Mini Project 1

Adarsh Raghupati NetID: axh190002
Contribution: Question 1 a) b) 1,2,3,4,5
Keerti Keerti NetID: kxk190012
Contribution: Question 2 and Question 1 b) 6 and c

Question 1:

It is given that *XA* and *XB* follow independent exponential distributions with mean 10 years.
$\lambda = 1/10$
$= 0.1$

Lifetime of satellite = Max(X_A, X_B)

Given E(T) = 15

(a) probability that the lifetime of the satellite exceeds 15 years = P(X > 15)

$$P(x > 15) = \int_{15}^{\infty} f_T(t)\, dt$$

$$= \int_{15}^{\infty} 0.2 e^{-0.1t} - 0.2 e^{-0.2t}$$

$$= 0.2 \left[\frac{e^{-1.5}}{0.1} - \frac{e^{-3}}{0.2} \right]$$

$$= 0.3964$$

(b) Monte Carlo approach to compute *E*(*T*) and *P*(*T* > 15):

i) One draw of the block lifetimes *XA* and *XB*

```
set.seed(100)          #Setting seed to 100
x_a = rexp(1,0.1)      #Draw for XA
x_b = rexp(1,0.1)      #Draw for XB
T = max(x_a,x_b)  #lifeTime = 9.242116
```

One line code for above operation:

Satellite life time T = max(rexp(1, 0.1), rexp(1, 0.1))

ii) Repeating the previous step 10,000 times.

T = replicate(10000, max(rexp(1, 0.1), rexp(1, 0.1)))

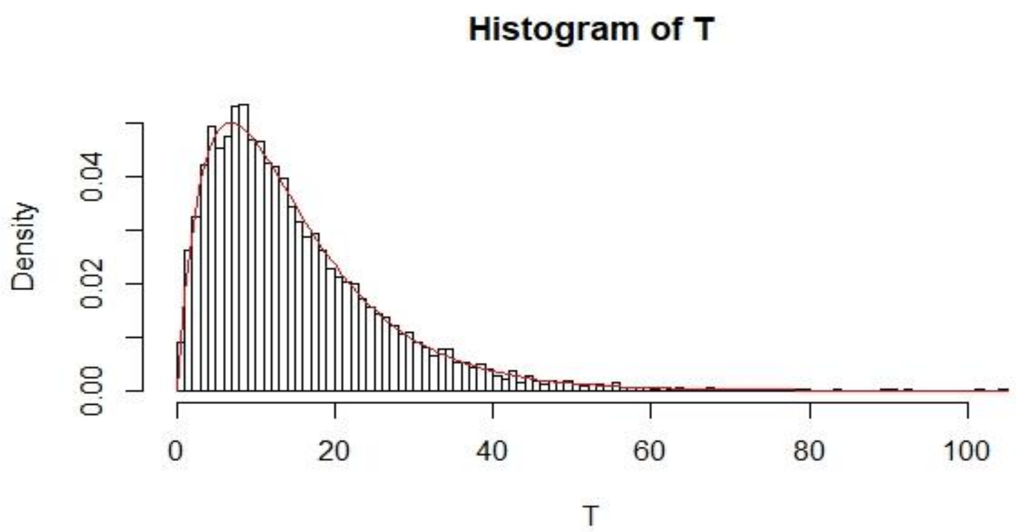
#Used replicate function of R to draw 10000 times

iii) Histogram of the draws. Using hist and curve functions
hist (T, probability = TRUE, breaks = 100)

Superimposing the density function given above

curve (0.2*exp(-0.1*x)-0.2*exp(-0.2*x), col ="red", add=TRUE)

Observation: From the below figure we can see that histogram values and density curve values are similar. Hence it can be said that there is good agreement between density and histogram



iv) Estimating E(T)
mean Value = mean(T) # R code to find mean
mean Value = 14.90889
Mean value from 10000 draws is very near to the given value 15

v) Estimate of P(x > 15) from saved samples:
P(x > 15) = 0.3921

Obtained from R code:
length(which(T>15))/length(T)

Probability value found in part (a) is 0.3964 which is very near to the computed value from the sample

vi) Repeating above process 4 more times:

Trial #	E(T)	P(X > 15)
1	15.16428	0.3982
2	14.88494	0.3933
3	14.9092	0.3971
4	14.96616	0.3979

(c) Repeating the similar process with 1000 draws

Trial #	E(T)	P(X > 15)
1	15.16085	0.407
2	14.96916	0.394
3	14.9932	0.386
4	14.52492	0.374
5	14.26848	0.368

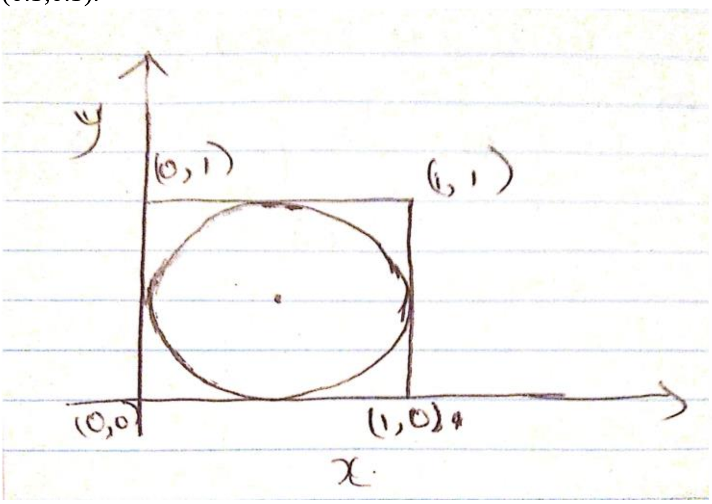
Repeating the similar process with 100000 draws

Trial #	E(T)	P(X > 15)
1	14.98211	0.39737
2	14.99169	0.39381
3	15.04843	0.3969
4	14.99765	0.39611
5	15.006	0.39532

Observation: From the above two tables we can observe that estimated value with 100000 draws is more accurate th an estimation with 1000 draws. When the values of 10000 draws is compared to the estimate of 100000 draws we ca n see that there is no significant change in the accuracy. Hence it can be concluded that accuracy of estimated value becomes better with the increase in number of draws upto certain limit.

Question 2: Use a Monte Carlo approach estimate the value of π based on 10,000 replications.

Firstly, considering a square with co-ordinates (0,0) (0,1) (1,0) (1,1) and inscribing a circle with the center at (0.5,0.5).



The probability of the points being inside the circle or on the circle is =
(Area of the circle) / (Area of the square)
(Π * radius ^2) / (side^2)
(Π * 0.5*0.5) / (1*1)
(Π * 0.25)
i.e., The probability of the points being inside the circle or on the circle is = (Π * 0.25)

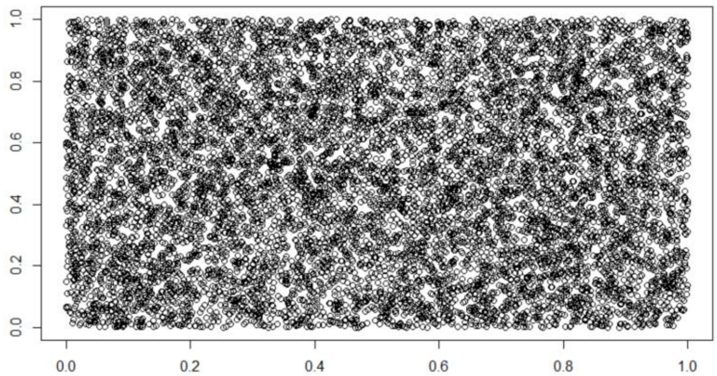
The probability of the points being inside the circle or on the circle is * 4 = (Π) -------equation 1

Estimating the above rules using R simulation:

// generate random uniformly distributed 10000 numbers from 0-1

```
> x = runif(10000)
> y = runif(10000)

> plot(x,y)
```




```
p = mean((x-0.5)^2+(y-0.5)^2 <= 0.25)

> print("Value of pi : ")
[1] "Value of pi : "

> print(4*p)
[1] 3.1324
```

Hence using the random uniformly generated points, the probability of any point lying inside the circle is 0.7831.

The value of pi then will be = 4*0.7831(using Equation 1) = 3.1324 ~ actual value of pi 3.14159.