 0 & 18 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 12 \\ 8 \end{bmatrix}$$

$\rho(A) = \rho(A:B)$ consistency

b. Q) For what values of λ and μ
$x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$
i) No solution ii) a unique solution iii) Infinity no of solution

Sol

$AX=B$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix} \quad A:B = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & \mu \end{bmatrix}$$

i) No solution $(\rho(A) \neq \rho(A:B)) = \begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{bmatrix}$
$\lambda = 3,\ \mu \neq 10$

ii) a unique solution $\rho(A) = \rho(A:B) = n$
$\lambda \neq 3$, many value

iii) Infinite solution $\rho(A) = \rho(A:B) < n$
$\lambda = 3,\ \mu = 10$

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$n \to$ no of unknows

x, y, z

(Q) Find for what value of λ.

$x + y + z = 1$

$x + 2y + 4z = \lambda$

$x + 4y + 10z = \lambda^2$

Have a solution & solve for each value of λ.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 4 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda \\ \lambda^2 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 3 & 9 \end{bmatrix} \quad A:B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & \lambda \\ 1 & 4 & 10 & \lambda^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 3 & 9 & \lambda^2 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & \lambda^2 - 1 - 3\lambda + 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & \lambda - 1 \\ 0 & 0 & 0 & \lambda^2 - 3\lambda + 2 \end{bmatrix}$$

$\rho(A) = 2$

$\rho[A:B) = 2$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda(\lambda - 1) - 2(\lambda - 1) = 0$

$(\lambda - 1)(\lambda - 2) = 0$

$\lambda = 1, 2$

$\rho(A) = \rho(A:B) \neq n$

3 $\to$ unknows

<u>Case - 1</u> $\lambda = 1$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 0$

$z = k$

$y = -3k$

$x = 1 - y - z$

$= 1 + 3k - k = 1 + 2k$

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Case - 2 $\quad \lambda = 2$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x + y + z = 1$

$y + 3z = 1$

$y = 1 - 3k, \; z = k$

$x = 1 - 1 + 3k - k = 2k$

d) ~~Find~~ Find the sol^n of the system of equations $x + 3y - 2z = 0, \; 2x - y + 4z = 0, \; x - 11y + 14z = 0.$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix} = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$$

$\cancel{122 = 0}$ $\quad \rho(A) = \rho(A:B) = 2 \neq n$

$\cancel{z = 0}$

Infinitesolution

$\cancel{-7y = 0}$

$\cancel{y = 0}$

$\cancel{x + 0}$

e. find for what values of λ the given equations $3x + y - \lambda z = 0, \; 4x - 2y - 3z = 0, \; 2\lambda x + 4y + \lambda z = 0$, may posses non-trivial solution and solve them completely in each case.

$$A = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda & 4 & \lambda \end{bmatrix} = \begin{bmatrix} 3 & +1 & -\lambda \\ 4 & -2 & -3 \\ 2\lambda+3 & 4+1 & 0 \end{bmatrix} \quad \text{Infinite}$$

$R_3 = R_3 + R_1$

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$$\begin{bmatrix} 12 & 4 & -4\lambda \\ 12 & -6 & -9 \\ 2\lambda+3 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 12 & 4 & -4\lambda \\ 0 & -10 & -9+4\lambda \\ 2\lambda+3 & 5 & 0 \end{bmatrix}$$

$R_1 = R_1 4 / R_2 = R_2 3$ $\qquad$ $R_2 = R_2 - R_1$

$$= \begin{bmatrix} 12 & 4 & -4\lambda & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 12 & -6 & -9 & 0 \\ 0 & -10 & -9+4\lambda & 0 \\ 2\lambda+3 & 5 & 0 & 0 \end{bmatrix}$$

$R_1 = R_1 + R_2$

$= 12x + 6y - 9z = 0$

$-10y + (-9+4\lambda)z = 0$

$(2\lambda+3)x + 5y = 0$

$-\frac{(2\lambda+3)}{2}x = y$

$12x - 6y - 9z = 0$

$12x + \frac{6(2\lambda+3)}{5}x - 9\left[\frac{10y}{(-9-4\lambda)}\right] = 0$

$12x + \frac{6(2\lambda+3)}{5} + \frac{18(2\lambda+3)}{(4\lambda-9)} = 0$

$12(4\lambda-9)5 + 6(2\lambda+3)(4\lambda-9) + 18(5)(2\lambda+3)$

$240\lambda - 540 + 180\lambda + 270 + 48\lambda^2 - 108\lambda + 72\lambda - 162 = 0$

$48\lambda^2 + 384\lambda - 432 = 0$

$\lambda^2 + 8\lambda - 9 = 0$

$\lambda^2 + 9\lambda - \lambda - 9 = 0$

$\lambda(\lambda-1) + 9(\lambda-1) = 0$

$(\lambda+9)(\lambda-1) = 0 \Rightarrow \lambda = 1, \lambda = -9$

$\underline{\lambda = 1}$

$-x = y, \; z = -2y$

$12x - 6y - 9z = 0$

$12(-y) - 6y - 9(-2y) = 0$

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$18y - 18y = 0$

$y = 0$ trivial solution

$\lambda = -9$

$y = \left[\frac{-(2\lambda + 3)x}{5}\right]$ $\quad z = \frac{10y}{(-9+4\lambda)}$

$= -\left(\frac{-18-3}{5}\right)x = +3x$ $\quad = \frac{10y}{-45} = \frac{2y}{-9}$

$12x - 6y - 9z = 0$

$\cancel{12}^{4}\left(\frac{y}{\cancel{3}}\right) - 6y - \cancel{9}\left(\frac{2y}{-\cancel{9}}\right) = 0$

$4y - 6y + 2y = 0$

$y = 0$ trivial solution

It has no trivial solution.

Assignment - 2

Q) i) $[1, 0, 0], [1, 1, 0], [1 1 1]$

$[C_1 + C_2 + C_3 \quad C_2 + C_3 \quad C_3] = [0\ 0\ 0]$

$C_3 = 0, C_2 = 0, C_1 = 0$

unique solution

Linearly independent

8) $[6\ 0\ 3\ 1\ 4\ 2], [0\ -1\ 2\ 7\ 0\ 5], [12\ 3\ 0\ -19\ 8\ -11]$

$[6C_1 + 12C_3 \quad -1C_2 + 3C_3 \quad 3C_1 + 2C_2 \quad C_1 + 7C_2 - 19C_3$
$4C_1 + 8C_3 \quad 5C_2 - 11C_3] = [0\ 0\ 0\ 0\ 0\ 0]$

$C_1 + 2C_3 = 0$ $\quad C_1 + 2C_3 = 0$ $\quad 3C_1 + 2C_2 = 0$

$-C_2 + 3C_3 = 0$ $\quad C_1 - 7C_2 - 19C_3 = 0$ $\quad 5C_2 - 11C_3 = 0$

$C_2 = 3C_3$ $\quad 15C_3 - 11C_3 = 0$

$C_2 = 0$ $\quad C_1 = 0$ $\quad 4C_3 = 0$

Unisolution linearly independent $\quad C_3 = 0$

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2) $[7 \;\; -3 \;\; 11 \;\; -6]$, $[-56 \;\; 24 \;\; -88 \;\; 48]$

$$[7C_1 - 56C_2 \quad -3C_1 + 24C_2 \quad 11C_1 - 88C_2 \quad -6C_1 + 48C_2]$$

$$= [0 \;\; 0 \;\; 0 \;\; 0]$$

$$\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} 8k \\ k \end{bmatrix} = k \begin{bmatrix} 8 \\ 2 \end{bmatrix}$$

$7C_1 = 56C_2$

$C_1 = 8C_2$ Linearely ~~in~~dependent

$C_1 = 8k$

3) $[-1 \;\; 5 \;\; 0]$, $[16 \;\; 8 \;\; -3]$, $[-64 \;\; 56 \;\; 9]$

$$[-1C_1 + 16C_2 - 64C_3 \quad 5C_1 + 8C_2 + 56C_3 \quad -3C_2 + 9C_3]$$
$$= [0 \;\; 0 \;\; 0]$$

$-3C_2 + 9C_3 = 0$

$3 \cdot \cancel{9}C_3 = \cancel{3}C_2$

$C_2 = 3C_3$

$= 3k$

$-C_1 + 16C_2 - 64C_3 = 0$

$16C_2 = 64C_3 + C_1$

$16[3C_3] = 64C_3 + C_1$

$C_1 = 0$

$$\text{Vector} = \begin{bmatrix} 0 \\ 3k \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$$ linearely dependent

4) $[1 \;\; -1 \;\; 1]$, $[1 \;\; 1 \;\; -1]$, $[-1 \;\; 1 \;\; 1]$, $[0 \;\; 1 \;\; 0]$

$$[C_1 + C_2 \cancel{2} - C_3 \quad -C_1 + C_2 + C_3 + C_4 \quad C_1 - C_2 + C_3]$$

$C_1 + C_2 - C_3 = 0$

$C_1 + C_2 = C_3$

$-C_1 + C_2 + C_3 + C_4 = 0$

$C_1 = C_2 + C_3 + C_4$

$C_4 = -2k$

$C_1 - C_2 + C_3 = 0$

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$\cancel{C_1} + C_1 + \cancel{C_3} = \cancel{C_3}$ $\qquad C_2 = C_1 + C_3$ linearly

$2C_1 = 0 \Rightarrow C_1 = 0$ $\qquad C_2 = C_3 = k$ dependent

$$\text{Vector} = k\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

5) $[2\ -4]$, $[1\ 9]$, $[3\ 5]$

$[2C_1 + C_2 + 3C_3 \quad -4C_1 + 9C_2 + 5C_3] = [0\ 0]$

$2C_1 + C_2 + 3C_3 = 0$ $\qquad 4C_2 = 9C_2 + 5C_3$

$4C_1 + C_2 + 3C_3 = 0$ $\qquad 4C_1 = 9(k) - 5k$

$\qquad 4C_1 = 4k$

$4C_1 + 2C_2 + 6C_3 = 0$ $\qquad C_1 = k$

$9C_2 + 5C_3 - 2C_2 + 6C_3 = 0$

$11C_2 + 11C_3 = 0$

$C_2 = -C_3 = -k$

$$\text{Vector} = \begin{bmatrix} k \\ k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$ linearly dependent

6) Q) $[3\ -2\ 0\ 4]$, $[5\ 0\ 0\ +1]$, $[-6\ 1\ 0\ 1]$

$[2\ 0\ 0\ 3]$

$[3C_1 + 5C_2 - 6C_3 + 2C_4 \quad -2C_1 + C_3 \quad 0 \quad 4C_1 + C_2 + C_3 + 3C_4] = [0\ 0\ 0\ 0]$

$-2C_1 + C_3$ $\qquad 4C_1 + C_2 + C_3 + 3C_4 = 0$

$2C_1 = 2C_3$ $\qquad 20C_1 + 5\cancel{C_2} + 5C_3 + 15C_4 = 0$

$C_1 = k$ $\qquad 3C_1 + 5\cancel{C_2} + 6C_3 + 2C_4 = 0$

$C_3 = 2C_1$

$= 2k$

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$4C_1 + C_2 + C_3 + 3C_4$ $\quad$ $17C_1 + 11C_3 - 13C_4 = 0$

$4(k_1) + C_2 + 2k + 3(3k) = 0$ $\quad$ $17k + 11(2k) = 13C_4$

$C_2 + 15k = 0$ $\quad$ $(17+22)k = 13C_4$

$C_2 = -15k$ $\quad$ $\frac{39}{13}k = C_4 = 3k$

$$\text{Vector} = \begin{bmatrix} k \\ -15k \\ 2k \\ 3k \end{bmatrix} = k \begin{bmatrix} 1 \\ -15 \\ 2 \\ 3 \end{bmatrix}$$ linearely dependent

7) $[3\ 4\ 7], [2\ 0\ 3], [8\ 2\ 3], [5\ 5\ 6]$

$[3C_1 + 2C_2 + 8C_3 + 5C_4 \quad 4C_1 + 2C_3 + 5C_4 \quad 7C_1 + 3C_2 + 3C_3] = [0\ 0\ 0]$

$3C_1 + 2C_2 + 8C_3 + 5C_4 = 0$ $\quad$ $7C_1 + 3C_2 + 3C_3 + 6C_4 = 0$

$4C_1 + 2C_3 + 5C_4 = 0$ $\quad$ $14C_1 + 6C_2 + 12C_4 = 0$

$\quad$ $9C_1 + 6C_2 + 24C_3 + 15C_4 = 0$

$-C_1 + 2C_2 + 6C_3 = 0$ $\quad$ $5C_1 - 18C_3 - 3C_4 = 0$

$C_1 = 2C_2 + 6C_3 = 0$ $\quad$ $5C_1 = 18C_3 + 3C_4$

$C_1 = 2C_2 + 6C_3$ $\quad$ $10C_2 + 30C_3 = 18C_3 + 3C_4$

$5C_1 = 10C_2 + 30C_3$ $\quad$ $12C_3 = 3C_4 - 10C_2$

$4C_1 + 2C_3 + 5C_4 = 0$

$\to 36C_1 + 18C_3 + 45C_4 = 0$

$5C_1 - 18C_3 - 3C_4 = 0$

$41C_1 + 42C_4 = 0$

$$= \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = k \begin{bmatrix} \frac{-42}{41} \\ \frac{10543}{492} \\ 1 \\ \frac{1606}{216} \end{bmatrix}$$

$41C_1 = -42C_4$

$C_1 = \frac{-42}{41}C_4 = \frac{-42}{41}k$

$18C_3 = 5C_1 - 3C_4$ $\quad$ Linearely dependent

$C_3 = 5\left(\frac{-42}{41}\right)k - 3(k)\left(\frac{1}{18}\right)$

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$$=\left(\frac{-210c_6-6(41)}{(41)c_6}\right)k = \frac{(1260+216)k}{216}$$

$$\frac{42}{41}k = 2c_2 + \cancel{6}\left(\frac{251}{\cancel{36}_6}\right)k = \frac{1506}{216}k$$

$$= c_2 = -\frac{42}{41}k - \frac{251}{6}k$$

$$= c_2 = -\left(\frac{252+10291}{2\times 246}\right)k = \frac{10543}{492}k$$

1) $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ $A - \lambda I = 0$

$$= \begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} = \begin{matrix} (-2-\lambda)(-\lambda(1-\lambda)+6(-2)) \\ -2(-\lambda(2)+6(-1)) \\ -3(-4+1-\lambda) \end{matrix}$$

$$= (-2-\lambda)(-\lambda+\lambda^2-12)+2(+\lambda+6)+3(+3+\lambda)$$
$$= +2\lambda-2\lambda^2-24+\lambda^2-\lambda^3+12\lambda+2\lambda+12+9+3\lambda$$
$$0 = 19\lambda-\lambda^2-\lambda^3+45$$
$$\lambda^3+\lambda^2-19\lambda-45=0$$

2) $\begin{bmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} = A - \lambda I = 0$

$$\begin{bmatrix} 4-\lambda & 0 & 1 \\ -2 & 1-\lambda & 0 \\ -2 & 0 & 1-\lambda \end{bmatrix} = (4-\lambda)(1-\lambda)(1-\lambda)+1(+2(1-\lambda))$$
$$= (1-\lambda)((4-\lambda)(1-\lambda)+2)$$
$$= 4-\lambda-4\lambda+\lambda^2+2)$$
$$= (\lambda^2-5\lambda+6)$$
$$= (1-\lambda)(\lambda-2)(\lambda-3)$$

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$\lambda = -1$

$$\begin{bmatrix} 5 & 0 & 1 \\ -2 & 2 & 0 \\ -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 5x + z = 0 \\ -2x + 2y = 0 \\ -2x + 2z = 0 \end{matrix}$$

$$\begin{array}{r} \overset{(-)}{}10x \overset{(-)}{+} 2z = 0 \\ \hline -12x = 0 \\ x = 0 \end{array}$$

$z = 0$

$y = 0$

Eigen vector $= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 2$

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & -1 & 0 \\ -2 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 2x + z = 0 \\ -2x - y = 0 \\ -2x - z = 0 \end{matrix}$$

$x = k$ $\quad y = -2k$ $\quad$ Eigen vector $= k \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}$

$z = 2k$

$\lambda = 3$

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & -2 & 0 \\ -2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} x + z = 0 \\ -2x - 2y = 0 \\ -2x - 2z = 0 \end{matrix}$$

$x = -z = -k$ $\quad y = -x$ $\quad$ vector $= \begin{bmatrix} -k \\ k \\ k \end{bmatrix}$

$z = k$ $\quad y = k$

$= +k \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

3)
$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

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$$A - \lambda I = \begin{bmatrix} 5-\lambda & 0 & 0 \\ 0 & -\lambda & 0 \\ -1 & 0 & 3-\lambda \end{bmatrix}$$

$$(5-\lambda)(-\lambda)(3-\lambda) = 0$$

$$\lambda = 0, 3, 5.$$

$\lambda = 0$

$$\begin{bmatrix} 5 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 3 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad = 5x = 0 \Rightarrow x = 0$$

$$-x + 3z = 0$$

$$z = 0$$

$$\text{Vector} = k \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad = 2x = 0$$

$$-3y = 0$$

$$-x = 0$$

$$\text{Vector} = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 5$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & -5 & 0 \\ 1 & 0 & -2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad -5y = 0$$

$$-x - 2z = 0$$

$$x = 2z = -2k$$

$$\text{Vector} = \begin{bmatrix} -2k \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

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4) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 0-\lambda & 0 & 0 \\ 0 & 3-\lambda & 4 \\ 0 & 0 & -2-\lambda \end{bmatrix}$

$+\lambda((3-\lambda)(+2+\lambda)=0$

$\lambda = 0, 3, -2$

$\lambda = 0$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & -2 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{matrix} 3y+4z=0 \\ -z=0 \\ z=0, y=0 \end{matrix}$

Vector $= k = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$\lambda = 3$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 4 \\ 0 & 0 & -5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \begin{matrix} 4z=0 \\ -5z=0 \end{matrix}$

Vector $= k = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\lambda = -2$

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 4 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0 \quad \begin{matrix} 5y+4z=0 \\ 5y=-4z=-4k \\ y=\frac{-4}{5}k \end{matrix}$

$K = \begin{bmatrix} 0 \\ \frac{-4}{5} \\ 1 \end{bmatrix}$

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Assignment

1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$R_2 \to R_2 - 2R_1 \quad R_4 \to R_4 - 4R_1 \quad R_4 \to R_4 - R_2$

$R_3 \to R_3 - 3R_1 \quad R_4 \to R_4 - R_3$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -8 & 3 \\ 0 & 0 & -3 & 2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ no of non zero rows

rank = 3

2) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \to A^{-1} = \frac{1}{4-1}\begin{bmatrix} 2 & +1 \\ +1 & 2 \end{bmatrix} = \begin{bmatrix} \frac{2}{3} + & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$\begin{bmatrix} \frac{2}{3} - \lambda & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} - \lambda \end{bmatrix} = \left[\frac{2}{3} - \lambda^2\right]^2 - \left(\frac{1}{3}\right)^2 = 0$

$a^2 - b^2 = [a+b][a-b]$

$\left[\frac{1}{3} - \lambda\right](1 - \lambda) = 0$

Eigen values $\lambda = 1, \frac{1}{3}$

$\lambda = 1$

$\begin{bmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & -\frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x\frac{1}{3} + y\left(\frac{1}{3}\right) = 0$

$x = y = k$ Eigen values $= k\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

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$$\lambda = \frac{1}{3}\begin{bmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

$x = -y$ Eigen vector $= \begin{bmatrix} k \\ -k \end{bmatrix} = k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$Z = A + 4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

$$Z - \lambda I = \begin{bmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{bmatrix}$$

$a^2 - b^2 = (a-b)(a+b)$

$$(6-\lambda)^2 - 1 = (6-\lambda-1)(6-\lambda+1)$$
$$= (5-\lambda)(7-\lambda) = 0$$

eigen value $\lambda = 5, 7$

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0 \quad x - y = 0$$
$$x = y$$

eigen vector $= k\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\lambda = 7$ $\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$ $x = y$ Eigen vector $= k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

4. $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

$$x = \frac{1}{3}[7.85 + 0.1y - 0.2z]$$

$$y = \frac{1}{7}[-19.3 - 0.1x + 0.3z]$$

$$z = \frac{1}{10}[71.4 - 0.3x + 0.2y]$$

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Iteration - 1

$z = y = y = 0$

$x = \frac{7.85}{3} = 2.6$

$z = 0$

$y = \frac{1}{7}\left(-19.3 - 0.1\left(\frac{7.85}{3}\right)\right) = 2.79$

$y(1) = 2.79$

$z = \frac{1}{10}\left[71.4 - 0.3(2.61) + 0.2(2.79)\right)$

$z = \frac{1}{10}(71.4 - 0.783 + 0.558)$

$z(1) = 7.1175$

Iteration - 2

$x_{(2)} = \frac{7.85 - 0.1(2.6167) - 0.2(7.1408)}{3}$

$= 2.9255$

$y_{(2)} = \frac{19.3 - 0.1(2.9255) - 0.3(7.1408)}{7}$

$= 3.0123$

$z_{(2)} = \frac{71.4 - 0.3(2.9255) - 0.2(3.0123)}{10}$

$= 7.0132$

Iteration - 3

$x^{(3)} = \frac{7.85 \cdot 0.1(2.9255) - 0.2(7.0132)}{3}$

≈ 3.0032

$$y^{(3)} = \frac{19.3 - 0.1(3.0032) - 0.3(7.0132)}{7}$$

$$\simeq 3.001$$

$$z^{(3)} = \frac{71.4 - 0.3(3.0032) - 0.2(3.0001)}{10}$$

$$= 7.00$$

$x = 3.0032$, $y = 3.0001$, $z \simeq 7.000$

5) Define consistent and inconsistent

$x + 3y + 2z = 0$, $2x - y + 3z = 0$, $3x - 5y + 4z = 0$
$x + 17y + 4z = 0$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -7 & -1 \\ 0 & -14 & -2 \\ 0 & 14 & 2 \end{bmatrix} \quad \begin{aligned} &R_2 \to R_2 - 2R_1 \\ &R_3 \to R_3 - 3R_1 \\ &R_4 \to R_4 - R_1 \end{aligned}$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = R_3 \to R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This corresponds to the system:

$x + 3y + 2z = 0$
$-7y - z = 0$

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let $y = t$
$x = -3t, \quad z = -7t$

$\because$ system has infinely many solutions

The system is consistent and dependent

2) $T : W \to P_2$

$$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$$

find the rank and nullity of T

$$\to T\left(\begin{bmatrix} a & b \\ b & c \end{bmatrix}\right) = (a-b) + (b-c)x + (c-d)x^2$$

let $A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$ then

$$T(A) = (a-b) + (b-c)x + (c-a)x^2$$
$$= a - bx + c(x^2 - x + 1)$$

$\therefore$ the image of T is the set of all polynomials of degree atmost 2, denoted as P_2

Rank of T:

The rank of T is the dimension of its image. since P_2 has a dimension of 3 (coefficients for x^0, x^1 and x^2) the $\rho(T) = 3$

$$T(A) = 0$$

$\Rightarrow a - b = 0, \; b - c = 0, \; c - a = 0$

$\therefore a = b = c$

$\therefore$ T is the set of symmetric matrices of the form.

$$\begin{bmatrix} t & t \\ t & -t \end{bmatrix}$$ where t is any scalar.

$\therefore$ Dimension = 1

$\because$ rank (T) = 3, nullity of T is 1

6. $T: P_2 \to P_2$ is linear transformation

$$T(a+bx+cx^2) = (a+1)+(b+1)x+(c+1)x^2$$

$\rightarrow T(a+bx+c) = (a+1)+(b+1)x+(c+1)x^2$ is a linear transformation, we need to check 2 properties.

1. Additivity $T(U+V) = T(U) + T(V)$
2. homogeneity of degree:

$T(kU) = KT(U)$ for all u in the domain of T and all scalars k

1) $T(U+V) = T((a_1+b_1x+c_1)+(a_2+b_2x+c_2))$
$= T((a_1+a_2)+(b_1+b_2)x+(c_1+c_2))$
$= (a_1+a_2+1)+(b_1+b_2+1)x+(c_1+c_2+1)x^2$
$= (a_1+1)+(b_1+1)x+(c_1+1)x^2+(a_2+1)+(b_2+1)x+(c_2+1)x^2$

$T(a_1+b_1x+c_1)+T(a_2+b_2x+c_2)$

$\therefore$ function is additive.

$\therefore$ Homogeneity of Degree 1.

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$$T(kU) = T(k(a+bx+c))$$

$$= T(ka+kbx+kc) = (ka+1)+(kb+1)+(kc+1)x$$

$$= k(a+1) + k(b+1)x + k(c+1)x^2$$

$$= kT(a+bx+c)$$

so, the function is homogeneous of degree 1.

$\therefore$ it is linearly transformed

7) $S = \{(1,2,3),(3,1,0),(-2,1,3)\}$ is a basis of $V_3(R)$. In cases S is not a basis determine subspace - spanned by S.

$$S = \{(1,2,3),(3,1,0),(-2,1,3)\}$$

$$A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & 1 & 1 \\ 3 & 0 & 3 \end{bmatrix}$$

$R_2 \to R - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & -9 & 9 \end{bmatrix} \quad R_3 \to R_3 + \frac{9}{5} R_3$$

$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \\ 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ Linearly independent.

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$$\begin{bmatrix} 1 & 3 & -2 \\ 0 & -5 & 5 \end{bmatrix}$$

(1, 3, 2) and (0, −5, 5) these vectors form a basis for the subspace spanned by S

∴ Dimension of subspace spanned by S = 2
∴ set S is not a basic of R^3 because the row reduced form has a row of zeros.
∴ The basis for the subspace spanned by S is {(1, 3, −2), (0, −5, 5)}
∴ The dimension of the subspace is 2.

8. Using Jacobi's method (perform 3 iterations) solve $3x - 6y + 2z = 23$, $-4x + y - z = -15$, $x - 3y + 7z = 16$ with initial values $x_0 = 1, y_0 = 1, z_0 = 1$

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

Iteration −1

$$x(1) = \frac{23 + 6y(0) - 2(z)(0)}{3} \approx 9.0$$

$$y(1) = \frac{-15 + 4(x)(1) + z(0)}{7} \approx 9.0$$

$$z(1) = \frac{16 - x(0) - 3(y)(1)}{7} = 2.0$$

Iteration − 2

$$x(2) = \frac{23 + 6y(1) - 2(z)(1)}{3} = 5.0$$

$$y(2) = \frac{-15 + 4(x)(1) + z(1)}{1} = -5.0$$

$$z(2) = \frac{16 - x(1) + 3y(1)}{7} \approx 3.0$$

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Iteration - 3

$$x(3) = \frac{23 + 6y(2) - 2z(2)}{3} = 6.0$$

$$y(3) = \frac{-15 + 4x(2) + z(2)}{1} \approx -6.0$$

$$z(3) = \frac{16 - x(2) + 3y(2)}{7} \approx 2.0$$