specific algorithms

W2. Discrete Classification I - Logistic Regression

Guang Cheng

University of California, Los Angeles

guangcheng@ucla.edu

Week 2

Classification

"averaged 0-1 loss" 是指对分类模型进行评估时使用的一种损失函数。在机器学习和统计学中，0-1损失函数是一种常用的损失函数，用于评估分类模型的性能。

0-1损失函数定义如下：如果预测正确，则损失为0，否则为1。

- A typical dataset in classification $\mathcal{D} = \{(\boldsymbol{x}_i, y_i)\}_{i=1}^n$.
  - $\boldsymbol{x}_i$: the covariate vector of i-th instance
  - $y_i \in \{-1, 1\}$: binary label of i-th instance

- **Question**: Can we directly minimize the averaged 0-1 loss?

$$\text{Training Error} : \frac{1}{n} \sum_{i=1}^{n} I(f(\boldsymbol{x}_i) \neq y_i)$$

指示函数. 因为不好求最小值.

- **Answer**: No, the 0-1 loss function is non-convex and discontinuous, so (sub)gradient methods cannot be applied.

Classification - surrogate loss

替代损失函数试图近似0-1损失函数的行为，同时还要保持可导和凸性质，以便利用梯度下降法等优化算法。常见的替代损失函数包括Logistic损失、交叉熵损失等。

为计算和逼近0-1等量下 loss function的办法

- We can replace the 0-1 loss by other loss functions, say surrogate loss

$$\frac{1}{n}\sum_{i=1}^{n} I(f(\boldsymbol{x}_i) \neq y_i) \Rightarrow \frac{1}{n}\sum_{i=1}^{n} L(f(\boldsymbol{x}_i), y_i) = \frac{1}{n}\sum_{i=1}^{n} \phi(f(\boldsymbol{x}_i)y_i)$$

两种 surrogate loss 的类型：

- Hinge loss: $\phi(x) = \max\{0, 1 - x\}$
- Logistic loss $\phi(x) = \log(1 + \exp(-x))$

两个方法求 loss

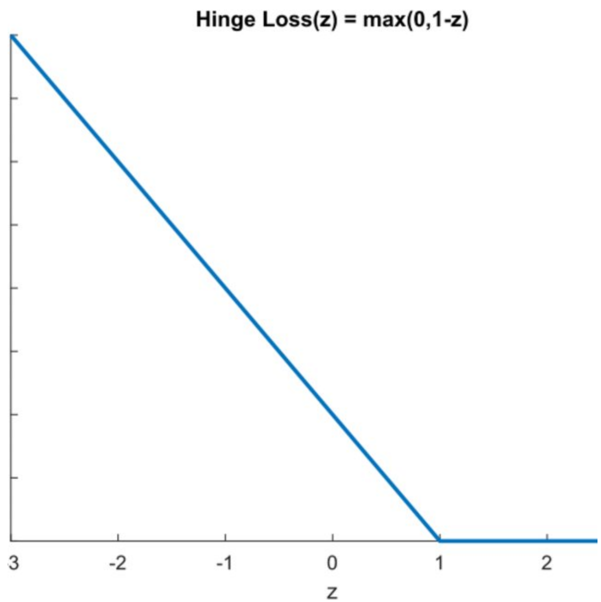
在整个定义域内可导

One surrogate loss - Hinge Loss

模型预测值与实际值之间的差异
$\hat{y}$与y之间的差异

- Definition of Hinge loss:

$$L_{hinge}(f(\boldsymbol{x}_i), y_i) = \begin{cases} 1 - f(\boldsymbol{x}_i)y_i & \text{if } f(\boldsymbol{x}_i)y_i \leq 1 \\ 0, & \text{if } f(\boldsymbol{x}_i)y_i > 1 \end{cases}$$



Why Hinge Loss?

- Let $\eta(\boldsymbol{x}) = \mathbb{P}(Y = 1|\boldsymbol{X} = \boldsymbol{x})$. The expected hinge loss: hinge risk.

$$\begin{aligned} R_{hinge}(f) =& \mathbb{E}_{\boldsymbol{X},Y}\left[L_{hinge}(f(\boldsymbol{X}), Y)\right] \\ =& \mathbb{E}_{X}\Big[\eta(\boldsymbol{X})(1 - f(\boldsymbol{X}))_+ + \big(1 - \eta(\boldsymbol{X})\big)(1 + f(\boldsymbol{X}))_+\Big] \end{aligned}$$

- Suppose that $f(\boldsymbol{X}) \in [-1, 1]$, for any $\boldsymbol{X}$, we have (pls verify in class)

$$\begin{aligned} & \eta(\boldsymbol{X})(1 - f(\boldsymbol{X})) + \big(1 - \eta(\boldsymbol{X})\big)(1 + f(\boldsymbol{X})) \\ =& \eta(\boldsymbol{X}) - 2\eta(\boldsymbol{X})f(\boldsymbol{X}) + 1 + f(\boldsymbol{X}) - \eta(\boldsymbol{X}) \\ =& f(\boldsymbol{X})\big(1 - 2\eta(\boldsymbol{X})\big) + 1. \end{aligned}$$

- The optimal function f^*_{hinge} minimizing $R_{hinge}(f)$ (why?)
  - If $\eta(\boldsymbol{X}) < 1/2$, hinge loss is minimized at $f(\boldsymbol{X}) = -1$
  - If $\eta(\boldsymbol{X}) > 1/2$, hinge loss is minimized at $f(\boldsymbol{X}) = 1$

Why Hinge Loss?

- The optimal classifier (i.e., Bayes classifier) of Binary loss is defined as

$$f^*(\boldsymbol{x}) = \text{sign}(\eta(\boldsymbol{x}) - 1/2) = \begin{cases} 1 & \text{if } \mathbb{P}(Y = 1|\boldsymbol{X} = \boldsymbol{x}) > 1/2 \\ 0 & \text{if } \mathbb{P}(Y = 1|\boldsymbol{X} = \boldsymbol{x}) < 1/2 \end{cases}$$

- **Observation**:
  (i) f^*_{hinge} is exactly the Bayes classifier defined above;
  (ii) The hinge loss is a convex function, which makes it possible to minimize the training error in practice.

Another surrogate loss - Logistic Loss

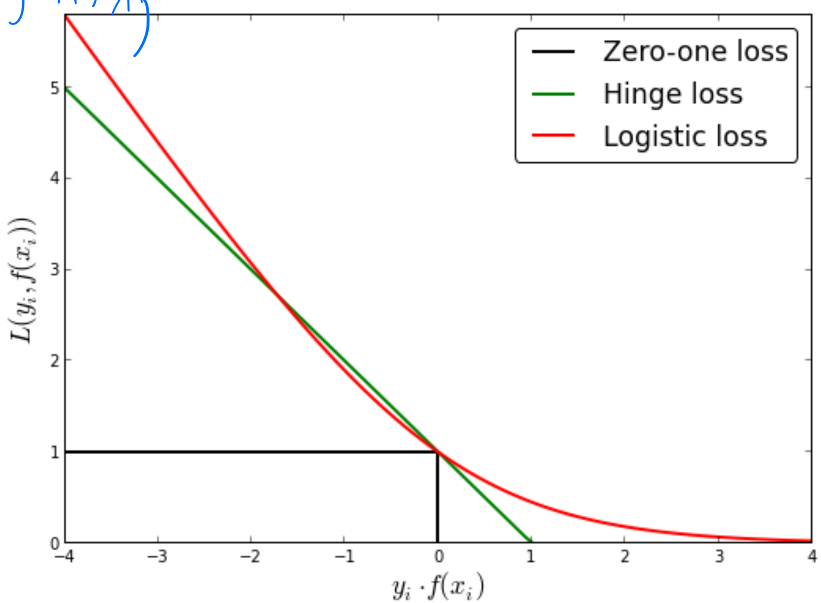
用于logistic回归

- Definition of Logistic loss:

$$L_{log}(f(\boldsymbol{x}_i), y_i) = \log\Big(1 + \exp(-f(\boldsymbol{x}_i)y_i)\Big)$$

Hinge : $\max(0, 1 - f(x_i) \cdot y_i)$ # $1 - f(x_i)y_i > 0$

Logistic : $\log(1 + e^{-f(x_i)y_i})$



Why Logistic Loss?

- The logistic risk:

$$R_{log}(f) = \mathbb{E}_{\boldsymbol{X},Y}\Big[\log\Big(1+\exp(-f(\boldsymbol{X})Y)\Big)\Big]$$
$$= \mathbb{E}_{\boldsymbol{X}}\Big[\eta(\boldsymbol{X})\log\Big(1+\exp(-f(\boldsymbol{X}))\Big) + (1-\eta(\boldsymbol{X}))\log\Big(1+\exp(f(\boldsymbol{X}))\Big)\Big]$$

- Take the derivative with respect to f, pls verify the following in class

$$-\eta(\boldsymbol{X})\frac{\exp(-f(\boldsymbol{X}))}{1+\exp(-f(\boldsymbol{X}))} + (1-\eta(\boldsymbol{X}))\frac{\exp(f(\boldsymbol{X}))}{1+\exp(f(\boldsymbol{X}))}$$
$$= -\eta(\boldsymbol{X})\frac{1}{1+\exp(f(\boldsymbol{X}))} + (1-\eta(\boldsymbol{X}))\frac{\exp(f(\boldsymbol{X}))}{1+\exp(f(\boldsymbol{X}))}$$
$$= \frac{\exp(f(\boldsymbol{X}))}{1+\exp(f(\boldsymbol{X}))} - \eta(\boldsymbol{X}) = 0 \longleftrightarrow f^*_{log}(\boldsymbol{X}) = \log\frac{\eta(\boldsymbol{X})}{1-\eta(\boldsymbol{X})}$$

Connection between binary loss and surrogate losses

- The Bayes classifier $f^*(\boldsymbol{x}) = \text{sign}(\eta(\boldsymbol{x}) - 1/2)$
- The optimal classifier of Hinge risk $f^*_{hinge}(\boldsymbol{x}) = \text{sign}(\eta(\boldsymbol{x}) - 1/2)$
- The optimal classifier of Logistic risk $f^*_{log}(\boldsymbol{x}) = \log \frac{\eta(\boldsymbol{X})}{1-\eta(\boldsymbol{X})}$
- **Question**: what is the connection between these optimal classifiers?
- **Answer**: They are consistent in sign in the sense that signs of $f^*, f^*_{hinge}, f^*_{log}$ are always the same, e.g., always positive as long as $\eta(\boldsymbol{x}) > 1/2$.

Logistic Regression – model details

- To estimate $f^*_{log}(\boldsymbol{x})$, we need to impose an assumption on the form of:

$$\eta(\boldsymbol{X}) = \mathbb{P}\Big(Y = 1 \Big| \boldsymbol{X}\Big)$$

- In logistic regression, it is often assumed that

$$\eta(\boldsymbol{x}) = \frac{\exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}{1 + \exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}, \tag{1}$$

where
- $\boldsymbol{x} = (x_1, \ldots, x_p)^T$ is a p-dimensional predictor
- β_0 and $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_p)$ are unknown parameters to be estimated
- $\boldsymbol{\beta}^T \boldsymbol{x} = \sum_{l=1}^{p} \beta_i x_i$

Rational behind logistic loss: log odds ratio

- By reformulating (1), we have obtained

$$\exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x}) = \frac{\mathbb{P}\Big(Y = 1 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)}{1 - \mathbb{P}\Big(Y = 1 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)} = \frac{\mathbb{P}\Big(Y = 1 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)}{\mathbb{P}\Big(Y = 0 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)},$$

  where the last term above is the ratio between the conditional probability of $Y = 1$ and that of $Y = 0$ on $\boldsymbol{X} = \boldsymbol{x}$, i.e., "odds ratio."
- In other words, we can claim that the log-odds is assumed to be linear with respect to $\boldsymbol{\beta}$:

β'

$$\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x} = \log\left(\frac{\mathbb{P}\Big(Y = 1 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)}{\mathbb{P}\Big(Y = 0 \Big| \boldsymbol{X} = \boldsymbol{x}\Big)}\right)$$

  Interpretability: β_i can then be interpreted as the average change in the log-odds ratio given by a one-unit increase in x_i

Maximum likelihood estimation

- Likelihood function $L(\beta_0, \boldsymbol{\beta})$:

The probability to find x and y giving β_0 , β

$$L(\beta_0, \boldsymbol{\beta}) = \prod_{i=1}^{n} \Big(\mathbb{P}\big(Y = 1 \big| \boldsymbol{X} = \boldsymbol{x}\big) \Big)^{y_i} \Big(\mathbb{P}\big(Y = 0 \big| \boldsymbol{X} = \boldsymbol{x}\big) \Big)^{1-y_i}$$

we have to maximize this prob.

- Logarithm of $L(\beta_0, \boldsymbol{\beta})$ (pls verify in class):

$$\log L(\beta_0, \boldsymbol{\beta}) = \sum_{i=1}^{n} \Big[y_i \log \Big(\mathbb{P}\big(Y = 1 \big| \boldsymbol{X} = \boldsymbol{x}\big) \Big) + (1 - y_i) \log \Big(\mathbb{P}\big(Y = 0 \big| \boldsymbol{X} = \boldsymbol{x}\big)$$

$$= \sum_{i=1}^{n} \Big[y_i (\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x}) - \log \Big(1 + \exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x}) \Big) \Big]$$

Gradient descent/ascent in the computation

Estimate β_0 and $\boldsymbol{\beta}$ (Gradient Ascent):

$$\beta_0^{(t+1)} \leftarrow \beta_0^{(t)} + \lambda \sum_{i=1}^{n} \left[y_i - \frac{\exp(\beta_0^{(t)} + \boldsymbol{\beta}^{(t)T}\boldsymbol{x})}{1+\exp(\beta_0^{(t)} + \boldsymbol{\beta}^{(t)}\boldsymbol{x})} \right]$$

$$\boldsymbol{\beta}^{(t+1)} \leftarrow \boldsymbol{\beta}^{(t)} + \lambda \sum_{i=1}^{n} \left[y_i - \frac{\exp(\beta_0^{(t)} + \boldsymbol{\beta}^{(t)T}\boldsymbol{x})}{1+\exp(\beta_0^{(t)} + \boldsymbol{\beta}^{(t)T}\boldsymbol{x})} \right] \boldsymbol{x}_i$$

Now, we are ready to do classification

We have obtained the estimate for β_0 and $\boldsymbol{\beta}$, denoted as $\widehat{\beta}_0$ and $\widehat{\boldsymbol{\beta}}$, based on which we can estimate $P(Y=1|\boldsymbol{X})$ as follows:

$$\widehat{\eta}(\boldsymbol{x}) = \frac{\exp(\widehat{\beta}_0 + \widehat{\boldsymbol{\beta}}^T \boldsymbol{x})}{1 + \exp(\widehat{\beta}_0 + \widehat{\boldsymbol{\beta}}^T \boldsymbol{x})}$$

Then we make predictions by (recall that $\eta(\boldsymbol{x}) = P(Y=1|\boldsymbol{X})$)

$$\widehat{f}(\boldsymbol{x}) = \begin{cases} 1, & \text{if } \widehat{\eta}(\boldsymbol{x}) > 1/2 \\ 0, & \text{if } \widehat{\eta}(\boldsymbol{x}) < 1/2 \end{cases}$$

If $\widehat{\eta}(\boldsymbol{x}) = 1/2$, then just randomly assign a label to it.

Example

Dataset $\mathcal{D} = \{(\boldsymbol{x}_i, y_i)\}_{i=1}^{5000}$, where $\boldsymbol{x}_i = (x_{i1}, x_{i2}, x_{i3}, x_{i4})$.

- Features are generated from uniform distribution $x_{il} \sim Unif(0, 2), l = 1, 2, 3, 4.$
- $\beta_0 = 0.5$ and $\boldsymbol{\beta} = (\beta_1, \beta_2, \beta_3, \beta_4)$ with $\beta_i \sim Unif(-1, 1)$
- Model:

$$Y_i \sim Bernoulli\left(\frac{\exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}{1 + \exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}\right),$$

which means

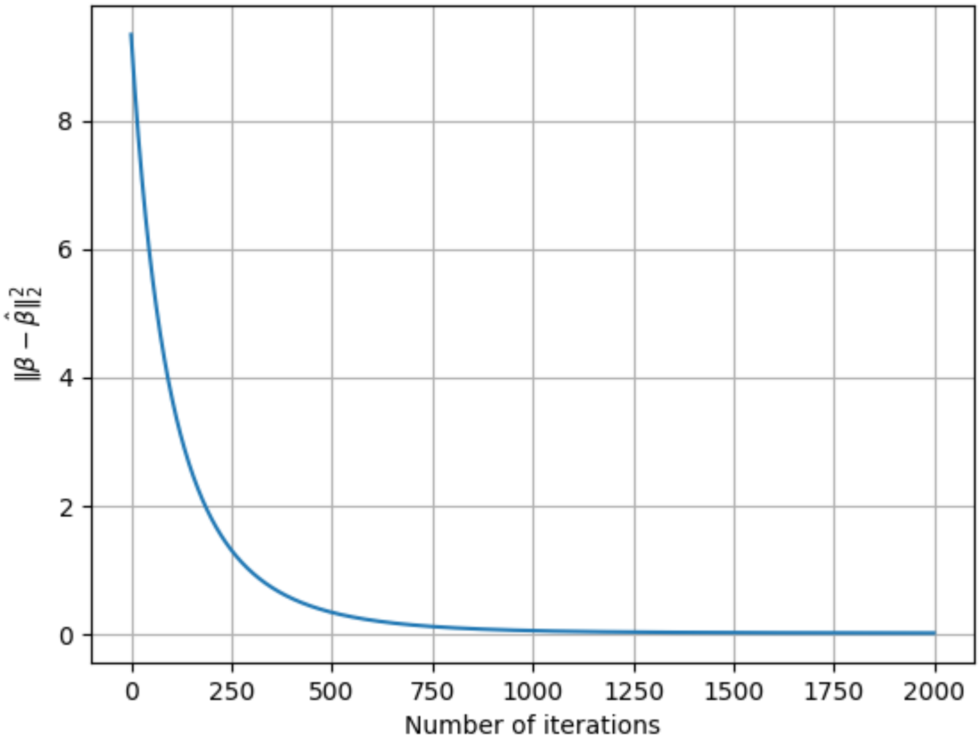
$$P(Y_i = 1 | \boldsymbol{X}) = \frac{\exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}{1 + \exp(\beta_0 + \boldsymbol{\beta}^T \boldsymbol{x})}$$

Python Codes – data generation

```
import numpy as np
np.random.seed(2)
n,p = 5000,4  # Set training datasize and dimension of features
X = np.random.uniform(-1,1,[n,p]) # Generation of features
beta = np.random.uniform(0,2,4) # Generation of parameters
beta_0 = 0.5 # Set the intercept term to 0.5
logOdd = (X * beta).sum(axis=1)+beta_0 # Log-odds
Prob = np.exp(logOdd)/(1+np.exp(logOdd)) # Probability
Y = np.array(Prob - np.random.uniform(0,1,n)>0,dtype=int) # Generate labels
```

```python
Beta_0_hat = 0. # Initialization of intercept term
Beta_hat = np.zeros(p) # Initialization of beta
lamb = 0.1 # Learning rate
Error = [] # Error set
for i in range(2000):# Iterations of gradient ascent
    logOdd_hat = (X * Beta_hat).sum(axis=1)+Beta_0_hat
    Beta_0_hat = Beta_0_hat + lamb * np.mean(Y - np.exp(logOdd_hat)/(1+np.exp(logOdd_hat)))
    Beta_hat = Beta_hat + lamb * ((Y - np.exp(logOdd_hat)/(1+np.exp(logOdd_hat))) * X.T).mean(axis=1)
    Error.append(np.linalg.norm(Beta_hat-beta)**2)
import matplotlib.pyplot as plt
plt.plot(np.arange(0,2000),Error)
plt.xlabel('Number of iterations')
plt.ylabel('$\Vert \\beta - \hat{\\beta}\Vert_2^2$')
plt.grid()
```

Example: gradient ascent for logistic regression

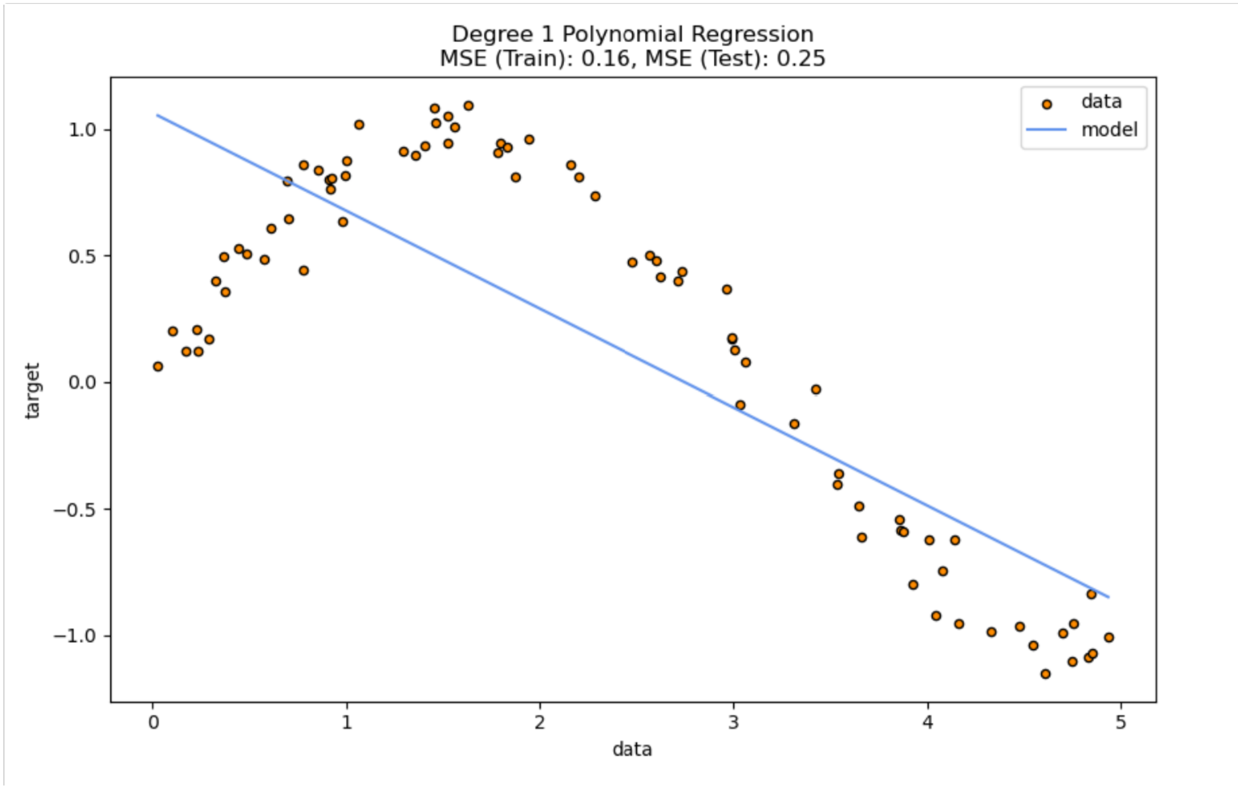


Over/under-fitting problem

- Overfitting occurs when a model learns the training data too well, capturing noise and making it perform poorly on new, unseen data.
- Underfitting, on the other hand, happens when a model is too simple to capture the underlying patterns in the data, resulting in poor performance on both the training and test data.
- Let's use a simple example with polynomial regression and visualize the above with the out-of-sample (OOS) metrics.

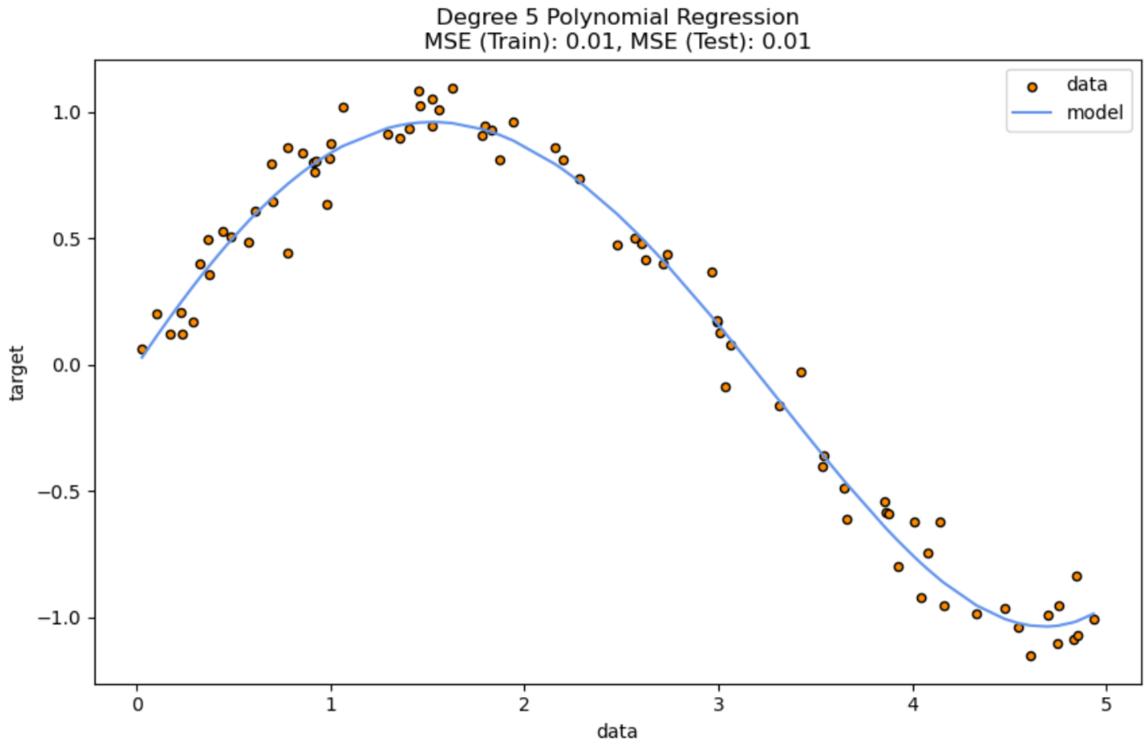
An example - underfitting

Degree 1: Underfitting (Too simple) - The model is not able to capture the underlying pattern in the data.



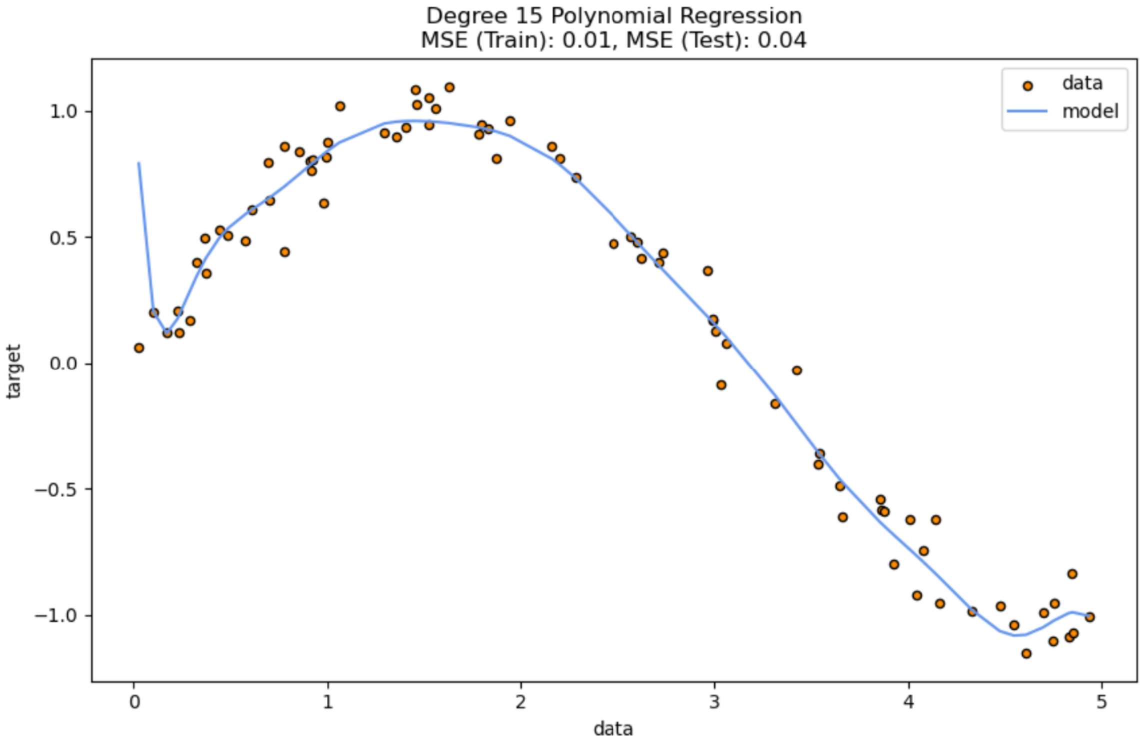
An example - good fit

Degree 5: Good fit - The model captures the underlying pattern well and generalizes to the test data.



An example - overfitting

Degree 15: Overfitting (Too complex) - The model fits the training data too closely, capturing noise and performing poorly on new data.



Confusion matrix – measure the performance of classification

- In Machine Learning, to measure the performance of the classification model, such as logistic regression, we use the confusion matrix.
- A confusion matrix is a matrix that displays the number of accurate and inaccurate classification outcomes for each input instance $\boldsymbol{X}$.

Classification outcomes

To measure the performance of classification, we have the following metrics:

- true positives (TP): occurs when the model accurately predicts a positive data point, i.e., $\hat{y} = y = 1$.
- true negatives (TN): occurs when the model accurately predicts a negative data point, i.e., $\hat{y} = y = -1$.
- false positives (FP): occurs when the model predicts a positive data point incorrectly, i.e., $\hat{y} = 1$ but $y = -1$.
- false negatives (FN): occurs when the model predicts a negative data point incorrectly, i.e., $\hat{y} = -1$ but $y = 1$.

An example : dog recognition

Dog: $Y = 1$ & Not Dog: $Y = -1$ (pls verify this table in class!)

index	actual	predicted	Result
1	Dog	Dog	TP
2	Dog	Not Dog	FN
3	Dog	Dog	TP
4	Not Dog	Not Dog	TN
5	Dog	Dog	TP
6	Not Dog	Dog	FP
7	Dog	Dog	TP
8	Dog	Dog	TP
9	Not Dog	Not Dog	TN
10	Not Dog	Not Dog	TN

An example - counts

Pls take a min to count....

- Actual Dog Counts = ?
- Actual Not Dog Counts = ?
- True Positive Counts = ?
- False Positive Counts = ?
- True Negative Counts = ?
- False Negative Counts = ?

An example - counts

- Actual Dog Counts $= 6$
- Actual Not Dog Counts $= 4$
- True Positive Counts $= 5$
- False Positive Counts $= 1$
- True Negative Counts $= 3$
- False Negative Counts $= 1$

An example - construct the confusion matrix

- Construct the Confusion Matrix

		Actual	
		Dog	**Not Dog**
Predicted	**Dog**	True Positive (TP =5)	False Positive (FP=1)
	Not Dog	False Negative (FN =1)	True Negative (TN=3)

- How to use the Confusion Matrix for assessing a classification model's performance ?

Classification metrics based on confusion matrix: Accuracy

- Accuracy is used to measure the performance of the model. It is the ratio of total correct instances to the total instances.
- $\text{Accuracy} = \frac{\text{TP}+\text{TN}}{\text{TP}+\text{TN}+\text{FP}+\text{FN}}$
- For the above case: Accuracy = ?
- For the above case: Accuracy $= (5+3)/(5+3+1+1) = 8/10 = 0.8$

Classification metrics based on confusion matrix: Precision

- Precision is a measure of how accurate a model's positive predictions are. Basically, it answers the question "What proportion of positive identifications was actually correct?"
- It is defined as the ratio of true positive predictions to the total number of positive predictions made by the model.
- Precision $= \frac{\text{TP}}{\text{TP}+\text{FP}}$
- For the above case: Precision = ?
- For the above case: Precision $= 5/(5+1) = 5/6 = 0.8333$

Classification metrics based on confusion matrix: Recall

- Recall measures the effectiveness of a classification model in identifying all relevant instances from a dataset. Basically, it answers the question "What proportion of actual positives was identified correctly?"
- It is defined as the ratio of the number of true positive (TP) instances to the sum of true positive and false negative (FN) instances.
- Recall $= \frac{\text{TP}}{\text{TP+FN}}$
- For the above case: Recall = ?
- For the above case: Recall $= 5/(5+1) = 5/6 = 0.8333$

Classification metrics based on confusion matrix: F-1 Score

- F1-score is used to evaluate the overall performance of a classification model. It is the harmonic mean of precision and recall
- The F1 score is named as such because it is a combination of two other metrics: precision (P) and recall (R). The "F" in F1 stands for "F-measure" or "F-score," and the "1" indicates that it is computed as the harmonic mean of precision and recall.
- The harmonic mean is often used to calculate the average of the ratios or rates.
- The harmonic mean can be expressed as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.
- For example, harmonic mean of $1, 4, 4$ is

$$\left(\frac{1^{-1} + 4^{-1} + 4^{-1}}{3}\right)^{-1} = 2$$

Classification metrics based on confusion matrix: F-1 Score

- Pls verify this in class: F1-score $= \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$
- For the above case: F1-Score=?
- For the above case: F1-Score = (2* 0.8333* 0.8333)/(0.8333+ 0.8333) = 0.8333

Exercise: Confusion Matrix and Classification Metrics Calculation

Problem statement

- You have a binary classification model used to predict whether an email is spam (positive class) or not spam (negative class). After testing the model on a dataset of 100 emails, you get the following results:
  - 40 emails are correctly identified as spam (True Positives).
  - 10 emails are incorrectly identified as spam (False Positives).
  - 30 emails are correctly identified as not spam (True Negatives).
  - 20 emails are incorrectly identified as not spam (False Negatives).

Exercise: Confusion Matrix and Classification Metrics Calculation

Your task

- Construct a confusion matrix from these results.
- Calculate the following metrics:
  - Accuracy
  - Precision
  - Recall
  - F-1 Score

Exercise: Confusion Matrix and Classification Metrics Calculation

Solution

- confusion matrix:
  - True Positives (TP): 40
  - False Positives (FP): 10
  - True Negatives (TN): 30
  - False Negatives (FN): 20
- Calculate Accuracy: Accuracy $= \frac{40+30}{40+30+10+20} = 0.7$
- Precision $= \frac{40}{40+10} = 0.8$
- Recall $= \frac{40}{40+20} = \frac{2}{3}$
- F-1 Score $= 2 \times \frac{0.8 \times 0.667}{0.8+0.667} \approx 0.727$

Application of logistic regression: Banknote authentication dataset

- The Banknote authentication dataset is used for the task of classifying whether a banknote is authentic or not based on certain features extracted from images.
- Variables meaning
  - β_0: the intercept term;
  - $\beta_1, \beta_2, \beta_3, \beta_4$ are the coefficients associated with each feature.
  - x_1 represents the Variance of the Wavelet Transformed image.
  - x_2 represents the Skewness of the Wavelet Transformed image.
  - x_3 represents the Curtosis of the Wavelet Transformed image.
  - x_4 represents the Entropy of the image.

Application of logistic regression: Banknote authentication dataset

- The logistic regression model makes the following assumption:

$$P(\text{authentic}) = \frac{1}{1 + \exp\{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4\}}$$

- This probability is then used to make a classification decision.
  - If $P(\text{authentic}) \geq 0.5$, the model predicts the banknote as authentic (Class 1)
  - If $P(\text{authentic}) < 0.5$, the model predicts the banknote as not authentic (Class 0)

Application of logistic regression: Results

Pls verify confusion matrix, precision, recall.... by hand after class

```
   Variance  Skewness  Curtosis  Entropy  Class
0   3.62160    8.6661   -2.8073 -0.44699      0
1   4.54590    8.1674   -2.4586 -1.46210      0
2   3.86600   -2.6383    1.9242  0.10645      0
3   3.45660    9.5228   -4.0112 -3.59440      0
4   0.32924   -4.4552    4.5718 -0.98880      0
Accuracy: 0.98
Confusion Matrix:
[[144   4]
 [  2 125]]
Classification Report:
              precision    recall  f1-score   support

           0       0.99      0.97      0.98       148
           1       0.97      0.98      0.98       127

    accuracy                           0.98       275
   macro avg       0.98      0.98      0.98       275
weighted avg       0.98      0.98      0.98       275
```

Statsmodels: Python package

- Statsmodels is primarily focused on statistical modeling and hypothesis testing. It provides tools for estimating and testing various statistical models, including linear regression, logistic regression, time-series analysis, and more.
- Statsmodels includes modules for performing hypothesis tests, constructing confidence intervals, and fitting different types of statistical models with an emphasis on providing detailed statistical information.
- Statsmodels is commonly used in academic research, econometrics, and any scenario where a detailed statistical analysis is required.

Statsmodels: Linear regression example

```python
import statsmodels.api as sm
import numpy as np

# Generate some random data for demonstration
np.random.seed(42)
X = np.random.rand(100, 2)
y = 3 * X[:, 0] + 2 * X[:, 1] + 1 + 0.1 * np.random.randn(100)

# Add a constant term to the independent variable
X = sm.add_constant(X)

# Create a linear model
model = sm.OLS(y, X)
results = model.fit()

# Print detailed statistical summary
print(results.summary())
```

Statsmodels: output

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.991
Model:                            OLS   Adj. R-squared:                  0.991
Method:                 Least Squares   F-statistic:                     5655.
Date:                Thu, 21 Dec 2023   Prob (F-statistic):          3.86e-101
Time:                        10:50:14   Log-Likelihood:                 89.304
No. Observations:                 100   AIC:                            -172.6
Df Residuals:                      97   BIC:                            -164.8
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.9772      0.026     37.651      0.000       0.926       1.029
x1             3.0339      0.033     91.615      0.000       2.968       3.100
x2             2.0355      0.035     57.426      0.000       1.965       2.106
==============================================================================
Omnibus:                        5.986   Durbin-Watson:                   2.104
Prob(Omnibus):                  0.050   Jarque-Bera (JB):                5.624
Skew:                           0.439   Prob(JB):                       0.0601
Kurtosis:                       3.761   Cond. No.                         5.22
==============================================================================
```

Scikit-learn: Python Lib (a collection of Python packages)

- Purpose: Scikit-learn is a versatile machine learning library that provides tools for various machine learning tasks, including classification, regression, clustering, dimensionality reduction, and more.
- Functionality: Scikit-learn focuses on providing a consistent interface for various machine learning algorithms, making it easy to train models, perform feature engineering, and evaluate model performance.
- Use Cases: Scikit-learn is widely used in industry for building and deploying machine learning models in areas such as image recognition, natural language processing, and predictive analytics.