Decision Trees

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Outline

- Introduction
- Criteria to Choose Expanding Attributes
- Decision Tree Learning

What is Decision Tree?

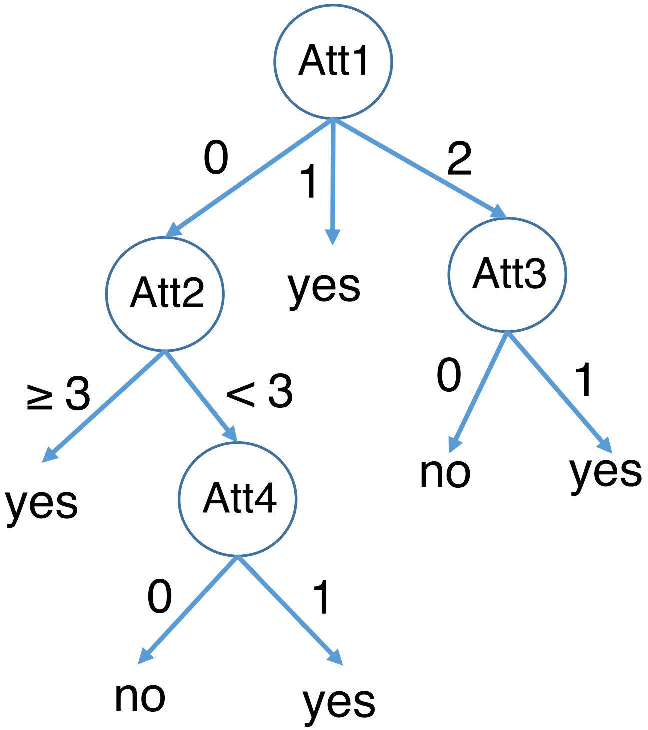
- Given a dataset below, we want to predict the outcome based on the attributes

Attributes Outcome

	Type	Length	Director	Famous actors	Liked?
1	Comedy	Short	Adamson	No	Yes
2	Animated	Short	Lasseter	No	No
3	Drama	Medium	Adamson	No	Yes
4	Animated	Long	Lasseter	Yes	No
5	Comedy	Long	Lasseter	Yes	No
6	Drama	Medium	Singer	Yes	Yes
7	Animated	Short	Singer	No	Yes
8	Comedy	Long	Adamson	Yes	Yes
9	Drama	Medium	Lasseter	No	Yes

- Different from the previous classifiers, decision tree classifies data into different categories by building a tree of attributes

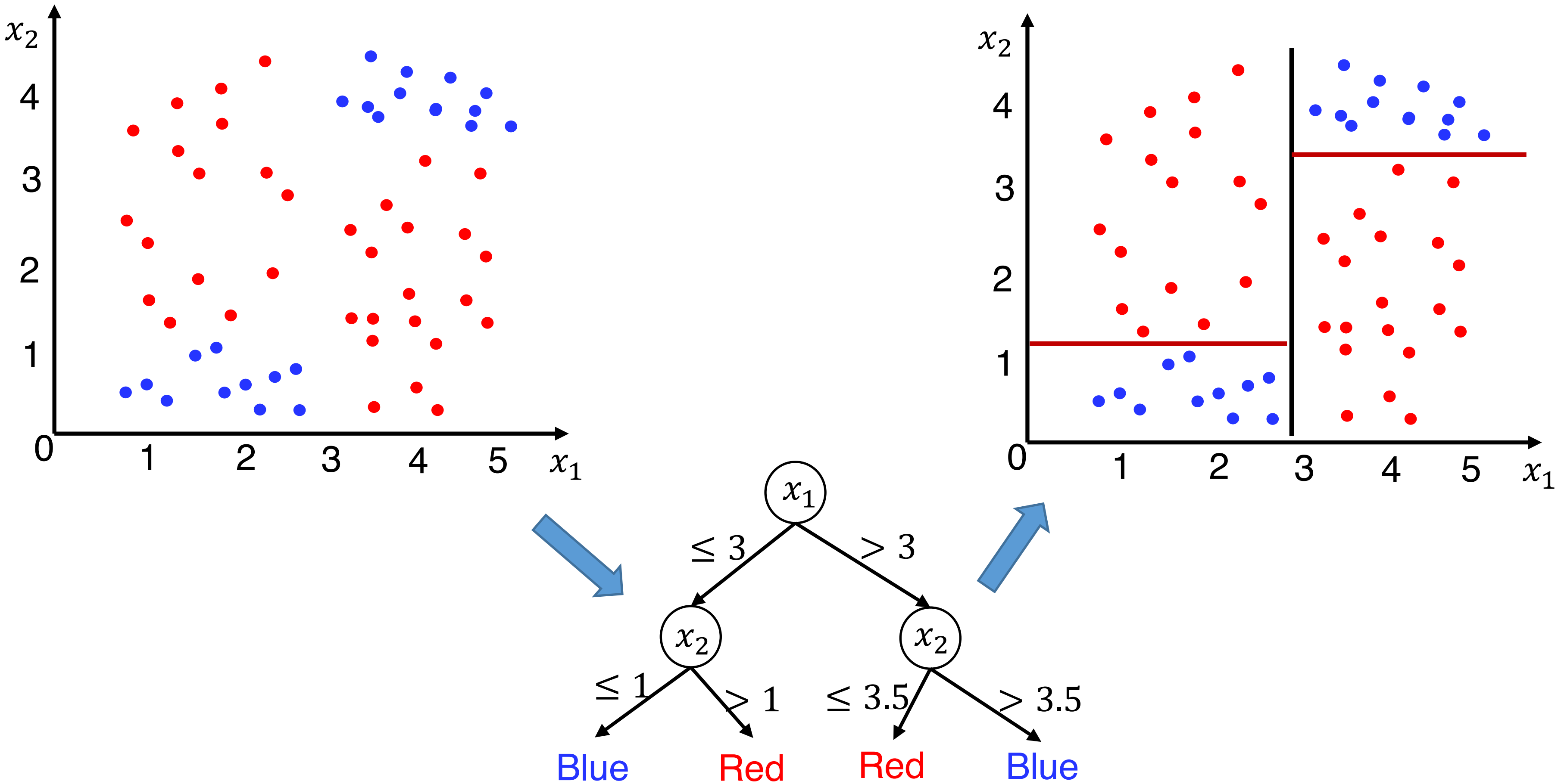
- Structure of a decision tree (DT)
  - Internal nodes correspond to attributes
  - Leaf nodes correspond to the outcome
  - Edges correspond to the attribute values



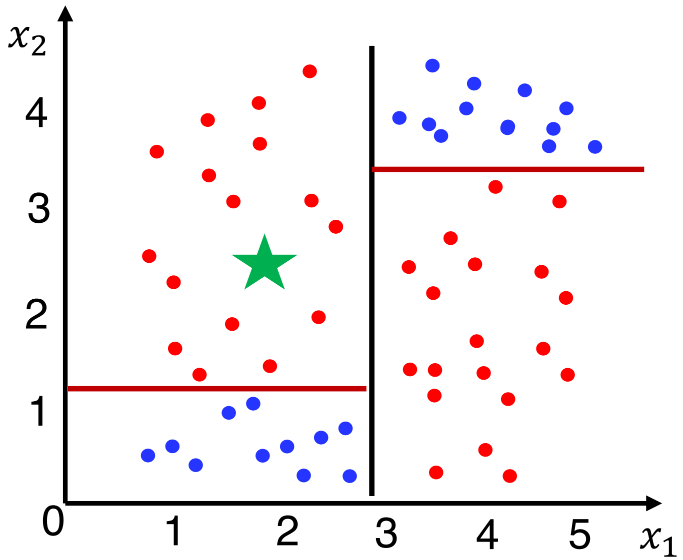
- Learning a decision tree
  - Which attribute to use at each node?
  - How to partition the attribute values?
  - When to stop expanding the tree?

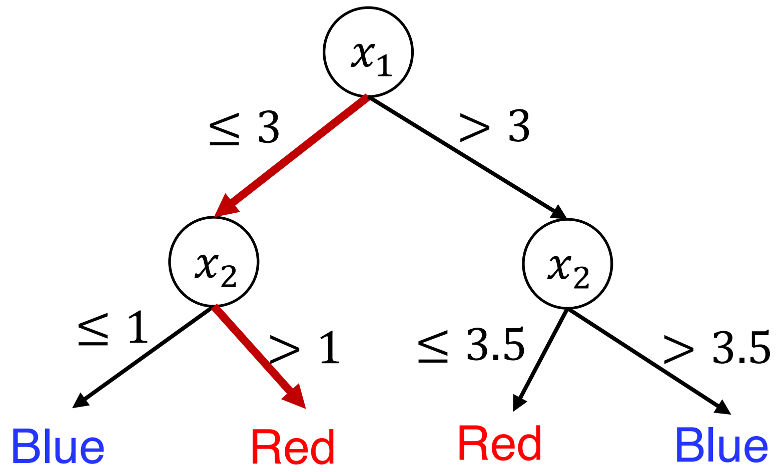
Decision Tree Learning: Example 1

- Given a dataset with two attributes x_1 and x_2, as shown below, we want to build a decision tree to classify the instances



- From the example, to build the tree, we need to decide
  - which attribute to use at each step
  - how to partition the attribute value
- For a new incoming data, a path from the root to the leaves can be found based on its attribute values, from which its category can be read out

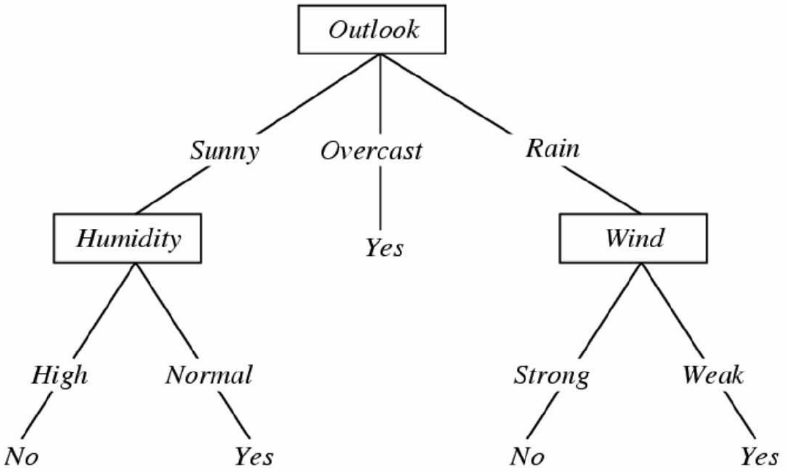




Decision Tree Learning: Example 2

- Given a dataset below, build a decision tree to decide whether to play or not based on the four attribute values

day	outlook	temperature	humidity	wind	play
1	sunny	hot	high	weak	no
2	sunny	hot	high	strong	no
3	overcast	hot	high	weak	yes
4	rain	mild	high	weak	yes
5	rain	cool	normal	weak	yes
6	rain	cool	normal	strong	no
7	overcast	cool	normal	strong	yes
8	sunny	mild	high	weak	no
9	sunny	cool	normal	weak	yes
10	rain	mild	normal	weak	yes
11	sunny	mild	normal	strong	yes
12	overcast	mild	high	strong	yes
13	overcast	hot	normal	weak	yes
14	rain	mild	high	strong	no



- *Questions*
  a) Why the outlook is chosen first, and then humidity and wind?
  b) Why not further expanding after the humidity and wind?
  c) Why the 'temperature' attribute does not appear in the tree?

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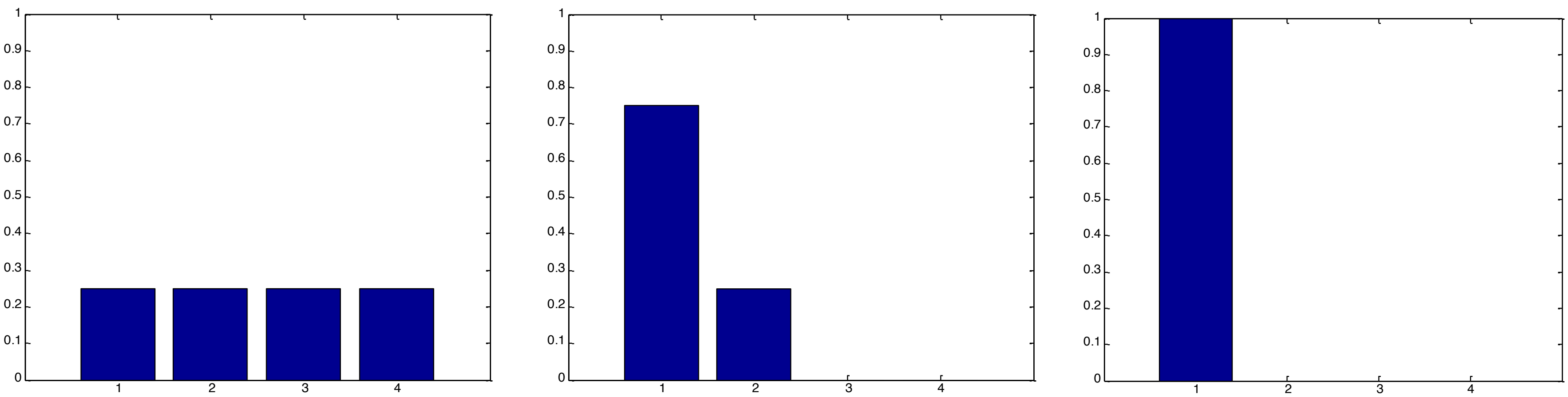
Entropy

- Among all attributes, choose the one that *contains most information* to expand
- How to measure the amount of information that an attribute contains?

we need to measure *the amount of uncertainties* of a random variable first



(a) (b) (c)

- The amount of uncertainties of a random variable in mathematics is measured by *entropy*

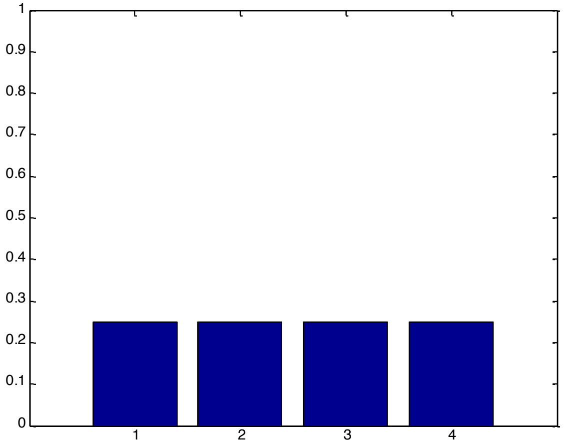
- Definition: Given a random variable Z that follows a distribution $p(z)$, its entropy is defined as

$$H(Z) = -\sum_{z \in \mathcal{C}} p(z) \log_2 p(z)$$

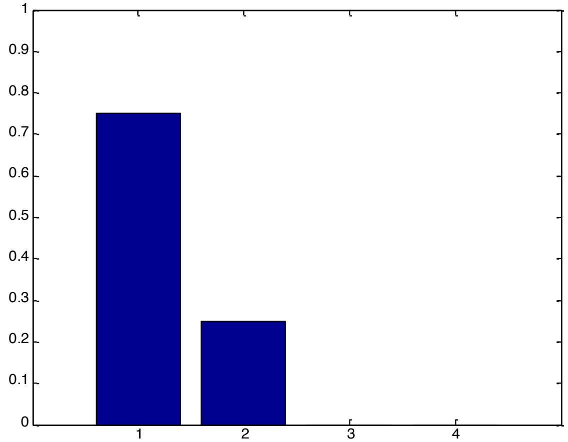
  - $\mathcal{C}$ is the set of possible values of random variable Z

Example

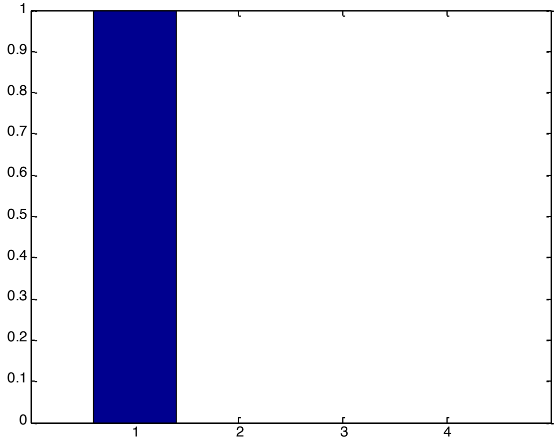
Which distribution below has the largest entropy?



(a)



(b)



(c)

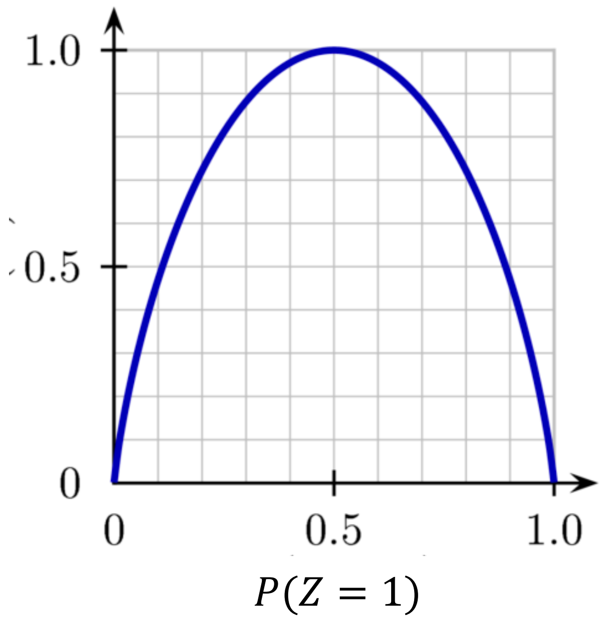
a) $H(Z) = -4 \times 0.25 \log_2 0.25 = 2$ bits

b) $H(Z) = -0.75 \log_2 0.75 - 0.25 \log_2 0.25 \approx 0.8133$ bits

c) $H(Z) = -1 \log_2 1 = 0$ bits

Distribution a) has the largest entropy, while c) has the smallest

- The entropy of a Bernoulli random variable as a function of the probability $P(Z = 1)$



- The entropy is consistent with our intuition, that is,

$$\boxed{\text{The more flat the distribution is, the larger the uncertainty will be}}$$

Conditional Entropy

- Conditional entropy $H(Z|Y)$: the entropy of random variable Z after knowing the values of random variable Y

$$H(Z|Y=y) = -\sum_{z \in \mathcal{C}} p(z|y) \log p(z|y)$$

$$H(Z|Y) = \sum_{y \in \mathcal{T}} P(Y=y) H(Z|Y=y)$$

Y	Z
t	t
t	t
t	t
t	t
f	t
f	f

➢ Example

$p(Z=t|Y=t)=1$ and $p(Z=f|Y=t)=0$ $\Longrightarrow$ $H(Z|Y=t)=0$

$p(Z=t|Y=f)=0.5$ and $p(Z=f|Y=f)=0.5$ $\Longrightarrow$ $H(Z|Y=f)=1$

$p(Y=t)=4/6$ and $p(Y=f)=2/6$

$$\Longrightarrow \quad H(Z|Y) = \frac{4}{6}\times 0 + \frac{2}{6}\times 1 = \frac{2}{6}$$

- The conditional entropy $H(Z|Y)$ is different from the entropy $H(Z)$

For the given example, it can be shown that

$$H(Z) = -p(z=t)\log p(z=t) - p(z=f)\log p(z=f)$$

$$= -\frac{5}{6}\log_2\frac{5}{6} - \frac{1}{6}\log_2\frac{1}{6}$$

$$= -\frac{5}{6}\log_2\frac{5}{6} - \frac{1}{6}\log_2\frac{1}{6}$$

$$\approx 0.65$$

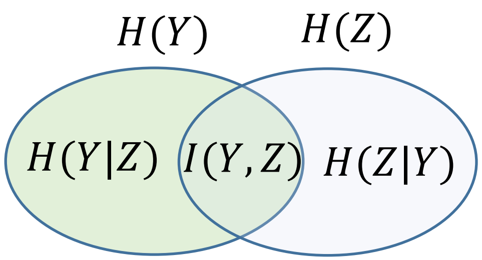
Obviously, it is larger than the conditional entropy $H(Z|Y) \approx 0.33$

Actually, the inequality $H(Z) \geq H(Z|Y)$ always holds

Information Gain

- The information gain of a random variable Y is the *amount of decreased entropy* of Z after knowing its values

$$IG(Y) = H(Z) - H(Z|Y)$$



$$IG(Y) = I(Y,Z)$$

- As for the example given above, the information gain of random variable Y is

$$IG(Y) = 0.65 - 0.33 = 0.32$$

- The information gain of Y means *the amount of uncertainties that can be reduced on average* if its value is known

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Choosing the Root Node

- The *entropy* of the outcome variable 'liked'

$$P(Like = yes) = 2/3 \text{ and } P(Like = no) = 1/3 \quad \Longrightarrow \quad H(Like) = 0.91$$

- The *conditional entropy* of the outcome given attributes Type, Length, Director and Actors

$$H(Like|Type) = 0.61$$

$$H(Like|Length) = 0.61$$

$$H(Like|Director) = 0.36$$

$$H(Like|Actor) = 0.85$$

	Type	Length	Director	Famous actors	Liked?
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8	Comedy	Long	Adamson	Yes	Yes
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- The information gain

$$IG(Type) = H(Like) - H(Like|Type) = 0.3$$

$$IG(Length) = H(Like) - H(Like|Length) = 0.3$$

$$IG(Actor) = H(Like) - H(Like|Actor) = 0.06$$

$$IG(Director) = H(Like) - H(Like|Director) = 0.55$$

$\Longrightarrow$ Director should be the root node

	Type	Length	Director	Famous actors	Liked?
1	Comedy	Short	Adamson	No	Yes
2	Animated	Short	Lasseter	No	No
3	Drama	Medium	Adamson	No	Yes
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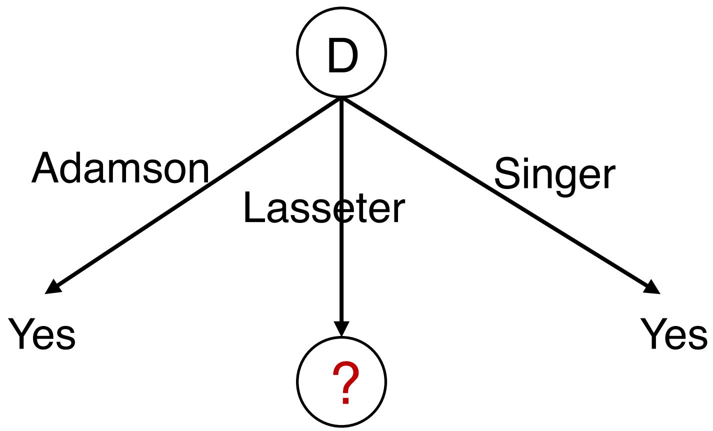
- Build the tree



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8	Comedy	Long	Adamson	Yes	Yes
9	Drama	Medium	Lasseter	No	Yes

- Since all outcomes from the branches of Adamson and Singer is Yes, we don't need to further expand the two branches
- The problem is how to choose the attribute for the branch of Lasseter

Continue to Expand

- After choosing the director of Lasseter, the remaining data is

	Type	Length	Director	Famous actors	Liked?
2	Animated	Short	Lasseter	No	No
4	Animated	Long	Lasseter	Yes	No
5	Comedy	Long	Lasseter	Yes	No
9	Drama	Medium	Lasseter	No	Yes

- Re-computing the entropy and conditional entropy gives

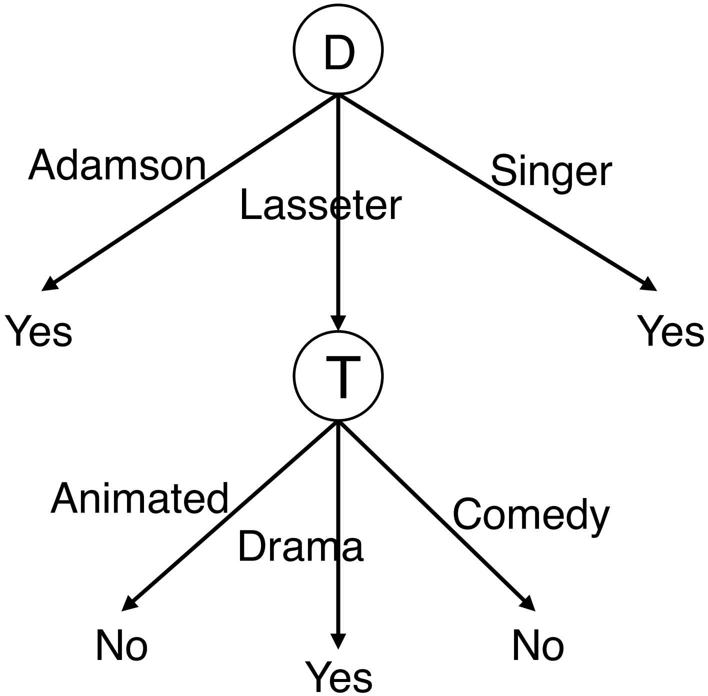
$$H(Like) = 0.81 \quad H(Like|Type) = 0 \quad H(Like|Length) = 0 \quad H(Like|Actor) = 0.5$$

- Thus, the information gains are

$$\text{IG}(Type) = 0.81 \quad IG(Length) = 0.81 \quad IG(Actor) = 0.31$$

Thus, we should choose the attribute of Type or Length to expand

- Build the tree



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This is the final decision tree!!

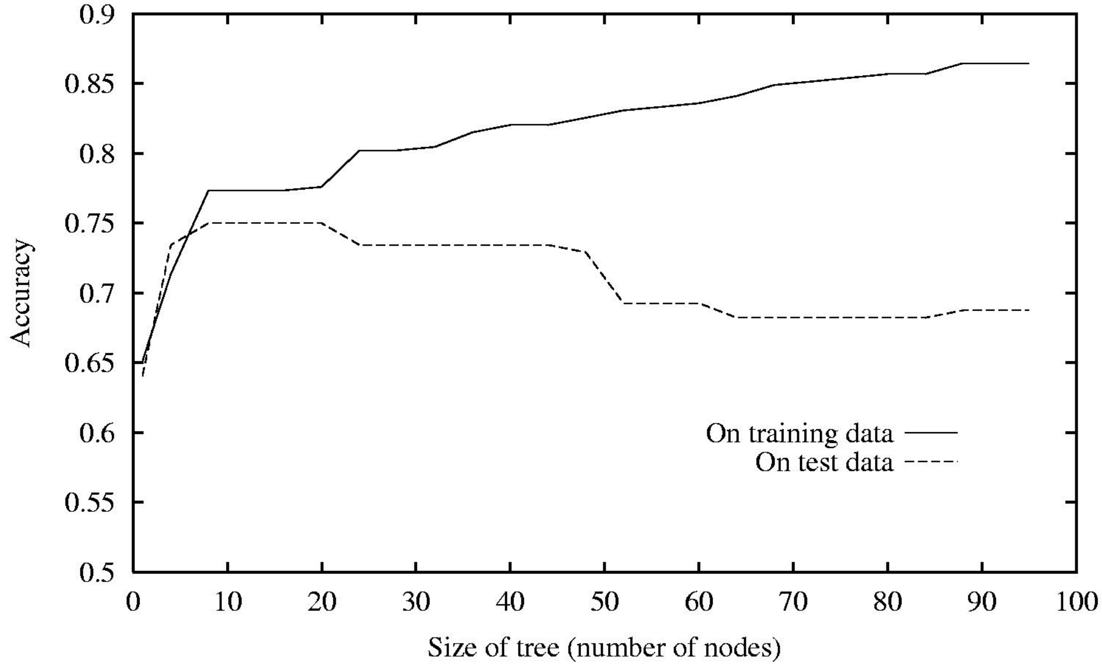
- We stop further expanding the tree because there is only one possible outcome *w.r.t.* every root-leaf path in the training data

Stop Expanding Criteria

- The tree cannot expand forever and should stop at some point. There are some stopping criteria as elaborated below
  - All the remaining instances have the same label
  - We run out of all the attributes
  - The depth of tree reaches the maximum limit
  - The information gain is smaller than a threshold
  - …….

Overfitting Issue

- If the tree is too large or too complex, it will work very well on the training data, but may perform poorly on the testing data



Tree Pruning

- To control the complexity of decision trees, we can
  - prune the branches as we learn the trees
  - prune the branches after learning the trees

- Basing on a validation dataset,
  - prune the nodes that doesn't hurt the accuracy on the validation set
  - Greedily remove the node that improves the validation accuracy least
  - Stop when the validation set accuracy starts to deteriorate