ChatGPT

Modeling Wear Patterns of Historic Stairs

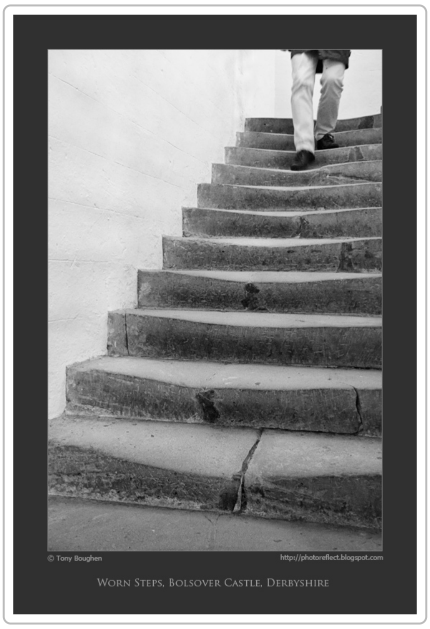

Figure: Historic stone steps (Bolsover Castle, UK) with pronounced concave wear in each tread from centuries of foot traffic. Such depressions provide measurable data (depth, width) to infer usage patterns and intensity over time.

Data Collection and Measurements

Non-destructive measurements: An archaeologist should first document the **3D profile of each stair tread**. This can be done with low-cost tools like digital photography for **photogrammetry** (to create a 3D model of the stairs) or simply a level and ruler to measure **wear depth** at various points (e.g. center vs edges). Key measurements include:

- **Vertical wear depth** at the most worn point of each step (depth of the concave "dish" worn into the tread).
- **Lateral wear profile** across each step's width (to see how wear is distributed from one side to the other). This could be a series of depth measurements across the tread or a contour profile.
- **Step dimensions and materials:** original tread thickness, material type (stone type or wood type) and hardness (if known or measured via a Schmidt hammer for stone hardness, for example).
- **Signs of repairs:** visual or tactile identification of any steps that look newer, have different material, or have **uneven wear compared to neighbors** (potential replacements or resurfacing).
- **Surface texture and polish:** noting if certain areas are more polished or smoother (indicating frequent foot contact) versus rough areas (little contact).

These measurements are non-destructive and only require a small team with measuring tools or cameras. They form the input for a mathematical wear model. We assume the original shape of the steps was flat and level, so any deviation (concavity or rounding) is due to wear.

Wear Rate Modeling with Archard's Law

A fundamental basis for the model is **Archard's wear law**, a classic engineering model for material wear. Archard's law states that the volume of material removed by wear is proportional to the normal force, the sliding distance, and inversely proportional to material hardness [1] [2]. In formula form:

$$V_{\text{wear}} = K \frac{F \cdot L}{H},$$

where F is the normal force (related to weight on the step), L is the total sliding distance of contact, H is the material hardness, and K is a dimensionless wear coefficient (depending on material and conditions). For footsteps on stairs, each footstep contributes a tiny wear volume. Over time, these add up to a measurable wear depth. We can simplify this into a linear **wear rate** relationship:

- **Linear wear model:** Assume each footstep removes an average volume ΔV of material. Then after N footsteps, total volume removed $V = N \cdot \Delta V$. Dividing by the contact area, the **average depth** worn is $D \approx \frac{V}{A} \propto N$. Thus **wear depth is roughly proportional to the total number of footfalls** on that step [2].

Using this, if we know or estimate the material's hardness and an average person's weight (to estimate force), we can **calibrate** K (perhaps by referencing known cases of wear) to compute how many footsteps produce the observed depth. For example, very hard stone (high H) wears slowly (small D per footstep), while softer wood wears faster. In practice, one might use known historic examples (like a staircase known to be ~500 years old with moderate use) to estimate K for that material. This provides a **mapping from measured wear to total foot traffic**.

Dynamic wear considerations: The model can be made time-dependent. Initially, wear might accumulate rapidly on fresh flat stone, then slow as a hollow forms (footsteps may distribute differently in a deep hollow or people adjust footing). If needed, we can express wear depth $h(t)$ as a differential equation. A simple form is:

$$\frac{dh}{dt} = \frac{KFv}{H}$$

where v is the rate of foot traffic (footsteps per time). Integrating over the usage period yields the total depth. If v is roughly constant over time and no behavioral change, this gives linear growth $h(t) \approx Ct$. More sophisticated models introduce a **correction coefficient** that accounts for changing contact area or foot placement as the step wears [3]. For example, as a dip forms, the contact area of a foot might increase (spreading force) which can reduce incremental wear – leading to a **saturation effect**. This can be modeled with a **logistic curve** (initial fast wear, then leveling off to a maximum depth) [4]. In fact, researchers have used a **two-phase logistic model** to distinguish natural wear progression from post-repair resets [4]. The logistic model is:

$$h(t) = \frac{D_{\max}}{1 + \exp(-r(t - t_0))},$$

where $D_{\max}$ is an asymptotic max depth (possibly when a step is so worn it's repaired or cannot wear much further), and r is a growth rate. This fits cases where stairs were **repaired and worn again**, by using multiple logistic segments (one per phase of use) [4].

Output (Usage Frequency): By inverting the wear model, we can estimate how often the stairs were used. For instance, if a particular stone tread has a **maximum wear depth** of 5 cm in the center, and our calibrated model says 1 million footsteps produce ~1 cm of wear on that stone, then roughly $5\text{ cm} \times 1\text{ million/cm} = 5$ million footfalls passed over that step. If the site is estimated to be in use for 500 years (~$500 \times 365 = 182,500$ days), that averages to about 27 footfalls per day. Such estimates answer the question **"How often were the stairs used?"** on average [2]. We can likewise compute an average daily or annual traffic from the model output. (In actual modeling, we would include error bars, as K and usage aren't perfectly known.) If historical records or population estimates suggest a very different daily usage, we then know the wear is **inconsistent or requires explanation** (e.g. perhaps a period of heavy use).

Lateral Wear Distribution and Traffic Lanes

By examining **where on the tread the wear is deepest**, we can deduce how people used the staircase laterally (side-to-side positioning). We model the **foot placement distribution** across the width of each step. Let x coordinate the width of a step. Define $f(x)$ as the probability density that a given footstep lands at lateral position x. Then, the wear depth profile across the step, $h(x)$, should reflect the cumulative foot traffic distribution: more footfalls at a location yield more wear. In a simple proportional model:

$$h(x) = C \cdot N \cdot f(x),$$

where C is a constant converting number of footfalls to depth (from the earlier wear calibration) [2]. In reality, the relationship may be smoothed by foot width (each footstep affects a small area), but fundamentally **peaks in the wear profile correspond to frequently trodden paths**.

Single vs. multiple lanes: If people generally traveled in **single file**, we expect $f(x)$ to have one dominant peak (perhaps near the center or one side) and the wear on each step will show a single concave groove. If, however, two people often walked **side-by-side**, the distribution might be **bimodal** – e.g. one peak on the left, one on the right. This would result in **two worn tracks** on each step. Archaeologists can detect this by looking at the cross-step wear pattern. For example, a Gaussian mixture model (GMM) can be fit to the measured wear profile to see if one or two (or more) clusters of wear exist [5] [6]. A **unimodal wear profile** (one central depression) suggests travel was mostly single-file. A **broad or twin-peaked profile** suggests at least two people abreast commonly used the stair. In one study, clustering analysis of wear patterns found about **82.4% of usage was side-by-side walking**, indicating the stairs frequently accommodated two people at once [7]. In practical terms, if the **width of the worn zone** exceeds what a single person's typical stride width would cover, that hints at multi-person breadth. Conversely, a narrow concentrated wear path implies single-file use.

Example: Imagine a 1.5-meter wide staircase. If people walked single-file, perhaps hugging the center, we might find a single wear groove ~0.5 m wide in the middle. If two people walked up together, we might see two grooves each ~0.5 m wide, perhaps 1.0 m apart (one closer to each side). Measurements of the **distance between wear maxima** or the **full width at half-maximum of the wear profile** can quantitatively distinguish these cases. This addresses **"How many people used the stairs simultaneously?"** – e.g. evidence of two side-by-side tracks means pairs walked together often, whereas one central track means single-file [2].

Directional Usage and Asymmetry

Another question is whether **up vs. down direction** was segregated or favored at certain times. In many staircases, especially wide ones, people might develop a keep-right or keep-left convention for passing. If so, ascending traffic predominantly uses one side of the stair and descending the other. This can create an **asymmetry in wear** from left to right. The model can interpret any left-right imbalance in $h(x)$:

- If one side of each step is systematically more worn than the other (e.g. the right half of each tread is deeper worn), it suggests that side had more foot traffic. Perhaps people mostly ascended/descended using that side, or the presence of a **handrail** or wall made one side more popular.
- If the wear pattern is symmetric about the center, that suggests either random usage across the width or that the stair was usually used in only one direction at a time (so people did not need to keep to sides to pass each other).

For instance, a spatial wear analysis of a busy stair found **heavier wear on the outer edges and right-hand side of steps**, correlating this to biomechanics (people often favor the railing side and place feet near step edges when descending) [7]. We can incorporate such logic: descending tends to scuff the step noses (front edges) more, whereas ascending might press more on the tread center. If we detect that front edge of each step on the right side is extremely worn, it might mean that **most people descended along the right rail**, dragging feet on edges, hence **predominant directionality**. On the other hand, uniformly distributed wear could mean the stair had **bidirectional mixing** (people going up and down freely intermingled) or was used in one direction predominantly (everyone uses the same path regardless of direction).

To answer **"Was a certain direction of travel favored?"**, we combine these observations. We might find, for example, that wear on the right side corresponds to downward traffic and is deeper than the left – implying that at any given time, people likely **kept right (two-way traffic)** [7]. Alternatively, lack of dual tracks might indicate the stair was one-way at a time (e.g. only used for going up during a certain period, then perhaps only down later, or generally so narrow that people took turns). Historical context (like monastic rules or building usage schedules) can inform this interpretation, but the wear model provides the physical evidence to confirm or challenge those assumptions.

Inferring Age and Usage Intensity

Using the wear models, we can estimate the **age of the stairway** or check consistency with a hypothesized construction date. If an estimate of age T (years since construction) is available, we plug that into the model: does N footsteps over T years (i.e. N/T per year) make sense for the site's known history? If a stair shows extreme wear that our model predicts would require, say, 10 million footfalls, but the site is only 100 years old, that would mean an improbable 100,000 footfalls per year unless the site was extremely busy. This could indicate either the age estimate is too low or our usage assumptions need revision. In essence, we solve for whichever variable is unknown:

- If age is unknown but we guess daily traffic from historical use, we can solve for T.
- If age is known, we solve for average daily traffic and compare it to historical population or visitor numbers.

Reliability: The reliability of an age estimate from wear depends on uncertainties in K and historical usage patterns. We can incorporate error margins by treating K as a range (since material properties vary) and usage as non-uniform. A **sensitivity analysis** would reveal how changes in assumed weight, hardness, etc., affect the outcome. If the wear-based age roughly agrees with archival records (within error bounds), that **validates the model's assumptions** (wear is consistent with information). If not, it flags a discrepancy: perhaps there were unrecorded periods of high usage or the stairs were built earlier than thought.

Advanced modeling can even attempt to infer **usage fluctuations over time**. For example, by segmenting the total timeframe into periods (e.g. different centuries or before/after a renovation) and treating each with a different average foot traffic rate, one can try to fit the wear profile in more detail [8] [9]. A procedure for this might be:

1. Assume a certain partition of the stair's history into segments (based on known events or hypothesized changes in usage).
2. Use the wear model to **predict a wear profile** given footfall rates in each segment (including any reset from repairs).
3. Adjust those rates (via least-squares or MLE fitting) until the **predicted wear matches the observed wear** as closely as possible [10] [9].
4. Examine the fitted footfall rates: if one period's rate is significantly higher than others, that indicates a **concentrated period of heavy use** [9]. If rates are steady, usage was relatively stable over time.

This approach can answer whether wear distribution suggests **"a large number of people over a short period or a small number over a long period."** However, it's important to note that purely from final wear, it's challenging to distinguish two scenarios that yield the same total footfall. External evidence (like historical records of a pilgrimage or an abandonment period) often guides such segmentation [8]. The model can then check consistency with those scenarios.

Detecting Repairs or Replacements

Stair wear models also help identify if certain steps were **repaired or replaced**. In the mathematical model, a repair is like a reset or change in parameters. For instance, if one step was replaced with new stone in 1800, its effective "age" (wear time) is lower than the others. We can detect this by anomalies in the wear data:

- If one step has **much shallower wear** than others above and below it (despite presumably similar traffic), it might have been replaced or resurfaced. The model would yield an extremely low footfall count for that step relative to neighbors, inconsistent with continuous use – pointing to a new stone with less wear.
- Different material: If a replacement stone came from a different quarry or a different wood was used in a repair, the hardness H might differ. The wear profile might show a sudden change at that step (either unusually high or low wear given the traffic). In our model, we could allow each step to have its own K, H parameters [11]. A significantly different fitted hardness or wear coefficient for one step suggests a **different source material or maintenance action** on that step.

Mathematically, one can extend the wear calculation to multiple steps by indexing steps $i = 1, 2, \ldots, s$. Each step i can have its own material parameters H_i, K_i. We then fit the model to the entire staircase's wear profile. If including a "repair factor" for a step dramatically improves the fit (e.g., step 10's wear is explained only if it had a later start or higher hardness), we conclude that step had a different history [12] [13]. This addresses **"What repairs or renovations have been conducted?"** by flagging steps with out-of-sequence wear. Often, physical inspection corroborates this (one might see a seam or newer tool marks on that step).

Material Consistency and Source

The **wear characteristics** can even hint at whether the material matches the presumed source (quarry or wood type). For example, if records say the stone came from Quarry A, and Quarry A's stone has a known hardness of, say, 7 on Mohs scale (or a certain Brinell hardness), our model would expect a certain wear rate. If the observed wear is too high given the foot traffic, perhaps the stone is softer than expected – maybe it actually came from a different source (Quarry B with softer stone) or has internal flaws. Similarly for wood: hardwood like oak should wear slower than pine. If an old wooden stair (supposedly oak) exhibits very deep wear not explainable by normal usage, perhaps it was actually a softer wood or weakened by rot over time, or simply much more traffic than assumed.

In practice, the model would output a fitted wear coefficient K (or an effective hardness H_{eff}) for the material given the traffic. We compare that to reference values for the supposed material. If there's a large discrepancy, we provide guidance that **either the material source might be different or additional factors (sand on shoes causing extra abrasion, etc.) are at play**. For instance, **is the wear consistent with the quarry's stone?** If not, archaeologists might re-examine the source hypothesis.

Another clue from wear is the **texture of wear**: some stones polish when worn (e.g., marble becomes smooth), others crumble. If the wear pattern (smooth vs rough) doesn't match the expected material's typical wear mode, it might indicate a different source or composition. While our model is primarily quantitative, it can incorporate a "correction factor" for material that is tuned to match the expected quarry source [3]. If a very different factor is needed to fit the data, that flags inconsistency.

Putting it Together: Model Conclusions

Using the above approaches, a comprehensive mathematical model of stair wear can yield multiple insights:

- **Usage frequency:** By calibrating wear depth to footfall count (using Archard's law), estimate how many people used the stairs per day or year on average [2]. For example, a model might reveal on the order of 10^5 uses per year for a heavily worn public stair, whereas a private stair in a home shows only 10^3 per year. One study even quantified an annual usage of ~75 million steps for a major public staircase [7] – an enormous number consistent with continuous heavy traffic. Such quantification helps validate or refute historical usage levels.

- **Travel configuration:** Determine if people moved mostly single-file or in groups. A single central wear path implies **single-file** usage, whereas dual wear paths imply **side-by-side** usage was common [7]. This answers how many people likely used the stair simultaneously. It also reflects the stair width: a narrow medieval turret stair will force single-file (and show one spiral path of wear), while a grand processional stair could accommodate pairs. Our model extracts that pattern from the wear distribution.

- **Directional patterns:** Identify any bias such as predominantly up vs down at certain times. This could be indicated by asymmetric wear (favoring one side) or differential wear on step edges (front lip wear might indicate downward use due to heel scuffing). If the wear is not symmetric, the model suggests a **preferred direction or a keep-left/keep-right system** was in effect [7]. If combined with historical context (e.g., a castle staircase where etiquette dictated ascending nobility had the right of way), the wear evidence either supports or contradicts those practices.

- **Consistency with age and records:** By inputting an estimated age and known daily routines of the site (e.g., a monastery might have 20 monks going up and down daily), the model predicts a certain wear depth. If the actual wear is far different, archaeologists learn that either the stair is older/younger than thought, or usage patterns were more intense than records suggest. For example, **"Is the wear consistent with available information?"** can be answered by comparing model-predicted wear vs. observed [9]. Discrepancies might reveal unrecorded usage (perhaps the stair was also used by visitors or pilgrims unbeknownst to historians) or error in the chronology.

- **Estimated age of the stairwell:** If no reliable construction date is known, the wear model can provide an age **estimate with uncertainty**. It might say, for instance, given typical usage rates for such a building, the wear depth indicates ~300–400 years of use. This would be labeled with caution (since usage rate assumptions are involved), but it offers a scientific basis for age when records are absent. The reliability of this estimate depends on how well we can pin down usage rate – which is why we incorporate all clues (width of wear, historical population, etc.) to refine the guess. If multiple scenarios (short intense use vs long sparse use) both explain the wear, we would present a range of possible ages and note the ambiguity.

- **Repairs and renovations:** The model can point out which steps deviate from the expected wear curve. Those are likely **repaired steps**. For example, if step 5 has half the wear depth of step 4 and 6, our model might only fit if step 5's "age" is half of the others, suggesting it was replaced midway through the stair's life. We provide that guidance, and archaeologists can then closely inspect step 5 for different tool marks or mortar lines. Additionally, if our fit needed a different material hardness for a section of steps, it could indicate a **partial renovation** (perhaps the upper staircase was rebuilt with new stone at a later date).

- **Material source confirmation:** Finally, by deriving an effective wear coefficient for the material from the data, we check it against known values for the presumed source. If they align, it reinforces the sourcing hypothesis (e.g., "the wear rate matches what we'd expect for hard limestone from Quarry X, supporting the idea that these stones came from there"). If not, it raises questions—maybe another quarry's stone (softer or harder) was used instead, or the stone was treated (sandstone hardened by salt, etc.). Similarly for wood: if a stair is said to be oak and our model shows a very high wear rate (which oak should resist), perhaps the wood is actually a softer species or has been affected by rot/termites historically.

In summary, **the mathematical modeling approach combines a physical wear model (Archard's law and its refinements) with statistical interpretation of wear patterns** [3]. It takes measurable wear features – depths, profiles, asymmetries – and translates them into answers about usage frequency, traffic configuration, time span, and material properties. Crucially, it also allows cross-verification with historical records: if the model's conclusions don't match the known historical context, it flags either an error in assumptions or an intriguing anomaly to investigate further. This way, the model provides a rigorous, quantitative **"reality check"** for archaeological hypotheses about the life of a stairway over centuries.

Sources:

- Cheng, J. *et al.* (2025). *Unlocking the Ancient Staircase "Code": Ancient Building Information Exploration Based on Wear and Tear Theory* – Mathematical models using Archard's wear law to link stair wear with usage frequency, directionality, and side-by-side use [3].

- Yao, Y. *et al.* (2025). *Modeling Staircase Wear Using Logistic Growth and Clustering Analysis* – Introduces logistic wear progression and cluster analysis of foot traffic patterns (e.g. identifying ~82% side-by-side usage and right-side bias in a case study) [4] [7].
- Highlights in Sci & Tech (2025). *Dynamic Wear Simulation of Steps* – Describes a multi-step wear model with material factors and an inverse fitting approach to deduce average users per year and periods of intensive use from wear data [8] [9].
- PhotoReflect Blog (2014). *Time-worn stone steps* – Discusses the visual impact of concave worn steps (e.g. Wells Cathedral) as evidence of long-term use [14] (supporting the qualitative premise of wear indicating passage of time and people).
- Historic Environment Case Studies – e.g., worn **Wells Cathedral** chapter house stairs and other heritage sites illustrate typical wear patterns (concave centers, etc.) consistent with our model assumptions. These real-world instances ground-truth the model's expectations.

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[1] [2] [3] Unlocking the Ancient Staircase "Code": Ancient Building Information Exploration Based on Wear and Tear Theory by Junhao Cheng, Minglang Yu, Erhua Chen :: SSRN
https://papers.ssrn.com/sol3/papers.cfm?abstract_id=5312839

[4] [7] Microsoft Word - 01-ICCIA7318™H.docx
https://bcpublication.org/index.php/SJISR/article/download/8746/8689/11486

[5] Quantifying Staircase Wear in Historic Buildings Using GMM and ...
https://dl.acm.org/doi/pdf/10.1145/3745533.3745639

[6] [PDF] Hierarchical Means Clustering | Semantic Scholar
https://www.semanticscholar.org/paper/Hierarchical-Means-Clustering-Vichi-Cavicchia/c7f276b813d1e32bb7213347879d5361428fc3a7

[8] [9] [10] [11] [12] [13] hsetdata.com
https://hsetdata.com/index.php/ojs/article/download/1014/943/1931

[14] PhotoReflect: Time-worn stone steps
https://photoreflect.blogspot.com/2014/07/time-worn-stone-steps.html