Pareto-Based Multi-objective Estimation of Distribution Algorithm with Gaussian Copulas and Application in RFID Network Planning

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Abstract—A Pareto-based multi-objective estimation of distribution algorithm with Gaussian copulas is proposed and applied to RFID network planning. The algorithm employs Pareto-based approach and multivariate Gaussian copulas to construct probability distribution model. By estimating Kendall's tau and using the relationship of Kendall's tau and correlation matrix, Gaussian copula parameters are firstly estimated, thus, joint distribution is estimated. Afterwards, the Monte Carte simulation is used to generate new individuals. An archive with maximum capacity is used to maintain the non-dominated solutions. The Pareto optimal solutions are selected from the archive on the basis of the diversity of the solutions, and the crowding-distance measure is used for the diversity measurement. The archive gets updated with the inclusion of the non-dominated solutions from the combined population and current archive, and the archive which exceeds the maximum capacity is cut using the diversity consideration. The proposed algorithm is applied to some benchmark and RFID network planning. The relative experimental results show that the algorithm has better performance and is effective.

I. INTRODUCTION

MULTI-OBJECTIVE optimization problems(MOOPs) are widely encountered in various fields of science and technology. The solution of a multi-objective optimization problem is usually a set of acceptable trade-off optimal solutions. This solution set is called a Pareto set[1]. Multi-objective optimization algorithms, especially those based on evolutionary principles, have seen wide acceptability because most engineering problems are NP-hard and therefore a quick computation of approximate solutions is often desirable. In recent years, a considerable amount of interest has been shown in multi-objective evolutionary algorithm (MOEA) and a number of different MOEAs have been suggested, such as Strength Pareto Evolutionary Algorithm (SPEA2)[2], Non-dominated Sorting Genetic Algorithm (NSGA-II) [3], and Fast Pareto Genetic Algorithm (FastPGA)[4].

Estimation of distribution algorithms(EDAs)[5] are a class of evolutionary algorithms based on probability distribution model. It has been proven that EDA has some special characteristics of good convergence, high efficiency, concise concept and been successfully extended to multi-objective optimization problems. The performance of an EDA highly depends on how well it estimates and samples the probability distribution. Copula[6] is a recently developed mathematical theory and a power tool for multivariate probability analysis. It separates joint probability distribution function into the product of marginal distributions and a copula function which represents the dependency structure of random variables. Through copula, we can clearly represent the dependent relation of variables and analysis multivariate distribution of the underlying components. It has beed applied to constitute the probabilistic model in EDAs[7].

In this paper, EDA is extended to multi-objective optimization problems by using a Pareto-based approach and Gaussian copulas. The extended multi-objective EDA employs multivariate Gaussian copulas to construct probability distribution model. Gaussian copula parameters are firstly estimated, thus, joint distribution is estimated by estimating Kendall's tau and using the relationship of Kendall's tau and correlation matrix. Afterwards, the Monte Carte simulation is used to generate new individuals. An archive is used to maintain the non-dominated solutions. The Pareto optimal solutions are selected from the archive on the basis of the diversity of the solutions, and the crowding-distance measure is used for the diversity measurement. The archive gets updated with the inclusion of the non-dominated solutions from the combined population and current archive, and the archive which exceeds the maximum capacity is cut using the diversity consideration. The proposed algorithm is applied to some benchmark and RFID network planning. The experimental results show that the algorithm has better performance and is effective.

II. MULTI-OBJECTIVE OPTIMIZATION PROBLEMS

The general MOOPs can be defined as follows:
$$\min F(\mathbf{x}) = \left\{ f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_k(\mathbf{x}) \right\}$$
$$\text{s.t. } g_i(\mathbf{x}) \le 0, \quad i = 1,2,...,m$$
$$h_i(\mathbf{x}) = 0, \quad i = 1,2,...,p$$

$g_i(\mathbf{x}) \le 0$ and $h_i(\mathbf{x}) = 0$ define the feasible region Ω and any point in Ω defines a feasible solution.

The concept of optimum commonly adopted in MOOPs is Pareto optimality. Pareto optimality is defined as:

A point $\mathbf{x}^* \in \Omega$ is Pareto optimal if $\forall \ \mathbf{x} \in \Omega$ and $I = \{1,2,\cdots,k\}$

either: $\forall i \in I \ \ f_i(\mathbf{x}^*) \le f_i(\mathbf{x})$

and, there is at least one $i \in I$ such that $f_i(\mathbf{x}^*) < f_i(\mathbf{x})$

Other important definitions associated with Pareto optimality are Pareto dominance.

A vector $\mathbf{x} = (x_1, x_2, \cdots, x_n)$ is said to dominate $\mathbf{y} = (y_1, y_2, \cdots, y_n)$, denoted by $\mathbf{x} \prec \mathbf{y}$, if and only if $\mathbf{x}$ is partially less than $\mathbf{y}$, i.e., $\forall i \in \{1, 2, \cdots, k\}$ $x_i \leq y_i$ and, at least for one i, $x_i < y_i$.

For a given multi-objective problem $F(\mathbf{x})$, Pareto optimal set P^* is defined as:

$$P^* = \{\mathbf{x} \in \Omega | \neg \exists \mathbf{x}' \in \Omega \quad F(\mathbf{x}') \prec F(\mathbf{x})\}$$

Pareto front PF^* is defined as:

$$PF^* = \left\{ F(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_k(\mathbf{x})) | \mathbf{x} \in P^* \right\}$$

III. GAUSSIAN COPULAS

A copula[6] is a distribution function with known marginals. Any continuous Multivariate joint distribution of n random variables x_1, x_2,..., x_n

$F(x_1, x_2, ..., x_n) = \text{Prob}\{X_1 \leq x_1, X_2 \leq x_2, ..., X_n \leq x_n\}$, can be represented by a copula C as a function of the marginal distribution $F_{X_i}(x_i) = \text{Prob}\{X_i \leq x_i\}, i = 1, 2, \cdots, n$; i.e.

$$F(x_1, x_2, ..., x_n) = C(F_1(x_1), F_2(x_2), ..., F_n(x_n))$$
$$\stackrel{\Delta}{=} C(u_1, u_2, ..., u_n) \qquad (1)$$

Where $u_i = F_{X_i}(x_i), i = 1, 2, \cdots, n$ and $C(u_1, u_2, ..., u_n)$ is the associated copula function. Furthermore, application of the chain rule shows that the corresponding density function $f(x_1, x_2, ..., x_n)$ can be decomposed as

$$f(x_1, x_2, ..., x_n) = \frac{\partial^n C(u_1, u_2, ..., u_n)}{\partial u_1 \partial u_2 ... \partial u_n}$$
$$\stackrel{\Delta}{=} c(u_1, u_2, ..., u_n) \cdot f_1(x_1) \cdot f_2(x_2) \cdot ... \cdot f_n(x_n) \quad (2)$$

From the above it may be seen that the joint density function is the product of the marginals $f_i(x_i), i = 1, 2, \cdots, n$ and copula densities function $c(u_1, u_2, ..., u_n)$.

Let $\mathbf{R} = (r_{i,j}), i = 1, 2, \cdots, n, j = 1, 2, \cdots, n$ be a symmetric, positive definite matrix with unit diagonal entries. The *multivariate Gaussian copula* is defined as

$$C(u_1, u_2, ..., u_n; \mathbf{R}) = \mathbf{\Phi_R}(\varphi^{-1}(u_1), \varphi^{-1}(u_2), \cdots, \varphi^{-1}(u_n)) \quad (3)$$

Where $\mathbf{\Phi_R}$ denotes the standardized multivariate normal distribution with correlation matrix $\mathbf{R} = (r_{i,j}), i = 1, 2, \cdots, n, j = 1, 2, \cdots, n$. $\varphi^{-1}(x)$ denotes the inverse of the univariate standard normal distribution $\varphi(x)$.

The corresponding density is

$$c(u_1, u_2, ..., u_n; \mathbf{R}) = \frac{1}{|\mathbf{R}|^{1/2}} \exp\left(-\frac{1}{2} \mathbf{\omega}^T (\mathbf{R}^{-1} - \mathbf{I}) \mathbf{\omega}\right) \quad (4)$$

with $\mathbf{\omega} = (\varphi^{-1}(u_1), \varphi^{-1}(u_2), \cdots, \varphi^{-1}(u_n))^T$

IV. PARETO-BASED MULTI-OBJECTIVE EDA WITH GAUSSIAN COPULAS

The algorithm steps of pareto-based multi-objective EDA with Gaussian copulas for solving MOOPs is as follows:

```
t=0, initialize a population
      S(0) ← Initialization ()；
      Evaluate(S(0))
      A(0) ← Non_Dominated(S(0))
WHILE  t < T  DO
      Estimate_Marginals_Distribution();
      Estimate_Correlation_Matrix();
      S′(t) ← Sample()；
      S(t + 1) ← Selection( S(t) ∪ S′(t))
    Evaluate(S(t+1))
      A(t+1) ← Non_Dominated(S(t+1)∪A(t))
      IF  |A(t+1)| > M  THEN  Cut_Archive(A(t+1))
      t ← t + 1
END WHILE
Output the obtained Pareto optimal front
```

Initialization() is used to initialize a population $S(0)$ with size N. The initial population $S(0)$ is chosen randomly and should uniformly cover the entire solution space based on the consideration of the requirement of population diversity.

Evaluate() is used to evaluate individual in population.

$A(0)$ has been initialized to contain the non-dominated solutions from S(0).

Non_Dominated() returns the non-dominated solutions from a population.

Estimate_Marginals_Distribution() is used to estimate the marginals distribution from population.

Estimate_Correlation_Matrix() is used to estimate the correlation matrix $\mathbf{R}$ in Gaussian copulas.

A simple method[7] based on Kendall's tau is applied for estimating the correlation matrix $\mathbf{R}$. The method consists of constructing an empirical estimate of Kendall's tau for each bivariate margin of the copula and then using relationship (5)

$$\tau(\mathbf{X}_k, \mathbf{X}_m) = \frac{2}{\pi} \arcsin(r_{k,m}) \qquad (5)$$

To infer an estimate of the relevant element of $\mathbf{R}$. More specifically we estimate $\tau(\mathbf{X}_k, \mathbf{X}_m)$ by calculating the standard sample c coefficient

$$\hat{\tau}(\mathbf{X}_k, \mathbf{X}_m) = \frac{2}{n(n-1)} \sum_{1 \leq i < j \leq n} sign\left[(X_{i,k} - X_{j,k})(X_{i,m} - X_{j,m})\right] \quad (6)$$

From the original data vectors $\mathbf{X}_1$, $\mathbf{X}_2$,..., $\mathbf{X}_N$, and write the jth component of the ith vector as $X_{i,j}$; this yields an unbiased and consistent. An estimator of $r_{k,m}$ is then given by $\hat{r}_{k,m} = \sin(\frac{\pi}{2} \hat{\tau}(\mathbf{X}_k, \mathbf{X}_m))$. In order to obtain an estimator of the entire matrix $\mathbf{R}$ we can collect all pairwise estimates $\hat{r}_{k,m}$ and then construct the estimator

$$\hat{\mathbf{R}} = \left(\sin(\frac{\pi}{2} \hat{\tau}(\mathbf{X}_k, \mathbf{X}_m)) \right), k = 1, \cdots N, m = 1, \cdots, N \quad (7)$$

Sample() is used to generate offspring population $S'(t)$ by sampling Gaussian copulas with correlation matrix $\mathbf{R}$ and the univariate standard normal distribution $\varphi(x)$ according to the following algorithm[7].

1) Find the Cholesky decomposition of $\mathbf{R}$, so that $\mathbf{A}\mathbf{A}^T = \mathbf{R}$, with $\mathbf{A}$ lower triangular;

2) Generate a sample of n independent random variables $(Z_1, Z_2, ..., Z_n)$ from $N(0, 1)$;

3) Set $\mathbf{Y} = \mathbf{A}\mathbf{Z}$ with $\mathbf{Z} = (Z_1, Z_2, ..., Z_n)^T$ and $\mathbf{Y} = (Y_1, Y_2, ..., Y_n)^T$;

4) Return
$\mathbf{X} = (X_1, X_2, ..., X_n) = (F_1^{-1}(\varphi(Y_1)), F_2^{-1}(\varphi(Y_2)), ... F_n^{-1}(\varphi(Y_n)))$.

Selection() is used to evaluate individual in the current population $S(t)$ and the offspring population $S'(t)$ and carry out the non-dominated sorting and ranking selection, developed by Deb et al. in [3] to select non-dominated individuals to constitute the next generation population $S(t+1)$.

The proposed algorithm maintains an archive $A(t)$ with maximum capacity M. The archive gets updated with the inclusion of the non-dominated solutions from the next generation population $S(t+1)$ and $A(t)$. If the size of the archive exceeds the maximum capacity M, it is cut using the diversity consideration. Cut_Archive() is used to cut the archive.

V. RFID NETWORK PLANNING

The goal of RFID systems is to establish a wireless network, including: select a reasonable reader antenna position so that the wireless network can have the required coverage, ensure that RFID systems can provide effective communication performance and capacity, and control equipment costs within an acceptable range. RFID network planning[8][9] is to set a proper reader to ensure that the communications between electronic tags and readers. The reader has a limit on its interrogation range, within which the tags can be read by the reader. The placement of readers in RFID network is modeled in the following way. Assume we have a $X \times Y$ room. A number of RFID tags are evenly distributed in the area that needs signal coverage. For a RFID reader i, its detection range Ri is a function proportional to its emitting power Pi. That is Ri =f(Pi). We simplify the model by claiming that if the distance between a tag and a reader i is smaller than R_i and there is no obstacle between them, then the tag can receive the signal sent by the reader. If a tag can receive the signal sent by exactly one reader, we say that the tag is detected by the reader. However, if a tag can receive the signal sent by more than one reader, it usually cannot respond to either of the readers because of the nature of passive RFID tags. We say that the tag is interfered. For each of the readers to be deployed in a placement, we need to consider following parameters:

(x, y): The position of the reader in the room, where x [0;X] and y [0; Y].

P: The emitting power of the reader which directly affect the detection range R of the reader. Depends on the model of the reader, emitting power could be adjusted but limited in a range.

Multi-Objective Functions: The cost, the ratio of covered area and the ratio of the interfered area are the major factors need to be considered.

The cost function can be expressed as:
$$f_1(\mathbf{x}) = C - N * p \quad (8)$$

Where C is the maximum cost or the budget, N is the number of readers, p is the unit price of reader. This function measures how much we can gain by deploying fewer readers.

The cover rate function is:
$$f_2(\mathbf{x}) = \frac{\text{DetectedTa gs}}{\text{TotalTags}} \quad (9)$$

The interference rate function is:
$$f_3(\mathbf{x}) = \frac{\text{Interfered Tags}}{\text{TotalTags}} \quad (10)$$

Encoding: A placement of readers needs to be encoded so that it can be used in the proposed algorithm. A RFID network planning $\mathbf{x}$ with n readers can be expressed as a vector $\mathbf{x} = (x_1, \cdots, x_n, y_1, \cdots, y_n, P_1, \cdots, P_n)$ where x1, y1 and P1 are the x-coordinate, y-coordinate, emitting power.

The overall optimal solution for RFID network planning is represented by $\min F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\}$. The Pareto-based multi-objective estimation of distribution algorithm with Gaussian copulas is used to solve the MOOP.

VI. EXPERIMENTAL RESULTS

In order to test efficiency of the proposed algorithm, the performance of the proposed algorithm is compared with that of some multi-objective optimization algorithms. The following benchmark functions[10] have been used to test.

1) ZDT1: $f_1(\mathbf{x}) = x_1$
$$f_2(\mathbf{x}) = g(\mathbf{x})(1 - \sqrt{x_1/g(\mathbf{x})}) \quad x_i \in [0,1]$$
$$g(\mathbf{x}) = 1 + 9(\sum_{i=2}^{D} x_i)/(D-1)$$

2) ZDT2: $f_1(\mathbf{x}) = x_1$
$$f_2(\mathbf{x}) = g(\mathbf{x})(1 - (x_1/g(\mathbf{x}))^2) \quad x_i \in [0,1]$$
$$g(\mathbf{x}) = 1 + 9(\sum_{i=2}^{D} x_i)/(D-1)$$

3) ZDT3: $f_1(\mathbf{x}) = x_1$
$$f_2(\mathbf{x}) = g(\mathbf{x})(1 - \sqrt{\frac{x_1}{g(\mathbf{x})}} - \frac{x_1}{g(\mathbf{x})}\sin(10\pi x_1)) \quad x_i \in [0,1]$$
$$g(\mathbf{x}) = 1 + 9(\sum_{i=2}^{D} x_i)/(D-1)$$

4) ZDT4: $f_1(\mathbf{x}) = x_1$
$$f_2(\mathbf{x}) = g(\mathbf{x})(1 - \sqrt{x_1/g(\mathbf{x})}) \quad x_1 \in [0,1], x_i \in [-5,5],$$
$$i = 2, ..., D$$
$$g(\mathbf{x}) = 1 + 10(D-1) + \sum_{i=2}^{D}(x_i^2 - 10\cos(4\pi x_i))$$

5) *ZDT6:* $f_1(\mathbf{x}) = 1 - \exp(-4x_1)\sin^6(6\pi x_1)$

$$f_2(\mathbf{x}) = g(\mathbf{x})(1 - (f_1(x)/g(\mathbf{x}))^2) \qquad x_i \in [0,1]$$

$$g(\mathbf{x}) = 1 + 9\left(\sum_{i=2}^{D} x_i \Big/ (D-1)\right)^{0.25}$$

Convergence metric γ[3] and Diversity metric Δ[3] were used. The smaller the value of these metrics is, the better the performance of the algorithm is. The initial population was generated from a uniform distribution in the ranges specified below. Population size *N*=100. All experiments were repeated for 20 runs. The maximum number of iterations is set to 2000 in each running. TABLE 1 listed the mean and variances of the convergence and diversity metrics obtained using SPEA2[2], NSGA-II[3], FastPGA[4] and the proposed algorithm (PMOEDAGC) on ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6. It can be known from TABLE 1, in both aspects of convergence and distribution of solutions, PMOEDAGC outperforms SPEA2, NSGA-II and FastPGA on the benchmark functions.

The algorithm is also applied RFID network planning problem. A simplified 30m×30m square working area with 60 tags is used for the simulation. 20 RFID readers, whose radiated power is adjustable in the range from 0 to 30 dBm, are considered to serve this area. $\min F(\mathbf{x}) = \{f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})\}$ is used as the multi-objective function. The population size in the algorithm is set to 200. The algorithm runs to a maximum number of 4000 iterations. A sample of Pareto optimal solution `for network planning` is listed in TABLE 2.

TABLE I
MEAN AND VARIANCES OF CONVERGENCE AND DIVERSITY METRICS FOR
MULTI-OBJECTIVE BENCHMARK FUNCTIONS

Problem	Algorithm	Convergence γ	Diversity Δ
ZDT1	SPEA2	0.00925± 0.00321	0.32092±0.00113
	NSGA-II	0.01004± 0.00052	0.30782±0.00472
	FastPGA	0.00093± 0.00003	0.44119± 0.00165
	PMOEDAGC	0.00023±1.4601e-8	0.1123±4.2311e-6
ZDT2	SPEA2	0.06004± 0.01331	0.30012± 0.00314
	NSGA-II	0.00293±0.00002	0.20631±0.00315
	FastPGA	0.00076±0.00001	0.39943±0.00096
	PMOEDAGC	0.00021±1.3436e-9	0.10542± 1.0001e-7
ZDT3	SPEA2	0.09963± 0.00376	0.49735± 0.01007
	NSGA-II	0.00942± 0.00291	0.50862± 0.00292
	FastPGA	0.00203± 0.00001	0.60238± 0.00491
	PMOEDAGC	0.00026±2.4861e-9	0.10065±1.6801e-7
ZDT4	SPEA2	0.27656± 0.06731	0.41697±0.03718
	NSGA-II	1.03452±1.00057	0.54586±0.00029
	FastPGA	2.87447±2.84581	0.66769±0.00047
	PMOEDAGC	0.08543±0.00812	0.11244±2.3776e-7
ZDT6	SPEA2	0.10934± 0.02104	0.45765±0.00904
	NSGA-II	0.01113±0.00001	0.33476±0.00087
	FastPGA	0.22343±0.00408	0.44876±0.00041
	PMOEDAGC	0.00147±5.1873e-5	0.16315±0.00004

VII. CONCLUSIONS

The proposed algorithm employs the multivariate Gaussian copulas to construct probability distribution model, and the new individuals are generated according to the model. An archive is used to maintain the non-dominated solutions. The Pareto optimal solutions are selected from the archive. The archive gets updated with the inclusion of the non-dominated solutions from the combined population and current archive.

The algorithm is applied to some benchmarks and RFID Network planning. The results show that the algorithm has better performance than SPEA2[2], NSGA-II[3], FastPGA[4] on the benchmarks and is effective in RFID networks planning.

TABLE II
A SAMPLE OF PARETO OPTIMAL SOLUTION FOR NETWORK PLANNING

Reader	x	y	Radiated Powers
1	2.2431	2.0376	20.7643
2	8.7123	4. 6375	15.6542
3	11.2796	4. 8223	22.1217
4	17.0451	3. 0294	21. 3451
5	2.1125	9. 1002	23.6732
6	8.3761	11. 3821	22.1782
7	27.2341	11. 2169	21. 1677
8	3.8612	26. 2438	22. 6722
9	9.5676	25.6657	26.1761
10	15.8764	16.2541	18.8938
11	23.5659	3.8652	21.6564
12	6.78766	28.0031	19.7854
13	5.38435	25.6154	22.4786
14	28.0965	3.4723	21.6551
15	25.1122	2.0895	23.5612
16	5.3321	7.8975	15.7654
17	20.5432	8.6654	16.8743
18	18.5421	18.8671	21.8321
19	26.7632	27.8734	19.6532
20	7.6541	19.7652	16.9836

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