

# Adapting Heuristic Mastermind Strategies to Evolutionary Algorithms

Thomas Philip Runarsson and Juan J. Merelo-Guervós

**Abstract.** The art of solving the Mastermind puzzle was initiated by Donald Knuth and is already more than thirty years old; despite that, it still receives much attention in operational research and computer games journals, not to mention the nature-inspired stochastic algorithm literature. In this paper we try to suggest a strategy that will allow nature-inspired algorithms to obtain results as good as those based on exhaustive search strategies; in order to do that, we first review, compare and improve current approaches to solving the puzzle; then we test one of these strategies with an estimation of distribution algorithm. Finally, we try to find a strategy that falls short of being exhaustive, and is then amenable for inclusion in nature inspired algorithms (such as evolutionary of particle swarm algorithms). This paper proves that by the incorporation of what we call *local entropy* into the fitness function of the evolutionary algorithm it becomes a better player than a random one, and gives a rule of thumb on how to incorporate the best heuristic strategies to evolutionary algorithms without incurring in an excessive computational cost.

**Keywords:** puzzles, games, Mastermind, bulls and cows, search strategies, oracle games, evolutionary algorithms, estimation of distribution algorithms.

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# 1 Introduction

Mastermind in its current version is a board game that was introduced by the telecommunications expert Mordecai Merowitz [15] and sold to the company Invicta Plastics, who renamed it to its actual name; in fact, Mastermind is a version of a traditional puzzle called *bulls and cows* that dates back to the Middle Ages. In any case, Mastermind is a puzzle (rather than a game) in which two persons, the *codemaker* and *codebreaker* try to outsmart each other in the following way:

- The codemaker sets a length  $\ell$  combination of  $\kappa$  symbols. In the classical version,  $\ell = 4$  and  $\kappa = 6$ , and color pegs are used as symbols over a board with rows of  $\ell = 4$  holes; however, in this paper we will use uppercase letters starting with A instead of colours.
- The codebreaker then tries to guess this secret code by producing a combination.
- The codemaker gives a response consisting on the number of symbols guessed in the right position (usually represented as black pegs) and the number of symbols in an incorrect position(usually represented as white pegs).
- The codebreaker then, using that information as a hint, produces a new combination until the secret code is found.

For instance, a game could go like this: The codemaker sets the secret code *ABBC*. The rest of the game is shown in Table 1.

**Table 1** Progress in a Mastermind game that tries to guess the secret combination *ABBC*. The player here is not particularly clueful, playing a third combination that is not *consistent* with the first one, not coinciding in two positions and one color (corresponding to the 2 black/1 white response given by the codemaker) with it

Combination	Response
AABB	2 black, 1 white
ACDE	1 black, 1 white
FFDA	1 white
ABBE	3 black
ABBC	4 black

Different variations of the game include giving information on which position has been guessed correctly, avoiding repeated symbols in the secret combination (*bulls and cows* is actually this way), or allowing the codemaker to change the code during the game (but only if this does not make responses made so far false).

In any case, the codebreaker is allowed to make a maximum number of combinations (usually fifteen, or more for larger values of  $\kappa$  and  $\ell$ ), and score corresponds to the number of combinations needed to find the secret code; after repeating the game a number of times with codemaker and codebreaker changing sides, the one with the lower score wins.

Since Mastermind is asymmetric, in the sense that the position of one of the players after setting the secret code is almost completely passive, and limited to

give hints as a response to the guesses of the codebreaker, it is rather a puzzle than a game, since the codebreaker is not really matching his skills against the codemaker, but facing a problem that must be solved with the help of hints, the implication being that playing Mastermind is more similar to solving a Sudoku than to a game of chess; thus, the solution to Mastermind, unless in a very particular situation (always playing with an opponent who has a particular bias for choosing codes, or maybe playing the dynamic code version), is a search problem with constraints.

What makes this problem interesting is its relation to other, generally called *oracle* problems such as circuit and program testing, differential cryptanalysis and other puzzle games (these similarities were reviewed in our previous paper [10]) is the fact that it has been proved to be NP-complete [5, 14] and that there are several open issues, namely, what is the lowest average number of guesses you can achieve, how to minimize the number of evaluations needed to find them (and thus the runtime of the algorithm), and obviously, how it scales when increasing  $\kappa$  and  $\ell$ . This paper will concentrate on the first issue.

This NP completeness implies that it is difficult to find algorithms that solve the problem in a reasonable amount of time, and that is why in our previous work [2, 9, 10] we introduced stochastic evolutionary and simulated annealing algorithms that solved the Mastermind puzzle in the general case, finding solutions in a reasonable amount of time that scaled roughly logarithmically with problem size. The strategy followed to play the game was optimal in the sense that it was guaranteed to find a solution after a finite number of combinations; however, there was no additional selection on the combination played other than the fact that it was consistent with the responses given so far.

In this paper, after reviewing how the state of the art in solving this puzzle has evolved in the last few years, we examine how we could improve the code-breaking skills of an evolutionary algorithm by using different techniques, and how these techniques can be further optimized. In order to do that we examine different ways of scoring combinations in the search space, how to choose one combination out of a set of combinations that have exactly the same score, and how all that can be applied to a simple estimation of distribution algorithm to improve results over a standard one. This paper presents for the first time an evolutionary algorithm that biases search so that combinations played have a better chance of reducing the size of the remaining search space, and adapt to an stochastic environment deterministic techniques that had been previously published; all techniques, unlike our former papers, have been tested over the whole code space, instead of a random sample, so that they can be compared and yield significant results.

The rest of the paper is organized as follows: next we establish terminology and examine the state of the art; then heuristic strategies for Mastermind are examined in Section 3; the way they could be adapted to an evolutionary algorithm is presented in Section 5, and finally, conclusions are drawn in the closing section 6.

## 2 State of the Art

Before presenting the state of the art, a few definitions are needed. We will use the term *response* for the return code of the codemaker to a played combination,  $c_{played}$ . A response is therefore a function of the combination,  $c_{played}$  and the secret combination  $c_{secret}$ , let the response be denoted by  $h(c_{played}, c_{secret})$ . A combination  $c$  is *consistent with*  $c_{played}$  iff

$$h(c_{played}, c_{secret}) = h(c_{played}, c) \quad (1)$$

that is, if the combination has as many black and white pins with respect to the played combination as the played combination with respect to the secret combination. Furthermore, a combination is *consistent* iff

$$h(c_i, c) = h(c_i, c_{secret}) \text{ for } i = 1..n \quad (2)$$

where  $n$  is the number of combinations,  $c_i$ , played so far; that is,  $c$  is *consistent with* all guesses made so far. A combination that is consistent is a candidate solution. The concept of consistent combination will be important for characterizing different approaches to the game of Mastermind.

One of the earliest strategies, by Knuth [6], is perhaps the most intuitive for Mastermind. In this strategy the player selects the guess that reduces the number of remaining consistent guesses and the opponent the return code leading to the maximum number of guesses. Using a complete minimax search Knuth shows that a maximum of 5 guesses are needed to solve the game using this strategy. This type of strategy is still the most widely used today: most algorithms for Mastermind start by searching for a consistent combination to play.

In some cases once a single consistent guess is found it is immediately played, in which case the object is to find a consistent guess as fast as possible. For example, in [10] an evolutionary algorithm is described for this purpose. These strategies are fast and do not need to examine a big part of the space. Playing a consistent combinations eventually produces a number of guesses that uniquely determine the code. However, the maximum, and average, number of combinations needed is usually high. Hence, some bias must be introduced in the way combinations are searched. If not, the guesses will be no better than a purely random approach, as solutions found (and played) are a random sample of the space of consistent guesses.

The alternative to discovering a single consistent guess is to collect a set of consistent guesses and select among them the best alternative. For this a number of heuristics have been developed over the years. Typically these heuristics require all consistent guesses to be first found. The algorithms then use some kind of search over the space of consistent combinations, so that only the guess that extracts the most information from the secret code is issued, or else the one that reduces as much as possible the set of remaining consistent combinations. However, this is obviously not known in advance. To each combination corresponds a partition of the rest of the space, according to their match (the number of blacks and white pegs that would be the response when matched with each other). Let us consider the first combination:

if the combination considered is AABB, there will be 256 combinations whose response will be 0b, 0w (those with other colors), 256 with 0b, 1w (those with either an A or a B), etc. Some partitions may also be empty, or contain a single element (4b, 0w will contain just AABB, obviously). For a more exhaustive explanation see [7]. Each combination is thus characterized by the features of these partitions: the number of non-empty ones, the average number of combinations in them, the maximum, and other characteristics one may think of.

The path leading to the most successful strategies to date include using the *worst case*, *expected case*, *entropy* [3, 13] and *most parts* [7] strategies. The *entropy* strategy selects the guess with the highest entropy. The entropy is computed as follows: for each possible response  $i$  for a particular consistent guess, the number of remaining consistent guesses is found. The ratio of reduction in the number of guesses is also the *a priori* probability,  $p_i$ , of the secret code being in the corresponding partition. The entropy is then computed as  $\sum_{i=1}^n p_i \log_2(1/p_i)$ , where  $\log_2(1/p_i)$  is the information in bit(s) per partition, and can be used to select the next combination to play in Mastermind [13]. The *worst case* is a one-ply version of Knuth's approach, but Irving [4] suggested using the *expected case* rather than the worst case. Kooi [7] noted, however, that the size of the partitions is irrelevant and that rather the number of non empty partitions created,  $n$ , was important. This strategy is called *most parts*. The strategies above require one-ply look-ahead and either determining the size of resulting partitions and/or the number of them. Computing the number of them is, however, faster than determining their size. For this reason the *most parts* strategy has a computational advantage.

The heuristic strategies described above use some form of look-ahead which is computationally expensive. If no look-ahead is used to guide the search a guess is selected purely at *random*. However, it may be possible to discriminate by using local information. If this were possible one could even dismiss searching for all consistent guesses and search for a single consistent guess with the bias. In section 3 these heuristic strategies are compared. In section 4 an EDA using only local information is compared with those that need to examine all consistent guessed in order to select the best one.

### 3 Comparison of Heuristic Strategies

As has been mentioned before, there have been a number of different strategies proposed over the years for selecting among consistent guesses in Mastermind. These heuristics do not consider an exhaustive minimax search, but rather one-ply search. What is, however, not clear in these research papers is how ties are broken, which probably implies that a *first come, first served* approach is taken, using the first combination in lexicographical order out of all tied combinations. For this reason we propose to perform a comparison of the heuristic methods here where the ties are broken randomly. Each strategy is, therefore, used on all possible secret combinations (they are  $6^4 = 1296$ ) using ten independent runs.

The heuristics compared are the *entropy*, *most parts* and *worst case* strategy, as performed by Bestavros and Belal [3]. The worst case refers to the fact that for each possible return code for a particular guess the smallest reduction in assumed, i.e. the worst case. The actual consistent guess chosen is the one which maximizes the worst case. Finally, the *expected size* strategy, [4] is also tested; in this strategy the expected case is used instead of the worst case. These strategies are compared with the *random* strategy.

**Table 2** A comparison of the mean number of games played using all  $6^4$  colour combinations and breaking ties randomly, ranked from best to worst average number of guesses needed. Statistics are given for 10 independent experiments. The maximum number of moves used for the  $10 \times 6^4$  games is also presented in the final column. Horizontal separators are given for statistically independent results

Strategy	min	mean	median	max	st.dev.	max guesses
Entropy	4.383	4.408	4.408	4.424	0.012	6
Most parts	4.383	4.410	4.412	4.430	0.013	7
Expected size	4.447	4.470	4.468	4.490	0.015	7
Worst case	4.461	4.479	4.473	4.506	0.016	6
Random	4.566	4.608	4.608	4.646	0.026	8

The results of the experiments are given in table 2. The first combination played is always AABC, as proposed by [4]. The Wilcoxon rank sum (used instead of t-test since the variable does not follow a normal distribution) with a 0.05 significance level is used to determine which results are statistically different from another. The horizontal lines are used to group together heuristics that are not statistically different from the other. From these results we can gather that there is no statistical difference between the *entropy* and *most parts* strategies. However, out of all games played the maximum number of guesses needed by the Entropy strategy was only 6 while for most parts it was 7. These strategies are also better than the *worst* and *expected* case, which are statistically equivalent. For the worst case strategy used, nevertheless, only a maximum of 6 guesses, unlike the expected case with 7. The worst performer is the *random* strategy which also required a maximum of 8 guesses. Finally, note that the optimal expected result on playing all secrets is 4.340 [8].

#### 4 Estimation of Distribution Algorithm Using Local Entropy

One of the common approaches to using evolutionary algorithms for Mastermind, is simply to search for a single consistent guess which is then immediately played. This is especially true for the generalized version of the game, for  $N > 6$  and  $L > 4$ , where the task of just finding a consistent guess can be difficult. The result of such an approach is likely to do as well as the random strategy discussed in the previous sections if there are no major biases in searching for these consistent solutions. For

steady state evolutionary algorithms it may, however, be the case that the consecutive consistent guesses found may be similar to others played before; that is, the strategy of play may not necessarily be purely random. In any case it is highly likely that evolutionary algorithms of this type will not do better than the random strategy, as seen above, since consistent combinations found are a random sample of the set of consistent combinations.

In this section we investigate the performance of strategies that find a single consistent guess and play it immediately. In this case we use an estimation of distribution algorithm [12] *EDA* included with the `Algorithm::Evolutionary Perl` module [11], with the whole *EDA*-solving algorithm available as `Algorithm::MasterMind::EDA` from CPAN (the comprehensive Perl Archive Network). This is an standard *EDA* that uses a population of 200 individuals and a replacement rate of 0.5; each generation, half the population is generated from the previously generated distribution. The first combination played was *AABB*, since it was not found significantly different from using *AABC*, as before.

The fitness function used previously [10] to find consistent guesses is as follows,

$$f(c_{guess}) = \sum_{i=1}^n |h(c_i, c_{guess}) - h(c_i, c_{secret})|$$

that is, the sum of the absolute difference of the number of white and black pegs needed to make the guess consistent. However, this approach is likely to perform as well as the random strategy discussed in the previous section. When finding a single consistent guess we cannot apply the heuristic strategies from the previous section. For this reason we introduce now a local entropy measure, which can be applied to non-consistent guesses and so bias our search. The local entropy assumes that the fact that some combinations are better than others depends on its informational content, and that in turn depends on the entropy of the combination along with the rest of the combinations played so far. To compute *local entropy*, the combination is concatenated with  $n$  combinations played so far and its Shannon entropy computed:

$$s(c_{guess}) = \sum_{g \in \{A, \dots, F\}} \frac{\#g}{(n+1)\ell} \log \left( \frac{(n+1)\ell}{\#g} \right) \quad (3)$$

with  $g$  being a symbol in the alphabet and  $\#$  denotes the number of them. Thus, the fitness function which includes the local entropy is defined as,

$$f_\ell(c_{guess}) = \frac{s(c_{guess})}{1 + f(c_{guess})}$$

In this way a bias is introduced to the fitness to as to select the guess with the highest local entropy. When a consistent combination is found, the combination with the highest entropy found in the generation is played (which might be the only one or one among several; however, no special provision is done to generate several).

The result of ten independent runs of the *EDA* over the whole search space are now compared with the results of the previous section. These results may be seen in

table 3. Two EDA experiments are shown, one using the fitness function designed to find a consistent guess only ( $f$ ) and ones using local entropy  $f_\ell$ . The EDA using local entropy is statistically better than playing pure random, whereas the other EDA is not. In order to confirm the usefulness of the local entropy, an additional experiment was performed. This time, as in the previous sections, all consistent guesses are found and the one with the highest local entropy played. This results is labelled *LocalEntropy* in table 3. The results are not statistically different from the EDA results using fitness function  $f_\ell$ .

**Table 3** A comparison of the mean number of games played using all  $6^4$  colour combinations and breaking ties randomly, ranked from best to worse mean number of combinations. Statistics are given for 10 independent experiments. The maximum number of moves used for the  $10 \times 6^4$  games is also presented in the final column. Horizontal separators are given for statistically independent results

Strategy	min	mean	median	max	st.dev.	max guesses
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Most parts	4.383	4.410	4.412	4.430	0.013	7
Expected size	4.447	4.470	4.468	4.490	0.015	7
Worst case	4.461	4.479	4.473	4.506	0.016	6
LocalEntropy	4.529	4.569	4.568	4.613	0.021	7
EDA+ $f_\ell$	4.524	4.571	4.580	4.600	0.026	7
EDA+ $f$	4.562	4.616	4.619	4.665	0.032	7
Random	4.566	4.608	4.608	4.646	0.026	8

As a local conclusion, the *Entropy* method seemed to perform the best on average, but the estimation of distribution algorithm is not statistically different from (admittedly naive) exhaustive search strategies such as *LocalEntropy* and performs significantly better than the *Random* algorithm on average.

We should remark that the objective of this paper is not to show which strategy is the best runtime-wise, or which one offers the best algorithmic performance/runtime trade-off; but in any case we should note that the algorithm with the least number of evaluations and lowest runtime is the EDA. However, its average performance as a player is not as good as the rest, so some improvement might be obtained by creating a set of possible solutions. It remains to be seen how many solutions would be needed, but that will be investigated in the next section.

## 5 Heuristics Based on a Subset of Consistent Guesses

Following a tip in one of our former papers, recently Berghman et al. [1] proposed an evolutionary algorithm which finds a number of consistent guesses and then uses a strategy to select which one of these should be played. The strategy they apply is not unlike the *expected size* strategy. However, it differs in some fundamental ways.



In their approach each consistent guess is assumed to be the secret in turn and each guess played against every different secret. The return codes are then used to compute the size of the set of remaining consistent guesses in the set. An average is then taken over the size of these sets. Here, the key difference between the *expected size* method is that only a subset of all possible consistent guesses is used and some return codes may not be considered or considered more frequently than once, which might lead to a bias in the result. Indeed they remark that their approach is computationally intensive which leads them to reduce the size of this subset further. Note that Berghman et al. only present the result of a single evolutionary run and so their results cannot be compared with those here.

Their approach is, however, interesting, and lead us to consider the case where an evolutionary algorithms has been designed to find a maximum of  $\mu$  consistent guesses within some finite time. It will be assumed that this subset is sampled uniformly and randomly from all possible consistent guesses. The question is, how do the heuristic strategies discussed above work on a randomly sampled subset of consistent guesses? The experiment performed in the previous sections are now repeated, but this time only using the four best one-ply look-ahead heuristic strategies on a random subset of guesses, bounded by size  $\mu$ . If there are many guesses that give the same number of partitions or similar entropy then perhaps taking a random subset would be a good representation for all guesses. This has implications not only with respect to the application of EAs but also to the common strategies discussed here.

The size of the subsets are fixed at 10, 20, 30, 40, and 50, in order to investigate the influence of the subset size. The results for these experiments and their statistics are presented in table 4. The results are presented as are expected better as the subset size,  $\mu$ , gets bigger. Noticeable is the fact that the *entropy* and *most parts* strategies perform the best as before, however, at  $\mu = 40$  and 50 the entropy strategy is better.

Is there a statistical difference between the different subset sizes? To answer this we look at only the two best strategies in more detail, *entropy* and *most parts*, and compare their performances for the different subset sizes,  $\mu$ , and using the complete set, case when  $\mu = \infty$ , as presented in table 3. These results are given in table 5 and 6. From this analysis it may be concluded that a set size of  $\mu = 20$  is sufficiently large and not statistically different from using the entire set of consistent guesses. This is actually quite a large reduction in the set size, which is about 250 on average after the first guess, then 55, followed by 12 [1].

This implies that, at least in this case, using a subset of the combination pool that is around 1/10th of the total size potentially yields a result that is as good as using the whole set; even as algorithmically finding 20 tentative solutions is harder than finding a single one, using this in stochastic search algorithms such as the EDA mentioned above or an evolutionary algorithm holds the promise of combining the accuracy of exhaustive search algorithms with the speed of an EDA or an EA. In any case, for spaces bigger than  $\kappa = 6, \ell = 4$  there is no other option, and this 1/10 gives at least a rule of thumb. How this proportion grows with search space size is still an open question.

**Table 4** Statistics for the average number of guesses for different maximum sizes  $\mu$  of sub-sets of consistent guesses. The horizontal lines are used as before to indicate statistical independent, with the exception of one case: for  $\mu = 10$  the expected size and worst case are not independent

Strategy	min	mean	median	max	st.dev.	max guesses
$\mu = 10$						
Most parts	4.429	4.454	4.454	4.477	0.016	7
Entropy	4.438	4.468	4.476	4.483	0.016	7
Expected size	4.450	4.472	4.474	4.493	0.014	7
Worst case	4.447	4.486	4.487	4.519	0.020	7
$\mu = 20$						
Entropy	4.394	4.423	4.426	4.455	0.021	7
Most parts	4.424	4.431	4.427	4.451	0.009	7
Expected size	4.427	4.454	4.455	4.481	0.017	7
Worst case	4.429	4.453	4.451	4.486	0.017	7
$\mu = 30$						
Entropy	4.380	4.413	4.410	4.443	0.020	6
Most parts	4.393	4.416	4.416	4.435	0.015	7
Expected size	4.426	4.453	4.456	4.491	0.019	7
Worst case	4.434	4.459	4.461	4.477	0.013	7
$\mu = 40$						
Entropy	4.372	4.398	4.399	4.426	0.017	7
Most parts	4.383	4.424	4.427	4.448	0.020	7
Expected size	4.418	4.457	4.455	4.491	0.023	7
Worst case	4.424	4.458	4.457	4.490	0.022	7
$\mu = 50$						
Entropy	4.365	4.397	4.393	4.438	0.020	6
Most parts	4.400	4.424	4.422	4.454	0.017	7
Expected size	4.419	4.453	4.453	4.495	0.022	7
Worst case	4.431	4.456	4.457	4.474	0.012	6

**Table 5** No statistical advantage is gained when using a set size larger than  $\mu = 30$  when using the *entropy* strategy. However, there is also no statistically difference between  $\mu = 20$  and both  $\mu = 30$  and  $\mu = \infty$  (the only cases not indicated by the horizontal lines)

$\mu =$	min	mean	median	max	st.dev.
10	4.438	4.468	4.476	4.483	0.016
20	4.394	4.423	4.426	4.455	0.021
30	4.380	4.413	4.410	4.443	0.020
40	4.372	4.398	4.399	4.426	0.017
50	4.365	4.397	4.393	4.438	0.020
$\infty$	4.383	4.408	4.408	4.424	0.012

**Table 6** No statistical advantage is gained when using a set size larger than  $\mu = 20$  for the *most parts* strategy. However, there is a statistical difference between  $\mu = 20$  and  $\mu = \infty$  (the only case not indicated by the horizontal lines)

$\mu =$	<i>min</i>	<i>mean</i>	<i>median</i>	<i>max</i>	<i>st.dev.</i>
10	4.429	4.454	4.454	4.477	0.016
20	4.424	4.431	4.427	4.451	0.009
30	4.393	4.416	4.416	4.435	0.015
40	4.383	4.424	4.427	4.448	0.020
50	4.400	4.424	4.422	4.454	0.017
$\infty$	4.383	4.410	4.412	4.430	0.013

6 Discussion and Conclusion

In this paper we have tried to study and compare the different heuristic strategies for the simplest version of Mastermind in order to come up with a nature-inspired algorithm that is able to beat them in terms of running time and scalability. The main problem with heuristic strategies is that they need to have the whole search space in memory; even the most advanced ones that run over it only once will become unwieldy as soon as  $\ell$  or  $\kappa$  increase. However, evolutionary algorithms have already been proved [10] to scale much better, the only problem being that their performance as players is no better than a random player.

In this paper, after improving (or maybe just clarifying) heuristic and deterministic algorithms with an random choice of a combination to play, we have incorporated the simplest of those strategies to an estimation of distribution algorithm (the so-called *local entropy*, which takes into account the amount of *surprise* the new combination implies); results are promising, but still fall short of the best heuristic strategies, which take into account the partition of search space created by each combination. That is why we have tried to compute the subset that would be able to obtain results that are indistinguishable, in the statistical sense, from those obtained with the whole set, coming up with a subset whose size is around 10% of the whole one, being thus less computational intensive and easily incorporated into an evolutionary algorithm.

However, how this is incorporated within the evolutionary algorithm remains to be seen, and will be one of our future lines of work. So far, distance to consistency and entropy are combined in an aggregative fitness function; the quality of partitions induced will also have to be taken into account; however, there are several ways of doing this: putting consistent solutions in an *archive*, in the same fashion that multiobjective optimization algorithms do, leave them into the population and take the quality of partitions as another objective, not to mention the evolutionary parameter issues themselves: population size, operator rate. Our objective, in this sense, will be not only to try and minimize the number of average/median games played, but also to minimize the proportion of the search space examined to find the final solution.

All the tests and algorithms have been implemented using the Matlab package, and are available as open source source software with a GPL licence from the

authors. The evolutionary algorithm and several mastermind strategies are also available from CPAN; most results and configuration files needed to compute them are available from the group's CVS server (at <http://sl.ugr.es/algmm>).

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