

Classification Scheme of Multi-objective Estimation of Distribution Algorithms

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Abstract— A variety of Estimation of Distribution Algorithms for multi-objective optimization (MOEDAs) has been reported, each of them with its own characteristics and techniques in their optimization process. In this research we present a classification scheme for these algorithms, based on ten characteristics: domain of the variables, relationships between the variables, probabilistic graphical model, estimation approach, restriction support, problem handling, sorting method, individuals' handling, selection approach, and replacement approach. These characteristics were extracted by analyzing all the 24 MOEDAs reported in the literature. The scheme presented here helps to identify the methods and techniques used in each algorithm, also, a useful method for the analysis of the optimization process of an EDA is proposed. This paper includes a brief analysis of the influence in the results of applying different selection/replacement percentages.

Keywords— *Estimation of Distribution Algorithms; Evolutionary Algorithms; Multi-objective optimization.*

I. INTRODUCTION

Applying Evolutionary Algorithms over optimization problems has been an interesting case of study for many researchers. Creativity has been the key to create new kind of algorithms. Estimation of Distribution Algorithms (EDAs) incorporate statistical knowledge into the basis of Genetic Algorithms (GA). The creative minds that work on EDAs have strengthened them, designing a variety of new algorithms. In this paper, a classification scheme for the Multi-objective Estimation of Distribution Algorithms (MOEDAs) based on ten criteria is presented. First of all, a deep analysis for all MOEDAs reported in the literature was made: 24 different algorithms were found. Afterwards, the techniques and methods used in their optimization process were identified by following a method described in this paper. The strategy of each algorithm were categorized in: domain of the variables, relationships between the variables, probabilistic graphical model, estimation approach, restriction support, problem handling, sorting method, individuals' handling, selection approach, and replacement approach.

The main contributions of this paper can be summarized as follows:

- A useful method of decomposition of an EDA.

- The presentation of a classification scheme of MOEDAs ruled by ten criteria.
- A useful tool to design, compare, and modify MOEDAs.
- The identification of the methods and techniques already applied in MOEDAs' optimization process.

This paper is presented as follows: Part 2 introduces the Estimation of Distribution Algorithms their basis and their general optimization process. Also, presents the basic concepts of multi-objective optimization. Following, in Part 3 an EDA analysis method is presented, including a proposal of EDA decomposition; the ten criteria of the classification scheme are explained in Part 4: their definitions, methods and techniques. Part 5 presents three useful applications for the taxonomic system. Finally, in Part 6 our conclusions are presented.

II. BASIC CONCEPTS

A. Multi-objective Optimization Problem

Multi-objective Optimization Problems (MOPs) have attracted plenty of interest of several areas, due to many real-world problems contain multiple competing objectives; i. e. not a single solution can optimize all the objectives simultaneously.

A general MOP can be expressed as:

$$\begin{aligned} \text{minimize } F(x) &= [f_1(x), \dots, f_m(x)] \\ \text{subject to } c_1(x), \dots, c_r(x) &\leq 0, \\ \text{with } x &\in D \end{aligned} \quad (1)$$

Where D is known as the decision space; the functions $f_1(x), \dots, f_m(x)$ are objective functions; the inequalities $c_1(x), \dots, c_r(x) \leq 0$ express the restrictions imposed to the values of x . Solving a MOP is the process of finding the Pareto-optimal front, which is defined as the set of solutions that can be improved with respect to any objective only at the expense of their quality with respect to at least one other objective. One of the approaches to multi-objective optimization is to solve it through evolutionary algorithms,

due to their capability of working with populations of solutions; this fact has prompted the creation of Multi-objective Evolutionary Algorithms (MOEAs).

TABLE 1
MOEDAS REPORTED IN LITERATURE

MOEDA	Original EDA
MIDEA	IDEA
MO-COIN	COIN
MhBOA	hBOA
MrBOA	rBOA
MBOA	BOA
MO-PBIL	PBIL
MO-CMA-ES	CMA-ES
mMARLEDA	MARLEDA

B. Estimation of Distribution Algorithms

Estimation of Distribution Algorithms (EDAs) are Evolutionary Algorithms for optimization that, at each iteration, explore the space of promising solutions by sampling the model of their distribution [1]. The general process of EDAs involves, at least, 8 operators: initialization, evaluation, sorting, selection, learning, sampling, replacement, and finalization.

The EDAs' optimization process begins generating N random solutions (individuals), the fitness is calculated for each one; after all the individuals in the population have been sorted by their adaptability (i. e. their fitness), a subset K of individuals, with $|K| = \tau(N)$ and $\tau = [0, 1]$, is selected. Afterwards, the model of the distribution of the selected individuals is learnt and offspring is created by sampling this model. The set of parents and the set of offspring are joined in order to form a new population. The process is repeated until a stopping condition is met; when this occurs, the algorithm returns the current best individual as the solution of the problem. A pseudocode of this process is presented in figure 1.

C. Multi-objective Estimation of Distribution Algorithms

Recently EDAs have been successfully implemented on optimizing problems that includes more than one objective, showing a good performance and comparable results with other Multi-objective-implemented Metaheuristics [2–4]. Most of these MOEDAs have their basis in single-objective EDAs; the table 1 shows the cases.

A MOEDA optimization process, also follows the 8 operator shown above, implementing strategies in each made for Multi-objective optimization: evaluates all the objectives, sorts the population according the Pareto dominance (see II, A), return the best approximation to the Pareto front, to name a few. A generic process of a MOEDA is pretty much the same

as the one shown in figure 1, but returns the current-best approximation of the true Pareto front. The literature indicates that there currently there are 24 MOEDAs, table 2 shows them by order of appearance.

```

1   $t \leftarrow 0$ 
2  generate population  $P_t$  of size  $N$ 
3  evaluation and fitness assignment of  $P_t$ 
4  While(stopping condition is not met){
5     $S_t \leftarrow$  select  $K$  individuals from  $P_t$  with  $K \leq N$ 
6     $M_t \leftarrow$  learn the probability model of  $S_t$ 
7    sample  $M_t$  to create a set  $Q_t$  of new solution
8    evaluation and fitness assignment of  $Q_t$ 
9    create  $P_{t+1}$  with best_of( $S_t \cup Q_t$ )
10    $t \leftarrow t + 1$ 
11 }
12 return (best individual)

```

Figure 1 Single-objective EDA

III. ALGORITHMS ANALYSIS

In order to analyze each of the 24 state-of-the-art MOEDAs, a carefully study of them must be made; a useful way to accomplish this task is to decompose each algorithm in its 8 basic operators (see II, B) and identify the strategy that follows in each one.

A. Decomposition of an EDA

An easy way to analyze an EDA is to see its generic process as independent blocks, each of them develops a defined task; the figure 2 shows this fact. Each block can be seen as a function or a procedure that accepts input-parameters and returns output-values, the hard-work of the analyzer is to identify where the algorithm develops the corresponding task of each block. Identify the main task for each block is the key:

- Initialization: A strategy to construct the first population.
- Evaluation: A strategy to evaluate the fitness function.
- Sorting: A strategy to sort the population.
- Selection: A strategy to select a part of the population that will be considered to build the probabilistic model.
- Learning: A strategy to build the probabilistic model.
- Sampling: A strategy to create new individuals from the probabilistic model.
- Replacement: A strategy to create the new population for the next generation.
- Finalization: A strategy to return the best approximation to the optima.

The result of applying this analysis over the 24 state-of-the-art MOEDAs, is a data base with the strategy that each algorithm follows for each operator. The size of the data base is considerably big; since the goal is to construct a classification scheme, a reduction of this information to the significant data must be done. In the next part, the classification scheme will be constructed and the significant data will be extracted.

TABLE 2
REPORTED MOEDAS

MOEDA	YR	MOEDA	YR	MOEDA	YR
MIDEA-D,C	2001	MhBOA	2005	MONEDA	2008
P- BOA	2001	Naive MIDEA	2005	RM-MEDA	2008
BMOA	2002	MrBOA	2006	MMEA	2009
MBOA	2002	mMARLEDA	2007	MO-COIN	2009
MOPED	2003	MOCMAES	2007	JGBN-EDA	2011
MOHEDA	2004	MO-PBIL	2007	MARTEDA	2011
VEDA	2004	RCMEDA	2007	MEDA/D	2012
MASO	2005	DT-MEDA	2007	MOEA-HCEDA	2012

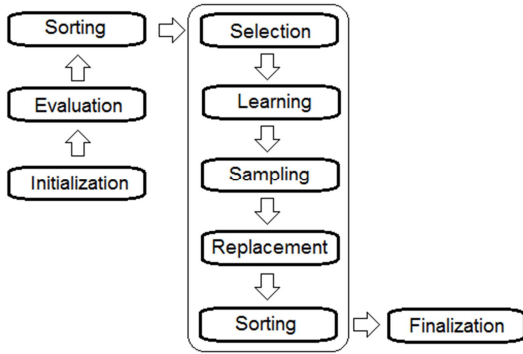


Figure 2 Blocks of the process of an EDA

IV. CLASSIFICATION SCHEME FOR MOEDAS

After the information of all the MOEDAs is collected, a research for the classification schemes for general EDAs must be done (i. e. not only for MOEDAs), literature shows that there are two common ways to classify EDAs.

A. Classification of EDAs

Some authors have made a useful classification of EDAs [1], [5], that have been adopted by others, these classifications were taken as a base for the extended to multi-objective scheme presented in this article.

Hauschild and Pelikan, categorize EDAs in a first level as: Discrete, Permutations and real-valued vectors. Larrañaga and Lozano groups them in two ways: EDAs for combinatorial optimization and EDAs for continuous domain. In both researches, it is considered the type of distribution they are able to encode: Discrete and Continuous, being the first one more specific in the Permutation category.

Then as a second level, the authors match by considering the complexity of the probabilistic model used to learn the interdependencies between the variables dividing each category of the first level: Without dependencies, multivariate dependencies and mixture models in [5]; and Univariate, tree-based models and multi-variate in [1].

The scheme of classification here proposed contains a more generalized criteria, that involves both approaches mentioned above:

1th criterion: Considering the domain of the variables

- Discrete (DIS)
- Continuous (CNT)
- Mixed (MIX)

5th criterion¹: Considering the relationships between variables

- Univariate (UNI)
- Bivariate (BIV)
- Multivariate (MUL)

B. Construction of the classification scheme criteria

2th criterion: Restriction support. This criterion arranges the algorithms that support constraint problems in one group (CON) and those do not in another group (NOC).

3th criterion: Problem handling. In [5], is shown how the combination of data clustering and evolutionary algorithms has some benefits, this idea was been taken by some authors, consequently, some MOEDAs cluster the objective space, some others do not, depending if the problem is decomposable or not. This criterion classifies the algorithms in two groups: those that implement clustering (CLT) and those do not (NCL).

In [11] it is mentioned another way to classify EDAs, but only applied to those EDAs that involves probabilistic graphical models (PGM) in its process, never the less, there are cases in the 24 state-of-the-art algorithms, that do not involves (PGM) [6–9], so that, a new category must be added in order to include this kind of algorithms:

4th criterion: Considering the PGM

- Independent (IND)
- Markovian (MK)
- Bayesian (BN)
- No graphical model (NG)

In this section, it will be specify the constitution of the rest of the eleven classification criteria that involves the whole scheme. All of them were extracted from the analysis shown in III applied to the 24 state-of-the-art MOEDAs shown in table 2. The reader can visualize the complete classification scheme in table 3.

6th criterion: Estimation approach. Larrañaga in [5], mentions that the learning process can be separated in two

¹ Please refer sec. IV,C for the explanation of why this is the 5th

sub-tasks structure learning and parametric learning; this criterion classifies MOEDAs considering how they learn the distribution: changing the structure (STR) at each iteration; changing only the parameters (PAR) or changing both.

7th criterion: Sorting method. There are several methods for sorting individuals in MOEDAs, nevertheless, we have group all of them in three categories: NSGA, Pareto dominance and specialized methods.

a) NSGA-II Sorting (NSG). The algorithms of this class, use the sorting method of NSGA-II [10]. It ranks the population considering the front where they belong, plus evaluating how crowded they are [11].

b) Pareto Dominance sorting (PDS). These algorithms use methods that sort the population considering the number of individuals dominated or that dominate each individual.

c) Specialized Sorting Methods (SSM). In this category *Specialized* must be understood as those methods that involve more complex theory than the traditional methods mentioned above, e. g. Hypervolume, Parzen sorting, Voronoi sorting, etc.

8th criterion: Individuals' Handling. MOEDAs evolve a set of individuals which are the possible solutions of the optimization problem. The algorithms of table 2 handle the individuals in two ways: evolving a set of solutions (population) [12] or using an external elitist file where the

non-dominated individuals are stored[13]; the category *file* is applied to the algorithms that use an Elitist File, and *pop* is applied to those that use only a population.

9th criterion: Selection approach. This criterion arranges the algorithms by the part of the population that is considered to estimate the probability distribution; i.e. the subset of the population that is selected.

a) All individuals (ALL). It means that the algorithms select the whole population to learn the model.

b) Best- τ individuals (B- τ). This class corresponds to the algorithms that selects only a percentile $\tau = [0,1]$ of the population from the learning operator will be applied.

c) Non-Dominated individuals (NDI). The part of the population selected, in these algorithms, for learning operator is only the current non-dominated individuals.

d) Best+Worst individuals (B+W). These algorithms select a part of the best individuals plus a part of the worst.

10th criterion: Replacement approach. This criterion refers of how the new solutions would be added to the next population. This process is commonly called replacement; this means that the algorithms are classified by the replacement methods that use: *Full replacement (FULL)*. These algorithms replace all the old population by the offspring generated.

a) ω - Replacement (ω - REP) . These algorithms replace a predefined percentile $\omega = [0,1]$ of the old population

TABLE 3
TAXONOMIC SYSTEM FOR MOEDAS

	<i>Problem-related</i>			<i>Learning -related</i>			<i>Solutions-related</i>			
MOEDA	1 th	2 th	3 th	4 th	5 th	6 th	7 th	8 th	9 th	10 th
BMOA	DIS	NOC	NCL	BN	MUL	STR	SSM	FILE	B- τ	ω -REP
DT-MEDA	CNT	NOC	NCL	NG	MUL	STR	NSG	FILE	B- τ	ω -REP
JGBN-EDA	CNT	NOC	NCL	BN	MUL	STR	NSG	POP	B- τ	ω -REP
MARTEDA	CNT	NOC	NCL	NG	MUL	STR	SSM	POP	B- τ	ω -REP
MASO	DIS	CON	NCL	IND	UNI	STR	SSM	FILE	NDI	FULL
MBOA	DIS	NOC	NCL	BN	MUL	STR	NSG	POP	ALL	ALL-W
MEDA/D	DIS	CON	CLT	NG	UNI	PAR	NSG	FILE	NDI	ALL-W
MhBOA	DIS	NOC	CLT	BN	MUL	STR	NSG	POP	B- τ	ALL-W
MIDEA-C	CNT	NOC	CLT	IND	UNI	PAR	PDS	POP	B- τ	ALL-W
MIDEA-D	DIS	NOC	CLT	IND	UNI	PAR	PDS	POP	B- τ	ALL-W
mMARLEDA	DIS	CON	NCL	MK	MUL	PAR	NSG	POP	B- τ	ω -REP
MMEA	CNT	NOC	CLT	IND	UNI	PAR	NSG	POP	NDI	ω -REP
MOCMAES	CNT	CON	NCL	NG	MUL	PAR	NSG	POP	B- τ	ALL-W
MO-COIN	DIS	NOC	NCL	NG	BIV	STR	NSG	POP	B+W	ALL
MOEAHCEDA	DIS	NOC	NCL	IND	UNI	PAR	PDS	FILE	NDI	ω -REP
MOHEDA	DIS	CON	CLT	IND	UNI	PAR	SSM	FILE	B- τ	FULL
MONEDA	CNT	NOC	CLT	NG	MUL	STR	NSG	POP	B- τ	ω -REP
MO-PBIL	DIS	NOC	NCL	IND	UNI	PAR	NSG	FILE	NDI	FULL
MOPED	CNT	NOC	NCL	NG	MUL	PAR	SSM	POP	ALL	ω -REP
MrBOA	CNT	NOC	NCL	BN	MUL	STR	PDS	POP	B- τ	ω -REP
P-BOA	DIS	NOC	NCL	BN	MUL	STR	PDS	FILE	B- τ	ALL-W
RCMEDA	CNT	CON	NCL	NG	MUL	PAR	NSG	POP	B- τ	ω -REP
RM-MEDA	CNT	NOC	CLT	IND	UNI	PAR	NSG	POP	ALL	FULL
VEDA	CNT	NOC	CLT	NG	MUL	STR	NSG	FILE	B+W	ALL-W

by the offspring.

b) *All- worst replacement (ALL-W)*. This class of algorithms replaces all the non-selected individuals by the new ones.

c) *Random-worst (R-W)*. These algorithms replace randomly the worst individuals in the old population by the new ones.

C. Groups of Criteria

With the purpose of maintain an order in the classification scheme; the 10 criteria have been group in categories: Learning-related, Problem-related and Solutions-related.

a) *Problem-related*. Involves the criteria that concern characteristics of the problem to be optimized.

b) *Learning-related*. Involves the criteria that concern (in some way) the task related with the process of learning the distribution.

c) *Solutions-Related*. Groups the criteria that involve how the algorithm manages the solutions in the whole process of optimization.

V. APPLYING THE CLASSIFICATION SCHEME

Besides the compendium of the methods and techniques already used in MOEDAs, and the *radiography* of the state-of-the-art MOEDAs, three possible applications of the Classification Scheme here proposed were found:

- The design of a MOEDA
- An easy way to select algorithms that will be part of a comparison.
- The improvement of an already-tested algorithm

A. Design of a MOEDA

The scheme of the table 3 can be used as a guide to select the characteristics of a new algorithm. The function to be optimized is shown in equation 2; this is a continuous multi-objective problem. The algorithm will use a Univariate Gaussian distribution.

$$\begin{aligned} f_1 &= f_1(x_1) = x_1 \\ g &= g(x_2, x_3, \dots, x_n) = 1 + \frac{9}{n-1} * \sum_{i=2}^n x_i \\ h(f_1, g) &= 1 - \sqrt{f_1/g} \\ \text{with } n &= 30 \text{ and } x_i \in [0,1] \end{aligned} \quad (2)$$

The classification scheme will be used in order to know what other characteristics would be consider in this design, for this purpose, the focus will be in the algorithms that match in this characteristics: continuous domain of the variables, univariate relationships between the variables, independent probabilistic graphical model. The analysis helps to select the next characteristics, due that the combination has not already tried, the values in to the scheme for this algorithm called GI-MOEDA (Gaussian Independent Multi-objective Estimation of Distribution Algorithm), are shown in table 4.

TABLE 4
CLASSIFICATION CRITERIA FOR GI-MOEDA

GI-MOEDA				
1 th	2 th	3 th	4 th	5 th
IND	UNI	PAR	CNT	NOC
6 th	7 th	8 th	9 th	10 th
NCL	NSG	FILE	B- τ	ω -REP

B. Modification of a MOEDA²

The characteristics of the GI-MOEDA still can be changed, in order to have a better combination, without altering neither the learning strategy nor the kind of problem optimizing; i. e. by the modification of the *Solutions-Related* characteristics. Note that this modification does not guarantee the improvement of the algorithm; this is only shown as a good design practice.

This modification would consist in testing the algorithm with all the possible combination in criteria 7th, 8th, 9th, 10th, generate at least 96 different algorithms, since this is not the main goal of this research, the test will try the combinations with the next values (which are the most common values) generating 16 algorithms:

- Criterion 7: NSG
- Criterion 8: FILE, POP
- Criterion 9: B- τ , NDI, ALL, B+W
- Criterion 10: ω -REP, FULL

There are three combinations that approximate in a better way to the optimum, (the four nearest to the (0,0) in figure 3), these correspond to:

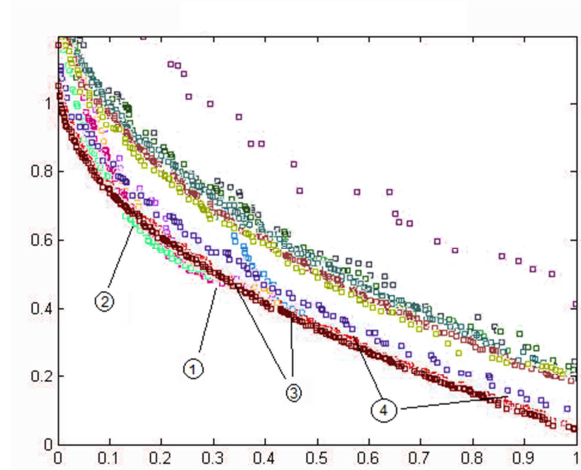


Figure 3: Pareto fronts of combinations for IG-MOEDA

1. Combination (NSG, POP, B- τ , ω -REP)
2. Combination (NSG, POP, NDI, ω -REP)

² This algorithm is only shown as an example for the application of the classification scheme

3. Combination (NSG, FILE, B- τ , ω -REP)
4. Combination (NSG, FILE, NDI, ω -REP)

The combination 3 is chosen (in this paper), because is the one that best covers the objective space.

C. Comparison of MOEDAs

The third application shown in this research of the Classification Scheme of MOEDAs is to compare the GI-MOEDA with other algorithms; the scheme helps to choose those algorithms.

The algorithms chosen to this test is going to be those that have the most coincidences in the scheme with GI-MOEDA: MIDEA-C and MMEA; they match in criteria 1th, 2th, 4th, 5th, 6th, and 7th.

The function to optimize is the one of equation 2, and for all the algorithms the next initial parameters were fixed:

- Maximum of generation:100
- Population size: 100
- 20 independent runs of each algorithms

In figure 4, the average Pareto fronts (the average of all Pareto approximations of all generations), after the 100 generations of the 20 runs, of each algorithm, are shown. This test is only shown as an application of the scheme, since the objective of this paper was not the design of an algorithm, the conclusion of this test is not focused on the quality of GI-MOEDA; nevertheless, good results were obtain with it.

CONCLUSIONS

A Classification Scheme for Multi-objective Estimation of Distribution Algorithms was presented in this paper; it involves the traditional classification criteria plus 7 new-added ones, which complement the comparable-characteristics of the algorithms. Due to the design of the scheme was made by a wide analysis of the state-of-the-art MOEDAs, the reader can obtain information of the methods and techniques that are currently applied. Finally, three ways of its application were analyzed: A design of a new MOEDA called GI-MOEDA guided by the scheme, a modification of the parameters of this algorithms that shown some improvement on it (also guided by the scheme), and a comparison of this algorithm with two of the reported ones in literature (chosen using the scheme).

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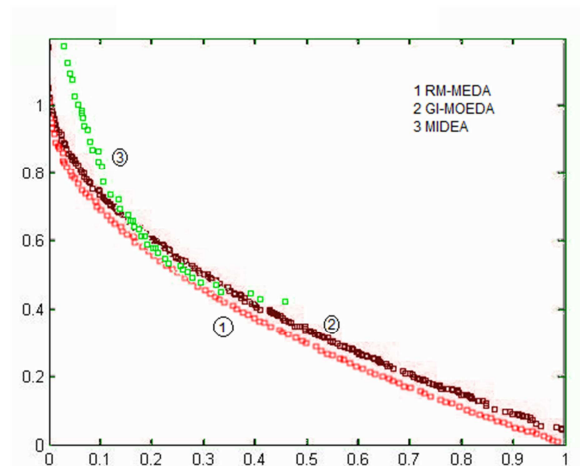


Figure 4: Comparison of Pareto Fronts

The complete list of algorithms and their reference is shown next: BMOA [14], DT-MEDA [15], JGBN-EDA [4], MARTEDA [9], MASO [16], MBOA [17], MEDA/D [3], MhBOA [11], MIDEA [19], mMARLEDA [20], MMEA [21], MO-CMA-ES [22], MO-COIN [23], MOEA-HCEDA [24], [8], MOHEDA [18], MONEDA [7], MO-PBIL [25], MOPED [8], MrBOA [26], P-BOA [27], RCMEDA [28], RM-MEDA [29], and VEDA [6].

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