SECA1201
DIGITAL LOGIC CIRCUITS

Unit – 2

DESIGN OF COMBINATIONAL LOGIC

Syllabus

UNIT II DESIGN OF COMBINATIONAL LOGIC

Design procedure of Combinational Logic

Design of two level gate networks

Sum of Products (SOP) and Product of Sums(POS)

Canonical SOP and Canonical POS

Karnaugh Map

Simplifications of Boolean functions using Karnaugh Map and implementation using Logic function

Advantages and limitations of K-Map

Tabulation method - Simplifications of Boolean functions using Tabulation method.

Min and Max Term

AND terms **or Product terms** is called a '***minterm***'

OR terms **or Sum terms** is called a '***maxterm***'

Complement of **min** term is **Max** term

3- variable function can be represented as in table below

Variables			**Minterms**	**Maxterms**
x	**y**	**z**	$\mathbf{m_i}$	$\mathbf{M_i}$
0	0	0	$x'y'z' = m_0$	$x+y+z = M_0$
0	0	1	$x'y'z = m_1$	$x+y+z' = M_1$
0	1	0	$x'yz' = m_2$	$x+y'+z = M_2$
0	1	1	$x'yz = m_3$	$x+y'+z' = M_3$
1	0	0	$xy'z' = m_4$	$x'+y+z = M_4$
1	0	1	$xy'z = m_5$	$x'+y+z' = M_5$
1	1	0	$xyz' = m_6$	$x'+y'+z = M_6$
1	1	1	$xyz = m_7$	$x'+y'+z' = M_7$

Design Procedure of Combinational Logic

- Problem definition
- Assign an input and output variables
- Simplify the boolean expression
- Implement Using the Logic gates

SOP and POS

Sum of Minterm: (Sum of Products)

The logical sum of two or more logical product terms is called sum of products expression. It is logically an OR operation of AND operated variables such as:

1. $Y = AB + BC + AC$

Sum

2. $Y = AB + \overline{B}C + A\overline{C}$

Product terms

Product of Maxterm: (Product of Sums)

A product of sums expression is a logical product of two or more logical sum terms. It is basically an AND operation of OR operated variables such as,

1. $Y = (A+B).(B+C).(A+C)$

Product

2. $Y = (A+B).(\overline{B}+C).(A+\overline{C})$

Sum terms

Canonical SOP or Standard SOP

If each term in SOP form contains all the literals then the SOP is known as standard (or) canonical SOP form. Each individual term in standard SOP form is called minterm canonical form.

F (A, B, C) = AB'C+ ABC+ ABC'

Steps to convert general SOP to standard SOP form:

1. Find the missing literals in each product term if any.
2. AND each product term having missing literals by ORing the literal and its complement.
3. Expand the term by applying distributive law and reorder the literals in the product term.
4. Reduce the expression by omitting repeated product terms if any.

Canonical POS or Standard POS

If each term in POS form contains all literals then the POS is known as standard (or) Canonical POS form. Each individual term in standard POS form is called Maxterm canonical form.

- $F(A, B, C) = (A + B + C).\ (A + B' + C).\ (A + B + C')$
- $F(x, y, z) = (x + y' + z').\ (x' + y + z).\ (x + y + z)$

<u>Steps to convert general POS to standard POS form</u>:

1. Find the missing literals in each sum term if any.
2. OR each sum term having missing literals by ANDing the literal and its complement.
3. Expand the term by applying distributive law and reorder the literals in the sum term.
4. Reduce the expression by omitting repeated sum terms if any.

Problem

Obtain the Canonical SOP

1. $Y(A, B) = A + B$

B A --- missing Literals

$= A.(B + B') + B(A + A')$

$= \underline{AB} + AB' + \underline{AB} + A'B$

$Y(A, B) = AB + AB' + A'B.$

3. $Y(A, B, C) = A + BC$

B,C A missing literals

$= A.(B + B').(C + C') + (A + A').BC$

$= (AB + AB').(C + C') + ABC + A'BC$

$= \underline{ABC} + ABC' + AB'C + AB'C' + \underline{ABC} + A'BC$

$= ABC + ABC' + AB'C + AB'C' + A'BC$

$= m_7 + m_6 + m_5 + m_4 + m_3$

$= \sum m(3, 4, 5, 6, 7).$

2. $Y(A, B, C) = A + ABC$

B,C missing literals

$= A.(B + B').(C + C') + ABC$

$= (AB + AB').(C + C') + ABC$

$= \underline{ABC} + ABC' + AB'C + AB'C' + \underline{ABC}$

$= ABC + ABC' + AB'C + AB'C'$

$Y(A, B, C) = m_7 + m_6 + m_5 + m_4 = \sum m(4, 5, 6, 7).$

4. $Y(A, B, C, D) = AB + ACD$

C,D B missing literals

$= AB(C + C')(D + D') + ACD(B + B')$

$= (ABC + ABC')(D + D') + ABCD + AB'CD$

$= \underline{ABCD} + ABCD' + ABC'D + ABC'D' + \underline{ABCD} + AB'CD$

$= ABCD + ABCD' + ABC'D + ABC'D' + AB'CD.$

$= \sum m(\mathbf{15, 14, 13, 12, 11})$

Problem

Obtain the Canonical POS

1. $Y = A + B'C$

$= (A + B')(A + C)$ $\boxed{[A + BC = (A+B)(A+C)]}$

C B missing literals

$= (A + B' + C.C')(A + C + B.B')$

$= \underline{(A + B' + C)}\,(A + B' + C')\,(A + B + C)\,\underline{(A + B' + C)}$

$= (A + B' + C).\,(A + B' + C').\,(A + B + C)$

$= M_2.\,M_3.\,M_0$

$= \prod M\,(0, 2, 3)$

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2. $Y = A.\,(B + C + A)$

B,C missing literals

$= (A + B.B' + C.C').\,(A + B + C)$

$= (A + B.B' + C)\,(A + B.B' + C')\,(A + B + C)$

$= \underline{(A+B+C)}\,(A+B'+C)\,(A+B+C')(A + B'+C')\,\underline{(A+B+C)}$

$= (A+B+C)\,(A+B+C')\,(A+B'+C)\,(A + B'+C')$

$= M_0.\,M_1.\,M_2.\,M_3$

$= \prod M\,(0, 1, 2, 3)$

Karnaugh Map (K – Map) Minimization

The simplification of the functions using Boolean laws and theorems becomes complex with the increase in the number of variables and terms. The map method, first proposed by Veitch and slightly improvised by Karnaugh, provides a simple, straightforward procedure for the simplification of Boolean functions. The method is called **Veitch diagram** or **Karnaugh map**, which may be regarded as a pictorial representation of a truth table.

The Karnaugh map technique provides a systematic method for simplifying and manipulation of Boolean expressions. **A** K-map is a diagram made up of squares, with each square representing one minterm of the function that is to be minimized. For n variables on a Karnaugh map there are 2^n numbers of squares. Each square or cell represents one of the minterms. It can be drawn directly from either minterm (sum-of-products) or maxterm (product-of-sums) Boolean expressions.

Two Variable, Three Variable, Four Variable K-Map

Karnaugh maps can be used for expressions with two, three, four and five variables. For n variables on a Karnaugh map there are 2^n numbers of squares.

For Two variables, the number of cells is $2^2 = 4$.

For three variables, the number of cells is $2^3 = 8$.

For four variables, the number of cells is $2^4 = 16$.

A	0
0	m_0
1	m_1

1 -Variable map

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

2 -Variable map

→ Gray code Sequence

A \ BC	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

3 -Variable map

	A
$\bar{A}$	$\bar{A}$
A	A

1 -Variable map

A \ B	$\bar{B}$	B
$\bar{A}$	$\bar{A}\bar{B}$	$\bar{A}B$
A	$A\bar{B}$	AB

2 -Variable map

A \ BC	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	$\bar{A}\bar{B}\bar{C}$	$\bar{A}\bar{B}C$	$\bar{A}BC$	$\bar{A}B\bar{C}$
A	$A\bar{B}\bar{C}$	$A\bar{B}C$	ABC	$AB\bar{C}$

3 -Variable map

Four Variable K-Map

CD → Gray code Sequence

Gray code Sequence ↓ (AB)

AB \ CD	00	01	11	10
00	m_0	m_1	m_3	m_2
01	m_4	m_5	m_7	m_6
11	m_{12}	m_{13}	m_{15}	m_{14}
10	m_8	m_9	m_{11}	m_{10}

4- Variable map

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	$\bar{A}\bar{B}\bar{C}\bar{D}$	$\bar{A}\bar{B}\bar{C}D$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}C\bar{D}$
$\bar{A}B$	$\bar{A}B\bar{C}\bar{D}$	$\bar{A}B\bar{C}D$	$\bar{A}BCD$	$\bar{A}BC\bar{D}$
AB	$AB\bar{C}\bar{D}$	$AB\bar{C}D$	$ABCD$	$ABC\bar{D}$
$A\bar{B}$	$A\bar{B}\bar{C}\bar{D}$	$A\bar{B}\bar{C}D$	$A\bar{B}CD$	$A\bar{B}C\bar{D}$

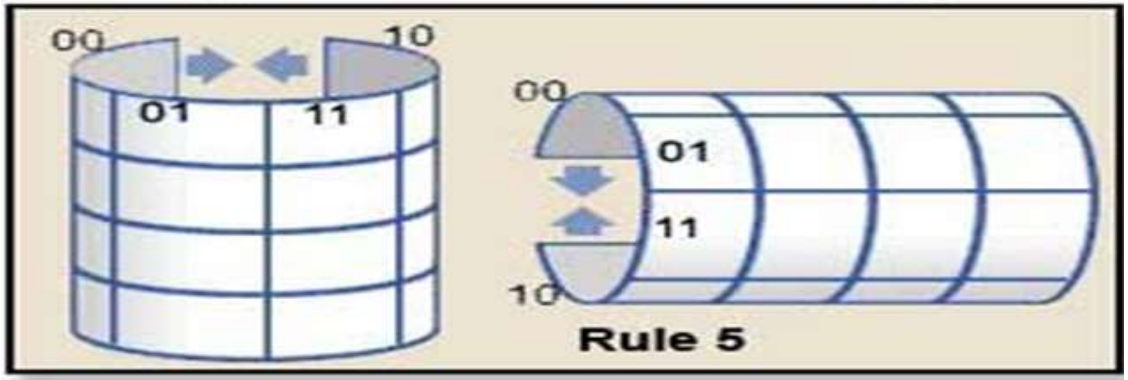
4 -Variable map

Grouping Cells for Simplification

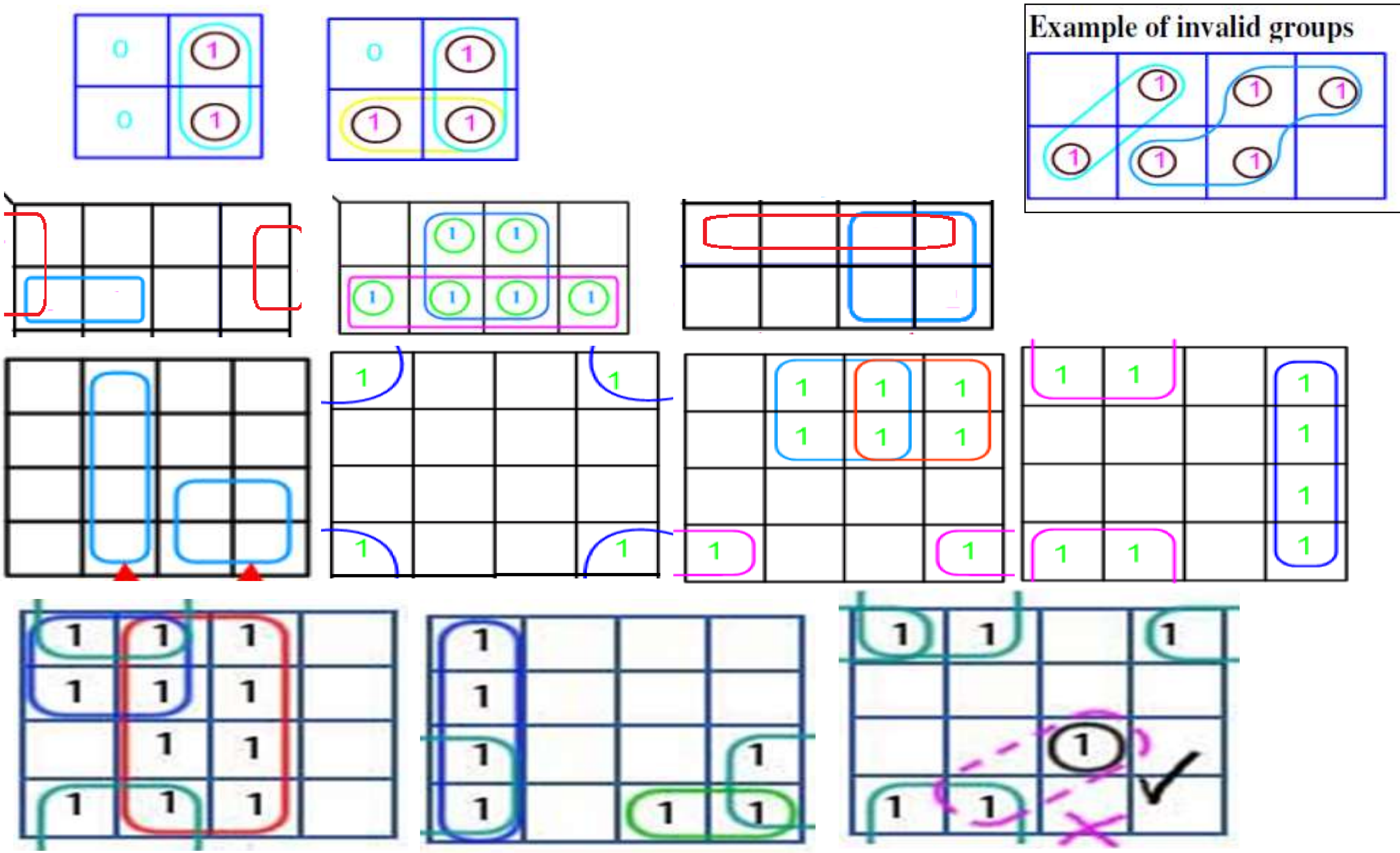
- **Karnaugh Map Rules**

1. **Groups should contain as many '1' cells (i.e. cells containing a logic 1) as possible and no blank cells.**
2. **Groups can only contain 1, 2 (Pair), 4 (Quad) , 8 (Octet) , 16 (Hexa) or 32... etc. cells (powers of 2).**
3. **A '1' cell can only be grouped with adjacent '1' cells that are immediately above, below, left or right of that cell; no diagonal grouping.**
4. **Groups of '1' cells can overlap. This helps make smaller groups as large as possible, which is an advantage in finding the simplest solution.**
5. **The top/bottom and left/right edges of the map are considered to be continuous, so larger groups can be made by grouping cells across the top and bottom or left and right edges of the map.**

6. **There should be as few groups as possible.**

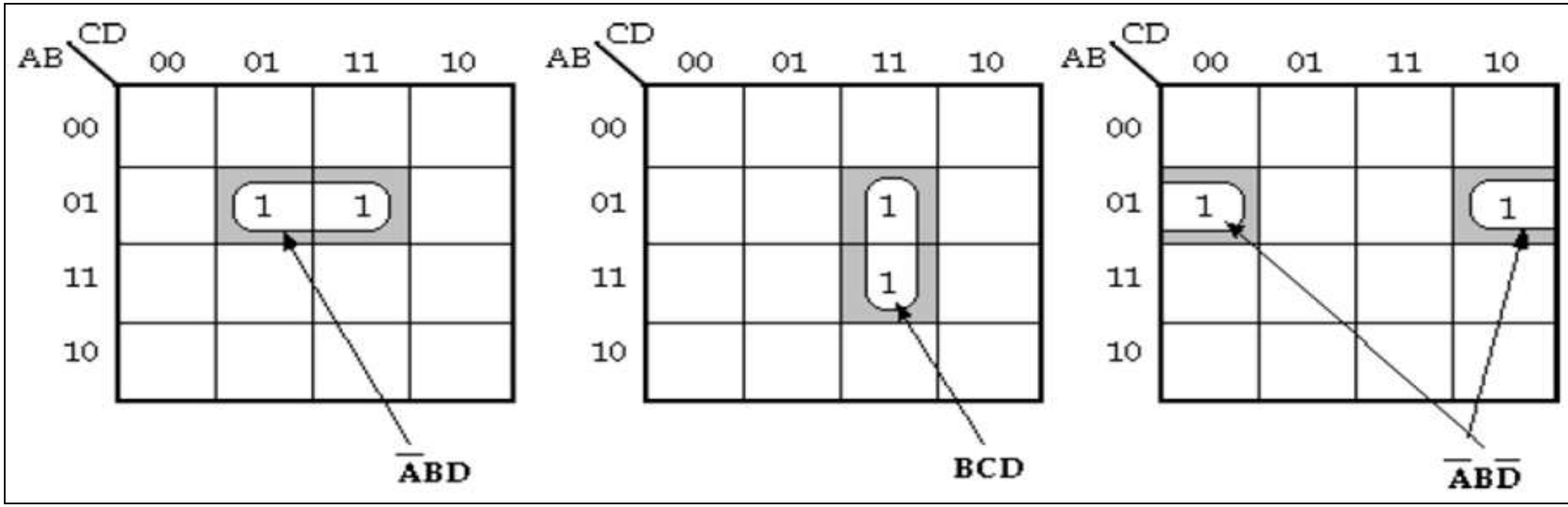


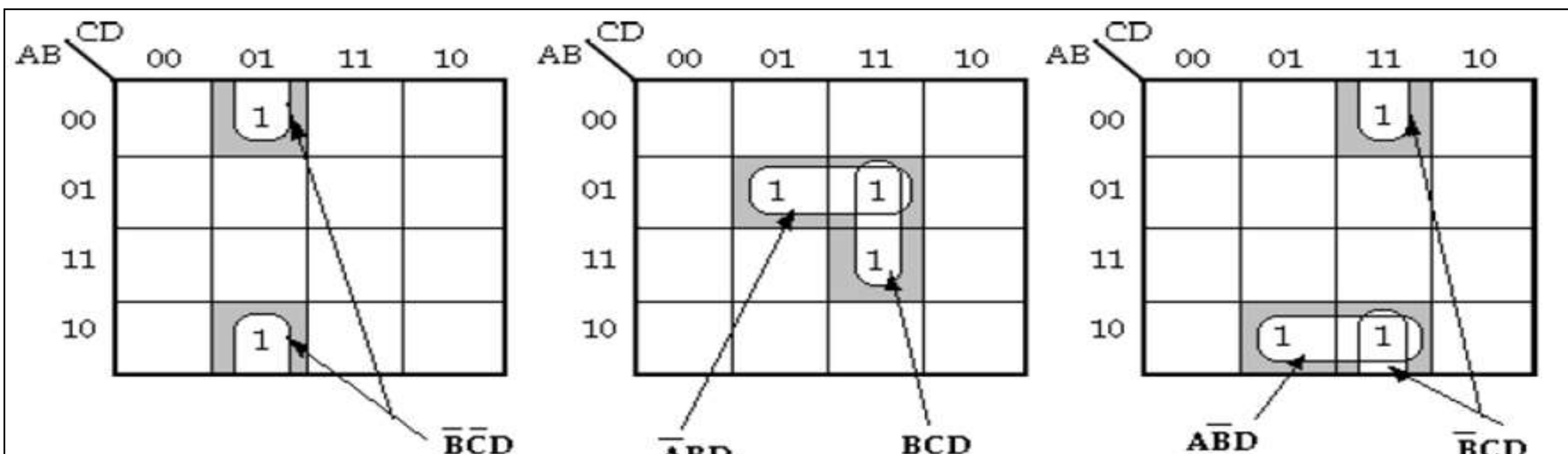
Grouping Cells for Simplification Cont..



Grouping Cells for Simplification Cont..

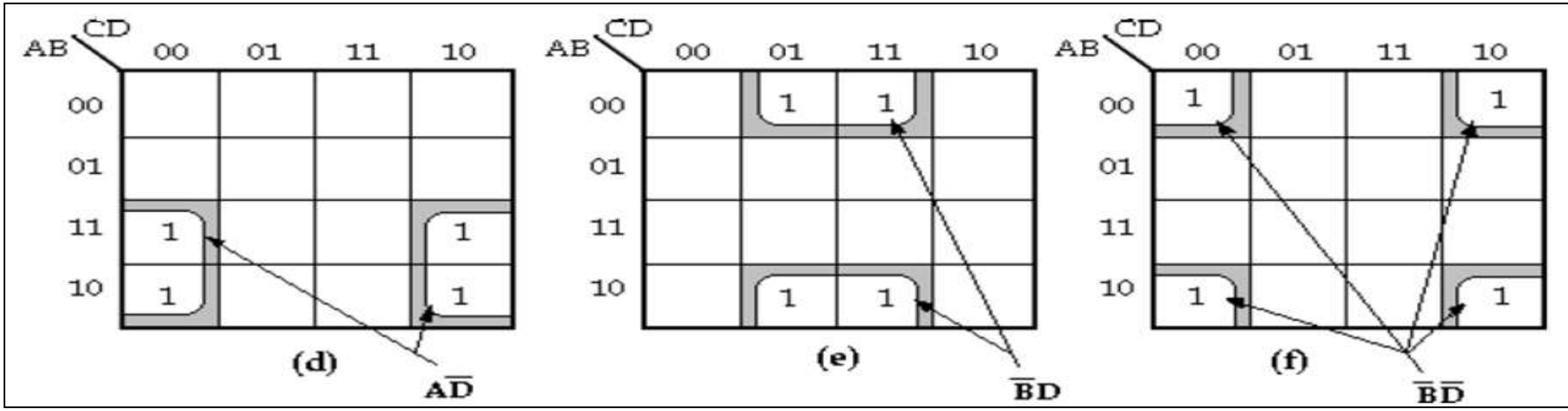
Grouping Two Adjacent 1's: (Pair)



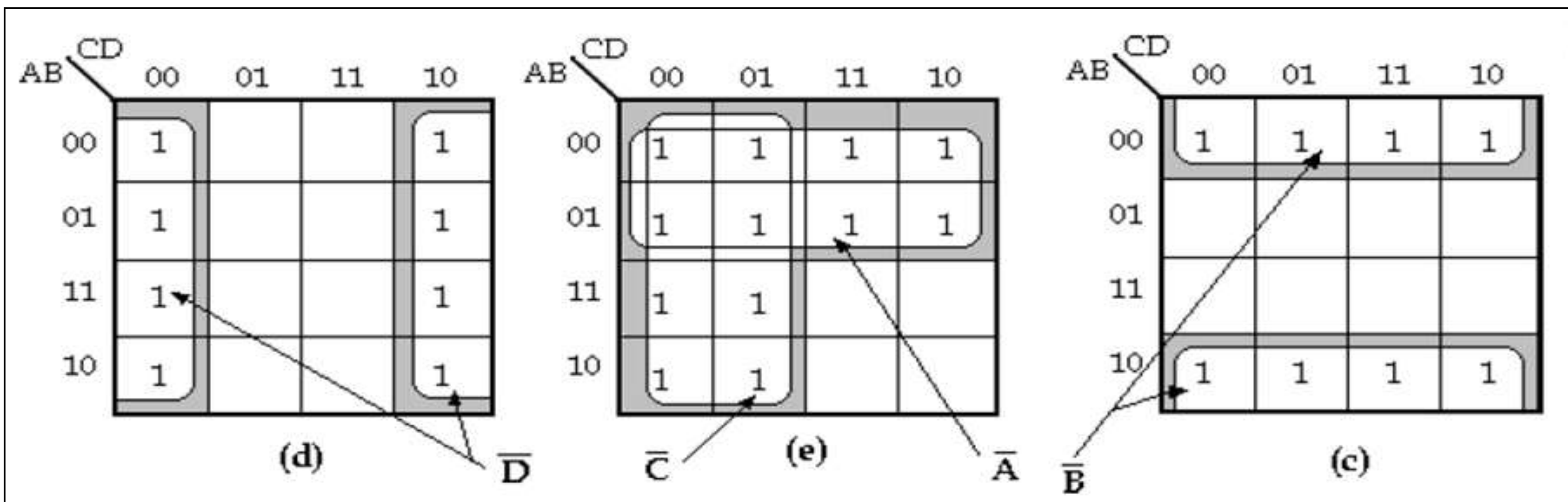


Grouping Cells for Simplification Cont..

Grouping Four Adjacent 1's: (Quad)



Grouping Eight Adjacent 1's: (Octet)



Simplification of Sum of Products Expressions:

The generalized procedure to simplify Boolean expressions as follows:

1. Plot the K-map and place 1's in those cells corresponding to the 1's in the sum of product expression. Place 0's in the other cells.
2. Check the K-map for adjacent 1's and encircle those 1's which are not adjacent to any other 1's. These are called **isolated 1's**.
3. Check for those 1's which are adjacent to only one other 1 and encircle such **pairs**.
4. Check for **quads** and **octets** of adjacent 1's even if it contains some 1's that have already been encircled. While doing this make sure that there are minimum number of groups.
5. Combine any pairs necessary to include any 1's that have not yet been grouped.
6. Form the simplified expression by summing product terms of all the groups.

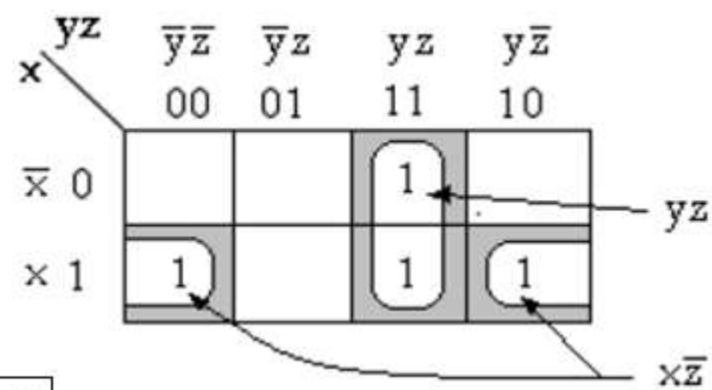
Problem

1. Simplify the Boolean expression,

$$F(x, y, z) = \sum m\,(3, 4, 6, 7).$$

Soln:

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	(0)	(1)	1 (3)	(2)
x 1	1 (4)	(5)	1 (7)	1 (6)

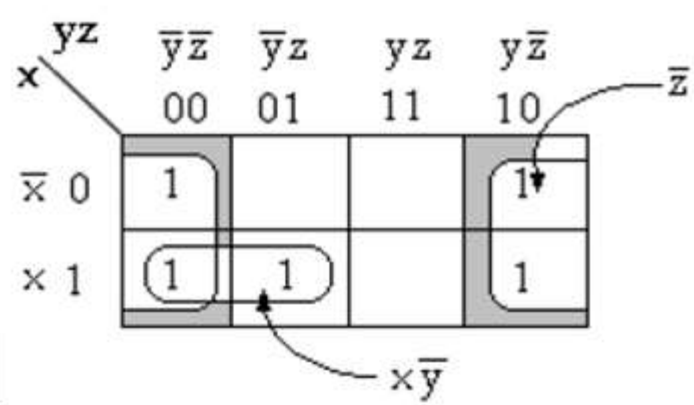


$$\boxed{F = yz + x\bar{z}}$$

2. $F(x, y, z) = \sum m\,(0, 2, 4, 5, 6).$

Soln:

$x \backslash yz$	$\bar{y}\bar{z}$ 00	$\bar{y}z$ 01	yz 11	$y\bar{z}$ 10
$\bar{x}$ 0	1 (0)	(1)	(3)	1 (2)
x 1	1 (4)	1 (5)	(7)	1 (6)



$$\boxed{F = \bar{z} + x\bar{y}}$$

3. $F = A'C + A'B + AB'C + BC$

Soln:

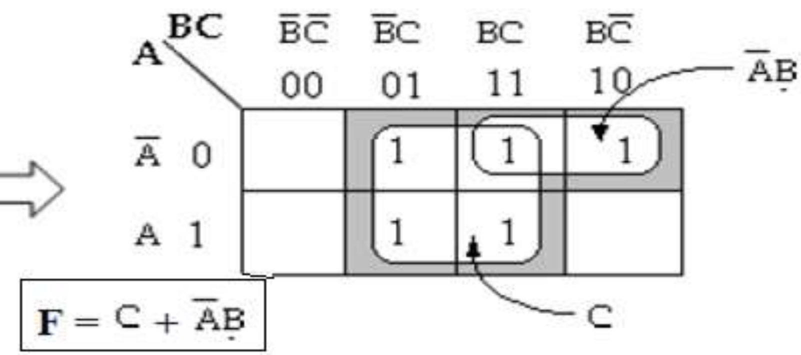
$= A'C\,(B + B') + A'B\,(C + C') + AB'C + BC\,(A + A')$

$= \underline{A'BC} + A'B'C + \underline{A'BC} + A'BC' + AB'C + ABC + \underline{A'BC}$

$= A'BC + A'B'C + A'BC' + AB'C + ABC$

$= m_3 + m_1 + m_2 + m_5 + = \sum m\,(1, 2, 3, 5, 7)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	0	1 (1)	1 (3)	1 (2)
A 1	4	1 (5)	1 (7)	6

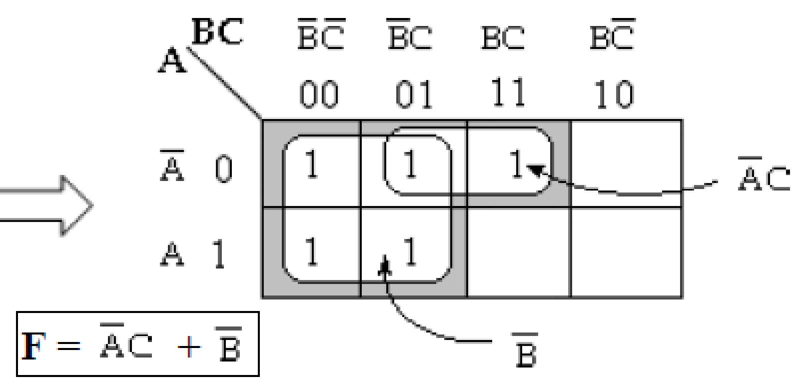


4. $AB'C + A'B'C + A'BC + AB'C' + A'B'C'$

Soln:

$= m5 + m1 + m3 + m4 + m0 \;= \sum m\,(0, 1, 3, 4, 5)$

A \ BC	$\bar{B}\bar{C}$ 00	$\bar{B}C$ 01	BC 11	$B\bar{C}$ 10
$\bar{A}$ 0	1 (0)	1 (1)	1 (3)	2
A 1	1 (4)	1 (5)	7	6

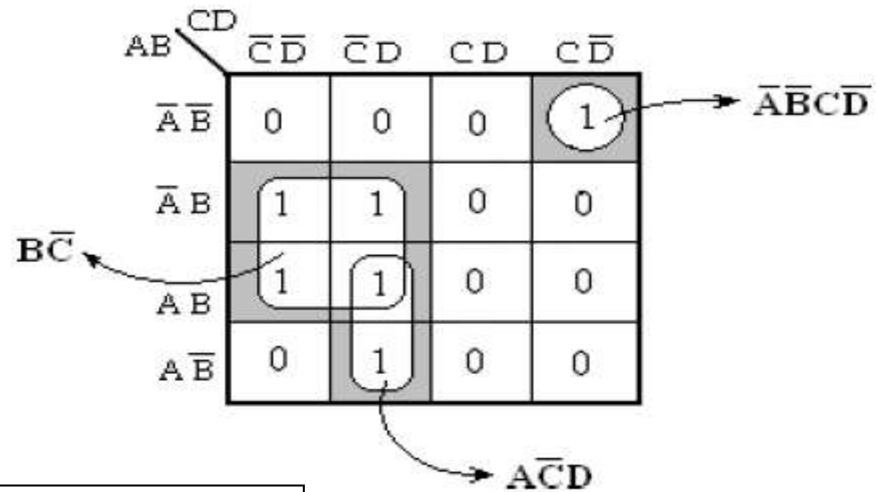


5. Simplify the Boolean expression,

$$Y = A'BC'D' + A'BC'D + ABC'D' + ABC'D + AB'C'D + A'B'CD'$$

Soln: $= \sum m(4, 5, 12, 13, 9, 2)$

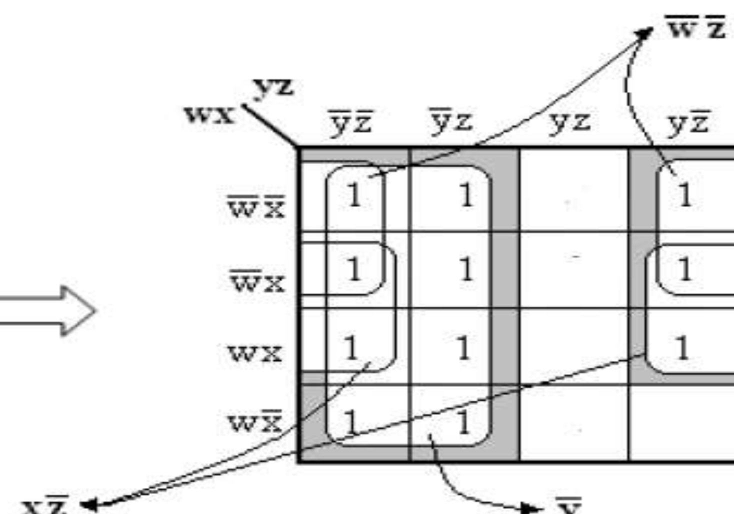
AB \ CD	00	01	11	10
00	0	1	3	1 2
01	1 4	1 5	7	6
11	1 12	1 13	15	14
10	8	1 9	11	10



$$\boxed{Y = \bar{A}\bar{B}C\bar{D} + A\bar{C}D + B\bar{C}}$$

6. $F(w, x, y, z) = \sum m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

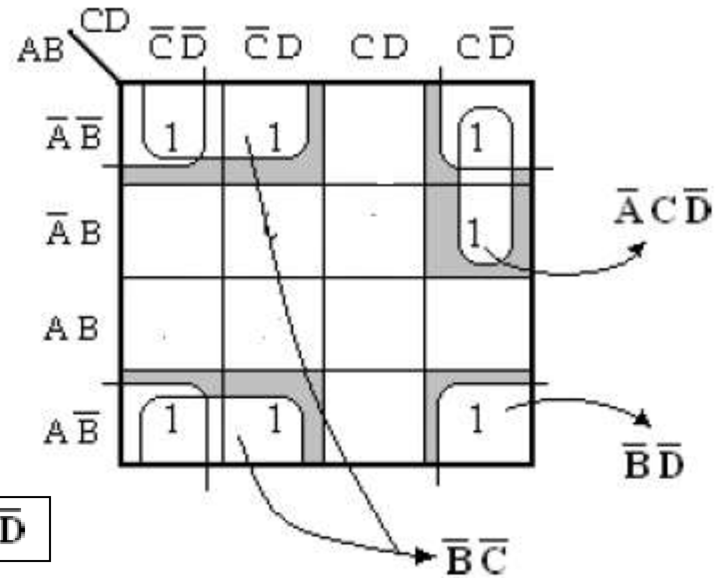
wx \ yz	00	01	11	10
00	1 0	1 1	3	1 2
01	1 4	1 5	7	1 6
11	1 12	1 13	15	1 14
10	1 8	1 9	11	10



$$\boxed{F = \bar{y} + \bar{w}\bar{z} + x\bar{z}}$$

7. **F= $\sum$ m (0, 1, 2, 6, 8, 9, 10)**

AB \ CD	00	01	11	10
00	1_{0}	1_{1}	$_{3}$	1_{2}
01	$_{4}$	$_{5}$	$_{7}$	1_{6}
11	$_{12}$	$_{13}$	$_{15}$	$_{14}$
10	1_{8}	1_{9}	$_{11}$	1_{10}

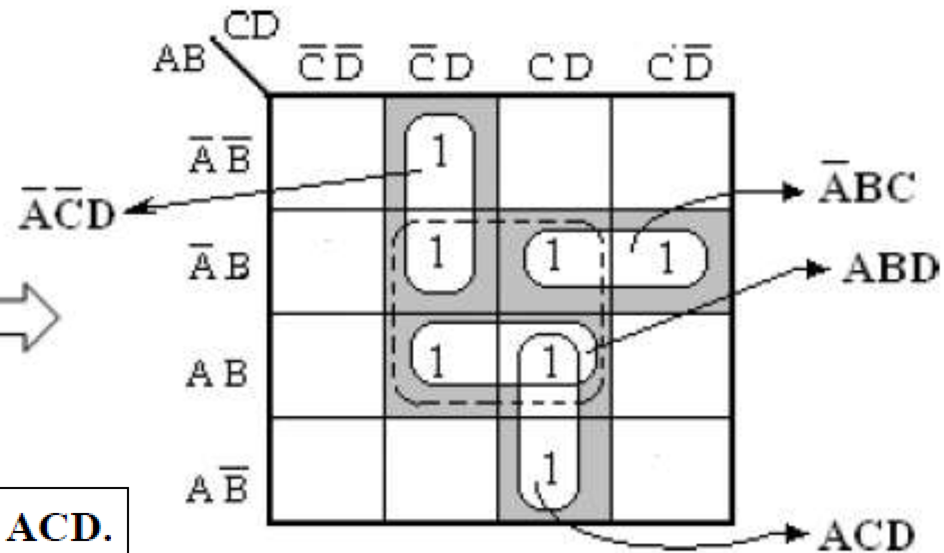


$$F = \bar{B}\bar{D} + \bar{B}\bar{C} + \bar{A}C\bar{D}$$

8. Y= A'B'C'D+ A'BC'D+ A'BCD+ A'BCD'+ ABC'D+ ABCD+ AB'CD

$= m_1 + m_5 + m_7 + m_6 + m_{13} + m_{15} + m_{11}$ $=$ **$\sum$ m (1, 5, 6, 7, 11, 13, 15)**

AB \ CD	00	01	11	10
00	$_{0}$	1_{1}	$_{3}$	$_{2}$
01	$_{4}$	1_{5}	1_{7}	1_{6}
11	$_{12}$	1_{13}	1_{15}	$_{14}$
10	$_{8}$	$_{9}$	1_{11}	$_{10}$



$$Y = \bar{A}\bar{C}D + \bar{A}BC + ABD + ACD.$$

Home work : Simplify using K - Map

1. Y= ABCD+ AB'C'D'+ AB'C+ AB

   Ans: Y= AB+ AC+ AD'.

2. Y (A, B, C, D)= $\sum$ m (7, 9, 10, 11, 12, 13, 14, 15)

   Ans: Y= AB+ AC+ AD+BCD.

3. Y= $\sum$ m (1, 5, 10, 11, 12, 13, 15)

   Ans: Y= A'C'D+ ABC'+ ACD+ AB'C.

4. Y= A'B'CD'+ ABCD'+ AB'CD'+ AB'CD+ AB'C'D'+ ABC'D'+ A'B'CD+ A'B'C'D'

   Ans: Y= AD'+ B'C+ B'D'

Simplification of Products of Sum Expressions:

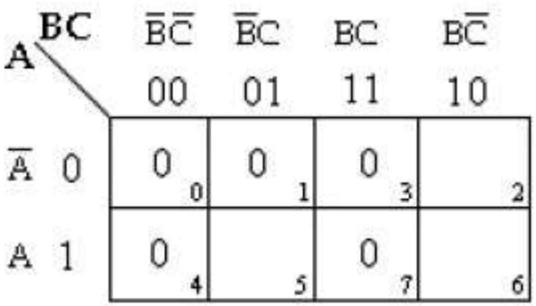
The generalized procedure to simplify Boolean expressions as follows:

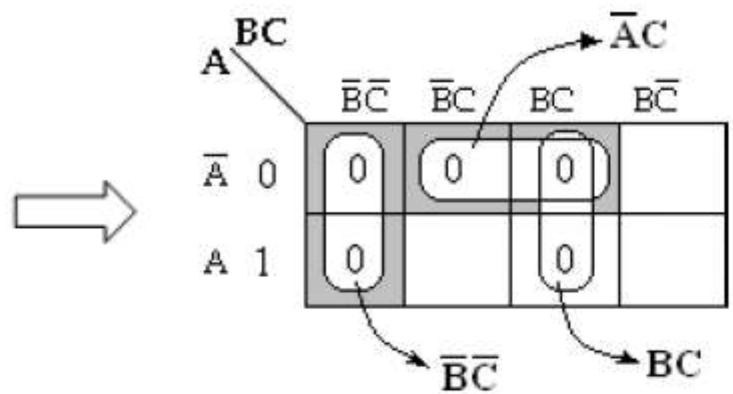
1. Plot the K-map and place 0's in those cells corresponding to the product sum of expression. Place 1's in the other cells.
2. Check the K-map for adjacent 0's and encircle those 0's which are not adjacent to any other 0's. These are called **isolated** 0's.
3. Check for those 0's which are adjacent to only one other 0 and encircle such **pairs**.
4. Check for **quads** and **octets** of adjacent 1's even if it contains some 0's that have already been encircled. While doing this make sure that there are minimum number of groups.
5. Combine any pairs necessary to include any 0's that have not yet been grouped.
6. Form the simplified expression by summing product terms of all the groups.
7. Find the complement by applying De Margon's Theorem to obtain the simplified product of sum expression.

Problem – Simplify using K - Map

1. $Y = (A+B+C')(A+B'+C')(A'+B'+C')(A'+B+C)(A+B+C)$

$$= M_1 + M_3 + M_7 + M_6 + M_0 = \Pi M(0,1,3,6,7)$$





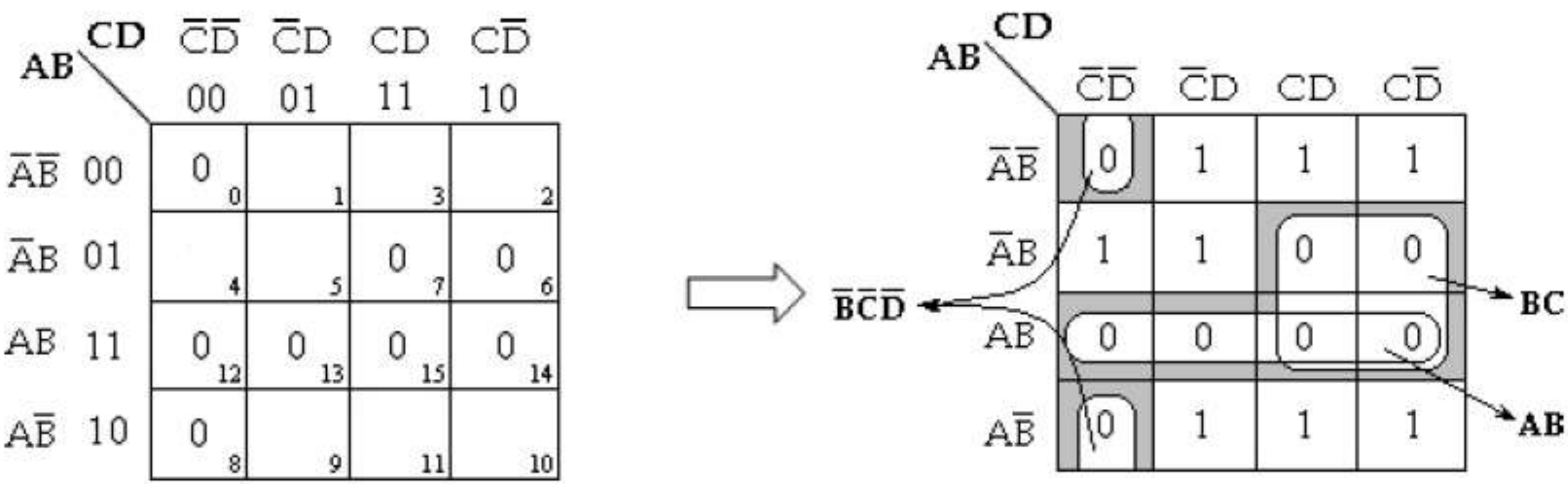
$$\overline{Y} = \overline{B}\,\overline{C} + \overline{A}C + BC.$$

$$Y = (B+C).(A+\overline{C}).(\overline{B}+\overline{C})$$

Problem – Simplify using K - Map

2. $Y = (A' + B' + C + D)(A' + B' + C' + D)(A' + B' + C' + D')(A' + B + C + D)(A + B' + C' + D)$
$(A + B' + C' + D')(A + B + C + D)(A' + B' + C + D')$

$= M_{12} . M_{14} . M_{15} . M_8 . M_6 . M_7 . M_0 . M_{13} = \prod M\,(0, 6, 7, 8, 12, 13, 14, 15)$



$$\overline{Y} = \bar{B}\,\bar{C}\,\bar{D} + AB + BC$$

$$Y = (B + C + D).(\bar{A} + \bar{B}).(\bar{B} + \bar{C})$$

Home work : Simplify using K - Map

5. $F(A, B, C, D) = \prod M\,(0, 2, 3, 8, 9, 12, 13, 14, 15)$

Ans: $Y = (A + B + D).\,(A + B + \overline{C}).\,(\overline{A} + \overline{B} + \overline{D}).\,(\overline{A} + C)$

6. $F(A, B, C, D) = \sum m\,(0, 1, 2, 5, 8, 9, 10)$

$= \prod M\,(3, 4, 6, 7, 11, 12, 13, 14, 15)$

Ans: $Y = (\overline{B} + D).\,(\overline{C} + \overline{D}).\,(\overline{A} + \overline{B})$

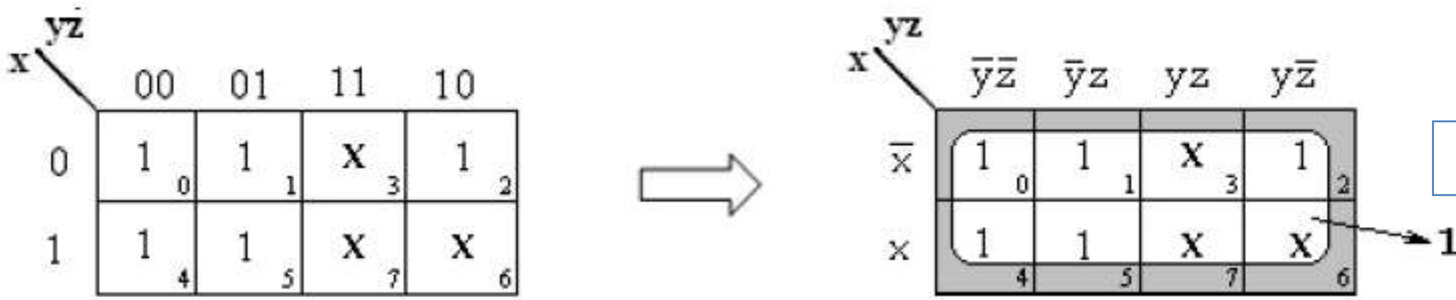
Don’t care Conditions:

A don’t care minterm is a combination of variables whose logical value is not specified. When choosing adjacent squares to simplify the function in a map, the don’t care minterms may be assumed to be either 0 or 1. When simplifying the function, we can choose to include each don’t care minterm with either the 1’s or the 0’s, depending on which combination gives the simplest expression.

Don’t can be represent by ‘d’ or ‘X’

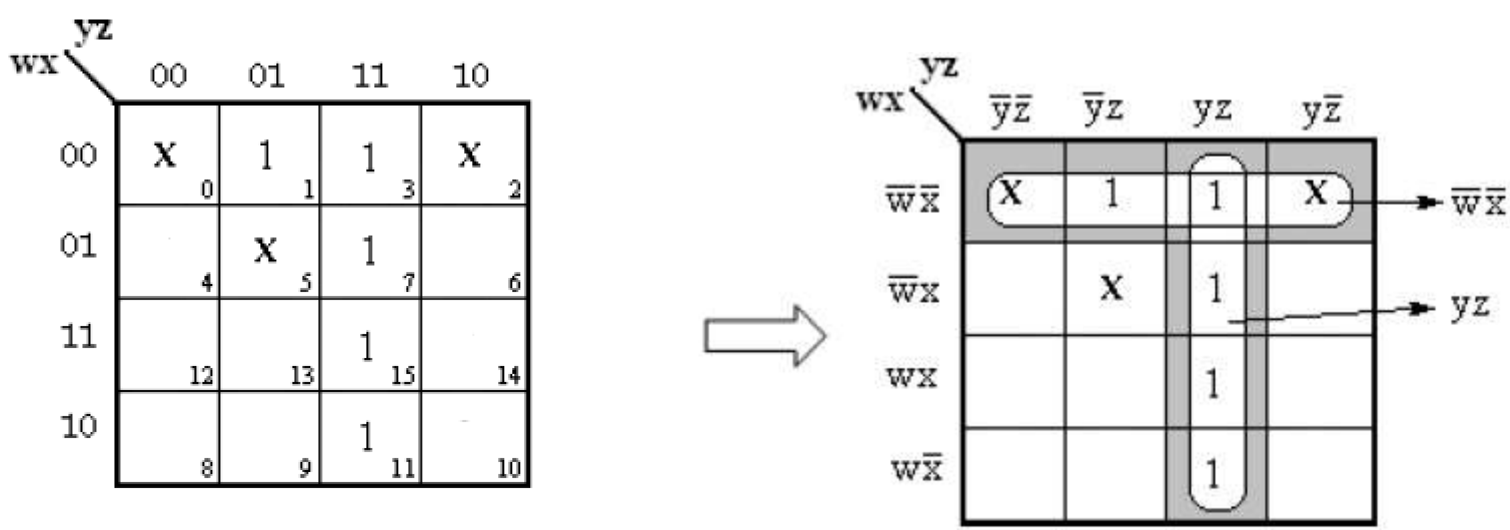
Simplify using K - Map

1. $F(x, y, z) = \sum m(0, 1, 2, 4, 5) + \sum d(3, 6, 7)$



$F(x, y, z) = 1$

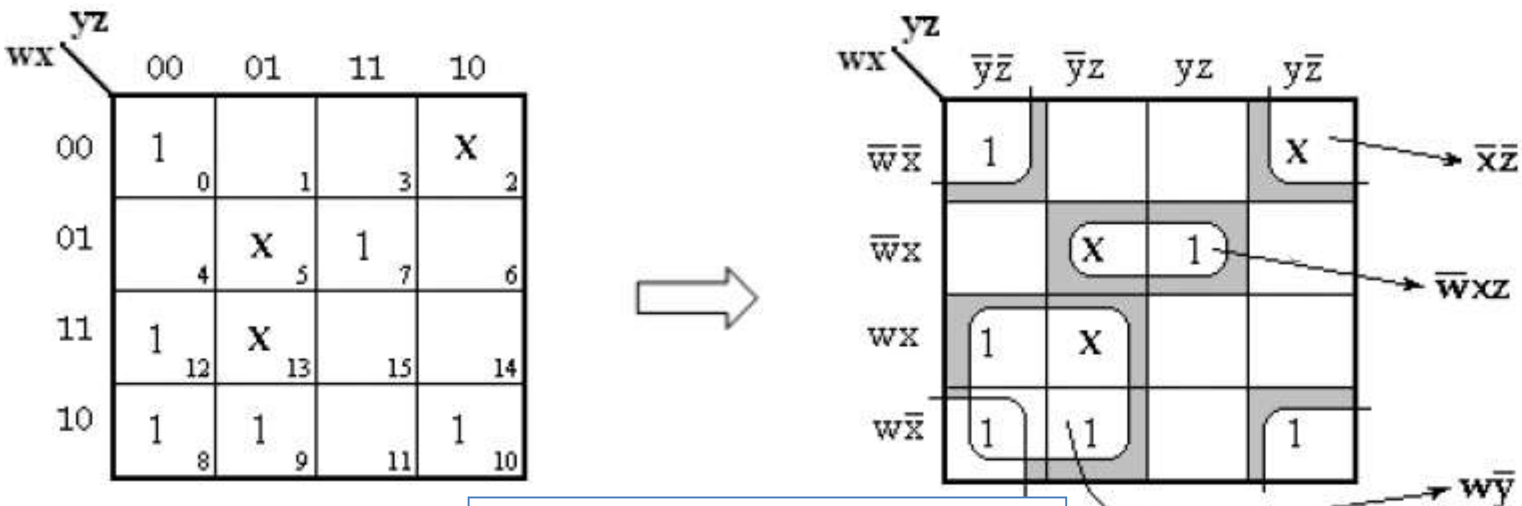
2. $F(w, x, y, z) = \sum m(1, 3, 7, 11, 15) + \sum d(0, 2, 5)$



$F(w, x, y, z) = \overline{w}\,\overline{x} + yz$

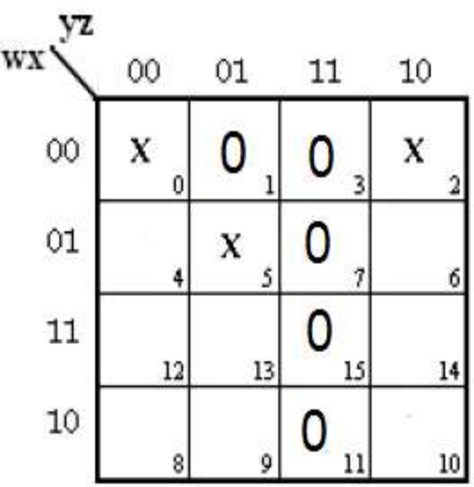
Simplify using K - Map

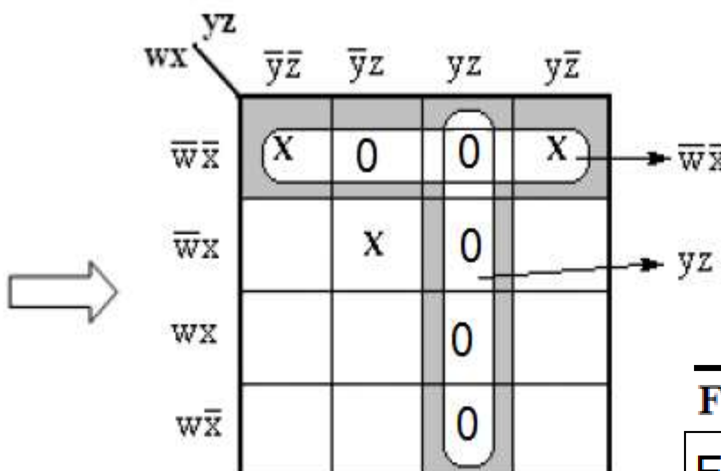
3. F (w, x, y, z) = $\sum m$ (0, 7, 8, 9, 10, 12)+ $\sum d$ (2, 5, 13)



F (w, x, y, z) = $\bar{w}$ xz+ w$\bar{y}$ + $\bar{x}$ $\bar{z}$.

4. F (w, x, y, z) = $\prod M$ (1, 3, 7, 11, 15)+ $\sum d$ (0, 2, 5)





$\bar{F}$ (w, x, y, z) = $\bar{w}$ $\bar{x}$ + yz

F (w, x,y,z) = (w + x) ($\bar{y}$ + $\bar{z}$)

Home work : Simplify using K - Map

7. $F(w, x, y, z) = \sum m(0, 1, 4, 8, 9, 10) + \sum d(2, 11)$

Ans: $F(w, x, y, z) = w\overline{x} + \overline{x}\,\overline{y} + \overline{w}\,\overline{y}\,\overline{z}$.

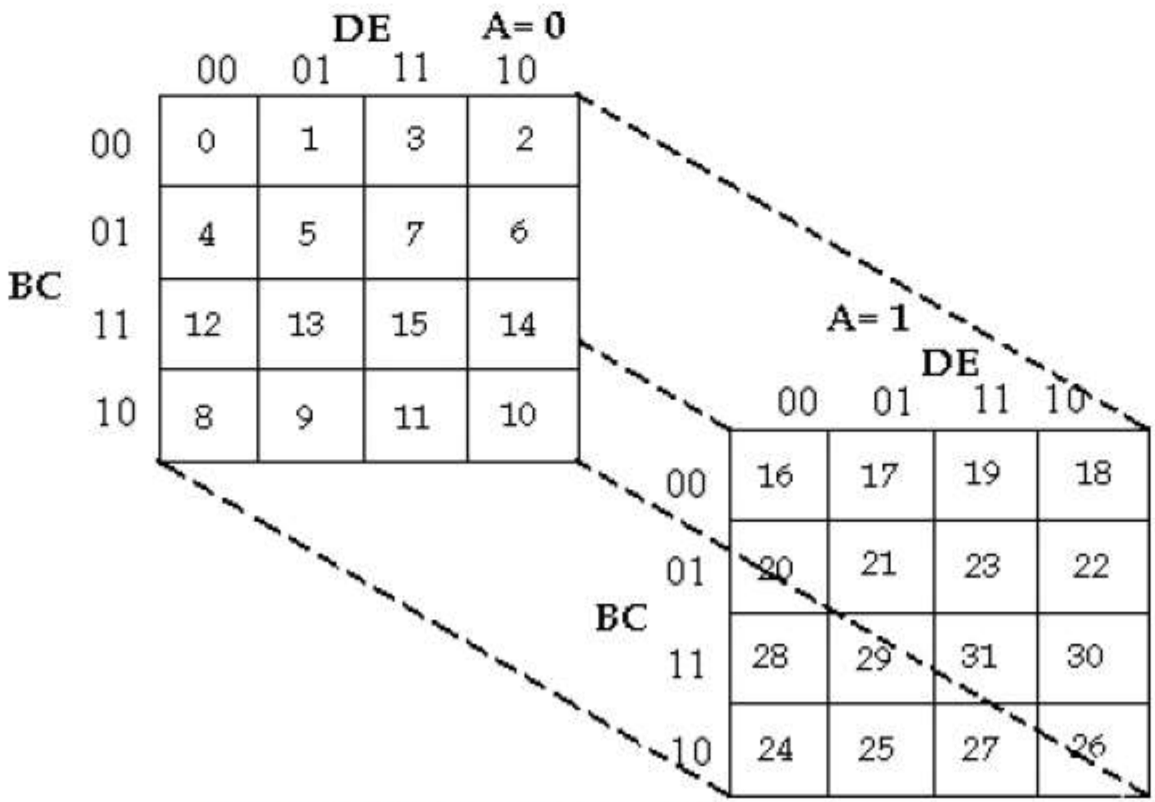
8. $F(A, B, C, D) = \sum m(0, 6, 8, 13, 14) + \sum d(2, 4, 10)$

Ans: $F(A, B, C, D) = C\overline{D} + \overline{B}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$

Five Variable K-Map

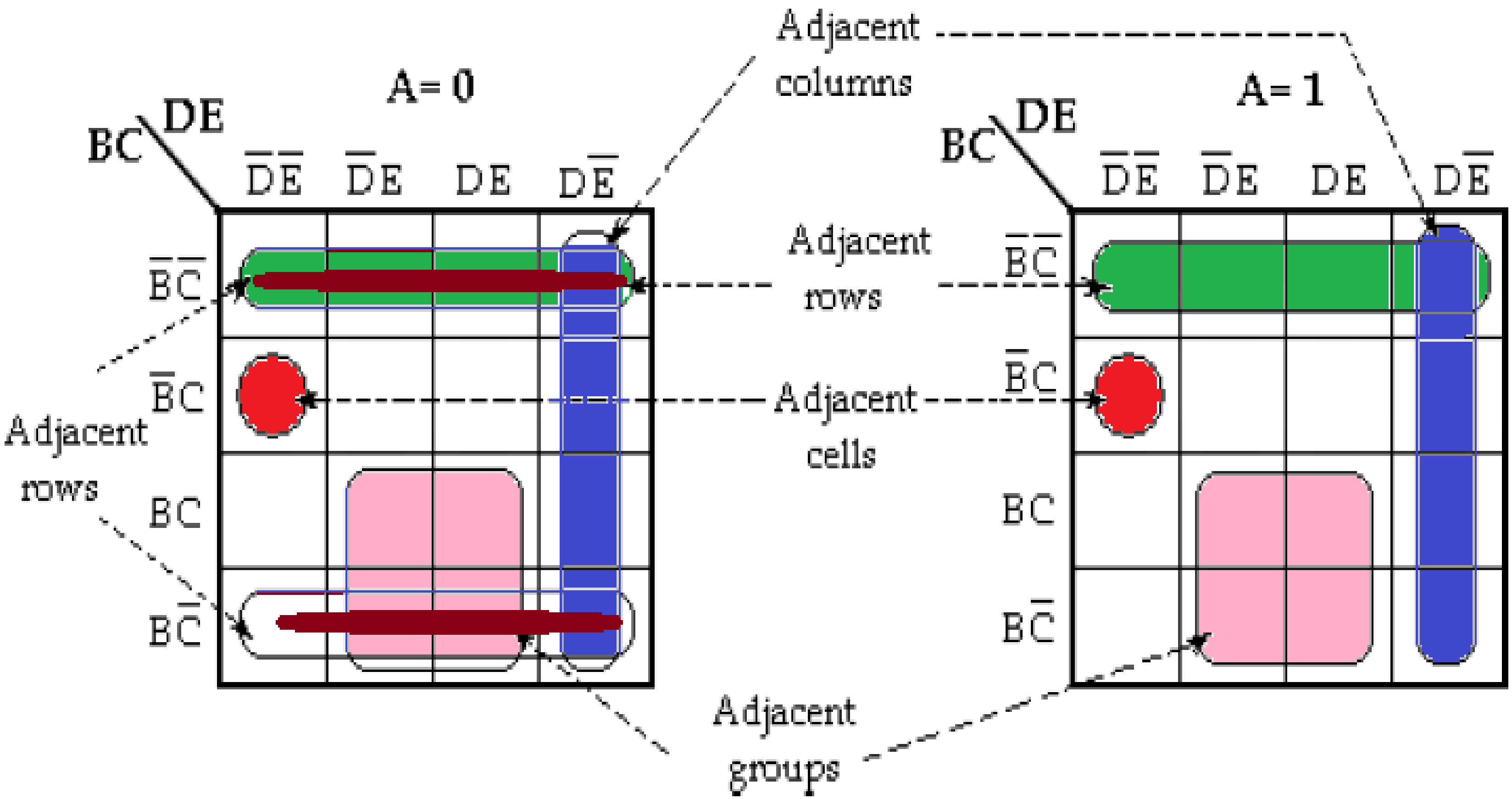
A 5- variable K- map requires 25= 32 cells, but adjacent cells are difficult to identify on a single 32-cell map. Therefore, two 16 cell K-maps are used.

If the variables are A, B, C, D and E, two identical 16- cell maps containing B, C, D and E can be constructed. One map is used for A and other for A’.



Five- Variable Karnaugh map (Layer Structure)

Grouping in 5 variable K-Map



Problem

1. Simplify the Boolean function
   F (A, B, C, D, E) = ∑m (0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)

A= 0

BC \ DE	D̄Ē	D̄E	DE	DĒ
B̄C̄	1 0	1	3	1 2
B̄C	1 4	5	7	1 6
BC	12	1 13	1 15	14
BC̄	8	1 9	1 11	10

A= 1

BC \ DE	D̄Ē	D̄E	DE	DĒ
B̄C̄	16	1 17	19	18
B̄C	20	1 21	23	22
BC	28	1 29	1 31	30
BC̄	24	1 25	1 27	26

A= 0

BC \ DE	D̄Ē	D̄E	DE	DĒ
B̄C̄	1 0	0 1	0 3	1 2
B̄C	1 4	0 5	0 7	1 6
BC	0 12	1 13	1 15	0 14
BC̄	0 8	1 9	1 11	0 10

A= 1

BC \ DE	D̄Ē	D̄E	DE	DĒ
B̄C̄	0 16	1 17	0 19	0 18
B̄C	0 20	1 21	0 23	0 22
BC	0 28	1 29	1 31	0 30
BC̄	0 24	1 25	1 27	0 26

F (A, B, C, D, E) = A'B'E'+ BE+ AD'E

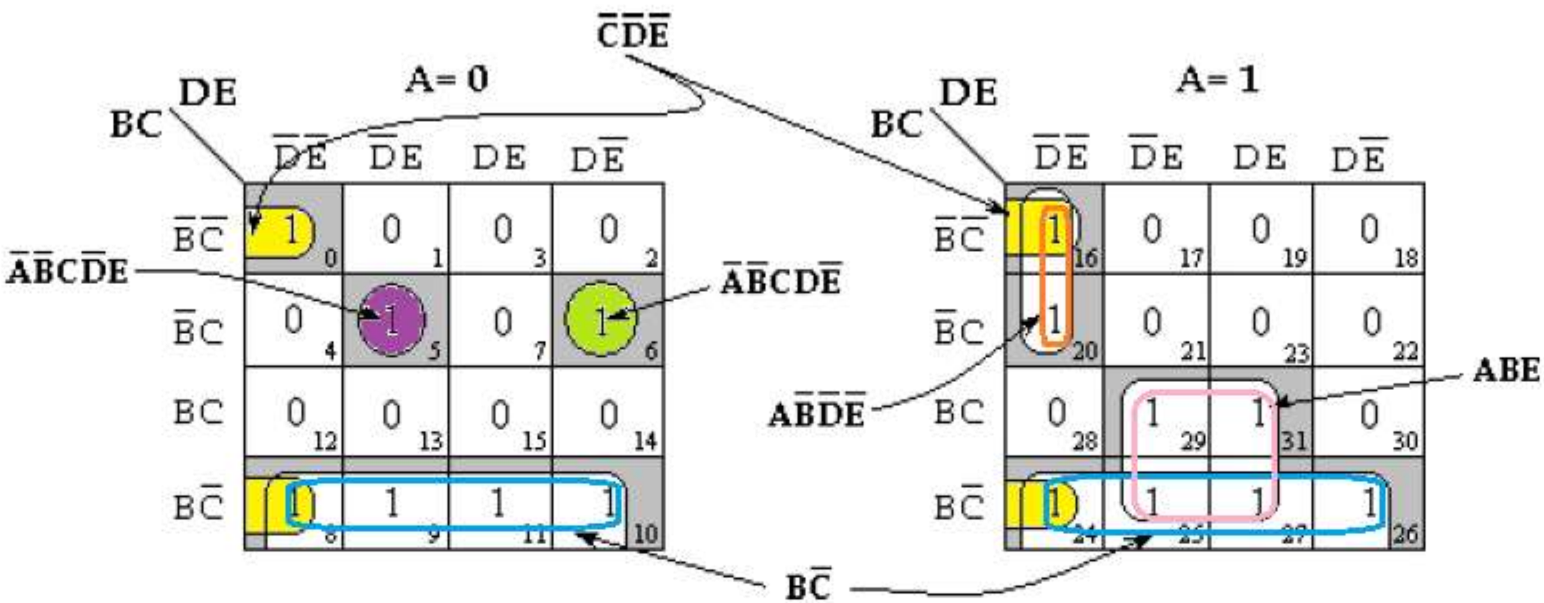
2. F (A, B, C, D, E) = $\sum m$ (0, 5, 6, 8, 9, 10, 11, 16, 20, 24, 25, 26, 27, 29, 31)

A= 0

BC \ DE	$\overline{D}\,\overline{E}$	$\overline{D}E$	DE	$D\overline{E}$
$\overline{B}\,\overline{C}$	1 (0)	(1)	(3)	(2)
$\overline{B}C$	(4)	1 (5)	(7)	1 (6)
BC	(12)	(13)	(15)	(14)
$B\overline{C}$	1 (8)	1 (9)	1 (11)	1 (10)

A= 1

BC \ DE	$\overline{D}\,\overline{E}$	$\overline{D}E$	DE	$D\overline{E}$
$\overline{B}\,\overline{C}$	1 (16)	(17)	(19)	(18)
$\overline{B}C$	1 (20)	(21)	(23)	(22)
BC	(28)	1 (29)	1 (31)	(30)
$B\overline{C}$	1 (24)	1 (25)	1 (27)	1 (26)



F (A, B, C, D, E) = C’D’E’+ A’B’CD’E+ A’B’CDE’+ AB’D’E’+ ABE+ BC’

Home Work

1. $F(A, B, C, D, E) = \sum m\,(0, 1, 2, 6, 7, 9, 12, 28, 29, 31)$

   Ans: **F (A, B, C, D, E) = BCD'E'+ ABCE+ A'B'C'E'+ A'C'D'E+ A'B'CD**

2. $F(A, B, C, D, E) = \sum m\,(1, 4, 8, 10, 11, 20, 22, 24, 25, 26) + \sum d\,(0, 12, 16, 17)$

   Ans: **F (A, B, C, D, E) = B'C'D'+ A'D'E'+ BC'E'+ A'BC'D+ AC'D'+ AB'CE'**

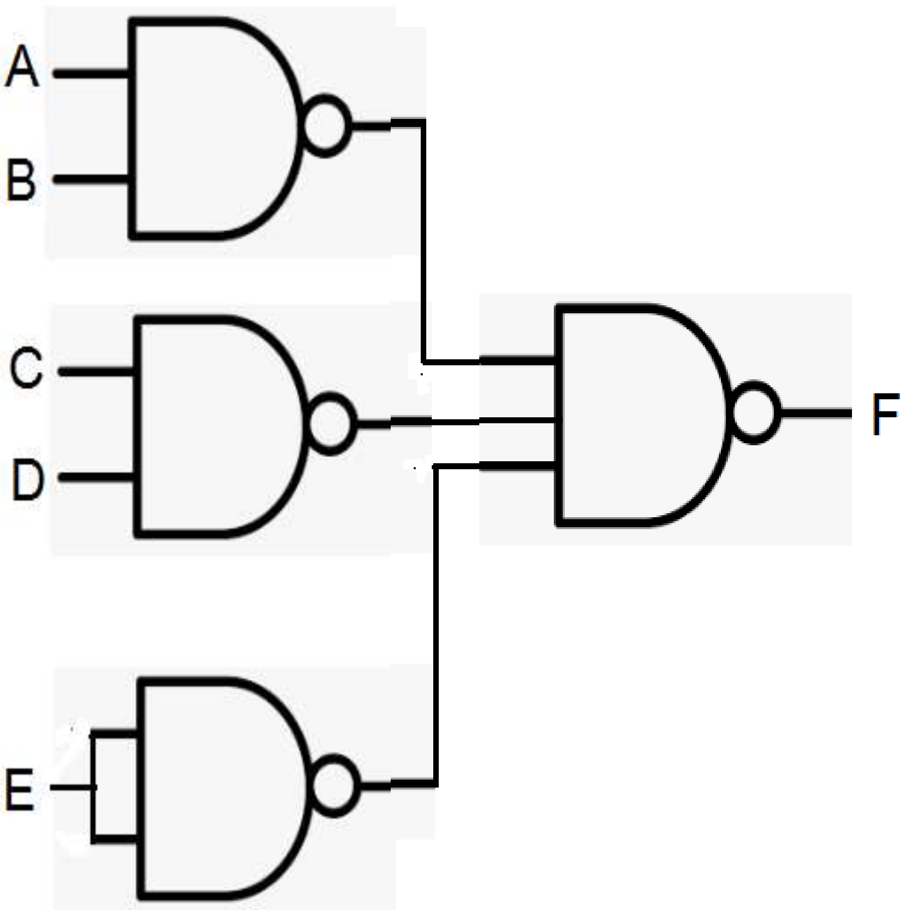
Two Level Gate Network

- **The SOP can be implemented using NAND – NAND logic**
  1. Each product term is connected to NAND gates in level 1
  2. One NAND is connected in the second level 2

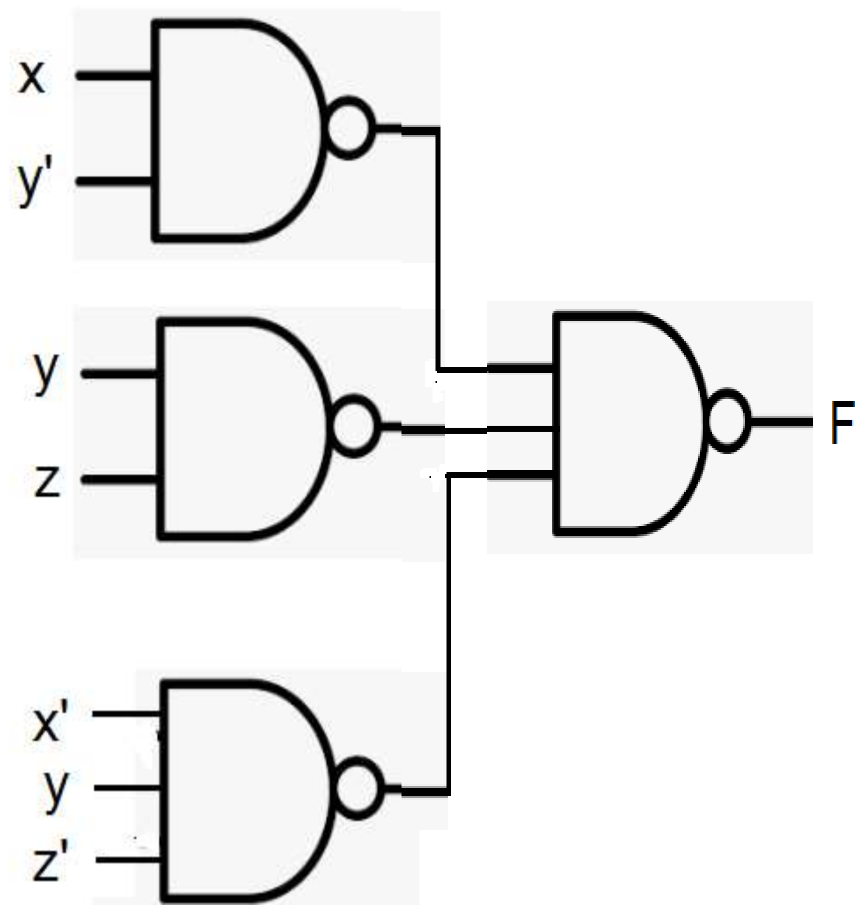
- **The POS can be implemented using NOR – NOR logic**
  1. Each sum term is connected to NOR gates in level 1
  2. One NOR is connected in the second level 2

Implement Using NAND – NAND logic

$F = A.B + C.D + E$

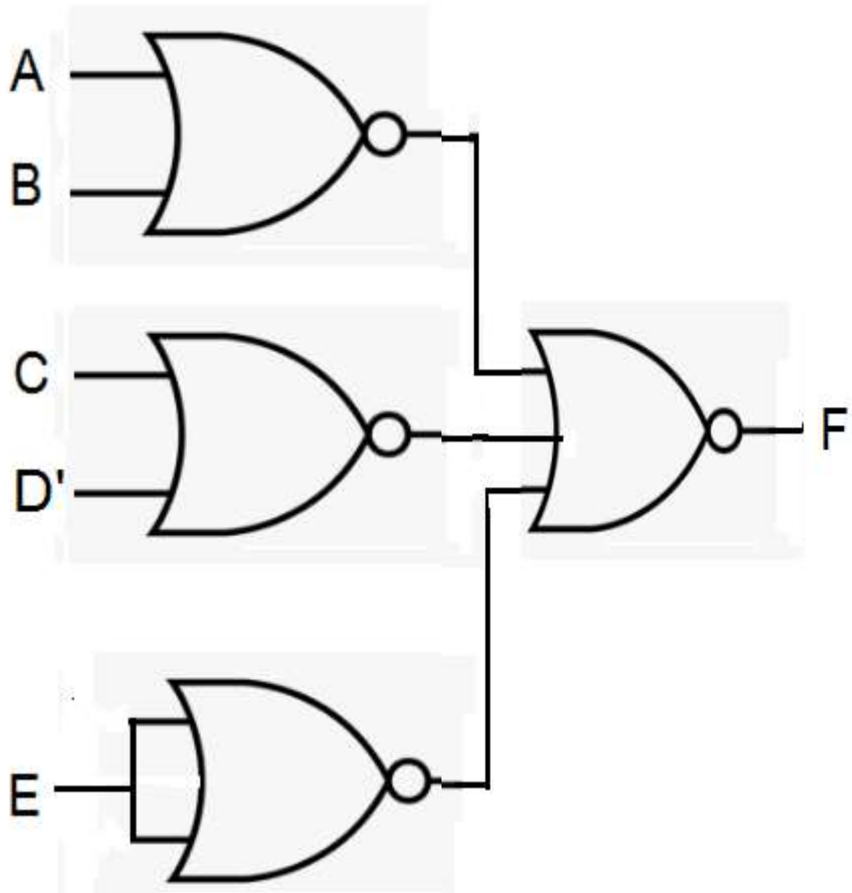


$F = x\,y' + y\,z + x'\,y\,z'$

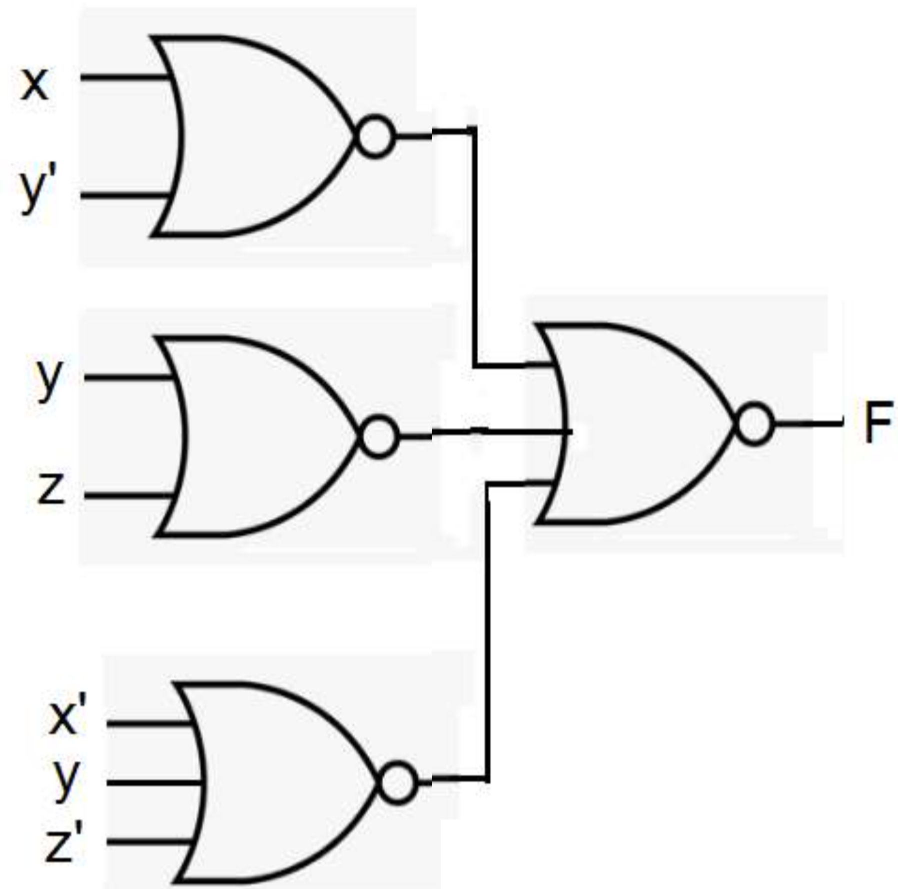


Implement Using NOR – NOR logic

$F = (A+B)(C+D')E$



$F = (x + y')(y + z)(x' + y + z')$



QUINE- MCCLUSKEY METHOD or TABULATION METHOD

The tabular method which is also known as the Quine-McCluskey method is particularly useful when minimising functions having a large number of variables, e.g. The six-variable functions. Computer programs have been developed employing this algorithm. The method reduces a function in standard sum of products form to a set of prime implicants from which as many variables are eliminated as possible. These prime implicants are then examined to see if some are redundant.

The tabular method makes repeated use of the law $A + \bar{A} = 1$. Note that Binary notation is used for the function, although decimal notation is also used for the functions. As usual a variable in true form is denoted by 1, in inverted form by 0, and the absence of a variable by a dash (-).

RULES OF TABULATION METHOD

1. List all minterms in the binary form.
2. Arrange the minterms according to number of 1's and separate by a horizontal line.
3. Compare each binary number with every term in the adjacent next higher category and if they differ only by one position, put a check mark and copy the term in the next column with '-' in the position that they differed.
4. Apply the same process described in step 3 for the resultant column and continue these until no further elimination of literals.
5. List all the prime implicants.
6. Select the minimum number of prime implicants which must cover all the minterms.

Simplify the given boolean expression using Tabulation method

$$Y(A, B, C, D) = \sum m(2, 4, 5, 9, 12, 13)$$

Table : 1

Min Term	A	B	C	D	No.of 1s	
2	0	0	1	0	1	✓
4	0	1	0	0	1	✓
5	0	1	0	1	2	✓
9	1	0	0	1	2	✓
12	1	1	0	0	2	✓
13	1	1	0	1	3	✓

Table : 2

Min Term	A	B	C	D	
2	0	0	1	0	*
4	0	1	0	0	✓
5	0	1	0	1	✓
9	1	0	0	1	✓
12	1	1	0	0	✓
13	1	1	0	1	✓

Table : 3

Min Term	A	B	C	D	
4,5	0	1	0	_	✓
4,12	_	1	0	0	✓
5,13	_	1	0	1	✓
9,13	1	_	0	1	*
12,13	1	1	0	_	✓

Table : 4

Min Term	A	B	C	D
4,5,12,13	_	1	0	_
4,12,5,13	_	1	0	_

Table : 4 Reduced

Min Term	A	B	C	D	
4,5,12,13	_	1	0	_	*

Table: 5 Prime Implicants Table

Min Term	A	B	C	D	Product Term		2	4	5	9	12	13
2	0	0	1	0	A' B' C D'	✓	X					
9,13	1	_	0	1	A C' D	✓				X		X
4, 5, 12, 13	_	1	0	_	B C'	✓		X	X		X	X
							✓	✓	✓	✓	✓	

Select Single X column for Essential Prime Implecants

$$Y = A'B'CD' + AC'D + BC'$$

Simplify using Tabulation Method

2. $f(A, B, C, D) = \sum m(0,1,2,3,5,7,8,10,12,13,15)$

Table : 1

Min Term	A	B	C	D	No.of 1s	
0	0	0	0	0	0	✓
1	0	0	0	1	1	✓
2	0	0	1	0	1	✓
3	0	0	1	1	2	✓
5	0	1	0	1	2	✓
7	0	1	1	1	3	✓
8	1	0	0	0	1	✓
10	1	0	1	0	2	✓
12	1	1	0	0	2	✓
13	1	1	0	1	3	✓
15	1	1	1	1	4	✓

(Binary Representation: A, B, C, D)

Table : 2

Min Term	A	B	C	D	
0	0	0	0	0	✓
1	0	0	0	1	✓
2	0	0	1	0	✓
8	1	0	0	0	✓
3	0	0	1	1	✓
5	0	1	0	1	✓
10	1	0	1	0	✓
12	1	1	0	0	✓
7	0	1	1	1	✓
13	1	1	0	1	✓
15	1	1	1	1	✓

(Binary Representation: A, B, C, D)

Table : 3

Min Term	A	B	C	D	
0,1	0	0	0	_	✓
0,2	0	0	_	0	✓
0,8	_	0	0	0	✓
1,3	0	0	_	1	✓
1,5	0	_	0	1	✓
2,3	0	0	1	_	✓
2,10	_	0	1	0	✓
8,10	1	0	_	0	✓
8,12	1	_	0	0	*
3,7	0	_	1	1	✓
5,7	0	1	_	1	✓
5,13	_	1	0	1	✓
12,13	1	1	0	_	*
7,15	_	1	1	1	✓
13,15	1	1	_	1	✓

(Binary Representation: A, B, C, D)

2. $f(A, B, C, D) = \sum m(0,1,2,3,5,7,8,10,12,13,15)$ Cont..

Table : 4

Min Term	A	B	C	D
0, 1, 2, 3	0	0	_	_
0, 2, 1, 3	0	0	_	_
0, 2, 8, 10	_	0	_	0
0, 8, 2, 3	_	0	_	0
1, 3, 5, 7	0	_	_	1
1, 5, 3, 7	0	_	_	1
5, 7, 13, 15	_	1	_	1
5, 13, 7, 15	_	1	_	1

Table : 4 Reduced

0, 1, 2, 3	0	0	_	_	*
0, 2, 8, 10	_	0	_	0	*
1, 3, 5, 7	0	_	_	1	*
5, 7, 13, 15	_	1	_	1	*

Table: 5 Prime Implicants Table

Min Term	A	B	C	D	Product Term	0	1	2	3	5	7	8	10	12	13	15
8,12	1	_	0	0	$A\bar{C}\bar{D}$							X		X		
12,13	1	1	0	_	$AB\bar{C}$									X	X	
0, 1, 2, 3	0	0	_	_	$\bar{A}\bar{B}$	X	X	X	X							
0, 2, 8, 10	_	0	_	0	$\bar{B}\bar{D}$ ✓	X		X				X	X			
1, 3, 5, 7	0	_	_	1	$\bar{A}D$		X		X	X	X					
5, 7, 13, 15	_	1	_	1	BD ✓					X	X				X	X
Select Single X column for Essential Prime Implecants													✓			✓

Missing min terms 1, 3, 12 so include 1, 3, 5, 7 & 12, 13 OR 1, 3, 5, 7 & 8, 12

$f(A, B, C, D) = \bar{B}\bar{D} + BD + \bar{A}D + AB\bar{C}$ OR $f(A, B, C, D) = \bar{B}\bar{D} + BD + \bar{A}D + A\bar{C}\bar{D}$

Simplify using Tabulation Method

3. $Y(A,B,C,D,E) = \sum m(0, 1, 9, 15, 24, 29, 30) + \sum d(8, 11, 31)$

Table : 1

Min Term	A	B	C	D	E	No.of 1s	
0	0	0	0	0	0	0	✓
1	0	0	0	0	1	1	✓
d8	0	1	0	0	0	1	✓
9	0	1	0	0	1	2	✓
d11	0	1	0	1	1	3	✓
15	0	1	1	1	1	4	✓
24	1	1	0	0	0	2	✓
29	1	1	1	0	1	4	✓
30	1	1	1	1	0	4	✓
d31	1	1	1	1	1	5	✓

Table : 2

Min Term	A	B	C	D	E	
0	0	0	0	0	0	✓
1	0	0	0	0	1	✓
8	0	1	0	0	0	✓
9	0	1	0	0	1	✓
24	1	1	0	0	0	✓
11	0	1	0	1	1	✓
15	0	1	1	1	1	✓
29	1	1	1	0	1	✓
30	1	1	1	1	0	✓
31	1	1	1	1	1	✓

Simplify using Tabulation Method

3. $Y(A,B,C,D,E) = \sum m(0, 1, 9, 15, 24, 29, 30) + \sum d(8, 11, 31)$ cont..

Table : 3

Min Term	Binary Representation					
	A	B	C	D	E	
0,1	0	0	0	0	_	✓
0,8	0	_	0	0	0	✓
1,9	0	_	0	0	1	✓
8,9	0	1	0	0	_	✓
8,24	_	1	0	0	0	*
9,11	0	1	0	_	1	*
11,15	0	1	_	1	1	*
15,31	_	1	1	1	1	*
29,31	1	1	1	_	1	*
30,31	1	1	1	1	_	*

Table : 4

Min Term	Binary Representation				
	A	B	C	D	E
0,1,8,9	0	_	0	0	_
0,8,1,9	0	_	0	0	_

Table : 4 Reduced

0,1,8,9	0	_	0	0	_	*

Simplify using Tabulation Method

3. $Y(A,B,C,D,E) = \sum m(0, 1, 9, 15, 24, 29, 30) + \sum d(8, 11, 31)$ cont..

Table: 5 Prime Implicants Table

Min Term	Binary Representation					Product Term		0	1	d8	9	d11	15	24	29	30	d31
	A	B	C	D	E												
8,24	_	1	0	0	0	B C' D' E'	✓			X				X			
9,11	0	1	0	_	1	A' B C' E					X	X					
11,15	0	1	_	1	1	A' B D E						X	X				
15,31	_	1	1	1	1	B C D E							X				X
29,31	1	1	1	_	1	A B C E	✓								X		X
30,31	1	1	1	1	_	A B C D	✓									X	X
0,1,8,9	0	_	0	0	_	A' C' D'	✓	X	X	X	X						
Select Single X column for Essential Prime Implecants								✓	✓					✓	✓	✓	

Missing min terms 15 so include 11, 15 OR 15,31

Y = B C' D' E' + A B C E + A B C D + A' C' D' + A' B D E OR

Y = B C' D' E' + A B C E + A B C D + A' C' D' + B C D E

Home work

1. $F(A, B, C, D) = \sum m(0, 2, 3, 6, 7, 8, 10, 12, 13)$

   Ans : $F(A, B, C, D) = A B C' + B' D' + A' C$

2. $f(w, x, y, z) = \sum m(1, 3, 4, 5, 6, 8, 9, 10, 11)$

   Ans : $f = w' x z' + x' z + w x' + w' x y'$