Designing, Visualizing and Understanding Deep Neural Networks

Lecture 4: Optimization

CS 182/282A Spring 2020

John Canny

Last Time: Bias-Variance Tradeoff

Bias at a point x is the expected difference between predictions and the true y.

i.e. $$Bias\left(\hat{f}_D(x)\right) = \mathrm{E}_D\left[\hat{f}_D(x) - f(x)\right] = \bar{f}(x) - f(x)$$

where $\bar{f}(x) = \mathrm{E}_D\left[\hat{f}_D(x)\right]$ is the expected prediction at x

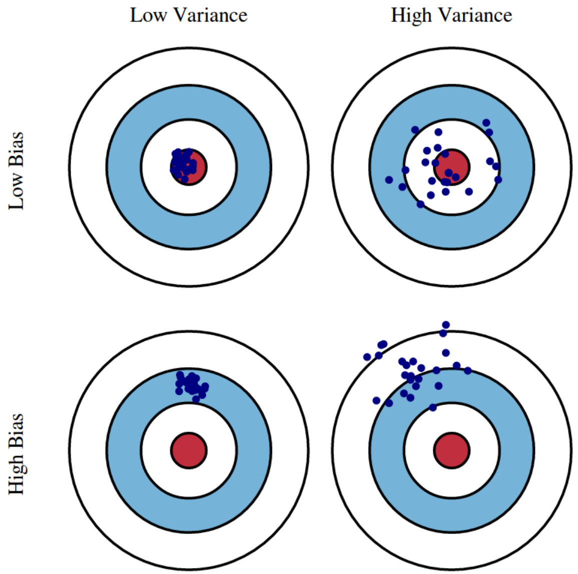
Variance is the variance of the predictions:

$$Variance\left(\hat{f}_D(x)\right) = \mathrm{E}_D\left[\left(\hat{f}_D(x) - \bar{f}(x)\right)^2\right]$$

The linear least-squares error decomposes as:

$$Bias^2 + Variance$$

Tradeoff: Can reduce bias at the expense of higher variance and vice versa.



Last Time: Regularization

L2 regularization is widely used with deep networks (its commonly implemented as "weight decay" in SGD optimizers). For multivariate linear regression, the regularized loss is:

$$L(A) = \sum_{i=1}^{n} (x_i^T A^T - y_i^T)(Ax_i - y_i) + \lambda \sum_{i,j} A_{ij}^2$$

Like other forms of regularization, L2 regularization allows you to trade off bias and variance.

Strong regularization (large λ) ➔ lower variance and higher bias

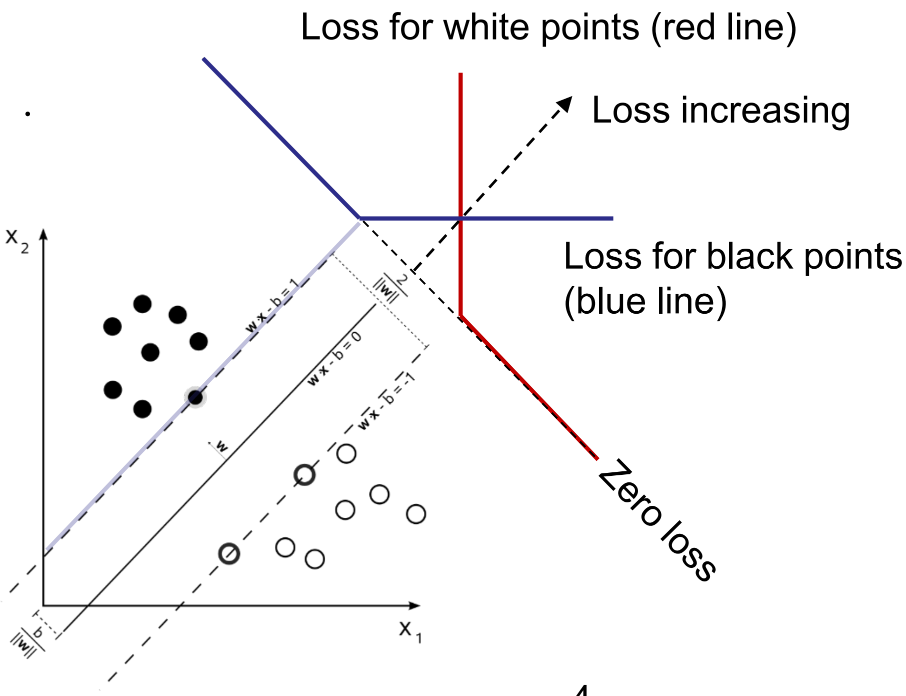
Weak regularization (small λ) ➔ higher variance and lower bias

Last Time: Hinge Loss and SVMs

Hinge loss penalizes data points that lie inside the margin around the decision boundary

$$L = \max(0, 1 - yf(x))$$

A linear classifier that minimizes hinge loss is called a ***Support Vector Machine or SVM*** .



Last Time: Multi-Class Classification

Multiclass Logistic regression estimates the class target probability with a softmax function:

$$f_j(x) = \frac{\exp(s_j)}{\exp(s_1) + \exp(s_2) + \cdots + \exp(s_k)}$$

where $s = Wx$.

And minimizes the cross-entropy loss which is $-y^T \log f(x)$

Reminder – Assignment 1

Assignment 1 is out, due on Feb 17th, 11pm.

- Uses python + ipython. Please check right away that you have a working installation of python 3 and ipython/jupyter, and that you can load and execute the assignment notebooks.
- Python virtualenv is your best bet, but there are tricks to doing it on a Mac.
- The assignment itself closely tracks the lecture material over the next couple of weeks, so you'll get best value by doing it in stages.

Issue Reporting

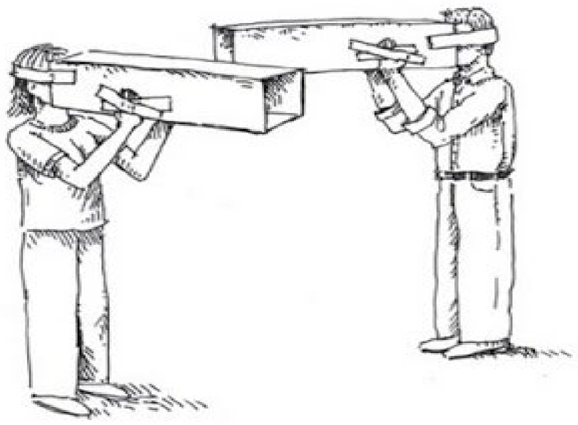
Please use best issue reporting practices for Python or other system issues:

https://bcourses.berkeley.edu/courses/1478831/pages/reporting-an-issue

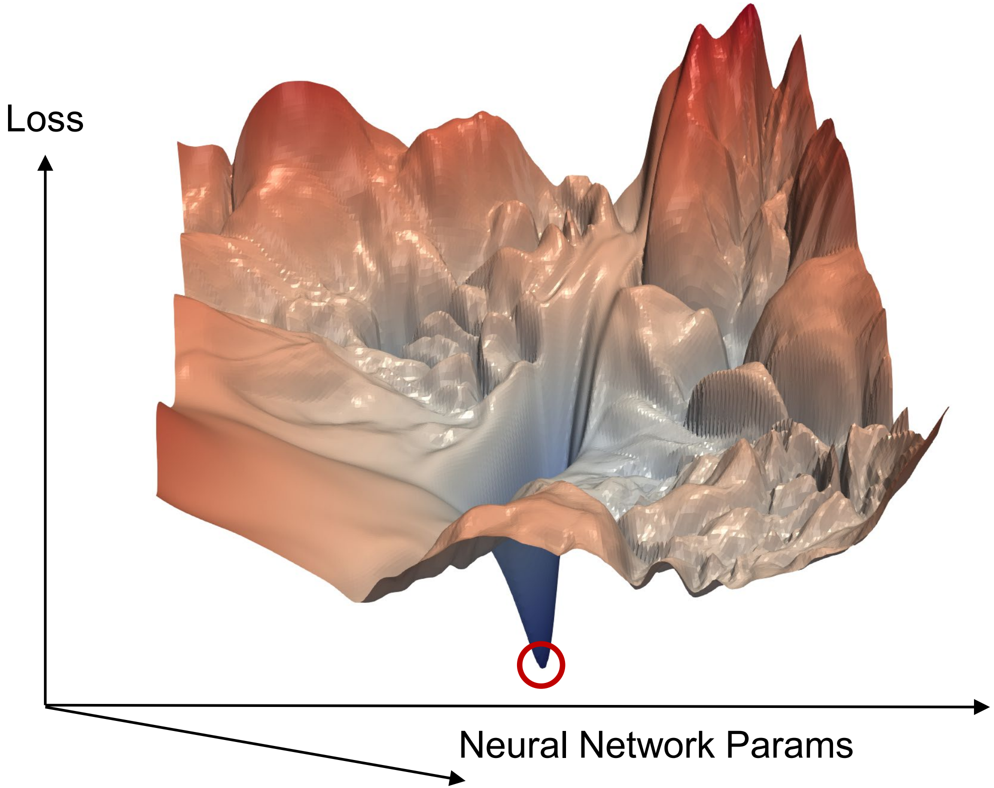
- Give background info: your system, OS version, software version.
- Show what you saw (complete error message, not summary), and what you expected.
- Exact steps to reproduce, not “I followed these directions <<link>>”.
- Structured issue reporting is required in many companies, and its an important skill to have.
- It will also help you to maintain good relationships with system support staff...

Issue Reporting Principles

- Complete issue reports reduce the diagnostic cycle by 2-5x.
- Most simple issues are resolved in a single cycle from good reports.
- Don't rely on "Hashing" i.e. that the error message uniquely specifies the problem – often it doesn't.
- Avoid "Tunnel Vision" – i.e. don't make assumptions about the cause of the error if you cant fix it. Provide *all* the relevant information.
- When *responding* to an issue report: The goal should be to help the poster solve their problem, not to solve it for them.



This Time: Optimization



Visualizing the loss landscape of neural nets
Hao Li, Zheng Xu, Gavin Taylor, Tom Goldstein
arXiv 1712.09913/

Method 1: Search

Take several random steps and measure the altitude (loss). Then go to the lowest one.

- Not totally crazy. This is one kind of "gradient-free" optimization. It makes sense if you can't compute gradients for some reason (e.g. loss is not continuous or differentiable).
- A slightly smarter version takes an average of the random points weighted by altitude (deeper points get higher weight). This in fact approximates the gradient.

See:

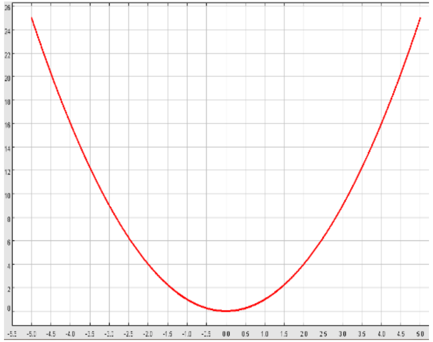
Salimans, Ho, Chen, Sidor and Sutskever "Evolution Strategies as a Scalable Alternative to Reinforcement Learning" arXiv 1703.03864, 2017.

But it's less efficient than gradient descent when gradients are available.

Losses we have seen so far

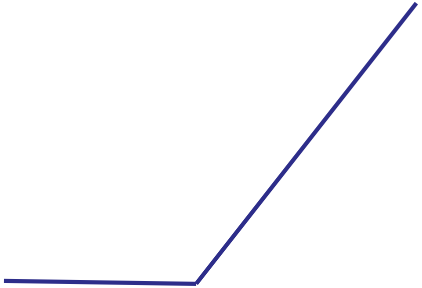
Squared Loss

$L = \left(y_i - f(x_i)\right)^2$



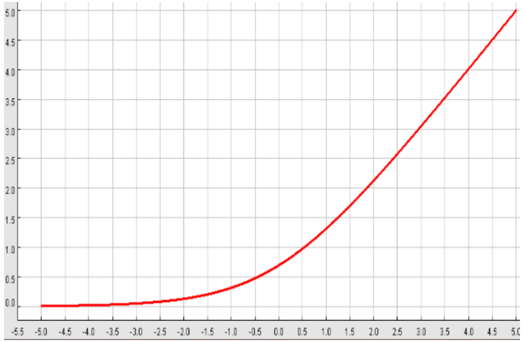
Hinge Loss, $y_i \in \{-1,1\}$

$L = \max(0, 1 - y_i f(x_i))$



Cross-entropy loss on logistic function, $y_i \in \{-1,1\}$

$L = \log(1 + \exp(-y_i f(x_i)))$



All three have "well behaved" derivatives. $f(x) = w^T x$ is a linear function of the weights W, so we can differentiate loss with respect to weights.

Gradients Again

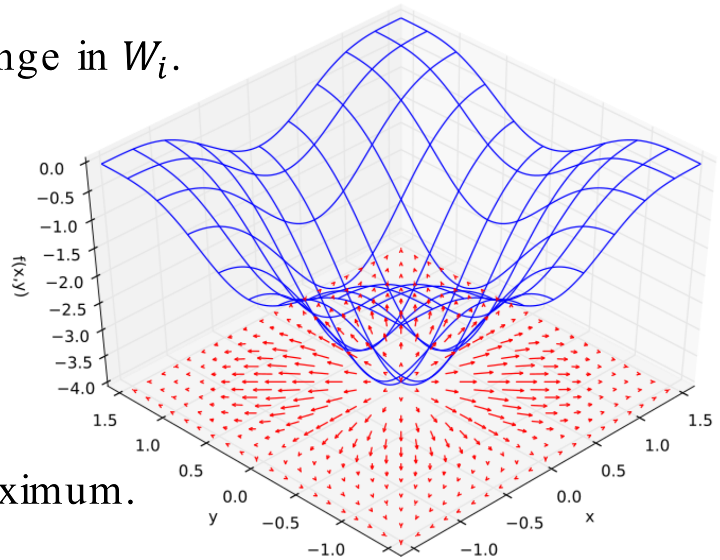
When we write $\nabla_W L(W)$, we mean the vector of partial derivatives wrt all coordinates of W:

$$\nabla_W L(W) = \left[\frac{\partial L}{\partial W_1}, \frac{\partial L}{\partial W_2}, \dots, \frac{\partial L}{\partial W_m}\right]^T$$

Where $\frac{\partial L}{\partial W_i}$ measures how fast the loss changes vs. change in W_i.

In figure: loss surface is blue, gradient vectors are red:

When $\nabla_W L(W) = 0$, it means all the partials are zero.
i.e. the loss is not changing in any direction.



Note: arrows point out from a minimum, in toward a maximum.

Gradient Descent

So to reach a minimum of loss, we should follow the negative gradient.

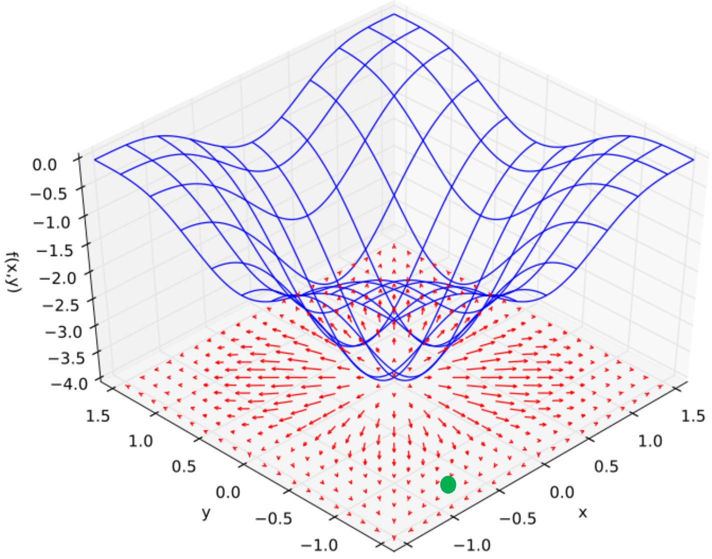
i.e. we should take small steps in direction

$$-\nabla_W L(W)$$

To be more concrete, let W^t denote the weights at step t of gradient descent. Then

$$W^{t+1} = W^t - \alpha \ \nabla_W L(W)$$

Where α is called the ***learning rate*** .



Vector Calculus

The gradient $\nabla_W L(W)$ vanishes at

A. Local maxima of $L(W)$

B. Local minima of $L(W)$

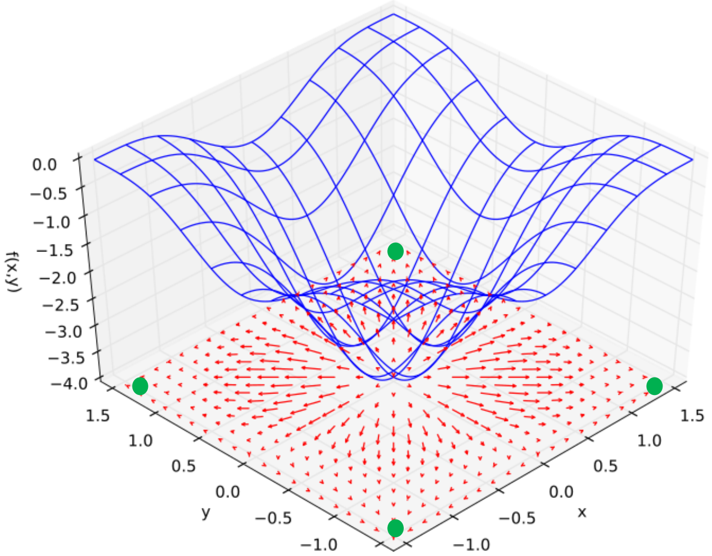
C. Saddle points of $L(W)$

D. All of the above

Yes, but not complete

A. ~~Local maxima of $L(W)$~~

Try Again



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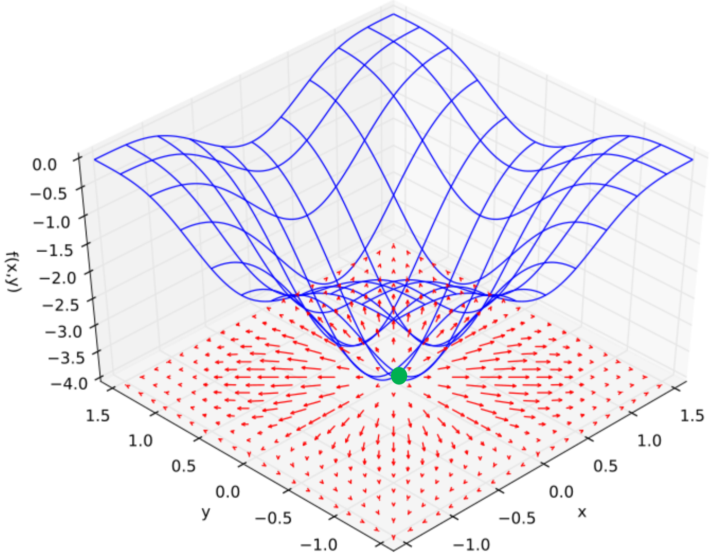
Yes, but not complete

A. ~~Local minima~~ of $L(W)$

Try Again



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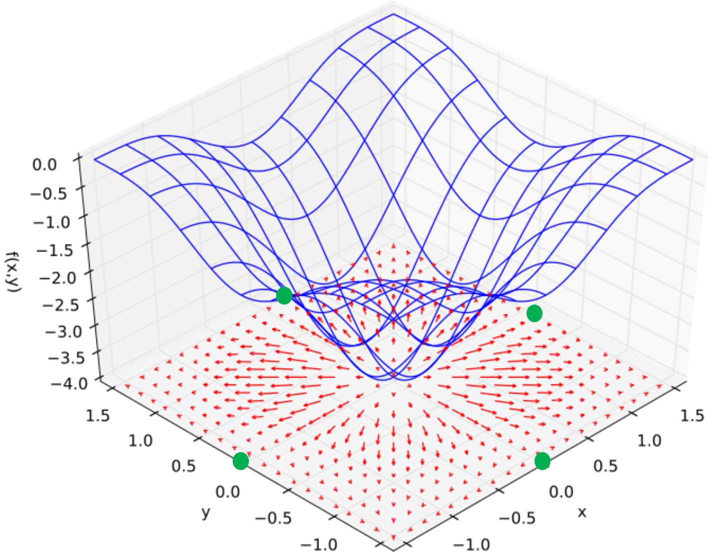
Yes, but not complete

A. ~~Saddle Points~~ of $L(W)$

Try Again



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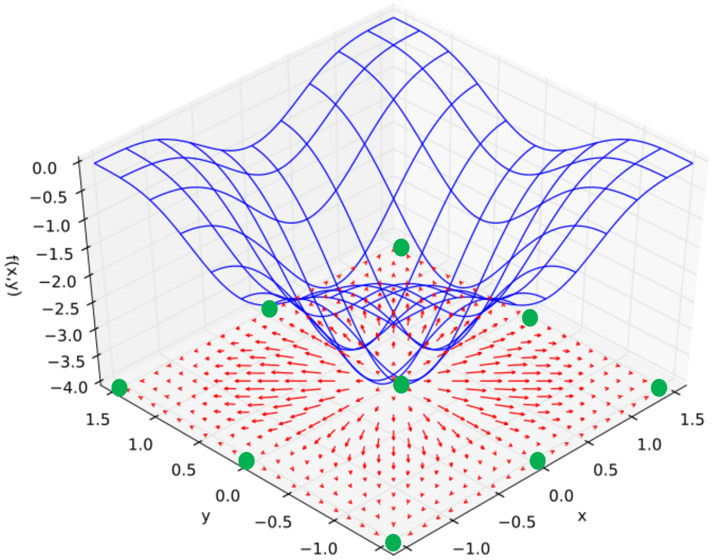
Yes!

A. All of the above

Try Again



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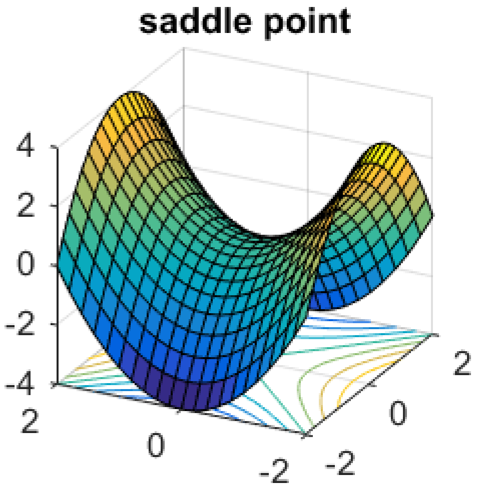
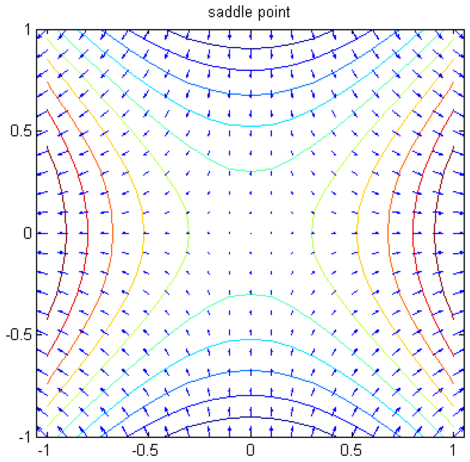
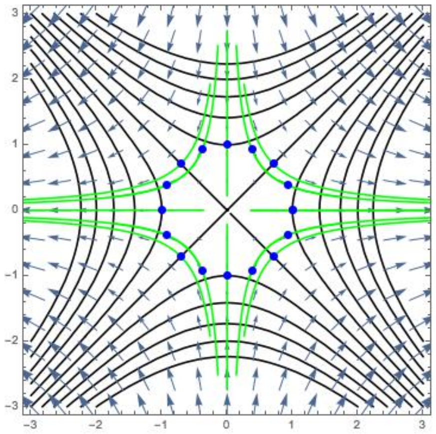
Caveat: Saddle Points

Following the negative gradient should eventually get you to a loss minimum. But it may take a long time. One reason is the presence of saddle points, where the gradient also vanishes:





Gradient vectors



Gradient flows (green)

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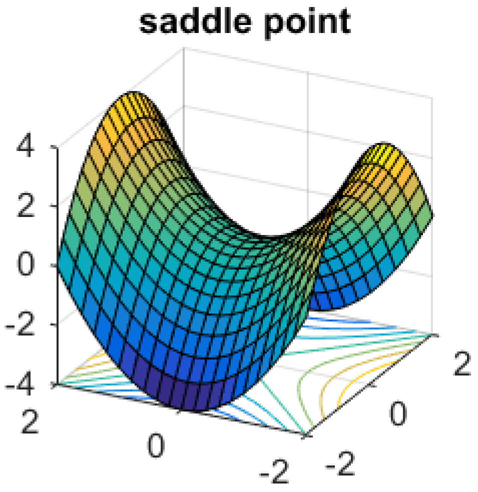
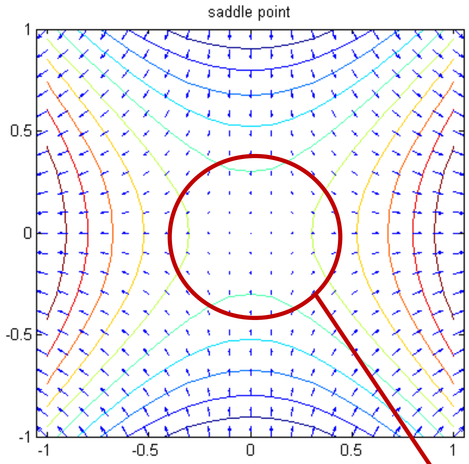
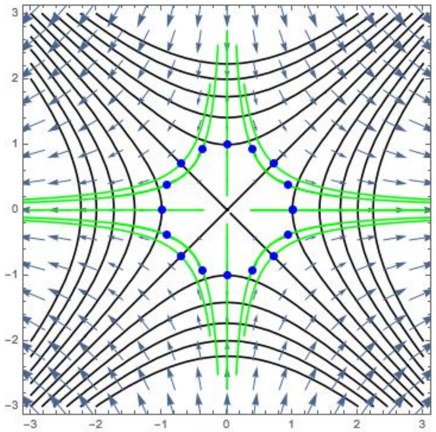




Gradient vectors



Gradient flows (green)

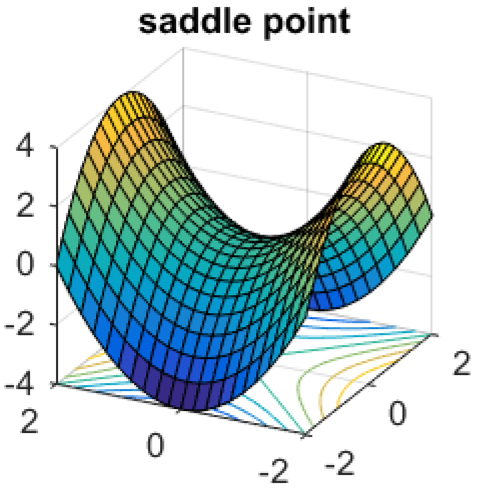
Gradients very small in here
– slow progress

Caveat: Saddle Points

There is a lot of evidence that ***most zeros of the loss gradient*** $\nabla_W L$ for neural networks ***are in fact saddles*** .

See e.g.

Dauphin et al. "Identifying and attacking the saddle point problem in high-dimensional non-convex optimization" arXiv 1406.2572, 2014.



Caveat: Efficiency

The loss L is a sum of the losses for all the data items:

$$L = \sum_{i=1}^{N} L(x_i, y_i, W)$$

and

$$\frac{dL}{dW} = \sum_{i=1}^{N} \frac{dL}{dW}(x_i, y_i, W)$$

so computing the gradient wrt W requires a ***full pass through the dataset***. Getting to the loss minimum may require millions of gradient steps, so this is ***very*** expensive.

Minibatches

Instead of computing a gradient across the entire dataset, we can compute it using a fixed-size subset of data samples called a ***minibatch*** . The minibatch size m is typically 32, 64, 128, 256,…

The minibatch is ideally a random sample of size m from the full dataset. In practice it may just be m consecutive samples.

So we compute this gradient (N is our dataset size, m is our minibatch size):

$$g^{(t)} = \frac{1}{m} \sum_{j = i_1,\ldots,i_m \in \{1,\ldots,N\}} \nabla_W L(x_j, y_j, W)$$

And then (superscripts are iteration numbers):

$$W^{(t+1)} = W^{(t)} - \alpha\, g^{(t)}$$

In this way, we perform N/m updates to the weights for one pass over the dataset.

Stochastic Gradient Descent

This approach is called *Stochastic Gradient Descent* or SGD. SGD and its variants are used almost universally in deep network training.

SGD uses $g^{(t)}$, the gradient of a minibatch instead of the true average gradient (call it g) on the full dataset. It should be clear that:

$$g = \mathbb{E}[g^{(t)}]$$

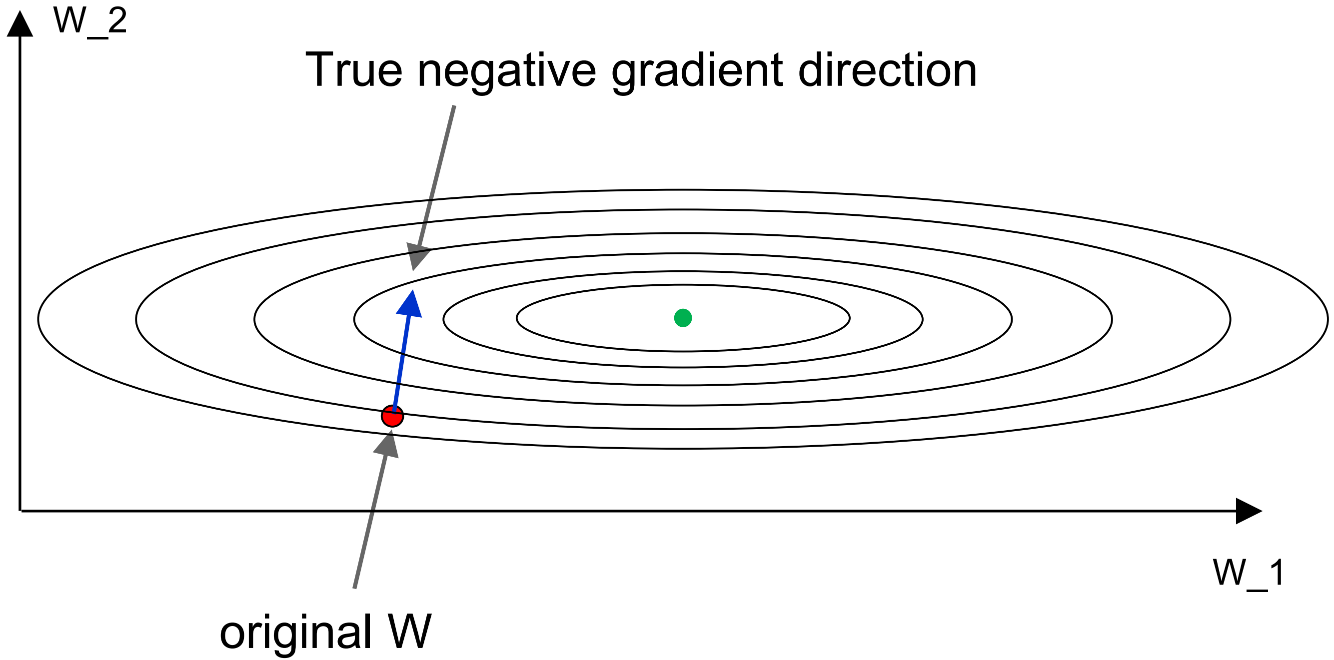
where expectation is over minibatches sampled from the full dataset.

So that $g^{(t)}$ is an unbiased estimate of g. But $g^{(t)}$ will typically have a lot of variance (is noisy) compared to g (if we considered the full dataset as a sample of an infinite dataset…).

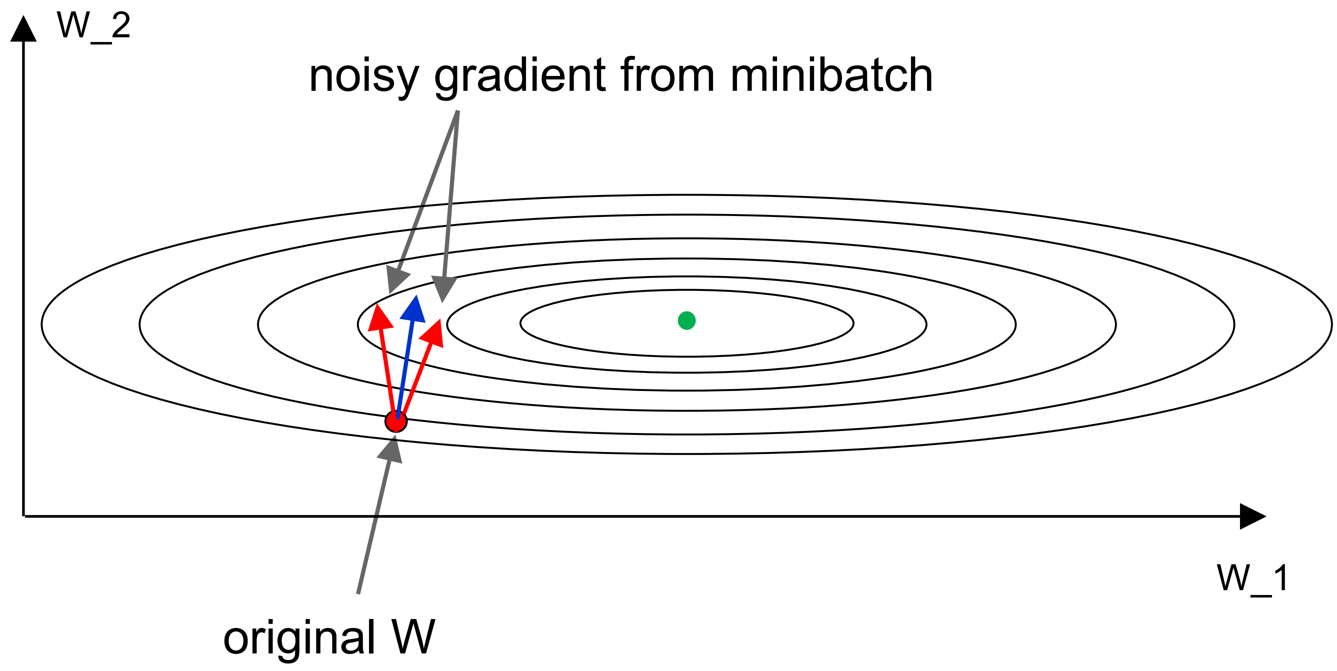
SGD is "stochastic" because it uses $g^{(t)}$, the gradient of a minibatch instead of the true gradient (call it g) on the full dataset.

Minibatch updates

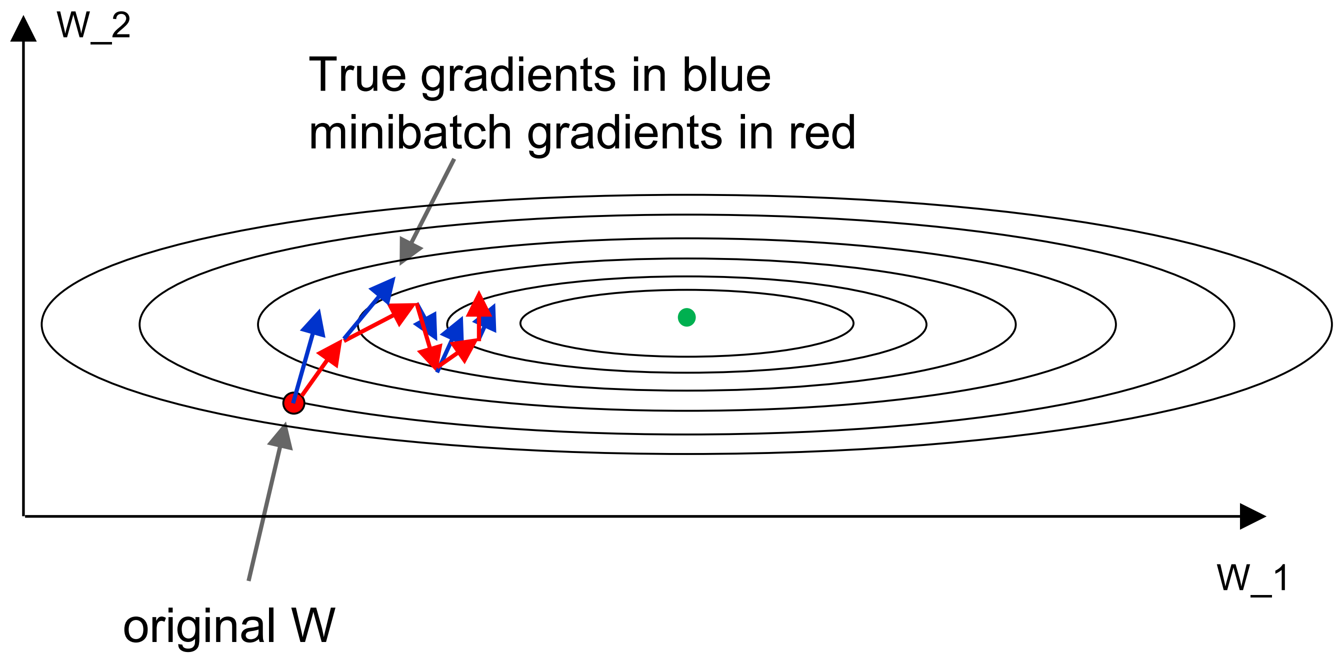
A contour plot represents a function with contours of equal value $f(x) = c$. The gradient of the function is always orthogonal to the contour.



Stochastic Gradient

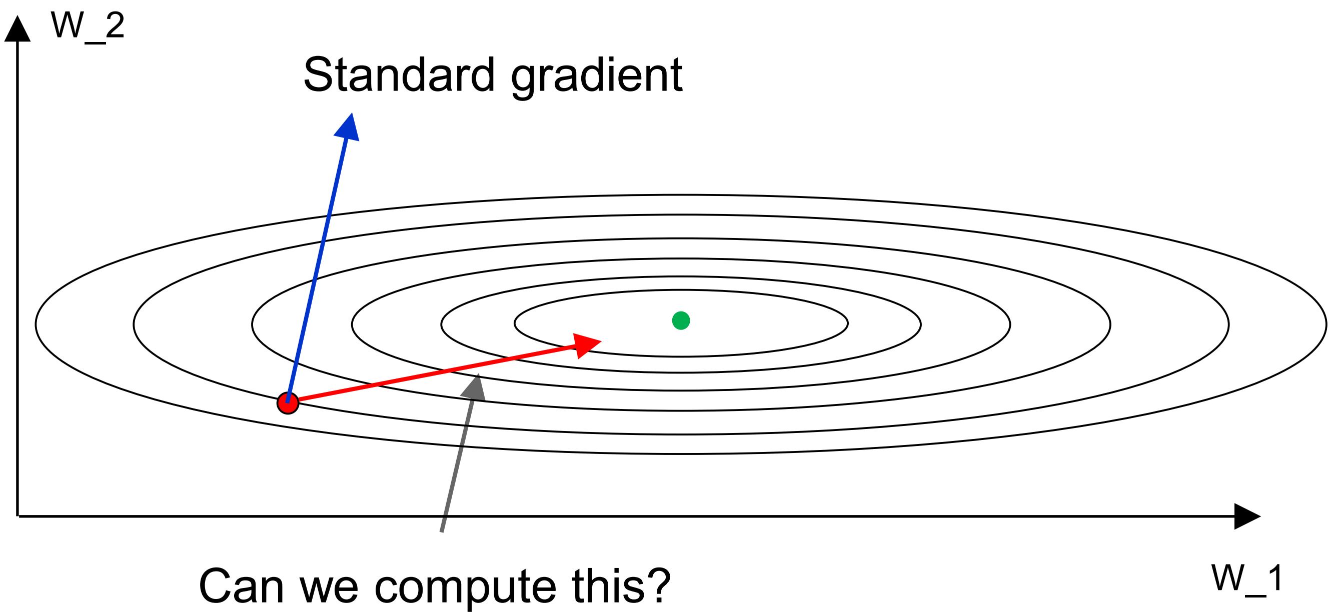


Stochastic Gradient Descent



Gradients are noisy but still make good progress on average

You might be wondering…



Yes: with Newton’s method

Newton's method for zeros of a function

Based on the Taylor Series for $f(x+h)$:

$$f(x+h) = f(x) + hf'(x) + O(h^2)$$

To find a zero of f, assume $f(x+h) = 0$, so

$$h \approx -\frac{f(x)}{f'(x)}$$

And as an iteration:

$$x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)}$$

Newton's method for optima of a scalar function

For zeros of $f(x)$:

$$x_{t+1} = x_t - \frac{f(x_t)}{f'(x_t)}$$

At a local optima, $f'(x) = 0$, so we use:

$$x_{t+1} = x_t - \frac{f'(x_t)}{f''(x_t)}$$

If $f''(x)$ is constant (f is quadratic), then Newton's method finds the optimum in ***one step***.

More generally, Newton's method has ***quadratic (very fast) converge.***

Newton's method for gradient zeros:

To find an optimum of a function $f(x)$ for high-dimensional x, we want zeros of its gradient: $\nabla f(x) = 0$

For zeros of $\nabla f(x)$ with a vector displacement h, Taylor's expansion is:

$$\nabla f(x+h) = \nabla f(x) + h^T H_f(x) + O\left(||h||^2\right)$$

where H_f is the Hessian matrix of second derivatives of f.

We can compute the "best" update by ignoring the $O(||h||^2)$ term, setting LHS to zero, and solving for h:

$$x_{t+1} = x_t - H_f(x_t)^{-1} \nabla f(x_t)$$

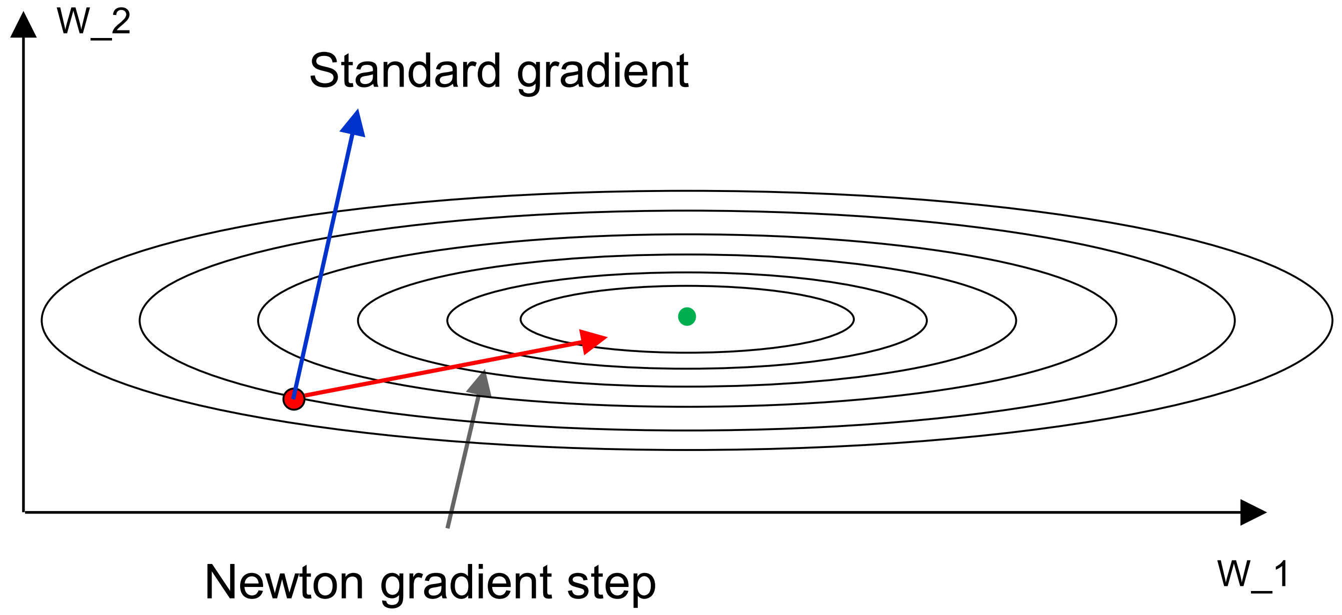
Aside: Hessian matrices

The Hessian for a function $f(x)$ is the matrix of 2nd order partial derivatives:

$$\mathbf{H} = \begin{bmatrix} \dfrac{\partial^2 f}{\partial x_1^2} & \dfrac{\partial^2 f}{\partial x_1\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_1\,\partial x_n} \\[2.2ex] \dfrac{\partial^2 f}{\partial x_2\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_2^2} & \cdots & \dfrac{\partial^2 f}{\partial x_2\,\partial x_n} \\[2.2ex] \vdots & \vdots & \ddots & \vdots \\[2.2ex] \dfrac{\partial^2 f}{\partial x_n\,\partial x_1} & \dfrac{\partial^2 f}{\partial x_n\,\partial x_2} & \cdots & \dfrac{\partial^2 f}{\partial x_n^2} \end{bmatrix}.$$

or $H = \nabla_x (\nabla_x f)^T$

Newton step



Newton's method for gradient zeros

The Newton update is:

$$x_{t+1} = x_t - H_f(x_t)^{-1} \nabla f(x_t)$$

Converges very fast, but rarely used in Deep Learning.

Why do you think this is?

Newton's method for gradient zeros:

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Too expensive: if x_t has dimension M, the Hessian $H_f(x_t)$ has dimension M^2 and takes $O(M^3)$ time to invert.

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We can address this to some extent with more advanced methods like L-BFGS which uses a K-dimensional approximation: $O(MK^2)$

Newton's method for gradient zeros:

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Converges very fast, but rarely used in Deep Learning. Why?

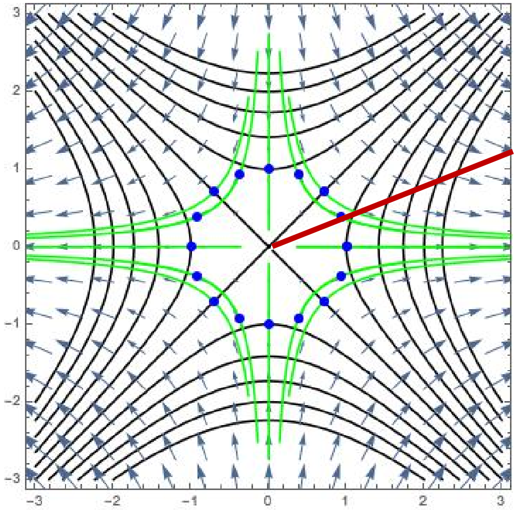
Too expensive: if x_t has dimension M, the Hessian $H_f(x_t)$ has dimension M^2 and takes $O(M^3)$ time to invert.

Too unstable: Because it involves a matrix inverse it can be unstable numerically. Again advanced methods like L-BFGS are more stable.

But there is another ***big*** problem…?

Newton’s method for gradient zeros:

Too clever **(gets stuck):** The second-order terms in Newton’s method allow it to quickly get to the nearest gradient zero, ***including saddle points***.



Newton gradient flow (red)

Newton's method for gradient zeros:

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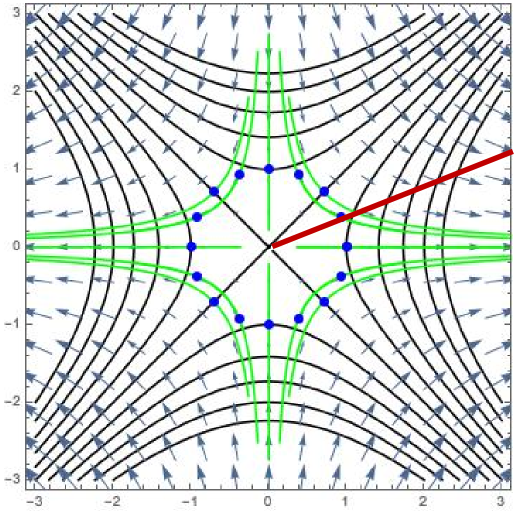
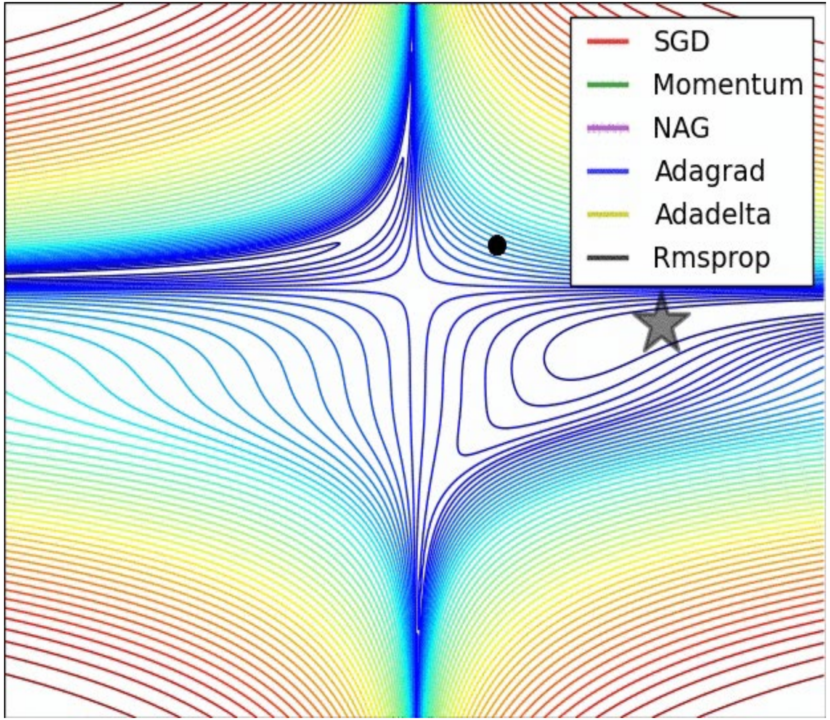


Newton gradient flow (red)

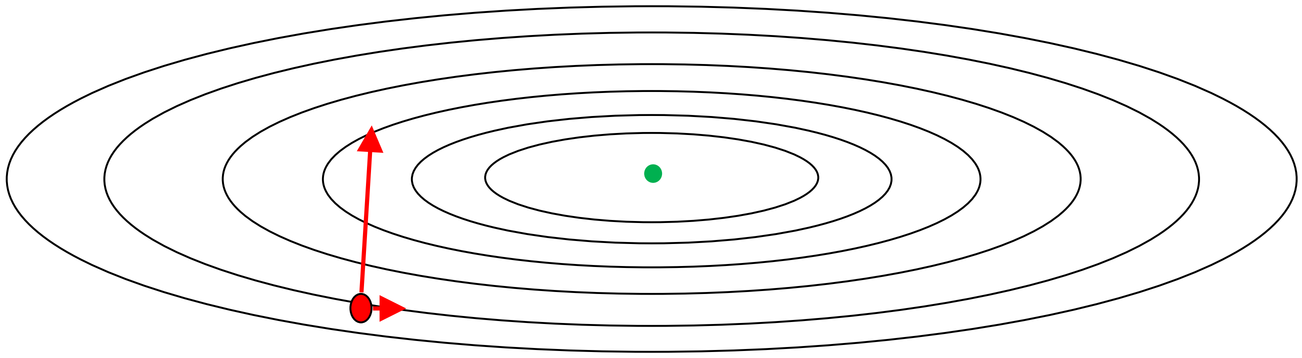
In fact we know that neural loss landscapes have lots of saddle points because people found them with Newton's method.

The effects of different update formulas



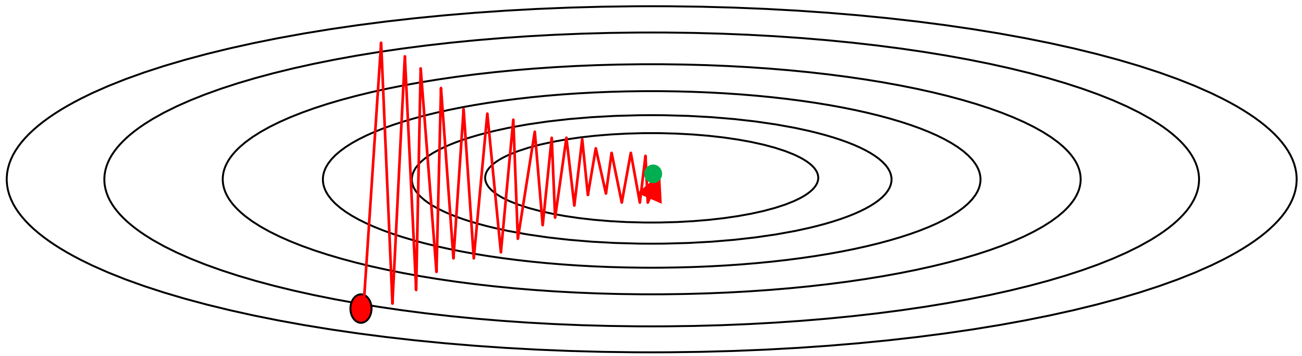
(Alec Radford animation)

Gradient $\neq$ Best direction to step



What happens when we take large steps in the gradient direction?

Gradient $\neq$ Best direction to step



The gradient is much larger across the bowl, so the directions of the gradient steps tends to be vertical.

Newton’s First Law

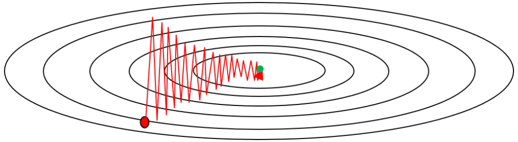
“Every body persists in its state of being at rest or of moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed”
– Isaac Newton

The object’s “memory” of its motion state is ***momentum***.

Usually expressed as $F = ma$

SGD with Momentum and Damping

However, a particle with momentum only would still oscillate up and down, in fact the oscillation would grow near the minimum:



To make the trajectory more stable we want a *damping* term, which reduces the energy of the system.

The hyperparameter commonly used by DNN optimizers called “momentum” is actually the “momentum decay rate”.

SGD with Momentum and Damping

Start with $F = ma$, or $F = m\frac{dv}{dt}$ where v is velocity.

Assume the force is given by a gradient + *a damping term (velocity dependent)* νv:

$$-\nabla L - \nu v = m\frac{dv}{dt} \qquad \text{and} \qquad \frac{dx}{dt} = v$$

So for a finite step (SGD):

$$dv = \frac{dt}{m}(-\nabla L - \nu v)$$

And as an update:

$$v' = v\left(1 - \frac{dt}{m}\nu\right) - \frac{dt}{m}\nabla L \qquad \text{and} \qquad x' = x + v\,dt$$

This is less than 1

SGD with Momentum

Momentum update for step i :

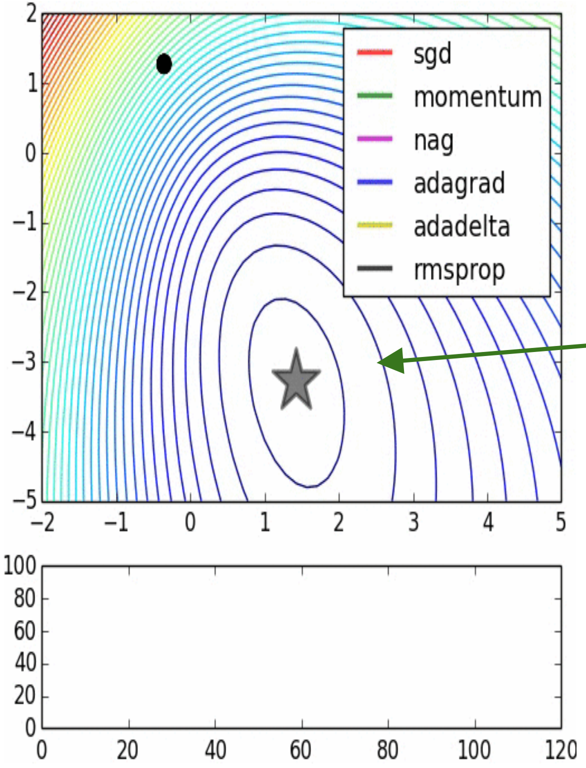
$$p^{(t+1)} = \mu p^{(t)} - \alpha g^{(t)}$$

Where $p^{(t)}$ is the momentum, $g^{(t)}$ is the minibatch gradient,
α is the learning rate, and
$\mu \in [0,1]$ is the “momentum” constant.

The weight update is:

$$W^{(t+1)} = W^{(t)} + p^{(t+1)}$$

SGD
vs
Momentum



notice momentum overshooting the target, but overall getting to the minimum much faster than vanilla SGD.

(Alec Radford animation)

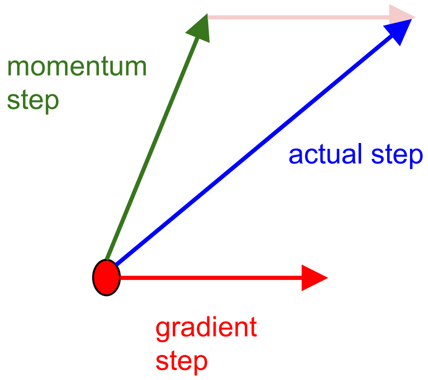
Nesterov Momentum update

Ordinary momentum update:

$$p^{(t+1)} = \mu p^{(t)} - \alpha g^{(t)}$$

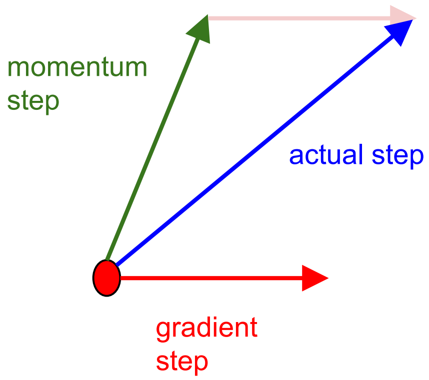
$$W^{(t+1)} = W^{(t)} + p^{(t+1)}$$

The update as a diagram:

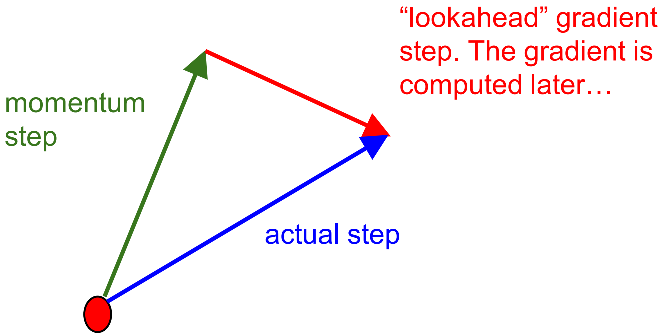


Nesterov Momentum update

Momentum update

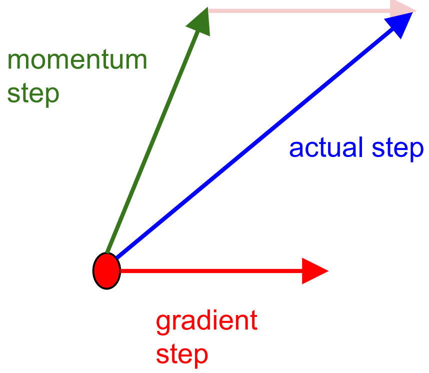


Nesterov momentum update

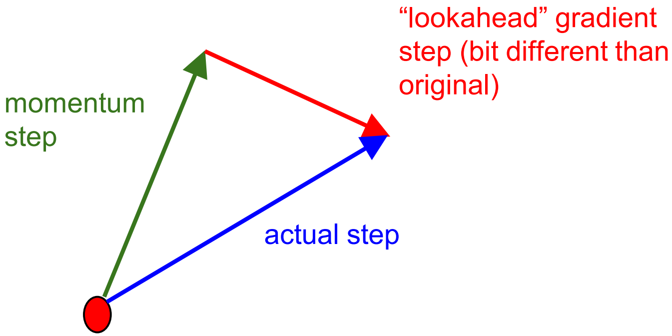


Nesterov Momentum update

Momentum update



Nesterov momentum update



New term

$$p^{(t+1)} = \mu p^{(t)} - \alpha \nabla_W L(W^{(t)} \boxed{+ \mu p^{(t)}})$$

$$W^{(t+1)} = W^{(t)} + p^{(t+1)}$$

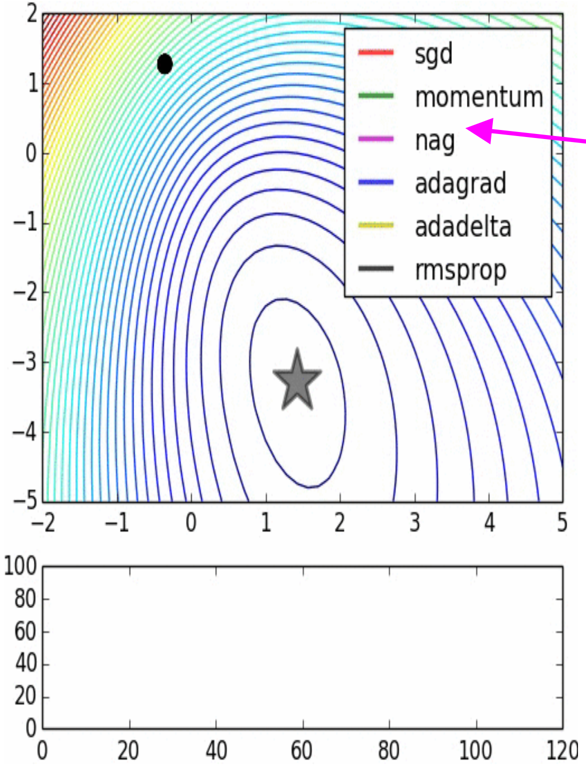
SGD vs Nesterov

A carefully implemented SGD has a convergence rate of $O\left(\frac{1}{t}\right)$. i.e. its error is bounded by a constant times 1/t, where t is the number of steps.

Nesterov has been shown to have a convergence rate of $O\left(\frac{1}{t^2}\right)$, quite a bit faster.

Note: Both these bounds apply to convex optimization problems.

See e.g.
Attouch and Peypouquet, “The rate of convergence of Nesterov's accelerated forward-backward method is actually faster than $1/k^2$” arXiv 1510.08740, 2015.



nag = Nesterov Accelerated Gradient

RMSprop

[Hinton et al., 2012]

Gradients can vary wildly even though parameters often have the same scale.

RMSprop scales the gradients by the inverse of a moving average, RMS (Root-Mean-Squared) gradient. Define

$$s^{(t)} = \beta s^{(t-1)} + (1-\beta)\left(g^{(t)}\right)^2$$

where $s^{(t)}$ is the (moving average) Mean-Squared Gradient at step t,

$g^{(t)}$ is the normal minibatch gradient at step t,

$\beta \in [0,1]$ is a moving-average decay factor, (close to 1)

$\left(g^{(t)}\right)^2$ is the element-wise square of $g^{(t)}$, so $s^{(t)}$ has same dims as $g^{(t)}$.

RMSprop:

$$W^{(t+1)} = W^{(t)} - \alpha \frac{g^{(t)}}{\sqrt{s^{(t)}}}$$

ADAGRAD

[Duchi et al., 2011]

ADAGRAD is similar to RMSprop, but uses the ***cumulative sum*** of squared gradients.

Define

$$c^{(t)} = \sum_{j=1}^{t} \left(g^{(j)}\right)^2$$

Where $c^{(t)}$ is the ***Cumulative*** Squared Gradient at step t,

ADAGRAD:

$$W^{(t+1)} = W^{(t)} - \alpha \frac{g^{(t)}}{\sqrt{c^{(t)}}}$$

Note: $c^{(t)}$ tends to grow linearly with time t, so ADAGRAD decreases its effective learning rate over time as $1/\sqrt{t}$

RMSprop and ADAGRAD

- Because they normalize gradient magnitudes, both RMSprop and ADAGRAD work very well on datasets with a wide range of gradient magnitudes.

- The most common example is text data. Word frequencies follow a power law: the j^{th} most common word has a relative frequency of $1/j$. So word gradients vary over 4-5 orders of magnitude.

- Using RMSprop/ADAGRAD can accelerate learning simple text models by 2-3 orders of magnitude.

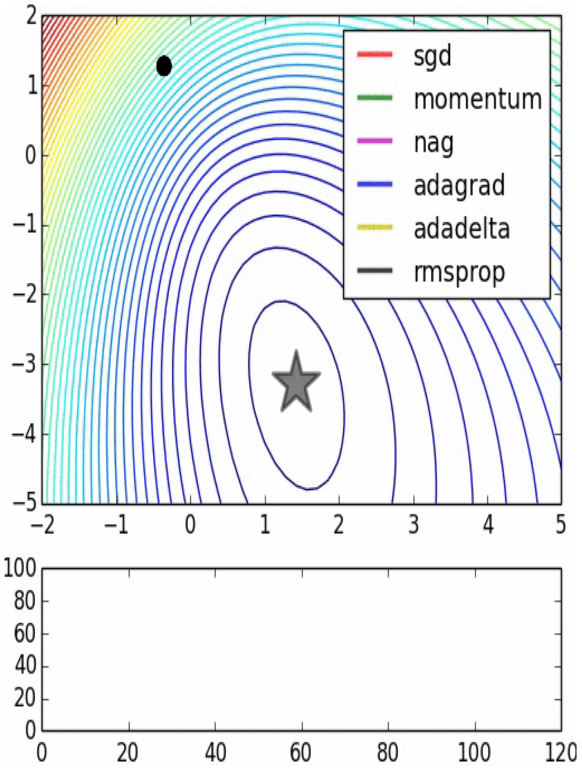
- But less effective with strong feature dependencies.

RMSprop vs. ADAGRAD

- RMSprop is a heuristic method, ADAGRAD has formal bounds on its convergence rate, although only for convex problems.

- The learning rate in RMSprop is fixed across time, and more suitable for long-running training tasks.

- The magnitude of ADAGRAD's sum of squared gradients grows linearly with time, so the learning rate of ADAGRAD decays as $1/\sqrt{T}$ which is quite aggressive. This is very good for short, easy-to-train models but too fast for long-running calculations.



adagrad
rmsprop

Momentum + RMSprop ≈ ADAM

[Kingma and Ba, 2014]

- Compute moving averages of the gradient and squared gradient.
- Treat them as moments and add a small-sample bias correction (next slide):

$$p^{(t)} = \beta_1 p^{(t-1)} + (1-\beta_1) g^{(t)}$$
$$s^{(t)} = \beta_2 s^{(t-1)} + (1-\beta_2)\left(g^{(t)}\right)^2$$

- Then normalize the momentum update (no bias correction):

$$W^{(t+1)} = W^{(t)} - \alpha \frac{p^{(t)}}{\sqrt{s^{(t)}}}$$

- Important practical point: β_1 typically 0.9, β_2 typically much closer to 1, e.g. 0.9999

ADAM Bias Correction

[Kingma and Ba, 2014]

- Moments are initialized to 0 at $t = 0$, so the early moving averages are biased toward zero.

- You can correct this bias (assuming $p^{(t)}$ and $s^{(t)}$ defined as before) like this:

$$p_{corr}^{(t)} = \frac{p^{(t)}}{1 - \beta_1^t}$$

$$s_{corr}^{(t)} = \frac{s^{(t)}}{1 - \beta_2^t}$$

- Then normalize the momentum update:

$$W^{(t+1)} = W^{(t)} - \alpha \frac{p_{corr}^{(t)}}{\sqrt{s_{corr}^{(t)}} + \epsilon}$$

Avoid divide by zero

ADAM Bias Correction

- Caveat: Many users report inconsistent performance with Adam.
- Adam assumes that $s^{(t)}$ and $p^{(t)}$ are *constant* when defining bias.
- Reasonable for $s^{(t)}$ but often false for $p^{(t)}$, since $g^{(t)}$ can oscillate.
- If $g^{(t)}$ oscillates, its true moving average will be smaller (possibly much smaller) than its first value.
- Since β close to 1, e.g. 0.99, the corrections increase $s^{(t)}$ and $p^{(t)}$ by 100x at t=1.

$$s_{corr}^{(t)} = \frac{s^{(t)}}{1-\beta_2^t}$$

Should be OK, since gradient magnitude roughly constant

$$p_{corr}^{(t)} = \frac{p^{(t)}}{1-\beta_1^t}$$

May significantly overestimate the moving average if g not constant

ADAM Bias Correction

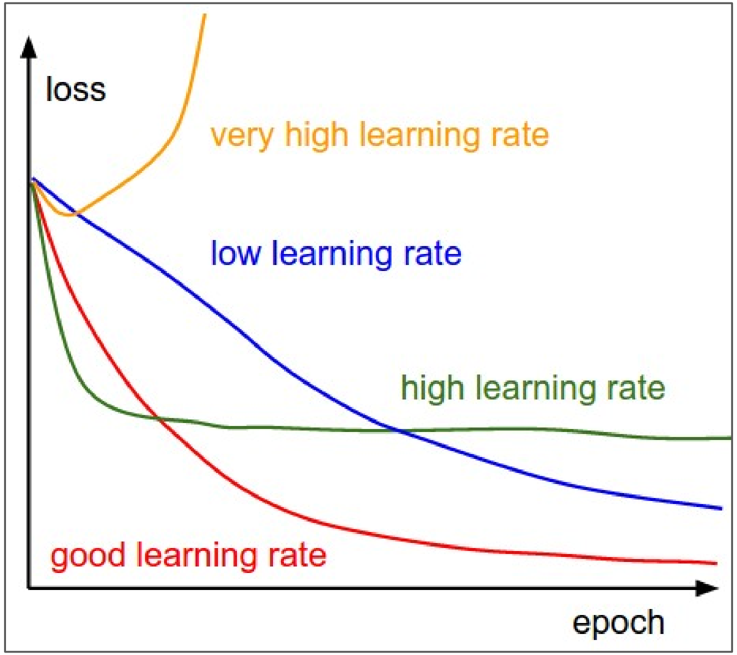
- The ADAM update again:

Possibly an over-estimate

$$W^{(t+1)} = W^{(t)} - \alpha \frac{p_{corr}^{(t)}}{\sqrt{s_{corr}^{(t)}} + \epsilon}$$

Should be a good estimate

Which Learning Rate?



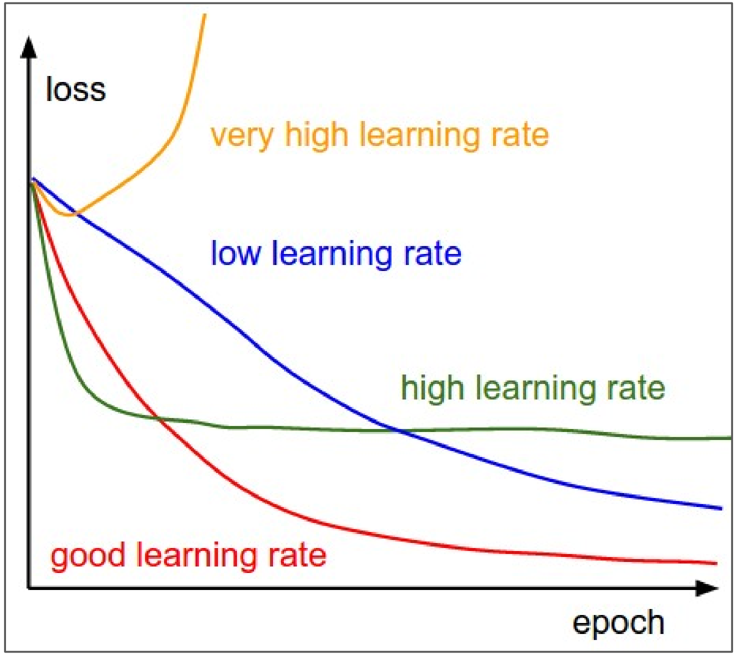
A. High Learning Rate

B. Low Learning Rate

C. Good Learning Rate

D. Decreasing Learning Rate

Which Learning Rate?



A. High Learning Rate

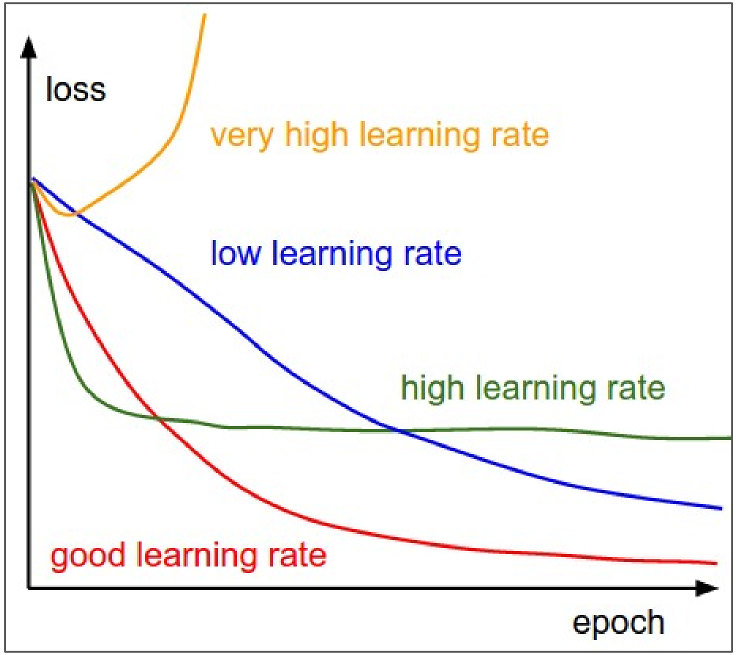
High learning rate improves fast early, but then stops improving.

Large SGD steps stop the parameters from settling into a local optimum.

Try Again

Continue

Which Learning Rate?



B. Low Learning Rate

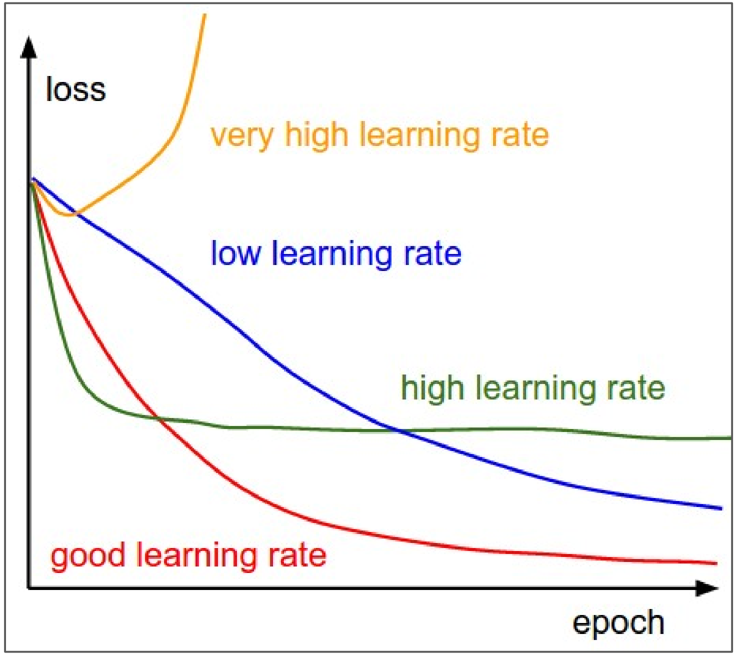
Low Learning rate learns slowly early, and takes a long time to reach an optimum.

For many problems, low learning rate fails to find an attracting basin for a good optimum – i.e. final solution is worse.

Try Again

Continue

Which Learning Rate?



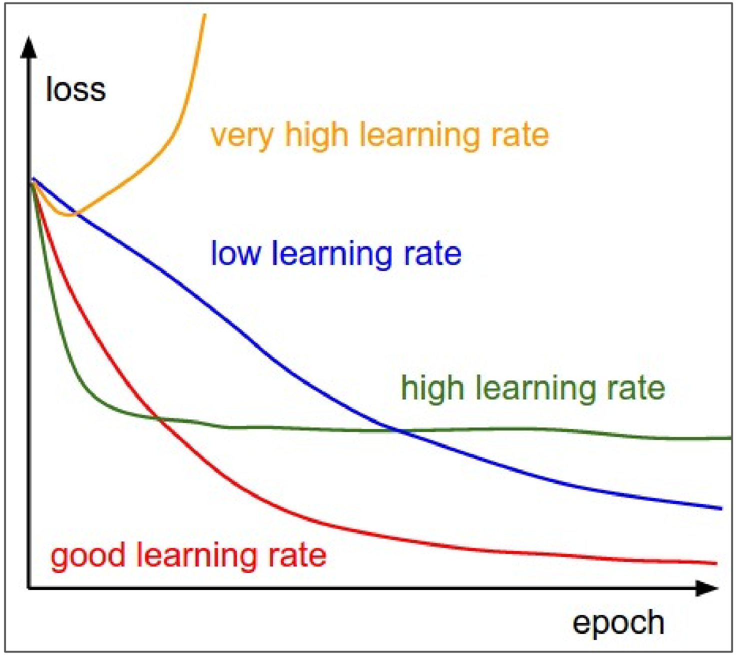
C. Good Learning Rate

A good compromise learning rate may exist, giving good progress early without too much variance later on.

Try Again

Continue

Which Learning Rate?



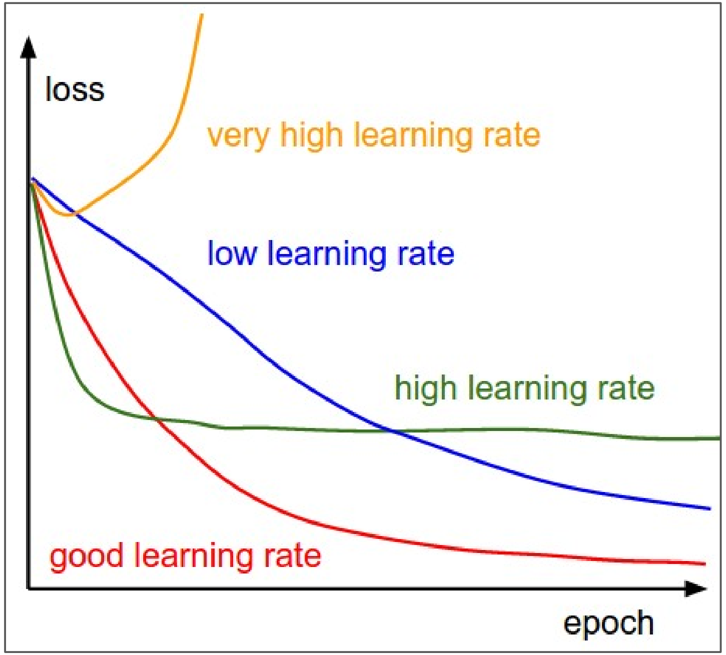
D. Decreasing Learning Rate

The best option for most problems is to start with a relatively high learning rate, and decay it over time.

Try Again

Continue

SGD, SGD+Momentum, Adagrad, RMSProp, Adam all have **learning rate** as a hyperparameter.



=> Learning rate decay over time!

step decay: e.g. decay learning rate by half every few epochs.

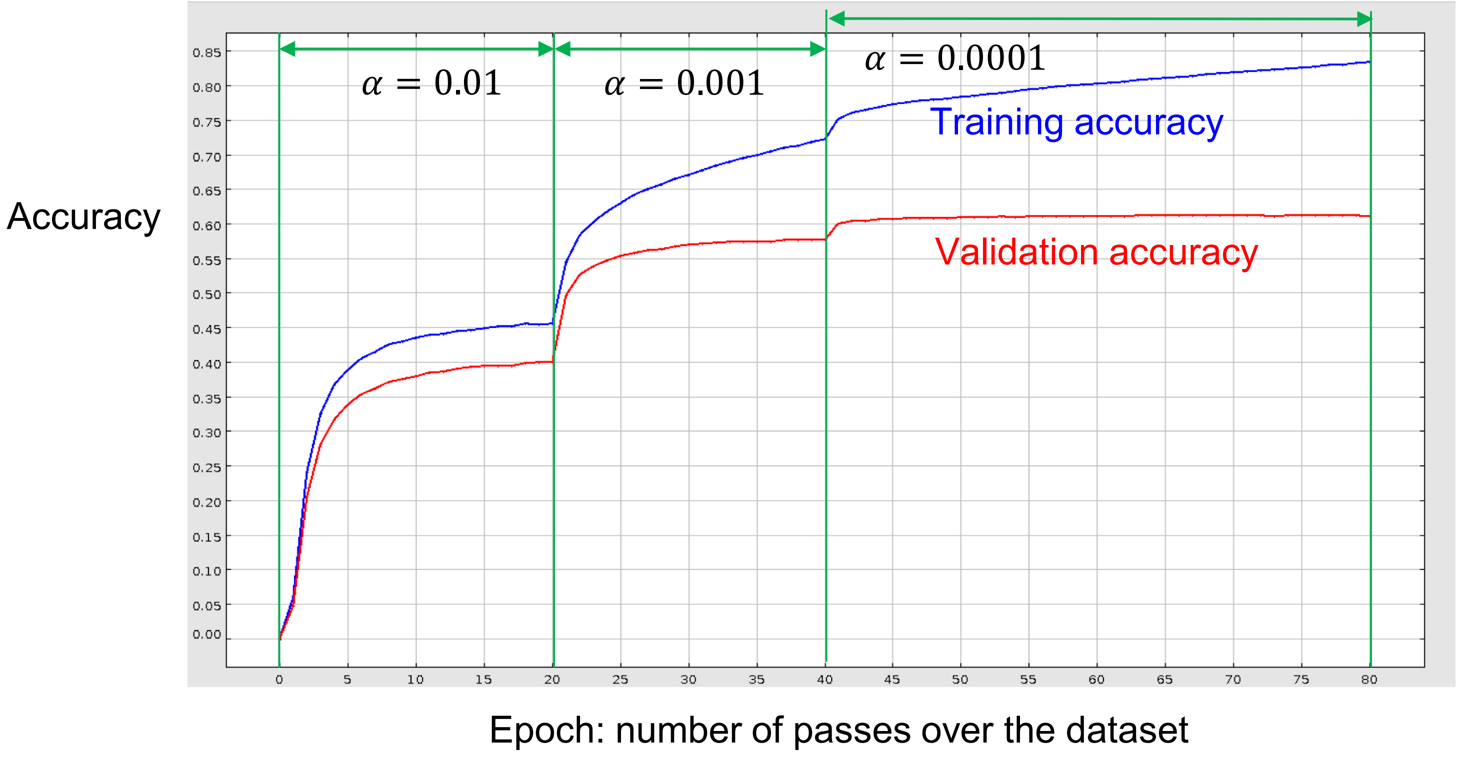
exponential decay: $\alpha = \alpha_0 e^{-kt}$

1/t decay: $\alpha = \alpha_0 / (1 + kt)$

$1/\sqrt{t}$ decay (ADAGRAD): $\alpha = \alpha_0 / \sqrt{1 + kt}$

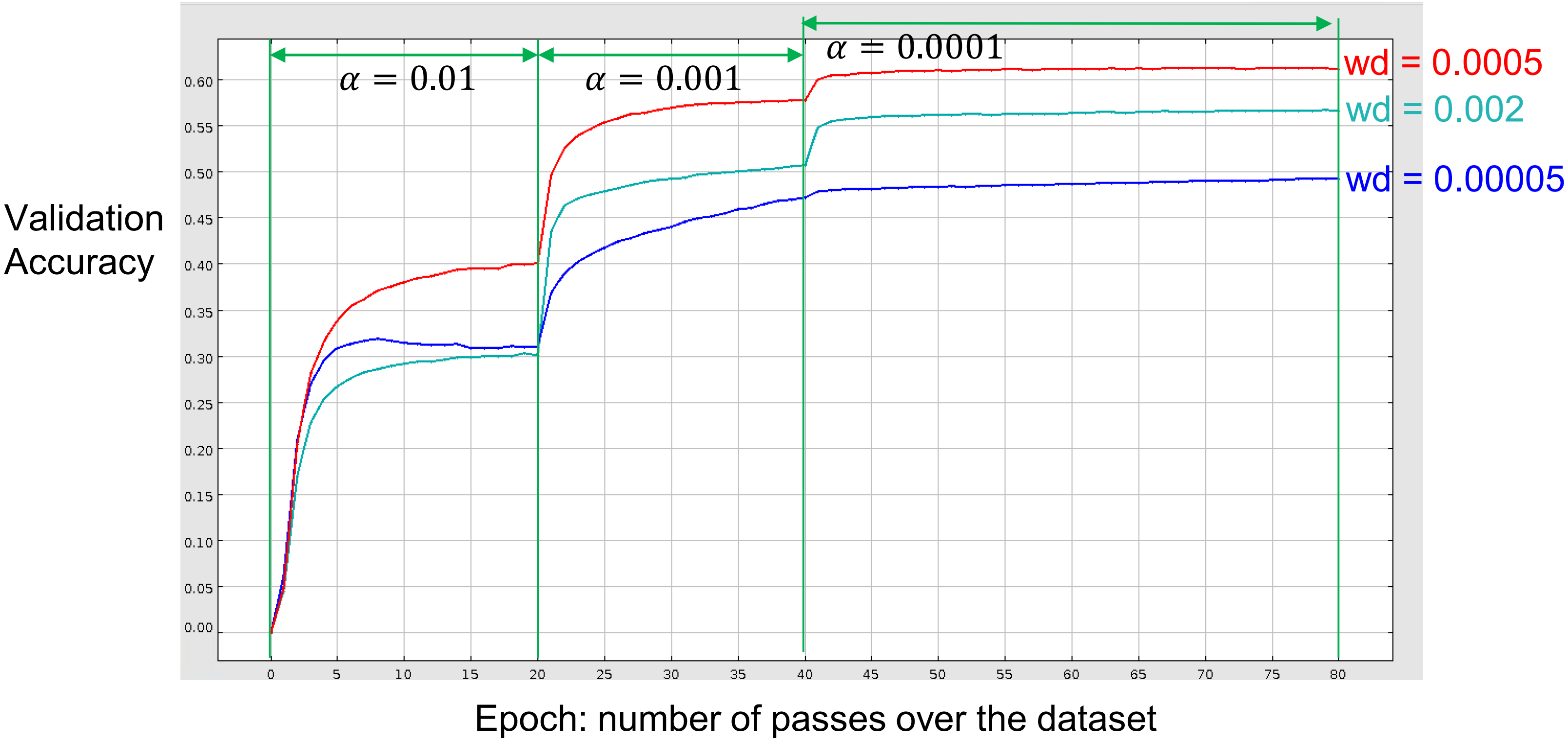
Step decay of Learning Rate

Alexnet trained on ImageNet data. α = learning rate.



Effects of L2 regularization (aka weight decay)

Alexnet trained on ImageNet data. Validation accuracy shown. wd = weight decay coefficient



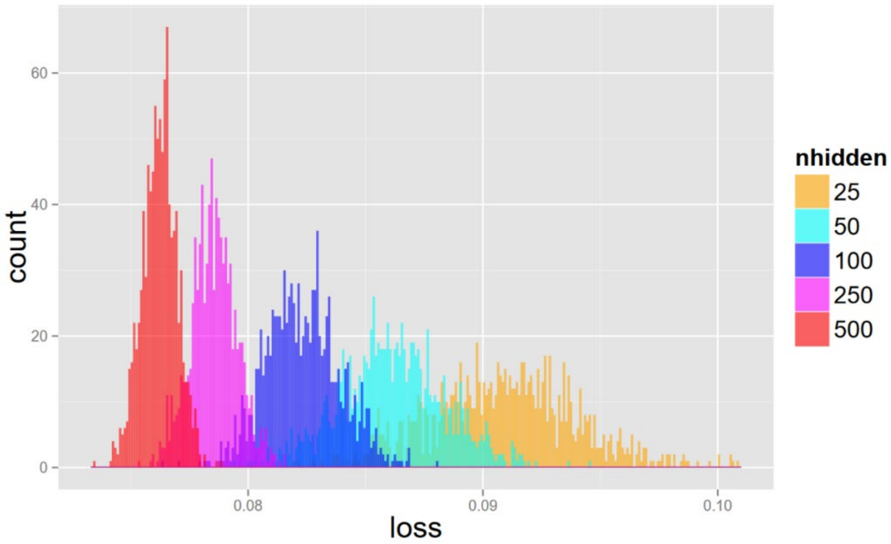
Are Local Minima a Problem?

They happen, but most local minima have similar loss to the global minimum.

e.g. MNIST digit recognition task:
images of 10x10 pixels

Authors built a two-layer network with nhidden units in the middle layer.

1000 networks are trained for each value of nhidden, and their final loss plotted at right.



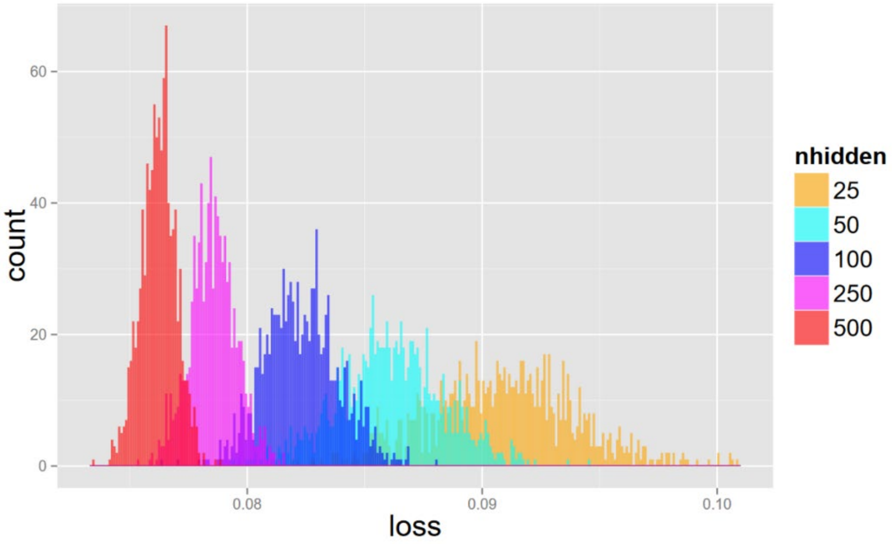
See Choromanska, Henaff, Mathieu, Ben Arous and Lecunn
“The Loss Surface of Multilayer Networks” arXiv 1412.0233, 2014.

Are Local Minima a Problem?

They happen, but most local minima have similar loss to the global minimum.

For a simple image network, as the network gets more complex (more hidden units), there are more local minima clustered even more closely near the global minimum loss.

So its fine, even desirable, to design very complex networks to mitigate the local minima problem.



See Choromanska, Henaff, Mathieu, Ben Arous and Lecunn "The Loss Surface of Multilayer Networks" arXiv 1412.0233, 2014.

Prospectus

- In the early days of deep network optimization, researchers borrowed ideas from convex optimization (e.g. ADAGRAD, Nesterov). These can work very well on certain problems that are,… well,… nearly convex.

- There was concern that existence of multiple local minima might make it hard to find good minima, but Choromanska et al. suggest that it's not a big problem.

- There has also been a lot of concern that saddle points slow down learning. However there are recent positive results:

First-order Methods Almost Always Avoid Saddle Points. Jason D. Lee, Ioannis Panageas, Georgios Piliouras, Max Simchowitz, Michael I. Jordan, and Benjamin Recht.

- As Choromaska et al. suggest "overparametrization" helps the local optima problem.

Gradient Descent Finds Global Minima of Deep Neural Networks Simon S. Du, Jason D. Lee, Haochuan Li, Liwei Wang, and Xiyu Zhai

Optimization Summary

- **Simple Gradient Methods** like SGD can make adequate progress to an optimum when used on minibatches of data.
- **Second-order** methods (quasi-Newton) make much better progress toward a gradient zero, but are more expensive and unstable. They also can't distinguish optima from saddle points, and get trapped in the latter.
- **Momentum:** is another method to produce better effective gradients.
- **ADAGRAD, RMSprop:** diagonally scale the gradient. **ADAM** diagonally scales and applies momentum.
- **Nesterov Momentum:** can improve over vanilla SGD by gradient "look-ahead"
- **Assignment 1** is out, due Feb 17. Please start soon.