



Restricted Boltzmann Machine-driven Interactive Estimation of Distribution Algorithm for personalized search

Lin Bao^{a,b}, Xiaoyan Sun^{a,*}, Yang Chen^c, Dunwei Gong^a, Yongwei Zhang^b

^a School of Information and Control Engineering, China University of Mining and Technology, Xuzhou, China

^b School of Electronics and Information, Jiangsu University of Science and Technology, Zhenjiang, China

^c School of Computer Science and Engineering, Nanyang Technological University, Singapore

ARTICLE INFO

Article history:

Received 3 October 2019

Received in revised form 11 May 2020

Accepted 12 May 2020

Available online 14 May 2020

Keywords:

Personalized search

Interactive Estimation of Distribution

Algorithm

Restricted Boltzmann Machine

Surrogate

ABSTRACT

Effective and efficient personalized search is one of the most pursued objectives in the era of big data. The challenge of this problem lies in its complex quantifying evaluations and dynamic user preferences. A user-involved interactive evolutionary algorithm is a good choice if it has reliable preference surrogate and powerful evolutionary strategies. A Restricted Boltzmann Machine (RBM) assisted Interactive Estimation of Distribution Algorithm (IEDA) is presented to enhance the IEDA in solving the personalized search. Specifically, a dual-RBM module is developed to simultaneously provide a preference surrogate and a probability model for conducting the individual selection and generation of the IEDA. Firstly, the positive and negative preferences of the currently involved user in IEDA are distinguished and combined to achieve a dual-RBM, and then the weighted energy functions of the RBM model together with social group information from users with similar preferences are designed as the preference surrogate. The probability of the trained positive RBM on the visible units is fetched as the reproduction model of EDA since it reflects the attribute distributions of more preferred items. Some benchmarks from the Movielens and Amazon datasets are applied to experimentally demonstrate the superiority of the proposed algorithm in improving the efficiency and effectiveness of the interactive evolutionary computations served personalized search.

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1. Introduction

The task of personalized search is to find items that meet a user's specific (can be changeable) preferences or requirements, therefore, its nature is an optimization problem. Evolutionary algorithms (EAs) will be effective on solving this problem supposing the user's preferences or intentions can be explicitly expressed with accurate mathematical models. Unfortunately, such an assumption is hard to be satisfied even if the user's preference is very certain and clear, not to mention the changeable scenarios.

The optimized objective of personalized search is based on users' qualitative evaluation, comparison and decisions with their experiential knowledge and preferences, i.e., it is subjective, variable and fuzzy compared with traditional mathematically defined objectives. Accordingly, traditional optimization methods as well as various successful nature-inspired EAs for explicitly defined mathematical functions are no longer applicable. It is of practical significance to develop suitable EAs to effectively solve personalized search problems.

In the family of EAs, interactive evolutionary computations (IECs) are powerful for optimizing problems with qualitative objectives and expected to be effective for the personalized search [1–3]. In the past decades, fruitful studies on IECs have been devoted to alleviate users' evaluation burdens in the evolutionary process, especially for complicated optimization tasks. The corresponding work can be classified into three groups: (1) Designing friendly interfaces, e.g., changing continuous evaluation mode to a discrete or fuzzy number ones [2,4]; (2) enhancing evolutionary operators to accelerate the evolution process, e.g., Chen et al. [5] presented a Bayesian model based IEC to effectively reduce initial decision space according to the historical search; (3) developing surrogate or learning assisted IECs to quantitatively approximate the preference or evaluation of a given user on a candidate, i.e., in such IECs, the fitness function of the qualitative objective is estimated to drive the evolutionary operations as traditional ones [4–6]. We here try to effectively solve the personalized search with surrogate-assisted IECs since they have been successfully applied to some complex design and multi-objective decision problems.

Surrogate assisted IECs are similar to that of EAs. A user is required to first evaluate some individuals along with the evolutionary search, and these individuals together with the evaluated scores are used to train or build a model to approximate the

* Corresponding author.

E-mail addresses: baolin_zj@163.com (L. Bao), xysun78@126.com (X. Sun), Yang.Chen@ntu.edu.sg (Y. Chen), dwgong@vip.163.com (D. Gong), yongwzhang@gmail.com (Y. Zhang).

user's preferences. Then, the model is applied as a fitness surrogate in the subsequent evolution process, and the user only needs to revise few wrongly evaluated estimations by the surrogate. The model will be managed or updated when the user finds that the estimation is far from his/her preferences. Clearly, the surrogate building, including data collection, model selection and training, is critical for developing a reliable surrogate assisted IEC [7–9]. Model selection and training have been greatly attracted in various applications. Sun et al. [1] presented a semi-supervised learning based surrogate when the training data of interactive genetic algorithms are difficult to be sufficiently collected in handling complicated design problems. Pan et al. [8] proposed a classification-based surrogate to improve interactive decisions when using many-objective EA for numerically defined expensive optimization problems. Integrating parallel computing with surrogate-based EAs, Akinsolu et al. [10] proposed a parallel surrogate assisted algorithm to enhance the mutation operators of differential evolution for electromagnetic design. We also used probabilistic conditional preference network as a surrogate for personalized book search [11]. As for collecting the training data, only few studies have been developed. Chen et al. [5] presented a Bayesian induced interactive Estimation of Distribution Algorithm (IEDA) for personalized laptop search, in which users' interactive time is used to construct an RBF-based surrogate model. Tian et al. [12] articulated granularity into a surrogate building to effectively collect the training data with relatively smaller computation cost when solving high-dimensional expensive optimization problems.

These surrogate-assisted EAs are effective on solving quantitatively or qualitatively defined complex problems. They endeavor to construct/manage the surrogate model with supervised or semi-supervised learning methods with evaluated individuals, and then use the model to approximate the individual fitness to perform evolutionary operators. The following three deficiencies of the exiting algorithms can be concluded. (1) A given user must provide initial interactions for constructing a surrogate, no matter by explicit or implicit ways, which inevitably conflicts with the motivation of alleviating user fatigue. To address this problem, unsupervised learning-based surrogates are more helpful and expectable. (2) The relationship among the evolutionary operators and the surrogate has rarely been considered, i.e., the information implicated in the surrogate construction may be valuable to strengthen the performance of the operators. (3) The intrinsic preference features of a user hidden in his/her historical interactions can greatly benefit to accurately reveal the user's preferences, however, which has not been concerned even such a technology has been well developed and used in personalized recommendation. Therefore, integrating the achievement of user interest model in personalized recommendation into surrogate-assisted IECs will greatly improve the performance of personalized search.

As for using an unsupervised learning model to construct a surrogate and further capturing the relationship among the evolutionary operators and the surrogate, we presented a Restricted Boltzmann Machine (RBM)-based Estimation of Distribution Algorithm (EDA) for complex numerical problems [13]. In this algorithm, EDA is first performed for some generations on real problems to obtain the training data, and then RBM is trained with those better individuals (without using specific fitness values). Both the probability model of EDA and the fitness function are simultaneously fetched from the trained RBM, i.e., the joint probability of the visible layer in RBM is calculated as the probability model in EDA for population reproduction, and the energy function of the RBM is used to estimate the individual fitness of the optimized complex problem. Experimental results demonstrate its superior in effectively reducing computational

complexity and improving the accuracy of fitness estimation. Inspired by these results, we here further study an unsupervised RBM surrogate-assisted IEC for personalized search since it figures out the shortages in existing surrogate-based ECs.

With regard to extract the intrinsic features of a user's preference, many interest models used in personalized recommendation will provide valuable references [14,15], e.g., Bayesian model [16,17], Factorization Machine [18,19], Multilayer Perceptron [20,21], RBM [22,23], Autoencoder model [24,25], Convolutional Neural Network (CNN) [26,27]. Rendle et al. [16] presented a Bayesian learning method for personalized ranking by maximizing the posterior estimator. In this method, the training data are grouped into evaluated items and unrated ones as positive and negative information. Cheng et al. [20] proposed a Wide & Deep learning based interest model by jointly training wide linear models and deep neural networks (DNN) to combine the benefits of memorization and generalization for recommendation. Kim et al. [26] presented a novel context-aware recommendation model, called as Convolutional Matrix Factorization (ConvMF), which makes full use of the positive and negative preferences to combine CNN with probabilistic matrix factorization for improving the prediction accuracy. Zhou et al. [28] proposed an attention-based user behavior modeling framework, which effectively integrates all of users' historical interactive behaviors. However, these models have not been well combined with the IEC process to further effectively improve the personalized evolutionary search.

Motivated by our previous work, we here expand it to interactive personalized search and present a dual-RBM-assisted IEDA by articulating interest model construction with historical interactive behaviors. The RBM-based surrogate in [13] is first enhanced by modifying it into a dual-module one according to the grouped historical information for precisely extracting the user preference features. After the dual-module RBM is trained, the probability model of EDA will be constructed using the critical features of the positive RBM. The surrogate for estimating the fitness of the searched items is ultimately obtained by using the energy functions of the RBMs. The probability and surrogate models will be applied in IEDA to effectively find the satisfied *TopN* items for the current user. Adequate experiments on typical real-world datasets demonstrate that the proposed algorithm can effectively not only enhance the performance of the personalized search but also alleviate users' evaluation burdens to improve user experiences in the searching process.

Accordingly, the main contributions of our work are as follows: (1) A dual-RBM module is presented by constructing two related RBM models, i.e., positive and negative ones. These two models are trained with dominant and inferior items evaluated by the current user to accurately track the user preference features. The module is then used to define the probability model and fitness surrogate for EDA; (2) the reproduction probability model of EDA for generating more preferred individuals is defined based on the probability of the visible layer in the positive RBM model by sufficiently using the positive preference features and effectively impairing the impacts of the negative ones; (3) the fitness surrogate is obtained by not only weighting the energy functions of the positive and negative RBM models but also social group knowledge.

The remainder of the paper is organized as follows. Section 2 introduces the notations of our study and the related preliminary work. The proposed dual RBM-driven IEDA is addressed in detail in Section 3. Section 4 presents the comparative experiments and corresponding experimental analysis. The conclusion is finally followed.

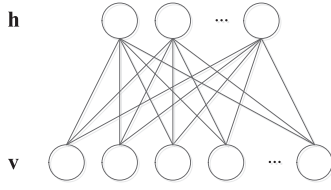


Fig. 1. Structure of RBM.

2. Notations and preliminary work

2.1. Notation of personalized search

Personalized search is a searching process in which a user finds out the satisfied items according to his/her interests and preferences. It can be described as a combinatorial optimization problem with qualitative index. An item (solution) with n attributes (decision variables) is expressed as $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$, and the objective function $f_u(\mathbf{x})$ of a user u in the personalized search can be formally expressed as:

$$\begin{cases} f_u(\mathbf{x}) \\ \text{s.t. } \mathbf{x} \in G \end{cases} \quad (1)$$

where $f_u(\mathbf{x})$ represents the preference of user u on the item \mathbf{x} and often changes in the searching process due to the dynamical interests and preferences of the user. Accordingly, such a function cannot be accurately described with a specific mathematical model, and is determined by user's empirical knowledge and cognitive preferences. G indicates a feasible solution space and is usually very large and sparse.

Apparently, traditional optimization algorithms are powerless for such a problem with variables in a very large even sparse space but without an accurate mathematical model. IECs by involving a user to evaluate the individuals in the evolutionary searching can be a more helpful choice. Traditional IECs, however, need a user to directly evaluate the displayed items to provide the individual fitness. It is not applicable for personalized search because the user is impossible to give specific numeric ranks on all items. On the other hand, the interactive process of IECs may result in users' fatigue and boredom, which will finally make the use abandon the personalized search. Therefore, we should deeply study the effective and efficient strategies to improve the capabilities of IECs for solving the personalized search problems.

2.2. Preliminary theory of restricted Boltzmann machine

Several correlated notations of RBM are simply given here since they are the foundations of our algorithm. RBM is a stochastic neural network with two layers, symmetric connections and no self-feedback [29–31]. A typical RBM is illustrated in Fig. 1.

In RBM, \mathbf{v} is the visible layer with n units and its input are the observed data. \mathbf{h} is the hidden layer with m units, working as a feature extractor. The states of the neurons in traditional RBM are usually changed between binary variables, and the entire state of the network is measured based on the value of its energy function. Given the state (\mathbf{v}, \mathbf{h}) and the parameter set $\theta = \{W, a, b\}$ with connection weights W and biases a and b , the energy function denoted as $E_\theta(\mathbf{v}, \mathbf{h})$ of a RBM is usually formulated as Eq. (2).

$$E_\theta(\mathbf{v}, \mathbf{h}) = - \sum_{i=1}^n \sum_{j=1}^m v_i w_{ij} h_j - \sum_{i=1}^n a_i v_i - \sum_{j=1}^m b_j h_j \quad (2)$$

where v_i and h_j are the states of the i th visible unit and the j th hidden one, respectively.

With a training dataset, all of the parameters in RBM can be obtained by Contrastive Divergence (CD) algorithm proposed by Hinton [32]. Since then, the researches on RBM have flourished. Salakhutdinov et al. [22] first used RBM to solve recommendation problem and proposed a RBM-based collaborative filtering model. Compared with deep learning models, RBM model has a simple structure and can well extract the nonlinear characteristics of decision variables in a faster training mode. Accordingly, RBM is more suitable to approximate user's preferences for the personalized search in an interactive environment. Furthermore, RBM provides the distribution probabilities of the features involved in those input nodes (items), which can be naturally applied in EDA to generate new individuals. Therefore, we will study a RBM enhanced IEDA to deal with the personalized search.

3. Restricted Boltzmann machine-driven interactive estimation of distribution algorithm

3.1. Framework

As aforementioned, RBM has been successfully used in approximating users' preferences for recommendation, but not applied in IECs for personalized search. Here, we propose a dual RBM-driven IEDA with social knowledge (shorted as SC_DRBMIEDA) for the personalized search. The framework of the proposed algorithm is presented in Fig. 2.

The SC_DRBMIEDA algorithm consists of three main contents:

(1) Construction of RBM

For getting a more reliable RBM to enhance the performance of IEDA, we here utilize the current user's historical searching behaviors to construct a dual-RBM model to approximate his/her preference for personalized search. Specifically, the items searched and evaluated by the user are selected and classified into two groups, i.e., dominant group whose items have longer interactive time or higher evaluations as mostly preferred ones and inferior group whose items have lower scores as unsatisfied ones. These two groups of items are used to train a dual-RBM to obtain the user's positive and negative preferences, respectively.

(2) Developing IEDA with the dual-RBM for evolving the search list

The probability model of EDA used to generate new individuals for the interactive evolutionary searching is derived from the positive preference RBM because it represents the preferred feature distributions of the searched items. Then, new population (items) is generated through sampling the probability model from the search space. On the other hand, by combining the trained dual-RBM model with social knowledge, a RBM-based surrogate model is constructed to estimate the fitness or preference of those new items for personalized ranking. N preferable items with higher estimations are selected as a *TopN* list and recommended to the user for the next iterative evolution.

(3) Management of RBM surrogate

The dominant and inferior groups are updated based on the interactively evaluated *TopN* list. The average approximation error of the surrogate on the evaluated items is calculated and used to control the surrogate management. If the error is larger than a given threshold, the RBM models will be updated. Accordingly, the probability and surrogate models together with the energy functions are updated along with the evolutionary searching process.

The above process is repeated until use's satisfactory items are found or the iterative number of evaluations is reached.

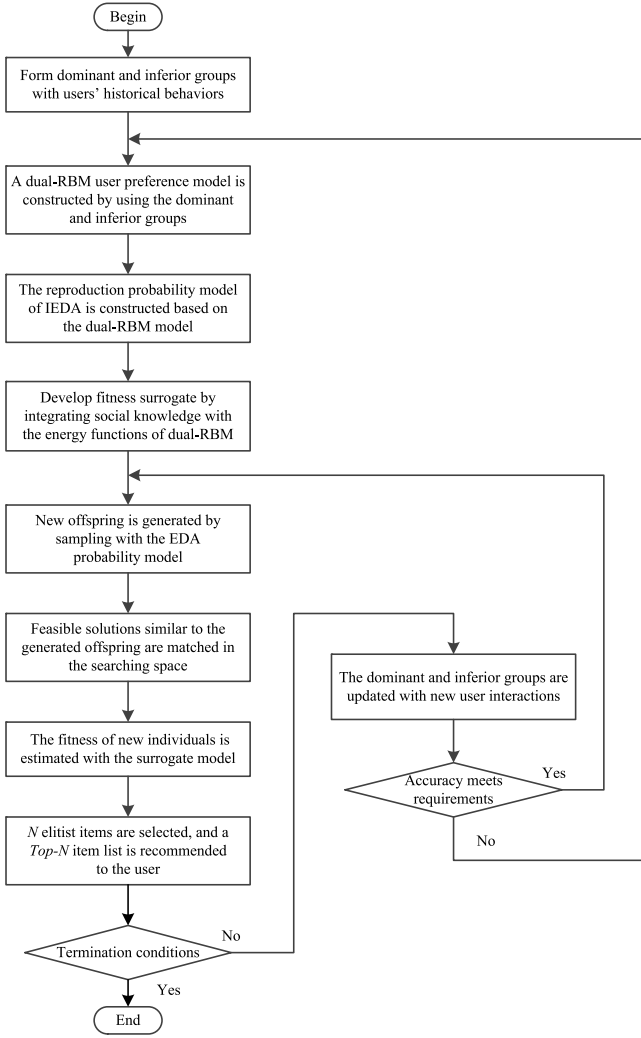


Fig. 2. Flowchart of SC_DRBMIEDA.

3.2. Construction of dual-RBM and EDA probability model

The dual-RBM model constructed to fetch the user's preference is shown in Fig. 3. It consists of two parallel sub-networks: the positive and negative RBM models. The inputs of the positive RBM are the dominant items with higher scores or longer browsing time, and that of the negative RBM are those inferior items most disliked by the user. With these grouped items, two sub-networks are simultaneously trained with CD learning algorithm. After training, the probability model of EDA will be constructed based on the visible layer's outputs of the positive RBM, and the fitness of new individuals will be estimated by weighting the energy functions of the positive and negative RBM models.

In personalized search, the items are considered as a combination of their attributes, e.g., an item \mathbf{x} is described as $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ with n attributes. If an item belongs to the dominant group, i.e., it has higher ranks or longer browsing time, then the attributes contained in this item are generally preferred by the user and should be emphasized. Motivated by this, we here use binary encoding $\mathbf{v} = [v_1, v_2, \dots, v_n]$ to represent an item. If the i th attribute is not included in \mathbf{x} , then the i th bit v_i of the binary chromosome \mathbf{v} is 0, otherwise it is 1. Accordingly, the input layers of the positive and negative RBM models have n visible units. The input of the dual-RBM user preference model is exactly the binary chromosomes of the attribute encoding of

dominant and inferior items. As for the hidden layers of those RBMs, supposing they have m units.

Given the state of visible units, the active probabilities $P_{\theta_p}(h_{pj} = 1|\mathbf{v}_p)$ and $P_{\theta_n}(h_{nj} = 1|\mathbf{v}_n)$ of the j th hidden unit in the positive and negative RBM models are calculated as follows:

$$P_{\theta_p}(h_{pj} = 1|\mathbf{v}_p) = \sigma \left(b_{pj} + \sum_{i=1}^n v_{pi} W_{pij} \right) \quad (3)$$

$$P_{\theta_n}(h_{nj} = 1|\mathbf{v}_n) = \sigma \left(b_{nj} + \sum_{i=1}^n v_{ni} W_{nij} \right) \quad (4)$$

where v_{pi} and v_{ni} are the states of the i th visible unit in the positive and negative RBM models; h_{pj} and h_{nj} are the states of the j th hidden one in the positive and negative RBM models, respectively. $\theta_p = \{W_p, a_p, b_p\}$ and $\theta_n = \{W_n, a_n, b_n\}$ represents the parameters of the positive and negative RBMs, respectively. $\sigma(x) = 1/(1 + \exp(-x))$ is the active function.

Given the state of hidden units, the active probabilities $P_{\theta_p}(v_{pi} = 1|h_p)$ and $P_{\theta_n}(v_{ni} = 1|h_n)$ of the i th visible unit in the positive and negative RBM models are calculated with the following equations:

$$P_{\theta_p}(v_{pi} = 1|h_p) = \sigma \left(a_{pi} + \sum_{j=1}^m W_{pij} h_{pj} \right) \quad (5)$$

$$P_{\theta_n}(v_{ni} = 1|h_n) = \sigma \left(a_{ni} + \sum_{j=1}^m W_{nij} h_{nj} \right) \quad (6)$$

After training the dual-RBM user preference model, the parameters $\theta_p = \{W_p, a_p, b_p\}$ and $\theta_n = \{W_n, a_n, b_n\}$ of the positive and negative RBM models have the preference features of the current user, and represent a preferable and dislike relationship between the current user and the item attributes. Therefore, the dual-RBM user preference model is more general to express the preference features of the current user.

Since the positive preference model can well express the user's preferences on items, we use it to calculate $p(x_i = 1)$. The value of $p(x_i = 1)$ indicates the preference probability on the i th attribute, and is obtained according to Eq. (7) [29]:

$$p(x_i = 1) = \frac{\sum_{l=1}^T \delta_l(x_i^+) + \phi}{\sum_{l=1}^T \delta_l(x_i^+) + \sum_{l=1}^T \delta_l(x_i^-) + 2\phi} \quad (7)$$

where T is the size of the dominant group; x_i is the i th attribute of an item (individual); for the l th sample in the dominant group,

$$\delta_l(x_i^+) = \sum_{j=1}^m e^{-E_{\theta_p}(x_i^+=1, h_{pj})} \text{ and } \delta_l(x_i^-) = \sum_{j=1}^m e^{-E_{\theta_p}(x_i^-=0, h_{pj})}$$

are the marginal distribution of x_i^l when $x_i^l = 1$ and that of x_i^l when $x_i^l = 0$ respectively; $\phi = (\sum_{l=1}^T \delta_l(x_i)) / T$ is the average distribution.

What we concern on is the emergence probability of each attribute in the searched item, therefore, the probability model expressed in Eq. (8) is calculated as that of EDA.

$$P(\mathbf{x}) = [p(x_1 = 1), p(x_2 = 1), \dots, p(x_n = 1)] \quad (8)$$

The probability model $P(\mathbf{x})$ of EDA is used to efficiently generate more preferred items for the current user. New individuals are first generated by randomly sampling as shown in Eq. (9):

$$x_i = \begin{cases} 1, & \text{if } \text{random}(0, 1) \leq p(x_i = 1) \\ 0, & \text{otherwise} \end{cases} \quad (9)$$

where x_i is the i th attribute value of a new individual and $\text{random}(0, 1)$ is a random variable in $[0, 1]$. Then, according to

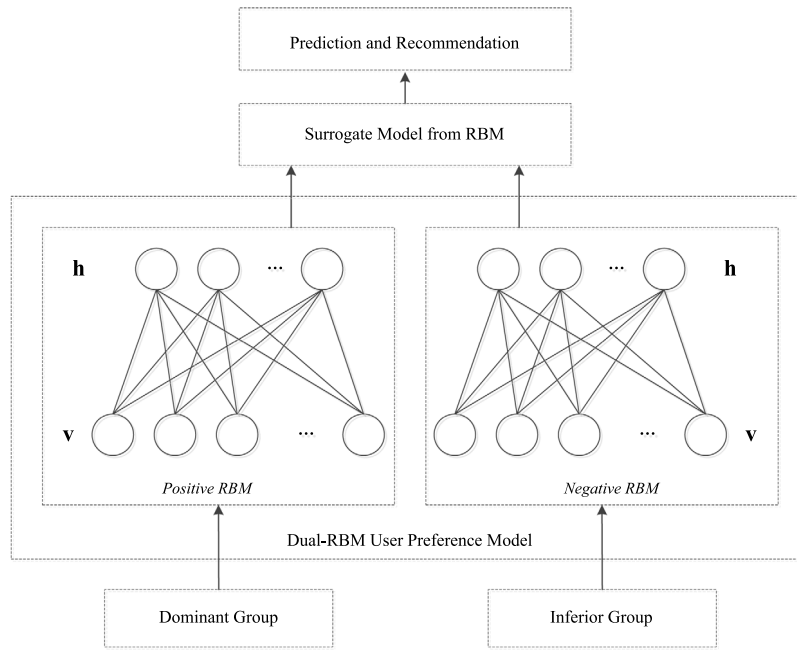


Fig. 3. Diagram of dual-RBM user preference model.

their chromosome similarity, new solutions in the searching space are matched to form a set of S feasible individuals (items).

Due to the diversity and changeability of a user's preference, the RBM-based preference model and preferred feature distributions are rough in the early stage of IEDA. Along with the user's interactions and searching, his/her requirements and interests are gradually clearer and clearer. To possibly obtain an accurate and reliable preference model, the RBM must be updated according to those newly evaluated items to timely track the preferences for effectively guiding the IEDA evolution.

3.3. Surrogate model for fitness approximation

In IEDA, the selection of elitist individuals is performed according to the estimated fitness which is obtained by a surrogate derived from the dual-RBM user preference model. In the trained dual-RBM, the obtained parameters represent the potential relationship between the current user's preferences and items' attributes. If an item is preferred by the user, then the corresponding energy functions of the positive and negative RBM models should reach their minimal and maximal values, respectively. In addition, the available behaviors of other users who have similar preferences to the current user can provide valuable information to accurately estimate the current user's ratings on the searched items since social preferences usually represent a fashion trend. Therefore, we here design the surrogate model by combining the dual-RBM user preference model with social knowledge.

According to the Pearson similarity, the Pearson correlation coefficient $Sim(u_i, u_j)$ between user u_i and u_j is calculated and used to find social group with similar preferences.

$$Sim(u_i, u_j) = \frac{\sum_{k \in I_{u_i, u_j}} (R_{u_i k} - \bar{R}_{u_i}) (R_{u_j k} - \bar{R}_{u_j})}{\sqrt{\sum_{k \in I_{u_i, u_j}} (R_{u_i k} - \bar{R}_{u_i})^2 \times \sum_{k \in I_{u_i, u_j}} (R_{u_j k} - \bar{R}_{u_j})^2}} \quad (10)$$

where I_{u_i, u_j} represents the item set rated by users u_i and u_j ; $R_{u_i k}$ and $R_{u_j k}$ are the scores of the k th item given by u_i and u_j ; \bar{R}_{u_i} and \bar{R}_{u_j} are the average scores.

According to Eq. (10), c similar users are selected. By incorporating the trained dual-RBM model and the social information of these c similar users, the surrogate model $\hat{f}(\mathbf{x})$ is developed in Eq. (11) to estimate the individual fitness.

$$\hat{f}(\mathbf{x}) = \sigma(\alpha \times social(\mathbf{x}) + preference(\mathbf{x})) \quad (11)$$

where $social(\mathbf{x})$ indicates the weighted average score of those c similar users on item \mathbf{x} and will be defined in Eq. (12); $preference(\mathbf{x})$ represents the predicted score of the dual-RBM model on item \mathbf{x} and can be calculated with Eq. (13). α is used to adjust the contributions of the social group. $\sigma(x) = 1/(1 + \exp(-x))$ is the normalized function.

The term $social(\mathbf{x})$ is defined in Eq. (12):

$$social(\mathbf{x}) = \sum_{j=1}^c Sim(u, u_j) \times R_{u_j \mathbf{x}} \quad (12)$$

where $Sim(u, u_j)$ is the Pearson similarity coefficient between the current user u and the neighbor u_j , and $R_{u_j \mathbf{x}}$ is the score of item \mathbf{x} given by the neighbor u_j .

The term $preference(\mathbf{x})$ in Eq. (11) is determined by the energy function difference between the positive and negative models. If the user has very clear and positive preference, then the energy function of the positive RBM will be very small and that of the negative one will be large, i.e., their difference should be great and be the $preference(\mathbf{x})$ great too. Otherwise, the value of $preference(\mathbf{x})$ will be smaller. Besides, the preference of the positive RBM is more important and should contribute more. Accordingly, the following equation is defined to calculate the $preference(\mathbf{x})$.

$$preference(\mathbf{x}) = \frac{\max(E_{\theta_p}) - E_{\theta_p}(\mathbf{x}, \mathbf{h})}{\max(E_{\theta_p}) - \min(E_{\theta_p})} - \beta \times \frac{\max(E_{\theta_n}) - E_{\theta_n}(\mathbf{x}, \mathbf{h})}{\max(E_{\theta_n}) - \min(E_{\theta_n})} \quad (13)$$

where $E_{\theta_p}(\mathbf{x}, \mathbf{h})$ and $E_{\theta_n}(\mathbf{x}, \mathbf{h})$ are the energy values of individual \mathbf{x} in the positive and negative RBM models, respectively; $\max(E_{\theta})$ and $\min(E_{\theta})$ are the maximum and minimum energy values of

the individuals in the evolutionary population. β is used to adjust the proportion of the contributions of the negative RBM.

According to the surrogate model defined in Eq. (11), the predicted value $\hat{f}(\mathbf{x})$ varies in $[0, 1]$, which represents the relative preference degree of the user on item \mathbf{x} . The larger the fitness $\hat{f}(\mathbf{x})$, the greater the user may be interested in the individual \mathbf{x} .

For the S new individuals generated in Section 3.2, the S' feasible items are matched in the itemset according to their chromosome similarity. Then, the surrogate model is used to estimate the fitness of the S' items. According to the individual fitness, N individuals with higher fitness are chosen as *TopN* item list and presented to the user for the next interaction and evolution.

According to the user's interactions or evaluations in IEDA, the average estimation error of the surrogate model is monitored. If the error is greater than a given threshold, the dual-RBM together with the corresponding probability model and the fitness estimation one are updated with the newly evaluated items. The model management guarantees the approximation accuracy of the surrogate model in the IEDA process for guiding the evolutionary direction.

3.4. Implementation

The pseudocode of the proposed SC_DRBMIEDA algorithm is presented in Algorithm 1.

Algorithm 1 Pseudocode of SC_DRBMIEDA

Input: Users' historical behavior data

Output: *TopN* item list

- 1: Initialization: All the items are screened out to form the social, dominant and inferior groups of the current user.
 - 2: **while** Termination conditions are not met **do**
 - 3: **Training:** The dual-RBM user preference model is constructed and trained with the methods given in Section 3.2 to comprehensively extract the preference features of the user.
 - 4: **Probability Model:** The RBM-based probability model $P(\mathbf{x})$ of EDA is constructed. Then, new individuals are generated by randomly sampling with $P(\mathbf{x})$, and they are matched in the searching space to generate feasible solutions.
 - 5: **Surrogate Model:** The surrogate model from RBM is designed with Equations from 11 to 13 as stated in Section 3.3, and used to estimate the fitness of feasible solutions.
 - 6: **TopN List:** N outstanding individuals with higher estimated fitness are selected to generate a *TopN* item recommendation list.
 - 7: **Interactive Evaluations:** The *TopN* list is submitted to the user for interactive evaluations.
 - 8: **Model Management:** The surrogate model is evaluated. If the average accuracy is lower than the threshold, update the models. Otherwise, keep using them.
 - 9: **end while**
-

3.5. Complexity analysis

The computational complexity of the proposed algorithm is determined by that of calculating social knowledge, training the model and selecting the feasible solutions. First, the calculation of the social information of the current user costs $O(|U|)$. $|U|$ is the number of social group. The computation complexity is $O(G_d + G_i)$ for training the dual-RBM user preference model. G_d and G_i are the scale of the dominant and inferior groups, respectively. The searching process spends $O(SD)$ on the calculation of the distance between S individuals and D feasible individuals to generate S' candidate items. The fitness calculation of S' candidate items costs $O(S)$. As a result, the total computational complexity of the proposed method is $O(|U| + G_d + G_i + SD + S)$. Since $SD \gg S$, the computational complexity of SC_DRBMIEDA can be simplified as $O(|U| + G_d + G_i + SD)$, which is very computationally efficient.

4. Experiments and results

4.1. Experimental settings

Two kinds of typical datasets used in personalized recommendation are employed here to objectively demonstrate the performance of the proposed algorithms. The MovieLens datasets [33], i.e., MovieLens-latest-small (ML-l-s) and Amazon datasets [34], i.e., Digital_Music (Music), Apps_for_Android (Apps) and Movies_and_TV (Movies) are selected as the benchmark tasks. The statistical information of the datasets is shown in Table 1.

In the experiments, we run Python 3.6 on a computer with an AMD Ryzen 5 CPU 3.60 GHz and 16.0 GB RAM, and conduct three groups of experiments: (1) Determining the number of hidden layers of the RBM model; (2) comparing with four popular recommendation algorithms to show the effectiveness of the RBM-based surrogate; (3) illustrating the performance of the presented SC_DRBMIEDA by comparing with other IECs in personalized search.

The following metrics are used to objectively measure the performance of the algorithms in the personalized search, which are quite different from the traditionally used ones in IECs as evolutionary generations (to show users' evaluation fatigue) and average fitness (to present users' satisfactory degree on the searched items).

(1) Prediction accuracy: Root Mean Square Error

Root Mean Square Error (RMSE) [35,36] is calculated as follows.

$$RMSE = \sqrt{\frac{\sum_{u, \mathbf{x} \in R_{test}} (r_{ux} - \hat{r}_{ux})^2}{|R_{test}|}} \quad (14)$$

where R_{test} indicates the test dataset with its scale being $|R_{test}|$, r_{ux} represents the real ranks of the item \mathbf{x} given by user u , and \hat{r}_{ux} is the estimated one.

(2) Search reliability: Hit Ratio

Hit Ratio (HR) is often used to measure the reliability and effectiveness of recommendation algorithms [37]. It refers to the ratio between the number of preferred items in the *TopN* list and that in the test dataset. The HR value of *TopN* for the user at each recommendation is expressed as follows:

$$HR = \frac{Hit}{|GT|} \quad (15)$$

where *Hit* is the number of preferred items in *TopN* with its scale as $|GT|$. Higher *HR* indicates a better reliability and effectiveness in the personalized search.

(3) Recommendation accuracy: Average Precision

Average Precision (AP) is used to show the recommendation accuracy and reflect the user satisfaction. Assuming a user is interested in L items of the *TopN* list, the AP value in the recommendation is calculated as:

$$AP = \frac{1}{L} \sum_{l=1}^L \frac{l}{position(l)} \quad (16)$$

where *position* (l) is the position of the l th preferred item in *TopN*.

Clearly, AP is sensitive to the order or position of the preferred items in *TopN*. The higher the preferred items rank, the larger the AP. Simultaneously, larger AP means that the user is more satisfied with the submitted results, otherwise, AP is 0 if no preferred items appear in *TopN*.

(4) Searching satisfaction: Mean Average Precision

Mean Average Precision (MAP) [36] is the average AP of all the users in the test dataset and expressed as follows:

$$MAP = \frac{1}{Q} \sum_{q=1}^Q AP(q) \quad (17)$$

Table 1
Statistical information of datasets.

Dataset	# of Users	# of Items	# of Ratings	Sparsity (%)
MovieLens-latest-small (ML-l-s)	671	9066	100,004	98.36
Digital_Music (Music)	5541	3568	64,706	99.67
Apps_for_Android (Apps)	87,271	13,209	752,937	99.93
Movies_and_TV (Movies)	123,960	50,052	1,697,533	99.97

where $AP(q)$ is the AP of the q th user in the test dataset with total Q members. MAP reflects the searching satisfaction of all the users in the test dataset. Larger MAP indicates higher searching satisfaction.

4.2. Experiment 1: Number of hidden units in RBM-based user preference model

The number of hidden units in the RBM-based user preference model will have great influence on the performance of the proposed algorithm because it affects both the extraction of the user preferences and the RBM-based surrogate. Therefore, we first investigate the impact of the number of hidden units in the RBM-based user preference model, and determine an appropriate value to achieve better recommendation accuracy and higher searching efficiency in the personalized search.

In the experimental datasets, items' text category tags together with the ratings varied from 0.5 to 5 points are selected as the attribute features of the user preference model. Ten users are randomly selected from the datasets to conduct the experiments. The rating timestamps are used to orderly get the training and testing datasets, i.e., the first 70% of all the items are used as the training dataset and the remaining 30% of those as the testing one.

Taking the Digital_Music of Amazon as an example, the encoding of the items fed into the RBM user preference model is presented as follows. In Digital_Music, all the items have 305 text category tags. An individual (item) \mathbf{x} in the EDA population is encoded into a binary chromosome as $\mathbf{x} = [x_1, x_2, \dots, x_{305}]$. If \mathbf{x} has the i th category tag, then $x_i = 1$, otherwise, $x_i = 0$. Accordingly, the visible layer of the RBM-based user preference model has 305 units for the Digital_Music. In addition, the items with score larger than 4 in the training set are selected as the dominant group while the items with score smaller than 2 are screened out as the inferior group. The dominant and inferior groups are used to train the positive and negative RBM models, respectively. The trained dual-RBM user preference model can represent the user's preference features. The RBM-based surrogate model defined by Eqs. (11)–(13) is used to estimate the item ratings in the test set.

In the experiments, 10 independent runs are performed for each parameter setting in our algorithm. The average RMSE, HR and MAP values are calculated to present the impacts of different numbers of hidden units in the dual-RBM model. The ratio of the number of hidden units to that of decision variables varies from 0.3 to 2 with step 0.1, and appears on the y-axis. The results in the ML-l-s and Music datasets are shown in Figs. 4 and 5, respectively.

As can be seen from Fig. 4, with increased values of the number, the average RMSE gradually increases, while the average HR and MAP firstly increase and then decrease. When the ratio for the ML-l-s dataset is 0.9, all these metrics reach better results. More hidden units may lead to the overfitting of RBM. On the other hand, if the number of hidden units is smaller, the extracted preference features are not enough to sufficiently express the user preference. Similarly, such a ratio for the Music dataset should be 0.8–1.2 as shown in Fig. 5 and possibly decreases the computational cost. Comparing Figs. 4 and 5, the number of Hidden Units in RBM is related with that of category features which are encoded as the input of RBM. According to the observed conclusions, the ratio between the number of hidden units to that of visible units is set to 0.8–1.2. The other parameters of the proposed algorithm are listed in Table 2.

Table 2
Experimental parameters of the proposed algorithm.

Parameter	Value
# of visible units	# of categories
# of hidden units	(0.8–1.2)# of categories
Learning rate	0.1
Momentum	0.5–0.9
# of training epochs in RBM	20
α	0.3
β	0.2
TopN	10
# of similar users (c)	6
Threshold γ	0.3
Population size	50

4.3. Experiment 2: Effectiveness of proposed dual-RBM surrogate model

In order to demonstrate the feasibility and effectiveness of the proposed dual-RBM surrogate model strengthened with social knowledge (named as SC_DRBM), six popular recommendation algorithms, i.e., user-based collaborative filtering (user-based CF) [38], BPRMF [16], Wide & Deep [20], ConvMF [26], ATRank [28] and RBMEDA [13], are compared on the metrics RMSE, HR and MAP. The consumed time (seconds) of these algorithms is also presented to show the computational cost. The user-based CF is the baseline method in recommendation system. The BPRMF is an effective BPR-based method by using the positive and negative user preferences. Wide & Deep is a novel DNN-based recommendation algorithm. ConvMF is a CNN-based recommendation algorithm by integrating the positive and negative preferences. ATRank is a hybrid algorithm. RBMEDA is a basic RBM-based EDA algorithm. In addition, our algorithm without using the social knowledge, shorted as DRBM, is also compared in the experiments.

In the comparative experiments, the training and testing samples are the same as that given in Section 4.2. As for the user-based CF, the number of the similar users is 20. The number of hidden factors of BPR is set as 20. For Wide & Deep, ConvMF and ATRank, the parameters set in the corresponding references are used here. We independently run all the algorithms on four different datasets for 10 times, and the average results are recorded in Table 3 with the best values being bolded.

The Mann–Whitney U test with confidence level 0.95 is used to show the significance of the proposed SC_DRBM algorithm, and the results marked with “*” are significantly different from the best results. From Table 3, we can observe the following conclusions:

(1) The proposed SC_DRBM significantly outperforms the user-based CF, BPRMF, ConvMF and DRBM algorithms on various datasets. The main reason is that the user-based CF and BPRMF algorithms cannot handle new items, i.e., suffering from the cold start problem, and therefore, about 20%–40% predicted items get zero scores in the test dataset. Although ConvMF and ATRank make full use of the available information to improve the prediction accuracy and recommendation effect, SC_DRBM is obviously superior to these algorithms. On the other hand, SC_DRBM is better than Wide&Deep in the HR, MAP and Time metrics, but its RMSE is a little worse than Wide&Deep. Therefore, SC_DRBM effectively takes full advantages of items' category tags and social

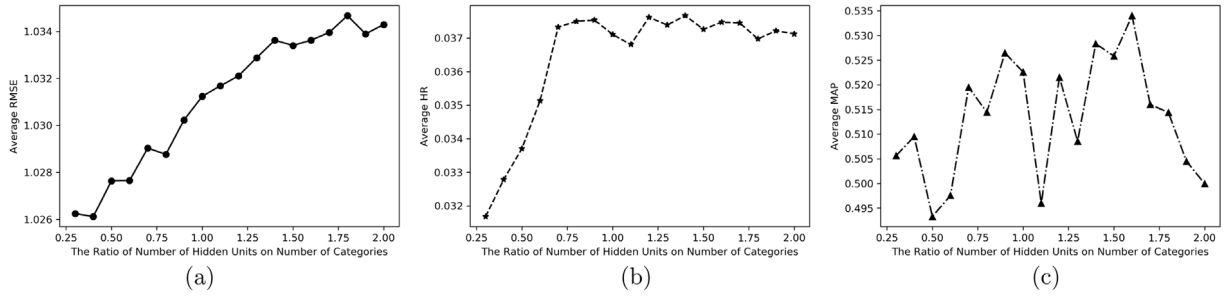


Fig. 4. # of hidden units vs. # of decision variables in ML-1-s.

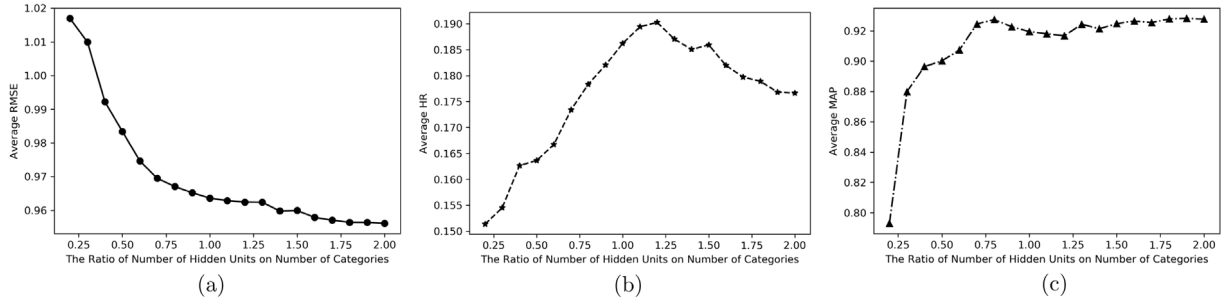


Fig. 5. # of hidden units vs. # of decision variables in Music.

Table 3

Results on effectiveness of surrogate model.

Algorithm	User-based CF	BPRMF	Wide&Deep	ConvMF	ATRank	RBMEDA	DRBM	SC_DRBM
ML-1-s	RMSE	1.522	1.425	1.038	3.023	2.256	1.016	0.995
	HR	0.0151	0.0246	0.0380*	0.0256	0.0287	0.0261	0.0550
	MAP	0.379	0.426	0.618*	0.589	0.603	0.428	0.639
	Time(s)	6.816	7.897	0.968	123.67	2.356	0.546	0.864
Music	RMSE	2.045	1.893	0.923	3.130	2.198	0.959	1.169*
	HR	0.0597	0.1302	0.1797	0.0742	0.0778	0.1709	0.2149
	MAP	0.550	0.778	0.808	0.728	0.778	0.864	0.934
	Time(s)	1.240	0.780	1.471	276.716	2.900	0.206	0.216
Apps	RMSE	3.271	2.251	1.516	3.119	2.699	1.649	1.680*
	HR	0.0522	0.0775	0.1079*	0.0701	0.0887	0.0915	0.1171
	MAP	0.642	0.614	0.630	0.688	0.759	0.640	0.672
	Time(s)	52.073	4.017	0.624	90.489	2.573	0.131	0.133
Movies	RMSE	2.664	2.137	1.061	3.029	2.271	1.167	1.200*
	HR	0.0173	0.0215*	0.0195	0.0183	0.0166	0.0145	0.0263
	MAP	0.684	0.803	0.768	0.838*	0.682	0.677	0.967
	Time(s)	6097.745	304.705	2.661	506.125	18.341	0.506	0.533

knowledge, which not only relaxes the cold start but also achieves higher prediction accuracy and better recommendation results.

(2) Putting more emphasis on the RMSE, HR and MAP for all the compared algorithms, it can be observed that SC_DRBM conducts the best in enhancing the HR and MAP and concerns on getting more accurate ranks of the item recommendation list instead of a specific score. An accurate rank list is just what we pursued since our algorithm is to improve the searching reliability and user satisfaction.

(3) Move to the RBMEDA, DRBM and SC_DRBM, i.e., three algorithms using RBM model as surrogate and deriving the probability model for the EDA process. We conclude that DRBM and SC_DRBM are superior to RBMEDA because they are the enhanced IEDA with the dual-RBM model. Taking the ML-1-s dataset as an example, DRBM is 1.18% lower in RMSE, 43.30% higher in HR and 23.60% higher in MAP than that of RBMEDA, and SC_DRBM is 2.07% lower in RMSE, 110.73% higher in HR and 49.30% higher in MAP than that of RBMEDA. The similar conclusions can be also derived for the other datasets. The main reason lies in that DRBM takes full advantage of the available information to construct the dual-RBM user preference model to comprehensively

extract the user preferences for guiding the personalized search. Furthermore, SC_DRBM integrates the dual-RBM user preference model with social group knowledge to achieve precise fitness approximation. The results sufficiently demonstrate the excellent performance of the proposed algorithms in improving the personalized search results.

(4) From the perspective of the computational cost, the time-consuming of the surrogate-assisted IEDA is often not high and entirely acceptable for the personalized search. Compared with the user-based CF, BPRMF, Wide&Deep, ConvMF and ATRank algorithms, the surrogate-assisted EDAs greatly reduce the computational cost, especially for the DNN-based methods, such as Wide&Deep, ConvMF and ATRank. Additionally, the advantages of the computational complexity of the surrogate-assisted IEDAs are more obvious in the cases that the datasets have larger size and dimensions. It shows that some complex models or DNN-based user preference models are even not applicable in the practical dynamic environment of the personalized search. Therefore, the proposed algorithms are more suitable for the personalized search with dynamical changes, and they will achieve better searching results with less time cost.

Table 4
Experimental results among compared IECs.

Algorithm		IEDA	RBMIGA	RBMIEDA	DRBMIEDA	SC_DRBMIEDA
ML-I-s	RMSE	–	0.945	0.943	1.003	1.071*
	HR	0.0128	0.0143*	0.0128	0.0136	0.0157
	AP	0.815	0.885	0.813	0.928*	0.946
Music	RMSE	–	0.734*	0.737	0.736	0.689
	HR	0.1571	0.1429	0.1381	0.1738*	0.1810
	AP	0.855	0.842	0.828	0.902*	0.971
Apps	RMSE	–	1.686	1.693	1.182	1.255*
	HR	0.0806	0.1032	0.0581	0.0645	0.0677*
	AP	0.730	0.654	0.739	0.983*	1.0
Movies	RMSE	–	0.984	0.982	0.916*	0.907
	HR	0.0131	0.0145	0.0122	0.0130	0.0132*
	AP	0.787	0.844*	0.796	0.717	1.0

In general, the feasibility and effectiveness of the proposed dual-RBM user preference model and surrogate model in SC_DRBM are well validated and demonstrated, and SC_DRBM balances the prediction accuracy and search efficiency in the personalized search. SC_DRBM constructs the dual-RBM preference model to sufficiently extract the user preference features by using available information, and integrates social group knowledge to further improve the performance of the personalized search. Therefore, SC_DRBM effectively deals with the personalized search task with appropriate computational cost, and is more suitable for improving the search efficiency in an interactive environment.

4.4. Experiment 3: Performance of the SC_DRBM enhanced IEDA

To illustrate the performance of the SC_DRBM enhanced IEDA in solving the personalized search, the proposed SC_DRBMIEDA algorithm is compared with four IECs, i.e., traditional IEDA, RBM assisted IGA (RBMIGA), RBMEDA [13] and DRBM assisted IEDA (DRBMIEDA). The traditional IEDA is chosen as the baseline algorithm. RBMIGA incorporates the RBM-based surrogate into the IGA framework. DRBMIEDA is a dual-RBM model assisted IEDA without social knowledge.

In these experiments, a user is randomly selected from the dataset to perform an interactive personalized searching to demonstrate the performance of the algorithms. Considering the evaluating time, the items related to the user are sorted according to their timestamps in the dataset. The first 50% items are regarded as the historical interactive items. The remaining items in the dataset are the feasible items in the test dataset. RMSE, HR and AP are used as evaluation metrics. Each algorithm evolves 10 generations, and gives 10 predictions and recommendations for the user. All of the algorithms are conducted 20 times, and the average experimental results are calculated and shown in Table 4.

In Table 4, the best values are marked in bold. The results marked with “*” represent that the corresponding algorithm is significantly different from the best results with confidence level 0.95 under the Mann–Whitney U test. Additionally, IEDA does not have the RMSE values since no surrogate model is used to predict items' scores. From Table 4, the following observations can be obtained:

(1) SC_DRBMIEDA outperforms almost all the other IECs on various datasets. For example, SC_DRBMIEDA gets 6.51% lower in RMSE than RBMEDA on Music while it improves 31.06% and 17.27% in HR and AP respectively. The experimental results illustrate that our algorithm has better recommendation results and higher searching reliability. Specially, SC_DRBMIEDA has obtained the highest AP in all the other compared algorithms on all datasets. It means that the proposed algorithm ranks the recommended items based on the user preferences and takes more favorite items with higher ranking in the *TopN* recommendation

list. Therefore, SC_DRBMIEDA achieves higher success results and better user satisfaction for the personalized search.

(2) DRBMIEDA without social knowledge has also reached better results in the personalized search. It further demonstrates the validity of the dual-RBM user preference model, the RBM-based probability model and RBM-based surrogate for the personalized search. By integrating social group knowledge into DRBMIEDA, SC_DRBMIEDA effectively improves the prediction accuracy of the searched items, and efficiently assists the user to find satisfactory solutions with less interactive evaluations for alleviates the user's fatigue.

(3) Although RBMEDA has the smallest RMSE in ML-I-s, its HR and AP are both the smallest values among all the algorithms. It indicates that RBMEDA gets higher prediction accuracy while it has lower search reliability and user satisfaction, which does not meet the practical needs and actual requirements of the personalized search. Moreover, it shows that it is difficult to improve the comprehensive performance of the personalized search algorithm when building the surrogate model in IECs only pursuing higher prediction accuracy.

Furthermore, the changes of the RMSE, HR and AP values along with the increasing iterations in the personalized searching on the ML-I-s and Music datasets are shown in Figs. 6 and 7, respectively.

According to Figs. 6 and 7, it can be concluded that:

(1) The RMSE values of all the surrogate assisted IECs decrease along with evolutionary iterations in the personalized searching, indicating that the prediction accuracy of these algorithms is increasing with more interactions. The HR and AP values of those algorithms increase gradually, which illustrates that both search effectiveness and user satisfaction are gradually improved in the searching process. Although the values of the evaluation metrics of each algorithm have fluctuations in the iteration searching process, the overall trend of the performance of the algorithms is getting better and better along with the personalized searching.

(2) SC_DRBMIEDA is significantly superior to other compared algorithms in the personalized search. In specific, it nearly obtains the best results after about 5 rounds interactive iterations. It means that the user has gotten the satisfied items with less evaluations. Therefore, our algorithm is very suitable for the personalized search of an interactive evaluation environment.

In summary, compared with the other IECs, the proposed algorithm has a good balance between effectiveness and efficiency in the personalized search. It achieves better searching results and higher user satisfaction while alleviating users' evaluation burden and psychological fatigue. Such a work provides another way for the personalized search by applying ECs and also extends the applications of ECs.

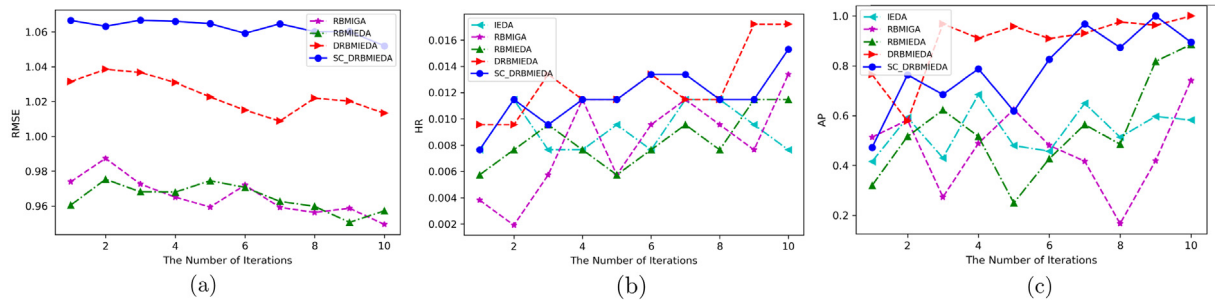


Fig. 6. Metric variations vs. iterations on ML-1-s.

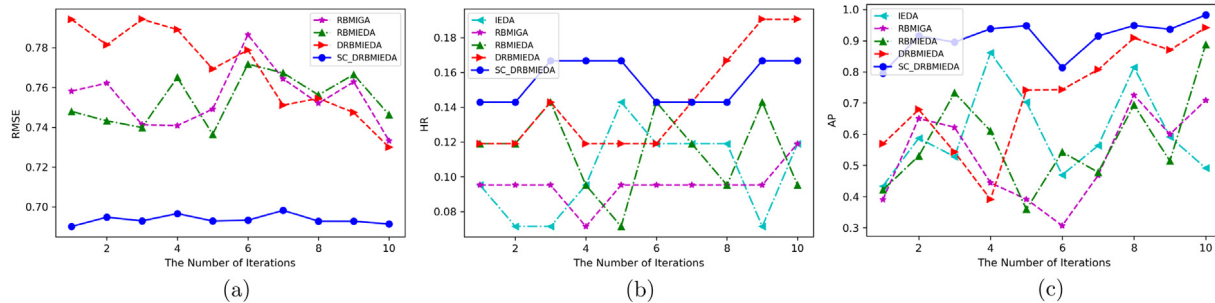


Fig. 7. Metric variations vs. iterations on music.

5. Conclusions

Personalized search is an optimization problem from the viewpoint of finding users' satisfied items, and few ECs have been developed to solve such problems. Motivated by the researches of the user interest model in recommender system and the surrogate model in IECs, we present an enhanced RBM assisted IEDA by integrating social knowledge with a dual-RBM user preference model, in which the energy functions of RBMs are designed as a user's preference surrogate to approximate the individual fitness and the probability model of the visible units in the positive RBM is carried out to drive the EDA process. The methods of constructing a dual-RBM user preference model and designing the probability and surrogate models are presented in detail. The performance of the proposed algorithm is empirically demonstrated by comparing with popular recommendation algorithms and IECs on the Movielens and Amazon datasets.

A text-based encoding will be further studied in the future, and the social communication network should be involved in the construction of surrogate model to further improve the prediction accuracy and searching results in personalized search.

CRediT authorship contribution statement

Lin Bao: Methodology, Software, Validation, Writing-original draft preparation. **Xiaoyan Sun:** Conceptualization, Funding acquisition, Resources, Writing-reviewing and editing. **Yang Chen:** Investigation. **Dunwei Gong:** Formal analysis, Project administration, Supervision. **Yongwei Zhang:** Data curation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments

This work was jointly supported by the National Natural Science Foundation of China under grants No. 61876184 and No. 61473298. We also thank the anonymous reviewers for their valuable suggestions for helping improve the quality of this manuscript.

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