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D.1.a. Decision Variables:

We may adopt X and Y as the variables in this case.

Let X represents the weekly production goal for Collegiate backpacks.

Let Y represents the weekly target for Mini backpack production.

$$\rightarrow X \geq 0$$

$$\rightarrow Y \geq 0$$

b. Objective Function:

The objective in this scenario is to maximize profit since the owners want to achieve so.

It is represented by the letter Z:

$$\rightarrow Z = 32X + 24Y$$

c. Constraints:**1. Sales Constraints:**

The Collegiate backpack sales are limited to 1,000 units per week, whereas

Mini backpack sales are limited to 1,200 units per week. As a result, we can write the constraint as:

$$\rightarrow X \leq 1000$$

$$\rightarrow Y \leq 1200$$

2. Material Constraints:

In this case, Back Savers has a weekly supply of 5000 square feet of nylon fabric.

Each Collegiate requires 3 square feet and each Mini requires 2 square feet.

Therefore, the material constraint can be expressed as:

$$\rightarrow 3X + 2Y \leq 5000$$

3. Labor Constraints:

Here Back Savers has 35 laborers, each providing 40 hours of labor week. The Labor required to produce each Collegiate in 45 minutes and the Labor required to produce each Mini in 40 minutes. Therefore, the Labor constraints can be written as:

$$\rightarrow 45X + 40Y \leq 35 \times 40 \times 60$$

d. Mathematical Formulation:

The Linear Programming problem can be written as follows:

- Maximize Z: $Z = 32X + 24Y$

Constraints:

- Sales constraints: $X \leq 1,000$, $Y \leq 1,200$
- Material constraints: $3X + 2Y \leq 5000$
- Labor constraints: $45X + 40Y \leq 35 \times 40 \times 60$

Here, $X \geq 0$, $Y \geq 0$.

D.2.a. Decision Variables:

Here, we can define three decision variables to represent the number of units of each size i.e., large, medium, and small produced by each of the three plants.

Let,

* P_{1l} , P_{1m} and P_{1n} are the number of large, medium and small units produced by Plant1.

* P_{2l} , P_{2m} and P_{2n} are the number of large, medium and small units produced by Plant2.

* P_{3l} , P_{3m} and P_{3n} are the number of large, medium and small units produced by Plant3.

These variables will represent the production quantities of each size at each plant.

b. Linear Programming Model:

Objective Function: Here the objective is to maximize the total profit, it means the sum of the profits from each size of each plant. The profit for each unit of each size at each plant is given in the problem. So, the objective function can be written as:

$$\rightarrow Z = 420(P_{1l} + P_{2l} + P_{3l}) + 360(P_{1m} + P_{2m} + P_{3m}) + 300(P_{1n} + P_{2n} + P_{3n})$$

1. Capacity Constraints:

Plant1, Plant2, Plant3 capacity constraints:

$$\rightarrow P_{1l} + P_{1m} + P_{1n} \leq 750$$

$$\rightarrow P_{2l} + P_{2m} + P_{2n} \leq 900$$

$$\rightarrow P_{3l} + P_{3m} + P_{3n} \leq 450$$

2. Sales Forecasts Constraints:

Large, medium and small size sales constraints:

$$\rightarrow P_{1l} + P_{2l} + P_{3l} \leq 900$$

$$\rightarrow P_{1m} + P_{2m} + P_{3m} \leq 1200$$

$$\rightarrow P_{1n} + P_{2n} + P_{3n} \leq 750$$

3. Storage Space Constraints:

Plant1, Plant2 and Plant3 storage space constraints:

$$\rightarrow 20P_{1l} + 15P_{1m} + 12P_{1n} \leq 13,000$$

$$\rightarrow 20P_{2l} + 15P_{2m} + 12P_{2n} \leq 12,000$$

$$\rightarrow 20P_{3l} + 15P_{3m} + 12P_{3n} \leq 5,000$$

c. Here all the variables must be non-negative:

→ $P_{al} + P_{am} + P_{an} \geq 0$, $a=1,2,3$ and l, m, n =large, medium, small.

In order to avoid the employees lay off due to excess production capacity the management have made a decision using the same percentage of excess capacity to produce a new product,

Therefore,

$$(P_{1l} + P_{1m} + P_{1n}) * 100 / 750 = (P_{2l} + P_{2m} + P_{2n}) * 100 / 900 = (P_{3l} + P_{3m} + P_{3n}) * 100 / 450.$$