Note on the functions

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To implement the formulas in the paper, it is necessary to evaluate the integrals. The following are the results of the evaluation.

For screen-detected cancer, we need to evaluate the integrals in $P_i^{LP}(t_i)$ and $P_i^{LP}(t_i)Q_i^{LP}(t_r - t_i)$.

$$P_i^{LP}(t_i) = A \times I[i = j] + B \times I[i > 0, j < i],$$

where

$$\begin{aligned} A =& \frac{w\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_3 t_i} \Big[\frac{1}{\lambda_3}(e^{\lambda_3 t_i} - e^{\lambda_3 t_{i-1}}) \\ & - e^{(\lambda_1+\lambda_2)t_{i-1}} \frac{1}{\lambda_3 - \lambda_1 - \lambda_2}(e^{(\lambda_3-\lambda_1-\lambda_2)t_i} - e^{(\lambda_3-\lambda_1-\lambda_2)t_{i-1}}) \Big] \end{aligned}$$

and

$$\begin{aligned} B =& \frac{w\lambda_2}{\lambda_1 + \lambda_2} e^{-\lambda_3 t_i} \frac{1}{\lambda_3 - \lambda_1 - \lambda_2} \{ e^{(\lambda_3-\lambda_1-\lambda_2)t_i} - e^{(\lambda_3-\lambda_1-\lambda_2)t_{i-1}} \} \\ & \times \sum_{j=0}^{i-1} (1 - \beta_E)^{i-j} [e^{(\lambda_1+\lambda_2)t_j} - e^{(\lambda_1+\lambda_2)t_{j-1}}] \end{aligned}$$

$P_i^{LP}(t_i)$ and $P_i^{LP}(t_i)Q_i^{LP}(t_r - t_i)$ are similar to each other:

$$P_i^{LP}(t_i)Q_i^{LP}(t_r - t_i) = e^{-\lambda_3(t_r - t_i)} P_i^{LP}(t_i)$$

For the interval cancer:

$$I_r^L = \int_{t_{r-1}}^{t_r} I_r^L(t)dt = \sum_{i=0}^{r-1} (1 - \beta_L)^{r-i} C + I[j = r] \times D_1 + I[j < r] \times D_2,$$

where

$$C = e^{\lambda_3 t_i} P_i^{LP}(t_i)(e^{-\lambda_3 t_{r-1}} - e^{-\lambda_3 t_r})$$

and

$$\begin{aligned} D_1 =& \frac{w\lambda_2}{\lambda_1 + \lambda_2} \Big\{ (t_r - t_{r-1}) + \frac{1}{\lambda_3}(e^{-\lambda_3(t_r - t_{r-1})} - 1) \\ & + \frac{\lambda_3}{\lambda_3 - \lambda_2 - \lambda_1} \{ \frac{1}{\lambda_1 + \lambda_2}(e^{-(\lambda_1+\lambda_2)(t_r - t_{r-1})} - 1) - \frac{1}{\lambda_3}(e^{-\lambda_3(t_r - t_{r-1})} - 1) \} \Big\} \end{aligned}$$

and

$$
\begin{aligned}
D_2 = &\sum_{j=0}^{r-1}(1-\beta_E)^{r-j}\frac{w\lambda_2}{\lambda_1+\lambda_2}(e^{(\lambda_1+\lambda_2)t_j}-e^{(\lambda_1+\lambda_2)t_{j-1}})\frac{\lambda_3}{\lambda_3-\lambda_2-\lambda_1}\\
&\times[\frac{1}{\lambda_2+\lambda_1}(e^{-(\lambda_1+\lambda_2)t_{r-1}}-e^{-(\lambda_1+\lambda_2)t_r})+\frac{1}{\lambda_3}(e^{-\lambda_3 t_r}-e^{-\lambda_3 t_{r-1}})e^{(\lambda_3-\lambda_2-\lambda_1)t_{r-1}}]
\end{aligned}
$$