1 Number System

FUNDAMENTAL CONCEPTS

I. Numbers

In Hindu-Arabic system, we have ten digits, namely 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.

A number is denoted by a group of digits, called ***numeral****.*

For denoting a numeral, we use the place-value chart, given below.

	Ten-Crores	Crores	Ten-Lakhs	Lakhs	Ten-Thousands	Thousands	Hundreds	Tens	Ones
(*i*)				5	2	8	6	7	9
(*ii*)			4	3	8	0	9	6	7
(*iii*)		3	5	2	1	8	0	0	9
(*iv*)	5	6	1	3	0	7	0	9	0

The four numerals shown above may be written in words as:

(*i*) Five lakh twenty-eight thousand six hundred seventy-nine

(*ii*) Forty-three lakh eighty thousand nine hundred sixty-seven

(*iii*) Three crore fifty-two lakh eighteen thousand nine

(*iv*) Fifty-six crore thirteen lakh seven thousand ninety

Now, suppose we are given the following four numerals in words:

(*i*) Nine crore four lakh six thousand two

(*ii*) Twelve crore seven lakh nine thousand two hundred seven

(*iii*) Four lakh four thousand forty

(*iv*) Twenty-one crore sixty lakh five thousand fourteen

Then, using the place-value chart, these may be written in figures as under:

	Ten-Crores	Crores	Ten-Lakhs	Lakhs	Ten-Thousands	Thousands	Hundreds	Tens	Ones
(*i*)		9	0	4	0	6	0	0	2
(*ii*)	1	2	0	7	0	9	2	0	7
(*iii*)				4	0	4	0	4	0
(*iv*)	2	1	6	0	0	5	0	1	4

II. Face value and Place value (or Local Value) of a Digit in a Numeral

(*i*) The face value of a digit in a numeral is its own value, at whatever place it may be.

Ex. ***In the numeral 6872, the face value of 2 is 2, the face value of 7 is 7, the face value of 8 is 8 and the face value of 6 is 6.***

(*ii*) In a given numeral:

Place value of ones digit = (ones digit) × 1,

Place value of tens digit = (tens digit) × 10,

Place value of hundreds digit = (hundreds digit) × 100 and so on.

Ex. ***In the numeral 70984, we have***

Place value of 4 = (4 × 1) = 4,

Place value of 8 = (8 × 10) = 80,

Place value of 9 = (9 × 100) = 900,

Place value of 7 = (7 × 10000) = 70000.

Note: Place value of 0 in a given numeral is 0, at whatever place it may be.

III. Various Types of Numbers

1. Natural Numbers: *Counting numbers are called natural numbers.*

Thus, 1, 2, 3, 4, ... are all natural numbers.

2. Whole Numbers: *All counting numbers, together with 0, form the set of whole numbers.*

Thus, 0, 1, 2, 3, 4, ... are all whole numbers.

3. Integers: *All counting numbers, zero and negatives of counting numbers, form the set of integers.*
Thus, ..., – 3, – 2, – 1, 0, 1, 2, 3, ... are all integers.
Set of positive integers = {1, 2, 3, 4, 5, 6, ...}
Set of negative integers = { – 1, – 2, – 3, – 4, – 5, – 6, ...}
Set of all non-negative integers = {0, 1, 2, 3, 4, 5, ...}

4. Even Numbers: *A counting number divisible by 2 is called an even number.*
Thus, 0, 2, 4, 6, 8, 10, 12, ... etc. are all even numbers.

5. Odd Numbers: *A counting number not divisible by 2 is called an odd number.*
Thus, 1, 3, 5, 7, 9, 11, 13, ... etc. are all odd numbers.

6. Prime Numbers: *A counting number is called a prime number if it has exactly two factors, namely itself and 1.*
Ex. All prime numbers less than 100 are:
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

7. Composite Numbers: *All counting numbers, which are not prime, are called composite numbers.*
A composite number has more than 2 factors.

8. Perfect Numbers: *A number, the sum of whose factors (except the number itself), is equal to the number, is called a perfect number,* e.g. 6, 28, 496
The factors of 6 are 1, 2, 3 and 6. And, 1 + 2 + 3 = 6.
The factors of 28 are 1, 2, 4, 7, 14 and 28. And, 1 + 2 + 4 + 7 + 14 = 28.

9. Co-primes (or Relative Primes): *Two numbers whose H.C.F. is 1 are called co-prime numbers,*
Ex. (2, 3), (8, 9) are pairs of co-primes.

10. Twin Primes: *Two prime numbers whose difference is 2 are called twin-primes,*
Ex. (3, 5), (5, 7), (11, 13) are pairs of twin-primes.

11. Rational Numbers: *Numbers which can be expressed in the form* $\frac{p}{q}$, *where p and q are integers and* $q \neq 0$, *are called rational numbers.*
Ex. $\frac{1}{8}, \frac{-8}{11}, 0, 6, 5\frac{2}{3}$ etc.

12. Irrational Numbers: *Numbers which when expressed in decimal would be in non-terminating and non-repeating form, are called irrational numbers.*
Ex. $\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi, e, 0.231764735......$

IV. Important Facts:

1. All natural numbers are whole numbers.
2. All whole numbers are not natural numbers.
   0 is a whole number which is not a natural number.
3. Even number + Even number = Even number
   Odd number + Odd number = Even number
   Even number + Odd number = Odd number
   Even number – Even number = Even number
   Odd number – Odd number = Even number
   Even number – Odd number = Odd number
   Odd number – Even number = Odd number
   Even number × Even number = Even number
   Odd number × Odd number = Odd number
   Even number × Odd number = Even number
4. The smallest prime number is 2.
5. The only even prime number is 2.
6. The first odd prime number is 3.
7. 1 is a unique number – neither prime nor composite.
8. The least composite number is 4.
9. The least odd composite number is 9.
10. **Test for a Number to be Prime:**
    Let p be a given number and let n be the smallest counting number such that $n^2 \geq p$.
    Now, test whether p is divisible by any of the prime numbers less than or equal to n. If yes, then p is not prime otherwise, p is prime.

Ex. ***Test, which of the following are prime numbers?***

(i) 137 ***(ii) 173*** ***(iii) 319*** ***(iv) 437*** ***(v) 811***

Sol. (*i*) We know that $(12)^2 > 137$.
Prime numbers less than 12 are 2, 3, 5, 7, 11.
Clearly, none of them divides 137.
$\therefore$ 137 is a prime number.

(*ii*) We know that $(14)^2 > 173$.
Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.
Clearly, none of them divides 173.
$\therefore$ 173 is a prime number.

(*iii*) We know that $(18)^2 > 319$.
Prime numbers less than 18 are 2, 3, 5, 7, 11, 13, 17.
Out of these prime numbers, 11 divides 319 completely.
$\therefore$ 319 is not a prime number.

(*iv*) We know that $(21)^2 > 437$.
Prime numbers less than 21 are 2, 3, 5, 7, 11, 13, 17, 19.
Clearly, 437 is divisible by 19.
$\therefore$ 437 is not a prime number.

(*v*) We know that $(30)^2 > 811$.
Prime numbers less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.
Clearly, none of these numbers divides 811.
$\therefore$ 811 is a prime number.

V. Important Formulae:

(*i*) $(a + b)^2 = a^2 + b^2 + 2ab$

(*ii*) $(a - b)^2 = a^2 + b^2 - 2ab$

(*iii*) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(*iv*) $(a + b)^2 - (a - b)^2 = 4ab$

(*v*) $(a + b)^3 = a^3 + b^3 + 3ab\ (a + b)$

(*vi*) $(a - b)^3 = a^3 - b^3 - 3ab\ (a - b)$

(*vii*) $a^2 - b^2 = (a + b)\ (a - b)$

(*viii*) $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

(*ix*) $a^3 + b^3 = (a + b)\ (a^2 + b^2 - ab)$

(*x*) $a^3 - b^3 = (a - b)\ (a^2 + b^2 + ab)$

(*xi*) $a^3 + b^3 + c^3 - 3abc = (a + b + c)\ (a^2 + b^2 + c^2 - ab - bc - ca)$

(*xii*) If $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

TESTS OF DIVISIBILITY

1. Divisibility By 2:

A number is divisible by 2 if its unit digit is any of 0, 2, 4, 6, 8.

Ex. 58694 is divisible by 2, while 86945 is not divisible by 2.

2. Divisibility By 3:

A number is divisible by 3 only when the sum of its digits is divisible by 3.

Ex. (*i*) In the number 695421, the sum of digits = 27, which is divisible by 3.
$\therefore$ 695421 is divisible by 3.

(*ii*) In the number 948653, the sum of digits = 35, which is not divisible by 3.
$\therefore$ 948653 is not divisible by 3.

3. Divisibility By 9:

A number is divisible by 9 only when the sum of its digits is divisible by 9.

Ex. (*i*) In the number 246591, the sum of digits = 27, which is divisible by 9.
$\therefore$ 246591 is divisible by 9.

(*ii*) In the number 734519, the sum of digits = 29, which is not divisible by 9.
$\therefore$ 734519 is not divisible by 9.

4. Divisibility By 4:

A number is divisible by 4 if the number formed by its last two digits is divisible by 4.

Ex. (*i*) 6879376 is divisible by 4, since 76 is divisible by 4.

(*ii*) 496138 is not divisible by 4, since 38 is not divisible by 4.

5. Divisibility By 8:

A number is divisible by 8 if the number formed by its last three digits is divisible by 8.

Ex. (*i*) In the number 16789352, the number formed by last 3 digits, namely 352 is divisible by 8.
$\therefore$ 16789352 is divisible by 8.
(*ii*) In the number 576484, the number formed by last 3 digits, namely 484 is not divisible by 8.
$\therefore$ 576484 is not divisible by 8.

6. Divisibility By 10:
A number is divisible by 10 only when its unit digit is 0.
Ex. (*i*) 7849320 is divisible by 10, since its unit digit is 0.
(*ii*) 678405 is not divisible by 10, since its unit digit is not 0.

7. Divisibility By 5:
A number is divisible by 5 only when its unit digit is 0 or 5.
Ex. (*i*) Each of the numbers 76895 and 68790 is divisible by 5.

8. Divisibility By 11:
A number is divisible by 11 if the difference between the sum of its digits at odd places and the sum of its digits at even places is either 0 or a number divisible by 11.
Ex. (*i*) Consider the number 29435417.
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (7 + 4 + 3 + 9) – (1 + 5 + 4 + 2) = (23 – 12) = 11, which is divisible by 11.
$\therefore$ 29435417 is divisible by 11.
(*ii*) Consider the number 57463822.
(Sum of its digits at odd places) – (Sum of its digits at even places)
= (2 + 8 + 6 + 7) – (2 + 3 + 4 + 5) = (23 – 14) = 9, which is not divisible by 11.
$\therefore$ 57463822 is not divisible by 11.

9. Divisibility By 25:
A number is divisible by 25 if the number formed by its last two digits is either 00 or divisible by 25.
Ex. (*i*) In the number 63875, the number formed by last 2 digits, namely 75 is divisible by 25.
$\therefore$ 63875 is divisible by 25.
(*ii*) In the number 96445, the number formed by last 2 digits, namely 45 is not divisible by 25.
$\therefore$ 96445 is not divisible by 25.

10. Divisibility By 7 or 13:
Divide the number into groups of 3 digits (starting from right) and find the difference between the sum of the numbers in odd and even places. If the difference is 0 or divisible by 7 or 13 (as the case may be), it is divisible by 7 or 13.
Ex. (*i*) 4537792 $\rightarrow$ 4 / 537 / 792
(792 + 4) – 537 = 259, which is divisible by 7 but not by 13.
$\therefore$ 4537792 is divisible by 7 and not by 13.
(*ii*) 579488 $\rightarrow$ 579 / 488
579 – 488 = 91, which is divisible by both 7 and 13.
$\therefore$ 579488 is divisible by both 7 and 13.

11. Divisibility By 16:
A number is divisible by 16, if the number formed by its last 4 digits is divisible by 16.
Ex. (*i*) In the number 463776, the number formed by last 4 digits, namely 3776, is divisible by 16.
$\therefore$ 463776 is divisible by 16.
(*ii*) In the number 895684, the number formed by last 4 digits, namely 5684, is not divisible by 16.
$\therefore$ 895684 is not divisible by 16.

12. Divisibility By 6: *A number is divisible by 6, if it is divisible by both 2 and 3.*

13. Divisibility By 12: *A number is divisible by 12, if it is divisible by both 3 and 4.*

14. Divisibility By 15: *A number is divisible by 15, if it is divisible by both 3 and 5.*

15. Divisibility By 18: *A number is divisible by 18, if it is divisible by both 2 and 9.*

16. Divisibility By 14: *A number is divisible by 14, if it is divisible by both 2 and 7.*

17. Divisibility By 24: *A given number is divisible by 24, if it is divisible by both 3 and 8.*

18. Divisibility By 40: *A given number is divisible by 40, if it is divisible by both 5 and 8.*

19. Divisibility By 80: *A given number is divisible by 80, if it is divisible by both 5 and 16.*

Note: *If a number is divisible by p as well as q, where p and q are co-primes, then the given number is divisible by pq.*

If p and q are not co-primes, then the given number need not be divisible by pq, even when it is divisible by both p and q.

Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4 \times 6) = 24$, since 4 and 6 are not co-primes.

VI. Factorial of a Number

Let n be a positive integer.

Then, the continued product of first n natural numbers is called factorial n, denoted by $n\,!$ or $\underline{|n}$.

Thus, $n\,! = n\,(n-1)\,(n-2)$ 3.2.1

Ex. $5\,! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Note: $0\,! = 1$

VII. Modulus of a Number

$$|x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$$

Ex. $|-5| = 5$, $|4| = 4$, $|-1| = 1$, etc.

VIII. Greatest Integral Value

The greatest integral value of an integer x, denoted by [x], is defined as the greatest integer not exceeding x.

Ex. $[1.35] = 1$, $\left[\dfrac{11}{4}\right] = \left[2\dfrac{3}{4}\right] = 2$, etc.

IX. Multiplication BY Short cut Methods

1. Multiplication By Distributive Law:

(*i*) $a \times (b + c) = a \times b + a \times c$ (*ii*) $a \times (b - c) = a \times b - a \times c$

Ex. (*i*) $567958 \times 99999 = 567958 \times (100000 - 1) = 567958 \times 100000 - 567958 \times 1$

$= (56795800000) - 567958) = 56795232042$.

(*ii*) $978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000$.

2. Multiplication of a Number By 5^n: Put n zeros to the right of the multiplicand and divide the number so formed by 2^n.

Ex. $975436 \times 625 = 975436 \times 5^4 = \dfrac{9754360000}{16} = 609647500$.

X. Division Algorithm or Euclidean Algorithm

If we divide a given number by another number, then:

Dividend = (Divisor × Quotient) + Remainder

Important Facts:

1. (*i*) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n.
   (*ii*) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n.
   (*iii*) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n.
2. **To find the highest power of a prime number p in $n\,!$**

Highest power of p in $n! = \left[\dfrac{n}{p}\right] + \left[\dfrac{n}{p^2}\right] + \left[\dfrac{n}{p^3}\right] + \ldots\ldots + \left[\dfrac{n}{p^r}\right]$, where $p^r \leq n < p^{r+1}$

SOLVED EXAMPLES

Ex. 1. ***Simplify:*** (*i*) 8888 + 888 + 88 + 8

(*ii*) 715632 – 631104 – 9874 – 999 **(LIC, ADO, 2007)**

Sol. (*i*)

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ +\quad 8 \\ \hline 9872 \\ \hline \end{array}$$

(*ii*) Given exp = 715632 – (631104 + 9874 + 99)

= 715632 – 641077 = 74555.

$$\begin{array}{r} 631104 \\ 9874 \\ +\quad 99 \\ \hline 641077 \\ \hline \end{array} \qquad \begin{array}{r} 715632 \\ -\ 641077 \\ \hline 74555 \\ \hline \end{array}$$

50. Let $\frac{x}{1776} = \frac{111}{x}$. Then, $x^2 = 111 \times 1776 = 111 \times 111 \times 16$

$\Rightarrow x = \sqrt{(111)^2 \times (4)^2} = 111 \times 4 = 444.$

51. Let $\frac{4\frac{1}{2}}{x} = \frac{x}{32}$. Then, $x^2 = 32 \times \frac{9}{2} = 144 \Leftrightarrow x = \sqrt{144} = 12.$

52. Let $\frac{x}{\sqrt{128}} = \frac{\sqrt{162}}{x}$. Then, $x^2 = \sqrt{128 \times 162} = \sqrt{64 \times 2 \times 18 \times 9}$

$= \sqrt{8^2 \times 6^2 \times 3^2} = 8 \times 6 \times 3 = 144.$

$\therefore \quad x = \sqrt{144} = 12.$

53. $\sqrt{x + \frac{x}{y}} = x\sqrt{\frac{x}{y}} \Rightarrow x + \frac{x}{y} = x^2 \cdot \frac{x}{y} \Rightarrow \frac{xy + x}{y} = \frac{x^3}{y}$

$\Rightarrow \quad xy + x = x^3$

$\Rightarrow \quad y + 1 = x^2$

$\Rightarrow \quad y = x^2 - 1.$

54. $n^2 = (25)^{64} \times (64)^{25} = (5^2)^{64} \times (2^6)^{25}$

$= 5^{128} \times 2^{150} = 5^{128} \times 2^{128} \times 2^{22}$

$\Rightarrow n = 5^{64} \times 2^{64} \times 2^{11} = (5 \times 2)^{64} \times 2^{11} = 10^{64} \times 2048.$

$\therefore$ Sum of digits of $n = 2 + 0 + 4 + 8 = 14.$

55. $\frac{0.13}{p^2} = 13$

$\Leftrightarrow \quad p^2 = \frac{0.13}{13} = \frac{1}{100}$

$\Leftrightarrow \quad p = \sqrt{\frac{1}{100}} = \frac{1}{10} = 0.1.$

56. Let the required number be x. Then, $\frac{x}{\sqrt{0.25}} = 25$

$\Leftrightarrow \quad \frac{x}{0.5} = 25$

$\Leftrightarrow \quad x = 25 \times 0.5 = 12.5.$

57. $\sqrt{3^n} = 729 = 3^6 \Leftrightarrow (\sqrt{3^n})^2 = (3^6)^2 \Leftrightarrow 3^n = 3^{12} \Leftrightarrow n = 12.$

58. $\sqrt{18 \times 14 \times x} = 84 \Leftrightarrow 18 \times 14 \times x = 84 \times 84$

$\Leftrightarrow x = \frac{84 \times 84}{18 \times 14} = 28.$

59. Let $28\sqrt{x} + 1426 = 3 \times 718.$

Then, $28\sqrt{x} = 2154 - 1426 \Leftrightarrow 28\sqrt{x} = 728 \Leftrightarrow \sqrt{x} = 26$

$\Leftrightarrow x = (26)^2 = 676.$

60. Let $\sqrt{\frac{x}{169}} = \frac{54}{39}$. Then, $\frac{\sqrt{x}}{13} = \frac{54}{39} \Leftrightarrow \sqrt{x} = \left(\frac{54}{39} \times 13\right) = 18$

$\Leftrightarrow \quad x = (18)^2 = 324.$

61. $\frac{\sqrt{x}}{\sqrt{441}} = 0.02 \quad \Leftrightarrow \quad \frac{\sqrt{x}}{21} = 0.02$

$\Leftrightarrow \quad \sqrt{x} = 0.02 \times 21 = 0.42 \quad \Leftrightarrow \quad x = (0.42)^2 = 0.1764.$

62. Let $\sqrt{\frac{.0196}{x}} = 0.2$. Then, $\frac{.0196}{x} = 0.04$

$\Leftrightarrow \quad x = \frac{.0196}{.04} = \frac{1.96}{4} = .49.$

63. Let $\sqrt{0.0169 \times x} = 1.3$. Then, $0.0169x = (1.3)^2 = 1.69$

$\Rightarrow x = \frac{1.69}{0.0169} = 100.$

64. $37 + \sqrt{.0615 + x} = 37.25 \quad \Leftrightarrow \quad \sqrt{.0615 + x} = 0.25$

$\Leftrightarrow .0615 + x = (0.25)^2 = 0.0625$

$\Leftrightarrow x = .001 = \frac{1}{10^3} = 10^{-3}.$

65. $\sqrt{(x-1)(y+2)} = 7 \Rightarrow (x - 1)(y + 2) = (7)^2 \Rightarrow (x - 1) = 7$ and $(y + 2) = 7 \Rightarrow x = 8$ and $y = 5.$

66. $\frac{\sqrt{a}}{\sqrt{b}} = \frac{.004 \times .4}{\sqrt{.04 \times .4}} \Rightarrow \frac{a}{b} = \frac{.004 \times .4 \times .004 \times .4}{.04 \times .4} = \frac{.0000064}{.04}$

$\Rightarrow \quad \frac{a}{b} = \frac{.00064}{4} = .00016 = \frac{16}{10^5} = 16 \times 10^{-5}.$

67. Let the number be x. Then,

$\frac{3}{5}x^2 = 126.15 \quad \Leftrightarrow \quad x^2 = \left(126.15 \times \frac{5}{3}\right) = 210.25$

$\Leftrightarrow \quad x = \sqrt{210.25} = 14.5.$

68. $\sqrt{\frac{0.361}{0.00169}} = \sqrt{\frac{0.36100}{0.00169}} = \sqrt{\frac{36100}{169}} = \frac{190}{13}.$

69. $\sqrt{\frac{48.4}{0.289}} = \sqrt{\frac{48.400}{0.289}} = \sqrt{\frac{48400}{289}} = \frac{220}{17} = 12\frac{16}{17}.$

70. $\sqrt{1 + \frac{x}{169}} = \frac{14}{13} \quad \Rightarrow \quad 1 + \frac{x}{169} = \frac{196}{169}$

$\Rightarrow \quad \frac{x}{169} = \left(\frac{196}{169} - 1\right) = \frac{27}{169} \quad \Rightarrow \quad x = 27.$

71. $\sqrt{1 + \frac{55}{729}} = 1 + \frac{x}{27} \quad \Rightarrow \quad \sqrt{\frac{784}{729}} = \frac{27 + x}{27}$

$\Rightarrow \quad \frac{28}{27} = \frac{27 + x}{27} \quad \Rightarrow \quad 27 + x = 28 \quad \Rightarrow \quad x = 1.$

72. $\sqrt{\frac{4}{3}} - \sqrt{\frac{3}{4}} = \frac{\sqrt{4}}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{4} \times \sqrt{4} - \sqrt{3} \times \sqrt{3}}{\sqrt{12}} = \frac{4 - 3}{2\sqrt{3}} = \frac{1}{2\sqrt{3}}.$

73.

1	$\overline{2}.\overline{00}\ \overline{00}\ \overline{00}$ (1.414
	1
24	100
	96
281	400
	281
2824	11900
	11296

$\therefore \quad \sqrt{2} = 1.414$

74. $2\sqrt{27} - \sqrt{75} + \sqrt{12} = 2\sqrt{9 \times 3} - \sqrt{25 \times 3} + \sqrt{4 \times 3}$

$= 6\sqrt{3} - 5\sqrt{3} + 2\sqrt{3} = 3\sqrt{3}.$

12 Profit and Loss

IMPORTANT FACTS AND FORMULAE

Cost Price: The price at which an article is purchased, is called its ***cost price***, abbreviated as ***C.P.***

Selling Price: The price at which an article is sold, is called its ***selling price***, abbreviated as ***S.P.***

Profit or Gain: If S.P. is greater than C.P., the seller is said to have a ***profit*** or ***gain***.

Loss: If S.P. is less than C.P., the seller is said to have incurred a ***loss***.

I. Gain = (S.P.) – (C.P.)

II. Loss = (C.P.) – (S.P.)

III. Loss or gain is always reckoned on C.P.

IV. $\text{Gain\%} = \left(\dfrac{\text{Gain} \times 100}{\text{C.P.}}\right)$

V. $\text{Loss\%} = \left(\dfrac{\text{Loss} \times 100}{\text{C.P.}}\right)$

VI. $\text{S.P.} = \dfrac{(100 + \text{Gain\%})}{100} \times \text{C.P.}$

VII. $\text{S.P.} = \dfrac{(100 - \text{Loss\%})}{100} \times \text{C.P.}$

VIII. $\text{C.P.} = \dfrac{100}{(100 + \text{Gain\%})} \times \text{S.P.}$

IX. $\text{C.P.} = \dfrac{100}{(100 - \text{Loss\%})} \times \text{S.P.}$

X. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.

XI. If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.

XII. When a person sells two similar items, one at a gain of say, x%, and the other at a loss of x%, then the seller always incurs a loss given by:

$$\text{Loss\%} = \left(\frac{\text{Common Loss and Gain\%}}{10}\right)^2 = \left(\frac{x}{10}\right)^2.$$

XIII. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain\%} = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\%.$$

XIV. If a trader professes to sell his goods at a profit of x% but uses false weight which is y% less than the actual weight, then

$$\text{Gain \%} = \left\{\left(\frac{x + y}{100 - y}\right) \times 100\right\}\%$$

XV. If a trader professes to sell his goods at a loss of x% but uses false weight which is y% less than the actual weight, then

$$\text{Gain or Loss \%} = \left\{\left(\frac{y - x}{100 - y}\right) \times 100\right\}\%$$

according as the sign is + ve or – ve.

13 Ratio and Proportion

IMPORTANT CONCEPTS

I. Ratio: The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $\boldsymbol{a : b}$.

In the ratio $a : b$, we call a as the **first term** or **antecedent** and b, the **second term** or **consequent**.

Ex. The ratio 5 : 9 represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.

Rule: The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.

Ex. 4 : 5 = 8 : 10 = 12 : 15 etc. Also, 4 : 6 = 2 : 3.

II. Proportion: *The equality of two ratios is called proportion.*

If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion.

Here a and d are called **extremes**, while b and c are called **mean terms**.

Product of means = Product of extremes.

Thus, $\boldsymbol{a : b :: c : d \;\Leftrightarrow\; (b \times c) = (a \times d)}$.

III. (*i*) **Fourth Proportional:** If $a : b = c : d$, then d is called the fourth proportional to a, b, c.

(*ii*) **Third Proportional:** If $a : b = b : c$, then c is called the third proportional to a and b.

(*iii*) **Mean Proportional:** Mean proportional between a and b is $\sqrt{ab}$.

IV. (*i*) **Comparison of Ratios:** We say that $(a : b) > (c : d) \;\Leftrightarrow\; \frac{a}{b} > \frac{c}{d}$.

(*ii*) **Compounded Ratio:**

The compounded ratio of the ratios $(a : b)$, $(c : d)$, $(e : f)$ is $(ace : bdf)$.

V. (*i*) **Duplicate ratio** of $(a : b)$ is $(a^2 : b^2)$.

(*ii*) **Sub-duplicate ratio** of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.

(*iii*) **Triplicate ratio** of $(a : b)$ is $(a^3 : b^3)$.

(*iv*) **Sub-triplicate ratio** of $(a : b)$ is $\left(a^{\frac{1}{3}} : b^{\frac{1}{3}}\right)$.

(*v*) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. (componendo and dividendo)

VI. Variation:

(*i*) We say that x is directly proportional to y, if $x = ky$ for some constant k and we write, $x \propto y$.

(*ii*) We say that x is inversely proportional to y, if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

VII. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid in the final mixture = $\left[x\left(1-\frac{y}{x}\right)^n\right]$ units.

17 Time and Work

IMPORTANT FACTS AND FORMULAE

I. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.

II. If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.

II. If A is thrice as good a workman as B, then :

Ratio of work done by A and B = 3 : 1.

Ratio of times taken by A and B to finish a work = 1 : 3.

ILLUSTRATIVE EXAMPLES

Ex. 1. ***If Roger can do a piece of work in 8 days and Antony can complete the same work in 5 days, in how many days will both of them together complete it?*** **(L.I.C., 2008)**

Sol. Roger's 1 day's work = $\frac{1}{8}$; Antony's 1 day's work = $\frac{1}{5}$.

(Roger + Antony)'s 1 day's work = $\left(\frac{1}{8}+\frac{1}{5}\right)=\frac{13}{40}$.

$\therefore$ Both Roger and Antony will complete the work in $\frac{40}{13}=3\frac{1}{13}$ days.

Ex. 2. ***A and B together can complete a piece of work in 15 days and B alone in 20 days. In how many days can A alone complete the work?*** **(S.S.C., 2010)**

Sol. (A + B)'s 1 day's work = $\frac{1}{15}$; B's 1 day's work = $\frac{1}{20}$.

$\therefore$ A's 1 day's work = $\left(\frac{1}{15}-\frac{1}{20}\right)=\frac{1}{60}$.

Hence, A alone can complete the work in 60 days.

Ex. 3. ***A alone can complete a piece of work of ₹ 300 in 6 days; but by engaging an assistant, the work is completed in 4 days. Find the share to be received by the assistant.*** **(Section Officer's, 2008)**

Sol. Assistant's 1 day's work = $\frac{1}{4}-\frac{1}{6}=\frac{1}{12}$.

$\therefore$ A's share : Assistant's share = Ratio of their 1 day's work = $\frac{1}{6}:\frac{1}{12}=2:1$.

Hence, assistant's share = ₹ $\left(300\times\frac{1}{3}\right)$ = ₹ 100.

Ex. 4. ***A can do a work in 4 days, B in 5 days and C in 10 days. Find the time taken by A, B and C to do the work together.*** **(P.C.S., 2006)**

Sol. A's 1 day's work = $\frac{1}{4}$; B's 1 day's work = $\frac{1}{5}$; C's 1 day's work = $\frac{1}{10}$.

(A + B + C) 's 1 day's work = $\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{10}\right)=\frac{11}{20}$.

18 Time and Distance

IMPORTANT FACTS AND FORMULAE

I. Speed $= \left(\frac{\text{Distance}}{\text{Time}}\right)$, Time $= \left(\frac{\text{Distance}}{\text{Speed}}\right)$, Distance $=$ (Speed $\times$ Time)

II. x km/hr $= \left(x \times \frac{5}{18}\right)$ m/sec

III. x m/sec $= \left(x \times \frac{18}{5}\right)$ km/hr

IV. If the ratio of the speeds of A and B is $a : b$, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or $b : a$.

V. Suppose a man covers a certain distance at x km/hr and an equal distance at y km/hr. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$ km/hr.

VI. Suppose two men are moving in the same direction at u m/s and v m/s respectively, where $u > v$, then their relative speed $= (u - v)$ m/s.

VII. Suppose two men are moving in opposite directions at u m/s and v m/s respectively, then their relative speed $= (u + v)$ m/s.

VIII. If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journeys in a and b hours respectively, then
A's speed: B's speed $= \sqrt{b} : \sqrt{a}$.

SOLVED EXAMPLES

Ex. 1. *A train travels 82.6 km/hr. How many metres will it travel in 15 minutes?* **(E.S.I.C., 2006)**

Sol. Distance travelled in 1 min $= \left(\frac{82.6}{60}\right)$ km.

$\therefore$ Distance travelled in 15 min. $= \left(\frac{82.6}{60} \times 15\right)$ km $= 20.65$ km $= (20.65 \times 1000)$ m

$= 20650$ m.

Ex. 2. *How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?*

Sol. Aditya's speed $= 20$ km/hr $= \left(20 \times \frac{5}{18}\right)$ m/sec $= \frac{50}{9}$ m/sec.

$\therefore$ Time taken to cover 400 m $= \left(400 \times \frac{9}{50}\right)$ sec $= 72$ sec $= 1\frac{12}{60}$ min $= 1\frac{1}{5}$ min.

Ex. 3. *A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?*

Sol. Speed $= \left(\frac{750}{150}\right)$ m/sec $= 5$ m/sec $= \left(5 \times \frac{18}{5}\right)$ km/hr $= 18$ km/hr.

[$\because$ 2 min 30 sec = 150 sec]

Distance covered by second person in time $t = \frac{3}{8}$ round.

Speed of first person $= \frac{1}{4t}$;

Speed of second person $= \frac{3}{8t}$.

Since the two persons start from A and B respectively, so they shall meet each other when there is a difference of $\frac{7}{8}$ round between the two.

Relative speed of A and B $= \left(\frac{3}{8t} - \frac{1}{4t}\right) = \frac{1}{8t}$.

Time taken to cover $\frac{7}{8}$ round at this speed $= \left(\frac{7}{8} \times 8t\right) = 7t$.

124. Suppose after x km from the start B catches up with A. Then, the difference in the time taken by A to cover x km and that taken by B to cover x km is 4 hours.

$\therefore \frac{x}{4} - \frac{x}{10} = 4$ or $x = 26.7$ km.

125. Suppose the two trains meet x km from Delhi.

Then, $\frac{x}{60} - \frac{x}{80} = 2 \Leftrightarrow x = 480.$

126. Relative speed of the thief and policeman

$= (11 - 10)$ km/hr $= 1$ km/hr.

Distance covered in 6 minutes $= \left(\frac{1}{60} \times 6\right)$ km

$= \frac{1}{10}$ km $= 100$ m.

$\therefore$ Distance between the thief and policeman

$= (200 - 100)$ m $= 100$ m.

127. Relative speed of the car w.r.t. bus $= (50 - 30)$ km/hr

$= 20$ km/hr.

Required distance = Distance covered in 15 min at relative speed $= \left(20 \times \frac{1}{4}\right)$ km $= 5$ km.

128. Relative speed $= (10 - 8)$ km/hr $= 2$ km/hr.

Required time = Time taken to cover 100 m at relative speed

$= \left(\frac{100}{2000}\right)$ hr $= \frac{1}{20}$ hr $= \left(\frac{1}{20} \times 60\right)$ min $= 3$ min.

129. Suppose the thief is overtaken x hrs after 2.30 p.m.

Then, Distance covered by the thief in x hrs = Distance covered by the owner in $\left(x - \frac{1}{2}\right)$ hrs.

$\therefore\ 60x = 75\left(x - \frac{1}{2}\right) \Leftrightarrow 15x = \frac{75}{2} \Leftrightarrow x = \frac{5}{2}$ hrs.

So, the thief is overtaken at 5 p.m.

130. Distance covered by Aryan in 5 min $= (40 \times 5)$ m $= 200$ m.

Relative speed of Rahul w.r.t. Aryan $= (50 - 40)$ m/min

$= 10$ m/min.

Time taken to cover 200 m at relative speed

$= \left(\frac{200}{10}\right)$ min $= 20$ min.

Distance covered by the dog in 20 min $= (60 \times 20)$ m

$= 1200$ m.

131. Let the thief be caught x metres from the place where the policeman started running.

Let the speed of the policeman and the thief be $5y$ m/s and $4y$ m/s respectively.

Then, time taken by the policeman to cover x metres

= time taken by the thief to cover $(x - 100)$ m

$\Rightarrow \frac{x}{5y} = \frac{(x - 100)}{4y} \Rightarrow 4x = 5(x - 100) \Rightarrow x = 500.$

So, the thief ran (500 – 100) i.e. 400 m before being caught.

132. Distance covered by A in 4 hrs $= (4 \times 4)$ km $= 16$ km.

Relative speed of B w.r.t. A $= (10 - 4)$ km/hr $= 6$ km/hr.

Time taken to cover 16 km at relative speed

$= \left(\frac{16}{6}\right)$ hrs $= \frac{8}{3}$ hrs.

Distance covered by B in $\frac{8}{3}$ hrs $= \left(10 \times \frac{8}{3}\right)$ km $= \left(\frac{80}{3}\right)$ km

$= 26.7$ km.

133. Let the speed of the goods train be x km/hr. Then, relative speed $= (80 - x)$ km/hr.

Distance covered by goods train in 6 hrs at x km/hr

= Distance covered by passenger train in 4 hrs at $(80 - x)$ km/hr

$\Rightarrow 6x = 4(80 - x) \Rightarrow 10x = 320 \Rightarrow x = 32$ km/hr.

134. Error = Time taken to cover 10 m at 300 m/sec

$= \left(\frac{10}{300}\right)$ sec $= \frac{1}{30}$ sec ≈ 0.03 sec.

135. Clearly, the two persons would be maximum distance apart when they stand in opposite directions to the point at which sound is produced, and minimum distance apart when they stand in the same direction.

$\therefore$ Maximum distance between the two persons

= Distance covered by sound in (6 + 5) seconds, i.e. 11 sec

$= (300 \times 11)$ m $= 3300$ m $= 3.3$ km.

And, minimum distance between the two persons

= Distance covered by sound in (6 – 5) sec., i.e. 1 sec

$= 300$ m $= 0.3$ km.

136. Let the speed of the man be x m/sec.

Then, Distance travelled by the man in 5 min 52 sec

= Distance travelled by sound in 8 sec

$\Leftrightarrow x \times 352 = 330 \times 8$

$\Leftrightarrow x = \left(\frac{330 \times 8}{352}\right)$ m/sec.

$= \left(\frac{330 \times 8}{352} \times \frac{18}{5}\right)$ km/hr $= 27$ km/hr.

137. To be (18 + 20) km apart, they take 1 hour.

To be 47.5 km apart, they take $\left(\frac{1}{38} \times 47.5\right)$ hrs $= 1\frac{1}{4}$ hrs.

138. Suppose they meet after x hours. Then,

20 Problems on Trains

IMPORTANT FACTS AND FORMULAE

I. $a \text{ km/hr} = \left(a \times \frac{5}{18}\right) \text{m/s}.$ **II.** $\boxed{a \text{ m/s} = \left(a \times \frac{18}{5}\right) \text{km/hr}.}$

III. Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.

IV. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.

V. Suppose two trains or two bodies are moving in the same direction at u m/s and v m/s, where $u > v$, then their relative speed = $(u - v)$ m/s.

VI. Suppose two trains or two bodies are moving in opposite directions at u m/s and v m/s, then their relative speed = $(u + v)$ m/s.

VII. If two trains of length a metres and b metres are moving in opposite directions at u m/s and v m/s, then time taken by the trains to cross each other = $\frac{(a+b)}{(u+v)}$ sec.

VIII. If two trains of length a metres and b metres are moving in the same direction at u m/s and v m/s, then the time taken by the faster train to cross the slower train = $\frac{(a+b)}{(u-v)}$ sec.

IX. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then **(*A's speed*) : (*B's speed*) = $(\sqrt{b} : \sqrt{a})$.**

SOLVED EXAMPLES

Ex. 1. ***A 100-m long train is running at the speed of 30 km/hr. Find the time taken by it to pass a man standing near the railway line.***

Sol. Speed of the train = $\left(30 \times \frac{5}{18}\right) \text{m/sec} = \left(\frac{25}{3}\right) \text{m/sec}.$

Distance moved in passing the standing man = 100 m.

Required time taken = $\frac{100}{\left(\frac{25}{3}\right)} = \left(100 \times \frac{3}{25}\right) \text{sec} = 12 \text{ sec}.$

Ex. 2. ***A train is moving at a speed of 132 km/hr. If the length of the train is 110-m, how long will it take to cross a railway platform 165-m long?***

Sol. Speed of train = $\left(132 \times \frac{5}{18}\right) \text{m/sec} = \left(\frac{110}{3}\right) \text{m/sec}.$

Distance covered in passing the platform = (110 + 165) m = 275 m.

$\therefore$ Time taken = $\left(275 \times \frac{3}{110}\right) \text{sec} = \frac{15}{2} \text{sec} = 7\frac{1}{2} \text{sec}.$

Ex. 3. ***A 160-m long train crosses a 160-m long platform in 16 seconds. Find the speed of the train.*** **(R.R.B., 2009)**

Sol. Distance covered in passing the platform = (160 + 160) m = 320 m.

21 Alligation or Mixture

IMPORTANT FACTS AND FORMULAE

I. Alligation: It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.

II. Mean Price: The cost price of a unit quantity of the mixture is called the mean price.

III. Rule of Alligation: If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}}\right) = \frac{\text{(C.P. of dearer)} - \text{(Mean price)}}{\text{(Mean price)} - \text{(C.P. of cheaper)}}.$$

We present as under :

C.P. of a unit quantity of cheaper (c) | C.P. of a unit quantity of dearer (d)

Mean price (m)

$(d - m)$ | $(m - c)$

$\therefore$ (Cheaper quantity) : (Dearer quantity) = $(d - m) : (m - c)$.

IV. Suppose a container contains x units of liquid from which y units are taken out and replaced by water.

After n operations, the quantity of pure liquid $= \left[x\left(1 - \frac{y}{x}\right)^n\right]$ units.

SOLVED EXAMPLES

Ex. 1. *In what ratio must rice at ₹ 9.30 per kg be mixed with rice at ₹ 10.80 per kg so that the mixture be worth ₹ 10 per kg ?*

Sol. By the rule of alligation, we have :

C.P. of 1 kg rice of 1st kind (in paise) 930 | C.P. of 1 kg rice of 2nd kind (in paise) 1080

Mean price (in paise) 1000

80 | 70

$\therefore$ Required ratio = 80 : 70 = 8 : 7.

Ex. 2. *How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for ₹ 20 so as to have a mixture worth ₹ $10\frac{2}{3}$ a litre ?*

Sol. C.P. of 1 litre of milk = ₹ $\left(20 \times \frac{2}{3}\right)$ = ₹ $\frac{40}{3}$.

22 Simple Interest

IMPORTANT FACTS AND FORMULAE

I. Principal: The money borrowed or lent out for a certain period is called the ***principal*** or the ***sum.***

II. Interest: Extra money paid for using other's money is called ***interest.***

III. Simple Interest (S.I.): If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called ***simple interest.***

Let Principal = P, Rate = $R\%$ per annum (p.a.) and Time = T years.

Then, (*i*) S.I. = $\left(\dfrac{P \times R \times T}{100}\right)$.

(*ii*) $P = \left(\dfrac{100 \times S.I.}{R \times T}\right)$; $R = \left(\dfrac{100 \times S.I.}{P \times T}\right)$ and $T = \left(\dfrac{100 \times S.I.}{P \times R}\right)$.

SOLVED EXAMPLES

Ex. 1. ***Find the simple interest on ₹ 68000 at*** $16\frac{2}{3}\%$ ***per annum for 9 months.***

Sol. P = ₹ 68000, $R = \dfrac{50}{3}\%$ p.a. and $T = \dfrac{9}{12}$ years $= \dfrac{3}{4}$ years.

$\therefore$ S.I. $= \left(\dfrac{P \times R \times T}{100}\right) =$ ₹ $\left(68000 \times \dfrac{50}{3} \times \dfrac{3}{4} \times \dfrac{1}{100}\right) =$ ₹ 8500.

Ex. 2. ***Find the simple interest on ₹ 3000 at*** $6\frac{1}{4}\%$ ***per annum for the period from 4th Feb., 2009 to 18th April, 2009.***

Sol. Time = (24 + 31 + 18) days = 73 days = $\dfrac{73}{365}$ year $= \dfrac{1}{5}$ year.

P = ₹ 3000 and $R = 6\frac{1}{4}\%$ p.a. $= \dfrac{25}{4}\%$ p.a.

$\therefore$ S.I. = ₹ $\left(3000 \times \dfrac{25}{4} \times \dfrac{1}{5} \times \dfrac{1}{100}\right) =$ ₹ 37.50.

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted.

Ex. 3. ***A sum at simple interest at*** $13\frac{1}{2}\%$ ***per annum amounts to ₹ 2502.50 after 4 years. Find the sum.***

Sol. Let sum be ₹ x. Then, S.I. = ₹ $\left(x \times \dfrac{27}{2} \times 4 \times \dfrac{1}{100}\right) =$ ₹ $\dfrac{27x}{50}$.

$\therefore$ Amount = ₹ $\left(x + \dfrac{27x}{50}\right) =$ ₹ $\dfrac{77x}{50}$.

$\therefore \dfrac{77x}{50} = 2502.50 \Leftrightarrow x = \dfrac{2502.50 \times 50}{77} = 1625.$

Hence, sum = ₹ 1625.

23 Compound Interest

Compound Interest: Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the* **Compound Interest** (*abbreviated as C.I.*) *for that period*.

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compounded Annually:

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^n$$

II. When interest is compounded Half-yearly:

$$\text{Amount} = P\left[1+\frac{(R/2)}{100}\right]^{2n}$$

III. When interest is compounded Quarterly:

$$\text{Amount} = P\left[1+\frac{(R/4)}{100}\right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P\left(1+\frac{R}{100}\right)^3 \times \left(1+\frac{\frac{2}{5}R}{100}\right)$$

V. When rates are different for different years, say R_1%, R_2%, R_3% for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right).$$

VI. Present worth of ₹ x due n years hence is given by:

$$\text{Present Worth} = \frac{x}{\left(1+\frac{R}{100}\right)^n}.$$

SOLVED EXAMPLES

Ex. 1. *After 3 years, how much compound interest will be obtained on ₹ 7800 at the interest rate of 5% per annum?* **(R.R.B., 2009)**

Sol. $\text{Amount} = ₹\left[7800\times\left(1+\frac{5}{100}\right)^3\right] = ₹\left(7800\times\frac{21}{20}\times\frac{21}{20}\times\frac{21}{20}\right)$

$= ₹\left(\frac{361179}{40}\right) = ₹\ 9029.475.$

$\therefore$ C.I. = ₹ (9029.475 – 7800) = ₹ 1229. 475.

24 Area

FUNDAMENTAL CONCEPTS

I. Results on Triangles:

1. Sum of the angles of a triangle is 180°.
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras' Theorem:** In a right-angled triangle,

$$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$$

4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the **median.**
5. The point where the three medians of a triangle meet, is called **centroid.** The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The line joining the mid-points of any two sides of a triangle is parallel to the third side and equal to half of it.
9. The four triangles formed by joining the mid-points of the sides of a given triangle are equal in area, each equal to one-fourth of the given triangle.
10. The ratio of the areas of two similar triangles is equal to the ratio of the squares of their

    (i) corresponding sides (ii) corresponding altitudes

II. Results on Quadrilaterals:

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelograms of given sides, the parallelogram which is a rectangle has the greatest area.
8. The line joining the mid-points of the non-parallel sides of a trapezium is parallel to each of the parallel sides and equal to half of their sum.
9. The line joining the mid-points of the diagonals of a trapezium is parallel to each of the parallel sides and equal to half of their difference.

IMPORTANT FORMULAE

I. 1. Area of a rectangle = (Length × Breadth).

$$\therefore \text{ Length } = \left(\frac{\text{Area}}{\text{Breadth}}\right) \text{and Breath} = \left(\frac{\text{Area}}{\text{Lengdth}}\right)$$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square $= (\text{side})^2 = \frac{1}{2}(\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) × Height.

IV. 1. Area of a triangle $= \frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle $= \sqrt{s(s-a)(s-b)(s-c)}$,

where a, b, c are the sides of the triangle and s = $\frac{1}{2}(a+b+c)$

3. Area of an equilateral triangle = $\frac{\sqrt{3}}{4} \times (\text{side})^2$.

4. Area of a triangle = $\frac{1}{2}ab\sin\theta$, where a and b are the lengths of any two sides of the triangle and θ is the angle between them.

5. Radius of incircle of an equilateral triangle of side a = $\frac{a}{2\sqrt{3}}$.

6. Radius of circumcircle of an equilateral triangle of side a = $\frac{a}{\sqrt{3}}$.

7. Radius of incircle of a triangle of area Δ and semi-perimeter s = $\frac{\Delta}{s}$.

8. Radius of circumcircle of a triangle = $\frac{\text{Product of sides}}{4\Delta}$.

V. 1. Area of a parallelogram = (Base × Height).

2. Area of a rhombus = $\frac{1}{2} \times (\text{Product of diagonals})$.

3. Area of a trapezium = $\frac{1}{2} \times (\text{sum of parallel sides}) \times (\text{distance between them})$

VI. 1. Area of a circle = πR^2, where R is the radius

2. Circumference of a circle = $2\pi R$

3. Length of an arc = $\frac{2\pi R\theta}{360}$, where θ is the central angle

4. Area of a sector = $\frac{1}{2}(\text{arc length} \times R) = \frac{\pi R^2\theta}{360}$.

VII. 1. Area of a semi-circle = $\frac{\pi R^2}{2}$.

2. Circumference of semi-circle = πR.

3. Perimeter of a semi-circle = $\pi R + 2R$.

VIII. 1. Area of a regular polygon of N sides, with a as the length of each side = $\frac{a^2 N}{4\tan\left(\frac{180}{N}\right)}$.

2. Area of a regular hexagon of side a = $\frac{3\sqrt{3}}{2}a^2$.

3. Area of a regular pentagon of side a = 1.72 a^2.

4. The area enclosed between the circumcircle and incircle of a regular polygon of side a = $\frac{\pi a^2}{4}$.

SOLVED EXAMPLES

Ex. 1. ***Find the maximum distance between two points on the perimeter of a rectangular garden whose length and breadth are 100 m and 50 m.*** **(Hotel Management, 2007)**

Sol. Clearly, the two points which are maximum distance apart are the end-points of a diagonal.

$\therefore$ Reqd. distance = Length of the diagonal = $\sqrt{(100)^2 + (50)^2}$ m

$= \sqrt{1000 + 2500}\text{ m} = \sqrt{12500}\text{ m}$

$= 50\sqrt{5}\text{ m} = (50 \times 2.236) = 111.8\text{ m}.$

Ex. 2. *One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.*

Sol. Other side = $\sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8$ m.

$\therefore$ Area = (15×8) m^2 = 120 m^2.

Ex. 3. *A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.*

Sol. Let length = $2x$ metres and breadth = $3x$ metres.

Now, area = $\left(\frac{1}{6} \times 1000\right) \text{m}^2 = \left(\frac{5000}{3}\right) \text{m}^2$.

So, $2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right)$.

$\therefore$ Length = $2x = \frac{100}{3} \text{ m} = 33\frac{1}{3} \text{ m}$ and Breadth $= 3x = \left(3 \times \frac{50}{3}\right) \text{m} = 50$ m.

Ex. 4. *Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of ₹ 12.40 per square metre.* **(M.B.A., 2011)**

Sol. Area of the carpet = Area of the room = (13×9) m^2 = 117 m^2.

Length of the carpet = $\left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right) \text{m} = 156$ m.

$\therefore$ Cost of carpeting = ₹ (156×12.40) = ₹ 1934.40.

Ex. 5. *The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.*

Sol. Let breadth = x. Then, length = $2x$. Then,

$(2x - 5)(x + 5) - 2x \times x = 75 \Leftrightarrow 5x - 25 = 75 \Leftrightarrow x = 20$.

$\therefore$ Length of the rectangle = 20 cm.

Ex. 6. *A rectangular carpet has an area of 120 sq. metres and a perimeter of 46 metres. Find the length of its diagonal.* **(L.I.C. A.A.O., 2007)**

Sol. Let the length and breadth of the rectangle be l and b metres respectively.

Then, $2(l + b) = 46 \Rightarrow l + b = 23 \Rightarrow b = (23 - l)$.

And, $lb = 120 \Rightarrow l(23 - l) = 120 \Rightarrow 23l - l^2 = 120 \Rightarrow l^2 - 23l + 120 = 0$

$\Rightarrow l^2 - 15l - 8l + 120 = 0$

$\Rightarrow l(l - 15) - 8(l - 15) = 0$

$\Rightarrow (l - 15)(l - 8) = 0 \Rightarrow l = 15$.

So, $l = 15$ and $b = 8$.

$\therefore$ Length of diagonal = $\sqrt{l^2 + b^2} = \sqrt{(15)^2 + 8^2} \text{ m} = \sqrt{289} \text{ m} = 17$ m.

Ex. 7. *The length of a rectangle is increased by 30%. By what percent would the breadth have to be decreased to maintain the same area?* **(M.B.A., 2008)**

Sol. Let the length and breadth of the rectangle be l and b units respectively.

Then, area of rectangle = (lb) sq. units.

New length = 160% of $l = \frac{8l}{5}$ units.

Desired breadth = $\frac{\text{Area}}{\text{New length}} = \frac{lb}{\left(\frac{8l}{5}\right)} = \frac{5b}{8}$ units.

Decrease in breadth = $\left(b - \frac{5b}{8}\right) \text{units} = \frac{3b}{8}$ units.

$\therefore$ Decrease% = $\left(\frac{3b}{8} \times \frac{1}{b} \times 100\right)\% = \frac{75}{2}\% = 37.5\%$.

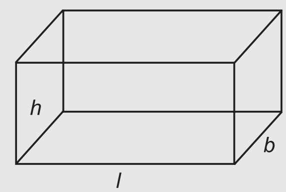
25 Volume and Surface Areas

IMPORTANT FORMULAE

I. Cuboid

Let length = l, breadth = b and height = h units. Then,

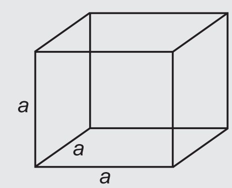
1. **Volume** = $(l \times b \times h)$ cubic units.
2. **Surface area** = $2\,(lb + bh + lh)$ sq. units.
3. **Diagonal** = $\sqrt{l^2 + b^2 + h^2}$ units.



II. Cube

Let each edge of a cube be of length a. Then,

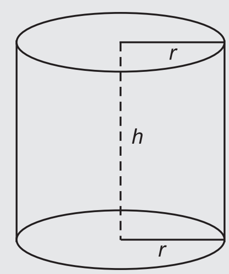
1. **Volume** = a^3 cubic units.
2. **Surface area** = $6a^2$ sq. units.
3. **Diagonal** = $\sqrt{3}\,a$ units.



III. Cylinder

Let radius of base = r and Height (or length) = h. Then,

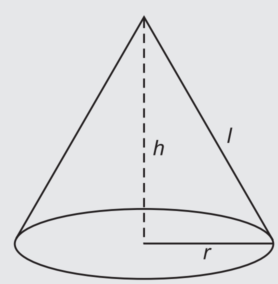
1. **Volume** = $(\pi r^2 h)$ cubic units.
2. **Curved surface area** = $(2\pi rh)$ sq. units.
3. **Total surface area** = $(2\pi rh + 2\pi r^2)$ sq. units
   = $2\pi r\,(h + r)$ sq. units.



IV. Cone

Let radius of base = r and Height = h. Then,

1. **Slant height,** $l = \sqrt{h^2 + r^2}$ units.
2. **Volume** = $\left(\frac{1}{3}\pi r^2 h\right)$ cubic units.
3. **Curved surface area** = (πrl) sq. units.
4. **Total surface area** = $(\pi rl + \pi r^2)$ sq. units.

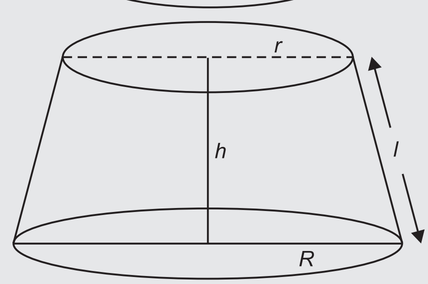


V. Frustum of a Cone

When a cone is cut by a plane parallel to the base of the cone then the portion between the plane and the base is called the frustum of the cone.

Let radius of base = R, radius of top = r, and height = h. Then,

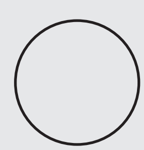
1. **Volume** = $\frac{\pi h}{3}(R^2 + r^2 + Rr)$ cubic units.
2. **Slant height,** $l = \sqrt{(R - r)^2 + h^2}$ units.
3. **Lateral (or curved) surface area** = $\pi.l\,(R + r)$ sq. units.
4. **Total surface area** = $\pi\,[R^2 + r^2 + 1\,(R + r)]$ sq. units.



VI. Sphere

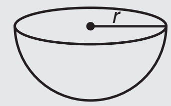
Let the radius of the sphere be r. Then,

1. **Volume** = $\left(\frac{4}{3}\pi r^3\right)$ cubic units.
2. **Surface area** = $(4\pi r^2)$ sq. units.



VII. Hemisphere

Let the radius of a hemisphere be r. Then,



1. **Volume** = $\left(\frac{2}{3}\pi r^3\right)$ cubic units.
2. **Curved surface area** = $(2\pi r^2)$ sq. units.
3. **Total surface area** = $(3\pi r^2)$ sq. units.

VIII. Pyramid

1. **Volume** = $\frac{1}{3} \times$ area of base $\times$ height.
2. **Whole surface area** = Area of base + Area of each of the lateral faces

Remember : 1 litre = 1000 cm^3.

SOLVED EXAMPLES

Ex. 1. ***Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.***

Sol. Volume = $(16 \times 14 \times 7)$ m^3 = 1568 m^3.

Surface area = $[2\ (16 \times 14 + 14 \times 7 + 16 \times 7)]$ cm^2 = (2×434) cm^2 = 868 cm^2.

Ex. 2. ***A room is 12 metres long, 9 metres broad and 8 metres high. Find the length of the longest bamboo pole that can be placed in it.*** **(P.C.S., 2008)**

Sol. Length of the longest pole = Length of the diagonal of the room

$$= \sqrt{(12)^2 + 9^2 + 8^2}\text{ m} = \sqrt{289}\text{ m} = 17\text{m}.$$

Ex. 3. ***The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.***

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and Length = $40x$ metres.

$$\therefore\ x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}.$$

$$\text{So, } x = \frac{4}{10}\text{ m} = \left(\frac{4}{10} \times 100\right)\text{ cm} = 40\text{ cm}.$$

Ex. 4. ***Find the number of bricks, each measuring 24 cm × 12 cm × 8 cm, required to construct a wall 24 m long, 8m high and 60 cm thick, if 10% of the wall is filled with mortar?*** **(M.B.A., 2010)**

Sol. Volume of the wall = $(2400 \times 800 \times 60)$ cu. cm.

Volume of bricks = 90% of the volume of the wall

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right)\text{cu. cm.}$$

Volume of 1 brick = $(24 \times 12 \times 8)$ cu. cm.

$$\therefore\ \text{Number of bricks} = \left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$$

Ex. 5. ***A rectangular sheet of paper, 10 cm long and 8 cm wide has squares of side 2 cm cut from each of its corners. The sheet is then folded to form a tray of depth 2 cm. Find the volume of this tray.*** **(R.R.B., 2006)**

Sol. Clearly, we have:

Length of the tray = $(10 - 2 \times 2)$ cm = 6 cm.

Breadth of the tray = $(8 - 2 \times 2)$ cm = 4 cm.

Depth of the tray = 2 cm.

$\therefore$ Volume of the tray = $(6 \times 4 \times 2)$ cm^3 = 48 cm^3.

Ex. 6. ***Water flows into a tank 200 m × 150 m through a rectangular pipe 1.5 m × 1.25 m @ 20 kmph. In what time (in minutes) will the water rise by 2 metres?***

Sol. Volume required in the tank = $(200 \times 150 \times 2)$ m^3 = 60000 m^3.

$$= \frac{16}{9}\pi^2 \times \frac{27}{8\pi^3}$$

$$= \frac{16 \times 27}{9 \times 8\pi}$$

$$= \frac{6}{\pi} = 6 : \pi$$

297. Let the radius of cone and the sphere be R and the height of the cone be H.

Volume of sphere = $\frac{4}{3}\pi r^3$

Volume of cone = $\frac{1}{3}\pi r^2 h$

According to given information = $\frac{4}{3}\pi R^3 = 2 \times \frac{1}{3}\pi R^2 H$

$\Rightarrow 4R = 2h$

$\Rightarrow \frac{H}{R} = \frac{4}{2} = 2:1$

298. Length of rectangle paper = circumference of the base of cylinder

If r is the radius of the cylinder $44 = 2\pi r$

$\Rightarrow r = \frac{44 \times 7}{2 \times 22} = 7$ cm.

299. Given Raddi of three metallic spheres be r_1, r_2 r_3 are 6cm, 8cm and 10cm respectively.

Let the radius of the new sphere be R.

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(r_1^3 + r_2^3 + r_3^3\right)$$

$$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(6^3 + 8^3 + 10^3\right)$$

$$R^3 = (216 + 512 + 1000) = 1728$$

$\Rightarrow R = \sqrt[3]{12 \times 12 \times 12}$

$R = 12$

Diameter = 24cm

300. Let the radius of a right circular cone be R cm and height be H cm.

Volume of right circular cone = $\frac{1}{3}\pi R^2 H$ cu.cm.

When height of right circular cone is increased by 200% and radius of the base is reduced by 50%.

New volume $= \frac{1}{3}\pi\left(\frac{R}{2}\right)^2 \cdot 3H$

$$= \frac{1}{3}\pi\frac{R^2 4}{4} \cdot 3H = \frac{\pi R^2 H}{4}$$

Difference $= \pi R^2 H\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{12}\pi R^2 H$

Decrease percentage = $\dfrac{\frac{1}{12}\pi R^2 H}{\frac{1}{3}\pi R^2 H} \times 100 = 25\%$

301. If R is the radius of sphere, volume of the sphere $= \frac{4}{3}\pi R^3$.

When radius of sphere is increased by 10%.

New volume $= \frac{4}{3}\pi(1.1R)^3$

$$= \frac{4}{3}\pi R^3(1.331)$$

Difference $= \frac{4}{3}\pi R^3(1.331) - \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R^3(1.331 - 1)$

$$= \frac{4}{3}\pi R^3(0.331)$$

Increase% $= \dfrac{\frac{4}{3}\pi R^3(0.331)}{\frac{4}{3}\pi R^3} \times 100 = 33.1\%$

302. Ball is dropped from the height of 36m when the ball will rise at the third bounce

Required height = $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} \times 36$

$= \frac{32}{3} = 10\frac{2}{3}$ m.

303. Given length of width of swimming pool is 9m and 12m respectively.

Volume of swimming pool = $9 \times 12 \times \left(\frac{1+4}{2}\right)$

$= 9 \times 12 \times \frac{5}{2} = 270$ cu. meter.

304. Let Edge of third small cube be x cm

Volume of cube = (edge)^3

According to the question, $6^3 + 8^3 + x^3 = 12^3$

$\Rightarrow 216 + 512 + x^3 = 1728$

$x^3 = 1728 - 728 = 1000$

$\Rightarrow x = \sqrt[3]{1000} = 10$ cm

EXERCISE

(DATA SUFFICIENCY TYPE QUESTIONS)

Directions (*Questions 1 to 10*): *Each of the questions given below consists of a statement and/or a question and two statements numbered I and II given below it. You have to decide whether the data provided in the statement(s) is/are sufficient to answer the given question. Read both the statements and*

Give answer (a) *if the data in Statement I alone are sufficient to answer the question, while the data in Statement II alone are not sufficient to answer the question;*

Give answer (b) *if the data in Statement II alone are sufficient to answer the question, while the data in Statement I alone are not sufficient to answer the question;*

Give answer (c) *if the data either in Statement I or in Statement II alone are sufficient to answer the question;*

28 Clocks

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

(*i*) In 60 minutes, the minute hand gains 55 minutes on the hour hand.

(*ii*) In every hour, both the hands coincide once.

(*iii*) The hands are in the same straight line when they are coincident or opposite to each other.

(*iv*) When the two hands are at right angles, they are 15 minute spaces apart.

(*v*) When the hands are in opposite directions, they are 30 minute spaces apart.

(*vi*) Angle traced by hour hand in 12 hrs = 360°.

(*vii*) Angle traced by minute hand in 60 min. = 360°.

Too Fast and Too Slow **:** *If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.*

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

Both the hands of a clock are together after every $65\frac{5}{11}$ min. So, if both the hands are meeting after an interval less than $65\frac{5}{11}$ min, the clock is running fast and if they meet after an interval greater than $65\frac{5}{11}$ min, the clock is running slow.

Interchange of Hands: Whenever the hands of the clock interchange positions (i.e., the minute hand takes the place of hour hand and the hour hand and takes the place of minute hand), the sum of the angles traced by hour hand and minute hand is 360°.

Suppose this happens after x minutes.

Angle traced by minute hand in x min = $(6x)°$.

Angle traced by hour hand in x min = $(0.5x)°$.

$\therefore\ 0.5x + 6x = 360 \Leftrightarrow 6.5x = 360 \Leftrightarrow x = \frac{3600}{65} = 55\frac{5}{13}$.

Thus, the hands of a clock interchange positions after every $55\frac{5}{13}$ minutes.

SOLVED EXAMPLES

Ex. 1. ***Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.***

Sol. Angle traced by the hour hand in 12 hours = 360°.

Angle traced by it in 3 hrs 25 min., i.e.

$\frac{41}{12}$ hrs $= \left(\frac{360}{12} \times \frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$.

Angle traced by minute hand in 60 min. = 360°.

Angle traced by it in 25 min. = $\left(\frac{360}{60} \times 25\right)^\circ = 150°$.

$\therefore$ Required angle = $\left(150° - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$.

29 Stocks and Shares

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them ***shares*** of the company.

IMPORTANT FACTS AND FORMULAE

I. Stock-capital: *The total amount of money needed to run the company is called the **stock-capital.***

II. Shares or Stock: *The whole capital is divided into small units, called **shares** or **stock**.*

For each investment, the company issues a *share-certificate*, showing the value of each share and the number of shares held by a person.

*The person who subscribes in shares or stock is called a **shareholder** or **stockholder**.*

III. Dividend: *The annual profit distributed among shareholders is called **dividend.***

Dividend is paid annually as per share or as a percentage.

IV. Face Value: *The value of a share or stock printed on the share-certificate is called its **Face Value** or **Nominal Value** or **Par Value.***

V. Market Value: The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

(*i*) ***At premium*** or ***Above par,*** if its market value is more than its face value.
(*ii*) ***At par***, if its market value is the same as its face value.
(*iii*) ***At discount*** or ***Below par,*** if its market value is less than its face value.

Thus, if a ₹ 100 stock is quoted at a premium of 16, then market value of the stock

$$= ₹\ (100 + 16) = ₹\ 116.$$

Likewise, if a ₹ 100 stock is quoted at a discount of 7, then market value of the stock

$$= ₹\ (100 - 7) = ₹\ 93.$$

VI. Brokerage: *The broker's charge is called **brokerage**.*

(*i*) When stock is purchased, brokerage is added to the cost price.
(*ii*) When stock is sold, brokerage is subtracted from the selling price.

Remember:

(*i*) The face value of a share always remains the same.
(*ii*) The market value of a share changes from time to time.
(*iii*) Dividend is always paid on the face value of a share.
(*iv*) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 Share}} = \frac{\text{Total Income}}{\text{Income from 1 Share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 Share}}.$$

Thus, by a ₹ 100, 9% stock at 120, we mean that :

(*i*) Face Value (N.V.) of stock = ₹ 100.
(*ii*) Market Value (M.V.) of stock = ₹ 120.
(*iii*) Annual dividend on 1 share = 9% of face value = 9% of ₹ 100 = ₹ 9.
(*iv*) An investment of ₹ 120 gives an annual income of ₹ 9.
(*v*) Rate of interest p.a. = Annual income from an investment of ₹ 100.

$$= \left(\frac{9}{120} \times 100\right)\% = 7\frac{1}{2}\%.$$